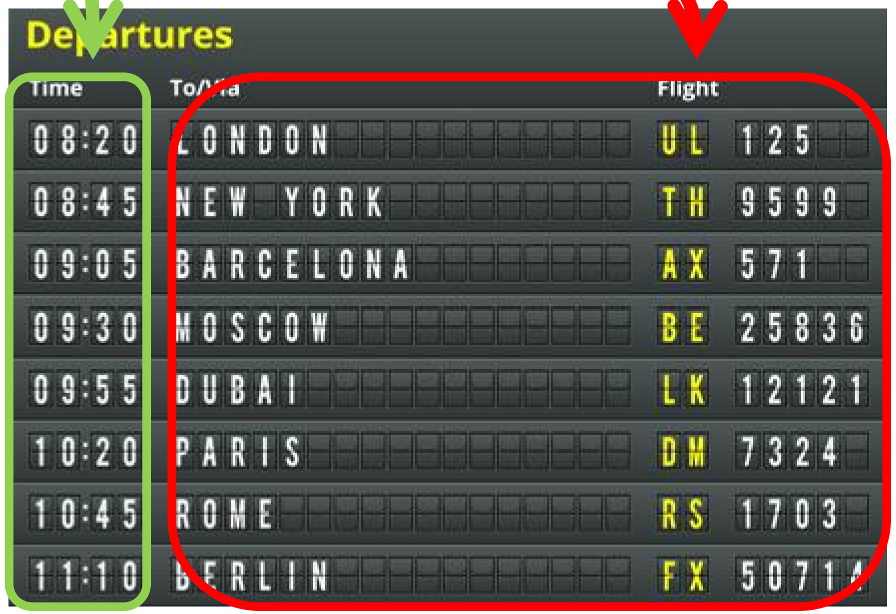
Trees

Problem: Pokemon Database

- One record of data:
  - Key
  - Data
- Dynamic collections
  - Can add or remove any one at anytime
- Can query the database
  - Find a particular record by the key
    - E.g. what is the flight at 09:05?
    - Or check the existence
      - Is there any flight between 09:00 to 09:20?
  - Find the one before or after
    - successor or predecessor



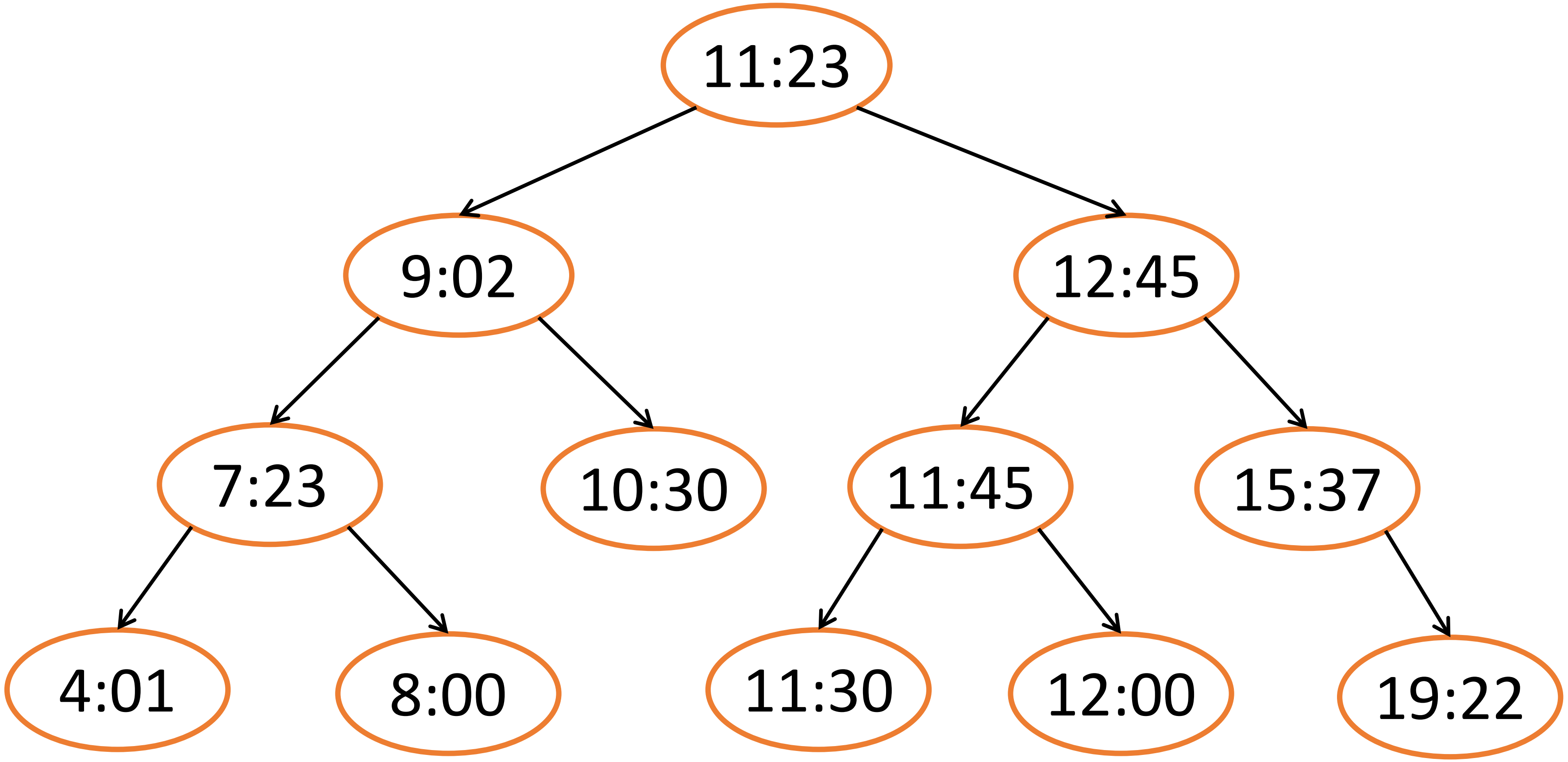
Dictionary ADT

`void`	`insert(Key k, Value v)`	*insert (k,v) into table*
`Value`	`search(Key k)`	*get value paired with k*
`Key`	**`successor(Key k)`**	*find next key > k*
`Key`	**`predecessor(Key k)`**	*find next key < k*
`void`	`delete(Key k)`	*remove key k (and value)*
`boolean`	`contains(Key k)`	*is there a value for k?*
`int`	`size()`	*number of (k,v) pairs*

Dictionary Implementation

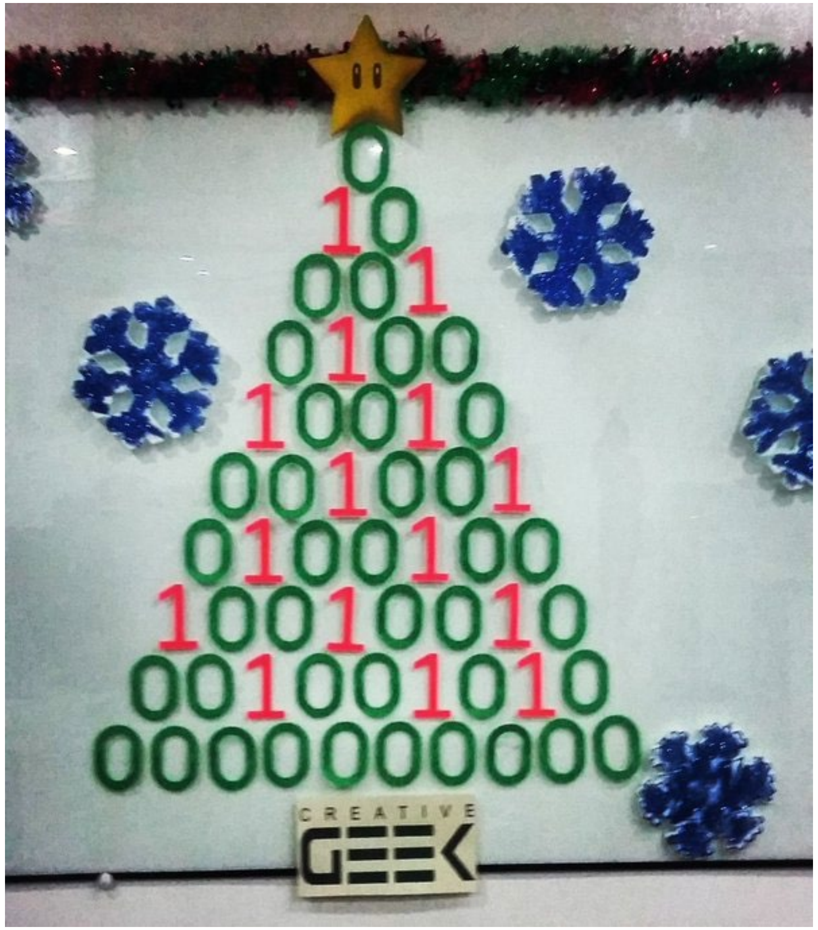
- Option 1: Sorted array
  - insert: add to middle of array --- O(n)
  - search : binary search through array --- O(log n)
- Option 2: Linked list
  - insert: add to middle of array --- O(n)
  - search : no binary search in array --- O(n)

