

```
> restart;alias(y=y(x),phi=phi(x),psi=psi(z)):with(PDEtools):with
(plots):with(LinearAlgebra):with(linalg):
> P6:=diff(y,x,x)-1/2*(1/y+1/(y-1)+1/(y-x))*diff(y,x)^2+(1/x+1/
(x-1)+1/(y-x))*diff(y,x)-y*(y-1)*(y-x)/x^2/(x-1)^2*(alpha+beta*
x/y^2+Gamma*(x-1)/(y-1)^2+delta*x*(x-1)/(y-x)^2):
> alpha:= (1/2)*(a)^2;beta:=- (1/2)*(c-b-n-1)^2;Gamma:= (1/2)*(a-n-
c)^2;delta:= 1/2-(1/2)*(b)^2;
```

$$\alpha := \frac{1}{2} a^2$$

$$\beta := -\frac{1}{2} (c-b-n-1)^2$$

$$\Gamma := \frac{1}{2} (a-n-c)^2$$

$$\delta := \frac{1}{2} - \frac{1}{2} b^2 \quad (1)$$

```
> y:=A*x+B+C/x;
```

$$y := Ax + B + \frac{C}{x} \quad (2)$$

```
> collect(numer(expand(P6)),[diff,x],factor):solve([op(1,%),op(2,
%),op(3,%)],[A,B,C]);%[3];%[1];factor(%%[2]);simplify(%%[3]);
```

$$\{A=0, B=B, C=C\}, \{A=1, B=0, C=0\}, \left\{A = \frac{b+a}{a}, B =$$

$$\begin{aligned} & -\frac{b(-b-ac-an+2cn-1+cb+ba-bn+a^2+c-n)}{a(2ba+b^2+a^2-1)}, C = -(b(-2cb \\ & +2bn+2an-2ac+2b^2+2ba+6c^2n^2a+10c^2n^2b+10c^2b^2n-6a^2c^2n \\ & -6a^2n^2c-10b^2n^2c+b^5-b^5c-2b^5a+b^5n+a^6c+a^6n+a^6b+b^5a^2+4b^4a^3 \\ & +6b^3a^4+4b^2a^5-a^5n^2-a^5c^2+4b^4+6c^2na^4+6cn^2a^4-6c^2n^2a^3-6c^2na^3 \\ & +6cn^2a^3-4bn^2a^4+2c^2n^2b^3+2c^2nb^4-2cn^2b^4+2c^2nb^3-2cn^2b^3+3cb^5a \\ & -2cb^5n-3b^4a^2n-b^5an-b^4n^2a+10c^2bn-10bn^2c-6an^2c+6ac^2n \\ & -6a^5cn-a^5cb+3a^5bn+4c^2b^2a^3-2c^2b^3a^2-2cb^3a^3-6cb^2a^4-6b^2n^2a^3 \\ & -2b^3na^3-4b^3n^2a^2+2b^2na^4-3c^2b^4a+5cb^4a^2+2c^2ba^4-7cba^4+bn^4a^4 \\ & +8cb^3a^2-2cb^2a^3-10b^3na^2-8b^2na^3+4cb^4a-2b^4na-6n^2b^2a^2 \\ & -8n^2ba^3-2c^2b^3a-4c^2b^2a^2+2c^2ba^3-4cnb^4+11cb^2a^2-11b^2na^2+cb^3a \\ & +3b^3na+3cba^3+12cna^3-11bn^2a^3+7n^2b^2a+2n^2ba^2-3c^2b^2a-4c^2ba^2 \\ & -12cnb^3+8n^2ba-2c^2ba-20cnb^2+5b^3+3c^2a^4-3n^2a^4+2a^5n-2a^5c \\ & +4b^2a^4+2a^5b+n^2b^4+c^2b^4-4b^4a^2-5cb^4-4b^4a+5b^4n-10b^2a^3-2a^4b \\ & +4n^2b^3+4c^2b^3-13b^3a^2+5n^2b^2+5c^2b^2-c^2a^3-n^2a^3-2a^4n-2a^4c+2n^2b \\ & +2c^2b-3c^2a^2+3n^2a^2+2c^2a+2n^2a+a^2b-7cb^2-4a^3n+4a^3c-6b^2a^2 \\ & -4a^3b-9cb^3+2b^3a+9b^3n+a^2c+a^2n+6b^2a+7b^2n+2cn^2b^2a \\ & -4c^2ban-16cn^2ab-10cba^4n+4c^2ba^3n+16cn^2a^3b-10c^2n^2a^2b \\ & -10c^2na^2b+10cn^2a^2b-2c^2n^2b^2a-2c^2nb^2a+4cb^2a^3n-8c^2b^2a^2n \end{aligned}$$

