```
> restart;
> with(PDEtools):with(linalg):with(LinearAlgebra):with(plots):alias
  (w=w(z),sigma=simga(z),phi=phi(t),q=q(z),p=p(z)):
> P5:=(diff(w, z, z))-((1/(2*w)+1/(w-1))*(diff(w,z)^2)-1/z*diff(w,
  z)+(w-1)^2/z^2*(A*w+B/w)+C*w/z+d*w*(w+1)/(w-1));S5:=z^2*(diff
  (sigma, z, z))^2-(2*(diff(sigma, z))^2-z*(diff(sigma, z))+sigma)
  ^2+(4*(diff(sigma, z)+k[0]))*(diff(sigma, z)+k[1])*(diff(sigma,
  z)+k[2])*(diff(sigma, z)+k[3]);
```

$$P5 := \frac{\partial^2}{\partial z^2}\, w - \left(\frac{1}{2w} + \frac{1}{w-1}\right)\left(\frac{\partial}{\partial z}\, w\right)^2 + \frac{\frac{\partial}{\partial z}\, w}{z} - \frac{(w-1)^2\left(A w + \frac{B}{w}\right)}{z^2} - \frac{C w}{z}$$

$$- \frac{d\, w\, (w+1)}{w-1}$$

$$S5 := z^2\left(\frac{\partial^2}{\partial z^2}\, \sigma\right)^2 - \left(2\left(\frac{\partial}{\partial z}\, \sigma\right)^2 - z\left(\frac{\partial}{\partial z}\, \sigma\right) + \sigma\right)^2 + 4\left(\frac{\partial}{\partial z}\, \sigma + k_0\right)\left(\frac{\partial}{\partial z}\, \sigma\right.$$ (1)

$$\left. + k_1\right)\left(\frac{\partial}{\partial z}\, \sigma + k_2\right)\left(\frac{\partial}{\partial z}\, \sigma + k_3\right)$$

```
> H:=q*(q-1)^2*p^2-(((b+theta)*q^2-(2*b+theta-z)*q+b))*p-(a^2-(b+
  theta)^2)/4*q;
```

$$H := q\,(q-1)^2\, p^2 - \left((b+\theta)\, q^2 - (2b + \theta - z)\, q + b\right) p - \frac{1}{4}\left(a^2 - (b+\theta)^2\right) q$$ (2)

```
> H1:=diff(q,z)=(2*q*(q-1)^2*p-(b+theta)*q^2+(2*b+theta-z)*q-b)/z;
  H2:=diff(p,z)=(-(3*q-1)*(q-1)*p^2+2*(b+theta)*q*p-(2*b+theta-z)*
  p+(a^2-(b+theta)^2)/4)/z;
```

$$H1 := \frac{\partial}{\partial z}\, q = \frac{2q\,(q-1)^2\, p - (b+\theta)\, q^2 + (2b + \theta - z)\, q - b}{z}$$

$$H2 := \frac{\partial}{\partial z}\, p$$ (3)

$$= \frac{-(3q-1)\,(q-1)\, p^2 + 2\,(b+\theta)\, q\, p - (2b + \theta - z)\, p + \frac{1}{4}\, a^2 - \frac{1}{4}\,(b+\theta)^2}{z}$$

```
> S:=sigma=simplify(H+(2*b+theta)*z/4-1/8*(2*b+theta)^2);
```

$$S := \sigma = q^3\, p^2 - 2\, q^2\, p^2 + q\, p^2 - q^2\, p\, b - q^2\, p\, \theta + 2\, q\, p\, b + q\, p\, \theta - q\, p\, z - p\, b - \frac{1}{4}\, q\, a^2$$ (4)

$$+ \frac{1}{4}\, q\, b^2 + \frac{1}{2}\, q\, b\, \theta + \frac{1}{4}\, q\, \theta^2 + \frac{1}{2}\, z\, b + \frac{1}{4}\, z\, \theta - \frac{1}{2}\, b^2 - \frac{1}{2}\, b\, \theta - \frac{1}{8}\, \theta^2$$

```
> S1:=simplify(subs(H1,H2,diff(S,z)));S2:=collect((subs(H1,H2,diff
  (S1,z))),[q,p],factor);
```

$$S1 := \frac{\partial}{\partial z}\, \sigma = -q\, p + \frac{1}{2}\, b + \frac{1}{4}\, \theta$$

$$S2 := \frac{\partial^2}{\partial z^2}\, \sigma = \frac{p^2\, q^3}{z} - \frac{(b+\theta)\, p\, q^2}{z} + \left(-\frac{p^2}{z} - \frac{1}{4}\, \frac{(b+\theta+a)\,(-b-\theta+a)}{z}\right) q$$ (5)