Tree!

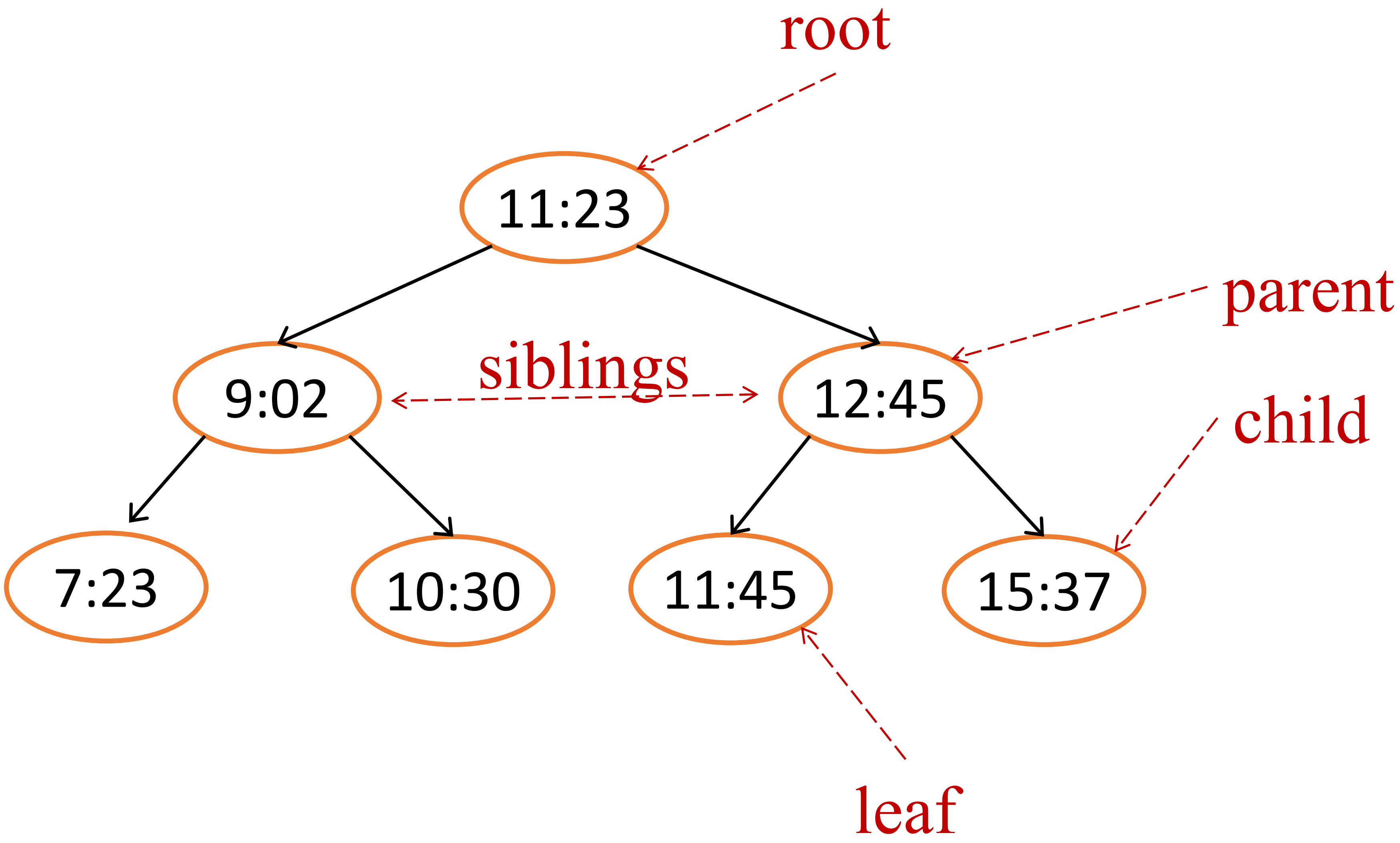


There are many types of trees

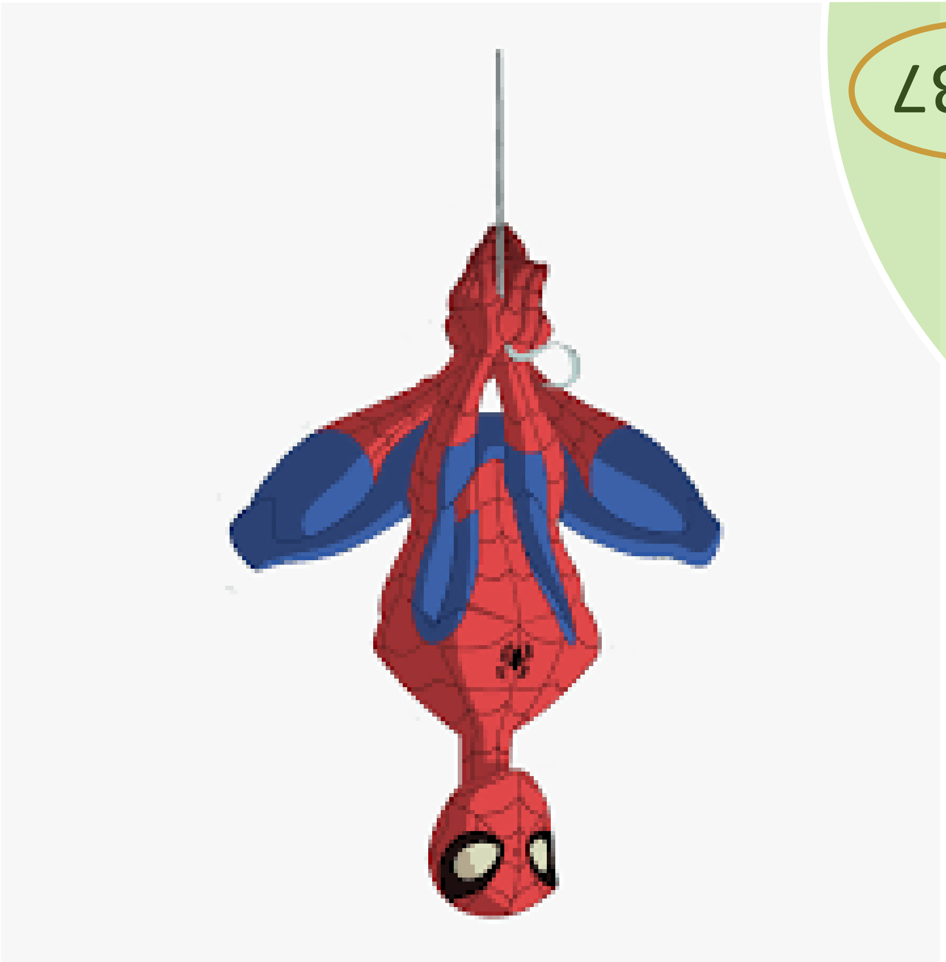
- Binary Tree

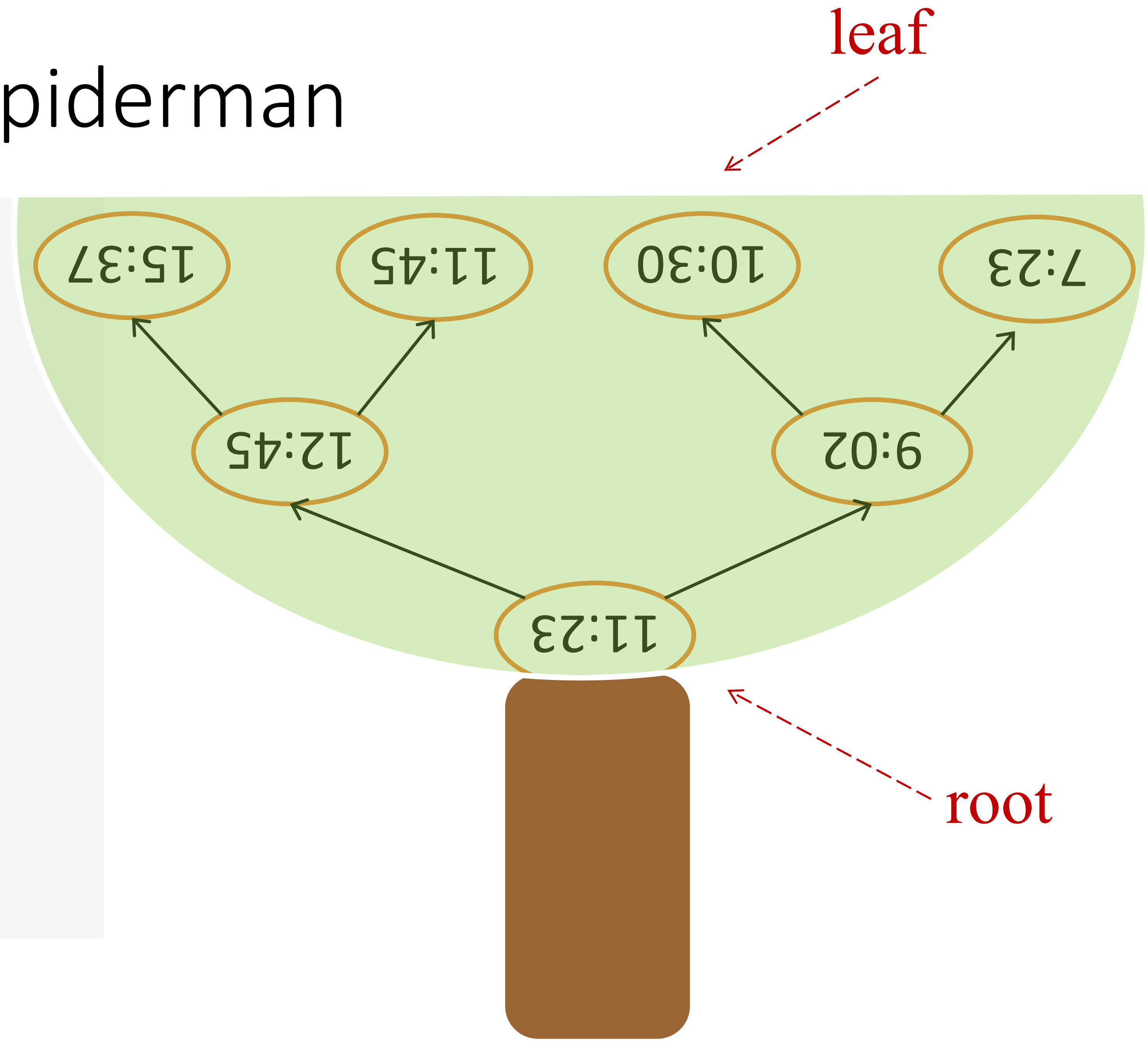


