

```

> restart;alias(sigma=sigma(x),phi=phi(x),psi=psi(z),w=w(x)):with
(PDEtools):with(plots):with(LinearAlgebra):with(linalg):
> a:=-3;b:=-23;c:=-8;n:=3;Digits:=50;
      a := -3
      b := -23
      c := -8
      n := 3
      Digits := 50

```

(1)

```

> P6:=diff(w,x,x)-1/2*(1/w+1/(w-1)+1/(w-x))*diff(w,x)^2+(1/x+1/
(x-1)+1/(w-x))*diff(w,x)-w*(w-1)*(w-x)/x^2/(x-1)^2*(alpha+beta*
x/w^2+Gamma*(x-1)/(w-1)^2+delta*x*(x-1)/(w-x)^2):
> alpha:= (1/2)*(a)^2;beta:=- (1/2)*(c-b-n-1)^2;Gamma:= (1/2)*(a-n-
c)^2;delta:= 1/2-(1/2)*(b)^2;
      α := 9/2
      β := -121/2
      Γ := 2
      δ := -264

```

(2)

```

> phi:=(a,b,c)->factor(simplify(sort(hypergeom([a,b],[c],x)))*x^
(b)):
> simplify(hypergeom([a,b],[c],x))=simplify((-a)!/pochhammer(c,-a)
*JacobiP(-a,c-1,a+b-c,-2*x+1));
      1 - 69/8 x + 759/28 x^2 - 253/8 x^3 = -1/56 JacobiP(3, -9, -18, -2x+1)

```

(3)

```

> phi(a+1,b+1,c+1):for K from 1 to n+1 do;l[K]:=diff(%,x)*x*(x-1);
od:wronskian([phi(a+1,b+1,c+1),seq(l[k],k=1..n)],x):for K from 1
to n+1 do;h[K]:=Row(%,1);row(%,2);wronskian(%,x*(x-1),x)
:od:simplify(<seq(simplify(h[k]),k=1..n+1)>):tau[n+1]:=factor
(collect(simplify(det(%) * x^(-(n+1)*(b+1)) * (x*(x-1))^( (-n)*(n+1)
/2))),x,factor));
τ4 := 9133172928000 x8 - 146130766848000 x7 + 140318747712000 x6
      - 73502377344000 x5 + 22869348480000 x4 - 4311673344000 x3
      + 480693312000 x2 - 29132928000 x + 766656000

```

(4)

```

> phi(a-1,b+1,c):for K from 1 to n do;l[K]:=diff(%,x)*x*(x-1);
od:wronskian([phi(a-1,b+1,c),seq(l[k],k=1..n-1)],x):for K from 1
to n do;h[K]:=Row(%,1);row(%,2);wronskian(%,x*(x-1),x)
:od:simplify(<seq(simplify(h[k]),k=1..n)>):tau[n]:=factor(collect
(simplify(det(%) * x^(-n*(b+1)) * (x*(x-1))^( (-n+1)*(n/2))),x,
factor));
τ3 := -12571086033 x12 + 39698166420 x11 - 57782886678 x10 + 51114632580 x9
      - 30564124470 x8 + 13015963224 x7 - 4053573810 x6 + 932833044 x5 - 158182695 x4
      + 19405980 x3 - 1646568 x2 + 87120 x - 2178
> w:=1/a*(n+c-(2*n+b+1)*x)-convert(simplify(x*(x-1)/a*diff(ln(tau

```

(5)

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[n+1]/tau[n]),x)),parfrac,x):
```

```
> collect(numer(P6),x,factor);
```

0

(6)

```
> C:=(-b+a)/a: E:=b*(-b-a*c-a*n+2*c*n-1+c*b-b*a-b*n+a^2+c-n)/(a*  
(-2*b*a+b^2+a^2-1)):
```

