> restart:with(linalg):with(plots):with(PDEtools):with(plots):with (LinearAlgebra):with(linalg):alias(w=w(z),phi=phi(z),sigma=sigma(z)):

 $> P2 := diff(w,z,z) - (2*w^3+z*w+alpha);$

$$P2 := \frac{\partial^2}{\partial z^2} w - 2 w^3 - z w - \alpha \tag{1}$$

> S2:=(diff(sigma,z,z))^2+4*diff(sigma,z)^3+2*diff(sigma,z)*(z*diff(sigma,z)-sigma)-1/4*(alpha+1/2)^2;

$$S2 := \left(\frac{\partial^2}{\partial z^2} \sigma\right)^2 + 4\left(\frac{\partial}{\partial z} \sigma\right)^3 + 2\left(\frac{\partial}{\partial z} \sigma\right)\left(z\left(\frac{\partial}{\partial z} \sigma\right) - \sigma\right) - \frac{1}{4}\left(\alpha + \frac{1}{2}\right)^2$$
 (2)

> alpha:=n+1/2;

$$\alpha := n + \frac{1}{2} \tag{3}$$

 $> w:=-(n+1/2)/z+B/z^2;$

$$w := -\frac{n + \frac{1}{2}}{z} + \frac{B}{z^2} \tag{4}$$

> collect(expand(P2),z,factor);

$$-\frac{B}{z} + \frac{1}{4} \frac{(2n+3)(2n+1)(2n-1)}{z^3} - \frac{3}{2} \frac{B(2n+3)(2n-1)}{z^4} + \frac{3B^2(2n+1)}{z^5}$$

$$-\frac{2B^3}{6}$$
(5)