```
> restart;alias(H[n]=H[n](z),sigma=sigma(z)):
> S5:=(z*diff(H[n],z,z))^2-(n*(n-beta-2*n)-H[n]+(-alpha-beta-2*n+z)
  *diff(H[n],z))^2-4*diff(H[n],z)*(z*diff(H[n],z)-H[n])*(alpha-diff
  (H[n],z));
```

$$S5 := z^2\left(\frac{\partial^2}{\partial z^2}H_n\right)^2-\left(n\left(-n-\beta\right)-H_n+\left(-\alpha-\beta-2\,n+z\right)\left(\frac{\partial}{\partial z}H_n\right)\right)^2 \tag{1}$$
$$-4\left(\frac{\partial}{\partial z}H_n\right)\left(z\left(\frac{\partial}{\partial z}H_n\right)-H_n\right)\left(\alpha-\left(\frac{\partial}{\partial z}H_n\right)\right)$$

```
> JMOeq:=(z*diff(sigma,z,z))^2-(2*diff(sigma,z)^2-z*diff(sigma,z)+
  sigma)^2+4*product(diff(sigma,z)+k[j],j=0..3);
```

$$JMOeq := z^2\left(\frac{\partial^2}{\partial z^2}\sigma\right)^2-\left(2\left(\frac{\partial}{\partial z}\sigma\right)^2-z\left(\frac{\partial}{\partial z}\sigma\right)+\sigma\right)^2+4\left(\frac{\partial}{\partial z}\sigma+k_0\right)\left(\frac{\partial}{\partial z}\sigma\right. \tag{2}$$
$$\left.+k_1\right)\left(\frac{\partial}{\partial z}\sigma+k_2\right)\left(\frac{\partial}{\partial z}\sigma+k_3\right)$$

```
> eq1:=collect(expand(subs(H[n]=sigma+b*z+c,S5)-JMOeq),[diff,sigma,
  z],factor):
> coeff(eq1,diff(sigma,z)):bc:=solve({op(1,%),op(2,%)},{b,c}):
> S[n]=collect(subs(%,sigma+b*z+c),z,factor):collect(%-sigma,[n],
  factor);
```

$$S_n-\sigma=-\frac{1}{2}n^2+\left(-\frac{1}{2}\alpha-\frac{1}{2}z-\frac{1}{2}\beta\right)n+\frac{1}{8}\left(\alpha-\beta\right)\left(\alpha+2\,z-\beta\right) \tag{3}$$

```
> eq2:=collect(expand(subs(bc,eq1)),[diff,sigma,z],factor):
> solve({seq(coeff(eq2,diff(sigma,z),j),j=0..3)},{k[0],k[1],k[2],k
  [3]}):A:=%[1];
```

$$A := \left\{k_0=-\frac{1}{2}n+\frac{1}{4}\alpha-\frac{1}{4}\beta,\,k_1=\frac{3}{4}\beta+\frac{1}{4}\alpha+\frac{1}{2}n,\,k_2=-\frac{1}{4}\beta+\frac{1}{4}\alpha+\frac{1}{2}n,\,k_3=\right. \tag{4}$$
$$\left.-\frac{1}{4}\beta-\frac{3}{4}\alpha-\frac{1}{2}n\right\}$$

```
> restart;with(PDEtools):with(linalg):with(VectorCalculus):with
  (LinearAlgebra):alias(H[n]=H[n](t),sigma=sigma(t),phi=phi(t)):
> S5:=(t*diff(H[n],t,t))^2-(n*(n-beta-2*n)-H[n]+(-alpha-beta-2*n+t)
  *diff(H[n],t))^2-4*diff(H[n],t)*(t*diff(H[n],t)-H[n])*(alpha-diff
  (H[n],t));
```

$$S5 := t^2\left(\frac{\partial^2}{\partial t^2}H_n\right)^2-\left(n\left(-n-\beta\right)-H_n+\left(-\alpha-\beta-2\,n+t\right)\left(\frac{\partial}{\partial t}H_n\right)\right)^2 \tag{5}$$
$$-4\left(\frac{\partial}{\partial t}H_n\right)\left(t\left(\frac{\partial}{\partial t}H_n\right)-H_n\right)\left(\alpha-\left(\frac{\partial}{\partial t}H_n\right)\right)$$

```
> K2 := diff(phi, t, t) = (alpha+1)*phi/t-(alpha+beta+2-t)*(diff
  (phi, t))/t:K3:=diff(K2,t):K4:=diff(K3,t):
> n:=2;
```

$$n := 2 \tag{6}$$

```
> tau[n]:=collect(subs(K3,K2,det(Wronskian([exp(-t)*phi,seq(diff
  (exp(-t)*phi,t$j),j=1..n-1)],t))),diff(phi,t),factor):
> H[n]:=convert(simplify(subs(K4,K3,K2,t*diff(ln(tau[n]),t))),
  parfrac,diff(phi,t));
```

$$H_2(t) := -2 - t - \alpha - \beta + \frac{\phi\left(-2\left(\frac{\partial}{\partial t}\phi\right) - \left(\frac{\partial}{\partial t}\phi\right)\alpha - \left(\frac{\partial}{\partial t}\phi\right)\beta + \phi + \phi\,\alpha\right)}{\left(\frac{\partial}{\partial t}\phi\right)^2 t + \left(-t\,\phi + 2\,\phi + \phi\,\alpha + \beta\,\phi\right)\left(\frac{\partial}{\partial t}\phi\right) - \phi^2 - \phi^2\,\alpha} \quad (7)$$

```
> subs(K3,K2,S5):collect(numer(%),[diff(phi, t)],factor);
```

$$0 \quad (8)$$