\sigma=t diff(ln(M))

```
> restart;with(PDEtools):with(linalg):with(LinearAlgebra):alias(S
  [n]=S[n](t),sigma=sigma(t),phi=phi(t),psi=psi(t),Phi=Phi(t));
```

$$S_n, \sigma, \phi, \psi, \Phi \tag{1}$$

```
> S5:=t^2*(diff(sigma, t, t))^2-(2*(diff(sigma, t))^2-t*(diff
  (sigma, t))+sigma)^2+(4*(diff(sigma, t)+k[0]))*(diff(sigma, t)+k
  [1])*(diff(sigma, t)+k[2])*(diff(sigma, t)+k[3]);
```

$$S5 := t^2 \left(\frac{\partial^2}{\partial t^2}\,\sigma\right)^2 - \left(2\left(\frac{\partial}{\partial t}\,\sigma\right)^2 - t\left(\frac{\partial}{\partial t}\,\sigma\right) + \sigma\right)^2 + 4\left(\frac{\partial}{\partial t}\,\sigma + k_0\right)\left(\frac{\partial}{\partial t}\,\sigma + k_1\right)\left(\frac{\partial}{\partial t}\,\sigma + k_2\right)\left(\frac{\partial}{\partial t}\,\sigma + k_3\right) \tag{2}$$

```
> K2a2:=diff(phi, t, t)=(alpha)*phi/t-(beta-t)*diff(phi,t)/t;K3a2:=
  diff(K2a2, t):K4a2:=diff(K3a2, t):
```

$$K2a2 := \frac{\partial^2}{\partial t^2}\,\phi = \frac{\alpha\,\phi}{t} - \frac{(\beta - t)\left(\frac{\partial}{\partial t}\,\phi\right)}{t} \tag{3}$$

```
> A:=-(5/8)*(n+1)^2+((1/4)*beta+1/4+(1/2)*alpha+(3/4)*t)*(n+1)-
  (1/8*(2*alpha+1-beta))*(2*alpha+1-beta+2*t);k[0]:= -(1/4)*(2*
  alpha-3*beta-n+2);k[1]:=-1/4*(2*alpha+beta-n-2);k[2]:= 1/4*(2*
  alpha-beta+n+2);k[3]:= 1/4*(2*alpha-beta-3*n-2);
```

$$A := -\frac{5}{8}\,(n+1)^2 + \left(\frac{1}{4}\,\beta + \frac{1}{4} + \frac{1}{2}\,\alpha + \frac{3}{4}\,t\right)(n+1) - \frac{1}{8}\,(2\,\alpha + 1 - \beta)\,(2\,\alpha + 1 - \beta + 2\,t)$$

$$k_0 := -\frac{1}{2}\,\alpha + \frac{3}{4}\,\beta + \frac{1}{4}\,n - \frac{1}{2}$$

$$k_1 := -\frac{1}{2}\,\alpha - \frac{1}{4}\,\beta + \frac{1}{4}\,n + \frac{1}{2}$$

$$k_2 := \frac{1}{2}\,\alpha - \frac{1}{4}\,\beta + \frac{1}{4}\,n + \frac{1}{2}$$

$$k_3 := \frac{1}{2}\,\alpha - \frac{1}{4}\,\beta - \frac{3}{4}\,n - \frac{1}{2} \tag{4}$$

epsilon[3]=1;

```
> n:=0;
```

$$n := 0 \tag{5}$$

```
> phi*exp(-t):for K from 1 to n+1 do;l[K]:=diff(%,t)*t;od:wronskian
  ([phi*exp(-t),seq(l[j],j=1..n)],t):for j from 1 to n+1 do;h[j]:=
  Row(%,1);row(%%,2);wronskian(%*t,t):od:subs(K4a2,K3a2,K2a2,<seq
  (simplify(h[j]),j=1..n+1)>):W:=det(%):
```

