P4

```
> restart;Digits:=20;with(ListTools):with(orthopoly):with(linalg)
  :alias(sigma=sigma(z),psi=psi(z),phi=phi(z),w=w(z)):P4:=-diff(w,
  z,z)+(diff(w,z)^2/2/w+3/2*w^3+4*z*w^2+2*(z^2-A)*w+B/w);
```

$$Digits := 20$$

$$P4 := -\left(\frac{\partial^2}{\partial z^2}\,w\right) + \frac{1}{2}\,\frac{\left(\frac{\partial}{\partial z}\,w\right)^2}{w} + \frac{3}{2}\,w^3 + 4\,z\,w^2 + 2\,(z^2 - A)\,w + \frac{B}{w} \tag{1}$$

```
> n:=0;epsilon:=1;psi:=exp(epsilon*z^2/2)*((C[1]*CylinderD(nu,sqrt
  (2)*z))+C[2]*CylinderD(nu,-sqrt(2)*z));
```

$$n := 0$$

$$\varepsilon := 1$$

$$\psi := e^{\frac{1}{2}\,z^2}\left(C_1\,\text{CylinderD}\left(\nu, \sqrt{2}\,z\right) + C_2\,\text{CylinderD}\left(\nu, -\sqrt{2}\,z\right)\right) \tag{2}$$

```
> tau[nu]:=(n)->simplify(det(wronskian([psi,seq(diff(psi,z$j),j=1..
  n-1)],z)));tau[nu-1]:=(n)->simplify(det(wronskian([seq(diff(psi,
  z$j),j=1..n)],z)));
```

$$\tau_\nu := n \rightarrow simplify\left(linalg\text{-}det\left(linalg\text{-}wronskian\left(\left[\psi, seq\left(\frac{\partial^j}{\partial z^j}\,\psi, j = 1..n - 1\right)\right], z\right)\right)\right)$$

$$\tau_{\nu-1} := n \rightarrow simplify\left(linalg\text{-}det\left(linalg\text{-}wronskian\left(\left[seq\left(\frac{\partial^j}{\partial z^j}\,\psi, j = 1..n\right)\right], z\right)\right)\right) \tag{3}$$

```
> P1:=tau[nu](n+1);
```

$$P1 := e^{\frac{1}{2}\,z^2}\left(C_1\,\text{CylinderD}\left(\nu, \sqrt{2}\,z\right) + C_2\,\text{CylinderD}\left(\nu, -\sqrt{2}\,z\right)\right) \tag{4}$$

```
> P2:=tau[nu](n);
```

$$P2 := e^{\frac{1}{2}\,z^2}\left(C_1\,\text{CylinderD}\left(\nu, \sqrt{2}\,z\right) + C_2\,\text{CylinderD}\left(\nu, -\sqrt{2}\,z\right)\right) \tag{5}$$
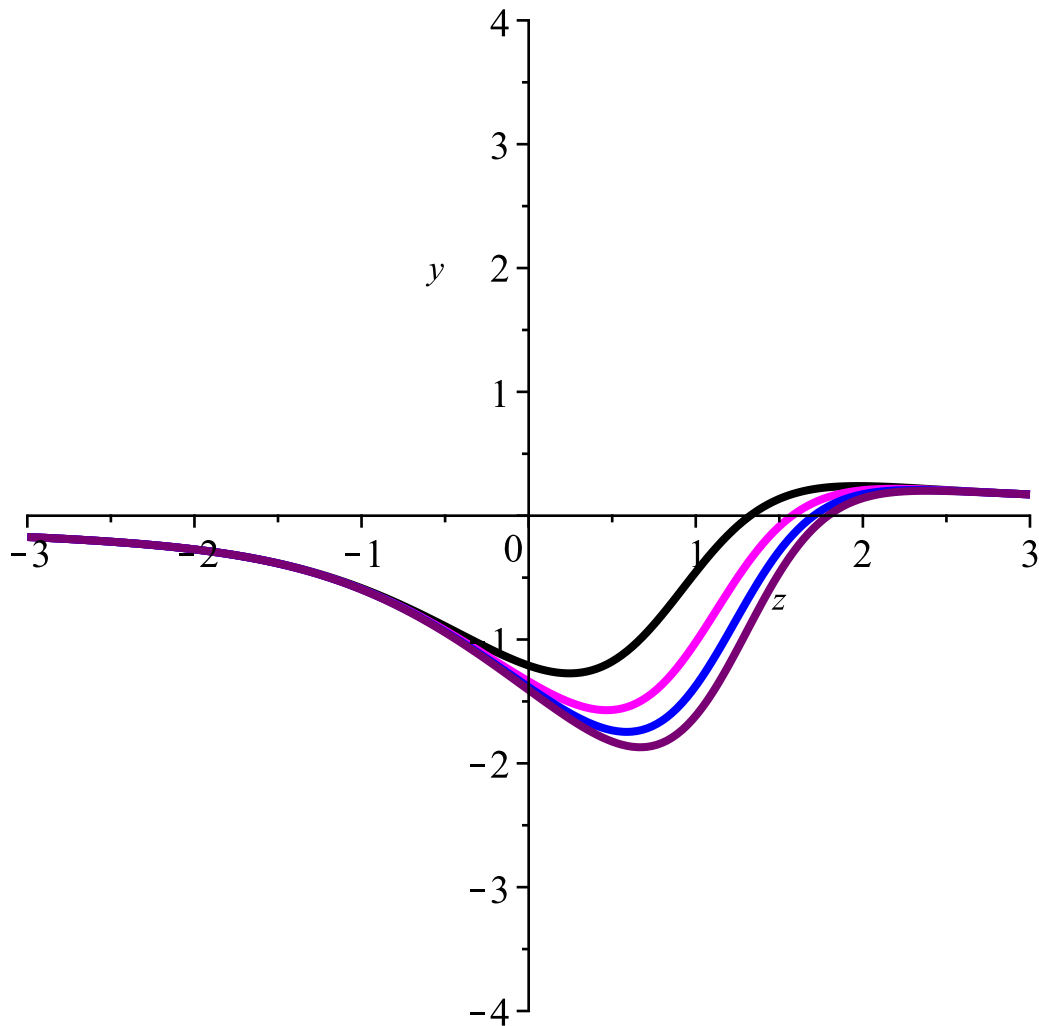
```
> tau[nu-1](0):=1:tau[nu](0):=1:tau[nu](0):=1:P1/P2;
```

