```
> restart;alias(sigma=sigma(x),phi=phi(x),psi=psi(z)):with
  (PDEtools):with(plots):with(LinearAlgebra):with(linalg):
> S6:=diff(sigma,x)*(x*(x-1)*diff(sigma,x,x))^2+(diff(sigma,x)*(2*
  sigma-(2*x-1)*diff(sigma,x))+nu[1]*nu[2]*nu[3]*nu[4])^2-product
  (diff(sigma,x)+nu[k]^2,k=1..4);
```

$$S6 := \left(\frac{\partial}{\partial x}\sigma\right) x^2 (x-1)^2 \left(\frac{\partial^2}{\partial x^2}\sigma\right)^2 + \left(\left(\frac{\partial}{\partial x}\sigma\right)\left(2\sigma - (2x-1)\left(\frac{\partial}{\partial x}\sigma\right)\right) + \nu_1\nu_2\nu_3\nu_4\right)^2 - \left(\frac{\partial}{\partial x}\sigma + \nu_1^2\right)\left(\frac{\partial}{\partial x}\sigma + \nu_2^2\right)\left(\frac{\partial}{\partial x}\sigma + \nu_3^2\right)\left(\frac{\partial}{\partial x}\sigma + \nu_4^2\right) \tag{1}$$

```
> #a:=-2;b:=10;c:=3;n:=2;
> #nu[1] := 1/2-n-(1/2)*b+(1/2)*a;nu[2] := -1/2+c-(1/2)*b-(1/2)*a;
  nu[3] := 1/2+(1/2)*a-(1/2)*b;nu[4] := 1/2-(1/2)*a-(1/2)*b;
> nu[1]:=-(a+1-b-2*(n+1))/2;nu[2]:=(2*c-1-a-b)/2;nu[3]:=-(1+a-b)/2;
  nu[4]:=(1-a-b)/2;
```

$$\nu_1 := -\frac{1}{2}a + \frac{1}{2} + \frac{1}{2}b + n$$

$$\nu_2 := c - \frac{1}{2} - \frac{1}{2}a - \frac{1}{2}b$$

$$\nu_3 := -\frac{1}{2} - \frac{1}{2}a + \frac{1}{2}b$$

$$\nu_4 := \frac{1}{2} - \frac{1}{2}a - \frac{1}{2}b \tag{2}$$

```
> sigma:=A*x+B+C/x;
```

$$\sigma := Ax + B + \frac{C}{x} \tag{3}$$

```
> N:=4;
```

$$N := 4 \tag{4}$$

```
> collect(expand(S6),[diff,x],factor);solve([op(1,%),op(2,%),op(3,
  %)],{A,B,C}):factor(op(1,%[N]));convert(4*factor(op(2,%%[N])),
  parfrac,b);factor(op(3,%%%[N]));
```

$$\frac{1}{16}A\,(-a^6+3a^5c+3a^5n+a^4b^2-a^4bc+a^4bn-3a^4c^2-8a^4cn-3a^4n^2-2a^3b^2c$$
$$-2a^3b^2n+4a^3bc^2-4a^3bn^2+8a^3c^2n+8a^3cn^2+a^2b^4-2a^2b^3c+2a^2b^3n$$
$$-2a^2b^2c^2-2a^2b^2n^2-8a^2bc^2n+8a^2bcn^2-8a^2c^2n^2-ab^4c-ab^4n+4ab^3c^2$$
$$-4ab^3n^2+8ab^2c^2n+8ab^2cn^2-b^6+3b^5c-3b^5n-3b^4c^2+8b^4cn-3b^4n^2$$
$$-8b^3c^2n+8b^3cn^2-8b^2c^2n^2-4Aa^4+8Aa^3c+8Aa^3n-8Aa^2b^2+8Aa^2bc$$
$$-8Aa^2bn-12Aa^2c^2-8Aa^2cn-12Aa^2n^2+8Aab^2c+8Aab^2n+8Aabc^2$$
$$-8Aabn^2+16Aac^2n+16Aacn^2-4Ab^4+8Ab^3c-8Ab^3n-12Ab^2c^2$$
$$+8Ab^2cn-12Ab^2n^2-16Abc^2n+16Abcn^2-16Ac^2n^2+4Ba^4-8Ba^3c$$
$$-8Ba^3n-8Ba^2b^2+8Ba^2bc-8Ba^2bn+16Ba^2cn+8Bab^2c+8Bab^2n$$
$$+4Bb^4-8Bb^3c+8Bb^3n-16Bb^2cn-a^4c+a^4n-4a^3bc-4a^3bn+4a^3c^2$$
$$-4a^3n^2+10a^2b^2c-10a^2b^2n-4a^2bc^2+32a^2bcn-4a^2bn^2-8a^2c^2n$$
$$+8a^2cn^2-4ab^3c-4ab^3n-4ab^2c^2+4ab^2n^2-16abc^2n-16abcn^2-b^4c$$