Terminology



Look at it like Spiderman





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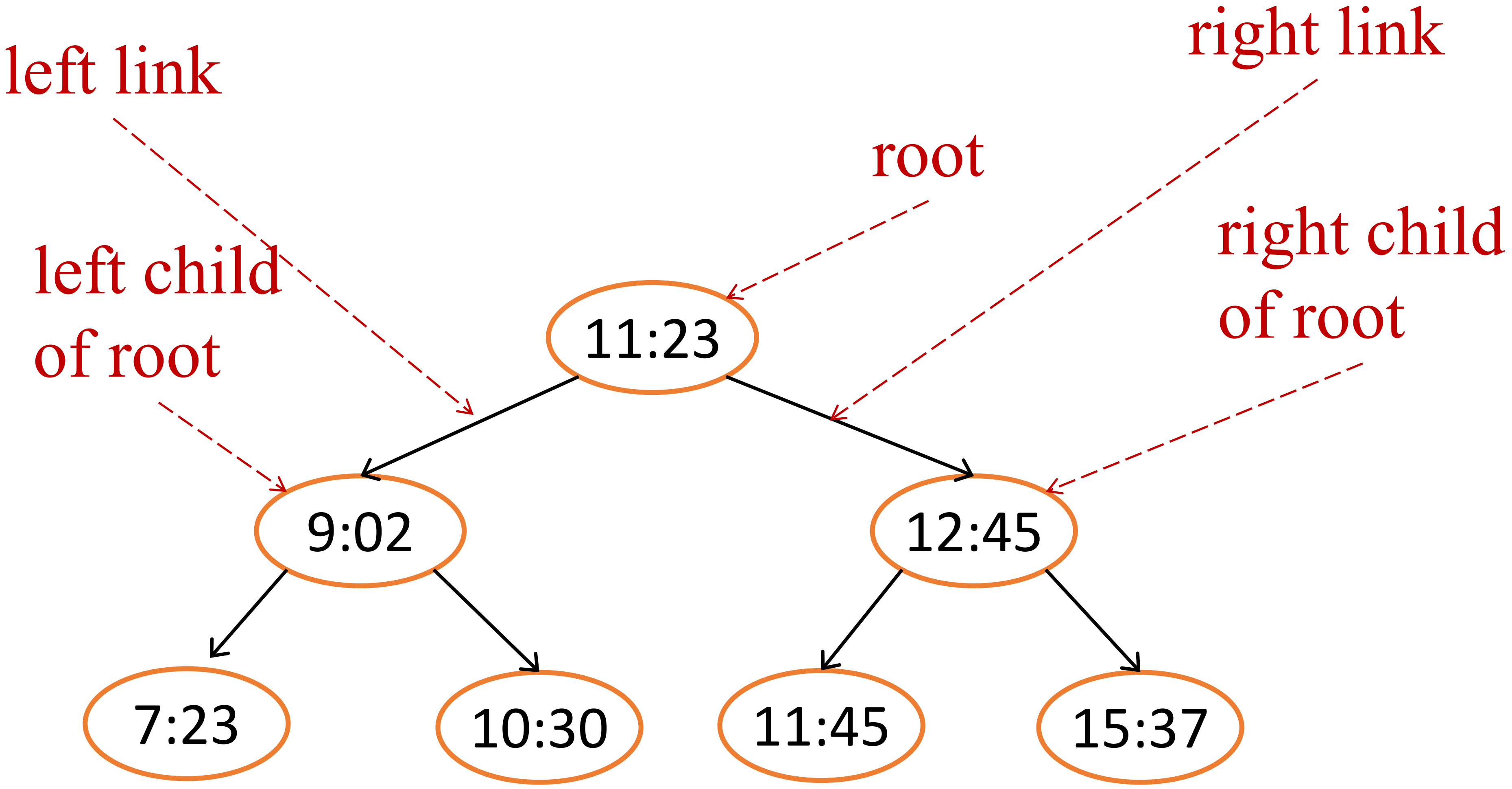

