```
> restart;with(PDEtools):with(linalg):with(LinearAlgebra):with
  (plots):
> alpha:=2;beta:=3:n:=2;
```

$$\alpha := 2$$

$$n := 2 \tag{1}$$

```
> J:=sort(simplify(GAMMA(alpha+1)*(-1)^beta*beta!*LaguerreL(beta,-
  alpha-beta-1,z))):
> tau:=(n)->factor(collect(subs(det(Wronskian([J/z^(alpha+beta+1),
  seq(diff(J/z^(alpha+beta+1),z$j),j=1..n-1)],z))),z,factor)):
> Delta:=(n)->tau(n)*z^(n*(alpha+beta+n));Deltahat:=(n)->-diff(tau
  (n),z)*z^(n*(alpha+beta+n)+1);
```

$$\Delta := n \to \tau(n)\, z^{n(\alpha+\beta+n)}$$

$$Deltahat := n \to -\left(\frac{\partial}{\partial z}\,\tau(n)\right) z^{n(\alpha+\beta+n)+1} \tag{2}$$

```
> a[n]:=convert(simplify(Deltahat(n+1)/Delta(n+1)-Deltahat(n)/Delta
  (n)),parfrac,z);b[n]:=simplify(Delta(n+1)*Delta(n-1)/Delta(n)^2);
```

$$a_2 := 7 + \left(45 z^8 + 1800 z^7 + 31140 z^6 + 302400 z^5 + 1803600 z^4 + 6782400 z^3 + 15876000 z^2 + 21772800 z + 13608000\right) / \left(z^9 + 45 z^8 + 900 z^7 + 10380 z^6 + 75600 z^5 + 360720 z^4 + 1130400 z^3 + 2268000 z^2 + 2721600 z + 1512000\right) + \frac{-24 z^5 - 504 z^4 - 4320 z^3 - 18720 z^2 - 43200 z - 43200}{z^6 + 24 z^5 + 252 z^4 + 1440 z^3 + 4680 z^2 + 8640 z + 7200}$$