$$+ b^4 n + 4 b^3 c^2 + 4 b^3 n^2 + 8 b^2 c^2 n - 8 b^2 c n^2 + 16 b c^2 n^2 - 16 A^2 a^2 + 16 A^2 a c$$
$$+ 16 A^2 a n - 16 A^2 b^2 + 16 A^2 b c - 16 A^2 b n - 16 A^2 c^2 - 16 A^2 n^2 + 8 A a^2 c - 8 A a^2 n$$
$$- 32 A a b c - 32 A a b n + 8 A a c^2 - 8 A a n^2 + 8 A b^2 c - 8 A b^2 n + 8 A b c^2$$
$$+ 48 A b c n + 8 A b n^2 - 16 A c^2 n + 16 A c n^2 + 8 B a^2 c - 8 B a^2 n - 16 B a b c$$
$$- 16 B a b n + 8 B b^2 c - 8 B b^2 n + 32 B b c n + a^4 - 2 a^3 c - 2 a^3 n - 10 a^2 b^2$$
$$+ 10 a^2 b c - 10 a^2 b n - 2 a^2 c^2 - 2 a^2 n^2 + 10 a b^2 c + 10 a b^2 n - 4 a b c^2 + 4 a b n^2$$
$$+ 8 a c^2 n + 8 a c n^2 + b^4 - 2 b^3 c + 2 b^3 n - 2 b^2 c^2 - 16 b^2 c n - 2 b^2 n^2 + 8 b c^2 n$$
$$- 8 b c n^2 - 8 c^2 n^2 + 64 A^2 B + 16 A^2 c - 16 A^2 n + 64 A B^2 - 8 A a^2 + 8 A a c + 8 A a n$$
$$- 8 A b^2 + 8 A b c - 8 A b n - 12 A c^2 + 8 A c n - 12 A n^2 - 8 B a^2 + 8 B a c + 8 B a n$$
$$- 8 B b^2 + 8 B b c - 8 B b n - 16 B c n - 2 a^2 c + 2 a^2 n - 4 a b c - 4 a b n + 4 a c^2$$
$$- 4 a n^2 - 2 b^2 c + 2 b^2 n + 4 b c^2 + 4 b n^2 - 8 c^2 n + 8 c n^2 - 16 A^2 + 8 A c - 8 A n$$
$$- 8 B c + 8 B n + a^2 - a c - a n + b^2 - b c + b n - 3 c^2 + 8 c n - 3 n^2 - 4 A + 4 B + 3 c$$
$$- 3 n - 1) + \frac{1}{2} \frac{1}{x} (A C (a^4 - 2 a^3 c - 2 a^3 n - 2 a^2 b^2 + 2 a^2 b c - 2 a^2 b n$$
$$+ 4 a^2 c n + 2 a b^2 c + 2 a b^2 n + b^4 - 2 b^3 c + 2 b^3 n - 4 b^2 c n + 2 a^2 c - 2 a^2 n$$
$$- 4 a b c - 4 a b n + 2 b^2 c - 2 b^2 n + 8 b c n + 16 A^2 + 32 A B - 2 a^2 + 2 a c + 2 a n$$
$$- 2 b^2 + 2 b c - 2 b n - 4 c n - 2 c + 2 n + 1)) - \frac{1}{16} \frac{1}{x^2} (C (-a^6 + 3 a^5 c + 3 a^5 n$$
$$+ a^4 b^2 - a^4 b c + a^4 b n - 3 a^4 c^2 - 8 a^4 c n - 3 a^4 n^2 - 2 a^3 b^2 c - 2 a^3 b^2 n + 4 a^3 b c^2$$