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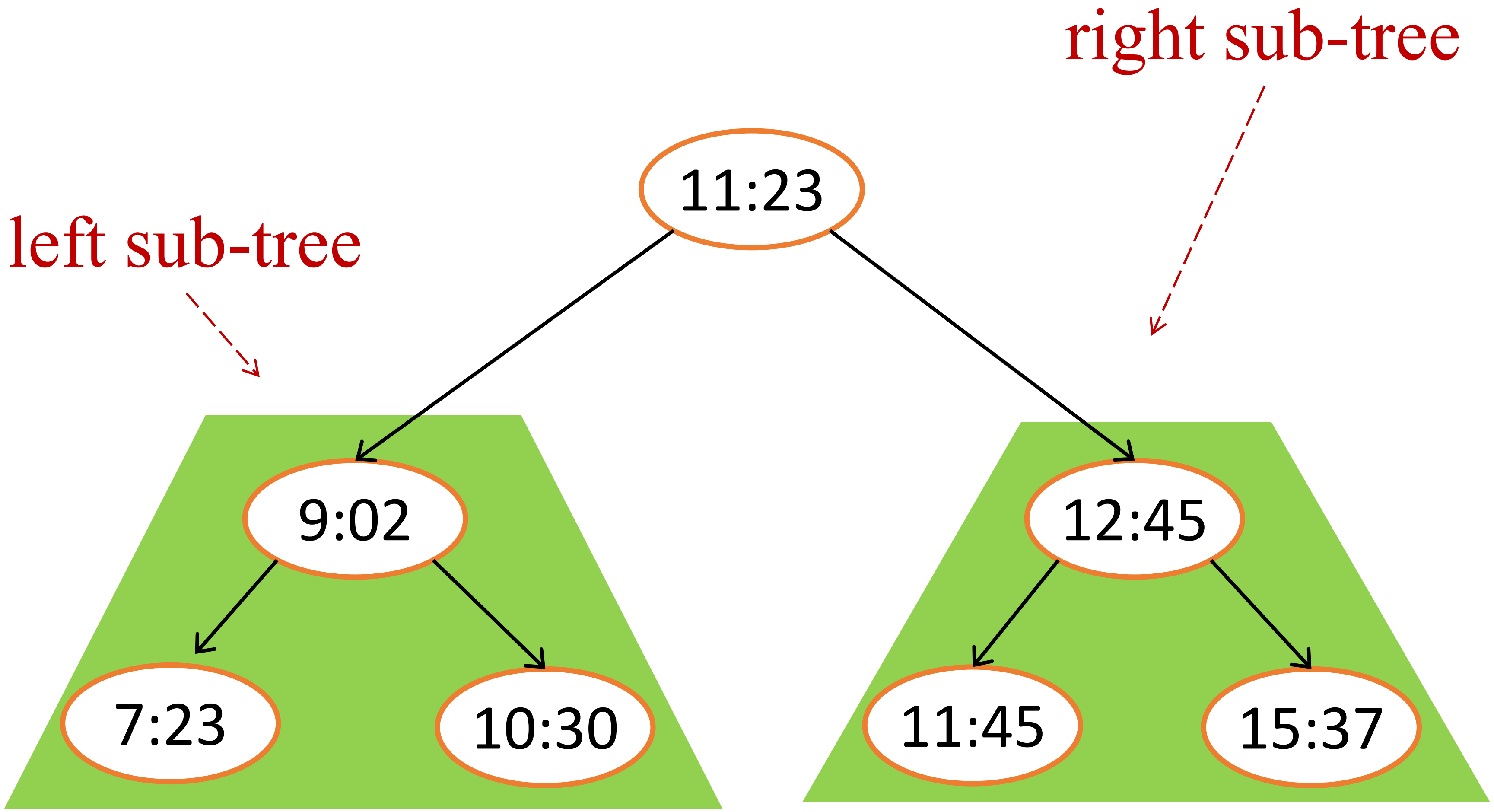
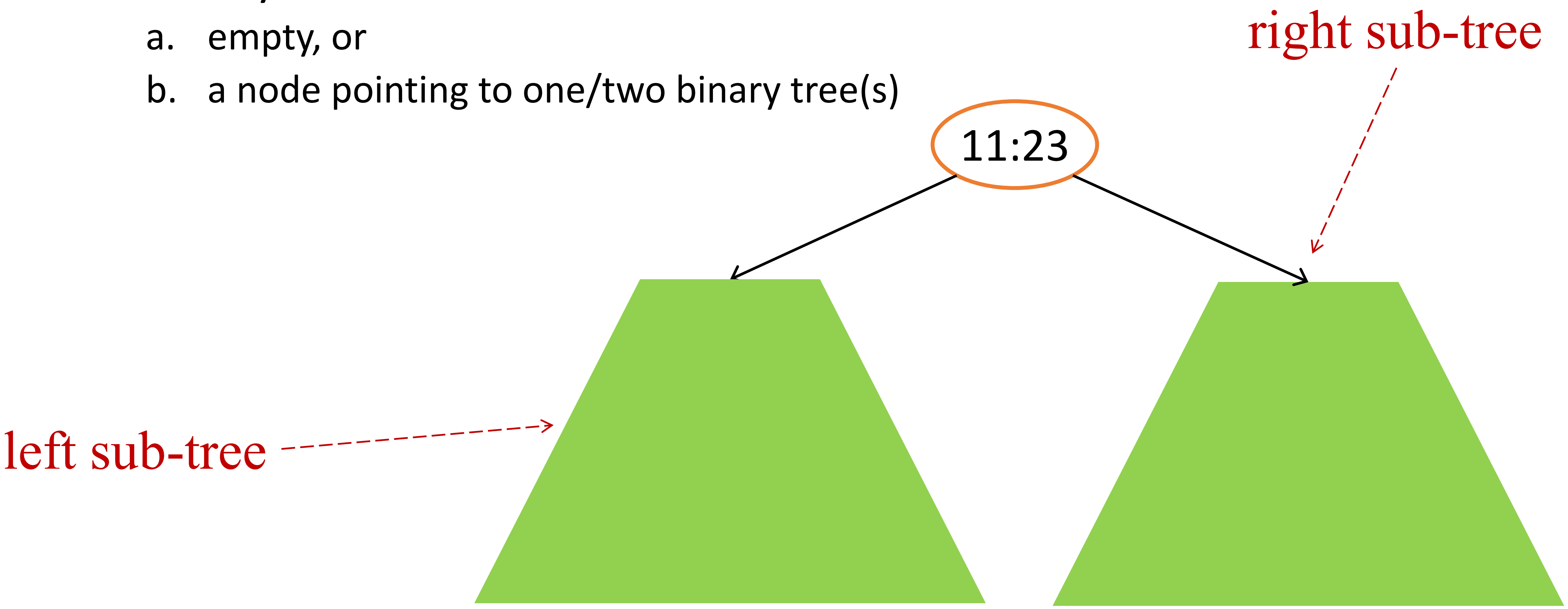


Binary Tree (Recursive) Definition

- A binary tree is either:
  a. empty, or
  b. a node pointing to one/two binary tree(s)



