P6 Rational Solution Generator (Okamoto)

```
> restart;
> with(plots):with(orthopoly):with(linalg):alias(w=w(z)):
> P4:=(diff(w,z)^2/2/w+3/2*w^3+4*z*w^2+2*(z^2-alpha)*w+beta/w)-diff
  (w,z,z);
```

$$P4 := \frac{1}{2}\,\frac{\left(\frac{\partial}{\partial z}\,w\right)^2}{w} + \frac{3}{2}\,w^3 + 4\,z\,w^2 + 2\left(z^2-\alpha\right)w + \frac{\beta}{w} - \left(\frac{\partial^2}{\partial z^2}\,w\right) \tag{1}$$

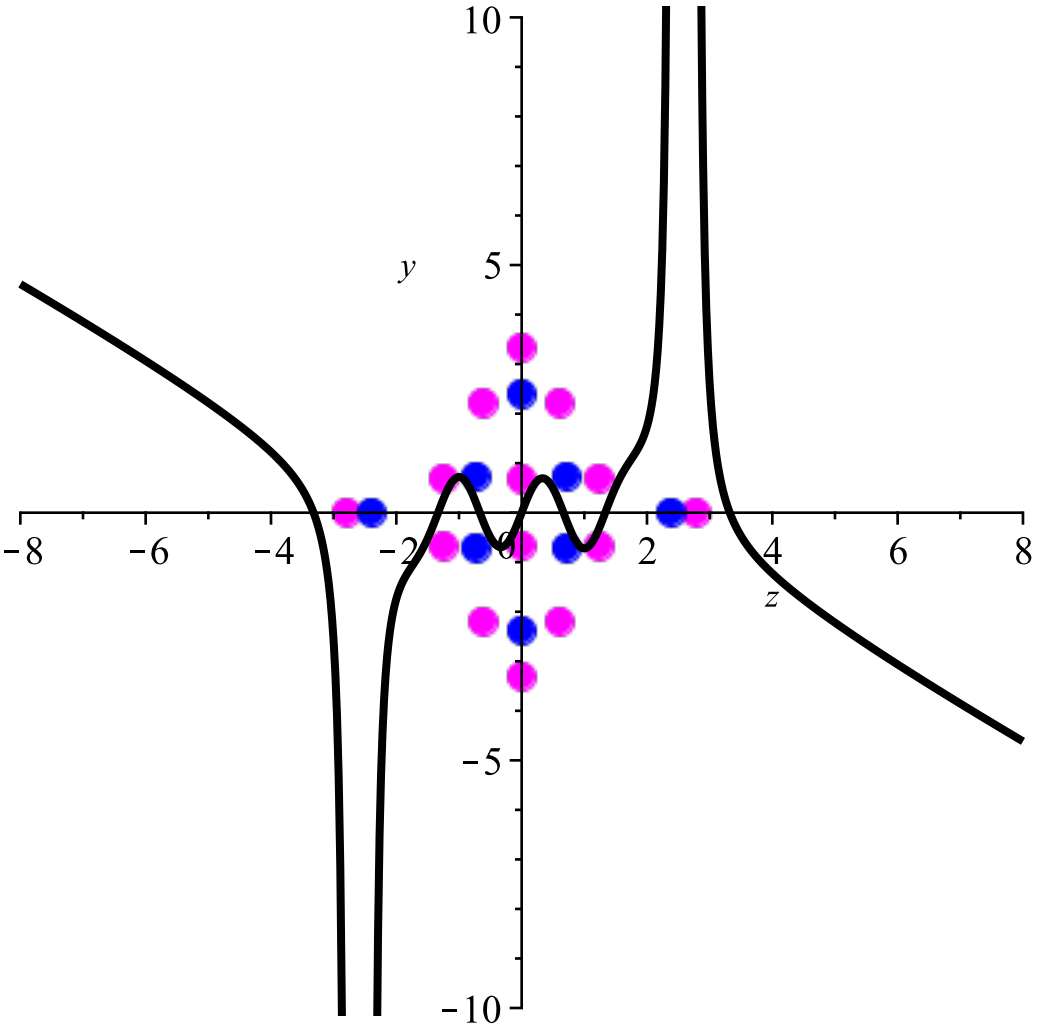
```
> PP:=taylor(exp(2*z*lambda+3*lambda^2),lambda=0,72):
> for n from 1 to 70 do phi[n]:=coeff(PP,lambda,n); od:n:='n':
> Q:=(m,n)->det(Wronskian([seq(phi[3*j-2],j=1..m+n-1),seq(phi[3*
  k-1],k=1..n-1)],z)):
> m:=2;n:=2;
```

$$m := 2$$
$$n := 2 \tag{2}$$

```
> w:=(m,n)->convert(diff(ln((Q(m+1,n))/(Q(m,n))),z),parfrac,z):
> alpha:=2*m+n:beta:=-2*(n-1/3)^2:
> w:=-2/3*z+w(m,n):
> simplify(P4);
```