$$\begin{aligned}
& + 12 c b^3 a^2 n + 12 b^2 n^2 a^2 c - 4 c^2 b^3 a n + 2 c b^4 a n + 12 c n a^3 b + 4 c n b^3 a \\
& + 20 c n a^2 b^2 + 20 c n a^2 b - 4 c n b^2 a - 12 c n a b - 2 c b a - 6 c n a + 8 b n a \\
& - 10 c n b + 9 c b a^2 - 3 b n a^2 + 10 b^2 n a) ) / ( a ( -4 a + 21 b^5 a^2 + 7 b^6 a + b^7 \\
& + 21 b^2 a^5 + 35 b^3 a^4 + 7 a^6 b - 30 a^4 b + 35 b^4 a^3 + a^7 - 6 a^5 - 30 b^4 a - 60 b^3 a^2 \\
& - 60 b^2 a^3 - 6 b^5 + 27 b^2 a + 9 b^3 + 27 a^2 b - 4 b + 9 a^3) ) ), \left\{ A = \frac{-b+a}{a}, B \right. \\
& = \frac{b (-b - a c - a n + 2 c n - 1 + c b - b a - b n + a^2 + c - n)}{a (-2 b a + b^2 + a^2 - 1)}, C = - (b (-2 c b \\
& + 2 b n - 2 a n + 2 a c + 2 b^2 - 2 b a - 6 c^2 n^2 a + 10 c^2 n^2 b + 10 c^2 b^2 n + 6 a^2 c^2 n \\
& + 6 a^2 n^2 c - 10 b^2 n^2 c + b^5 - b^5 c + 2 b^5 a + b^5 n - a^6 c - a^6 n + a^6 b + b^5 a^2 - 4 b^4 a^3 \\
& + 6 b^3 a^4 - 4 b^2 a^5 + a^5 n^2 + a^5 c^2 + 4 b^4 - 6 c^2 n a^4 - 6 c n^2 a^4 + 6 c^2 n^2 a^3 + 6 c^2 n a^3 \\
& - 6 c n^2 a^3 + 2 b n^2 a^4 + 2 c^2 n^2 b^3 + 2 c^2 n b^4 - 2 c n^2 b^4 + 2 c^2 n b^3 - 2 c n^2 b^3 - c b^5 a \\
& - 2 c b^5 n - 5 b^4 a^2 n + 3 b^5 a n + 3 b^4 n^2 a + 10 c^2 b n - 10 b n^2 c + 6 a n^2 c - 6 a c^2 n \\
& + 6 a^5 c n + 3 a^5 c b - a^5 b n + 6 c^2 b^2 a^3 - 4 c^2 b^3 a^2 - 2 c b^3 a^3 - 2 c b^2 a^4 - 4 b^2 n^2 a^3 \\
& - 2 b^3 n a^3 - 2 b^3 n^2 a^2 + 6 b^2 n a^4 + c^2 b^4 a + 3 c b^4 a^2 - 4 c^2 b a^4 - c b a^4 + 7 b n a^4 \\
& + 10 c b^3 a^2 - 8 c b^2 a^3 - 8 b^3 n a^2 - 2 b^2 n a^3 - 2 c b^4 a + 4 b^4 n a + 2 n^2 b^3 a \\
& - 4 n^2 b^2 a^2 - 2 n^2 b a^3 - 6 c^2 b^2 a^2 + 8 c^2 b a^3 - 4 c n b^4 + 11 c b^2 a^2 - 11 b^2 n a^2 \\
& + 3 c b^3 a + b^3 n a - 11 c b a^3 - 12 c n a^3 + 3 b n a^3 + 3 n^2 b^2 a - 4 n^2 b a^2 - 7 c^2 b^2 a \\
& + 2 c^2 b a^2 - 12 c n b^3 + 2 n^2 b a - 8 c^2 b a - 20 c n b^2 + 5 b^3 - 3 c^2 a^4 + 3 n^2 a^4 - 2 a^5 n \\