C++ Code

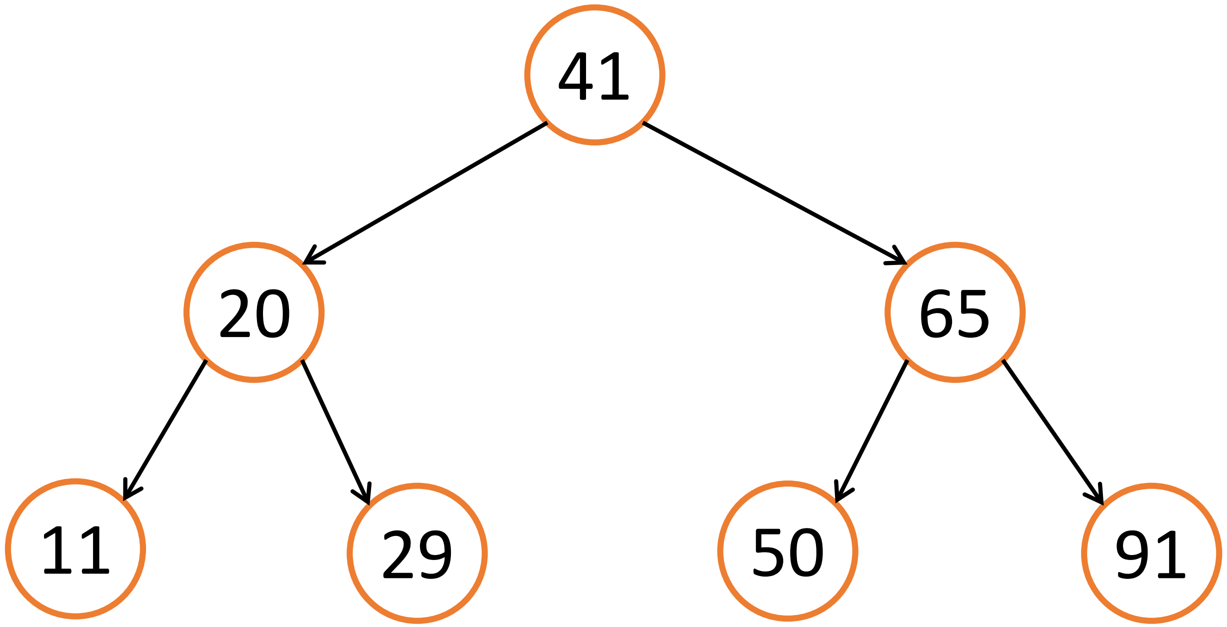
```
template <class T>
class TreeNode {
   private:
     T _item;
     TreeNode<T>* _left;
     TreeNode<T>* _right;

   public:
   TreeNode(T x)
   { _left = _right = NULL; _item = x; _height = 0; };

   friend BinarySearchTree<T>;
};
```

C++ Code

```
template <class T>
class BinarySearchTree {
   private:
     TreeNode<T>* _root;

   public:
   BinarySearchTree() { _root = NULL; }
   void insert(T);

};
```

Binary Search Trees (BST)

- BST Property:
  - all in left sub-tree < key < all in right sub-right

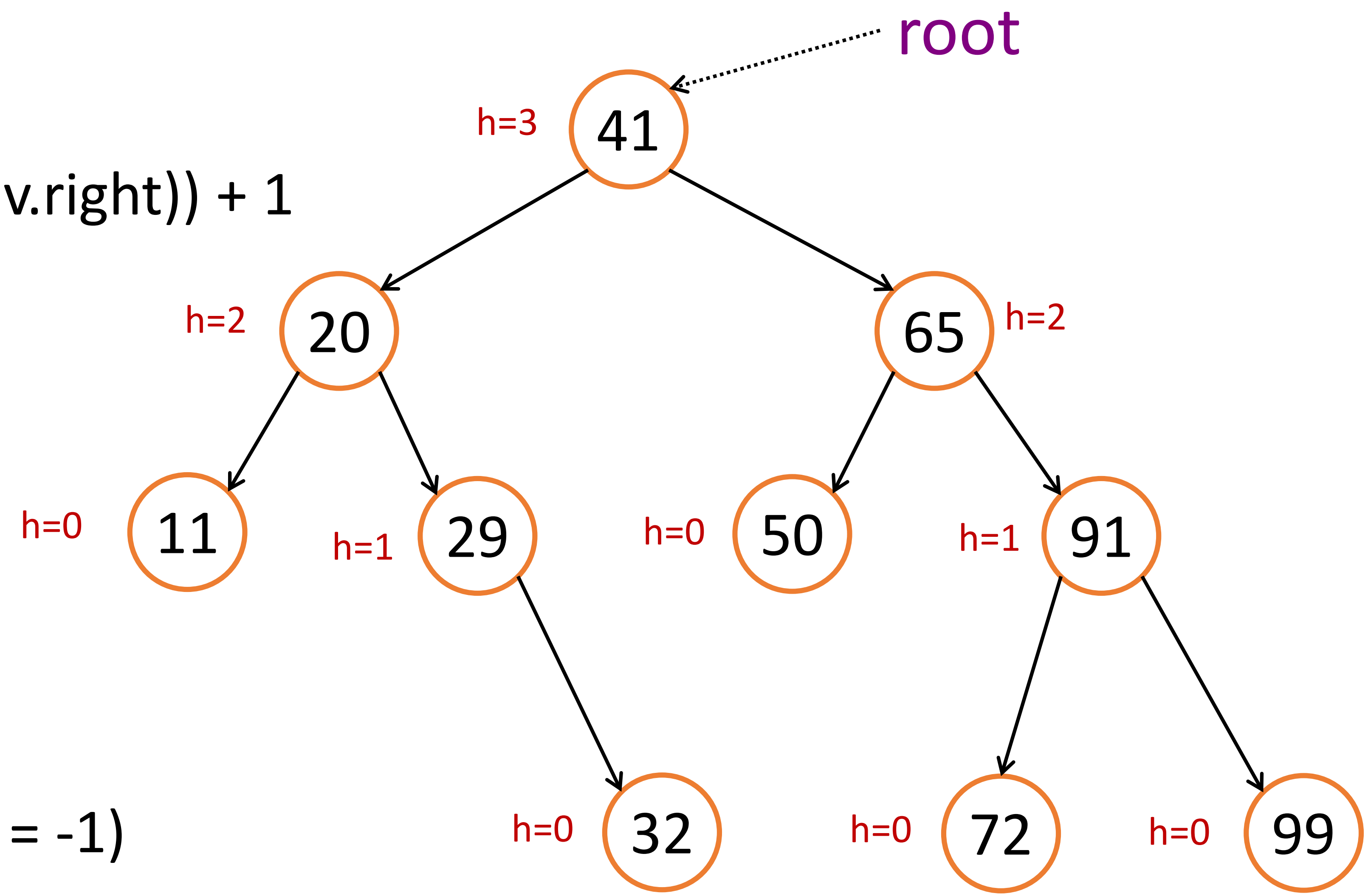


Basic Operations

- height
- search, insert
- searchMin, searchMax

Heights

- For a tree node v
- h(v) = 0 if v is a leaf
- h(v) = max(h(v.left), h(v.right)) + 1



- (For simplicity: h(null) = -1)

Computing the Height of a Node

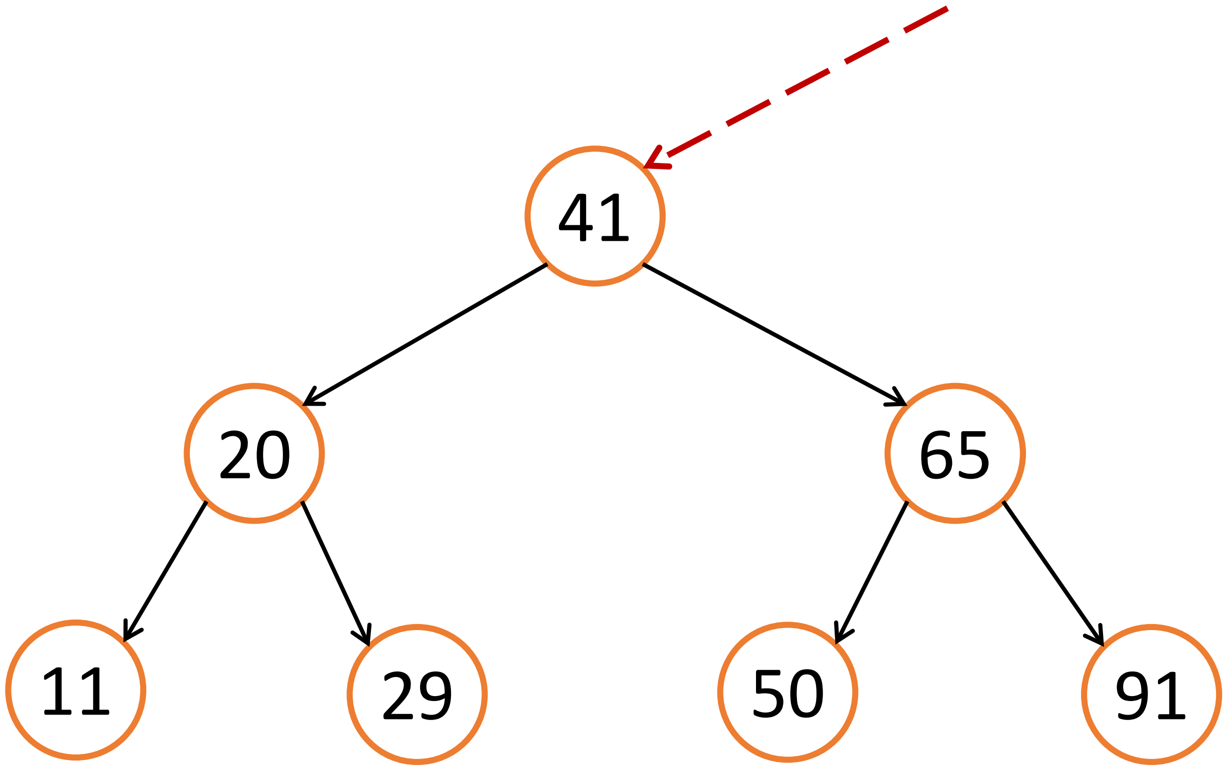
```
int TreeNode<T>::height(){
     int leftHeight = -1;
     int rightHeight = -1;

     if (_left != null)
          leftHeight = _left->height();
     if (_right != null)
          rightHeight = _right->height();

     return max(leftHeight, rightHeight) +  1;
}
```