$$0 \tag{3}$$

```
> RootOf(Q(m+1,n),z):J1:=evalf(allvalues(%)):RootOf(Q(m,n),z):J2:=
  (allvalues(%)):
> A:=complexplot([J1],style=point,symbol=solidcircle,color=magenta,
  symbolsize=20):
> B:=complexplot([J2],style=point,symbol=solidcircle,color=blue,
  symbolsize=20):
> C:=plot(w(m,n),z=-8..8,y=-10..10,colour=black,thickness=3,
  discont=true):display(A,B,C);
```




```
> w:=(m,n)->convert(diff(ln((Q(m,n))/(Q(m,n+1))),z),parfrac,z):
> alpha:=-m-2*n;beta:=-2*(m-1/3)^2;w:=-2/3*z+w(m,n);simplify(P4);
```

$$\alpha := -6$$

$$\beta := -\frac{50}{9}$$

$$w := -\frac{2}{3} z - \left(28 z \left(64 z^{12} - 576 z^{10} - 2160 z^{8} + 21600 z^{6} - 34020 z^{4} - 34020 z^{2} - 76545\right)\right) / \left(128 z^{14} - 1344 z^{12} - 6048 z^{10} + 75600 z^{8} - 158760 z^{6} - 238140 z^{4} - 1071630 z^{2} + 535815\right) + \frac{32 z^{3} \left(4 z^{4} - 63\right)}{16 z^{8} - 504 z^{4} - 567}$$

$$0 \qquad \textbf{(4)}$$

```
> w:=(m,n)->convert(diff(ln((Q(m,n+1))/(Q(m+1,n))),z),parfrac,z):
> alpha:=n-m;beta:=-2*(m+n+1/3)^2;w:=-2/3*z+w(m,n);simplify(P4);
```

$$\alpha := 0$$

$$\beta := -\frac{338}{9}$$

$$w := -\frac{2}{3} z - \left(28 z \left(64 z^{12} + 576 z^{10} - 2160 z^8 - 21600 z^6 - 34020 z^4 + 34020 z^2 - 76545\right)\right) / \left(128 z^{14} + 1344 z^{12} - 6048 z^{10} - 75600 z^8 - 158760 z^6 + 238140 z^4 - 1071630 z^2 - 535815\right) + \left(28 z \left(64 z^{12} - 576 z^{10} - 2160 z^8 + 21600 z^6 - 34020 z^4 - 34020 z^2 - 76545\right)\right) / \left(128 z^{14} - 1344 z^{12} - 6048 z^{10} + 75600 z^8 - 158760 z^6 - 238140 z^4 - 1071630 z^2 + 535815\right)$$

$$0 \tag{5}$$

S6 Rational Solution Generator (Okamoto)

```
> restart;with(plots):with(orthopoly):with(linalg):alias(w=w(z),
  sigma=sigma(z)):
> S4:=diff(sigma,z$2)^2-4*(z*diff(sigma,z)-sigma)^2+4*diff(sigma,z)
  *(diff(sigma,z)+2*theta[0])*(diff(sigma,z)+2*theta[infinity]);
```

$$S4 := \left(\frac{\partial^2}{\partial z^2}\sigma\right)^2 - 4\left(z\left(\frac{\partial}{\partial z}\sigma\right) - \sigma\right)^2 + 4\left(\frac{\partial}{\partial z}\sigma\right)\left(\frac{\partial}{\partial z}\sigma + 2\theta_0\right)\left(\frac{\partial}{\partial z}\sigma + 2\theta_\infty\right) \tag{6}$$

```
> PP:=taylor(exp(2*z*lambda+3*lambda^2),lambda=0,72):
> for n from 1 to 70 do phi[n]:=coeff(PP,lambda,n); od:n:='n':
> Q:=(m,n)->det(Wronskian([seq(phi[3*j-2],j=1..m+n-1),seq(phi[3*
  k-1],k=1..n-1)],z)):
> m:=2;n:=2;
```

$$m := 2$$
$$n := 2 \tag{7}$$

```
> sigma:=4/27*z^3+2/3*(n-m)*z+diff(ln(Q(m,n)),z);theta[0]:=1/3-n;
  theta[infinity]:=m-1/3;simplify(S4);
```

$$\sigma := \frac{4}{27} z^3 + \frac{-4608 z^3 + \frac{2048}{7} z^7}{-1152 z^4 + \frac{256}{7} z^8 - 1296}$$

$$\theta_0 := -\frac{5}{3}$$

$$\theta_\infty := \frac{5}{3}$$

$$0 \tag{8}$$

```
> sigma:=4/27*z^3+2/3*(2*m+n-1)*z+diff(ln(Q(m,n)),z);theta[0]:=1/3-
  m;theta[infinity]:=-n-m+2/3;simplify(S4);
```

$$\sigma := \frac{4}{27} z^3 + \frac{10}{3} z + \frac{-4608 z^3 + \frac{2048}{7} z^7}{-1152 z^4 + \frac{256}{7} z^8 - 1296}$$

$$\theta_0 := -\frac{5}{3}$$

$$\theta_\infty := -\frac{10}{3}$$

$$0 \tag{9}$$

```
> sigma:=4/27*z^3-2/3*(2*n+m-1)*z+diff(ln(Q(m,n)),z);theta[0]:=m+
  n-2/3;theta[infinity]:=n-1/3;simplify(S4);
```

