```
> restart;
> with(plots):with(linalg):alias(w=w(z)):
> P3:=numer(simplify(diff(w,z)^2/w-diff(w,z)/z+(A*w^2+B)/z+C*w^3+
  delta/w-diff(w,z,z)));Digits := 50;
```

$$P3 := C\,w^4\,z + A\,w^3 + \left(\frac{\partial}{\partial z}\,w\right)^2 z - \left(\frac{\partial^2}{\partial z^2}\,w\right) w\,z + B\,w - \left(\frac{\partial}{\partial z}\,w\right) w + \delta\,z$$

$$Digits := 50 \qquad (1)$$

```
> A:=2*n+2*mu+3;B:=2*n-2*mu+1;C:=1;delta:=-1;n:=2;
```

$$A := 2\,n + 2\,\mu + 3$$

$$B := 2\,n - 2\,\mu + 1$$

$$C := 1$$

$$\delta := -1$$

$$n := 2 \qquad (2)$$

```
> #A:=-2*n+2*mu-1;B:=-2*n-2*mu-3;C:=1;delta:=-1;n:=7;
> phi:=(mu,a)->simplify(LaguerreL(2*a-1,mu-2*a+1,-z)):
> tau:=(mu,n)->det(Wronskian([phi(mu,n),seq(diff(phi(mu,n),z$(2*
  j-2))
  ,j=2..n)],z));
```

$$\tau := (\mu, n) \rightarrow linalg{:}\text{-}det\left(linalg{:}\text{-}Wronskian\left(\left[\phi(\mu, n), seq\left(\frac{\partial^{2j-2}}{\partial z^{2j-2}}\,\phi(\mu, n), j = 2..n\right)\right], z\right)\right) \qquad (3)$$

n=1

```
> #w:=convert(1+diff(ln(1/phi(mu,n)),z),parfrac,z);
> #w:=convert(1-diff(ln(1/phi(mu-1,n)),z),parfrac,z);
```

The rest

```
> w:=convert(1-diff(ln(tau(mu+1,n+1)/tau(mu,n)),z),parfrac,z):
> C:=plot(subs(mu=10,w),z=-25..5,y=-15..15,colour=black,thickness=
  2,discont=true):

> with(plots):with(linalg):alias(w=w(z)):
> #P3:=numer(simplify(diff(w,z)^2/w-diff(w,z)/z+(A*w^2+B)/z+C*w^3+
  delta/w-diff(w,z,z)));Digits := 10;
> #A:=2*n+2*mu+3;B:=2*n-2*mu+1;C:=1;delta:=-1;n:=6;
> #A:=-2*n+2*mu-1;B:=-2*n-2*mu-3;C:=1;delta:=-1;n:=7;
> #phi:=(mu,a)->simplify(LaguerreL(2*a-1,mu-2*a+1,-z)):#phi2:=(a)
  ->simplify((-1)^a/a!*KummerU(-a,mu-a+1,-z));
> #tau:=(mu,n)->det(Wronskian([phi(mu,n),seq(diff(phi(mu,n),z$(2*
  j-2))
  ,j=2..n)],z));
Error, `,` unexpected
```

