P4

```
> restart;with(ListTools):with(orthopoly):with(linalg):alias(sigma=
  sigma(z),psi=psi(z),phi=phi(z),w=w(z)):S4 := (diff(sigma, z, z))
  ^2-4*z^2*(diff(sigma, z))^2+8*z*(diff(sigma, z))*sigma-4*
  sigma^2+4*(diff(sigma, z))^3+8*(diff(sigma, z))^2*theta[infinity]
  +8*theta[0]*(diff(sigma, z))^2+16*(diff(sigma, z))*theta[0]*theta
  [infinity];
```

$$S4 := \left(\frac{\partial^2}{\partial z^2}\,\sigma\right)^2 - 4z^2\left(\frac{\partial}{\partial z}\,\sigma\right)^2 + 8z\left(\frac{\partial}{\partial z}\,\sigma\right)\sigma - 4\sigma^2 + 4\left(\frac{\partial}{\partial z}\,\sigma\right)^3 + 8\left(\frac{\partial}{\partial z}\,\sigma\right)^2\theta_\infty + 8\,\theta_0\left(\frac{\partial}{\partial z}\,\sigma\right)^2 + 16\left(\frac{\partial}{\partial z}\,\sigma\right)\theta_0\,\theta_\infty \tag{1}$$

```
> n:=2;epsilon:=1;psi:=exp(epsilon*z^2/2)*((C[1]*CylinderD(nu,sqrt
  (2)*z))+C[2]*CylinderD(nu,-sqrt(2)*z));
```

$$n := 2$$

$$\varepsilon := 1$$

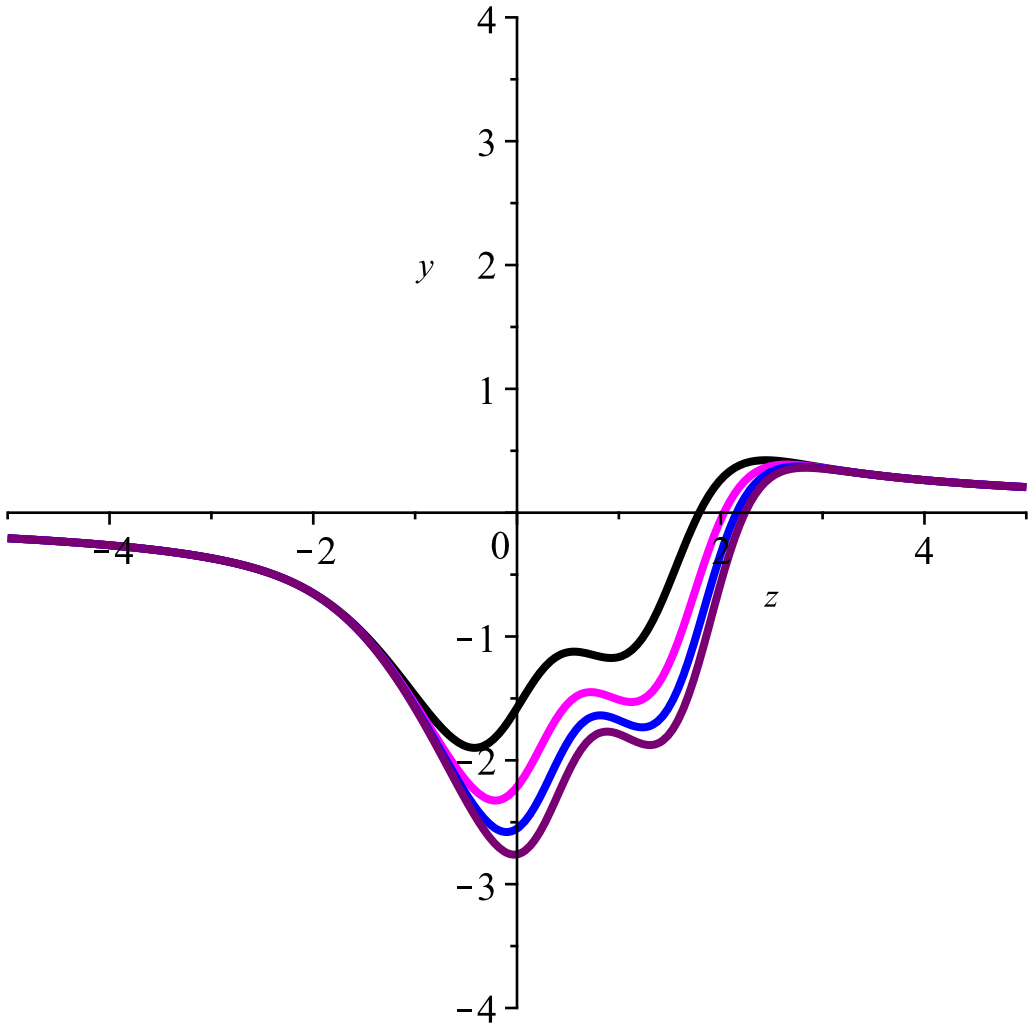
$$\psi := e^{\frac{1}{2}z^2}\left(C_1\,\mathrm{CylinderD}\left(\nu, \sqrt{2}\,z\right) + C_2\,\mathrm{CylinderD}\left(\nu, -\sqrt{2}\,z\right)\right) \tag{2}$$

```
> tau[nu]:=(n)->det(wronskian([psi,seq(diff(psi,z$j),j=1..n-1)],z))
  ;tau[nu-1]:=(n)->det(wronskian([seq(diff(psi,z$j),j=1..n)],z));
```