$$- 4 a^3 b n^2 + 8 a^3 c^2 n + 8 a^3 c n^2 + a^2 b^4 - 2 a^2 b^3 c + 2 a^2 b^3 n - 2 a^2 b^2 c^2 - 2 a^2 b^2 n^2$$
$$- 8 a^2 b c^2 n + 8 a^2 b c n^2 - 8 a^2 c^2 n^2 - a b^4 c - a b^4 n + 4 a b^3 c^2 - 4 a b^3 n^2$$
$$+ 8 a b^2 c^2 n + 8 a b^2 c n^2 - b^6 + 3 b^5 c - 3 b^5 n - 3 b^4 c^2 + 8 b^4 c n - 3 b^4 n^2 - 8 b^3 c^2 n$$
$$+ 8 b^3 c n^2 - 8 b^2 c^2 n^2 - 8 A a^4 + 16 A a^3 c + 16 A a^3 n - 16 A a^2 b^2 + 16 A a^2 b c$$
$$- 16 A a^2 b n - 24 A a^2 c^2 - 16 A a^2 c n - 24 A a^2 n^2 + 16 A a b^2 c + 16 A a b^2 n$$
$$+ 16 A a b c^2 - 16 A a b n^2 + 32 A a c^2 n + 32 A a c n^2 - 8 A b^4 + 16 A b^3 c - 16 A b^3 n$$
$$- 24 A b^2 c^2 + 16 A b^2 c n - 24 A b^2 n^2 - 32 A b c^2 n + 32 A b c n^2 - 32 A c^2 n^2 + 4 B a^4$$
$$- 8 B a^3 c - 8 B a^3 n - 8 B a^2 b^2 + 8 B a^2 b c - 8 B a^2 b n + 16 B a^2 c n + 8 B a b^2 c$$
$$+ 8 B a b^2 n + 4 B b^4 - 8 B b^3 c + 8 B b^3 n - 16 B b^2 c n - a^4 c + a^4 n - 4 a^3 b c$$
$$- 4 a^3 b n + 4 a^3 c^2 - 4 a^3 n^2 + 10 a^2 b^2 c - 10 a^2 b^2 n - 4 a^2 b c^2 + 32 a^2 b c n - 4 a^2 b n^2$$
$$- 8 a^2 c^2 n + 8 a^2 c n^2 - 4 a b^3 c - 4 a b^3 n - 4 a b^2 c^2 + 4 a b^2 n^2 - 16 a b c^2 n$$
$$- 16 a b c n^2 - b^4 c + b^4 n + 4 b^3 c^2 + 4 b^3 n^2 + 8 b^2 c^2 n - 8 b^2 c n^2 + 16 b c^2 n^2$$
$$- 48 A^2 a^2 + 48 A^2 a c + 48 A^2 a n - 48 A^2 b^2 + 48 A^2 b c - 48 A^2 b n - 48 A^2 c^2$$
$$- 48 A^2 n^2 + 16 A a^2 c - 16 A a^2 n - 64 A a b c - 64 A a b n + 16 A a c^2 - 16 A a n^2$$
$$+ 16 A b^2 c - 16 A b^2 n + 16 A b c^2 + 96 A b c n + 16 A b n^2 - 32 A c^2 n + 32 A c n^2$$
$$+ 8 B a^2 c - 8 B a^2 n - 16 B a b c - 16 B a b n + 8 B b^2 c - 8 B b^2 n + 32 B b c n + a^4$$
$$- 2 a^3 c - 2 a^3 n - 10 a^2 b^2 + 10 a^2 b c - 10 a^2 b n - 2 a^2 c^2 - 2 a^2 n^2 + 10 a b^2 c$$