n=1

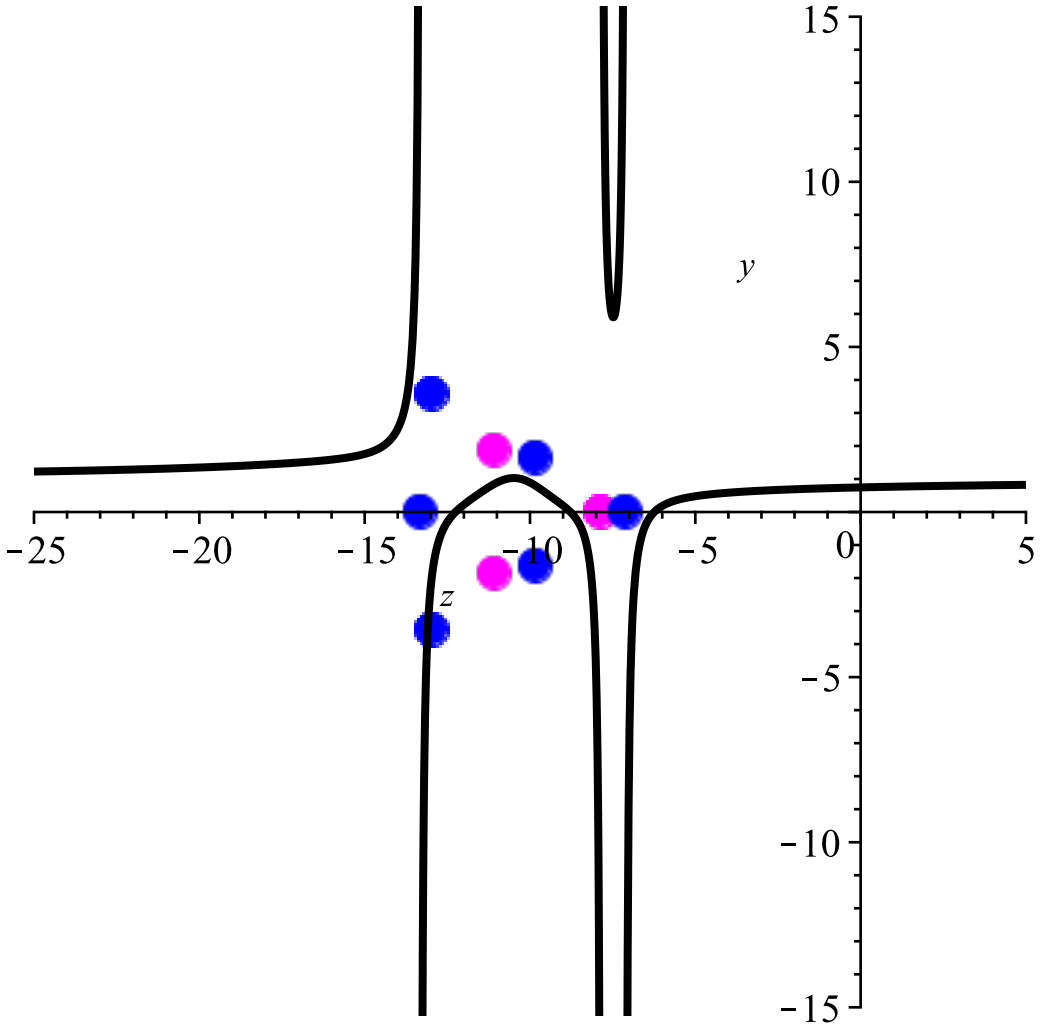
```
> #w:=convert(1+diff(ln(1/phi(mu,n)),z),parfrac,z);
> #w:=convert(1-diff(ln(1/phi(mu-1,n)),z),parfrac,z);
```

The rest

```
> #w:=convert(1-diff(ln(tau(mu+1,n+1)/tau(mu,n)),z),parfrac,z):
> #w:=convert(1+diff(ln(tau(mu,n+1)/tau(mu+1,n)),z),parfrac,z):
> #simplify(P3);
> nu:=10;
```

$$\nu := 10 \tag{4}$$

```
> RootOf(subs(mu=nu,tau(mu,n)),z):J1:=evalf(allvalues(%)):RootOf
  (subs(mu=nu,tau(mu+1,n+1)),z):J2:=(allvalues(%)):
> A:=complexplot([J1],style=point,symbol=solidcircle,color=magenta,
  symbolsize=25):
> B:=complexplot([J2],style=point,symbol=solidcircle,color=blue,
  symbolsize=25):
> C:=plot(subs(mu=nu,w),z=-25..5,y=-15..15,colour=black,thickness=
  3,discont=true):
> display(A,B,C);
```




```
> restart;with(linalg):with(LinearAlgebra):with(plots):alias(sigma=
  sigma(z),phi=phi(z)):
> S3:=(z*(diff(sigma, z, z))-(diff(sigma, z)))^2+4*(diff(sigma,z))
  ^2*(z*diff(sigma,z)-2*sigma)+4*z*theta[infinity]*diff(sigma,z)-
  z^2*(z*diff(sigma,z)-2*sigma+2*theta[0]);
```

