```
> restart;Digits:=10;with(PDEtools):with(linalg):with
  (LinearAlgebra):with(plots):
```

$$Digits := 10 \tag{1}$$

```
> p[-1]:=0;p[0]:=1;p[1]:=x;
```

$$p_{-1} := 0$$

$$p_0 := 1$$

$$p_1 := x \tag{2}$$

```
> alpha:=15;beta:=15;
```

$$\alpha := 15$$

$$\beta := 15 \tag{3}$$

```
> J:=simplify(alpha!/GAMMA(alpha+1)*LaguerreL(alpha,beta,-z));
```

$$J := 155117520 + 145422675\,z + 59879925\,z^2 + \frac{28831075}{2}\,z^3 + \frac{4552275}{2}\,z^4 + \frac{2003001}{8}\,z^5 + \frac{476905}{24}\,z^6 + \frac{130065}{112}\,z^7 + \frac{5655}{112}\,z^8 + \frac{1885}{1152}\,z^9 + \frac{377}{9600}\,z^{10} + \frac{29}{42240}\,z^{11} + \frac{29}{3421440}\,z^{12} + \frac{29}{415134720}\,z^{13} + \frac{1}{2905943040}\,z^{14} + \frac{1}{1307674368000}\,z^{15} \tag{4}$$

```
> Delta:=(n)->factor(collect(det(Wronskian([J,seq(diff(J,z$j),j=1..
  n-1)],z)),z,factor)):Deltahat:=(n)->diff(Delta(n),z):
> a:=(n)->convert(simplify(Deltahat(n+1)/Delta(n+1)-Deltahat(n)
  /Delta(n)),parfrac,z);b:=(n)->simplify(Delta(n+1)*Delta(n-1)
  /Delta(n)^2);
```

$$a := n \to convert\left(simplify\left(\frac{Deltahat(n+1)}{\Delta(n+1)} - \frac{Deltahat(n)}{\Delta(n)}\right), parfrac, z\right)$$

$$b := n \to simplify\left(\frac{\Delta(n+1)\,\Delta(n-1)}{\Delta(n)^2}\right) \tag{5}$$

```
> subs(z=0,Delta(1)/Delta(0));subs(z=0,Delta(1));
```

$$1$$

$$155117520 \tag{6}$$

```
> N:=3;
```

$$N := 3 \tag{7}$$

```
> for n from 1 to N do; p[n+1]:=collect(x*p[n]-a(n)*p[n]-b(n)*p
  [n-1],[z,x],factor);od:
> z:=-100;for j from 1 to N+1 do: P[j]=p[j];od:p[3];RootOf(%,x):A:=
  evalf(allvalues(%));complexplot([A],x=-1..1,y=-5000..5000,style=
  point,symbol=solidcircle,color=blue,symbolsize=25);#plot({p[4]},
  x=-8..7,y=-10..10,thickness=3);
```

$$z := -100$$

$$x^3 + 589771900215430721497622171104204584790839069323084214720341180595492809\backslash$$
$$1/166605463403415482178652855952051357161726228419391647222469684147639397202$$
$$x^2 - 848293794242751992814230818560010769264019243330213241292904456765758025\backslash$$

$$\frac{4006548116454113418049}{1134717156569835779684297585929655191562513643423706308469385414671478997\backslash 0734880}\,x - 1872545326151455898804573300795060988699580638961046510198138866171295843434714335437\backslash 4158860401919777009155801662030151726818751910079318883185713958054007059\backslash 581028832$$

$$A := 2.734193954\ 10^{6} - 0.1026\ \mathrm{I},\ -0.158 + 0.2050339324\ \mathrm{I},\ -2.734194150\ 10^{6} - 0.1020339324\ \mathrm{I}$$