$$+10\,a\,b^2\,n-4\,a\,b\,c^2+4\,a\,b\,n^2+8\,a\,c^2\,n+8\,a\,c\,n^2+b^4-2\,b^3\,c+2\,b^3\,n-2\,b^2\,c^2$$
$$-16\,b^2\,c\,n-2\,b^2\,n^2+8\,b\,c^2\,n-8\,b\,c\,n^2-8\,c^2\,n^2+192\,A^2\,B-256\,A^2\,C+48\,A^2\,c$$
$$-48\,A^2\,n+128\,A\,B^2-16\,A\,a^2+16\,A\,a\,c+16\,A\,a\,n-16\,A\,b^2+16\,A\,b\,c-16\,A\,b\,n$$
$$-24\,A\,c^2+16\,A\,c\,n-24\,A\,n^2-8\,B\,a^2+8\,B\,a\,c+8\,B\,a\,n-8\,B\,b^2+8\,B\,b\,c-8\,B\,b\,n$$
$$-16\,B\,c\,n-2\,a^2\,c+2\,a^2\,n-4\,a\,b\,c-4\,a\,b\,n+4\,a\,c^2-4\,a\,n^2-2\,b^2\,c+2\,b^2\,n+4\,b\,c^2$$
$$+4\,b\,n^2-8\,c^2\,n+8\,c\,n^2-48\,A^2-64\,A\,C+16\,A\,c-16\,A\,n-8\,B\,c+8\,B\,n+a^2-a\,c$$
$$-a\,n+b^2-b\,c+b\,n-3\,c^2+8\,c\,n-3\,n^2-8\,A+4\,B+3\,c-3\,n-1\big)\big)$$
$$-\frac{1}{2}\,\frac{1}{x^3}\big(C^2\,\big(a^4-2\,a^3\,c-2\,a^3\,n-2\,a^2\,b^2+2\,a^2\,b\,c-2\,a^2\,b\,n+4\,a^2\,c\,n+2\,a\,b^2\,c$$
$$+2\,a\,b^2\,n+b^4-2\,b^3\,c+2\,b^3\,n-4\,b^2\,c\,n+2\,a^2\,c-2\,a^2\,n-4\,a\,b\,c-4\,a\,b\,n+2\,b^2\,c$$
$$-2\,b^2\,n+8\,b\,c\,n+48\,A^2+64\,A\,B-2\,a^2+2\,a\,c+2\,a\,n-2\,b^2+2\,b\,c-2\,b\,n-4\,c\,n$$
$$+16\,A-2\,c+2\,n+1\big)\big)+\frac{1}{4}\,\frac{1}{x^4}\big(C^2\,\big(-a^4+2\,a^3\,c+2\,a^3\,n-2\,a^2\,b^2+2\,a^2\,b\,c$$
$$-2\,a^2\,b\,n-3\,a^2\,c^2-2\,a^2\,c\,n-3\,a^2\,n^2+2\,a\,b^2\,c+2\,a\,b^2\,n+2\,a\,b\,c^2-2\,a\,b\,n^2$$
$$+4\,a\,c^2\,n+4\,a\,c\,n^2-b^4+2\,b^3\,c-2\,b^3\,n-3\,b^2\,c^2+2\,b^2\,c\,n-3\,b^2\,n^2-4\,b\,c^2\,n$$
$$+4\,b\,c\,n^2-4\,c^2\,n^2-12\,A\,a^2+12\,A\,a\,c+12\,A\,a\,n-12\,A\,b^2+12\,A\,b\,c-12\,A\,b\,n$$
$$-12\,A\,c^2-12\,A\,n^2+2\,a^2\,c-2\,a^2\,n-8\,a\,b\,c-8\,a\,b\,n+2\,a\,c^2-2\,a\,n^2+2\,b^2\,c$$
$$-2\,b^2\,n+2\,b\,c^2+12\,b\,c\,n+2\,b\,n^2-4\,c^2\,n+4\,c\,n^2+48\,A\,B-128\,A\,C+12\,A\,c$$
$$-12\,A\,n+16\,B^2-2\,a^2+2\,a\,c+2\,a\,n-2\,b^2+2\,b\,c-2\,b\,n-3\,c^2+2\,c\,n-3\,n^2+4\,A$$
$$-16\,C+2\,c-2\,n-1\big)\big)+\frac{8\,C^3\,(3\,A+2\,B+1)}{x^5}$$
$$-\frac{C^3\,\left(-a^2+a\,c+a\,n-b^2+b\,c-b\,n-c^2-n^2+4\,B-16\,C+c-n+3\right)}{x^6}-\frac{8\,C^4}{x^7}$$