```
> r:=x=-0.5..1.4,y=-2..1.6;
```

$r:=x=-0.5..1.4,y=-2..1.6$

(7)

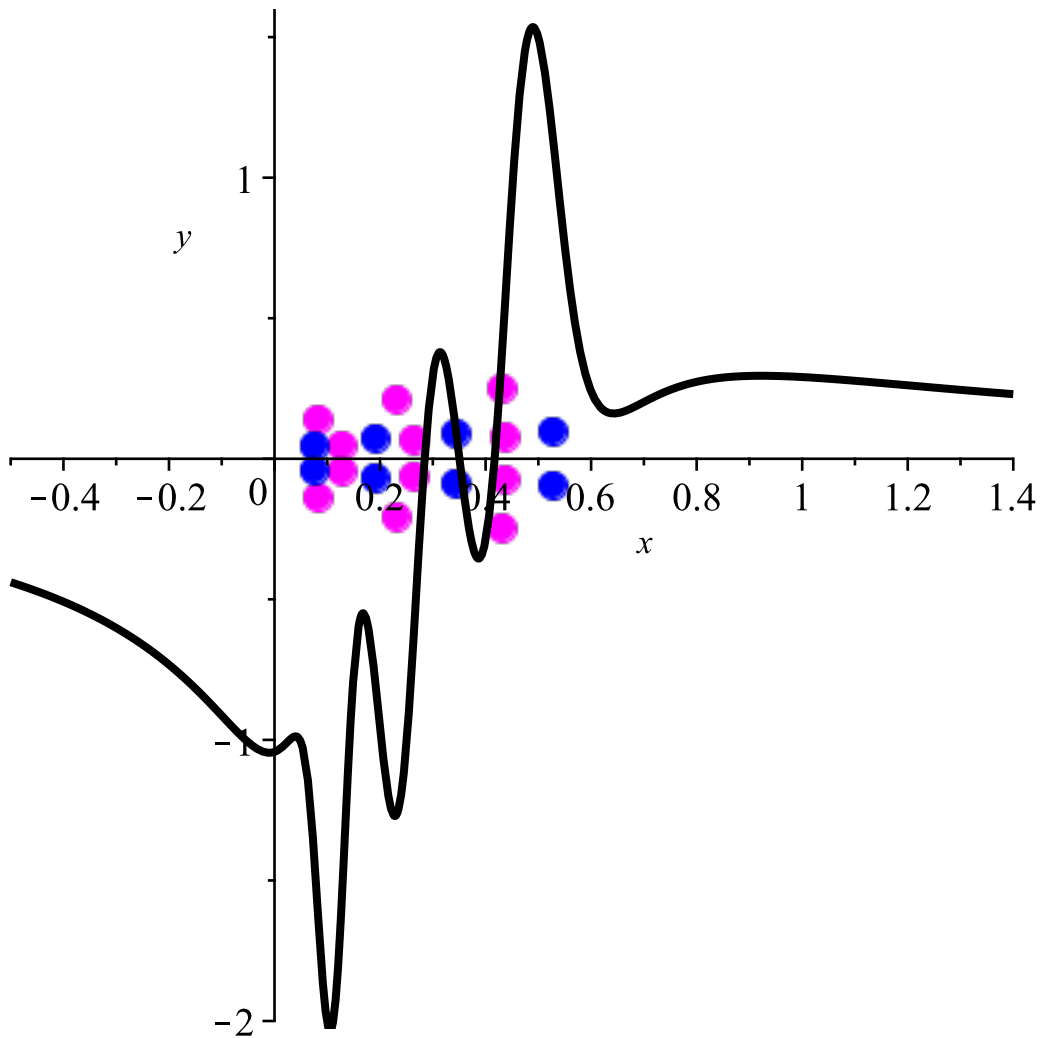
```
> RootOf(tau[n],x):J1:=evalf(allvalues(%)):RootOf(tau[n+1],x):J2:=  
evalf(allvalues(%)):
```

```
> A:=complexplot([J1],r,style=point,symbol=solidcircle,color=  
magenta,symbolsize=22):
```

```
> B:=complexplot([J2],r,style=point,symbol=solidcircle,color=blue,  
symbolsize=22):
```

