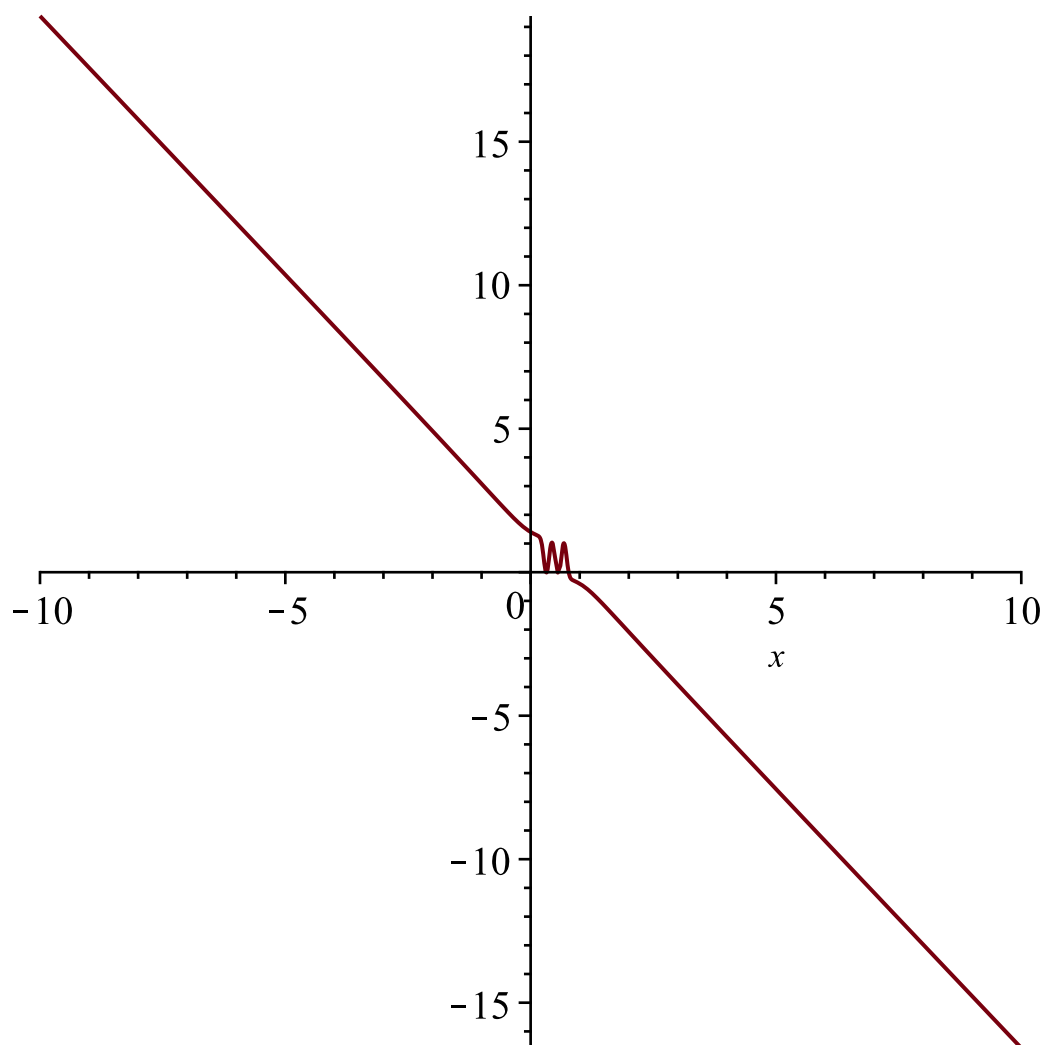```
> restart;alias(sigma=sigma(x),phi=phi(x),psi=psi(z),w=w(x)):with
  (PDEtools):with(plots):with(LinearAlgebra):with(linalg):
> a:=-5;b:=-14;c:=-9;n:=2;Digits:=50;
```

$$a := -5$$
$$b := -14$$
$$c := -9$$
$$n := 2$$
$$Digits := 50 \qquad \textbf{(1)}$$

```
> P6:=diff(w,x,x)-1/2*(1/w+1/(w-1)+1/(w-x))*diff(w,x)^2+(1/x+1/
  (x-1)+1/(w-x))*diff(w,x)-w*(w-1)*(w-x)/x^2/(x-1)^2*(alpha+beta*
  x/w^2+Gamma*(x-1)/(w-1)^2+delta*x*(x-1)/(w-x)^2):
> alpha:= (1/2)*(a)^2;beta:=-(1/2)*(c-b-n-1)^2;Gamma:= (1/2)*(a-n-
  c)^2;delta:= 1/2-(1/2)*(b)^2;
```

$$\alpha := \frac{25}{2}$$
$$\beta := -2$$
$$\Gamma := 2$$
$$\delta := -\frac{195}{2} \qquad \textbf{(2)}$$

```
> phi:=(a,b,c)->factor(simplify(sort(hypergeom([a,b], [c], x))))*x^
  (b);
```

$$\phi := (a, b, c) \rightarrow factor(simplify(sort(\text{hypergeom}([a, b], [c], x)))) \, x^b \qquad \textbf{(3)}$$

