```
> restart;
> with(PDEtools):with(linalg):with(LinearAlgebra):alias(w=w(t),phi=
  phi(t),psi=psi(t),psi2=psi2(t),Phi=Phi(t)):d:=-1/2:
> P5:=(diff(w, t, t))-((1/(2*w)+1/(w-1))*(diff(w,t)^2)-1/t*diff(w,
  t)+(w-1)^2/t^2*(A*w+B/w)+C*w/t+d*w*(w+1)/(w-1));
```

$$P5 := \frac{\partial^2}{\partial t^2} w - \left(\frac{1}{2\,w} + \frac{1}{w-1} \right) \left(\frac{\partial}{\partial t} w \right)^2 + \frac{\frac{\partial}{\partial t} w}{t} - \frac{(w-1)^2 \left(A\,w + \frac{B}{w} \right)}{t^2} - \frac{C\,w}{t} + \frac{1}{2} \frac{w\,(w+1)}{w-1} \tag{1}$$

```
> K2a:=diff(phi, t, t)=alpha*phi/t-(beta-t)*diff(phi,t)/t;K3a:=diff
  (K2a, t):K4a:=diff(K3a, t):
```

$$K2a := \frac{\partial^2}{\partial t^2} \phi = \frac{\alpha\,\phi}{t} - \frac{(\beta - t) \left(\frac{\partial}{\partial t} \phi \right)}{t} \tag{2}$$

```
> K2b:=diff(psi, t, t)=alpha*psi/t-(beta+1-n-t)*diff(psi,t)/t;K3b:=
  diff(K2b, t):K4b:=diff(K3b, t):
```

$$K2b := \frac{\partial^2}{\partial t^2} \psi = \frac{\alpha\,\psi}{t} - \frac{(\beta + 1 - n - t) \left(\frac{\partial}{\partial t} \psi \right)}{t} \tag{3}$$

```
> K2c:=diff(Phi, t, t)=(alpha+n-1)*Phi/t-(beta+n-1-t)*diff(Phi,t)
  /t;K3c:=diff(K2c, t):K4c:=diff(K3c, t):
```

$$K2c := \frac{\partial^2}{\partial t^2} \Phi = \frac{(\alpha + n - 1)\,\Phi}{t} - \frac{(\beta + n - 1 - t) \left(\frac{\partial}{\partial t} \Phi \right)}{t} \tag{4}$$

epsilon[3]=
1;

```
> n:=1;
```

$$n := 1 \tag{5}$$

```
> diff(exp(-t)*phi,t):for K from 1 to n+1 do;l[K]:=diff(%,t)*t;
  od:wronskian([diff(exp(-t)*phi,t),seq(l[k],k=1..n)],t):for K from
  1 to n+1 do;h[K]:=Row(%,1);row(%%,2);wronskian(%*t,t):od:simplify
  (subs(<seq(simplify(h[k]),k=1..n+1)>)):tau[n+1]:=det(%):
> exp(-t)*phi:for K from 1 to n do;l[K]:=diff(%,t)*t;od:wronskian(
  [exp(-t)*phi,seq(l[k],k=1..n-1)],t):for K from 1 to n do;h[K]:=
  Row(%,1);row(%%,2);wronskian(%*t,t):od:simplify(<seq(simplify(h
  [k]),k=1..n)>):tau[n]:=det(%):
Error, (in linalg:-row) row index out of bounds
> w:=convert(simplify(expand(subs(K4a,K3a,K2a,1+1/(alpha-beta-n)*
  (beta+t+t*diff(ln((tau[n+1])/(tau[n])),t))))),parfrac,diff(phi,t)
  ):
> H:={A=1/2*(beta-alpha+n)^2,B=-1/2*alpha^2,C=1+n-beta};
```

$$H := \left\{ A = \frac{1}{2} (-\alpha + \beta + 1)^2, B = -\frac{1}{2} \alpha^2, C = 2 - \beta \right\} \tag{6}$$

```
> coeff(collect(subs(K3a,K2a,numer(subs(H,P5))),[diff(phi,t),t],
  factor),diff(phi,t)^3);
```

$$0 \tag{7}$$

epsilon[3]=

-1

```
> diff(phi,t):for K from 1 to n+1 do;l[K]:=diff(%,t)*t;od:wronskian
  ([diff(phi,t),seq(l[k],k=1..n)],t):for K from 1 to n+1 do;h[K]:=
  Row(%,1);row(%%,2);wronskian(%*t,t):od:simplify(subs(K4a,K3a,K2a,
  <seq(simplify(h[k]),k=1..n+1)>)):tau1[n+1]:=det(%):
```

```
> phi:for K from 1 to n do;l[K]:=diff(%,t)*t;od:wronskian([phi,seq
  (l[k],k=1..n-1)],t):for K from 1 to n do;h[K]:=Row(%,1);row(%%,2)
  ;wronskian(%*t,t):od:simplify(subs(K4a,K3a,K2a,<seq(simplify(h[k]
  ),k=1..n)>)):tau1[n]:=det(%):
```