$$\tau_\nu := n \to linalg{:}\text{-}det\left(linalg{:}\text{-}wronskian\left(\left[\psi, seq\left(\frac{\partial^j}{\partial z^j}\,\psi, j = 1..n-1\right)\right], z\right)\right)$$

$$\tau_{\nu-1} := n \to linalg{:}\text{-}det\left(linalg{:}\text{-}wronskian\left(\left[seq\left(\frac{\partial^j}{\partial z^j}\,\psi, j = 1..n\right)\right], z\right)\right) \tag{3}$$

```
> tau[nu-1](0):=1:tau[nu](0):=1:
> w:=-2*n*z+simplify(expand(diff(ln(tau[nu](n)),z))):
> plot([subs(nu=-3/2,C[2]=1,C[1]=10,w),subs(nu=-3/2,C[2]=1,C[1]=20,
  w),subs(nu=-3/2,C[2]=1,C[1]=30,w),subs(nu=-3/2,C[2]=1,C[1]=40,w)
  ],z=-5..5,y=-4..4,thickness=3,color=[black,magenta,blue,purple]);
```



S4

```
> restart;with(ListTools):with(orthopoly):with(linalg):alias(sigma=
  sigma(z),psi=psi(z),phi=phi(z),w=w(z)):S4 := (diff(sigma, z, z))
  ^2-4*z^2*(diff(sigma, z))^2+8*z*(diff(sigma, z))*sigma-4*
  sigma^2+4*(diff(sigma, z))^3+8*(diff(sigma, z))^2*theta[infinity]
  +8*theta[0]*(diff(sigma, z))^2+16*(diff(sigma, z))*theta[0]*theta
  [infinity];
```

$$S4 := \left(\frac{\partial^2}{\partial z^2}\,\sigma\right)^2 - 4z^2\left(\frac{\partial}{\partial z}\,\sigma\right)^2 + 8z\left(\frac{\partial}{\partial z}\,\sigma\right)\sigma - 4\sigma^2 + 4\left(\frac{\partial}{\partial z}\,\sigma\right)^3 + 8\left(\frac{\partial}{\partial z}\,\sigma\right)^2\theta_\infty + 8\,\theta_0\left(\frac{\partial}{\partial z}\,\sigma\right)^2 + 16\left(\frac{\partial}{\partial z}\,\sigma\right)\theta_0\,\theta_\infty \quad \textbf{(4)}$$

