```
> restart;with(PDEtools):with(linalg):with(LinearAlgebra):
> alpha=-3;beta:=2;n:=2;
```

$$\alpha = -3$$

$$\beta := 2$$

$$n := 2 \tag{1}$$

```
> mu[k]:=(k)->sort(simplify(z^(1+alpha+beta+k)*GAMMA(alpha+k+1)*
  KummerU(alpha+k+1,alpha+k+beta+2,z)));mu[k](0);
```

$$\mu_k := k \to sort\left(simplify\left(z^{1+\alpha+\beta+k}\,\Gamma(\alpha+k+1)\ \mathrm{KummerU}(\alpha+k+1, \alpha+k+\beta+2, z)\right)\right)$$

$$\left(\alpha^2 + 2\,\alpha z + z^2 + 3\,\alpha + 2\,z + 2\right)\Gamma(\alpha+1) \tag{2}$$

```
> J:=sort(simplify(GAMMA(alpha+1)*(-1)^beta*beta!*LaguerreL(beta,-
  alpha-beta-1,z)));
```

$$J := \left(\alpha^2 + 2\,\alpha z + z^2 + 3\,\alpha + 2\,z + 2\right)\Gamma(\alpha+1) \tag{3}$$

```
> tau:=(n)->collect(subs(det(Wronskian([mu[k](0)/z^(alpha+beta+1),
  seq(diff(mu[k](0)/z^(alpha+beta+1),z$j),j=1..n-1)],z))),z,factor)
  :
> sort(simplify(expand(tau(n)*z^(n*(alpha+beta+n)))));
```

$$(\alpha+1)\left(\alpha^4 + 4\,\alpha^3 z + 6\,\alpha^2 z^2 + 4\,\alpha z^3 + z^4 + 8\,\alpha^3 + 24\,\alpha^2 z + 24\,\alpha z^2 + 8\,z^3 + 23\,\alpha^2 + 44\,\alpha z + 24\,z^2 + 28\,\alpha + 24\,z + 12\right)\Gamma(\alpha+1)^2 \tag{4}$$

```
> tau:=(n)->collect(subs(det(Wronskian([J/z^(alpha+beta+1),seq(diff
  (J/z^(alpha+beta+1),z$j),j=1..n-1)],z))),z,factor):
> sort(simplify(tau(n)*z^(n*(alpha+beta+n))));
```

$$(\alpha+1)\left(\alpha^4 + 4\,\alpha^3 z + 6\,\alpha^2 z^2 + 4\,\alpha z^3 + z^4 + 8\,\alpha^3 + 24\,\alpha^2 z + 24\,\alpha z^2 + 8\,z^3 + 23\,\alpha^2 + 44\,\alpha z + 24\,z^2 + 28\,\alpha + 24\,z + 12\right)\Gamma(\alpha+1)^2 \tag{5}$$

```
> simplify(sort(det(Matrix([[mu[k](0),mu[k](1)],[mu[k](1),mu[k](2)]
  ]))));
```

$$(\alpha+1)\left(\alpha^4 + 4\,\alpha^3 z + 6\,\alpha^2 z^2 + 4\,\alpha z^3 + z^4 + 8\,\alpha^3 + 24\,\alpha^2 z + 24\,\alpha z^2 + 8\,z^3 + 23\,\alpha^2 + 44\,\alpha z + 24\,z^2 + 28\,\alpha + 24\,z + 12\right)\Gamma(\alpha+1)^2 \tag{6}$$

```
> expand(%%-%);
```

$$0 \tag{7}$$

```
> tau:=(n)->collect(subs(det(Wronskian([J/z^(alpha+beta+1),seq(diff
  (J/z^(alpha+beta+1),z$j),j=1..n-1)],z))),z,factor):
> sort(simplify(-diff(tau(n),z)*z^(n*(alpha+beta+n)+1)));
```