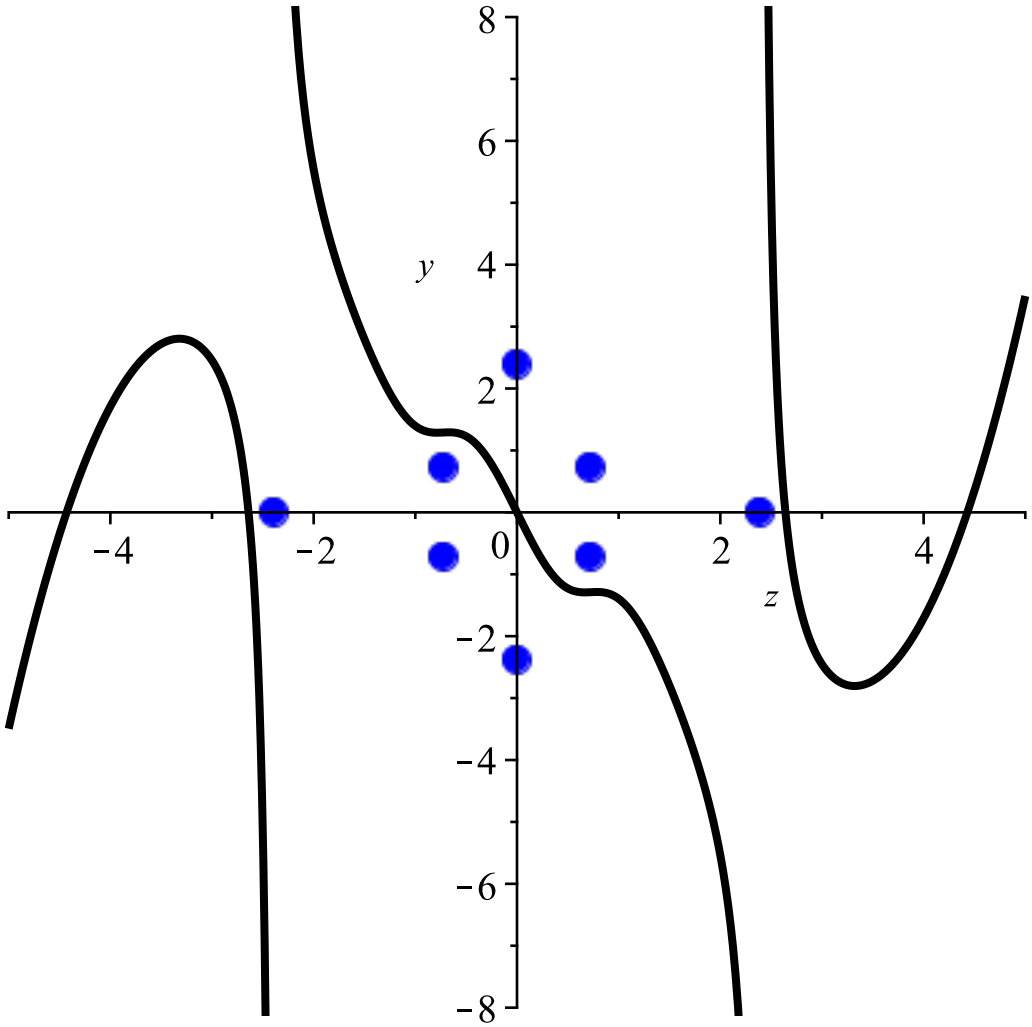
$$\sigma := \frac{4}{27} z^3 - \frac{10}{3} z + \frac{-4608 z^3 + \frac{2048}{7} z^7}{-1152 z^4 + \frac{256}{7} z^8 - 1296}$$

$$\theta_0 := \frac{10}{3}$$

$$\theta_\infty := \frac{5}{3}$$

$$0 \tag{10}$$

```
> RootOf(Q(m,n),z):J1:=evalf(allvalues(%)):
> A:=complexplot([J1],style=point,symbol=solidcircle,color=blue,
  symbolsize=20):
> C:=plot(sigma,z=-5..5,y=-8..8,colour=black,thickness=3,discont=
  true):display(A,C);
```




```
>
```