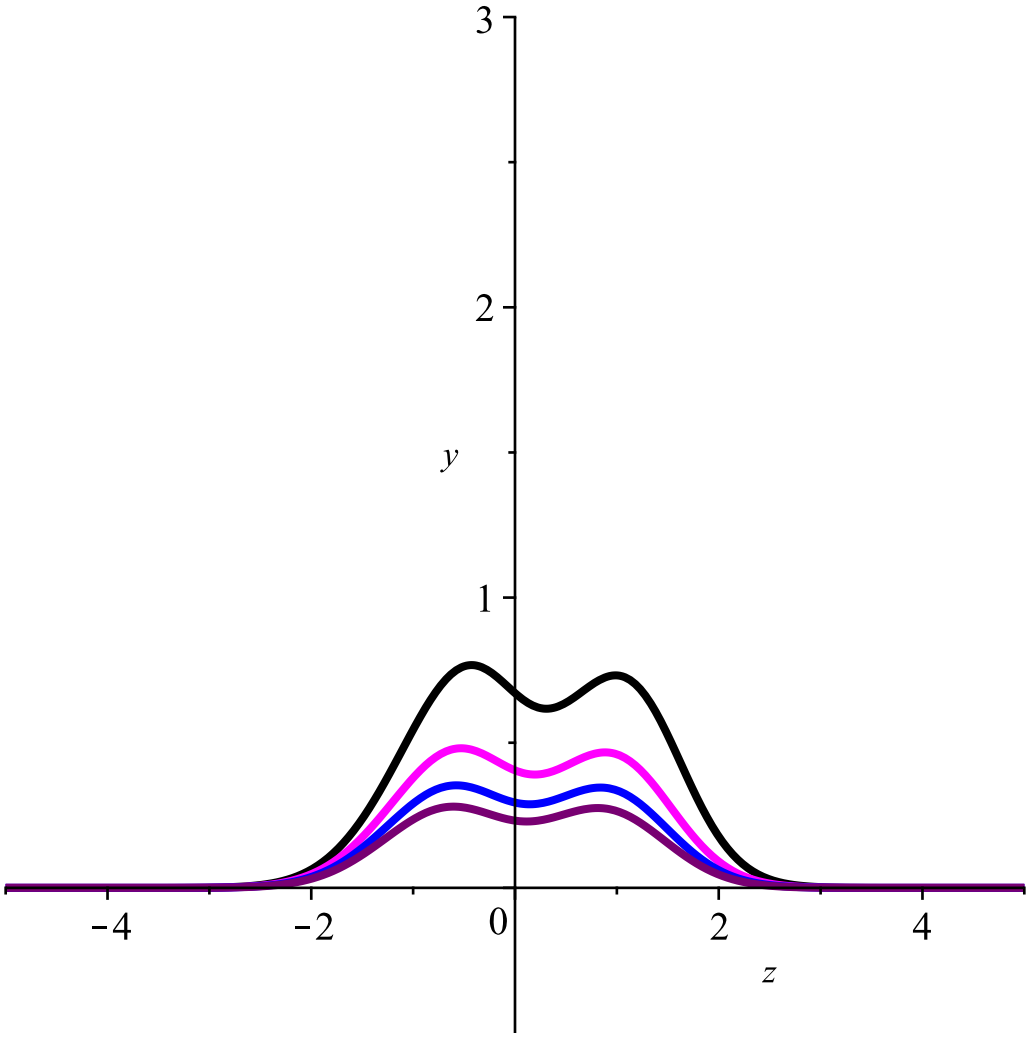
```
> n:=2;
```

$$n := 2 \quad \textbf{(5)}$$

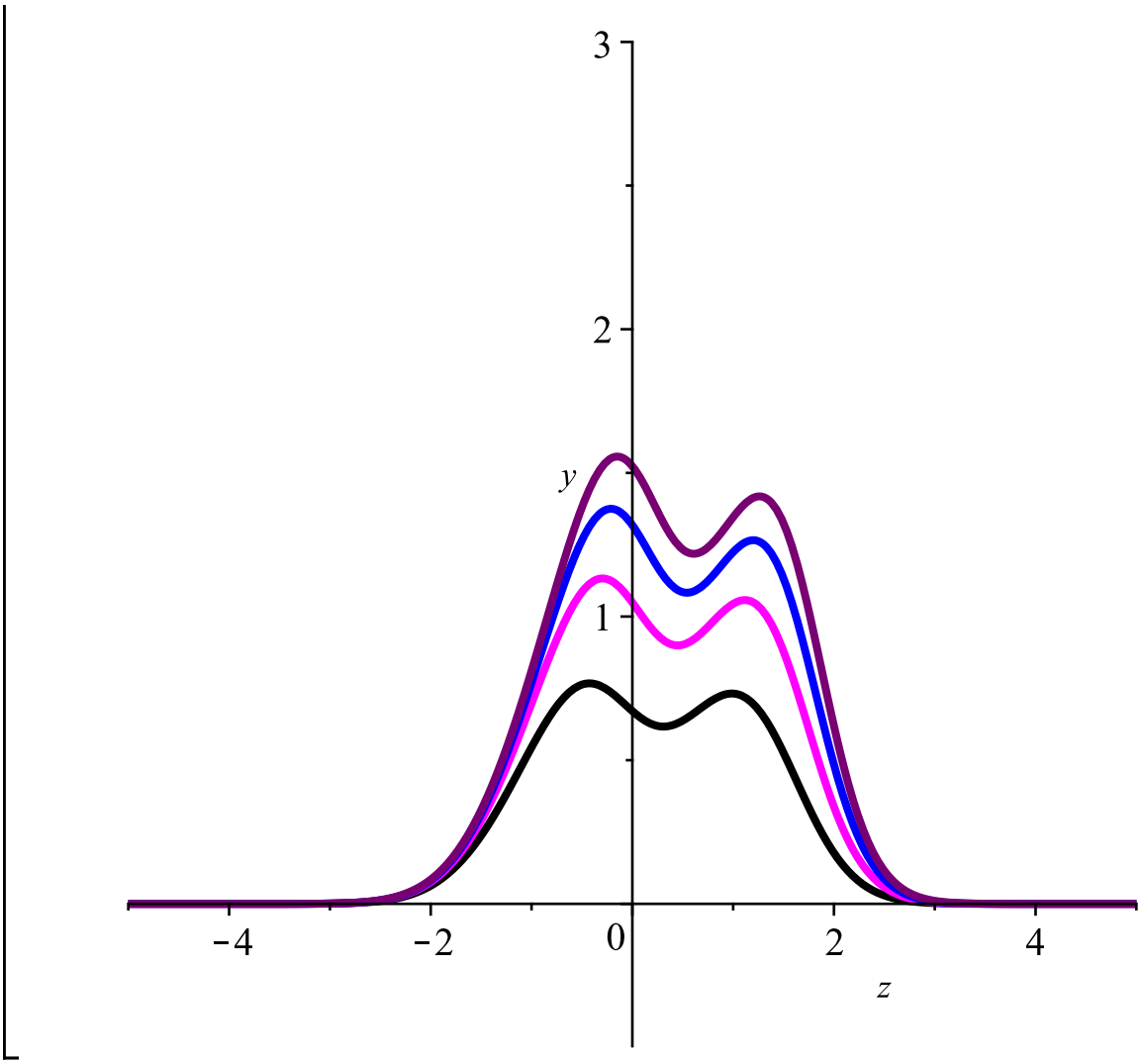
```
> nu:=n-1;epsilon:=1:psi:=simplify(exp(-z^2)*diff((C[1]+C[2]*erfc
  (z))*exp(z^2),z$nu)):
```

$$\nu := 1 \quad \textbf{(6)}$$

```
> tau:=simplify(det(wronskian([psi,seq(diff(psi,z$j),j=1..n-1)],z))
  ):
> sigma1:=simplify(expand(diff(ln(tau),z))):
> plot([subs(C[1]=1,C[2]=1,-sigma1),subs(C[1]=2,C[2]=1,-sigma1),
  subs(C[1]=3,C[2]=1,-sigma1),subs(C[1]=4,C[2]=1,-sigma1)],z=-5..5,
  y=-1/2..3,thickness=3,color=[black,magenta,blue,purple]);
```




```
> plot([subs(C[1]=1,C[2]=1,-sigma1),subs(C[1]=1,C[2]=2,-sigma1),
  subs(C[1]=1,C[2]=3,-sigma1),subs(C[1]=1,C[2]=4,-sigma1)],z=-5..5,
  y=-1/2..3,thickness=3,color=[black,magenta,blue,purple]);
```


3
2
y
1
-4
-2
0
2
4
z