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> restart;with(PDEtools):with(plots):with(LinearAlgebra):with
(linalg):with(PolynomialTools):with(ListTools):
> n:=4;a:=-2;c:=-10;
```

$$\begin{aligned}n &:= 4 \\a &:= -2 \\c &:= -10\end{aligned}$$

(1)

```
> m:=(m,n)->expand(diff((1)/x^(n-1)*pochhammer(b,n-1)/m!,b$m)):
> A:=map(factor,Matrix([<seq(factor(expand(1/x^(i-1)*pochhammer(b,
i-1))),i=1..n)>,seq(<seq(m(k,i),i=1..n)>,k=1..n-1)],n,shape=
triangular[lower]));
```

$$A := \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{b}{x} & \frac{1}{x} & 0 & 0 \\ \frac{b(b+1)}{x^2} & \frac{2b+1}{x^2} & \frac{1}{x^2} & 0 \\ \frac{b(b+1)(b+2)}{x^3} & \frac{3b^2+6b+2}{x^3} & \frac{3(b+1)}{x^3} & \frac{1}{x^3} \end{bmatrix}$$

(2)

```
> b:=-24:
> mu:=(b,k)->(simplify(x^b*simplify(sort(Beta(1-b-n+c, a+2-c-n+k)*
hypergeom([a,-2+b+2*n-k],[c+n-1-k],x)*pochhammer(c+n-1-k,-a)
/pochhammer(-2+b+2*n-k,-a))))):
> mu2 := (k) -> simplify(sort(x^b*simplify((-1)^a*Beta(-c-n+2+a+k,
-b-n+1+c)*x^(-a)*hypergeom([a,-c-n+2+a+k],[3+a+k-b-2*n],1/x)))
):
> phi:=expand(simplify(sort(hypergeom([a,b],[c],x))))*x^(b);mu(b-
n+1,n-1)*x^(n-1);
```

$$\phi := \frac{1 - \frac{24}{5}x + \frac{92}{15}x^2}{x^{24}}$$

$$\frac{1}{411863760} \frac{92x^2 - 72x + 15}{x^{24}}$$

(3)

