```
> restart;with(linalg):with(LinearAlgebra):alias(w=w(z),phi[nu]=phi
  [nu](z)):
> P3:=numer(simplify(diff(w,z)^2/w-diff(w,z)/z+(alpha*w^2+beta)/z+
  Gamma*w^3+delta/w-diff(w,z,z)));
```

$$P3 := \left(\frac{\partial}{\partial z} w\right)^2 z - \left(\frac{\partial}{\partial z} w\right) w + \alpha w^3 + w\beta + \Gamma w^4 z + \delta z - \left(\frac{\partial^2}{\partial z^2} w\right) w z \tag{1}$$

```
> K:=(n)->(ToeplitzMatrix(p,n)):
> n:=2;U:=seq(p[d+1]=psi[nu-n+d+1],d=0..2*n+1):
```

$$n := 2 \tag{2}$$

```
> Y:=seq(psi[nu-n+d]=psi(nu-n+d),d=0..2*n):
> psi:=(nu)->z^(nu)*(BesselJ(nu,sqrt(epsilon[1]*epsilon[2])*z)+
  BesselY(nu,sqrt(epsilon[1]*epsilon[2])*z)):
> subs(U,K(n));
```

$$\begin{bmatrix} \psi_\nu & \psi_{\nu-1} \\ \psi_{\nu+1} & \psi_\nu \end{bmatrix} \tag{3}$$

```
> A:=c*z^(n*(n-1))*det(subs(U,Y,K(n))):
> phi[nu]:for k from 1 to n do;l[k]:=diff(%,z)*z;od:wronskian([phi
  [nu],seq(l[j],j=1..n-1)],z):for j from 1 to n do;h[j]:=Row(%,1);
  row(%%,2);wronskian(%*z,z):od:<seq(h[j],j=1..n)>:B:=simplify(det
  (subs(U,phi[nu]=psi(nu),%))):simplify((subs(U,%%)));
```

$$\begin{bmatrix} \phi_\nu & \left(\frac{\partial}{\partial z}\phi_\nu\right) z \\ \left(\frac{\partial}{\partial z}\phi_\nu\right) z & \left(\left(\frac{\partial^2}{\partial z^2}\phi_\nu\right) z + \frac{\partial}{\partial z}\phi_\nu\right) z \end{bmatrix} \tag{4}$$

```
> collect(A-B,z,factor);
```

$$\begin{aligned} & c z^2 \left((z^\nu)^2 \,\mathrm{BesselJ}\left(\nu, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right)^2 + 2 (z^\nu)^2 \,\mathrm{BesselJ}\left(\nu, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \mathrm{BesselY}\left(\nu, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \right. \\ & + (z^\nu)^2 \,\mathrm{BesselY}\left(\nu, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right)^2 - z^{\nu-1} z^{\nu+1} \,\mathrm{BesselJ}\left(\nu-1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \mathrm{BesselJ}\left(\nu+1, \right. \\ & \left. \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) - z^{\nu-1} z^{\nu+1} \,\mathrm{BesselJ}\left(\nu-1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \mathrm{BesselY}\left(\nu+1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \\ & - z^{\nu-1} z^{\nu+1} \,\mathrm{BesselY}\left(\nu-1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \mathrm{BesselJ}\left(\nu+1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \\ & \left. - z^{\nu-1} z^{\nu+1} \,\mathrm{BesselY}\left(\nu-1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \mathrm{BesselY}\left(\nu+1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \right) \\ & + \varepsilon_1 \varepsilon_2 \left(\mathrm{BesselJ}\left(\nu, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right)^2 + 2 \,\mathrm{BesselJ}\left(\nu, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \mathrm{BesselY}\left(\nu, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \right. \\ & + \mathrm{BesselY}\left(\nu, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right)^2 + \mathrm{BesselJ}\left(\nu+1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right)^2 + 2 \,\mathrm{BesselJ}\left(\nu+1, \right. \\ & \left. \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \mathrm{BesselY}\left(\nu+1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) + \mathrm{BesselY}\left(\nu+1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right)^2 \Big) z^{2\nu+1} z \\ & - 2\nu \left(\mathrm{BesselJ}\left(\nu+1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) + \mathrm{BesselY}\left(\nu+1, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \right) \left(\mathrm{BesselJ}\left(\nu, \sqrt{\varepsilon_1 \varepsilon_2}\, z\right) \right. \end{aligned} \tag{5}$$

$$+ \mathrm{BesselY}\left(\nu, \sqrt{\varepsilon_1\,\varepsilon_2}\,z\right)\Big)\, z^{2\nu+1}\sqrt{\varepsilon_1\,\varepsilon_2}$$

```
> simplify(diff(ln(A),z)-diff(ln(B),z));
```

$$0 \tag{6}$$