```
> restart;
> alias(q=q(z),p=p(z),sigma=sigma(z)):
> L[PETER]:=(z*(diff(sigma, z, z))-(diff(sigma, z)))^2+4*(diff
  (sigma,z))^2*(z*diff(sigma,z)-2*sigma)+4*z*theta[infinity]*diff
  (sigma,z)-z^2*(z*diff(sigma,z)-2*sigma+2*theta[0]);
```

$$L_{PETER} := \left(z\left(\frac{\partial^2}{\partial z^2}\sigma\right)-\left(\frac{\partial}{\partial z}\sigma\right)\right)^2+4\left(\frac{\partial}{\partial z}\sigma\right)^2\left(z\left(\frac{\partial}{\partial z}\sigma\right)-2\sigma\right)+4z\theta_\infty\left(\frac{\partial}{\partial z}\sigma\right)-z^2\left(z\left(\frac{\partial}{\partial z}\sigma\right)-2\sigma+2\theta_0\right) \tag{1}$$

```
> theta[0]:=1/8*(lambda[infinity]^2+(lambda[0]-2)^2);theta
  [infinity]:=-1/4*lambda[infinity]*(lambda[0]-2);
```

$$\theta_0 := \frac{1}{8}\lambda_\infty^2+\frac{1}{8}\left(\lambda_0-2\right)^2$$

$$\theta_\infty := -\frac{1}{4}\lambda_\infty\left(\lambda_0-2\right) \tag{2}$$

```
> H:=q^2*p^2-z*p*q^2-(lambda[0]-1)*p*q+z*p+(lambda[0]-2-lambda
  [infinity])*z*q/2;
```

$$H := q^2p^2-zpq^2-\left(\lambda_0-1\right)pq+zp+\frac{1}{2}\left(\lambda_0-2-\lambda_\infty\right)zq \tag{3}$$

```
> H1:=diff(q,z)=(2*p*q^2-z*q^2-(lambda[0]-1)*q+z)/z;H2:=diff(p,z)=
  (-2*p^2*q+2*z*p*q+(lambda[0]-1)*p-(lambda[0]-2-lambda[infinity])*
  z/2)/z;
```

$$H1 := \frac{\partial}{\partial z}q=\frac{2pq^2-zq^2-\left(\lambda_0-1\right)q+z}{z}$$

$$H2 := \frac{\partial}{\partial z}p=\frac{-2p^2q+2zpq+\left(\lambda_0-1\right)p-\frac{1}{2}\left(\lambda_0-2-\lambda_\infty\right)z}{z} \tag{4}$$

```
> S:=sigma=1/2*H+p*q/2+1/8*(lambda[0]-2)^2-1/4*z^2;
```

$$S := \sigma=\frac{1}{2}q^2p^2-\frac{1}{2}zpq^2-\frac{1}{2}\left(\lambda_0-1\right)pq+\frac{1}{2}zp+\frac{1}{4}\left(\lambda_0-2-\lambda_\infty\right)zq+\frac{1}{2}pq+\frac{1}{8}\left(\lambda_0-2\right)^2-\frac{1}{4}z^2 \tag{5}$$

```
> S1:=simplify(subs(H1,H2,diff(S,z)));S2:=simplify(expand(subs(H1,
  H2,diff(S1,z))));
```

$$S1 := \frac{\partial}{\partial z}\sigma=p-\frac{1}{2}z$$

$$S2 := \frac{\partial^2}{\partial z^2}\sigma=-\frac{1}{2}\,\frac{4p^2q-4zpq-2p\lambda_0+z\lambda_0-z\lambda_\infty+2p-z}{z} \tag{6}$$

```
> solve({S1,S2},{q,p});
```

$$\left\{p=\frac{\partial}{\partial z}\sigma+\frac{1}{2}z,\ q=-\frac{2z\left(\frac{\partial^2}{\partial z^2}\sigma\right)-2\lambda_0\left(\frac{\partial}{\partial z}\sigma\right)-z\lambda_\infty+2\left(\frac{\partial}{\partial z}\sigma\right)}{4\left(\frac{\partial}{\partial z}\sigma\right)^2-z^2}\right\} \tag{7}$$

```
> collect(expand(subs(%,sigma-(1/2*H+p*q/2+1/8*(lambda[0]-2)^2-1/4*
```

```
z^2))),diff,factor):factor(%);simplify(-2*%*(-z+2*(diff(sigma, z)
))*(2*(diff(sigma, z))+z));simplify(L[PETER]-%);
```

$$-\frac{1}{8}\,\frac{1}{\left(-z+2\left(\frac{\partial}{\partial z}\,\sigma\right)\right)\left(2\left(\frac{\partial}{\partial z}\,\sigma\right)+z\right)}\left(4\,z^2\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)^2+16\left(\frac{\partial}{\partial z}\,\sigma\right)^3 z-4\,z^3\left(\frac{\partial}{\partial z}\,\sigma\right)\right.$$
$$-4\,\lambda_0\left(\frac{\partial}{\partial z}\,\sigma\right)z\,\lambda_\infty-z^2\,\lambda_0^2-z^2\,\lambda_\infty^2-8\,z\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)\left(\frac{\partial}{\partial z}\,\sigma\right)-32\left(\frac{\partial}{\partial z}\,\sigma\right)^2\sigma$$
$$\left.+8\,z\,\lambda_\infty\left(\frac{\partial}{\partial z}\,\sigma\right)+8\,\sigma\,z^2+4\,z^2\,\lambda_0+4\left(\frac{\partial}{\partial z}\,\sigma\right)^2-4\,z^2\right)$$

$$z^2\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)^2+4\left(\frac{\partial}{\partial z}\,\sigma\right)^3 z-z^3\left(\frac{\partial}{\partial z}\,\sigma\right)-\lambda_0\left(\frac{\partial}{\partial z}\,\sigma\right)z\,\lambda_\infty-\frac{1}{4}\,z^2\,\lambda_0^2-\frac{1}{4}\,z^2\,\lambda_\infty^2$$
$$-2\,z\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)\left(\frac{\partial}{\partial z}\,\sigma\right)-8\left(\frac{\partial}{\partial z}\,\sigma\right)^2\sigma+2\,z\,\lambda_\infty\left(\frac{\partial}{\partial z}\,\sigma\right)+2\,\sigma\,z^2+z^2\,\lambda_0+\left(\frac{\partial}{\partial z}\,\sigma\right)^2$$
$$-z^2$$

$$0 \tag{8}$$