

```
> restart; alias (H[n]=H[n](z), sigma=sigma(z)) :
> S5:=(z*diff(H[n], z, z))^2-(n*(n+alpha)+H[n]+(-alpha-beta-2*n-z)*
diff(H[n], z))^2-4*diff(H[n], z)*(z*diff(H[n], z)-H[n])*(-beta-diff
(H[n], z));
```

$$S5 := z^2 \left( \frac{\partial^2}{\partial z^2} H_n \right)^2 - \left( n(n + \alpha) + H_n + (-\alpha - \beta - 2n - z) \left( \frac{\partial}{\partial z} H_n \right) \right)^2 - 4 \left( \frac{\partial}{\partial z} H_n \right) \left( z \left( \frac{\partial}{\partial z} H_n \right) - H_n \right) \left( -\beta - \left( \frac{\partial}{\partial z} H_n \right) \right) \quad (1)$$

```
> JMOeq:=(z*diff(sigma, z, z))^2-(2*diff(sigma, z)^2-z*diff(sigma, z)+
sigma)^2+4*product(diff(sigma, z)+k[j], j=0..3);
```

$$JMOeq := z^2 \left( \frac{\partial^2}{\partial z^2} \sigma \right)^2 - \left( 2 \left( \frac{\partial}{\partial z} \sigma \right)^2 - z \left( \frac{\partial}{\partial z} \sigma \right) + \sigma \right)^2 + 4 \left( \frac{\partial}{\partial z} \sigma + k_0 \right) \left( \frac{\partial}{\partial z} \sigma + k_1 \right) \left( \frac{\partial}{\partial z} \sigma + k_2 \right) \left( \frac{\partial}{\partial z} \sigma + k_3 \right) \quad (2)$$

```
> eq1:=collect(expand(subs(H[n]=sigma+b*z+c, S5)-JMOeq), [diff, sigma,
z], factor):
```

```
> coeff(eq1, diff(sigma, z)):bc:=solve({op(1,%), op(2,%)}, {b, c}):
```

```
> S[n]=collect(subs(%, sigma+b*z+c), z, factor):collect(%, -sigma, [n],
factor);
```

$$S_n - \sigma = -\frac{1}{2} n^2 + \left( -\frac{1}{2} \alpha + \frac{1}{2} z - \frac{1}{2} \beta \right) n + \frac{1}{8} (\alpha - \beta) (\alpha + 2z - \beta) \quad (3)$$

```
> eq2:=collect(expand(subs(bc, eq1)), [diff, sigma, z], factor):
```

```
> solve({seq(coeff(eq2, diff(sigma, z), j), j=0..3)}, {k[0], k[1], k[2], k
[3]}): A:=%[1];
```

$$A := \left\{ k_0 = -\frac{1}{4} \beta + \frac{1}{4} \alpha - \frac{1}{2} n, k_1 = \frac{3}{4} \beta + \frac{1}{4} \alpha + \frac{1}{2} n, k_2 = \frac{1}{2} n + \frac{1}{4} \alpha - \frac{1}{4} \beta, k_3 = -\frac{1}{4} \beta - \frac{3}{4} \alpha - \frac{1}{2} n \right\} \quad (4)$$

```
> restart; with(PDEtools):with(linalg):with(VectorCalculus):with
(LinearAlgebra):alias(S[n]=S[n](t), sigma=sigma(t), phi=phi(t));
```

$$S_n, \sigma, \phi \quad (5)$$

```
> S5:=(t*diff(S[n], t, t))^2-(n*(n+alpha)+S[n]+(-alpha-beta-2*n-t)*
diff(S[n], t))^2-4*diff(S[n], t)*(t*diff(S[n], t)-S[n])*(-beta-diff
(S[n], t));collect(expand(W5), [diff, n], factor);
```

$$S5 := t^2 \left( \frac{\partial^2}{\partial t^2} S_n \right)^2 - \left( n(n + \alpha) + S_n + (-\alpha - \beta - 2n - t) \left( \frac{\partial}{\partial t} S_n \right) \right)^2 - 4 \left( \frac{\partial}{\partial t} S_n \right) \left( t \left( \frac{\partial}{\partial t} S_n \right) - S_n \right) \left( -\beta - \left( \frac{\partial}{\partial t} S_n \right) \right) \quad (6)$$

W5

```
> K2 := diff(phi, t, t) = (alpha+1)*phi/t-(alpha+beta+2-t)*(diff
(phi, t))/t:K3:=diff(K2, t):K4:=diff(K3, t):
```

```
> n:=2;
```

$$n := 2 \quad (7)$$

```
> tau[n]:=collect(subs(K3, K2, det(Wronskian([phi, seq(diff(phi, t$j),
j=1..n-1)], t)), diff(phi, t), factor):
```

```
> S[n]:=convert(simplify(subs(K4, K3, K2, t*diff(ln(tau[n]), t))),
```

```
parfrac,diff(phi,t));
```

$$S_2(t) := -\alpha - \beta - 2 + t + \frac{\phi \left( -2 \left( \frac{\partial}{\partial t} \phi \right) - \left( \frac{\partial}{\partial t} \phi \right) \alpha - \left( \frac{\partial}{\partial t} \phi \right) \beta + \phi \alpha + \phi \right)}{\left( \frac{\partial}{\partial t} \phi \right)^2 t + (-t \phi + 2 \phi + \phi \alpha + \beta \phi) \left( \frac{\partial}{\partial t} \phi \right) - \phi^2 \alpha - \phi^2}$$

(8)

```
> subs(K3,K2,S5):collect(numer(%),[diff(phi,t)],factor);
```

0

(9)