$$A=-\frac{1}{4}\,(-1+a+b)^2$$

$$4\,B=b^2+(-c+n)\,b+a^2+3\,a\,c-n\,a-2\,n\,c-4\,a-c+n+1$$
$$-\frac{4\,a\,(a\,c-c\,n-a-c+n+1)}{-1+a+b}$$

$$C=\frac{a\,(-1+b)\,(b+n)\,(c-1)\,(a-n-1)\,(a+b-c)}{(a+b)\,(a+b-2)\,(-1+a+b)^2} \qquad \textbf{(5)}$$

```
> solve([-a^6+3*a^5*c+3*a^5*n+a^4*b^2-a^4*b*c+a^4*b*n-3*a^4*c^2-8*
  a^4*c*n-3*a^4*n^2-2*a^3*b^2*c-2*a^3*b^2*n+4*a^3*b*c^2-4*a^3*b*
  n^2+8*a^3*c^2*n+8*a^3*c*n^2+a^2*b^4-2*a^2*b^3*c+2*a^2*b^3*n-2*
  a^2*b^2*c^2-2*a^2*b^2*n^2-8*a^2*b*c^2*n+8*a^2*b*c*n^2-8*a^2*c^2*
  n^2-a*b^4*c-a*b^4*n+4*a*b^3*c^2-4*a*b^3*n^2+8*a*b^2*c^2*n+8*a*
  b^2*c*n^2-b^6+3*b^5*c-3*b^5*n-3*b^4*c^2+8*b^4*c*n-3*b^4*n^2-8*
  b^3*c^2*n+8*b^3*c*n^2-8*b^2*c^2*n^2-4*A*a^4+8*A*a^3*c+8*A*a^3*
  n-8*A*a^2*b^2+8*A*a^2*b*c-8*A*a^2*b*n-12*A*a^2*c^2-8*A*a^2*c*
  n-12*A*a^2*n^2+8*A*a*b^2*c+8*A*a*b^2*n+8*A*a*b*c^2-8*A*a*b*
  n^2+16*A*a*c^2*n+16*A*a*c*n^2-4*A*b^4+8*A*b^3*c-8*A*b^3*n-12*A*
  b^2*c^2+8*A*b^2*c*n-12*A*b^2*n^2-16*A*b*c^2*n+16*A*b*c*n^2-16*A*
```

```
c^2*n^2+4*B*a^4-8*B*a^3*c-8*B*a^3*n-8*B*a^2*b^2+8*B*a^2*b*c-8*B*
a^2*b*n+16*B*a^2*c*n+8*B*a*b^2*c+8*B*a*b^2*n+4*B*b^4-8*B*b^3*c+8*
B*b^3*n-16*B*b^2*c*n-a^4*c+a^4*n-4*a^3*b*c-4*a^3*b*n+4*a^3*c^2-4*
a^3*n^2+10*a^2*b^2*c-10*a^2*b^2*n-4*a^2*b*c^2+32*a^2*b*c*n-4*a^2*
b*n^2-8*a^2*c^2*n+8*a^2*c*n^2-4*a*b^3*c-4*a*b^3*n-4*a*b^2*c^2+4*
