```
> restart;with(PDEtools):with(linalg):with(LinearAlgebra):
> alpha=-2;beta:=2;n:=3;
```

$$\alpha = -2$$

$$\beta := 2$$

$$n := 3 \tag{1}$$

```
> mu[k]:=(k)->sort(simplify(z^(1+alpha+beta+k)*GAMMA(alpha+k+1)*
  KummerU(alpha+k+1,alpha+k+beta+2,z)));
```

$$\mu_k := k \to sort\left(simplify\left(z^{1+\alpha+\beta+k}\,\Gamma(\alpha+k+1)\ \mathrm{KummerU}(\alpha+k+1, \alpha+k+\beta+2, z)\right)\right) \tag{2}$$

```
> J:=sort(simplify(LaguerreL(beta,-alpha-beta-1,z)));
```

$$J := \frac{1}{2}\,\alpha^2 + \alpha z + \frac{1}{2}\,z^2 + \frac{3}{2}\,\alpha + z + 1 \tag{3}$$

```
> tau:=(n)->collect(subs(det(Wronskian([J/z^(alpha+beta+1),seq(diff
  (J/z^(alpha+beta+1),z$j),j=1..n-1)],z)))*z^(n*(alpha+beta+n)),z,
  factor):
> factor(simplify(expand(((-1)^beta*beta!*GAMMA(alpha+1))^n*tau(n))
  ));
```

$$\frac{1}{(\alpha+2)^2\,(\alpha+1)}\Big(2\,\big(\alpha^6 + 6\,\alpha^5 z + 15\,\alpha^4 z^2 + 20\,\alpha^3 z^3 + 15\,\alpha^2 z^4 + 6\,\alpha z^5 + z^6 + 15\,\alpha^5 + 78\,\alpha^4 z + 162\,\alpha^3 z^2 + 168\,\alpha^2 z^3 + 87\,\alpha z^4 + 18\,z^5 + 91\,\alpha^4 + 390\,\alpha^3 z + 633\,\alpha^2 z^2 + 460\,\alpha z^3 + 126\,z^4 + 285\,\alpha^3 + 930\,\alpha^2 z + 1062\,\alpha z^2 + 408\,z^3 + 484\,\alpha^2 + 1044\,\alpha z + 648\,z^2 + 420\,\alpha + 432\,z + 144\big)\,\Gamma(\alpha+3)^3\Big) \tag{4}$$

```
> simplify(sort(det(Matrix([[mu[k](0),mu[k](1),mu[k](2)],[mu[k](1),
  mu[k](2),mu[k](3)],[mu[k](2),mu[k](3),mu[k](4)]]))));
```

$$\frac{1}{(\alpha+2)^2\,(\alpha+1)}\Big(2\,\big(\alpha^6 + 6\,\alpha^5 z + 15\,\alpha^4 z^2 + 20\,\alpha^3 z^3 + 15\,\alpha^2 z^4 + 6\,\alpha z^5 + z^6 + 15\,\alpha^5 + 78\,\alpha^4 z + 162\,\alpha^3 z^2 + 168\,\alpha^2 z^3 + 87\,\alpha z^4 + 18\,z^5 + 91\,\alpha^4 + 390\,\alpha^3 z + 633\,\alpha^2 z^2 + 460\,\alpha z^3 + 126\,z^4 + 285\,\alpha^3 + 930\,\alpha^2 z + 1062\,\alpha z^2 + 408\,z^3 + 484\,\alpha^2 + 1044\,\alpha z + 648\,z^2 + 420\,\alpha + 432\,z + 144\big)\,\Gamma(\alpha+3)^3\Big) \tag{5}$$

```
> simplify(expand(diff(ln(%%),z)-diff(ln(%),z)));
```

$$0 \tag{6}$$