& + 2 a^5 c + 4 b^2 a^4 - 2 a^5 b + n^2 b^4 + c^2 b^4 - 4 b^4 a^2 - 5 c b^4 + 4 b^4 a + 5 b^4 n + 10 b^2 a^3 \\
& - 2 a^4 b + 4 n^2 b^3 + 4 c^2 b^3 - 13 b^3 a^2 + 5 n^2 b^2 + 5 c^2 b^2 + c^2 a^3 + n^2 a^3 + 2 a^4 n + 2 a^4 c \\
& + 2 n^2 b + 2 c^2 b + 3 c^2 a^2 - 3 n^2 a^2 - 2 c^2 a - 2 n^2 a + a^2 b - 7 c b^2 + 4 a^3 n - 4 a^3 c \\
& - 6 b^2 a^2 + 4 a^3 b - 9 c b^3 - 2 b^3 a + 9 b^3 n - a^2 c - a^2 n - 6 b^2 a + 7 b^2 n - 2 c n^2 b^2 a \\
& - 16 c^2 b a n - 4 c n^2 a b - 10 c b a^4 n + 16 c^2 b a^3 n + 4 c n^2 a^3 b - 4 c n^2 b^3 a \\
& - 10 c^2 n^2 a^2 b - 10 c^2 n a^2 b + 10 c n^2 a^2 b + 2 c^2 n^2 b^2 a + 2 c^2 n b^2 a - 4 c b^2 a^3 n \\
& - 12 c^2 b^2 a^2 n + 12 c b^3 a^2 n + 8 b^2 n^2 a^2 c - 2 c b^4 a n - 12 c n a^3 b - 4 c n b^3 a \\
& + 20 c n a^2 b^2 + 20 c n a^2 b + 4 c n b^2 a + 12 c n a b + 8 c b a + 6 c n a - 2 b n a \\
& - 10 c n b + 3 c b a^2 - 9 b n a^2 + 10 c b^2 a) ) / ( a ( -4 a - 21 b^5 a^2 + 7 b^6 a - b^7 \\
& + 21 b^2 a^5 - 35 b^3 a^4 - 7 a^6 b + 30 a^4 b + 35 b^4 a^3 + a^7 - 6 a^5 - 30 b^4 a + 60 b^3 a^2 \\
& - 60 b^2 a^3 + 6 b^5 + 27 b^2 a - 9 b^3 - 27 a^2 b + 4 b + 9 a^3) ) ) \} \\
& \left\{ A = \frac{b+a}{a}, B = - \frac{b (-b - a c - a n + 2 c n - 1 + c b + b a - b n + a^2 + c - n)}{a (2 b a + b^2 + a^2 - 1)}, C = \right. \\
& - (b (-2 c b + 2 b n + 2 a n - 2 a c + 2 b^2 + 2 b a + 6 c^2 n^2 a + 10 c^2 n^2 b + 10 c^2 b^2 n \\
& - 6 a^2 c^2 n - 6 a^2 n^2 c - 10 b^2 n^2 c + b^5 - b^5 c - 2 b^5 a + b^5 n + a^6 c + a^6 n + a^6 b + b^5 a^2 \\
& + 4 b^4 a^3 + 6 b^3 a^4 + 4 b^2 a^5 - a^5 n^2 - a^5 c^2 + 4 b^4 + 6 c^2 n a^4 + 6 c n^2 a^4 - 6 c^2 n^2 a^3 \\
& - 6 c^2 n a^3 + 6 c n^2 a^3 - 4 b n^2 a^4 + 2 c^2 n^2 b^3 + 2 c^2 n b^4 - 2 c n^2 b^4 + 2 c^2 n b^3 - 2 c n^2 b^3 \\
& - 2 c n^2 b^3)
\end{aligned}$$