$$1 \tag{6}$$

```
> w:=simplify(-2*z+epsilon*diff(ln(P1/1),z)):
```

```
> plot([subs(nu=-3/2,C[2]=1,C[1]=10,w),subs(nu=-3/2,C[2]=1,C[1]=20,
  w),subs(nu=-3/2,C[2]=1,C[1]=30,w),subs(nu=-3/2,C[2]=1,C[1]=40,w)
  ],z=-3..3,y=-4..4,thickness=3,color=[black,magenta,blue,purple]);
```


```
> restart;with(ListTools):with(orthopoly):with(linalg):alias(sigma=
  sigma(z),psi=psi(z),phi=phi(z),w=w(z)):P4:=-diff(w,z,z)+(diff(w,
  z)^2/2/w+3/2*w^3+4*z*w^2+2*(z^2-A)*w+B/w);
```

$$P4 := -\left(\frac{\partial^2}{\partial z^2}\,w\right) + \frac{1}{2}\,\frac{\left(\frac{\partial}{\partial z}\,w\right)^2}{w} + \frac{3}{2}\,w^3 + 4\,z\,w^2 + 2\left(z^2 - A\right)w + \frac{B}{w} \tag{7}$$

```
> n:=2;nu:=-3;m:=-nu;epsilon:=1;psi:=simplify(HermiteH(m-1,I*z)*(-
  I)^(m-1)*exp(z^2)+diff(erfc(z)*exp(z^2),z$(m-1)));
```

$$n := 2$$
$$\nu := -3$$
$$m := 3$$
$$\varepsilon := 1$$

$$\psi := -\frac{2\,e^{z^2}\left(2\,z^2\,\sqrt{\pi}\,\mathrm{erf}(z) - 4\,z^2\,\sqrt{\pi} + \sqrt{\pi}\,\mathrm{erf}(z) + 2\,z\,e^{-z^2} - 2\,\sqrt{\pi}\right)}{\sqrt{\pi}} \tag{8}$$

```
> tau[nu]:=(n)->det(wronskian([psi,seq(diff(psi,z$j),j=1..n-1)],z))
```

```
  :tau[nu-1]:=(n)->det(wronskian([seq(diff(psi,z$j),j=1..n)],z)):
> tau[nu-1](0):=1:tau[nu](0):=1:
> w1:=simplify(-2*z+epsilon*diff(ln(tau[nu](n+1)/tau[nu](n)),z)):
> nu:=-5;m:=-nu;epsilon:=1;psi:=simplify(HermiteH(m-1,I*z)*(-I)^
  (m-1)*exp(z^2)+diff(erfc(z)*exp(z^2),z$(m-1)));
```

