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> restart;with(PDEtools):with(linalg):with(LinearAlgebra):assign
(Deltah=#mover(mi(Delta),mo("~"))):alias(sigma=sigma(x),phi=phi
(x),psi=psi(z)):
> a:=-1;b:=10;c:=20;n:=4;p[-1]:=0;p[0]:=1;p[1]:=x;
a := -1
b := 10
c := 20
n := 4
p-1 := 0
p0 := 1
p1 := x

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(1)

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> B:=a-n-c+2;C:=n+c-2;
B := -23
C := 22

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(2)

The moments $\mu_{[k]}$

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> mu:=(n,k)->simplify((-1)^(-a)*Beta(a+2-c-n+k,1+c-b-n)*z^(-a)*
hypergeom([a,2-c+a+k-n],[a+k+3-b-2*n],1/z)):

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> Delta:=(n)->Determinant(HankelMatrix(<seq(mu(n,i),i=0..2*n-2)>,n)
):`#mover(mi(Delta),mo("~"))`:=(n)->Determinant(<DeleteColumn
(HankelMatrix(<seq(mu(n,i),i=0..2*n-2)>,n),n)|<seq(mu(n,i+n),i=0.
.n-1)>>):

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> phi:=simplify(expand(sort(hypergeom([a,b],[c],z))))*z^(b)
:phi:for K from 1 to n do;l[K]:=diff(%,z)*z*(z-1);od:wronskian(
[phi,seq(l[k],k=1..n-1)],z):for K from 1 to n do;h[K]:=Row(%,1);
row(%,2);wronskian(%,z*(z-1),z):od:simplify(<seq(simplify(h[k]),
k=1..n)>):tau:=factor(expand(det(%))/z^(n*b)/(z*(z-1))^(n-1)*
(n/2)):
> Deltaha[n]:=factor(1/(b-1)*(Delta(n)*n*(1+C-n-B*z-C*z)-diff(Delta
(n),z)*z*(z-1))):Deltaha[n+1]:=factor(1/(b-1)*(Delta(n+1)*(n+1)*
(1+C-n-B*z-C*z)-diff(Delta(n+1),z)*z*(z-1))):

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Hankle determinant and modified wronskian connection

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> simplify(diff(ln(tau),z)-diff(ln(Delta(n)),z));
0

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(3)

Hankle determinant with tilde and modified wronskian connection

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> simplify(Deltah(n)-Deltaha[n]);
0

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(4)

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> alpha[n]:=convert(simplify(Deltah(n+1)/Delta(n+1)-Deltah(n)/Delta
(n)),parfrac,z);beta[n]:=convert(simplify(Delta(n+1)*Delta(n-1)
/Delta(n)^2),parfrac,z);

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$$\alpha_4 := 2 + \frac{1}{3} \frac{-650z^4 + 4550z^3 - 11550z^2 + 12650z - 5060}{143z^5 - 1430z^4 + 5460z^3 - 10010z^2 + 8855z - 3036} + \frac{1}{9} \frac{1456z^3 - 7644z^2 + 12936z - 7084}{143z^4 - 1144z^3 + 3276z^2 - 4004z + 1771}$$

(5)

$$\beta_4 := \frac{7}{18} + \frac{1}{99} \frac{-335335 z^3 + 1602237 z^2 - 2501499 z + 1276891}{(143 z^4 - 1144 z^3 + 3276 z^2 - 4004 z + 1771)^2} + \frac{1}{198} \frac{-2002 z^2 + 5005 z - 5677}{143 z^4 - 1144 z^3 + 3276 z^2 - 4004 z + 1771} \quad (5)$$

> p[n+1]=collect(x*p[n]-alpha[n]*p[n]-beta[n]*p[n-1],[z,x],factor);

$$p_5 = x p_4 - \frac{1}{18} (20469449 z^{13} p_3 + 105271452 z^{13} p_4 - 532205674 z^{12} p_3 - 2757261364 z^{12} p_4 + 6300868574 z^{11} p_3 + 32873403420 z^{11} p_4 - 44989988043 z^{10} p_3 - 236318913116 z^{10} p_4 + 216232726710 z^9 p_3 + 1143313688088 z^9 p_4 - 738985002756 z^8 p_3 - 3932671876848 z^8 p_4 + 1848206923836 z^7 p_3 + 9898664694360 z^7 p_4 - 3426263066226 z^6 p_3 - 18467253505272 z^6 p_4 + 4709819311197 z^5 p_3 + 25546803678396 z^5 p_4 - 4743096528170 z^4 p_3 - 25891183273956 z^4 p_4 + 3402611578366 z^3 p_3 + 18692927080828 z^3 p_4 - 1647174479951 z^2 p_3 - 9107653559004 z^2 p_4 + 482434808856 z p_3 + 2684994228224 z p_4 - 64585593072 p_3 - 361844925288 p_4) / ((143 z^5 - 1430 z^4 + 5460 z^3 - 10010 z^2 + 8855 z - 3036) (143 z^4 - 1144 z^3 + 3276 z^2 - 4004 z + 1771)^2) \quad (6)$$

> alpha[n]:=convert(1/(b-1)*simplify(c-a*z-1+z*(z-1)*diff(ln(Delta(n)),z)-z*(z-1)*diff(ln(Delta(n+1)),z)),parfrac,z);

$$\alpha_4 := 2 + \frac{1}{3} \frac{-650 z^4 + 4550 z^3 - 11550 z^2 + 12650 z - 5060}{143 z^5 - 1430 z^4 + 5460 z^3 - 10010 z^2 + 8855 z - 3036} + \frac{1}{9} \frac{1456 z^3 - 7644 z^2 + 12936 z - 7084}{143 z^4 - 1144 z^3 + 3276 z^2 - 4004 z + 1771} \quad (7)$$