max of subtrees

add 1

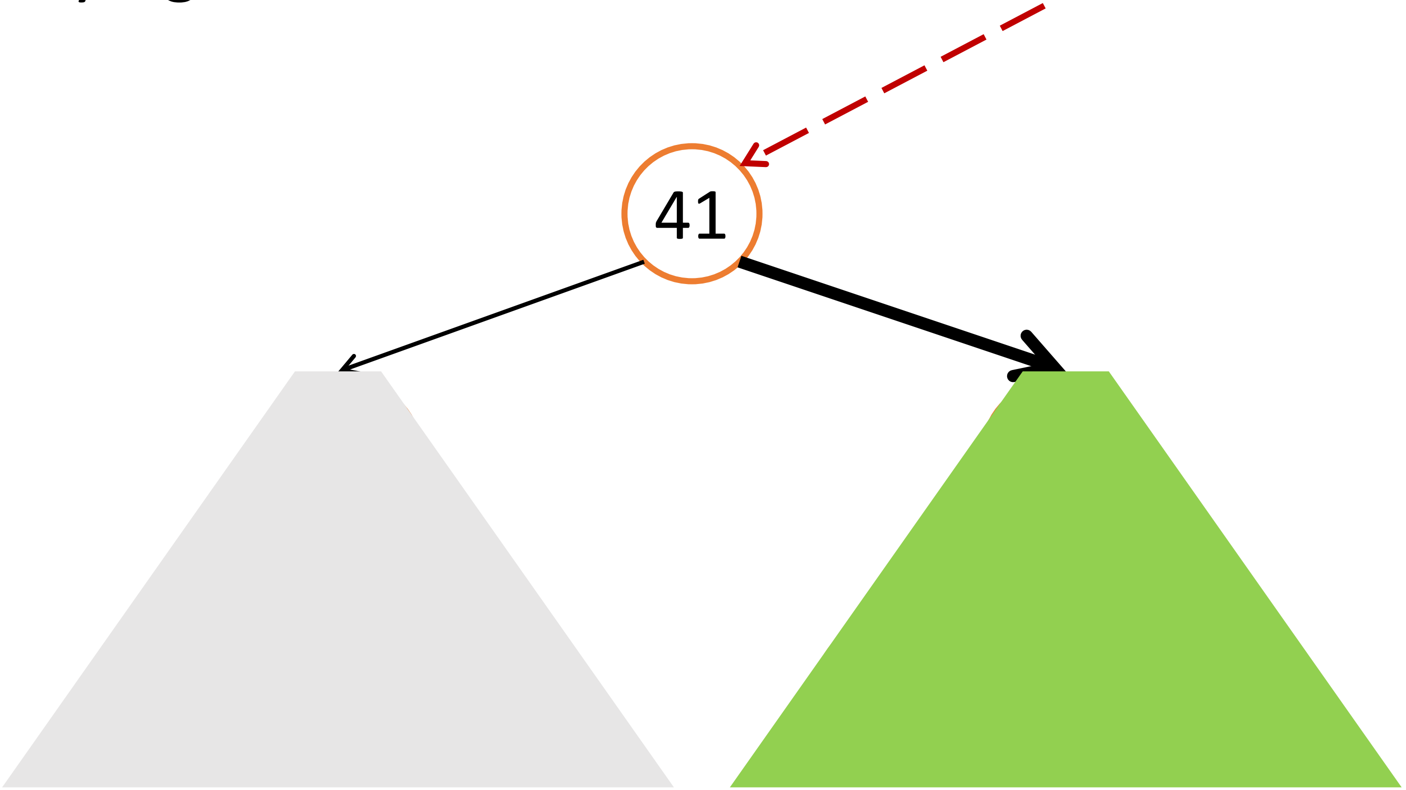
Search for the Maximum

- Starting from the root?



Idea

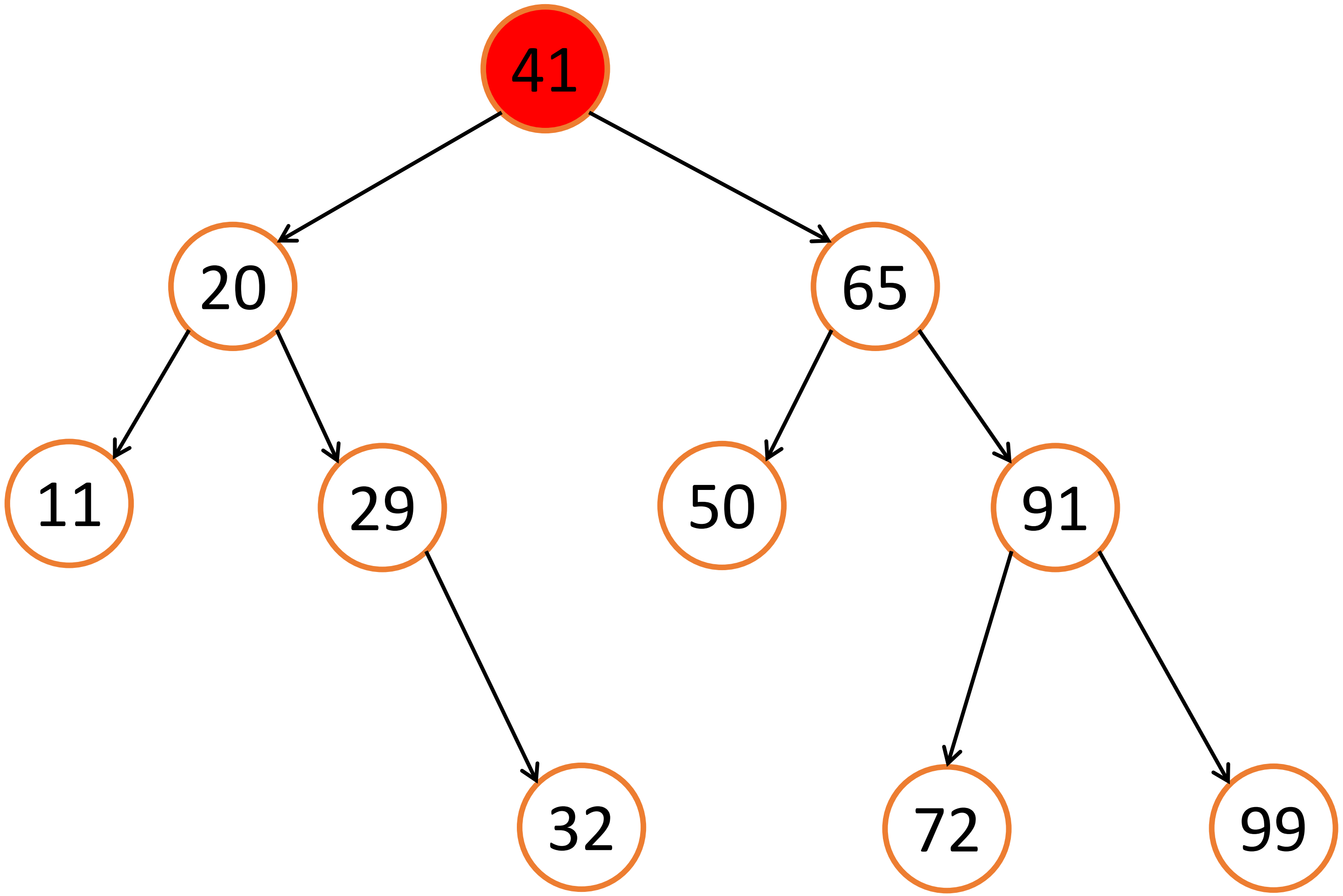
- For a node v, I know my right subtree contains elements that is greater than v



