

```
> restart; Digits:=30;
                                Digits := 30 (1)
```

```
> with(PDEtools):with(linalg):with(LinearAlgebra):with(plots):alias
(w=w(z), phi=phi(t), psi=psi(t)):d:=-1/2:epsilon[3]:=1;
                                ε3 := 1 (2)
```

```
> P5:=(diff(w, z, z))-((1/(2*w)+1/(w-1))*(diff(w,z)^2)-1/z*diff(w,
z)+(w-1)^2/z^2*(A*w+B/w)+C*w/z+d*w*(w+1)/(w-1));
P5 := \frac{\partial^2}{\partial z^2} w - \left( \frac{1}{2w} + \frac{1}{w-1} \right) \left( \frac{\partial}{\partial z} w \right)^2 + \frac{\frac{\partial}{\partial z} w}{z} - \frac{(w-1)^2 \left( A w + \frac{B}{w} \right)}{z^2} - \frac{C w}{z}
+ \frac{1}{2} \frac{w(w+1)}{w-1} (3)
```

```
> n:=3;a:=-10;b:=-30;phi:=simplify(KummerU(a,b,z));
                                n := 3
                                a := -10
                                b := -30
φ := z10 + 210 z9 + 20790 z8 + 1275120 z7 + 53555040 z6 + 1606651200 z5 + 34810776000 z4
+ 537080544000 z3 + 5639345712000 z2 + 36342450144000 z + 109027350432000 (4)
```

```
> diff(phi,z):for K from 1 to n+1 do;l[K]:=diff(%,z)*z;od:wronskian
([diff(phi,z),seq(l[k],k=1..n)],z):for K from 1 to n+1 do;h[K]:=
Row(%,1);row(%,2);wronskian(%*z,z):od:simplify(subs(<seq
(simplify(h[k]),k=1..n+1)>)):tau[n+1]:=det(%) :
> phi:for K from 1 to n do;l[K]:=diff(%,z)*z;od:wronskian([phi,seq
(l[k],k=1..n-1)],z):for K from 1 to n do;h[K]:=Row(%,1);row(%,2)
;wronskian(%*z,z):od:simplify(<seq(simplify(h[k]),k=1..n)>):tau
[n]:=det(%) :
> w:=convert(simplify(expand(1+1/(a+n)*(z-b-z*diff(ln((tau[n+1]))/
(tau[n])),z))),parfrac,z):
> A:=plot(w-((z+3*n+2*a+1-b)/(a+n)),z=-100..10,y=-15..15,colour=
black,thickness=3):
> b:=-40;phi:=simplify(KummerU(a,b,z));
                                b := -40
```

```
φ := z10 + 310 z9 + 44640 z8 + 3928320 z7 + 233735040 z6 + 9816871680 z5
+ 294506150400 z4 + 6226701465600 z3 + 88730495884800 z2 + 768997631001600 z
+ 3075990524006400 (5)
```

```
> diff(phi,z):for K from 1 to n+1 do;l[K]:=diff(%,z)*z;od:wronskian
([diff(phi,z),seq(l[k],k=1..n)],z):for K from 1 to n+1 do;h[K]:=
Row(%,1);row(%,2);wronskian(%*z,z):od:simplify(subs(<seq
(simplify(h[k]),k=1..n+1)>)):tau[n+1]:=det(%) :
> phi:for K from 1 to n do;l[K]:=diff(%,z)*z;od:wronskian([phi,seq
(l[k],k=1..n-1)],z):for K from 1 to n do;h[K]:=Row(%,1);row(%,2)
;wronskian(%*z,z):od:simplify(<seq(simplify(h[k]),k=1..n)>):tau
[n]:=det(%) :
> w:=convert(simplify(expand(1+1/(a+n)*(z-b-z*diff(ln((tau[n+1]))/
(tau[n])),z))),parfrac,z):
> B:=plot(w-((z+3*n+2*a+1-b)/(a+n)),z=-100..10,y=-15..15,colour=
magenta,thickness=3):
```