$$2\,(\alpha+2)(\alpha+1)\left(\alpha^4 + 4\,\alpha^3 z + 6\,\alpha^2 z^2 + 4\,\alpha z^3 + z^4 + 10\,\alpha^3 + 30\,\alpha^2 z + 30\,\alpha z^2 + 10\,z^3 + 35\,\alpha^2 + 68\,\alpha z + 36\,z^2 + 50\,\alpha + 42\,z + 24\right)\Gamma(\alpha+1)^2 \tag{8}$$

```
> simplify(sort(det(Matrix([[mu[k](0),mu[k](2)],[mu[k](1),mu[k](3)]
  ]))));
```

$$2\,(\alpha+2)(\alpha+1)\left(\alpha^4 + 4\,\alpha^3 z + 6\,\alpha^2 z^2 + 4\,\alpha z^3 + z^4 + 10\,\alpha^3 + 30\,\alpha^2 z + 30\,\alpha z^2 + 10\,z^3 + 35\,\alpha^2 + 68\,\alpha z + 36\,z^2 + 50\,\alpha + 42\,z + 24\right)\Gamma(\alpha+1)^2 \tag{9}$$

```
> expand(%%-%);
```

$$0 \tag{10}$$

```
> n:=3;
```

$$n := 3 \tag{11}$$

```
> mu[k]:=(k)->sort(simplify(z^(1+alpha+beta+k)*GAMMA(alpha+k+1)*
  KummerU(alpha+k+1,alpha+k+beta+2,z)));mu[k](0);
```

$$\mu_k := k \to sort\left(simplify\left(z^{1+\alpha+\beta+k}\,\Gamma(\alpha+k+1)\ \mathrm{KummerU}(\alpha+k+1,\alpha+k+\beta+2,z)\right)\right)$$

$$\left(\alpha^2+2\,\alpha z+z^2+3\,\alpha+2\,z+2\right)\Gamma(\alpha+1) \tag{12}$$

```
> J:=sort(simplify(GAMMA(alpha+1)*(-1)^beta*beta!*LaguerreL(beta,-
  alpha-beta-1,z)));
```

$$J := \left(\alpha^2+2\,\alpha z+z^2+3\,\alpha+2\,z+2\right)\Gamma(\alpha+1) \tag{13}$$

```
> tau:=(n)->collect(subs(det(Wronskian([mu[k](0)/z^(alpha+beta+1),
  seq(diff(mu[k](0)/z^(alpha+beta+1),z$j),j=1..n-1)],z))),z,factor)
  :
> sort(simplify(expand(tau(n)*z^(n*(alpha+beta+n)))));
```

$$2\,(\alpha+2)\,(\alpha+1)^2\left(\alpha^6+6\,\alpha^5 z+15\,\alpha^4 z^2+20\,\alpha^3 z^3+15\,\alpha^2 z^4+6\,\alpha z^5+z^6+15\,\alpha^5\right. \tag{14}$$
$$+78\,\alpha^4 z+162\,\alpha^3 z^2+168\,\alpha^2 z^3+87\,\alpha z^4+18\,z^5+91\,\alpha^4+390\,\alpha^3 z+633\,\alpha^2 z^2$$
$$\left.+460\,\alpha z^3+126\,z^4+285\,\alpha^3+930\,\alpha^2 z+1062\,\alpha z^2+408\,z^3+484\,\alpha^2+1044\,\alpha z\right.$$
$$\left.+648\,z^2+420\,\alpha+432\,z+144\right)\Gamma(\alpha+1)^3$$

```
> tau:=(n)->collect(subs(det(Wronskian([J/z^(alpha+beta+1),seq(diff
  (J/z^(alpha+beta+1),z$j),j=1..n-1)],z))),z,factor):
> sort(simplify(tau(n)*z^(n*(alpha+beta+n))));
```

$$2\,(\alpha+2)\,(\alpha+1)^2\left(\alpha^6+6\,\alpha^5 z+15\,\alpha^4 z^2+20\,\alpha^3 z^3+15\,\alpha^2 z^4+6\,\alpha z^5+z^6+15\,\alpha^5\right. \tag{15}$$
$$+78\,\alpha^4 z+162\,\alpha^3 z^2+168\,\alpha^2 z^3+87\,\alpha z^4+18\,z^5+91\,\alpha^4+390\,\alpha^3 z+633\,\alpha^2 z^2$$
$$\left.+460\,\alpha z^3+126\,z^4+285\,\alpha^3+930\,\alpha^2 z+1062\,\alpha z^2+408\,z^3+484\,\alpha^2+1044\,\alpha z\right.$$
$$\left.+648\,z^2+420\,\alpha+432\,z+144\right)\Gamma(\alpha+1)^3$$

```
> simplify(sort(det(Matrix([[mu[k](0),mu[k](1),mu[k](2)],[mu[k](1),
  mu[k](2),mu[k](3)],[mu[k](2),mu[k](3),mu[k](4)]]))));
```

