S6 Rational Solution Generator for the first case

```
> restart;with(plots):with(orthopoly):with(linalg):alias(w=w(z),
  sigma=sigma(z)):
> S4:=diff(sigma,z$2)^2-4*(z*diff(sigma,z)-sigma)^2+4*diff(sigma,z)
  *(diff(sigma,z)+2*theta[0])*(diff(sigma,z)+2*theta[infinity]);
```

$$S4 := \left(\frac{\partial^2}{\partial z^2} \sigma \right)^2 - 4 \left(z \left(\frac{\partial}{\partial z} \sigma \right) - \sigma \right)^2 + 4 \left(\frac{\partial}{\partial z} \sigma \right) \left(\frac{\partial}{\partial z} \sigma + 2 \theta_0 \right) \left(\frac{\partial}{\partial z} \sigma + 2 \theta_\infty \right) \qquad (1)$$

```
> c:=N->product(1/2^j*1/factorial(j),j=1..N-1):
> h:=(M,N)->simplify(c(N)*det(wronskian([seq(HermiteH(M+j,z),j=0..
  N-1)],z))):
> tau:=(m,n)->simplify(diff(ln(h(m,n)),z)):
> m:=6;n:=6;sigma1:=tau(m,n):C1:=plot(sigma1,z=-6..6,y=-27..27,
  colour=black,thickness=3):
```

$$m := 6$$
$$n := 6 \qquad (2)$$

```
> n:=4;sigma1:=tau(m,n):C2:=plot(sigma1,z=-6..6,y=-27..27,colour=
  blue,thickness=3):
```

$$n := 4 \qquad (3)$$

```
> n:=6;sigma1:=tau(m,n):C3:=plot(sigma1,z=-6..6,y=-27..27,colour=
  magenta,thickness=3):
```
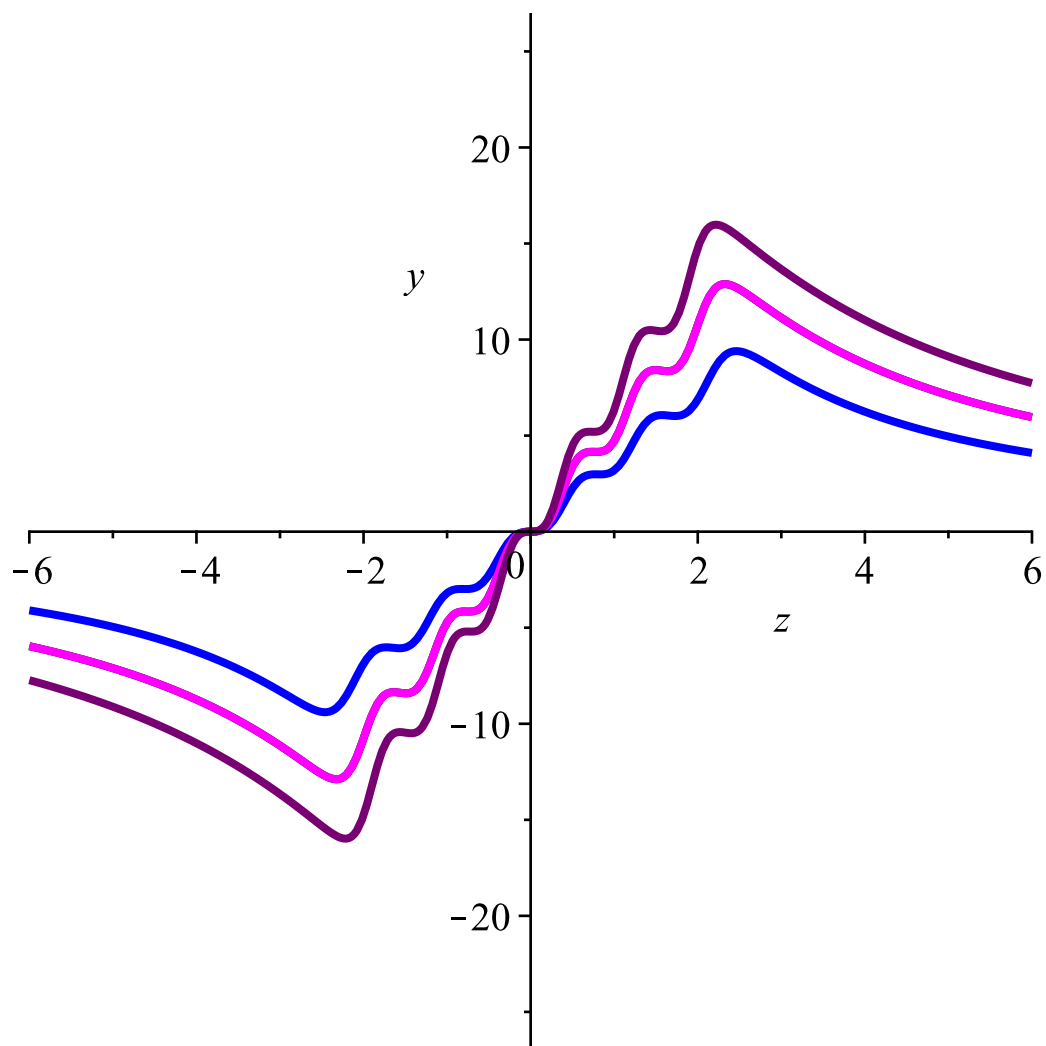
$$n := 6 \qquad (4)$$

```
> n:=8;sigma1:=tau(m,n):C4:=plot(sigma1,z=-6..6,y=-27..27,colour=
  purple,thickness=3):
```

$$n := 8 \qquad (5)$$

```
> display(C1,C2,C3,C4);
```

```
> RootOf(h(m,n),z):J1:=evalf(allvalues(%)):
> A:=complexplot([J1],style=point,symbol=solidcircle,color=blue,
  symbolsize=23):
> C1:=plot(sigma1/3.3,z=-8..8,y=-8..8,colour=black,thickness=3):
> display(C1,A);
```