a*b^2*n^2-16*a*b*c^2*n-16*a*b*c*n^2-b^4*c+b^4*n+4*b^3*c^2+4*b^3*
n^2+8*b^2*c^2*n-8*b^2*c*n^2+16*b*c^2*n^2-16*A^2*a^2+16*A^2*a*
c+16*A^2*a*n-16*A^2*b^2+16*A^2*b*c-16*A^2*b*n-16*A^2*c^2-16*A^2*
n^2+8*A*a^2*c-8*A*a^2*n-32*A*a*b*c-32*A*a*b*n+8*A*a*c^2-8*A*a*
n^2+8*A*b^2*c-8*A*b^2*n+8*A*b*c^2+48*A*b*c*n+8*A*b*n^2-16*A*c^2*
n+16*A*c*n^2+8*B*a^2*c-8*B*a^2*n-16*B*a*b*c-16*B*a*b*n+8*B*b^2*
c-8*B*b^2*n+32*B*b*c*n+a^4-2*a^3*c-2*a^3*n-10*a^2*b^2+10*a^2*b*
c-10*a^2*b*n-2*a^2*c^2-2*a^2*n^2+10*a*b^2*c+10*a*b^2*n-4*a*b*
c^2+4*a*b*n^2+8*a*c^2*n+8*a*c*n^2+b^4-2*b^3*c+2*b^3*n-2*b^2*c^2
-16*b^2*c*n-2*b^2*n^2+8*b*c^2*n-8*b*c*n^2-8*c^2*n^2+64*A^2*B+16*
A^2*c-16*A^2*n+64*A*B^2-8*A*a^2+8*A*a*c+8*A*a*n-8*A*b^2+8*A*b*
c-8*A*b*n-12*A*c^2+8*A*c*n-12*A*n^2-8*B*a^2+8*B*a*c+8*B*a*n-8*B*
b^2+8*B*b*c-8*B*b*n-16*B*c*n-2*a^2*c+2*a^2*n-4*a*b*c-4*a*b*n+4*a*
c^2-4*a*n^2-2*b^2*c+2*b^2*n+4*b*c^2+4*b*n^2-8*c^2*n+8*c*n^2-16*
A^2+8*A*c-8*A*n-8*B*c+8*B*n+a^2-a*c-a*n+b^2-b*c+b*n-3*c^2+8*c*
n-3*n^2-4*A+4*B+3*c-3*n-1,A*C*(a^4-2*a^3*c-2*a^3*n-2*a^2*b^2+2*
a^2*b*c-2*a^2*b*n+4*a^2*c*n+2*a*b^2*c+2*a*b^2*n+b^4-2*b^3*c+2*
b^3*n-4*b^2*c*n+2*a^2*c-2*a^2*n-4*a*b*c-4*a*b*n+2*b^2*c-2*b^2*
n+8*b*c*n+16*A^2+32*A*B-2*a^2+2*a*c+2*a*n-2*b^2+2*b*c-2*b*n-4*c*
n-2*c+2*n+1)],{A,B});
```