$$+ \frac{b\, p}{z}$$

```
> S:=simplify(algsubs(p*q=-diff(sigma, z) +(1/2)*b+(1/4)*theta,
```

```
  simplify(S)));S2:=simplify(algsubs(p*q=-diff(sigma, z) +(1/2)*b+
  (1/4)*theta,simplify(S2)));
```

$$S := \sigma = q\left(\frac{\partial}{\partial z}\,\sigma\right)^2 + \frac{1}{2}\,q\,\theta\left(\frac{\partial}{\partial z}\,\sigma\right) - \frac{1}{4}\,q\,a^2 + \frac{1}{16}\,q\,\theta^2 - p\left(\frac{\partial}{\partial z}\,\sigma\right) - \frac{1}{2}\,p\,b + \frac{1}{4}\,p\,\theta$$

$$-2\left(\frac{\partial}{\partial z}\,\sigma\right)^2 + z\left(\frac{\partial}{\partial z}\,\sigma\right)$$

$$S2 := \frac{\partial^2}{\partial z^2}\,\sigma \tag{6}$$

$$= \frac{1}{16}\,\frac{16\,q\left(\frac{\partial}{\partial z}\,\sigma\right)^2 + 8\,q\,\theta\left(\frac{\partial}{\partial z}\,\sigma\right) - 4\,q\,a^2 + q\,\theta^2 + 16\,p\left(\frac{\partial}{\partial z}\,\sigma\right) + 8\,p\,b - 4\,p\,\theta}{z}$$

```
> factor(solve({S,S2},{q,p}));
```

$$\left\{ p = -\frac{2\left(2\left(\frac{\partial}{\partial z}\,\sigma\right)^2 - z\left(\frac{\partial}{\partial z}\,\sigma\right) - \left(\frac{\partial^2}{\partial z^2}\,\sigma\right)z + \sigma\right)}{4\left(\frac{\partial}{\partial z}\,\sigma\right) + 2\,b - \theta},\ q \right. \tag{7}$$

$$\left. = \frac{8\left(2\left(\frac{\partial}{\partial z}\,\sigma\right)^2 - z\left(\frac{\partial}{\partial z}\,\sigma\right) + \left(\frac{\partial^2}{\partial z^2}\,\sigma\right)z + \sigma\right)}{\left(2\,a + 4\left(\frac{\partial}{\partial z}\,\sigma\right) + \theta\right)\left(-2\,a + 4\left(\frac{\partial}{\partial z}\,\sigma\right) + \theta\right)} \right\}$$

```
> expand(sigma-subs(%,H+(2*b+theta)*z/4-1/8*(2*b+theta)^2)):factor
  (%):
> k[0]:=1/4*(theta+2*a);k[1]:=1/4*(theta-2*a);k[2]:=-1/4*(theta+2*
  b);k[3]:=1/4*(2*b-theta);
```

$$k_0 := \frac{1}{2}\,a + \frac{1}{4}\,\theta$$

$$k_1 := -\frac{1}{2}\,a + \frac{1}{4}\,\theta$$

$$k_2 := -\frac{1}{2}\,b - \frac{1}{4}\,\theta$$

$$k_3 := \frac{1}{2}\,b - \frac{1}{4}\,\theta \tag{8}$$

```
> collect(1/64*(256*(diff(sigma, z))^3*z-64*(diff(sigma, z))^2*a^2
  -64*(diff(sigma, z))^2*b^2-32*(diff(sigma, z))^2*theta^2-64*z^2*
  (diff(sigma, z))^2+32*(diff(sigma, z))*a^2*theta-32*(diff(sigma,
  z))*b^2*theta+64*(diff(sigma, z, z))^2*z^2+16*a^2*b^2-4*a^2*
  theta^2-4*b^2*theta^2+theta^4-256*(diff(sigma, z))^2*sigma+128*z*
  (diff(sigma, z))*sigma-64*sigma^2),[diff,sigma],factor);
```

$$z^2\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)^2 + 4\left(\frac{\partial}{\partial z}\,\sigma\right)^3 z + \left(-\frac{1}{2}\,\theta^2 - a^2 - b^2 - 4\,\sigma - z^2\right)\left(\frac{\partial}{\partial z}\,\sigma\right)^2 + \left(2\,z\,\sigma + \frac{1}{2}\,\theta\,(a\right. \tag{9}$$

$$\left. - b)\,(a+b)\right)\left(\frac{\partial}{\partial z}\,\sigma\right) - \sigma^2 + \frac{1}{64}\,(2\,b - \theta)\,(2\,b + \theta)\,(2\,a - \theta)\,(2\,a + \theta)$$

```
> simplify(S5-%);
```

$$0 \tag{10}$$