> restart;

> alias(q=q(z),p=p(z),sigma=sigma(z),w=w(z)):

$$PI := \frac{\partial^2}{\partial z^2} w - 6 w^2 - z$$

$$\sigma I := \left(\frac{\partial^2}{\partial z^2} \sigma\right)^2 + 4 \left(\frac{\partial}{\partial z} \sigma\right)^3 + 2 z \left(\frac{\partial}{\partial z} \sigma\right) - 2 \sigma \tag{1}$$

> H:=p^2/2-2*q^3-z*q;

$$H := \frac{1}{2} p^2 - 2 q^3 - z q$$
 (2)

= > H1:=diff(q,z)=p;H2:=diff(p,z)=6*q^2+z;

$$H1 := \frac{\partial}{\partial z} q = p$$

$$H2 := \frac{\partial}{\partial z} p = 6 q^2 + z \tag{3}$$

> S:=sigma=H;

$$S := \sigma = \frac{1}{2} p^2 - 2 q^3 - z q$$
 (4)

> S1:=simplify(subs(H1,H2,diff(S,z)));S2:=simplify(expand(subs(H1, H2,diff(S1,z))));

$$S1 := \frac{\partial}{\partial z} \sigma = -q$$

$$S2 := \frac{\partial^2}{\partial r^2} \sigma = -p \tag{5}$$

(7)

> solve({S1,S2},{q,p});

$$\left\{ p = -\left(\frac{\partial^2}{\partial z^2} \sigma\right), q = -\left(\frac{\partial}{\partial z} \sigma\right) \right\}$$
 (6)

> expand(sigma-subs(%,H));expand(-%*2);sigma1-%;

$$\sigma - \frac{1}{2} \left(\frac{\partial^2}{\partial z^2} \sigma \right)^2 - 2 \left(\frac{\partial}{\partial z} \sigma \right)^3 - z \left(\frac{\partial}{\partial z} \sigma \right)$$

$$\left(\frac{\partial^2}{\partial z^2} \sigma\right)^2 + 4 \left(\frac{\partial}{\partial z} \sigma\right)^3 + 2 z \left(\frac{\partial}{\partial z} \sigma\right) - 2 \sigma$$