```
> restart;
> alias(q=q(z),p=p(z),sigma=sigma(z)):
> #kappa[0]:=1;kappa[infinity]:=1;
> S3[DLMF]:=(z*(diff(sigma, z, z))-(diff(sigma, z)))^2+(2*((diff
  (sigma, z))^2-kappa[0]^2*kappa[infinity]^2*z^2))*(z*(diff(sigma,
  z))-2*sigma)+8*kappa[0]*kappa[infinity]*theta[0]*theta[infinity]*
  z*(diff(sigma, z))-4*kappa[0]^2*kappa[infinity]^2*(theta[0]^2+
  theta[infinity]^2)*z^2;
```

$$S3_{DLMF} := \left(z\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)-\left(\frac{\partial}{\partial z}\,\sigma\right)\right)^2+2\left(\left(\frac{\partial}{\partial z}\,\sigma\right)^2-\kappa_0^2\kappa_\infty^2 z^2\right)\left(z\left(\frac{\partial}{\partial z}\,\sigma\right)-2\,\sigma\right) \tag{1}$$
$$+8\,\kappa_0\kappa_\infty\theta_0\theta_\infty z\left(\frac{\partial}{\partial z}\,\sigma\right)-4\,\kappa_0^2\kappa_\infty^2\left(\theta_0^2+\theta_\infty^2\right)z^2$$

```
> H:=(q^2*p^2-(kappa[infinity]*z*q^2+(2*theta[0]+1)*q-kappa[0]*z)*
  p+kappa[infinity]*(theta[0]+theta[infinity])*z*q)/z;
```

$$H := \frac{q^2p^2-\left(\kappa_\infty z q^2+\left(2\,\theta_0+1\right)q-\kappa_0 z\right)p+\kappa_\infty\left(\theta_0+\theta_\infty\right)z q}{z} \tag{2}$$

```
> H1:=diff(q,z)=(2*q^2*p-kappa[infinity]*z*q^2-(2*theta[0]+1)*q+
  kappa[0]*z)/z;H2:=diff(p,z)=-(2*q*p^2-(2*kappa[infinity]*z*q+2*
  theta[0]+1)*p+kappa[infinity]*(theta[0]+theta[infinity])*z)/z;
```

$$H1 := \frac{\partial}{\partial z}\,q = \frac{2\,q^2p-\kappa_\infty z q^2-\left(2\,\theta_0+1\right)q+\kappa_0 z}{z}$$

$$H2 := \frac{\partial}{\partial z}\,p = -\frac{2\,q p^2-\left(2\,\kappa_\infty z q+2\,\theta_0+1\right)p+\kappa_\infty\left(\theta_0+\theta_\infty\right)z}{z} \tag{3}$$

```
> S:=sigma=z*H+p*q+theta[0]^2-1/2*kappa[0]*kappa[infinity]*z^2;
```

$$S := \sigma = q^2p^2-\left(\kappa_\infty z q^2+\left(2\,\theta_0+1\right)q-\kappa_0 z\right)p+\kappa_\infty\left(\theta_0+\theta_\infty\right)z q+p\,q+\theta_0^2 \tag{4}$$
$$-\frac{1}{2}\,\kappa_0\kappa_\infty z^2$$

```
> S1:=simplify(subs(H1,H2,diff(S,z)));S2:=simplify(expand(subs(H1,
  H2,diff(S1,z))));
```

$$S1 := \frac{\partial}{\partial z}\,\sigma = \kappa_0\left(-\kappa_\infty z+2\,p\right)$$

$$S2 := \frac{\partial^2}{\partial z^2}\,\sigma = -\frac{\kappa_0\left(-4\,p\,\kappa_\infty z q+4\,q p^2+2\,\kappa_\infty z\theta_0+2\,\kappa_\infty z\theta_\infty-4\,p\,\theta_0+\kappa_\infty z-2\,p\right)}{z} \tag{5}$$

```
> solve({S1,S2},{q,p});
```

$$\left\{p=\frac{1}{2}\,\frac{\kappa_0\kappa_\infty z+\frac{\partial}{\partial z}\,\sigma}{\kappa_0},\ q=\right. \tag{6}$$
$$\left.-\frac{\kappa_0\left(2\,\kappa_0\kappa_\infty\theta_\infty z+z\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)-2\left(\frac{\partial}{\partial z}\,\sigma\right)\theta_0-\left(\frac{\partial}{\partial z}\,\sigma\right)\right)}{\left(\frac{\partial}{\partial z}\,\sigma\right)^2-\kappa_0^2\kappa_\infty^2 z^2}\right\}$$

```
> collect(expand(subs(%,sigma-(z*H+p*q+theta[0]^2-1/2*kappa[0]*
  kappa[infinity]*z^2))),diff,factor);factor(%);collect(simplify
  (-4*%*(-kappa[0]*kappa[infinity]*z+diff(sigma, z))*(kappa[0]*
  kappa[infinity]*z+diff(sigma, z))),diff,factor);simplify(S3[DLMF]
  -%);
```