Error, (in linalg:-row) row index out of bounds

```
> sigma:=convert(simplify(subs(K4a2,K3a2,K2a2,t*diff(ln(W),t)))+A,
  parfrac,diff(phi,t));
```

$$\sigma := \frac{t\left(\frac{\partial}{\partial t}\,\phi\right)}{\phi} - \frac{1}{4}\,t - \frac{3}{8} + \frac{1}{4}\,\beta + \frac{1}{2}\,\alpha - \frac{1}{8}\,(2\,\alpha + 1 - \beta)\,(2\,\alpha + 1 - \beta + 2\,t) \tag{6}$$

```
> simplify(subs(K4a2,K3a2,K2a2,S5)):simplify(%);
```

$$0 \tag{7}$$

```
> n:=1;
```

$$n := 1 \tag{8}$$

```
> K2a:=diff(phi, t, t)=(alpha+1)*phi/t-(beta+1-t)*diff(phi,t)/t;
  K3a:=diff(K2a, t):K4a:=diff(K3a, t):
```

$$K2a := \frac{\partial^2}{\partial t^2}\,\phi = \frac{(\alpha+1)\,\phi}{t} - \frac{(\beta+1-t)\left(\frac{\partial}{\partial t}\,\phi\right)}{t} \tag{9}$$

```
> K2b:=diff(psi, t, t)=(alpha+1)*psi/t-(beta+2-n-t)*diff(psi,t)/t;
  K3b:=diff(K2b, t):K4b:=diff(K3b, t):
```

$$K2b := \frac{\partial^2}{\partial t^2}\,\psi = \frac{(\alpha+1)\,\psi}{t} - \frac{(\beta+1-t)\left(\frac{\partial}{\partial t}\,\psi\right)}{t} \tag{10}$$

```
> K2c:=diff(Phi, t, t)=(alpha+n)*Phi/t-(beta+n-t)*diff(Phi,t)/t;
  K3c:=diff(K2c, t):K4c:=diff(K3c, t):
```

$$K2c := \frac{\partial^2}{\partial t^2}\,\Phi = \frac{(\alpha+1)\,\Phi}{t} - \frac{(\beta+1-t)\left(\frac{\partial}{\partial t}\,\Phi\right)}{t} \tag{11}$$

```
> A := -(5/8)*n^2+(-(3/4)*t+1/2-(1/2)*alpha+(3/4)*beta)*n-(1/8*(2*
  alpha-beta))*(2*alpha+2*t-beta);k[0] := -(1/4)*beta-(1/2)*alpha-
  (1/4)*n;k[1] := (1/2)*alpha+(3/4)*n-(1/4)*beta;k[2] := -(1/2)*
  alpha+(3/4)*beta-(1/4)*n;k[3] := -(1/4)*beta+(1/2)*alpha-(1/4)*n;
```

$$A := -\frac{1}{8} - \frac{3}{4}\,t - \frac{1}{2}\,\alpha + \frac{3}{4}\,\beta - \frac{1}{8}\,(2\,\alpha-\beta)\,(2\,\alpha+2\,t-\beta)$$

$$k_0 := -\frac{1}{4}\,\beta - \frac{1}{2}\,\alpha - \frac{1}{4}$$

$$k_1 := \frac{1}{2}\,\alpha + \frac{3}{4} - \frac{1}{4}\,\beta$$

$$k_2 := -\frac{1}{2}\,\alpha + \frac{3}{4}\,\beta - \frac{1}{4}$$

$$k_3 := -\frac{1}{4}\,\beta + \frac{1}{2}\,\alpha - \frac{1}{4} \tag{12}$$

```
> phi:for K from 1 to n do;l[K]:=diff(%,t)*t;od:wronskian([phi,seq
  (l[j],j=1..n-1)],t):for j from 1 to n do;h[j]:=Row(%,1);row(%%,2)
  ;wronskian(%*t,t):od:subs(K4a,K3a,K2a,<seq(simplify(h[j]),j=1..n)
  >):W:=det(%):
Error, (in linalg:-row) row index out of bounds
```

