```
> restart;alias(w=w(z),p[0]=p[0](z),p[1]=p[1](z),p[2]=p[2](z))
  :PDEtools[declare](prime = z):with(PolynomialTools):
```

derivatives with respect to z of functions of one variable will now be displayed with ' (1)

```
> R1:=p[2]*w^2+p[1]*w+p[0];
```

$$R1 := p_2 w^2 + p_1 w + p_0 \tag{2}$$

```
> R2:=diff(w,z,z)=collect(subs(iff(w,z)=R1,diff(R1,z)),w,factor);
```

$$R2 := w'' = p_2' w^2 + \left(2 p_2 w' + p_1'\right) w + p_1 w' + p_0' \tag{3}$$

```
> P1:=diff(w,z,z)-(6*w^2+z):collect(subs(diff(w,z)=R1,subs(R2,P1)),
  w,factor);
```

$$2 p_2^2 w^3 + \left(3 p_2 p_1 + p_2' - 6\right) w^2 + \left(2 p_2 p_0 + p_1^2 + p_1'\right) w + p_1 p_0 + p_0' - z \tag{4}$$

```
> P2:=diff(w,z,z)-(2*w^3+z*w+A):collect(subs(diff(w,z)=R1,subs(R2,
  P2)),w,factor);P:={p[2]=epsilon,p[1]=0,p[0]=epsilon*z/2,A=
  epsilon/2};subs(epsilon^2=1,expand(subs(P,%%)));diff(w,z)=subs(P,
  R1);
```

$$2\left(p_2 - 1\right)\left(p_2 + 1\right) w^3 + \left(3 p_2 p_1 + p_2'\right) w^2 + \left(2 p_2 p_0 + p_1^2 + p_1' - z\right) w + p_1 p_0 - A + p_0'$$

$$P := \left\{A = \frac{1}{2}\,\varepsilon, p_0 = \frac{1}{2}\,\varepsilon z, p_1 = 0, p_2 = \varepsilon\right\}$$

$$0$$

$$w' = \varepsilon w^2 + \frac{1}{2}\,\varepsilon z \tag{5}$$

```
> P3:=simplify(diff(w,z)^2/w-diff(w,z)/z+(A*w^2+B)/z+C*w^3+D/w-diff
  (w,z,z)):collect(numer(subs(diff(w,z)=R1,subs(R2,P3))),[w],
  factor);P:={p[2]=epsilon[1]/1,p[1]=(1*A-epsilon[1])/epsilon[1]/z,
  p[0]=epsilon[2]/1,D=-1,C=1,B=epsilon[2]*(2-A/epsilon[1])};subs
  (epsilon[1]^2=1,epsilon[2]^2=1,expand(subs(P,%%)));diff(w,z)=subs
  (P,R1);P3:=simplify(diff(w,z)^2/w-diff(w,z)/z+(A*w^2+B)/z+C*w^3+
  D/w-diff(w,z,z)):collect(numer(subs(diff(w,z)=R1,subs(R2,P3))),
  [w],factor);P:={p[2]=epsilon[1]/2,p[1]=(2*A-epsilon[1])/epsilon
  [1]/z,p[0]=epsilon[2]/2,D=-1/4,C=1/4,B=epsilon[2]*(1-A/epsilon[1]
  )};subs(epsilon[1]^2=1,epsilon[2]^2=1,expand(subs(P,%%)));diff(w,
  z)=subs(P,R1);
```

$$z\left(-p_2^2 + C\right) w^4 + \left(-p_2 p_1 z - p_2' z + A - p_2\right) w^3 + \left(-p_1' z - p_1\right) w^2 + \left(p_1 p_0 z - p_0' z + B - p_0\right) w + z\left(p_0^2 + \mathrm{D}\right)$$

$$P := \left\{B = \varepsilon_2\left(2 - \frac{A}{\varepsilon_1}\right), C = 1, \mathrm{D} = -1, p_0 = \varepsilon_2, p_1 = \frac{A - \varepsilon_1}{\varepsilon_1 z}, p_2 = \varepsilon_1\right\}$$

$$0$$

$$w' = \varepsilon_1 w^2 + \frac{\left(A - \varepsilon_1\right) w}{\varepsilon_1 z} + \varepsilon_2$$

$$z\left(-p_2^2 + C\right) w^4 + \left(-p_2 p_1 z - p_2' z + A - p_2\right) w^3 + \left(-p_1' z - p_1\right) w^2 + \left(p_1 p_0 z - p_0' z + B\right.$$