Error, (in linalg:-row) row index out of bounds

```
> tau2[n+1]:=collect(subs(K4b,K3b,K2b,det(Wronskian([t^(1-alpha)*
  diff(t^alpha*psi,t),seq(diff(t^(1-alpha)*diff(t^alpha*psi,t),
  t$j),j=1..n)],t))),t,factor):
```

```
> tau2[n]:=collect(subs(K4b,K3b,K2b,det(Wronskian([psi,seq(diff
  (psi,t$j),j=1..n-1)],t))),t,factor):tau2[0]:=1:
```

```
> tau3[n+1]:=collect(subs(K4c,K3c,K2c,det(Wronskian([t^(beta+n)*
  diff(Phi,t,t),seq(diff(t^(beta+n)*diff(Phi,t,t),t$j),j=1..n)],t))
  ),t,factor):
```

```
> tau3[n]:=collect(subs(K4c,K3c,K2c,det(Wronskian([t^(beta+n-2)*
  Phi,seq(diff(t^(beta+n-2)*Phi,t$j),j=1..n-1)],t))),t,factor):tau3
  [0]:=1:
```

```
> w:=convert(simplify(expand(subs(K4a,K3a,K2a,1+1/(alpha+n)*(t-
  beta-t*diff(ln((tau1[n+1])/(tau1[n])),t))))),parfrac,diff(phi,t,
  t)):
```

```
> H:={A=(alpha+n)^2/2,B=-(beta-alpha)^2/2,C=(beta-n-1)};
```

$$H := \left\{A = \frac{1}{2}\,(\alpha+1)^2,\ B = -\frac{1}{2}\,(\beta-\alpha)^2,\ C = \beta-2\right\} \tag{8}$$

```
> collect(subs(K3a,K2a,numer(subs(H,P5))),[diff(phi,t),t],factor);
```

$$0 \tag{9}$$

```
> w:=convert(simplify(expand(subs(K4b,K3b,K2b,1+1/(alpha+n)*(t-
  beta-n-t*diff(ln((tau2[n+1])/(tau2[n])),t))))),parfrac,diff(psi,
  t));
```

$$w := \frac{t\left(\frac{\partial}{\partial t}\,\psi\right)}{\psi\,(\alpha+1)} + \frac{\alpha^2+\alpha+\beta-t}{(\alpha+1)^2} - \left(\psi\,\alpha\left(\psi\,\alpha^2 - \psi\,\alpha\,\beta + \psi\,\alpha\,t - \left(\frac{\partial}{\partial t}\,\psi\right)\alpha\,\beta + 2\,\alpha\,t\left(\frac{\partial}{\partial t}\,\psi\right) + \left(\frac{\partial}{\partial t}\,\psi\right)\beta^2 - 2\,\beta\,t\left(\frac{\partial}{\partial t}\,\psi\right) + t^2\left(\frac{\partial}{\partial t}\,\psi\right) + \psi\,\alpha - \beta\left(\frac{\partial}{\partial t}\,\psi\right) + 2\,t\left(\frac{\partial}{\partial t}\,\psi\right)\right)\right) \Big/ \left(\left((\alpha\,t+t)\left(\frac{\partial}{\partial t}\,\psi\right)^2 + (\psi\,\alpha\,\beta - \psi\,\alpha\,t)\left(\frac{\partial}{\partial t}\,\psi\right) - \alpha^2\,\psi^2\right)(\alpha+1)^2\right) \tag{10}$$

```
> H:={A=(alpha+n)^2/2,B=-(beta-alpha)^2/2,C=(beta-n-1)};
```

$$H := \left\{A = \frac{1}{2}\,(\alpha+1)^2,\ B = -\frac{1}{2}\,(\beta-\alpha)^2,\ C = \beta-2\right\} \tag{11}$$

```
> collect(subs(K3b,K2b,numer(subs(H,P5))),[diff(psi,t),t],factor);
```

$$0 \tag{12}$$

```
> w:=convert(simplify(expand(subs(K4c,K3c,K2c,1+t/(alpha+n)*(1-diff
  (ln((tau3[n+1])/(tau3[n])),t))))),parfrac,diff(Phi,t));
```

$$w := \frac{t\left(\frac{\partial}{\partial t}\Phi\right)}{\Phi(\alpha+1)} + \frac{\alpha^2+\alpha+\beta-t}{(\alpha+1)^2} - \left(\Phi\,\alpha\left(\alpha^2\Phi - \alpha\Phi\beta + \Phi\alpha t - \alpha\left(\frac{\partial}{\partial t}\Phi\right)\beta + 2\alpha t\left(\frac{\partial}{\partial t}\Phi\right) + \beta^2\left(\frac{\partial}{\partial t}\Phi\right) - 2\beta t\left(\frac{\partial}{\partial t}\Phi\right) + t^2\left(\frac{\partial}{\partial t}\Phi\right) + \alpha\Phi - \beta\left(\frac{\partial}{\partial t}\Phi\right) + 2t\left(\frac{\partial}{\partial t}\Phi\right)\right)\right) \Big/ \left(\left((\alpha t + t)\left(\frac{\partial}{\partial t}\Phi\right)^2 + (\alpha\Phi\beta - \Phi\alpha t)\left(\frac{\partial}{\partial t}\Phi\right) - \Phi^2\alpha^2\right)(\alpha+1)^2\right) \tag{13}$$

```
> collect(subs(K3c,K2c,numer(subs(H,P5))),[diff(Phi,t),t],factor);
```

$$0 \tag{14}$$