$$\begin{aligned}
& + 3cb^5a - 2cb^5n - 3b^4a^2n - b^5an - b^4n^2a + 10c^2bn - 10bn^2c - 6an^2c \\
& + 6ac^2n - 6a^5cn - a^5cb + 3a^5bn + 4c^2b^2a^3 - 2c^2b^3a^2 - 2cb^3a^3 - 6cb^2a^4 \\
& - 6b^2n^2a^3 - 2b^3na^3 - 4b^3n^2a^2 + 2b^2na^4 - 3c^2b^4a + 5cb^4a^2 + 2c^2ba^4 \\
& - 7cba^4 + bna^4 + 8cb^3a^2 - 2cb^2a^3 - 10b^3na^2 - 8b^2na^3 + 4cb^4a - 2b^4na \\
& - 6n^2b^2a^2 - 8n^2ba^3 - 2c^2b^3a - 4c^2b^2a^2 + 2c^2ba^3 - 4cnb^4 + 11cb^2a^2 \\
& - 11b^2na^2 + cb^3a + 3b^3na + 3cba^3 + 12cn a^3 - 11bna^3 + 7n^2b^2a + 2n^2ba^2 \\
& - 3c^2b^2a - 4c^2ba^2 - 12cnb^3 + 8n^2ba - 2c^2ba - 20cnb^2 + 5b^3 + 3c^2a^4 \\
& - 3n^2a^4 + 2a^5n - 2a^5c + 4b^2a^4 + 2a^5b + n^2b^4 + c^2b^4 - 4b^4a^2 - 5cb^4 - 4b^4a \\
& + 5b^4n - 10b^2a^3 - 2a^4b + 4n^2b^3 + 4c^2b^3 - 13b^3a^2 + 5n^2b^2 + 5c^2b^2 - c^2a^3 \\
& - n^2a^3 - 2a^4n - 2a^4c + 2n^2b + 2c^2b - 3c^2a^2 + 3n^2a^2 + 2c^2a + 2n^2a + a^2b \\
& - 7cb^2 - 4a^3n + 4a^3c - 6b^2a^2 - 4a^3b - 9cb^3 + 2b^3a + 9b^3n + a^2c + a^2n \\
& + 6b^2a + 7b^2n + 2cn^2b^2a - 4c^2ban - 16cn^2ab - 10cba^4n + 4c^2ba^3n \\
& + 16cn^2a^3b - 10c^2n^2a^2b - 10c^2na^2b + 10cn^2a^2b - 2c^2n^2b^2a - 2c^2nb^2a \\
& + 4cb^2a^3n - 8c^2b^2a^2n + 12cb^3a^2n + 12b^2n^2a^2c - 4c^2b^3an + 2cb^4an \\
& + 12cn a^3b + 4cnb^3a + 20cn a^2b^2 + 20cn a^2b - 4cnb^2a - 12cnab - 2cba \\
& - 6cna + 8bna - 10cnb + 9cba^2 - 3bna^2 + 10b^2na) \bigg) / (a(-4a + 21b^5a^2 \\
& + 7b^6a + b^7 + 21b^2a^5 + 35b^3a^4 + 7a^6b - 30a^4b + 35b^4a^3 + a^7 - 6a^5 - 30b^4a \\
& - 60b^3a^2 - 60b^2a^3 - 6b^5 + 27b^2a + 9b^3 + 27a^2b - 4b + 9a^3)) \}
\end{aligned}$$

