```
> restart;alias(y=y(z),s=s(z),C=C(z),q=q(z)):
> eq:=1/2*diff(y,z,z)-(z^2/2+nu-1/2)*y;
```

$$eq := \frac{1}{2}\,\frac{\partial^2}{\partial z^2}\,y - \left(\frac{1}{2}\,z^2 + \nu - \frac{1}{2}\right) y \tag{1}$$

```
> expand(subs(y=exp(s),eq)/exp(s)):
> dsolve((1/2)*(diff(s, z))^2-(1/2)*z^2):
> s:=(1/2)*z^2;
```

$$s := \frac{1}{2}\,z^2 \tag{2}$$

```
> expand(subs(y=exp((1/2)*z^2+C),eq)/exp((1/2)*z^2+C)):
> dsolve(z*(diff(C, z))-nu+1):
> C:=(nu-1)*ln(z);
```

$$C := (\nu - 1)\,\ln(z) \tag{3}$$

```
> expand(subs(y=exp((1/2)*z^2+C)+a*exp((1/2)*z^2+(nu-3)*ln(z)),eq)*
  z^5/(exp((1/2)*z^2)*z^nu)):collect(factor(%),z):
> a:=factor(solve(coeff(%,z^2),a));
```

$$a := \frac{1}{4}\,(\nu - 1)\,(\nu - 2) \tag{4}$$

```
> expand(subs(y=exp((1/2)*z^2+C)+a*exp((1/2)*z^2+(nu-3)*ln(z))+b*
  exp((1/2)*z^2+(nu-5)*ln(z)),eq)*z^7/(exp((1/2)*z^2)*z^nu))
  :collect(factor(%),z):
> b:=factor(solve(coeff(%,z^2),b));
```

$$b := \frac{1}{32}\,(\nu - 1)\,(\nu - 2)\,(\nu - 3)\,(\nu - 4) \tag{5}$$

```
> expand(subs(y=exp((1/2)*z^2+C)+a*exp((1/2)*z^2+(nu-3)*ln(z))+b*
  exp((1/2)*z^2+(nu-5)*ln(z))+c*exp((1/2)*z^2+(nu-7)*ln(z)),eq)*
  z^9/(exp((1/2)*z^2)*z^nu)):collect(factor(%),z):
> c:=factor(solve(coeff(%,z^2),c));
```

$$c := \frac{1}{384}\,(\nu - 1)\,(\nu - 2)\,(\nu - 3)\,(\nu - 4)\,(\nu - 5)\,(\nu - 6) \tag{6}$$

```
> expand(subs(y=exp((1/2)*z^2+C)+a*exp((1/2)*z^2+(nu-3)*ln(z))+b*
  exp((1/2)*z^2+(nu-5)*ln(z))+c*exp((1/2)*z^2+(nu-7)*ln(z))+d*exp(
  (1/2)*z^2+(nu-9)*ln(z)),eq)*z^11/(exp((1/2)*z^2)*z^nu)):collect
  (factor(%),z):
> d:=factor(solve(coeff(%,z^2),d));
```

$$d := \frac{1}{6144}\,(\nu - 1)\,(\nu - 2)\,(\nu - 3)\,(\nu - 4)\,(\nu - 5)\,(\nu - 6)\,(\nu - 7)\,(\nu - 8) \tag{7}$$

```
> expand(subs(y=exp((1/2)*z^2+C)+a*exp((1/2)*z^2+(nu-3)*ln(z))+b*
  exp((1/2)*z^2+(nu-5)*ln(z))+c*exp((1/2)*z^2+(nu-7)*ln(z))+d*exp(
  (1/2)*z^2+(nu-9)*ln(z))+e*exp((1/2)*z^2+(nu-11)*ln(z)),eq)*z^13/
  (exp((1/2)*z^2)*z^nu)):collect(factor(%),z):
> e:=factor(solve(coeff(%,z^2),e));
```

$$e := \frac{1}{122880}\,(\nu - 1)\,(\nu - 2)\,(\nu - 3)\,(\nu - 4)\,(\nu - 5)\,(\nu - 6)\,(\nu - 7)\,(\nu - 8)\,(\nu - 9)\,(\nu - 10) \tag{8}$$

```
> expand(subs(y=exp((1/2)*z^2+C)+a*exp((1/2)*z^2+(nu-3)*ln(z))+b*
  exp((1/2)*z^2+(nu-5)*ln(z))+c*exp((1/2)*z^2+(nu-7)*ln(z))+d*exp(
  (1/2)*z^2+(nu-9)*ln(z))+e*exp((1/2)*z^2+(nu-11)*ln(z))+f*exp(
```

```
(1/2)*z^2+(nu-13)*ln(z)),eq)*z^15/(exp((1/2)*z^2)*z^nu)):collect
(factor(%),z):
> f:=factor(solve(coeff(%,z^2),f));
```

$$f := \frac{1}{2949120}(\nu-1)(\nu-2)(\nu-3)(\nu-4)(\nu-5)(\nu-6)(\nu-7)(\nu-8)(\nu-9)(\nu-10)(\nu-11)(\nu-12) \quad (9)$$

```
> expand(subs(y=exp((1/2)*z^2+C)+a*exp((1/2)*z^2+(nu-3)*ln(z))+b*
exp((1/2)*z^2+(nu-5)*ln(z))+c*exp((1/2)*z^2+(nu-7)*ln(z))+d*exp(
(1/2)*z^2+(nu-9)*ln(z))+e*exp((1/2)*z^2+(nu-11)*ln(z))+f*exp(
(1/2)*z^2+(nu-13)*ln(z))+g*exp((1/2)*z^2+(nu-15)*ln(z)),eq)*z^17/
(exp((1/2)*z^2)*z^nu)):collect(factor(%),z):
> g:=factor(solve(coeff(%,z^2),g));
```

