```
> restart;alias(Delta[n]=Delta[n](z),Delta[n+1]=Delta[n+1](z)):B:=
  a-n-c+2;C:=n+c-2;
```

$$B := a - n - c + 2$$

$$C := n + c - 2 \tag{1}$$

```
> alpha[n]:=`#mover(mi(Delta),mo("~"))`[n+1]/Delta[n+1]-`#mover(mi
  (Delta),mo("~"))`[n]/Delta[n];beta[n]:=Delta[n+1]*Delta[n-1]
  /Delta[n]^2;
```

$$\alpha_n := \frac{\tilde{\Delta}_{n+1}}{\Delta_{n+1}} - \frac{\tilde{\Delta}_n}{\Delta_n}$$

$$\beta_n := \frac{\Delta_{n+1}\,\Delta_{n-1}}{\Delta_n^2} \tag{2}$$

```
> `#mover(mi(Delta),mo("~"))`[n]:=collect(factor(1/(b-1)*(Delta[n]*
  n*(1+C-n-B*z-C*z)-diff(Delta[n],z)*z*(z-1))),diff,factor);`#mover
  (mi(Delta),mo("~"))`[n+1]:=collect(factor(1/(b-1)*(Delta[n+1]*
  (n+1)*(1+C-n-B*z-C*z)-diff(Delta[n+1],z)*z*(z-1))),diff,factor);
```

$$\tilde{\Delta}_n := -\frac{z\,(z-1)\left(\frac{\partial}{\partial z}\,\Delta_n\right)}{b-1} - \frac{\Delta_n\,n\,(a\,z-c+1)}{b-1}$$

$$\tilde{\Delta}_{n+1} := -\frac{z\,(z-1)\left(\frac{\partial}{\partial z}\,\Delta_{n+1}\right)}{b-1} - \frac{\Delta_{n+1}\,(n+1)\,(a\,z-c+1)}{b-1} \tag{3}$$

```
> collect(expand(alpha[n]),diff,factor);beta[n];
```

$$\frac{z\,(z-1)\left(\frac{\partial}{\partial z}\,\Delta_n\right)}{\Delta_n\,(b-1)} - \frac{z\,(z-1)\left(\frac{\partial}{\partial z}\,\Delta_{n+1}\right)}{\Delta_{n+1}\,(b-1)} - \frac{a\,z-c+1}{b-1}$$

$$\frac{\Delta_{n+1}\,\Delta_{n-1}}{\Delta_n^2} \tag{4}$$

```
> restart;B:=a-n-c+2;C:=n+c-2;
```

$$B := a - n - c + 2$$

$$C := n + c - 2 \tag{5}$$

```
> Delta[n]:=(n)->z^(n*(n-C-1))*(z-1)^((n*(1-B-n)))*H[n];H[n]:=(n)
  ->Delta[n]/(z^(n*(n-C-1))*(z-1)^((n*(1-B-n))));
```

$$\Delta_n := n \to z^{n\,(n-C-1)}\,(z-1)^{n\,(1-B-n)}\,H_n$$

$$H_n := n \to \frac{\Delta_n}{z^{n\,(n-C-1)}\,(z-1)^{n\,(1-B-n)}} \tag{6}$$

```
> Delta[n](n+1)*Delta[n](n-1)/Delta[n](n)^2;
```

$$\frac{z^{(n+1)\,(2-c)}\,(z-1)^{(n+1)\,(-2-a+c)}\,H_{n+1}\,z^{-(n-1)\,c}\,(z-1)^{(n-1)\,(-a+c)}\,H_{n-1}}{\left(z^{n\,(-c+1)}\right)^2\left((z-1)^{n\,(-1-a+c)}\right)^2 H_n^2} \tag{7}$$

```
> simplify(%);
```

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$$\frac{z^2 H_{n+1} H_{n-1}}{(z-1)^2 H_n^2} \tag{8}$$

```
> z^2/(z-1)^2*(z-1)^2*Diff((z-1)^2*Diff(ln(H[n](n)),z),z);
```

$$z^2 \left(\frac{\partial}{\partial z} \left((z-1)^2 \left(\frac{\partial}{\partial z} \ln\left(\frac{\Delta_n}{z^{n(-c+1)} (z-1)^{n(-1-a+c)}} \right) \right) \right) \right) \tag{9}$$

```
> diff(ln(z^(n*(n-C-1))*(z-1)^(n*(1-B-n))),z);simplify(%);
```

$$\frac{\dfrac{z^{n(-c+1)} n (-c+1) (z-1)^{n(-1-a+c)}}{z} + \dfrac{z^{n(-c+1)} (z-1)^{n(-1-a+c)} n (-1-a+c)}{z-1}}{z^{n(-c+1)} (z-1)^{n(-1-a+c)}} - \frac{n (a z - c + 1)}{z (z-1)} \tag{10}$$

```
> simplify(z^2/C*diff((h(z)-%)*(z-1)^2,z));collect(%,[diff,h],
  factor);
```

$$\frac{\left(\frac{d}{dz} h(z)\right) z^4 - 2 \left(\frac{d}{dz} h(z)\right) z^3 + 2 h(z) z^3 + n a z^2 + \left(\frac{d}{dz} h(z)\right) z^2 - 2 h(z) z^2 - n c + n}{n + c - 2}$$

$$\frac{z^2 (z-1)^2 \left(\frac{d}{dz} h(z)\right)}{n + c - 2} + \frac{2 z^2 (z-1) h(z)}{n + c - 2} + \frac{n (a z^2 - c + 1)}{n + c - 2} \tag{11}$$