

```
> restart;with(PDEtools):with(linalg):with(LinearAlgebra):with
(plots):
```

```
> a:=15;b:=15;
```

$$a := 15$$

$$b := 15$$

(1)

```
> J:=simplify(a!/GAMMA(a+1)*LaguerreL(a,b,-z)):Delta:=(n)->factor
(collect(det(Wronskian([J,seq(diff(J,z$j),j=1..n-1)],z)),z,
factor)):
```

```
> h[n]:=subs(z=0,Delta(2)/Delta(1));h[0]:=subs(z=0,Delta(1));
```

$$h_n := -\frac{265182525}{16}$$

$$h_0 := 155117520$$

(2)

```
> 1/Pi*Int(x*((1+exp(2*I*theta))^b*((1+exp(-2*I*theta))^a))*exp(z*
exp(2*I*theta)),theta=-Pi/2..Pi/2);
```

$$\frac{\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} x \left(1 + e^{2i\theta}\right)^{15} \left(1 + e^{-2i\theta}\right)^{15} e^{ze^{2i\theta}} d\theta}{\pi}$$

(3)

```
> z:=0;
```

$$z := 0$$

(4)

```
> x:=exp(2*I*theta);
```

$$x := e^{2i\theta}$$

(5)

```
> 1/Pi*int(((1+exp(2*I*theta))^b*((1+exp(-2*I*theta))^a))*exp(z*exp
(2*I*theta)),theta=-Pi/2..Pi/2)/h[0];
```

$$1$$

(6)

```
> restart;with(linalg):with(PDEtools):with(linalg):with
(LinearAlgebra):
```

```
> a:=12;b:=10;mu:=(k)->(simplify(GAMMA(a+b+1)/GAMMA(a-k+1)/GAMMA(b+
k+1)*KummerM(-a+k,b+k+1,-z)));
```

$$a := 12$$

$$b := 10$$

$$\mu := k \rightarrow \text{simplify} \left( \frac{\Gamma(a+b+1) \text{KummerM}(-a+k, b+k+1, -z)}{\Gamma(a-k+1) \Gamma(b+k+1)} \right)$$

(7)

Hankle Determinant

```
> n:=3;
```

$$n := 3$$

(8)

```
> k:=n-1;
```

$$k := 2$$

(9)

Hankel

```
> H:=HankelMatrix(<seq(mu(k),k=0..10)>,n):
```

```
> sort(factor(A*det(H)),t);
```

$$-\frac{1}{34691710959747072000000} A \left( z^{30} + 660 z^{29} + 208230 z^{28} + 41812320 z^{27} + 6002239320 z^{26} + 655986703680 z^{25} + 56758494592800 z^{24} + 3991707941760000 z^{23} \right)$$

(10)

$$\begin{aligned}
& + 232477588528416000 z^{22} + 11366348279711040000 z^{21} \\
& + 471285274032268819200 z^{20} + 16697862221607350784000 z^{19} \\
& + 508375782997532360064000 z^{18} + 13353572930233073319936000 z^{17} \\
& + 303429746783233298866176000 z^{16} + 5973276388743564544917504000 z^{15} \\
& + 101913371496288863815772160000 z^{14} + 1505988595010889234813173760000 z^{13} \\
& + 19240608569586043173754429440000 z^{12} \\
& + 211908343713816305992331427840000 z^{11} \\
& + 2003363216154924633797236162560000 z^{10} \\
& + 16163019829953526302698294476800000 z^9 \\
& + 110423107422722559715222133145600000 z^8 \\
& + 632262646241259747844165101158400000 z^7 \\
& + 2992703318262736720017121699430400000 z^6 \\
& + 11492989817070061879244129776435200000 z^5 \\
& + 34879936614422189140047407874048000000 z^4 \\
& + 80455459644011917439257024659456000000 z^3 \\
& + 132475878808974316104779792646144000000 z^2 \\
& + 138666340435561900969489128751104000000 z \\
& + 69333170217780950484744564375552000000)
\end{aligned}$$

Delta Determinant

> phi:=mu(0):phi:factor(A\*det(wronskian([phi,diff(phi,z),diff(phi,z,z)],z)));

$$\begin{aligned}
& - \frac{1}{34691710959747072000000} A (z^{30} + 660 z^{29} + 208230 z^{28} + 41812320 z^{27} \\
& + 6002239320 z^{26} + 655986703680 z^{25} + 56758494592800 z^{24} + 3991707941760000 z^{23} \\
& + 232477588528416000 z^{22} + 11366348279711040000 z^{21} \\
& + 471285274032268819200 z^{20} + 16697862221607350784000 z^{19} \\
& + 508375782997532360064000 z^{18} + 13353572930233073319936000 z^{17} \\
& + 303429746783233298866176000 z^{16} + 5973276388743564544917504000 z^{15} \\
& + 101913371496288863815772160000 z^{14} + 1505988595010889234813173760000 z^{13} \\
& + 19240608569586043173754429440000 z^{12} \\
& + 211908343713816305992331427840000 z^{11} \\
& + 2003363216154924633797236162560000 z^{10} \\
& + 16163019829953526302698294476800000 z^9 \\
& + 110423107422722559715222133145600000 z^8 \\
& + 632262646241259747844165101158400000 z^7 \\
& + 2992703318262736720017121699430400000 z^6 \\
& + 11492989817070061879244129776435200000 z^5
\end{aligned} \tag{11}$$

$$\begin{aligned}
& + 34879936614422189140047407874048000000 z^4 \\
& + 80455459644011917439257024659456000000 z^3 \\
& + 132475878808974316104779792646144000000 z^2 \\
& + 138666340435561900969489128751104000000 z \\
& + 69333170217780950484744564375552000000)
\end{aligned}$$

> **phi:=simplify(LaguerreL(a,b,-z)):phi:factor(A\*(a!/GAMMA(a+1))^n\*  
det(wronskian([phi,diff(phi,z),diff(phi,z,z)],z))) ;**

$$\begin{aligned}
& - \frac{1}{34691710959747072000000} A (z^{30} + 660 z^{29} + 208230 z^{28} + 41812320 z^{27} \\
& + 6002239320 z^{26} + 655986703680 z^{25} + 56758494592800 z^{24} + 3991707941760000 z^{23} \\
& + 232477588528416000 z^{22} + 11366348279711040000 z^{21} \\
& + 471285274032268819200 z^{20} + 16697862221607350784000 z^{19} \\
& + 508375782997532360064000 z^{18} + 13353572930233073319936000 z^{17} \\
& + 303429746783233298866176000 z^{16} + 5973276388743564544917504000 z^{15} \\
& + 101913371496288863815772160000 z^{14} + 1505988595010889234813173760000 z^{13} \\
& + 19240608569586043173754429440000 z^{12} \\
& + 211908343713816305992331427840000 z^{11} \\
& + 2003363216154924633797236162560000 z^{10} \\
& + 16163019829953526302698294476800000 z^9 \\
& + 110423107422722559715222133145600000 z^8 \\
& + 632262646241259747844165101158400000 z^7 \\
& + 2992703318262736720017121699430400000 z^6 \\
& + 11492989817070061879244129776435200000 z^5 \\
& + 34879936614422189140047407874048000000 z^4 \\
& + 80455459644011917439257024659456000000 z^3 \\
& + 132475878808974316104779792646144000000 z^2 \\
& + 138666340435561900969489128751104000000 z \\
& + 69333170217780950484744564375552000000)
\end{aligned} \tag{12}$$