$$
\begin{aligned}
&-\frac{1}{4}\,\frac{z^2\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)^2}{\left(-\kappa_0\,\kappa_\infty\,z+\frac{\partial}{\partial z}\,\sigma\right)\left(\kappa_0\,\kappa_\infty\,z+\frac{\partial}{\partial z}\,\sigma\right)}+\frac{1}{2}\,\frac{z\left(\frac{\partial}{\partial z}\,\sigma\right)\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)}{\left(-\kappa_0\,\kappa_\infty\,z+\frac{\partial}{\partial z}\,\sigma\right)\left(\kappa_0\,\kappa_\infty\,z+\frac{\partial}{\partial z}\,\sigma\right)}\\
&\quad-\frac{1}{4}\,\frac{1}{\left(-\kappa_0\,\kappa_\infty\,z+\frac{\partial}{\partial z}\,\sigma\right)\left(\kappa_0\,\kappa_\infty\,z+\frac{\partial}{\partial z}\,\sigma\right)}\left(-2\,z^3\left(\frac{\partial}{\partial z}\,\sigma\right)\kappa_0^2\,\kappa_\infty^2-4\,z^2\,\kappa_0^2\,\kappa_\infty^2\,\theta_0^2\right.\\
&\quad-4\,\kappa_0^2\,\kappa_\infty^2\,\theta_\infty^2\,z^2+4\,\sigma\,z^2\,\kappa_0^2\,\kappa_\infty^2+8\,\kappa_0\,\kappa_\infty\,\theta_0\,\theta_\infty\,z\left(\frac{\partial}{\partial z}\,\sigma\right)+2\,z\left(\frac{\partial}{\partial z}\,\sigma\right)^3-4\left(\frac{\partial}{\partial z}\,\sigma\right)^2\sigma\\
&\quad\left.+\left(\frac{\partial}{\partial z}\,\sigma\right)^2\right)\\
&-\frac{1}{4}\,\frac{1}{\left(-\kappa_0\,\kappa_\infty\,z+\frac{\partial}{\partial z}\,\sigma\right)\left(\kappa_0\,\kappa_\infty\,z+\frac{\partial}{\partial z}\,\sigma\right)}\left(-2\,z^3\left(\frac{\partial}{\partial z}\,\sigma\right)\kappa_0^2\,\kappa_\infty^2-4\,z^2\,\kappa_0^2\,\kappa_\infty^2\,\theta_0^2-4\,\kappa_0^2\,\kappa_\infty^2\right.\\
&\quad\theta_\infty^2\,z^2+4\,\sigma\,z^2\,\kappa_0^2\,\kappa_\infty^2+8\,\kappa_0\,\kappa_\infty\,\theta_0\,\theta_\infty\,z\left(\frac{\partial}{\partial z}\,\sigma\right)+z^2\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)^2+2\,z\left(\frac{\partial}{\partial z}\,\sigma\right)^3\\
&\quad\left.-2\,z\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)\left(\frac{\partial}{\partial z}\,\sigma\right)-4\left(\frac{\partial}{\partial z}\,\sigma\right)^2\sigma+\left(\frac{\partial}{\partial z}\,\sigma\right)^2\right)\\
&z^2\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)^2-2\,z\left(\frac{\partial^2}{\partial z^2}\,\sigma\right)\left(\frac{\partial}{\partial z}\,\sigma\right)+2\,z\left(\frac{\partial}{\partial z}\,\sigma\right)^3+(-4\,\sigma+1)\left(\frac{\partial}{\partial z}\,\sigma\right)^2\\
&\quad-2\,\kappa_0\,\kappa_\infty\,z\left(z^2\,\kappa_0\,\kappa_\infty-4\,\theta_0\,\theta_\infty\right)\left(\frac{\partial}{\partial z}\,\sigma\right)+4\,\kappa_0^2\,\kappa_\infty^2\,z^2\left(-\theta_0^2-\theta_\infty^2+\sigma\right)\\
&\qquad 0
\end{aligned}
\tag{7}
$$