```
> restart;
> alias(q=q(z),p=p(z),sigma=sigma(z),w=w(z)):
> P4:=diff(w,z,z)-(diff(w,z)^2/2/w+3/2*w^3+4*z*w^2+2*(z^2-alpha)*w+
  beta/w);sigma1:=diff(sigma, z, z)^2-4*(z*diff(sigma,z)-sigma)
  ^2+4*diff(sigma,z)*(diff(sigma,z)+2*kappa[0])*(diff(sigma,z)+2*
  kappa[infinity]);
```

$$P4 := \frac{\partial^2}{\partial z^2} w - \frac{1}{2} \frac{\left(\frac{\partial}{\partial z} w\right)^2}{w} - \frac{3}{2} w^3 - 4 z w^2 - 2 \left(z^2 - \alpha\right) w - \frac{\beta}{w}$$

$$\sigma 1 := \left(\frac{\partial^2}{\partial z^2} \sigma\right)^2 - 4 \left(z \left(\frac{\partial}{\partial z} \sigma\right) - \sigma\right)^2 + 4 \left(\frac{\partial}{\partial z} \sigma\right) \left(\frac{\partial}{\partial z} \sigma + 2 \kappa_0\right) \left(\frac{\partial}{\partial z} \sigma + 2 \kappa_\infty\right) \tag{1}$$

```
> H:=2*q*p^2-(q^2+2*z*q+2*kappa[0])*p+kappa[infinity]*q;
```

$$H := 2 q p^2 - \left(q^2 + 2 z q + 2 \kappa_0\right) p + \kappa_\infty q \tag{2}$$

```
> H1:=diff(q,z)=4*q*p-q^2-2*z*q-2*kappa[0];H2:=diff(p,z)=-2*p^2+2*
  p*q+2*z*p-kappa[infinity];
```

$$H1 := \frac{\partial}{\partial z} q = 4 q p - q^2 - 2 z q - 2 \kappa_0$$

$$H2 := \frac{\partial}{\partial z} p = -2 p^2 + 2 q p + 2 z p - \kappa_\infty \tag{3}$$

```
> S:=sigma=H;
```

$$S := \sigma = 2 q p^2 - \left(q^2 + 2 z q + 2 \kappa_0\right) p + \kappa_\infty q \tag{4}$$

```
> S1:=simplify(subs(H1,H2,diff(S,z)));S2:=collect((expand(subs(H1,
  H2,diff(S1,z)))),p,factor);
```

$$S1 := \frac{\partial}{\partial z} \sigma = -2 q p$$

$$S2 := \frac{\partial^2}{\partial z^2} \sigma = -4 q p^2 + \left(-2 q^2 + 4 \kappa_0\right) p + 2 \kappa_\infty q \tag{5}$$

```
> -2*q*p=diff(sigma, z):solve({algsubs(%,expand(S)),algsubs(%,
  expand(S2))},{q,p});
```

$$\left\{p = -\frac{1}{4} \frac{-2 z \left(\frac{\partial}{\partial z} \sigma\right) + 2 \sigma - \left(\frac{\partial^2}{\partial z^2} \sigma\right)}{\frac{\partial}{\partial z} \sigma + 2 \kappa_0}, q = \frac{1}{2} \frac{-2 z \left(\frac{\partial}{\partial z} \sigma\right) + 2 \sigma + \frac{\partial^2}{\partial z^2} \sigma}{\frac{\partial}{\partial z} \sigma + 2 \kappa_\infty}\right\} \tag{6}$$

```
> expand(sigma-subs(%,H)):factor(%);sigma1-%:
```

$$-\frac{1}{8} \frac{1}{\left(\frac{\partial}{\partial z} \sigma + 2 \kappa_\infty\right)^2 \left(\frac{\partial}{\partial z} \sigma + 2 \kappa_0\right)^2} \left(\left(2 \kappa_0 z \left(\frac{\partial}{\partial z} \sigma\right) + 2 \left(\frac{\partial}{\partial z} \sigma\right) \kappa_\infty z + 8 \kappa_0 \kappa_\infty z\right.\right. \tag{7}$$