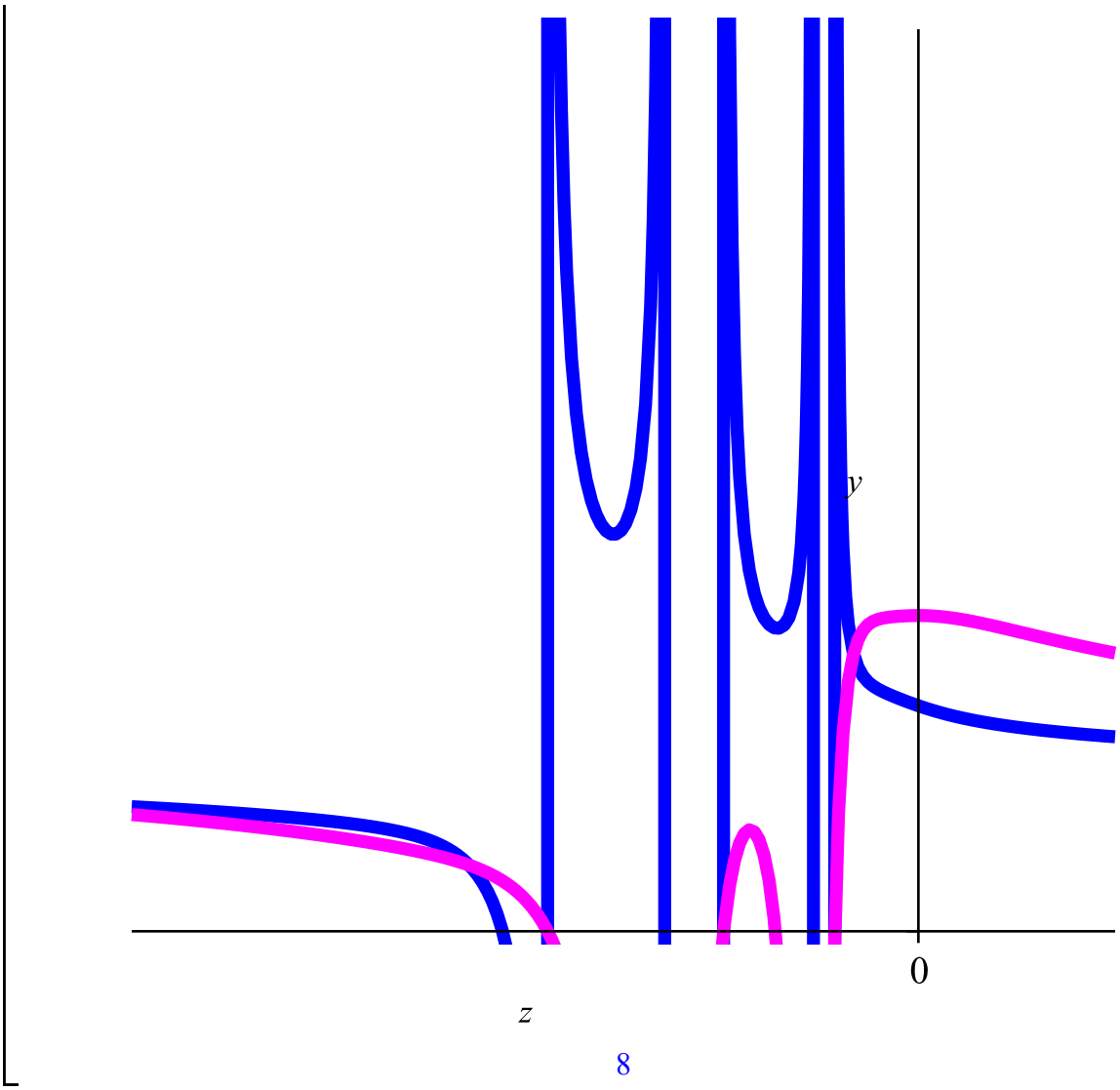
$$b_2 := \left(8 \left(z^9 + 45 z^8 + 900 z^7 + 10380 z^6 + 75600 z^5 + 360720 z^4 + 1130400 z^3 + 2268000 z^2 + 2721600 z + 1512000\right) \left(z^3 + 9 z^2 + 36 z + 60\right)\right) / \left(z^6 + 24 z^5 + 252 z^4 + 1440 z^3 + 4680 z^2 + 8640 z + 7200\right)^2 \tag{3}$$

```
> a[n]:=convert(simplify(z*diff(ln((tau(n))/(tau(n+1))),z)),
  parfrac,z);b[n]:=factor(z^2*diff(ln(tau(n)),z,z));
```

$$a_2 := 7 + \left(45 z^8 + 1800 z^7 + 31140 z^6 + 302400 z^5 + 1803600 z^4 + 6782400 z^3 + 15876000 z^2 + 21772800 z + 13608000\right) / \left(z^9 + 45 z^8 + 900 z^7 + 10380 z^6 + 75600 z^5 + 360720 z^4 + 1130400 z^3 + 2268000 z^2 + 2721600 z + 1512000\right) + \frac{-24 z^5 - 504 z^4 - 4320 z^3 - 18720 z^2 - 43200 z - 43200}{z^6 + 24 z^5 + 252 z^4 + 1440 z^3 + 4680 z^2 + 8640 z + 7200}$$

$$b_2 := \left(8 \left(z^9 + 45 z^8 + 900 z^7 + 10380 z^6 + 75600 z^5 + 360720 z^4 + 1130400 z^3 + 2268000 z^2 + 2721600 z + 1512000\right) \left(z^3 + 9 z^2 + 36 z + 60\right)\right) / \left(z^6 + 24 z^5 + 252 z^4 + 1440 z^3 + 4680 z^2 + 8640 z + 7200\right)^2 \tag{4}$$

```
> plot([a[n],b[n]],z=-20..5,y=0..40,thickness=5,color=[blue,
  magenta],tickmarks=[[0],[0]]);2*n*(1+alpha+beta-4);
```



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(5)