$$\left\{A=0, B=\frac{1}{4}\left(a^6-3\,a^5\,c-3\,a^5\,n-a^4\,b^2+a^4\,b\,c-a^4\,b\,n+3\,a^4\,c^2+8\,a^4\,c\,n+3\,a^4\,n^2\right.\right. \tag{6}$$
$$+2\,a^3\,b^2\,c+2\,a^3\,b^2\,n-4\,a^3\,b\,c^2+4\,a^3\,b\,n^2-8\,a^3\,c^2\,n-8\,a^3\,c\,n^2-a^2\,b^4+2\,a^2\,b^3\,c$$
$$-2\,a^2\,b^3\,n+2\,a^2\,b^2\,c^2+2\,a^2\,b^2\,n^2+8\,a^2\,b\,c^2\,n-8\,a^2\,b\,c\,n^2+8\,a^2\,c^2\,n^2+a\,b^4\,c$$
$$+a\,b^4\,n-4\,a\,b^3\,c^2+4\,a\,b^3\,n^2-8\,a\,b^2\,c^2\,n-8\,a\,b^2\,c\,n^2+b^6-3\,b^5\,c+3\,b^5\,n+3\,b^4\,c^2$$
$$-8\,b^4\,c\,n+3\,b^4\,n^2+8\,b^3\,c^2\,n-8\,b^3\,c\,n^2+8\,b^2\,c^2\,n^2+a^4\,c-a^4\,n+4\,a^3\,b\,c+4\,a^3\,b\,n$$
$$-4\,a^3\,c^2+4\,a^3\,n^2-10\,a^2\,b^2\,c+10\,a^2\,b^2\,n+4\,a^2\,b\,c^2-32\,a^2\,b\,c\,n+4\,a^2\,b\,n^2$$
$$+8\,a^2\,c^2\,n-8\,a^2\,c\,n^2+4\,a\,b^3\,c+4\,a\,b^3\,n+4\,a\,b^2\,c^2-4\,a\,b^2\,n^2+16\,a\,b\,c^2\,n$$
$$+16\,a\,b\,c\,n^2+b^4\,c-b^4\,n-4\,b^3\,c^2-4\,b^3\,n^2-8\,b^2\,c^2\,n+8\,b^2\,c\,n^2-16\,b\,c^2\,n^2-a^4$$
$$+2\,a^3\,c+2\,a^3\,n+10\,a^2\,b^2-10\,a^2\,b\,c+10\,a^2\,b\,n+2\,a^2\,c^2+2\,a^2\,n^2-10\,a\,b^2\,c$$
$$-10\,a\,b^2\,n+4\,a\,b\,c^2-4\,a\,b\,n^2-8\,a\,c^2\,n-8\,a\,c\,n^2-b^4+2\,b^3\,c-2\,b^3\,n+2\,b^2\,c^2$$
$$+16\,b^2\,c\,n+2\,b^2\,n^2-8\,b\,c^2\,n+8\,b\,c\,n^2+8\,c^2\,n^2+2\,a^2\,c-2\,a^2\,n+4\,a\,b\,c+4\,a\,b\,n$$
$$-4\,a\,c^2+4\,a\,n^2+2\,b^2\,c-2\,b^2\,n-4\,b\,c^2-4\,b\,n^2+8\,c^2\,n-8\,c\,n^2-a^2+a\,c+a\,n$$
$$\left.-b^2+b\,c-b\,n+3\,c^2-8\,c\,n+3\,n^2-3\,c+3\,n+1\right)\Big/\left(a^4-2\,a^3\,c-2\,a^3\,n-2\,a^2\,b^2\right.$$
$$+2\,a^2\,b\,c-2\,a^2\,b\,n+4\,a^2\,c\,n+2\,a\,b^2\,c+2\,a\,b^2\,n+b^4-2\,b^3\,c+2\,b^3\,n-4\,b^2\,c\,n$$
$$+2\,a^2\,c-2\,a^2\,n-4\,a\,b\,c-4\,a\,b\,n+2\,b^2\,c-2\,b^2\,n+8\,b\,c\,n-2\,a^2+2\,a\,c+2\,a\,n$$
$$\left.\left.-2\,b^2+2\,b\,c-2\,b\,n-4\,c\,n-2\,c+2\,n+1\right)\right\},\left\{A=-\frac{1}{4}+\frac{1}{2}\,a+\frac{1}{2}\,b-\frac{1}{4}\,a^2\right.$$
$$\left.-\frac{1}{2}\,a\,b-\frac{1}{4}\,b^2, B=\frac{1}{4}\,\frac{1}{-1+a+b}\left(a^3+a^2\,b-a^2\,c-n\,a^2+a\,b^2+2\,a\,c\,b\right.\right.$$
$$\left.\left.+2\,a\,c\,n+b^3-b^2\,c+n\,b^2-2\,n\,c\,b-a^2-4\,a\,b-2\,n\,a-b^2+2\,n\,c+a+b+c-n\right.\right.$$