$$S3 := \left(z\left(\frac{\partial^2}{\partial z^2}\,\sigma\right) - \left(\frac{\partial}{\partial z}\,\sigma\right)\right)^2 + 4\left(\frac{\partial}{\partial z}\,\sigma\right)^2\left(z\left(\frac{\partial}{\partial z}\,\sigma\right) - 2\,\sigma\right) + 4\,z\,\theta_\infty\left(\frac{\partial}{\partial z}\,\sigma\right) - z^2\left(z\left(\frac{\partial}{\partial z}\,\sigma\right) - 2\,\sigma + 2\,\theta_0\right) \tag{5}$$

```
> theta[infinity]:=mu^2-(n+1/2)^2;theta[0]:=mu^2+(n+1/2)^2;
```

$$\theta_\infty := \mu^2 - \left(n + \frac{1}{2}\right)^2$$

$$\theta_0 := \mu^2 + \left(n + \frac{1}{2}\right)^2 \quad (6)$$

```
> n:=3;nu:=10;
```

$$n := 3$$

$$\nu := 10 \quad (7)$$

```
> phi:=(mu,a)->simplify(LaguerreL(2*a-1,mu-2*a+1,-z)):#phi2:=(a)
  ->simplify((-1)^a/a!*KummerU(-a,mu-a+1,-z));
> tau:=(mu,n)->det(Wronskian([phi(mu,n),seq(diff(phi(mu,n),z$(2*
  j-2))
  ,j=2..n)],z));
```

$$\tau := (\mu, n) \rightarrow linalg{:}\text{-}det\left(linalg{:}\text{-}Wronskian\left(\left[\phi(\mu, n), seq\left(\frac{\partial^{2j-2}}{\partial z^{2j-2}}\,\phi(\mu, n), j = 2..n\right)\right], z\right)\right) \quad (8)$$