$$2\,(\alpha+2)\,(\alpha+1)^2\left(\alpha^6+6\,\alpha^5 z+15\,\alpha^4 z^2+20\,\alpha^3 z^3+15\,\alpha^2 z^4+6\,\alpha z^5+z^6+15\,\alpha^5\right. \tag{16}$$
$$+78\,\alpha^4 z+162\,\alpha^3 z^2+168\,\alpha^2 z^3+87\,\alpha z^4+18\,z^5+91\,\alpha^4+390\,\alpha^3 z+633\,\alpha^2 z^2$$
$$\left.+460\,\alpha z^3+126\,z^4+285\,\alpha^3+930\,\alpha^2 z+1062\,\alpha z^2+408\,z^3+484\,\alpha^2+1044\,\alpha z\right.$$
$$\left.+648\,z^2+420\,\alpha+432\,z+144\right)\Gamma(\alpha+1)^3$$

```
> expand(%%-%);
```

$$0 \tag{17}$$

```
> tau:=(n)->collect(subs(det(Wronskian([J/z^(alpha+beta+1),seq(diff
  (J/z^(alpha+beta+1),z$j),j=1..n-1)],z))),z,factor):
> sort(simplify(-diff(tau(n),z)*z^(n*(alpha+beta+n)+1)));
```

$$\frac{1}{(\alpha+3)^2(\alpha+2)^2(\alpha+1)}\left(6\left(\alpha^6+6\,\alpha^5 z+15\,\alpha^4 z^2+20\,\alpha^3 z^3+15\,\alpha^2 z^4+6\,\alpha z^5+z^6\right.\right. \tag{18}$$
$$+17\,\alpha^5+88\,\alpha^4 z+182\,\alpha^3 z^2+188\,\alpha^2 z^3+97\,\alpha z^4+20\,z^5+115\,\alpha^4+490\,\alpha^3 z$$
$$\left.\left.+789\,\alpha^2 z^2+568\,\alpha z^3+154\,z^4+395\,\alpha^3+1280\,\alpha^2 z+1438\,\alpha z^2+544\,z^3+724\,\alpha^2\right.\right.$$

$$+ 1544\,\alpha z + 936\,z^2 + 668\,\alpha + 672\,z + 240\Big)\,\Gamma(\alpha+4)^3\Big)$$

```
> simplify(sort(det(Matrix([[mu[k](0),mu[k](1),mu[k](3)],[mu[k](1),
  mu[k](2),mu[k](4)],[mu[k](2),mu[k](3),mu[k](5)]]))));
```

$$\frac{1}{(\alpha+3)^2(\alpha+2)^2(\alpha+1)}\Big(6\Big(\alpha^6 + 6\,\alpha^5 z + 15\,\alpha^4 z^2 + 20\,\alpha^3 z^3 + 15\,\alpha^2 z^4 + 6\,\alpha z^5 + z^6 \tag{19}$$
$$+ 17\,\alpha^5 + 88\,\alpha^4 z + 182\,\alpha^3 z^2 + 188\,\alpha^2 z^3 + 97\,\alpha z^4 + 20\,z^5 + 115\,\alpha^4 + 490\,\alpha^3 z$$
$$+ 789\,\alpha^2 z^2 + 568\,\alpha z^3 + 154\,z^4 + 395\,\alpha^3 + 1280\,\alpha^2 z + 1438\,\alpha z^2 + 544\,z^3 + 724\,\alpha^2$$
$$+ 1544\,\alpha z + 936\,z^2 + 668\,\alpha + 672\,z + 240\Big)\,\Gamma(\alpha+4)^3\Big)$$

```
> expand(%%-%);
```

$$0 \tag{20}$$