```
> Asymp:=plot((-b*(-2*c*b+2*b*n-2*a*n+2*a*c+2*b^2-2*b*a-6*c^2*n^2*  
a+10*c^2*n^2*b+10*c^2*b^2*n+6*a^2*c^2*n+6*a^2*n^2*c-10*b^2*n^2*c+  
b^5-b^5*c+2*b^5*a+b^5*n-a^6*c-a^6*n+a^6*b+b^5*a^2-4*b^4*a^3+6*  
b^3*a^4-4*b^2*a^5+a^5*n^2+a^5*c^2+4*b^4-6*c^2*n*a^4-6*c*n^2*  
a^4+6*c^2*n^2*a^3+6*c^2*n*a^3-6*c*n^2*a^3+2*b*n^2*a^4+2*c^2*n^2*  
b^3+2*c^2*n*b^4-2*c*n^2*b^4+2*c^2*n*b^3-2*c*n^2*b^3-c*b^5*a-2*c*  
b^5*n-5*b^4*a^2*n+3*b^5*a*n+3*b^4*n^2*a+10*c^2*b*n-10*b*n^2*c+6*  
a*n^2*c-6*a*c^2*n+6*a^5*c*n+3*a^5*c*b-a^5*b*n+6*c^2*b^2*a^3-4*  
c^2*b^3*a^2-2*c*b^3*a^3-2*c*b^2*a^4-4*b^2*n^2*a^3-2*b^3*n*a^3-2*  
b^3*n^2*a^2+6*b^2*n*a^4+c^2*b^4*a+3*c*b^4*a^2-4*c^2*b*a^4-c*b*  
a^4+7*b*n*a^4+10*c*b^3*a^2-8*c*b^2*a^3-8*b^3*n*a^2-2*b^2*n*a^3-2*  
c*b^4*a+4*b^4*n*a+2*n^2*b^3*a-4*n^2*b^2*a^2-2*n^2*b*a^3-6*c^2*  
b^2*a^2+8*c^2*b*a^3-4*c*n*b^4+11*c*b^2*a^2-11*b^2*n*a^2+3*c*b^3*  
a+b^3*n*a-11*c*b*a^3-12*c*n*a^3+3*b*n*a^3+3*n^2*b^2*a-4*n^2*b*a^2  
-7*c^2*b^2*a+2*c^2*b*a^2-12*c*n*b^3+2*n^2*b*a-8*c^2*b*a-20*c*n*  
b^2+5*b^3-3*c^2*a^4+3*n^2*a^4-2*a^5*n+2*a^5*c+4*b^2*a^4-2*a^5*b+  
n^2*b^4+c^2*b^4-4*b^4*a^2-5*c*b^4+4*b^4*a+5*b^4*n+10*b^2*a^3-2*  
a^4*b+4*n^2*b^3+4*c^2*b^3-13*b^3*a^2+5*n^2*b^2+5*c^2*b^2+c^2*a^3+  
n^2*a^3+2*a^4*n+2*a^4*c+2*n^2*b+2*c^2*b+3*c^2*a^2-3*n^2*a^2-2*  
c^2*a-2*n^2*a+a^2*b-7*c*b^2+4*a^3*n-4*a^3*c-6*b^2*a^2+4*a^3*b-9*  
c*b^3-2*b^3*a+9*b^3*n-a^2*c-a^2*n-6*b^2*a+7*b^2*n-2*c*n^2*b^2*  
a-16*c^2*b*a*n-4*c*n^2*a*b-10*c*b*a^4*n+16*c^2*b*a^3*n+4*c*n^2*  
a^3*b-4*c*n^2*b^3*a-10*c^2*n^2*a^2*b-10*c^2*n*a^2*b+10*c*n^2*a^2*  
b+2*c^2*n^2*b^2*a+2*c^2*n*b^2*a-4*c*b^2*a^3*n-12*c^2*b^2*a^2*  
n+12*c*b^3*a^2*n+8*b^2*n^2*a^2*c-2*c*b^4*a*n-12*c*n*a^3*b-4*c*n*  
b^3*a+20*c*n*a^2*b^2+20*c*n*a^2*b+4*c*n*b^2*a+12*c*n*a*b+8*c*b*  
a+6*c*n*a-2*b*n*a-10*c*n*b+3*c*b*a^2-9*b*n*a^2+10*c*b^2*a)/(a*  
(-4*a-21*b^5*a^2+7*b^6*a-b^7+21*b^2*a^5-35*b^3*a^4-7*a^6*b+30*  
a^4*b+35*b^4*a^3+a^7-6*a^5-30*b^4*a+60*b^3*a^2-60*b^2*a^3+6*  
b^5+27*b^2*a-9*b^3-27*a^2*b+4*b+9*a^3)))/x,r,color=grey,  
thickness=3,discont=true):
```