(6)

```

> b:=-50;phi:=simplify(KummerU(a,b,z));
      b:=-50

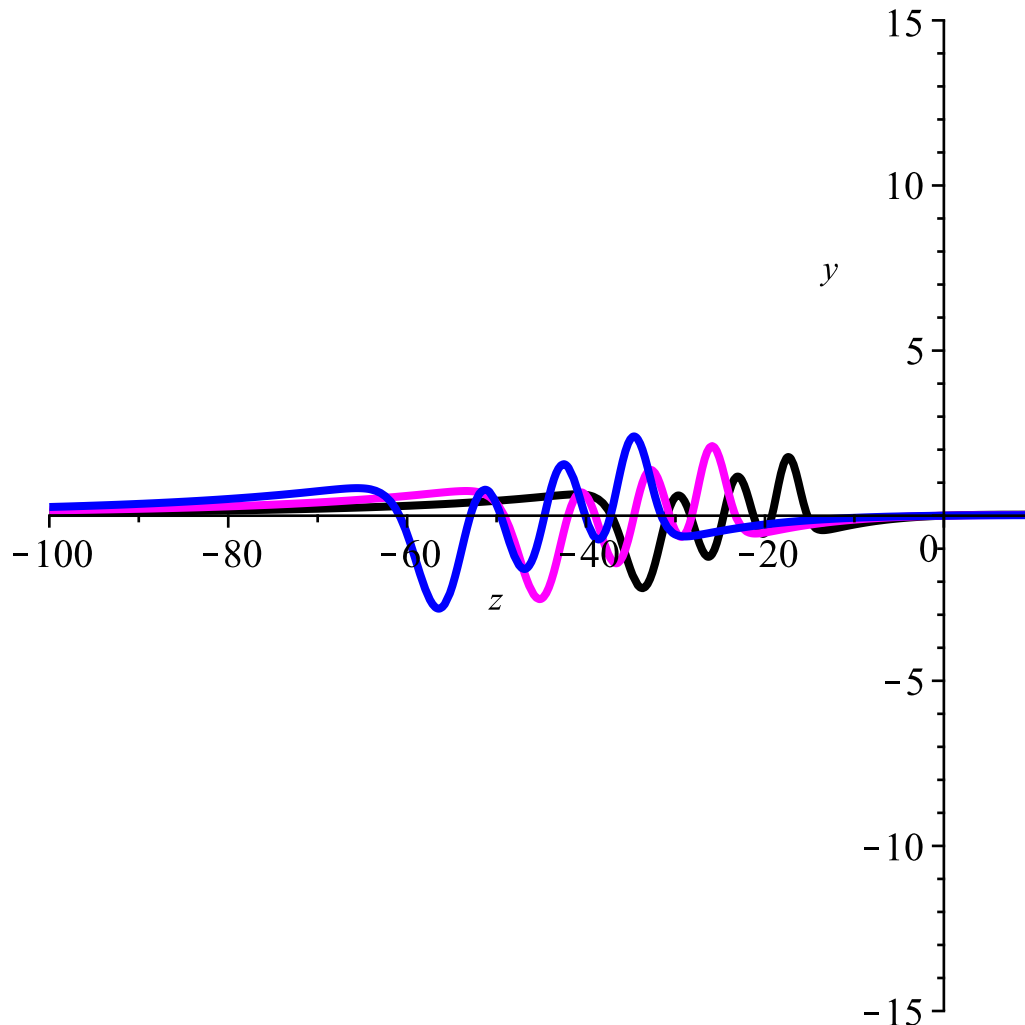
$$\phi := z^{10} + 410z^9 + 77490z^8 + 8885520z^7 + 684185040z^6 + 36945992160z^5$$


$$+ 1416263032800z^4 + 38036778595200z^3 + 684662014713600z^2$$


$$+ 7455208604659200z + 37276043023296000$$

> diff(phi,z):for K from 1 to n+1 do;l[K]:=diff(%,z)*z;od:wronskian
([diff(phi,z),seq(l[k],k=1..n)],z):for K from 1 to n+1 do;h[K]:=
Row(%,1);row(%,2);wronskian(%*z,z):od:simplify(subs(<seq
(simplify(h[k]),k=1..n+1)>)):tau[n+1]:=det(%):
> phi:for K from 1 to n do;l[K]:=diff(%,z)*z;od:wronskian([phi,seq
(l[k],k=1..n-1)],z):for K from 1 to n do;h[K]:=Row(%,1);row(%,2)
;wronskian(%*z,z):od:simplify(<seq(simplify(h[k]),k=1..n)>):tau
[n]:=det(%):
> w:=convert(simplify(expand(1+1/(a+n)*(z-b-z*diff(ln((tau[n+1]))/
(tau[n])),z))),parfrac,z):
> C:=plot(w-((z+3*n+2*a+1-b)/(a+n)),z=-100..10,y=-15..15,colour=
blue,thickness=3):
> display(A,B,C);