```
> phi(a+1,b+1,c+1):for K from 1 to n+1 do;l[K]:=diff(%,x)*x*(x-1);
  od:wronskian([phi(a+1,b+1,c+1),seq(l[k],k=1..n)],x):for K from 1
  to n+1 do;h[K]:=Row(%,1);row(%%,2);wronskian(%*x*(x-1),x)
  :od:simplify(<seq(simplify(h[k]),k=1..n+1)>):tau[n+1]:=collect
  (simplify(det(%)*x^(-(n+1)*(b+1))*(x*(x-1))^((-n)*((n+1)/2))),x,
  factor):
> phi(a-1,b+1,c):for K from 1 to n do;l[K]:=diff(%,x)*x*(x-1);
  od:wronskian([phi(a-1,b+1,c),seq(l[k],k=1..n-1)],x):for K from 1
  to n do;h[K]:=Row(%,1);row(%%,2);wronskian(%*x*(x-1),x)
  :od:simplify(<seq(simplify(h[k]),k=1..n)>):tau[n]:=collect
  (simplify(det(%)*x^(-(n)*(b+1))*(x*(x-1))^((-n+1)*((n)/2))),x,
  factor):
> w:=1/a*(n+c-(2*n+b+1)*x)-convert(simplify(x*(x-1)/a*diff(ln(tau
  [n+1]/tau[n]),x)),parfrac,x):
> collect(numer(P6),x,factor);
```

$$0 \qquad \textbf{(4)}$$

```
> plot(w);
```

```
> restart;alias(sigma=sigma(x),phi=phi(x),psi=psi(z)):with
  (PDEtools):with(plots):with(LinearAlgebra):with(linalg):
> S6:=diff(sigma,x)*(x*(x-1)*diff(sigma,x,x))^2+(diff(sigma,x)*(2*
  sigma-(2*x-1)*diff(sigma,x))+nu[1]*nu[2]*nu[3]*nu[4])^2-product
  (diff(sigma,x)+nu[k]^2,k=1..4);
```

$$S6 := \left(\frac{\partial}{\partial x}\, \sigma \right) x^2\, (x-1)^2 \left(\frac{\partial^2}{\partial x^2}\, \sigma \right)^2 + \left(\left(\frac{\partial}{\partial x}\, \sigma \right) \left(2\, \sigma - (2\, x - 1) \left(\frac{\partial}{\partial x}\, \sigma \right) \right) \right. \tag{5}$$

$$\left. + \nu_1\, \nu_2\, \nu_3\, \nu_4 \right)^2 - \left(\frac{\partial}{\partial x}\, \sigma + \nu_1^2 \right) \left(\frac{\partial}{\partial x}\, \sigma + \nu_2^2 \right) \left(\frac{\partial}{\partial x}\, \sigma + \nu_3^2 \right) \left(\frac{\partial}{\partial x}\, \sigma + \nu_4^2 \right)$$

```
> n:=1;a:=-3;b:=3;c:=1;
```

$$n := 1$$
$$a := -3$$
$$b := 3$$
$$c := 1 \tag{6}$$

```
> nu[1]:=-(a+1-b-2*(n+1))/2;nu[2]:=(2*c-1-a-b)/2;nu[3]:=-(1+a-b)/2;
  nu[4]:=(1-a-b)/2;
```

$$\nu_1 := \frac{9}{2}$$

$$\nu_2 := \frac{1}{2}$$

$$\nu_3 := \frac{5}{2}$$

$$\nu_4 := \frac{1}{2} \tag{7}$$

```
> phi2 := diff(phi, x, x) = (a*b*phi-(c-(a+b+1)*x)*(diff(phi, x)))/
  (x*(1-x));phi3:=diff(phi2,x):phi4:=diff(phi3,x):
```

$$\phi 2 := \frac{\partial^2}{\partial x^2}\,\phi = \frac{-9\,\phi - (1-x)\left(\dfrac{\partial}{\partial x}\,\phi\right)}{x\,(1-x)} \tag{8}$$

```
> phi*x^b:for K from 1 to n+1 do;l[K]:=diff(%,x)*x*(x-1);
  od:wronskian([phi*x^b,seq(l[k],k=1..n)],x):for K from 1 to n+1
  do;h[K]:=Row(%,1);row(%%,2);wronskian(%*x*(x-1),x):od:simplify
  (<seq(simplify(h[k]),k=1..n+1)>):tau:=collect(simplify(det(%)*x^
  (-(n+1)*b)*(x*(x-1))^((-n)*((n+1)/2))),x,factor):
```

```
> sigma:=convert(subs(phi4,phi3,phi2,simplify(x*(x-1)*diff(ln(tau),
  x))+(n+1)/4*(4*a*x-2*c-a+1+b)-(1/4)*(1+a-b)^2*x+1/4*(-b+b^2+c-c*
  a-c*b+a+a^2)),parfrac,diff(phi, x));
```

$$\sigma := \frac{27}{4} - \frac{53}{4}\,x - \frac{\left(7\left(\dfrac{\partial}{\partial x}\,\phi\right)x^2 + 21\,\phi\,x - 7\left(\dfrac{\partial}{\partial x}\,\phi\right)x - 12\,\phi\right)\phi}{(x^2 - x)\left(\dfrac{\partial}{\partial x}\,\phi\right)^2 - \left(\dfrac{\partial}{\partial x}\,\phi\right)\phi\,x - 12\,\phi^2} \tag{9}$$

```
> collect(numer(expand(subs(phi2,expand(subs(phi2,S6))))),[diff,x],
  factor);
```

$$0 \tag{10}$$