Log-linearization in the FEER model

Jamel Saadaoui

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1 Differentiation in the vicinity of equilibrium

The real effective exchange misalignment is the difference between the observed real effective exchange rate and the equilibrium real effective exchange rate expressed of as a percentage of the equilibrium real effective exchange rate.

$$\frac{(R-R^e)}{R^e} = r \tag{1}$$

$$R - R^e = rR^e$$

$$R = rR^e + R^e$$

$$R = (1+r)R^e$$

$$R^e = \frac{R}{(1+r)}$$

$$lnR^e = lnR - ln(1+r)$$

$$R^e = e^{lnR - ln(1+r)} \tag{2}$$

Where R stands for the real effective exchange rate, R^e for the equilibrium effective exchange rate and r for the real effective misalignment.

2 Logarithmic differentiation

The real effective exchange misalignment is the difference between the observed real effective exchange rate and the equilibrium real effective exchange rate expressed of as a percentage of the equilibrium real effective exchange rate.

$$\partial lnR = r \tag{3}$$

$$\partial lnR = lnR - lnR^e$$

$$r = lnR - lnR^e$$

$$lnR^e = lnR - r$$

$$R^e = e^{lnR - r} \tag{4}$$

Where R stands for the real effective exchange rate, R^e for the equilibrium real effective exchange rate and r for the real effective misalignment.

3 Maclaurin Series

We use a Maclaurin series to show that equation 2 can be approximated by equation 4. In particular, we note that :

$$\frac{1}{1+t} = 1 - t + t^2 - t^3 + \cdots \tag{5}$$

if $|t| < 1$ (infinite geometric series). Then we note that :

$$\ln(1+x) = \int_0^x \frac{1}{1+t}\, dt \tag{6}$$

Then we integrate the right-hand side of 5 term by term. We get

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \tag{7}$$

In figure 1, we can see that the approximation is valid at $x = 0$ (the Taylor expansion gives $x + O(x^2)$). In these conditions, it appears that $ln(1+x) \approx x$.

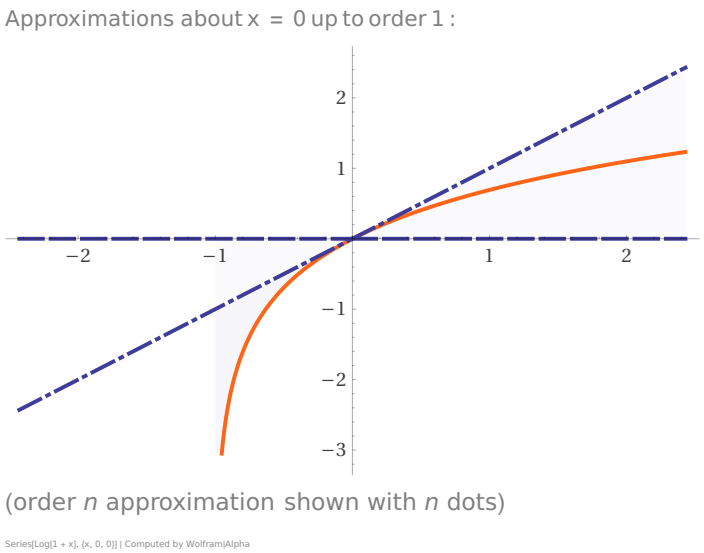

Figure 1: Maclaurin series for $ln(1+x)$ at $x = 0$

4 References

Eric W. Weisstein. Maclaurin Series. *From MathWorld – A Wolfram Web Resource*. Accessed 22 April 2020. Retrieved from URL.

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