

```
> restart:Digits:=100;with(linalg):with(plots):with(PDEtools):with
(plots):with(LinearAlgebra):with(linalg):alias(w=w(z),phi=phi(z),
sigma=sigma(z)):
```

$$Digits := 100$$

(1)

```
> P2:=diff(w,z,z)-(2*w^3+z*w+alpha);
```

$$P2 := \frac{\partial^2}{\partial z^2} w - 2 w^3 - z w - \alpha$$

(2)

```
> S2:=(diff(sigma,z,z))^2+4*diff(sigma,z)^3+2*diff(sigma,z)*(z*diff
(sigma,z)-sigma)-1/4*(alpha+1/2)^2;
```

$$S2 := \left(\frac{\partial^2}{\partial z^2} \sigma \right)^2 + 4 \left(\frac{\partial}{\partial z} \sigma \right)^3 + 2 \left(\frac{\partial}{\partial z} \sigma \right) \left(z \left(\frac{\partial}{\partial z} \sigma \right) - \sigma \right) - \frac{1}{4} \left(\alpha + \frac{1}{2} \right)^2$$

(3)

```
> alpha:=n+1/2;
```

$$\alpha := n + \frac{1}{2}$$

(4)

```
> n:=1;phi2:=diff(phi,z,z)=-phi/2*z;phi3:=diff(phi2,z);phi4:=diff
(phi3,z);phi5:=diff(phi4,z);
```

$$n := 1$$

$$\phi2 := \frac{\partial^2}{\partial z^2} \phi = -\frac{1}{2} \phi z$$

$$\phi3 := \frac{\partial^3}{\partial z^3} \phi = -\frac{1}{2} \left(\frac{\partial}{\partial z} \phi \right) z - \frac{1}{2} \phi$$

$$\phi4 := \frac{\partial^4}{\partial z^4} \phi = -\frac{1}{2} \left(\frac{\partial^2}{\partial z^2} \phi \right) z - \left(\frac{\partial}{\partial z} \phi \right)$$

$$\phi5 := \frac{\partial^5}{\partial z^5} \phi = -\frac{1}{2} \left(\frac{\partial^3}{\partial z^3} \phi \right) z - \frac{3}{2} \frac{\partial^2}{\partial z^2} \phi$$

(5)

```
> tau:=(n)->det(wronskian([phi,seq(diff(phi,z$j),j=1..n-1)],z));
```

$$\tau := n \rightarrow \text{linalg:-det} \left(\text{linalg:-wronskian} \left(\left[\phi, \text{seq} \left(\frac{\partial^j}{\partial z^j} \phi, j = 1 \dots n-1 \right) \right], z \right) \right)$$

(6)

```
> simplify(diff(ln(tau(n)/tau(n+1)),z)):w:=simplify(expand(subs
(phi2,expand(subs(phi3,expand(subs(phi4,expand(subs(phi5,expand
(%)))))))));convert(simplify(subs(diff(phi,z)=Phi*phi,%)),
parfrac,Phi);
```

$$w := - \frac{-\phi^2 \left(\frac{\partial}{\partial z} \phi \right) z + \phi^3 - 2 \left(\frac{\partial}{\partial z} \phi \right)^3}{\left(\phi^2 z + 2 \left(\frac{\partial}{\partial z} \phi \right)^2 \right) \phi}$$

