```
> restart;
> alias(q=q(z),p=p(z),sigma=sigma(z),w=w(z)):
> P2:=diff(w,z,z)-(2*w^3+z*w+alpha)=0;S2:=(diff(sigma,z,z))^2+4*
  diff(sigma,z)^3+2*diff(sigma,z)*(z*diff(sigma,z)-sigma)-1/4*
  (alpha+1/2)^2;
```

$$P2 := \frac{\partial^2}{\partial z^2} w - 2 w^3 - z w - \alpha = 0$$

$$S2 := \left(\frac{\partial^2}{\partial z^2}\sigma\right)^2 + 4\left(\frac{\partial}{\partial z}\sigma\right)^3 + 2\left(\frac{\partial}{\partial z}\sigma\right)\left(z\left(\frac{\partial}{\partial z}\sigma\right) - \sigma\right) - \frac{1}{4}\left(\alpha + \frac{1}{2}\right)^2 \quad (1)$$

```
> H:=p^2/2-(q^2+z/2)*p-(alpha+1/2)*q;
```

$$H := \frac{1}{2}p^2 - \left(q^2 + \frac{1}{2}z\right)p - \left(\alpha + \frac{1}{2}\right)q \quad (2)$$

```
> H1:=diff(q,z)=p-q^2-1/2*z;H2:=diff(p,z)=2*q*p+alpha+1/2;
```

$$H1 := \frac{\partial}{\partial z} q = p - q^2 - \frac{1}{2}z$$

$$H2 := \frac{\partial}{\partial z} p = 2 q p + \alpha + \frac{1}{2} \quad (3)$$

```
> S:=sigma=H;
```

$$S := \sigma = \frac{1}{2}p^2 - \left(q^2 + \frac{1}{2}z\right)p - \left(\alpha + \frac{1}{2}\right)q \quad (4)$$

```
> s1:=simplify(subs(H1,H2,diff(S,z)));s2:=simplify(expand(subs(H1,
  H2,diff(s1,z))));
```

$$s1 := \frac{\partial}{\partial z}\sigma = -\frac{1}{2}p$$

$$s2 := \frac{\partial^2}{\partial z^2}\sigma = -q p - \frac{1}{2}\alpha - \frac{1}{4} \quad (5)$$

```
> solve({s1,s2},{q,p});
```

$$\left\{p = -2\left(\frac{\partial}{\partial z}\sigma\right), q = \frac{1}{8}\,\frac{4\left(\frac{\partial^2}{\partial z^2}\sigma\right) + 2\alpha + 1}{\frac{\partial}{\partial z}\sigma}\right\} \quad (6)$$

```
> collect(expand(sigma-subs(%,H)),diff,factor);collect(simplify(-%*
  2*(diff(sigma, z))),diff,factor);expand(S2-%);
```

$$-\frac{1}{2}\,\frac{\left(\frac{\partial^2}{\partial z^2}\sigma\right)^2}{\frac{\partial}{\partial z}\sigma} - 2\left(\frac{\partial}{\partial z}\sigma\right)^2 - z\left(\frac{\partial}{\partial z}\sigma\right) + \sigma + \frac{1}{32}\,\frac{(2\alpha+1)^2}{\frac{\partial}{\partial z}\sigma}$$

$$\left(\frac{\partial^2}{\partial z^2}\sigma\right)^2 + 4\left(\frac{\partial}{\partial z}\sigma\right)^3 + 2z\left(\frac{\partial}{\partial z}\sigma\right)^2 - 2\sigma\left(\frac{\partial}{\partial z}\sigma\right) - \frac{1}{16}(2\alpha+1)^2$$

$$0 \quad (7)$$