```
> restart;
> alias(q=q(t),p=p(t),sigma=sigma(t)):
> sigma3d:=(t*diff(sigma,t,t))^2+(4*diff(sigma,t)^2-1)*(t*diff
  (sigma,t)-sigma)+theta[0]*theta[infinity]*diff(sigma,t)-1/4*
  (theta[0]^2+theta[infinity]^2);
```

$$sigma3d := t^2\left(\frac{\partial^2}{\partial t^2}\,\sigma\right)^2+\left(4\left(\frac{\partial}{\partial t}\,\sigma\right)^2-1\right)\left(t\left(\frac{\partial}{\partial t}\,\sigma\right)-\sigma\right)+\theta_0\,\theta_\infty\left(\frac{\partial}{\partial t}\,\sigma\right)-\frac{1}{4}\,\theta_0^2 \quad (1)$$

$$-\frac{1}{4}\,\theta_\infty^2$$

```
> H:=(q^2*p^2-(q^2+theta[0]*q-t)*p+(theta[0]+theta[infinity])*q/2)
  /t;
```

$$H := \frac{q^2\,p^2-\left(q^2+\theta_0\,q-t\right)p+\frac{1}{2}\left(\theta_0+\theta_\infty\right)q}{t} \quad (2)$$

```
> H1:=diff(q,t)=(2*q^2*p-q^2-theta[0]*q+t)/t;H2:=diff(p,t)=(-2*q*
  p^2+2*q*p+theta[0]*p-(theta[0]+theta[infinity])/2)/t;
```

$$H1 := \frac{\partial}{\partial t}\,q = \frac{2\,q^2\,p-q^2-\theta_0\,q+t}{t}$$

$$H2 := \frac{\partial}{\partial t}\,p = \frac{-2\,q\,p^2+2\,q\,p+\theta_0\,p-\frac{1}{2}\,\theta_0-\frac{1}{2}\,\theta_\infty}{t} \quad (3)$$

```
> S:=sigma=t*H-t/2+1/4*theta[0]^2;
```

$$S := \sigma = q^2\,p^2-\left(q^2+\theta_0\,q-t\right)p+\frac{1}{2}\left(\theta_0+\theta_\infty\right)q-\frac{1}{2}\,t+\frac{1}{4}\,\theta_0^2 \quad (4)$$

```
> S1:=simplify(subs(H1,H2,diff(S,t)));S2:=simplify(expand(subs(H1,
  H2,diff(S1,t))));
```

$$S1 := \frac{\partial}{\partial t}\,\sigma = -\frac{1}{2}+p$$

$$S2 := \frac{\partial^2}{\partial t^2}\,\sigma = -\frac{1}{2}\,\frac{4\,q\,p^2-4\,q\,p-2\,\theta_0\,p+\theta_0+\theta_\infty}{t} \quad (5)$$

```
> solve({S1,S2},{q,p});
```

$$\left\{p=\frac{\partial}{\partial t}\,\sigma+\frac{1}{2},\ q=-\frac{2\left(\frac{\partial^2}{\partial t^2}\,\sigma\right)t-2\,\theta_0\left(\frac{\partial}{\partial t}\,\sigma\right)+\theta_\infty}{4\left(\frac{\partial}{\partial t}\,\sigma\right)^2-1}\right\} \quad (6)$$

```
> factor(subs(%,sigma-(t*H-t/2+1/4*theta[0]^2)));collect(simplify(-
  %*(2*(diff(sigma, t))-1)*(2*(diff(sigma, t))+1)),diff,factor);
  simplify(sigma3d-%);
```

$$-\frac{1}{4}\,\frac{1}{\left(2\left(\frac{\partial}{\partial t}\,\sigma\right)-1\right)\left(2\left(\frac{\partial}{\partial t}\,\sigma\right)+1\right)}\left(4\,t^2\left(\frac{\partial^2}{\partial t^2}\,\sigma\right)^2+16\left(\frac{\partial}{\partial t}\,\sigma\right)^3 t-16\left(\frac{\partial}{\partial t}\,\sigma\right)^2\sigma\right.$$

$$\left.+4\,\theta_0\,\theta_\infty\left(\frac{\partial}{\partial t}\,\sigma\right)-4\,t\left(\frac{\partial}{\partial t}\,\sigma\right)-\theta_0^2-\theta_\infty^2+4\,\sigma\right)$$

$$\left[t^2 \left(\frac{\partial^2}{\partial t^2} \sigma \right)^2 + 4 \left(\frac{\partial}{\partial t} \sigma \right)^3 t - 4 \left(\frac{\partial}{\partial t} \sigma \right)^2 \sigma + \left(\theta_0 \, \theta_\infty - t \right) \left(\frac{\partial}{\partial t} \sigma \right) - \frac{1}{4} \theta_0^2 - \frac{1}{4} \theta_\infty^2 + \sigma \right] \quad \textbf{(7)}$$

$$0$$