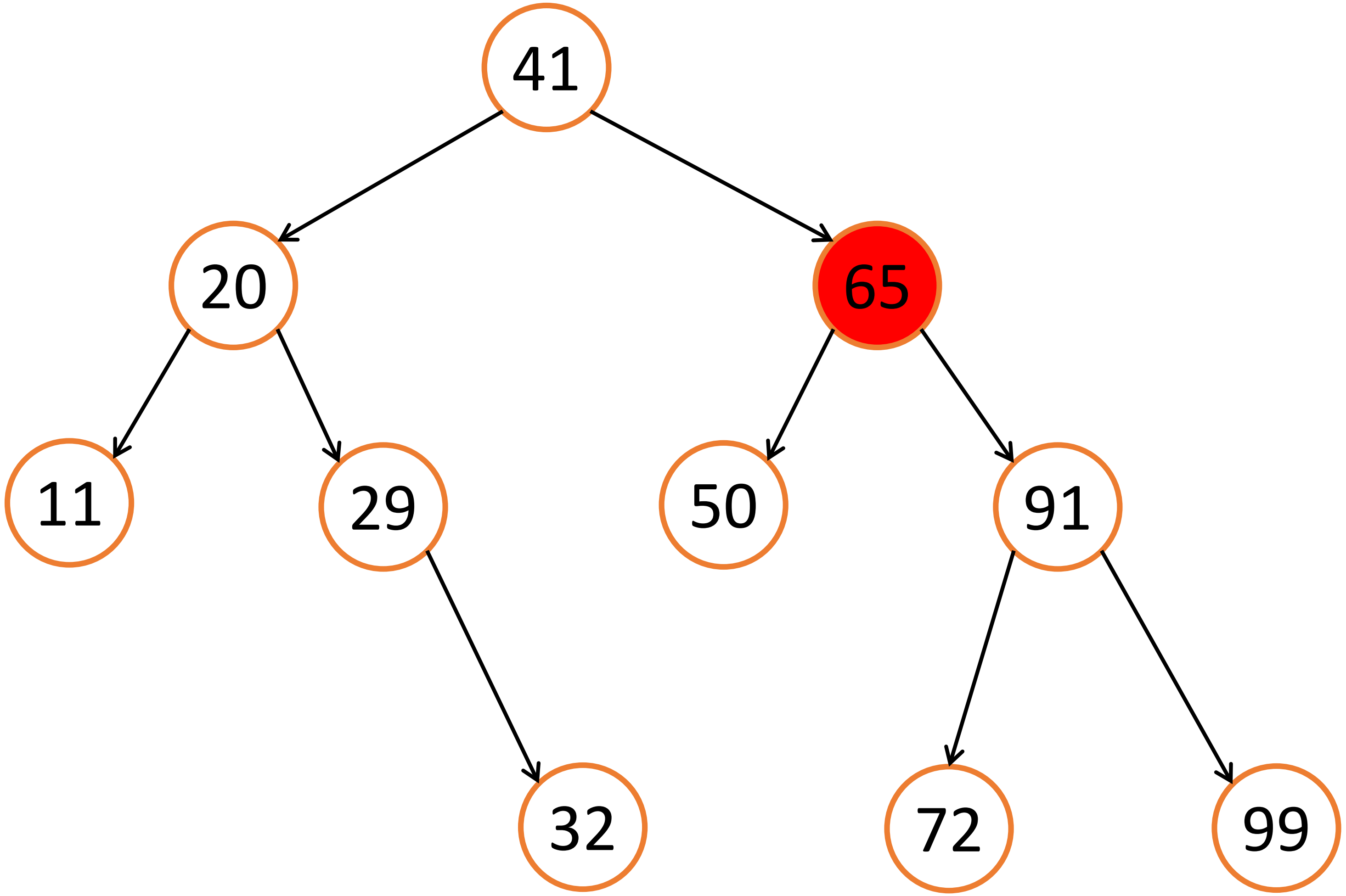
Searching For Maximum

```
template <class T>
T BinarySearchTree<T>::searchMax() {

  TreeNode<T>* current = _root;

  while (current->_right)
    current = current->_right;
  return current->_item;

}
```

