\sigma=t diff(ln(M))

```
> restart;with(PDEtools):with(linalg):with(LinearAlgebra):alias(S
  [n]=S[n](t),sigma=sigma(t),phi=phi(t),psi=psi(t),Phi=Phi(t));
```

$$S_n, \sigma, \phi, \psi, \Phi \tag{1}$$

```
> S5:=t^2*(diff(sigma, t, t))^2-(2*(diff(sigma, t))^2-t*(diff
  (sigma, t))+sigma)^2+(4*(diff(sigma, t)+k[0]))*(diff(sigma, t)+k
  [1])*(diff(sigma, t)+k[2])*(diff(sigma, t)+k[3]);
```

$$S5 := t^2 \left(\frac{\partial^2}{\partial t^2} \sigma \right)^2 - \left(2 \left(\frac{\partial}{\partial t} \sigma \right)^2 - t \left(\frac{\partial}{\partial t} \sigma \right) + \sigma \right)^2 + 4 \left(\frac{\partial}{\partial t} \sigma + k_0 \right) \left(\frac{\partial}{\partial t} \sigma + k_1 \right) \left(\frac{\partial}{\partial t} \sigma \right) \tag{2}$$

$$+ k_2 \right) \left(\frac{\partial}{\partial t} \sigma + k_3 \right)$$

```
> A:=-(5/8)*(n+1)^2+((1/4)*beta+1/4+(1/2)*alpha+(3/4)*t)*(n+1)-
  (1/8*(2*alpha+1-beta))*(2*alpha+1-beta+2*t);
```

$$A := -\frac{5}{8} (n+1)^2 + \left(\frac{1}{4} \beta + \frac{1}{4} + \frac{1}{2} \alpha + \frac{3}{4} t \right) (n+1) - \frac{1}{8} (2\alpha + 1 - \beta) (2\alpha + 1 - \beta \tag{3}$$

$$+ 2t)$$

```
> alpha:=-3;beta:=1/2;n:=1;
```

$$\alpha := -3$$

$$\beta := \frac{1}{2}$$

$$n := 1 \tag{4}$$

```
> C[1]:=1;C[2]:=-1;
```

$$C_1 := 1$$

$$C_2 := -1 \tag{5}$$

```
> phi:=simplify(C[1]*KummerM(alpha,beta,t)+C[2]*KummerU(alpha,beta,
  t));
```

$$\phi := \frac{23}{2} t^2 - \frac{69}{4} t + \frac{23}{8} - \frac{23}{15} t^3 \tag{6}$$

```
> phi*exp(-t):for K from 1 to n+1 do:l[K]:=diff(%,t)*t;od:wronskian
  ([phi*exp(-t),seq(l[j],j=1..n)],t):for j from 1 to n+1 do:h[j]:=
  Row(%,1);row(%%,2);wronskian(%*t,t):od:<seq(simplify(h[j]),j=1..
  n+1)>:W:=det(%):
```

```
> sigma:=convert(simplify(t*diff(ln(W),t))+A,parfrac,t):plot(%-op
  (1,%)-op(2,%),t=-30..30,thickness=3,color=[blue]);
```