$$-p_0\Big)\,w+z\left(p_0^2+\mathrm{D}\right)$$

$$P:=\left\{B=\varepsilon_2\left(1-\frac{A}{\varepsilon_1}\right),\ C=\frac{1}{4},\ \mathrm{D}=-\frac{1}{4},\ p_0=\frac{1}{2}\,\varepsilon_2,\ p_1=\frac{2A-\varepsilon_1}{\varepsilon_1 z},\ p_2=\frac{1}{2}\,\varepsilon_1\right\}$$

$$0$$

$$w'=\frac{1}{2}\,\varepsilon_1 w^2+\frac{\left(2A-\varepsilon_1\right)w}{\varepsilon_1 z}+\frac{1}{2}\,\varepsilon_2 \tag{6}$$

```
> P3dash:=simplify(diff(w,z,z)-diff(w,z)^2/w/z+1/z*diff(w,z)-A*
  w^2/2/z^2-B/2/z-w^3/z^2+1/w):collect(numer(subs(diff(w,z)=R1,subs
  (R2,P3))),[w],factor);P:={C=1,D=-1,p[2]=epsilon[1],p[0]=epsilon
  [2],p[1]=nu/z,A=epsilon[1]*(nu+1),B=epsilon[2]*(1-nu)};collect
  (subs(epsilon[1]^2=1,epsilon[2]^2=1,expand(subs(P,%%))),w,factor)
  ;diff(w,z)=subs(P,R1);
```

$$z\left(-p_2^2+C\right)w^4+\left(-p_2p_1z-p_2'z+A-p_2\right)w^3+\left(-p_1'z-p_1\right)w^2+\left(p_1p_0z-p_0'z+B\right.$$

$$-p_0\Big)\,w+z\left(p_0^2+\mathrm{D}\right)$$

$$P:=\left\{A=\varepsilon_1\left(\nu+1\right),\ B=\varepsilon_2\left(1-\nu\right),\ C=1,\ \mathrm{D}=-1,\ p_0=\varepsilon_2,\ p_1=\frac{\nu}{z},\ p_2=\varepsilon_1\right\}$$

$$0$$

$$w'=\varepsilon_1 w^2+\frac{\nu\,w}{z}+\varepsilon_2 \tag{7}$$

```
> P4:=diff(w,z,z)-(diff(w,z)^2/2/w+3/2*w^3+4*z*w^2+2*(z^2-A)*w+B/w)
  :collect(numer(subs(diff(w,z)=R1,subs(R2,P4))),[w],factor);P:={p
  [2]=epsilon,p[1]=2*epsilon*z,p[0]=2*nu,A=-epsilon*(nu+1),B=-2*
  nu^2};subs(epsilon^2=1,expand(subs(P,%%)));diff(w,z)=subs(P,R1);
```

$$3\left(p_2-1\right)\left(p_2+1\right)w^4+\left(4p_2p_1+2p_2'-8z\right)w^3+\left(2p_2p_0+p_1^2-4z^2+4A+2p_1'\right)w^2$$

$$+2p_0'w-p_0^2-2B$$

$$P:=\left\{A=-\varepsilon\left(\nu+1\right),\ B=-2\nu^2,\ p_0=2\nu,\ p_1=2\varepsilon z,\ p_2=\varepsilon\right\}$$

$$0$$

$$w'=\varepsilon w^2+2\varepsilon z w+2\nu \tag{8}$$

```
> P5:=(diff(w, z, z))-((1/(2*w)+1/(w-1))*(diff(w,z)^2)-1/z*diff(w,
  z)+(w-1)^2/z^2*(A*w+B/w)+C*w/z+d*w*(w+1)/(w-1)):collect(numer
  (subs(diff(w,z)=R1,subs(R2,P5))),[w],factor);P:={p[2]=a/z,p[1]=
  (b-a+epsilon[3]*z)/z,p[0]=-b/z,A=a^2/2,B=-b^2/2,d=-1/2,C=epsilon
  [3]*(1-a-b)};collect(subs(epsilon[1]^2=1,epsilon[2]^2=1,epsilon
  [3]^2=1,expand(subs(P,%%))),[w,z],factor);diff(w,z)=subs(P,R1);
```

