```
> restart;
> epsilon[3]:=1;
```

$$\varepsilon_3 := 1 \tag{1}$$

```
> w(z)=-z/alpha*diff(ln(psi(z)),z);z*diff(w(z),z)-(alpha*w(z)^2+
  (beta-alpha+epsilon[3]*z)*w(z)-beta);
```

$$w(z) = -\frac{z\left(\frac{d}{dz}\psi(z)\right)}{\alpha\,\psi(z)}$$

$$z\left(\frac{d}{dz}w(z)\right) - \alpha\,w(z)^2 - \left(-\alpha+\beta+z\right)w(z) + \beta \tag{2}$$

```
> collect(simplify(numer(expand(subs(%%,%)))),[diff,z],factor);
```

$$-z^2\left(\frac{d^2}{dz^2}\psi(z)\right) + \left(z^2 + \left(-\alpha+\beta-1\right)z\right)\left(\frac{d}{dz}\psi(z)\right) + \beta\,\alpha\,\psi(z) \tag{3}$$

```
> simplify(convert(dsolve(%,psi(z)),Kummer));
```

$$\psi(z) = z^{\beta}\left(\text{KummerU}\left(\beta, 1+\alpha+\beta, z\right)\_C2 + \text{KummerM}\left(\beta, 1+\alpha+\beta, z\right)\_C1\right) \tag{4}$$

```
> epsilon[3]:=-1;
```

$$\varepsilon_3 := -1 \tag{5}$$

```
> w(z)=-z/alpha*diff(ln(psi(z)),z);z*diff(w(z),z)-(alpha*w(z)^2+
  (beta-alpha+epsilon[3]*z)*w(z)-beta);
```

$$w(z) = -\frac{z\left(\frac{d}{dz}\psi(z)\right)}{\alpha\,\psi(z)}$$

$$z\left(\frac{d}{dz}w(z)\right) - \alpha\,w(z)^2 - \left(-\alpha+\beta-z\right)w(z) + \beta \tag{6}$$

```
> collect(simplify(numer(expand(subs(%%,%)))),[diff,z],factor);
```

$$-z^2\left(\frac{d^2}{dz^2}\psi(z)\right) + \left(-z^2 + \left(-\alpha+\beta-1\right)z\right)\left(\frac{d}{dz}\psi(z)\right) + \beta\,\alpha\,\psi(z) \tag{7}$$

```
> simplify(convert(dsolve(%,psi(z)),Kummer));
```

$$\psi(z) = z^{\beta}e^{-z}\left(\text{KummerU}\left(\alpha+1, 1+\alpha+\beta, z\right)\_C2 + \text{KummerM}\left(\alpha+1, 1+\alpha+\beta, z\right)\_C1\right) \tag{8}$$