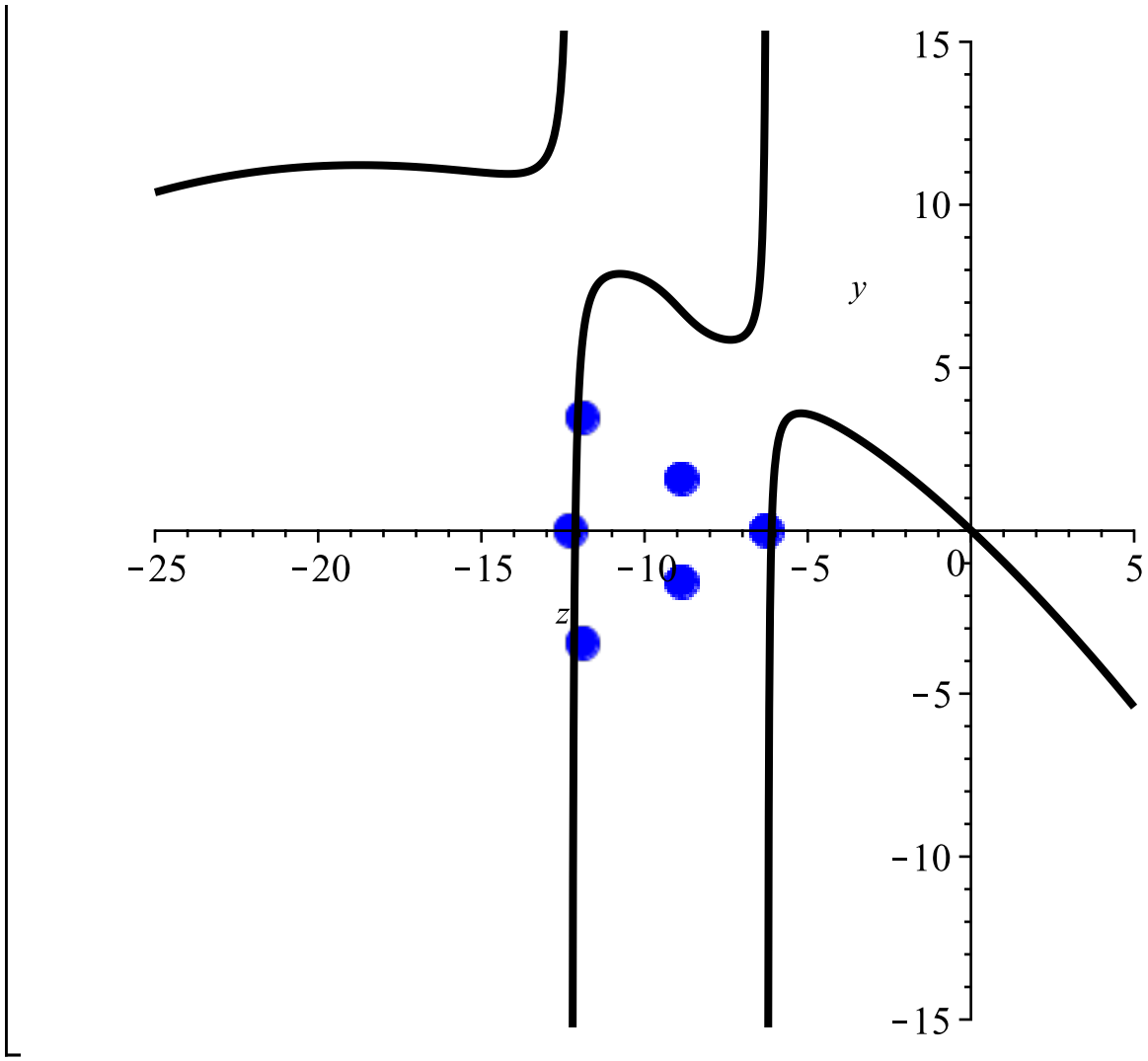
```
> sigma:=-z^2/4-mu*z+1/8+z*diff(ln(tau(mu,n)),z);
```

$$\sigma := -\frac{1}{4}z^2 - \mu z + \frac{1}{8} + \left(z\left(-\frac{1}{5}\mu + \frac{1}{3}\mu^3 + \frac{2}{3}\mu^2 z + \frac{1}{3}\mu z^2 - \frac{2}{15}z^5 - \frac{2}{15}\mu^5 - \frac{2}{3}\mu^4 z - \frac{4}{3}\mu^3 z^2 - \frac{4}{3}\mu^2 z^3 - \frac{2}{3}\mu z^4\right)\right) \Big/ \left(-\frac{1}{5}\mu z + \frac{1}{3}\mu^3 z + \frac{1}{3}\mu^2 z^2 + \frac{1}{9}\mu z^3 - \frac{4}{45}\mu^2 + \frac{1}{9}\mu^4 - \frac{1}{45}\mu^6 - \frac{1}{45}z^6 - \frac{2}{15}\mu^5 z - \frac{1}{3}\mu^4 z^2 - \frac{4}{9}\mu^3 z^3 - \frac{1}{3}\mu^2 z^4 - \frac{2}{15}\mu z^5\right) \quad (9)$$

```
> simplify(S3);
```

$$0 \quad (10)$$

```
> RootOf(subs(mu=nu,tau(mu,n)),z):J1:=evalf(allvalues(%)):
> A:=complexplot([J1],style=point,symbol=solidcircle,color=blue,
  symbolsize=25):
> C:=plot(subs(mu=nu,sigma/10),z=-25..5,y=-15..15,colour=black,
  thickness=3,discont=true):
> display(A,C);
```


15
10
y
5
−25
−20
−15
−10
−5
0
5
z
−5
−10
−15