```
> F:=plot(w-C*x-E,r,colour=black,thickness=3):display(A,B,F);
```



```
> restart; alias(sigma=sigma(x), phi=phi(x), psi=psi(z)):with
(PDEtools):with(plots):with(LinearAlgebra):with(linalg):Digits:=
60:
```

```
> a:=-2;b:=-15;c:=-8;n:=-a;
```

```
a:=-2
```

```
b:=-15
```

```
c:=-8
```

```
n:=2
```

(8)

```
> S6:=diff(sigma,x)*(x*(x-1)*diff(sigma,x,x))^2+(diff(sigma,x)*(2*
sigma-(2*x-1)*diff(sigma,x))+nu[1]*nu[2]*nu[3]*nu[4])^2-product
(diff(sigma,x)+nu[k]^2,k=1..4):
```

```
> phi:=factor(expand(simplify(hypergeom([a,b],[c],x))))*x^b;
```

$$\phi := \frac{1 - \frac{15}{4}x + \frac{15}{4}x^2}{x^{15}}$$

(9)

```
> #phi:=factor(expand(simplify(hypergeom([a-c+1,b-c+1],[2-c],x)*
x^(b+1-c))));
```

```
> #phi:=factor(expand(simplify(hypergeom([c-a,c-b],[c],x)*x^b*(1-
x)^(c-a-b))));
```

```
> simplify(hypergeom([a,b],[c],x))=simplify((-a)!/pochhammer(c,-a)
```

```
*JacobiP(-a,c-1,a+b-c,-2*x+1));
```

$$1 - \frac{15}{4}x + \frac{15}{4}x^2 = \frac{1}{28} \text{JacobiP}(2, -9, -9, -2x + 1) \quad (10)$$

```
> phi:=for K from 1 to n+1 do;l[K]:=diff(%,x)*x*(x-1);od:wronskian(
[phi,seq(l[k],k=1..n)],x):for K from 1 to n+1 do;h[K]:=Row(%,1);
row(%,2);wronskian(%,x*(x-1),x):od:simplify(<seq(simplify(h[k]),
k=1..n+1)>):tau:=collect(simplify(det(%) *x^(-(n+1)*b) * (x*(x-1)) ^ (
(-n)*((n+1)/2))),x,factor):
```

```
> sigma:=convert(simplify(x*(x-1)*diff(ln(tau),x))+(n+1)/4*(4*a*
x-2*c-a+1+b)-(1/4)*(1+a-b)^2*x+1/4*(-b+b^2+c-c*a-c*b+a+a^2),
parfrac,x);L1:=op(1,%);L2:=op(2,%);
```

$$\sigma := -49x + \frac{49}{2} + \frac{-1144x^5 + 2860x^4 - 2808x^3 + 1352x^2 - 320x + 30}{845x^6 - 2535x^5 + 3107x^4 - 1989x^3 + 702x^2 - 130x + 10}$$

$$L1 := -49x$$

$$L2 := \frac{49}{2} \quad (11)$$

```
> nu[1]:=-(a+1-b-2*(n+1))/2;nu[2]:=(2*c-1-a-b)/2;nu[3]:=-(1+a-b)/2;
nu[4]:=(1-a-b)/2;
```

$$v_1 := -4$$

$$v_2 := 0$$

$$v_3 := -7$$

$$v_4 := 9$$

(12)

```
> simplify(expand(S6));
```

$$0$$

(13)

```
> -(1/4)*(1+a-b)^2*x+(1/4)*(a^3-a^2*b-a^2*c-n*a^2+a*b^2-2*n*a*b+2*
a*c*n-b^3+b^2*c-n*b^2+2*n*c*b+a^2-4*a*b+2*a*c+b^2-2*n*c+a-b-c+
n+1)/(1+a-b);
```

$$-49x + \frac{49}{2}$$

(14)

```
> RootOf(tau,x):J1:=evalf(allvalues(%)):
```

```
> A:=complexplot([J1],style=point,symbol=solidcircle,color=blue,
symbolsize=20):
```

```
> F:=plot((sigma-(L1+L2))/15,x=-1..2,y=-1..1,colour=black,
thickness=3):display(A,F);
```