```



```

> restart;Digits:=100:with(PDEtools):with(linalg):with
(LinearAlgebra):with(plots):alias(S[n]=S[n](z),sigma=sigma(z),

```

```
phi=phi(z));
```

$$S_n, \sigma, \phi \quad (7)$$

```
> S5:=z^2*(diff(sigma, z, z))^2-(2*(diff(sigma, z))^2-z*(diff
(sigma, z))+sigma)^2+(4*(diff(sigma, z)+k[0]))*(diff(sigma, z)+k
[1])*(diff(sigma, z)+k[2])*(diff(sigma, z)+k[3]);
```

$$S5 := z^2 \left( \frac{\partial^2}{\partial z^2} \sigma \right)^2 - \left( 2 \left( \frac{\partial}{\partial z} \sigma \right)^2 - z \left( \frac{\partial}{\partial z} \sigma \right) + \sigma \right)^2 + 4 \left( \frac{\partial}{\partial z} \sigma + k_0 \right) \left( \frac{\partial}{\partial z} \sigma + k_1 \right) \left( \frac{\partial}{\partial z} \sigma + k_2 \right) \left( \frac{\partial}{\partial z} \sigma + k_3 \right) \quad (8)$$

```
> A:=-5/8*(n+1)^2+1/4*(2*alpha+1+beta+3*z)*(n+1)-1/8*(-2*alpha-1+
beta)*(-2*alpha-1-2*z+beta);
```

$$A := -\frac{5}{8} (n+1)^2 + \frac{1}{4} (2\alpha+1+\beta+3z) (n+1) - \frac{1}{8} (-2\alpha-1+\beta) (-2\alpha-1-2z + \beta) \quad (9)$$

```
> n:=4;alpha:=-7;beta:=20;phi:=simplify(KummerM(alpha,beta,z))
:phi:=simplify(LaguerreL(-alpha,beta-1,z)):
```

$n := 4$

$\alpha := -7$

$\beta := 20$

(10)

```
> phi:=simplify(KummerM(alpha,beta,z)):phi:=simplify(LaguerreL(-
alpha-1,beta,z)):
```

```
> phi:for K from 1 to n do;l[K]:=diff(%,z)*z;od:wronskian([phi,seq
(l[j],j=1..n-1)],z):for j from 1 to n do;h[j]:=Row(%,1);row(%,2)
;wronskian(%*z,z):od:<seq(simplify(h[j]),j=1..n)>:W:=factor(det
(%)):
```

```
> sigma:=convert(simplify(z*diff(ln(W),z))+A,parfrac,z):
```

```
> A:=plot(sigma-A-((3*n-1)*n/2),z=-20..120,y=-40..80,colour=black,
thickness=3):
```

```
> beta:=30;phi:=simplify(KummerM(alpha,beta,z)):phi:=simplify
(LaguerreL(-alpha,beta-1,z)):
```

$\beta := 30$

(11)

```
> phi:=simplify(KummerM(alpha,beta,z)):phi:=simplify(LaguerreL(-
alpha-1,beta,z)):
```

```
> phi:for K from 1 to n do;l[K]:=diff(%,z)*z;od:wronskian([phi,seq
(l[j],j=1..n-1)],z):for j from 1 to n do;h[j]:=Row(%,1);row(%,2)
;wronskian(%*z,z):od:<seq(simplify(h[j]),j=1..n)>:W:=factor(det
(%)):
```

```
> sigma:=convert(simplify(z*diff(ln(W),z))+A,parfrac,z):
```

```
> B:=plot(sigma-A-((3*n-1)*n/2),z=-20..120,y=-40..80,colour=
magenta,thickness=3):
```

```
> beta:=40;phi:=simplify(KummerM(alpha,beta,z)):phi:=simplify
(LaguerreL(-alpha,beta-1,z)):
```

$\beta := 40$

(12)

```
> phi:=simplify(KummerM(alpha,beta,z)):phi:=simplify(LaguerreL(-
alpha-1,beta,z)):
```

```
> phi:for K from 1 to n do;l[K]:=diff(%,z)*z;od:wronskian([phi,seq
(l[j],j=1..n-1)],z):for j from 1 to n do;h[j]:=Row(%,1);row(%,2)
```

```

;wronskian(%*z,z):od:<seq(simplify(h[j]),j=1..n)>:W:=factor(det
(%)):
> sigma:=convert(simplify(z*diff(ln(W),z))+A,parfrac,z):
> C:=plot(sigma-A-((3*n-1)*n/2),z=-20..120,y=-40..80,colour=blue,
thickness=3):
> display(A,B,C);

```