```
> H:=map(factor,HankelMatrix(<seq(mu2(i),i=0..2*n-2)>,n));phi:for K
from 1 to n do;l[K]:=diff(%,x)*x*(x-1);od:wronskian([phi,seq(l
[k],k=1..n-1)],x):for K from 1 to n do;h[K]:=Row(%,1);row(%,2);
wronskian(%,x*(x-1),x):od:T:=simplify(<seq(simplify(h[k]),k=1..n)
>);
```

$$H := \left[\left[\frac{1}{2450448} \frac{51x^2 - 36x + 7}{x^{24}}, \frac{1}{23279256} \frac{171x^2 - 133x + 28}{x^{24}}, \right. \right.$$

$$\left. \frac{1}{33256080} \frac{95x^2 - 80x + 18}{x^{24}}, \frac{1}{58198140} \frac{70x^2 - 63x + 15}{x^{24}} \right],$$

$$\left[\frac{1}{23279256} \frac{171x^2 - 133x + 28}{x^{24}}, \frac{1}{33256080} \frac{95x^2 - 80x + 18}{x^{24}}, \right.$$

$$\begin{aligned}
& \left[\frac{1}{58198140} \frac{70x^2 - 63x + 15}{x^{24}}, \frac{1}{38798760} \frac{21x^2 - 20x + 5}{x^{24}} \right], \\
& \left[\frac{1}{33256080} \frac{95x^2 - 80x + 18}{x^{24}}, \frac{1}{58198140} \frac{70x^2 - 63x + 15}{x^{24}}, \right. \\
& \left. \frac{1}{38798760} \frac{21x^2 - 20x + 5}{x^{24}}, \frac{1}{89237148} \frac{23x^2 - 23x + 6}{x^{24}} \right], \\
& \left[\frac{1}{58198140} \frac{70x^2 - 63x + 15}{x^{24}}, \frac{1}{38798760} \frac{21x^2 - 20x + 5}{x^{24}}, \right. \\
& \left. \frac{1}{89237148} \frac{23x^2 - 23x + 6}{x^{24}}, \frac{1}{356948592} \frac{46x^2 - 48x + 13}{x^{24}} \right] \Bigg] \\
T := & \left[\left[\frac{1}{15} \frac{92x^2 - 72x + 15}{x^{24}}, -\frac{8}{15} \frac{(253x^2 - 207x + 45)(x-1)}{x^{24}}, \right. \right. \\
& \frac{8}{15} \frac{(5313x^3 - 10120x^2 + 5796x - 1080)(x-1)}{x^{24}}, \\
& \left. -\frac{8}{15} \frac{(106260x^4 - 324093x^3 + 350152x^2 - 158148x + 25920)(x-1)}{x^{24}} \right], \\
& \left[-\frac{8}{15} \frac{(253x^2 - 207x + 45)(x-1)}{x^{24}}, \right. \\
& \frac{8}{15} \frac{(5313x^3 - 10120x^2 + 5796x - 1080)(x-1)}{x^{24}}, \\
& \left. -\frac{8}{15} \frac{(106260x^4 - 324093x^3 + 350152x^2 - 158148x + 25920)(x-1)}{x^{24}}, \right. \\
& \frac{8}{15} \frac{1}{x^{24}} ((2018940x^5 - 8607060x^4 + 14159145x^3 - 11182600x^2 + 4233564x \\
& - 622080)(x-1)) \Bigg], \\
& \left[\frac{8}{15} \frac{(5313x^3 - 10120x^2 + 5796x - 1080)(x-1)}{x^{24}}, \right. \\
& \left. -\frac{8}{15} \frac{(106260x^4 - 324093x^3 + 350152x^2 - 158148x + 25920)(x-1)}{x^{24}}, \right. \\
& \frac{8}{15} \frac{1}{x^{24}} ((2018940x^5 - 8607060x^4 + 14159145x^3 - 11182600x^2 + 4233564x \\
& - 622080)(x-1)), -\frac{8}{15} \frac{1}{x^{24}} ((x-1)(36340920x^6 - 201894000x^5 \\
& + 455324100x^4 - 532176645x^3 + 339155608x^2 - 111679812x + 14929920)) \Bigg],
\end{aligned} \tag{4}$$

$$\left[-\frac{8}{15} \frac{(106260 x^4 - 324093 x^3 + 350152 x^2 - 158148 x + 25920)(x-1)}{x^{24}}, \right. \\ \left. \frac{8}{15} \frac{1}{x^{24}} ((2018940 x^5 - 8607060 x^4 + 14159145 x^3 - 11182600 x^2 + 4233564 x - 622080)(x-1)), -\frac{8}{15} \frac{1}{x^{24}} ((x-1)(36340920 x^6 - 201894000 x^5 \right. \\ \left. + 455324100 x^4 - 532176645 x^3 + 339155608 x^2 - 111679812 x + 14929920)), \right. \\ \left. \frac{8}{15} \frac{1}{x^{24}} ((x-1)(617795640 x^7 - 4288228560 x^6 + 12487143900 x^5 - 19750014900 x^4 \right. \\ \left. + 18297977313 x^3 - 9918379240 x^2 + 2912023836 x - 358318080)) \right] \Bigg]$$

> B:=map(factor,Matrix(n, n, (i,j)->expand(binomial(n-j,n-i)/pochhammer(c-b-i+1,i-j)/(x-1)^(i-1))));

$$B := \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{3}{13(x-1)} & \frac{1}{x-1} & 0 & 0 \\ \frac{1}{52(x-1)^2} & \frac{1}{6(x-1)^2} & \frac{1}{(x-1)^2} & 0 \\ \frac{1}{1716(x-1)^3} & \frac{1}{132(x-1)^3} & \frac{1}{11(x-1)^3} & \frac{1}{(x-1)^3} \end{bmatrix} \quad (5)$$

> E:=Matrix(n, n, (i,j)->expand(Stirling1(j-1,i-1)));

$$E := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

> map(factor,A.T.E.B);

$$\left[\left[-\frac{1}{13} \frac{46 x^2 - 48 x + 13}{x^{24}}, -\frac{4(23 x^2 - 23 x + 6)}{x^{24}}, -\frac{552}{5} \frac{21 x^2 - 20 x + 5}{x^{24}}, \right. \right. \\ \left. \left. -\frac{4048}{5} \frac{70 x^2 - 63 x + 15}{x^{24}} \right], \right. \\ \left[\frac{32}{13} \frac{23 x^2 - 23 x + 6}{x^{24}}, \frac{368}{5} \frac{21 x^2 - 20 x + 5}{x^{24}}, \frac{2944}{5} \frac{70 x^2 - 63 x + 15}{x^{24}}, \right. \\ \left. \frac{56672}{5} \frac{95 x^2 - 80 x + 18}{x^{24}} \right], \\ \left[-\frac{2576}{65} \frac{21 x^2 - 20 x + 5}{x^{24}}, -\frac{5152}{15} \frac{70 x^2 - 63 x + 15}{x^{24}}, -\frac{36064}{5} \frac{95 x^2 - 80 x + 18}{x^{24}}, \right. \\ \left. \right]$$

$$\left[-\frac{113344 \left(171 x^2 - 133 x + 28 \right)}{x^{24}} \right], \\
 \left[\frac{10304}{65} \frac{70 x^2 - 63 x + 15}{x^{24}}, \frac{18032}{5} \frac{95 x^2 - 80 x + 18}{x^{24}}, \right. \\
 \left. \frac{61824 \left(171 x^2 - 133 x + 28 \right)}{x^{24}}, \frac{6460608 \left(51 x^2 - 36 x + 7 \right)}{x^{24}} \right] \Bigg]$$