$$\nu := -5$$

$$m := 5$$

$$\varepsilon := 1$$

$$\psi := -\frac{1}{\sqrt{\pi}}\left(4\,e^{z^2}\left(4\sqrt{\pi}\,\mathrm{erf}(z)\,z^4 - 8\sqrt{\pi}\,z^4 + 12\,z^2\sqrt{\pi}\,\mathrm{erf}(z) + 4\,e^{-z^2}z^3 - 24\,z^2\sqrt{\pi}\right. \tag{9}$$

$$\left.\left. + 3\sqrt{\pi}\,\mathrm{erf}(z) + 10\,z\,e^{-z^2} - 6\sqrt{\pi}\right)\right)$$

```
> tau[nu]:=(n)->det(wronskian([psi,seq(diff(psi,z$j),j=1..n-1)],z))
  :tau[nu-1]:=(n)->det(wronskian([seq(diff(psi,z$j),j=1..n)],z)):
> tau[nu-1](0):=1:tau[nu](0):=1:
> w2:=simplify(-2*z+epsilon*diff(ln(tau[nu](n+1)/tau[nu](n)),z)):
> nu:=-7;m:=-nu;epsilon:=1;psi:=simplify(HermiteH(m-1,I*z)*(-I)^
  (m-1)*exp(z^2)+diff(erfc(z)*exp(z^2),z$(m-1)));
```

$$\nu := -7$$

$$m := 7$$

$$\varepsilon := 1$$

$$\psi := -\frac{1}{\sqrt{\pi}}\left(8\,e^{z^2}\left(8\,z^6\sqrt{\pi}\,\mathrm{erf}(z) - 16\,z^6\sqrt{\pi} + 60\sqrt{\pi}\,\mathrm{erf}(z)\,z^4 + 8\,z^5\,e^{-z^2} - 120\sqrt{\pi}\,z^4\right.\right. \tag{10}$$

$$\left.\left. + 90\,z^2\sqrt{\pi}\,\mathrm{erf}(z) + 56\,e^{-z^2}z^3 - 180\,z^2\sqrt{\pi} + 15\sqrt{\pi}\,\mathrm{erf}(z) + 66\,z\,e^{-z^2} - 30\sqrt{\pi}\right)\right)$$

```
> tau[nu]:=(n)->det(wronskian([psi,seq(diff(psi,z$j),j=1..n-1)],z))
  :tau[nu-1]:=(n)->det(wronskian([seq(diff(psi,z$j),j=1..n)],z)):
> tau[nu-1](0):=1:tau[nu](0):=1:
> w3:=simplify(-2*z+epsilon*diff(ln(tau[nu](n+1)/tau[nu](n)),z)):
> nu:=-9;m:=-nu;epsilon:=1;psi:=simplify(HermiteH(m-1,I*z)*(-I)^
  (m-1)*exp(z^2)+diff(erfc(z)*exp(z^2),z$(m-1)));
```

$$\nu := -9$$

$$m := 9$$

$$\varepsilon := 1$$

$$\psi := -\frac{1}{\sqrt{\pi}}\left(16\,e^{z^2}\left(16\sqrt{\pi}\,\mathrm{erf}(z)\,z^8 - 32\sqrt{\pi}\,z^8 + 224\,z^6\sqrt{\pi}\,\mathrm{erf}(z) + 16\,e^{-z^2}z^7\right.\right. \tag{11}$$

$$ - 448\,z^6\sqrt{\pi} + 840\sqrt{\pi}\,\mathrm{erf}(z)\,z^4 + 216\,z^5\,e^{-z^2} - 1680\sqrt{\pi}\,z^4 + 840\,z^2\sqrt{\pi}\,\mathrm{erf}(z)$$

$$\left.\left. + 740\,e^{-z^2}z^3 - 1680\,z^2\sqrt{\pi} + 105\sqrt{\pi}\,\mathrm{erf}(z) + 558\,z\,e^{-z^2} - 210\sqrt{\pi}\right)\right)$$

```
> tau[nu]:=(n)->det(wronskian([psi,seq(diff(psi,z$j),j=1..n-1)],z))
  :tau[nu-1]:=(n)->det(wronskian([seq(diff(psi,z$j),j=1..n)],z)):
> tau[nu-1](0):=1:tau[nu](0):=1:
> w4:=simplify(-2*z+epsilon*diff(ln(tau[nu](n+1)/tau[nu](n)),z)):

> plot([w1,w2,w3,w4],z=-3..3,y=-4..4,thickness=3,color=[black,
```

magenta,blue,purple]);