$$\Phi - \frac{1}{2 \Phi^2 + z}$$

(7)

```
> simplify(expand(subs(phi2,expand(subs(phi3,expand(P2)))));
```

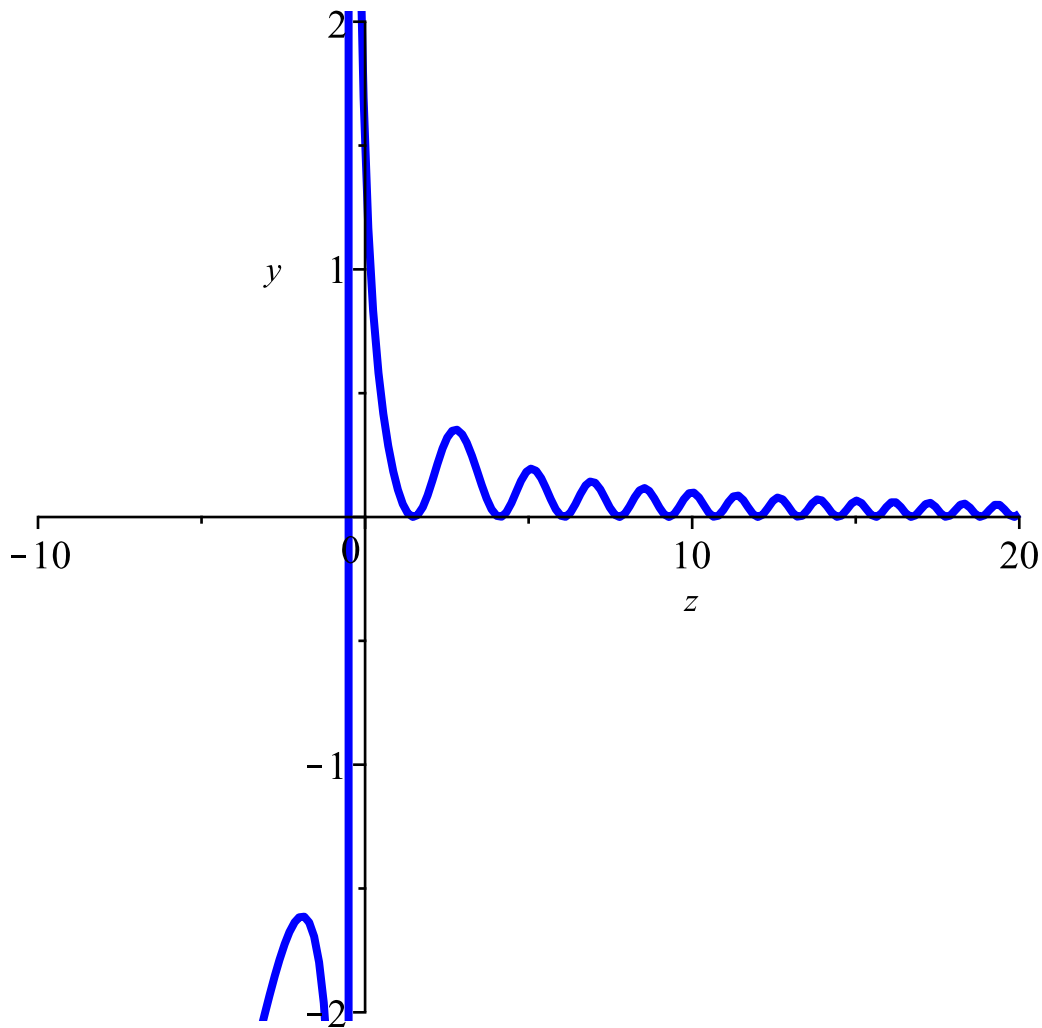
$$0$$

(8)

```
> simplify(diff(ln(tau(n+1)),z)):sigma:=simplify(expand(subs(phi2,
expand(subs(phi3,expand(subs(phi4,expand(subs(phi5,expand(%))))
)))));
```

$$\sigma := \frac{\phi^2}{\phi^2 z + 2 \left(\frac{\partial}{\partial z} \phi \right)^2} \quad (9)$$

```
> simplify(expand(subs(phi2,expand(subs(phi3,expand(S2)))));
0
(10)
> simplify(subs(phi=cos(Pi/2)*AiryAi(-2^(-1/3)*z)+sin(Pi/2)*AiryBi
(-2^(-1/3)*z),sigma)):
> plot(%,z=-10..20,y=-2..2,color=blue,thickness=3);
```



```
> restart:with(linalg):with(plots):with(PDEtools):with(plots):with
(LinearAlgebra):with(linalg):alias(w=w(z),phi=phi(z),sigma=sigma
(z)):
> P2:=diff(w,z,z)-(2*w^3+z*w+alpha);
```