$$g := \frac{1}{82575360}(\nu-1)(\nu-2)(\nu-3)(\nu-4)(\nu-5)(\nu-6)(\nu-7)(\nu-8)(\nu-9)(\nu-10)(\nu-11)(\nu-12)(\nu-13)(\nu-14) \quad (10)$$

```
> expand(subs(y=exp((1/2)*z^2+C)+a*exp((1/2)*z^2+(nu-3)*ln(z))+b*
exp((1/2)*z^2+(nu-5)*ln(z))+c*exp((1/2)*z^2+(nu-7)*ln(z))+d*exp(
(1/2)*z^2+(nu-9)*ln(z))+e*exp((1/2)*z^2+(nu-11)*ln(z))+f*exp(
(1/2)*z^2+(nu-13)*ln(z))+g*exp((1/2)*z^2+(nu-15)*ln(z))+h*exp(
(1/2)*z^2+(nu-17)*ln(z)),eq)*z^19/(exp((1/2)*z^2)*z^nu)):collect
(factor(%),z):
> h:=factor(solve(coeff(%,z^2),h));
```

$$h := \frac{1}{2642411520}(\nu-1)(\nu-2)(\nu-3)(\nu-4)(\nu-5)(\nu-6)(\nu-7)(\nu-8)(\nu-9)(\nu-10)(\nu-11)(\nu-12)(\nu-13)(\nu-14)(\nu-15)(\nu-16) \quad (11)$$

```
> expand(subs(y=exp((1/2)*z^2+C)+a*exp((1/2)*z^2+(nu-3)*ln(z))+b*
exp((1/2)*z^2+(nu-5)*ln(z))+c*exp((1/2)*z^2+(nu-7)*ln(z))+d*exp(
(1/2)*z^2+(nu-9)*ln(z))+e*exp((1/2)*z^2+(nu-11)*ln(z))+f*exp(
(1/2)*z^2+(nu-13)*ln(z))+g*exp((1/2)*z^2+(nu-15)*ln(z))+h*exp(
(1/2)*z^2+(nu-17)*ln(z))+j*exp((1/2)*z^2+(nu-19)*ln(z)),eq)*z^21/
(exp((1/2)*z^2)*z^nu)):collect(factor(%),z):
> j:=factor(solve(coeff(%,z^2),j));
```

$$j := \frac{1}{95126814720}(\nu-1)(\nu-2)(\nu-3)(\nu-4)(\nu-5)(\nu-6)(\nu-7)(\nu-8)(\nu-9)(\nu-10)(\nu-11)(\nu-12)(\nu-13)(\nu-14)(\nu-15)(\nu-16)(\nu-17)(\nu-18) \quad (12)$$

```
> 1,lcoeff(a),lcoeff(b),lcoeff(c),lcoeff(d),lcoeff(e),lcoeff(f),
lcoeff(h),lcoeff(j);
```

$$1, \frac{1}{4}, \frac{1}{32}, \frac{1}{384}, \frac{1}{6144}, \frac{1}{122880}, \frac{1}{2949120}, \frac{1}{2642411520}, \frac{1}{95126814720} \quad (13)$$

```
> seq(1/(n!*4^n),n=0..15);
```

$$1, \frac{1}{4}, \frac{1}{32}, \frac{1}{384}, \frac{1}{6144}, \frac{1}{122880}, \frac{1}{2949120}, \frac{1}{82575360}, \frac{1}{2642411520}, \frac{1}{95126814720}, \frac{1}{3805072588800}, \frac{1}{167423193907200}, \frac{1}{8036313307545600}, \frac{1}{417888291992371200}, \frac{1}{23401744351572787200}, \frac{1}{1404104661094367232000} \quad (14)$$

```
> p:=(n)->product((nu-2*k+1)*(nu-2*k),k=1..n):psi:=(nu,n)->z^(nu-1)
  *exp(z^2)*(1+sum((p(k))/(4^k*k!*z^(2*k)),k=1..n));
```

$$\psi := (\nu, n) \to z^{\nu - 1}\, e^{z^2} \left(1 + \sum_{k=1}^{n} \frac{p(k)}{4^k\, k!\, z^{2k}} \right) \tag{15}$$

```
> psi2:=(nu,n)->z^(nu-1)*exp(z^2)*(1+sum((1/(z^(2*k)*k!*(GAMMA(1/2-
  (1/2)*nu)*GAMMA(1-(1/2)*nu)))*GAMMA((2*k+1)/2-(1/2)*nu)*GAMMA(
  (k+1)-(1/2)*nu),k=1..n)));
```

$$\psi 2 := (\nu, n) \to z^{\nu - 1}\, e^{z^2} \left(1 + \sum_{k=1}^{n} \frac{\Gamma\left(k + \frac{1}{2} - \frac{1}{2}\,\nu\right) \Gamma\left(k + 1 - \frac{1}{2}\,\nu\right)}{z^{2k}\, k!\, \Gamma\left(\frac{1}{2} - \frac{1}{2}\,\nu\right) \Gamma\left(1 - \frac{1}{2}\,\nu\right)} \right) \tag{16}$$