epsilon[3]=

-1;

```
> sigma:=convert(simplify(subs(K4a,K3a,K2a,t*diff(ln(W),t)))+A,
  parfrac,diff(phi,t));
```

$$\sigma := \frac{t\left(\frac{\partial}{\partial t}\,\phi\right)}{\phi} - \frac{1}{8} - \frac{3}{4}\,t - \frac{1}{2}\,\alpha + \frac{3}{4}\,\beta - \frac{1}{8}\,(2\,\alpha-\beta)\,(2\,\alpha+2\,t-\beta) \tag{13}$$

```
> simplify(subs(K4a,K3a,K2a,S5)):simplify(%);
```

$$0 \tag{14}$$

```
> phi:for K from 1 to n do;l[K]:=diff(%,t)*t;od:wronskian([phi,seq
```

```
(l[j],j=1..n-1)],t):for j from 1 to n do;h[j]:=Row(%,1);row(%%,2)
;wronskian(%*t,t):od:subs(K4a,K3a,K2a,<seq(simplify(h[j]),j=1..n)
>):W:=det(%):
```

Error, (in linalg:-row) row index out of bounds

```
> tau[n]:=collect(subs(K4b,K3b,K2b,det(Wronskian([psi,seq(diff(psi,
t$j),j=1..n-1)],t))),t,factor):
> tau2[n]:=collect(subs(K4c,K3c,K2c,det(Wronskian([t^(beta+n-2)*
Phi,seq(diff(t^(beta+n-2)*Phi,t$j),j=1..n-1)],t))),t,factor):
```

Solution using delta operator

```
> sigma:=convert(simplify(subs(K4a,K3a,K2a,t*diff(ln(W),t)))+A,
parfrac,diff(phi,t));
```

$$\sigma := \frac{t\left(\frac{\partial}{\partial t}\,\phi\right)}{\phi} - \frac{1}{8} - \frac{3}{4}\,t - \frac{1}{2}\,\alpha + \frac{3}{4}\,\beta - \frac{1}{8}\,(2\,\alpha - \beta)\,(2\,\alpha + 2\,t - \beta) \tag{15}$$

```
> simplify(subs(K4a,K3a,K2a,S5)):simplify(%);
```

$$0 \tag{16}$$

Solution using normal Wronskian

```
> sigma:=convert(simplify(subs(K4b,K3b,K2b,A-n/2*(1-n)+simplify(t*
diff(ln(tau[n]),t)))),parfrac,diff(psi,t));
```

$$\sigma := \tag{17}$$

$$-\frac{1}{8}\,\frac{1}{\psi}\left(\psi + 6\,t\,\psi + 4\,\alpha\,\psi - 6\,\beta\,\psi + 4\,\psi\,\alpha^2 + 4\,\psi\,\alpha\,t - 4\,\psi\,\alpha\,\beta - 2\,\psi\,\beta\,t + \psi\,\beta^2 - 8\,t\left(\frac{\partial}{\partial t}\,\psi\right)\right)$$

```
> simplify(subs(K4b,K3b,K2b,S5)):simplify(%);
```

$$0 \tag{18}$$

Solution using normal Wronskian with seed t^(beta+n-2)(M+U)

```
> sigma:=convert(simplify(subs(K4c,K3c,K2c,+A+(1/2)*n^2+(1/2-beta)*
n+t*diff(ln(tau2[n]),t))),parfrac,diff(Phi,t));
```

$$\sigma := \tag{19}$$

$$-\frac{1}{8}\,\frac{1}{\Phi}\left(\Phi + 6\,\Phi\,t + 4\,\alpha\,\Phi - 6\,\Phi\,\beta + 4\,\Phi\,\alpha^2 + 4\,\alpha\,\Phi\,t - 4\,\Phi\,\alpha\,\beta - 2\,\beta\,\Phi\,t + \Phi\,\beta^2 - 8\left(\frac{\partial}{\partial t}\,\Phi\right)t\right)$$

```
> simplify(subs(K4c,K3c,K2c,S5)):simplify(%);
```

$$0 \tag{20}$$