$$-1)\},\left\{A=-\frac{1}{4}a^2+\frac{1}{2}ab-\frac{1}{4}b^2-\frac{1}{2}a+\frac{1}{2}b-\frac{1}{4},B=\frac{1}{4}\frac{1}{1+a-b}(a^3 -a^2b-a^2c-na^2+ab^2-2nab+2acn-b^3+b^2c-nb^2+2ncb+a^2-4ab +2ac+b^2-2nc+a-b-c+n+1)\},\left\{A=-\frac{1}{4}a^2+\frac{1}{2}ab+na-\frac{1}{4}b^2-nb -n^2+\frac{1}{2}a-\frac{1}{2}b-n-\frac{1}{4},B=\frac{1}{4}\frac{1}{-2n-1+a-b}(a^3-a^2b-a^2c-3na^2 +ab^2+6nab+6an^2-b^3+b^2c-3nb^2-6bn^2-4n^3-a^2+4ab+6na-b^2 -2bc-6nb-6n^2+a-b+c-3n-1)\},\left\{A=-\frac{1}{4}a^2-\frac{1}{2}ab+ac-\frac{1}{4}b^2 +bc-c^2-\frac{1}{2}a-\frac{1}{2}b+c-\frac{1}{4},B=\frac{1}{4}\frac{1}{-2c+1+a+b}(a^3+a^2b-3a^2c -na^2+ab^2-6acb+6ac^2+b^3-3b^2c+nb^2+6bc^2-4c^3+a^2+4ab-6ac +b^2-6bc-2nb+6c^2+a+b-3c+n+1)\}$$

```
> factor(-1/4+(1/2)*a+(1/2)*b-(1/4)*a^2-(1/2)*a*b-(1/4)*b^2);
```

$$-\frac{1}{4}(-1+a+b)^2 \quad (7)$$

```
> factor(-(1/4)*a^2+(1/2)*a*b-(1/4)*b^2-(1/2)*a+(1/2)*b-1/4);factor
  ((1/4)*(a^3-a^2*b-a^2*c-n*a^2+a*b^2-2*n*a*b+2*a*c*n-b^3+b^2*c-n*
  b^2+2*n*c*b+a^2-4*a*b+2*a*c+b^2-2*n*c+a-b-c+n+1)/(1+a-b));
```

$$-\frac{1}{4}(1+a-b)^2$$

$$\frac{1}{4}\frac{1}{1+a-b}(a^3-a^2b-a^2c-na^2+ab^2-2nab+2acn-b^3+b^2c-nb^2+2ncb +a^2-4ab+2ac+b^2-2nc+a-b-c+n+1) \quad (8)$$

```
> factor(-(1/4)*a^2+(1/2)*a*b+n*a-(1/4)*b^2-n*b-n^2+(1/2)*a-(1/2)*
  b-n-1/4);
```

$$-\frac{1}{4}(-2n-1+a-b)^2 \quad (9)$$

```
> factor(-(1/4)*a^2-(1/2)*a*b+a*c-(1/4)*b^2+b*c-c^2-(1/2)*a-(1/2)*
  b+c-1/4);
```

$$-\frac{1}{4}(-2c+1+a+b)^2 \quad (10)$$