$$\left(p_2^2z^2-2A\right)w^5+\left(-3p_2^2z^2+2p_2'z^2+2p_2z+6A\right)w^4+\left(-4p_2p_1z^2-2p_2p_0z^2-p_1^2z^2\right.$$

$$-2p_2'z^2+2p_1'z^2-2dz^2-2Cz-2p_2z+2p_1z-6A-2B\Big)w^3+\left(-2p_2p_0z^2-p_1^2z^2\right.$$

$$-4p_1p_0z^2-2p_1'z^2+2p_0'z^2-2dz^2+2Cz-2p_1z+2p_0z+2A+6B\Big)w^2+\left(-3\right.$$

$$\left.p_0^2z^2-2p_0'z^2-2p_0z-6B\right)w+p_0^2z^2+2B$$

$$P := \left\{A = \frac{1}{2}\,a^2, B = -\frac{1}{2}\,b^2, C = \varepsilon_3\,(1-a-b), d = -\frac{1}{2}, p_0 = -\frac{b}{z}, p_1 = \frac{z\,\varepsilon_3 - a + b}{z}, p_2 = \frac{a}{z}\right\}$$

$$0$$

$$w' = \frac{a\,w^2}{z} + \frac{\left(z\,\varepsilon_3 - a + b\right)\,w}{z} - \frac{b}{z} \tag{9}$$

```
> P6:=diff(w,z,z)-1/2*(1/w+1/(w-1)+1/(w-z))*diff(w,z)^2+(1/z+1/
  (z-1)+1/(w-z))*diff(w,z)-w*(w-1)*(w-z)/z^2/(z-1)^2*(A+B*z/w^2+C*
  (z-1)/(w-1)^2+D*z*(z-1)/(w-z)^2):collect(numer(subs(diff(w,z)=R1,
  subs(R2,P6))),[w],factor):P:={p[2]=a/(z*(z-1)),p[1]=((b+1-a)*z-c)
  /(z*(z-1)),p[0]=(c-b-1)/(z-1),A=a^2/2,B=-(b+1-c)^2/2,C=(c-a)^2/2,
  D=1/2*(1-b^2)};collect(expand(subs(P,%%)),[w,z],factor);diff(w,z)
  =subs(P,R1);
```

$$P := \left\{A = \frac{1}{2}\,a^2, B = -\frac{1}{2}\,(b+1-c)^2, C = \frac{1}{2}\,(c-a)^2, \mathrm{D} = -\frac{1}{2}\,b^2 + \frac{1}{2}, p_0 = \frac{c-b-1}{z-1}, p_1 = \frac{(b+1-a)\,z-c}{z\,(z-1)}, p_2 = \frac{a}{z\,(z-1)}\right\}$$

$$0$$

$$w' = \frac{a\,w^2}{z\,(z-1)} + \frac{((b+1-a)\,z-c)\,w}{z\,(z-1)} + \frac{c-b-1}{z-1} \tag{10}$$

DLMF Way

```
> P6:=diff(w,z,z)-1/2*(1/w+1/(w-1)+1/(w-z))*diff(w,z)^2+(1/z+1/
  (z-1)+1/(w-z))*diff(w,z)-w*(w-1)*(w-z)/z^2/(z-1)^2*(A+B*z/w^2+C*
  (z-1)/(w-1)^2+D*z*(z-1)/(w-z)^2):collect(numer(subs(diff(w,z)=R1,
  subs(R2,P6))),[w],factor):P:={p[2]=a/(z*(z-1)),p[1]=((b+c)*z-a-c)
  /(z*(z-1)),p[0]=-b/(z-1),A=a^2/2,B=-b^2/2,C=c^2/2,D=-(1/2*(c+b+a)
  )*(c+b-2+a)};collect(expand(subs(P,%%)),[w,z],factor);diff(w,z)=
  subs(P,R1);
```

$$P := \left\{A = \frac{1}{2}\,a^2, B = -\frac{1}{2}\,b^2, C = \frac{1}{2}\,c^2, \mathrm{D} = -\frac{1}{2}\,(c+b+a)\,(c+b-2+a), p_0 = -\frac{b}{z-1}, p_1 = \frac{(b+c)\,z-a-c}{z\,(z-1)}, p_2 = \frac{a}{z\,(z-1)}\right\}$$

$$0$$

$$w' = \frac{a\,w^2}{z\,(z-1)} + \frac{((b+c)\,z-a-c)\,w}{z\,(z-1)} - \frac{b}{z-1} \tag{11}$$