$$P2 := \frac{\partial^2}{\partial z^2} w - 2 w^3 - z w - \alpha \quad (11)$$

```
> S2:=(diff(sigma,z,z))^2+4*diff(sigma,z)^3+2*diff(sigma,z)*(z*diff
(sigma,z)-sigma)-1/4*(alpha+1/2)^2;
```

$$S2 := \left(\frac{\partial^2}{\partial z^2} \sigma \right)^2 + 4 \left(\frac{\partial}{\partial z} \sigma \right)^3 + 2 \left(\frac{\partial}{\partial z} \sigma \right) \left(z \left(\frac{\partial}{\partial z} \sigma \right) - \sigma \right) - \frac{1}{4} \left(\alpha + \frac{1}{2} \right)^2 \quad (12)$$

> alpha:=- (n+1/2) ;

$$\alpha := -n - \frac{1}{2} \quad (13)$$

> n:=1;phi2:=diff(phi, z, z)=-phi/2*z;phi3:=diff(phi2,z);phi4:=diff(phi3,z);phi5:=diff(phi4,z);

$$\begin{aligned} n &:= 1 \\ \phi_2 &:= \frac{\partial^2}{\partial z^2} \phi = -\frac{1}{2} \phi z \\ \phi_3 &:= \frac{\partial^3}{\partial z^3} \phi = -\frac{1}{2} \left(\frac{\partial}{\partial z} \phi \right) z - \frac{1}{2} \phi \\ \phi_4 &:= \frac{\partial^4}{\partial z^4} \phi = -\frac{1}{2} \left(\frac{\partial^2}{\partial z^2} \phi \right) z - \left(\frac{\partial}{\partial z} \phi \right) \\ \phi_5 &:= \frac{\partial^5}{\partial z^5} \phi = -\frac{1}{2} \left(\frac{\partial^3}{\partial z^3} \phi \right) z - \frac{3}{2} \frac{\partial^2}{\partial z^2} \phi \end{aligned} \quad (14)$$

> tau:=(n)->det(wronskian([phi,seq(diff(phi,z\$j),j=1..n-1)],z));

$$\tau := n \rightarrow \text{linalg:-det} \left(\text{linalg:-wronskian} \left(\left[\phi, \text{seq} \left(\frac{\partial^j}{\partial z^j} \phi, j=1..n-1 \right) \right], z \right) \right) \quad (15)$$

> simplify(diff(ln(tau(n+1)/tau(n)),z)):w:=simplify(expand(subs(phi2,expand(subs(phi3,expand(subs(phi4,expand(subs(phi5,expand(%)))))))));convert(simplify(subs(diff(phi,z)=Phi*phi,%)),parfrac,Phi);

$$\begin{aligned} w &:= \frac{-\phi^2 \left(\frac{\partial}{\partial z} \phi \right) z + \phi^3 - 2 \left(\frac{\partial}{\partial z} \phi \right)^3}{\phi \left(\phi^2 z + 2 \left(\frac{\partial}{\partial z} \phi \right)^2 \right)} \\ &\quad - \Phi + \frac{1}{2 \Phi^2 + z} \end{aligned} \quad (16)$$

> simplify(expand(subs(phi2,expand(subs(phi3,expand(P2))))));

$$0 \quad (17)$$

> alpha:=-alpha;

$$\alpha := \frac{3}{2} \quad (18)$$

> simplify(diff(ln(tau(n+1)),z)):sigma:=simplify(expand(subs(phi2,expand(subs(phi3,expand(subs(phi4,expand(subs(phi5,expand(%)))))))));convert(simplify(subs(diff(phi,z)=Phi*phi,%)),parfrac,Phi);

$$\begin{aligned} \sigma &:= \frac{\phi^2}{\phi^2 z + 2 \left(\frac{\partial}{\partial z} \phi \right)^2} \\ &\quad \frac{1}{2 \Phi^2 + z} \end{aligned} \quad (19)$$

```

> simplify(expand(subs(phi2,expand(subs(phi3,expand(S2)))));
0
> Phi:=diff(ln(cos(z)*AiryAi(z)+sin(z)*AiryBi(z)),z):
> #plot(subs(%%),z=-100..100);

```

(20)