$$+ 2 \sigma \left(\frac{\partial}{\partial z} \sigma\right) + 2 \kappa_0 \sigma + 2 \sigma \kappa_\infty + \kappa_0 \left(\frac{\partial^2}{\partial z^2} \sigma\right) - \left(\frac{\partial^2}{\partial z^2} \sigma\right) \kappa_\infty \Big) \left(4 z^2 \left(\frac{\partial}{\partial z} \sigma\right)^2\right.$$

$$\left. - 8 z \left(\frac{\partial}{\partial z} \sigma\right) \sigma - 4 \left(\frac{\partial}{\partial z} \sigma\right)^3 - 8 \kappa_0 \left(\frac{\partial}{\partial z} \sigma\right)^2 - 8 \left(\frac{\partial}{\partial z} \sigma\right)^2 \kappa_\infty - 16 \kappa_0 \left(\frac{\partial}{\partial z} \sigma\right) \kappa_\infty\right.$$

$$+4\,\sigma^2-\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)^2\Big)\Big)$$

```
> collect(2*kappa[0]*z*(diff(sigma, z))+2*(diff(sigma, z))*kappa
  [infinity]*z+8*kappa[0]*kappa[infinity]*z+2*sigma*(diff(sigma, z)
  )+2*kappa[0]*sigma+2*sigma*kappa[infinity]+kappa[0]*(diff(sigma,
  z, z))-(diff(sigma, z, z))*kappa[infinity],[diff,z],factor)/(
  (diff(sigma, z)+2*kappa[infinity])^2*(diff(sigma, z)+2*kappa[0])
  ^2*8);
```

$$\frac{1}{8}\,\frac{\left(\kappa_0-\kappa_\infty\right)\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)+\left(\left(2\,\kappa_0+2\,\kappa_\infty\right)z+2\,\sigma\right)\left(\frac{\partial}{\partial z}\,\sigma\right)+8\,\kappa_0\,\kappa_\infty\,z+2\,\sigma\left(\kappa_0+\kappa_\infty\right)}{\left(\frac{\partial}{\partial z}\,\sigma+2\,\kappa_\infty\right)^2\left(\frac{\partial}{\partial z}\,\sigma+2\,\kappa_0\right)^2} \tag{8}$$

```
> collect(4*z^2*(diff(sigma, z))^2-8*z*(diff(sigma, z))*sigma-4*
  (diff(sigma, z))^3-8*kappa[0]*(diff(sigma, z))^2-8*(diff(sigma,
  z))^2*kappa[infinity]-16*kappa[0]*(diff(sigma, z))*kappa
  [infinity]+4*sigma^2-(diff(sigma, z, z))^2,diff,factor);
```

$$-\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)^2-4\left(\frac{\partial}{\partial z}\,\sigma\right)^3+\left(4\,z^2-8\,\kappa_0-8\,\kappa_\infty\right)\left(\frac{\partial}{\partial z}\,\sigma\right)^2+\left(-8\,\sigma\,z-16\,\kappa_\infty\,\kappa_0\right)\left(\frac{\partial}{\partial z}\,\sigma\right)+4\,\sigma^2 \tag{9}$$

```
> collect(-sigma1,diff,factor);simplify(%%-%);
```

$$-\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)^2-4\left(\frac{\partial}{\partial z}\,\sigma\right)^3+\left(4\,z^2-8\,\kappa_0-8\,\kappa_\infty\right)\left(\frac{\partial}{\partial z}\,\sigma\right)^2+\left(-8\,\sigma\,z-16\,\kappa_\infty\,\kappa_0\right)\left(\frac{\partial}{\partial z}\,\sigma\right)+4\,\sigma^2$$

$$0 \tag{10}$$