$$A = \frac{b+a}{a}$$

$$B = - \frac{b(-b - ac - an + 2cn - 1 + cb + ba - bn + a^2 + c - n)}{a(b+1+a)(a+b-1)}$$

$$\begin{aligned}
C = & - (b(-2cb + 2bn + 2an - 2ac + 2b^2 + 2ba + 6c^2n^2a + 10c^2n^2b + 10c^2b^2n \\
& - 6a^2c^2n - 6a^2n^2c - 10b^2n^2c + b^5 - b^5c - 2b^5a + b^5n + a^6c + a^6n + a^6b + b^5a^2 \\
& + 4b^4a^3 + 6b^3a^4 + 4b^2a^5 - a^5n^2 - a^5c^2 + 4b^4 + 6c^2na^4 + 6cn^2a^4 - 6c^2n^2a^3 \\
& - 6c^2na^3 + 6cn^2a^3 - 4bn^2a^4 + 2c^2n^2b^3 + 2c^2nb^4 - 2cn^2b^4 + 2c^2nb^3 - 2cn^2b^3 \\
& + 3cb^5a - 2cb^5n - 3b^4a^2n - b^5an - b^4n^2a + 10c^2bn - 10bn^2c - 6an^2c \\
& + 6ac^2n - 6a^5cn - a^5cb + 3a^5bn + 4c^2b^2a^3 - 2c^2b^3a^2 - 2cb^3a^3 - 6cb^2a^4 \\
& - 6b^2n^2a^3 - 2b^3na^3 - 4b^3n^2a^2 + 2b^2na^4 - 3c^2b^4a + 5cb^4a^2 + 2c^2ba^4 \\
& - 7cba^4 + bna^4 + 8cb^3a^2 - 2cb^2a^3 - 10b^3na^2 - 8b^2na^3 + 4cb^4a - 2b^4na \\
& - 6n^2b^2a^2 - 8n^2ba^3 - 2c^2b^3a - 4c^2b^2a^2 + 2c^2ba^3 - 4cnb^4 + 11cb^2a^2 \\
& - 11b^2na^2 + cb^3a + 3b^3na + 3cba^3 + 12cn a^3 - 11bna^3 + 7n^2b^2a + 2n^2ba^2 \\
& - 3c^2b^2a - 4c^2ba^2 - 12cnb^3 + 8n^2ba - 2c^2ba - 20cnb^2 + 5b^3 + 3c^2a^4 \\
& - 3n^2a^4 + 2a^5n - 2a^5c + 4b^2a^4 + 2a^5b + n^2b^4 + c^2b^4 - 4b^4a^2 - 5cb^4 - 4b^4a \\
& + 5b^4n - 10b^2a^3 - 2a^4b + 4n^2b^3 + 4c^2b^3 - 13b^3a^2 + 5n^2b^2 + 5c^2b^2 - c^2a^3 \\
& - n^2a^3 - 2a^4n - 2a^4c + 2n^2b + 2c^2b - 3c^2a^2 + 3n^2a^2 + 2c^2a + 2n^2a + a^2b)
\end{aligned} \tag{3}$$

$$\begin{aligned}
& -7cb^2 - 4a^3n + 4a^3c - 6b^2a^2 - 4a^3b - 9cb^3 + 2b^3a + 9b^3n + a^2c + a^2n \\
& + 6b^2a + 7b^2n + 2cn^2b^2a - 4c^2ban - 16cn^2ab - 10cb^4n + 4c^2ba^3n \\
& + 16cn^2a^3b - 10c^2n^2a^2b - 10c^2na^2b + 10cn^2a^2b - 2c^2n^2b^2a - 2c^2nb^2a \\
& + 4cb^2a^3n - 8c^2b^2a^2n + 12cb^3a^2n + 12b^2n^2a^2c - 4c^2b^3an + 2cb^4an \\
& + 12cna^3b + 4cnb^3a + 20cna^2b^2 + 20cna^2b - 4cnb^2a - 12cnab - 2cba \\
& - 6cna + 8bna - 10cnb + 9cba^2 - 3bna^2 + 10b^2na) \Big/ (a(-4a + 21b^5a^2 \\
& + 7b^6a + b^7 + 21b^2a^5 + 35b^3a^4 + 7a^6b - 30a^4b + 35b^4a^3 + a^7 - 6a^5 - 30b^4a \\
& - 60b^3a^2 - 60b^2a^3 - 6b^5 + 27b^2a + 9b^3 + 27a^2b - 4b + 9a^3))
\end{aligned}$$

```
> collect(-b*(-a*n-a*c+2*c*n-b-1+b*a-b*n+c*b+a^2-n+c)/(a*(b+1+a)*(a+b-1)), [a,b,c], factor); latex(%);
```

$$-\frac{b(a^2 + (-n - c + b)a + (-1 + c - n)b + (1 + 2n)c - 1 - n)}{a(b + 1 + a)(a + b - 1)}$$

```
-{\frac {b \left( {a}^{2}+ \left( -n-c+b \right) a+ \left( -1+c-n \right) b+ \left( 1+2\right) c-1-n \right) }{a \left( b+1+a \right) \left( a+b-1 \right) }}
```