Search Max



Search Max



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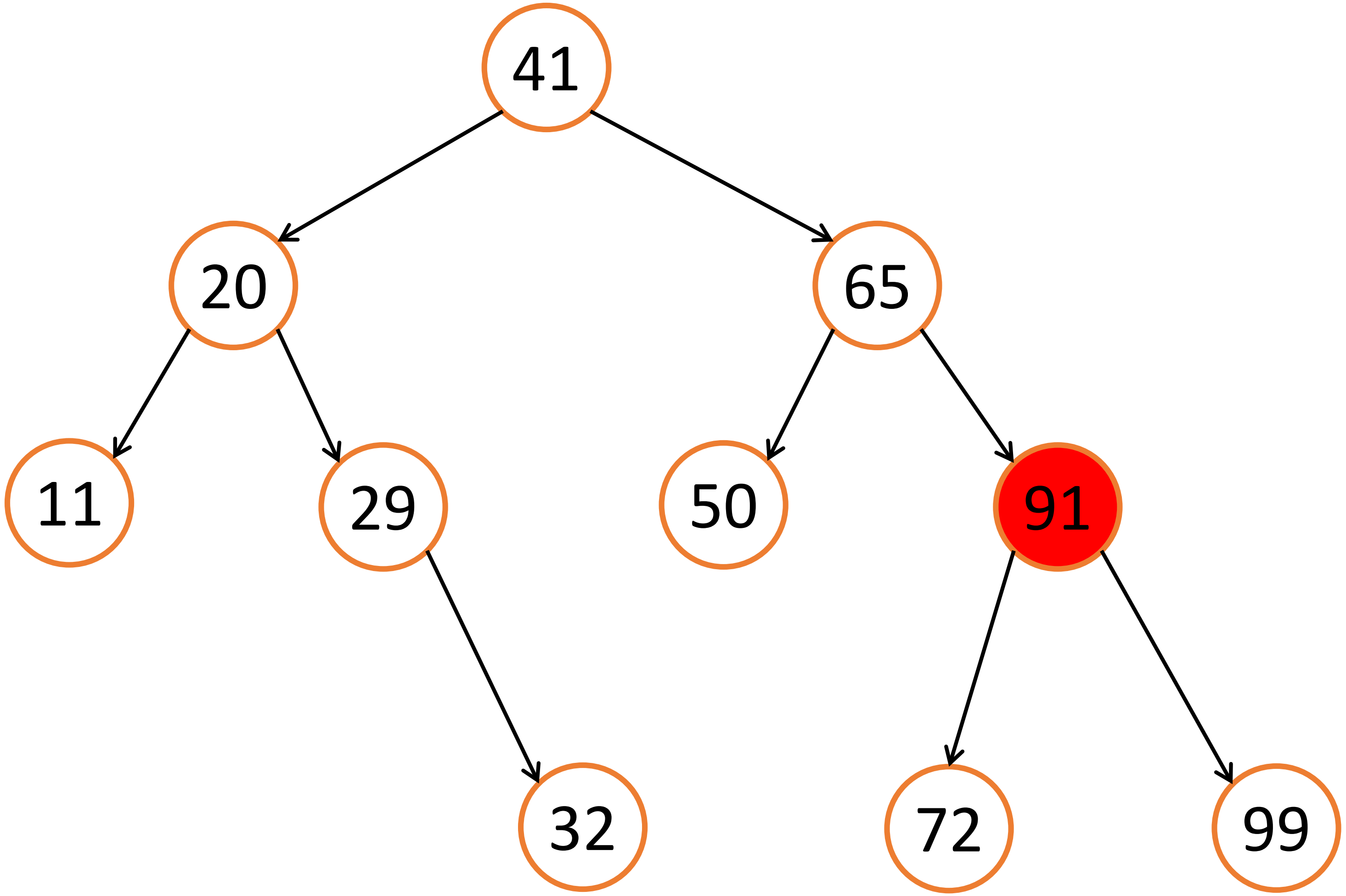
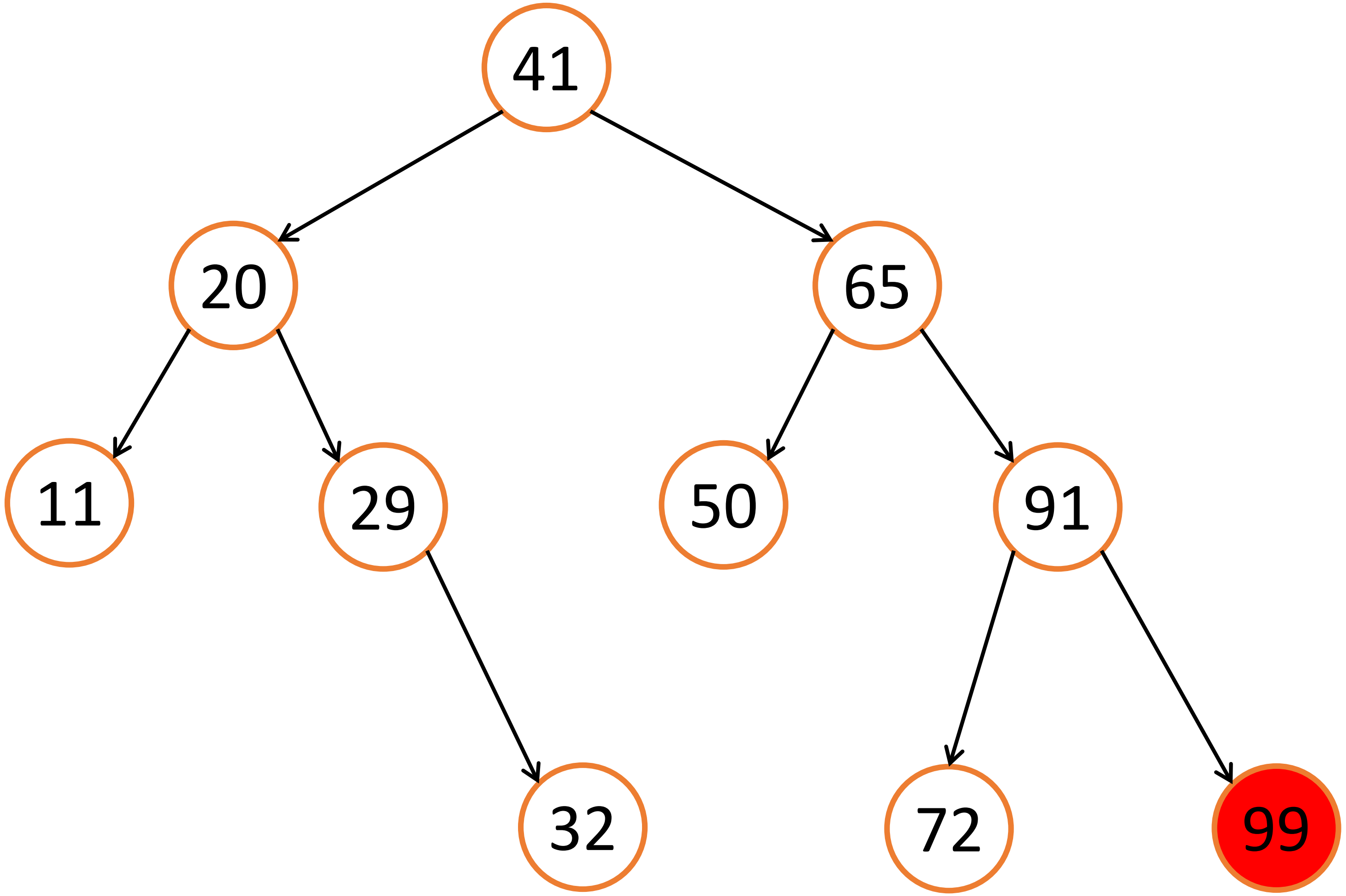


Search Max



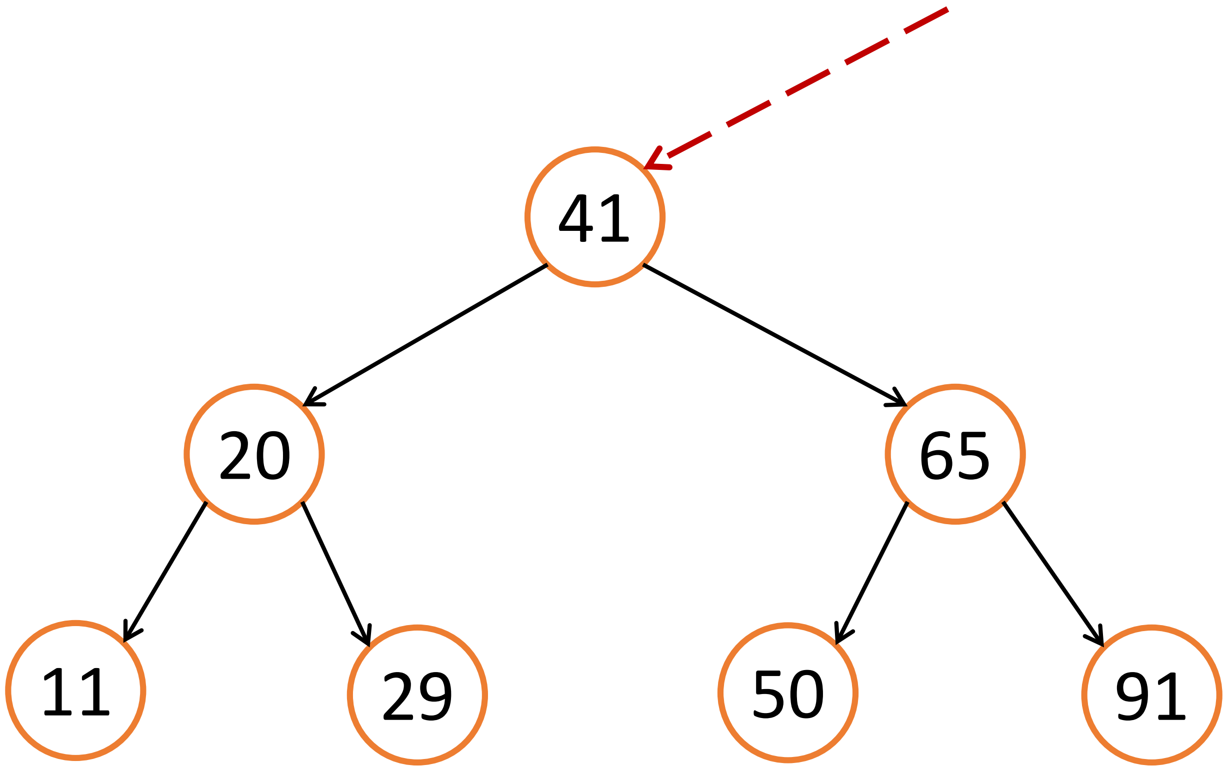
Searching For Maximum

```
template <class T>
T BinarySearchTree<T>::searchMax() {

  TreeNode<T>* current = _root;

  while (current->_right)
    current = current->_right;
  return current->_item;

}
```

Search for Minimum?

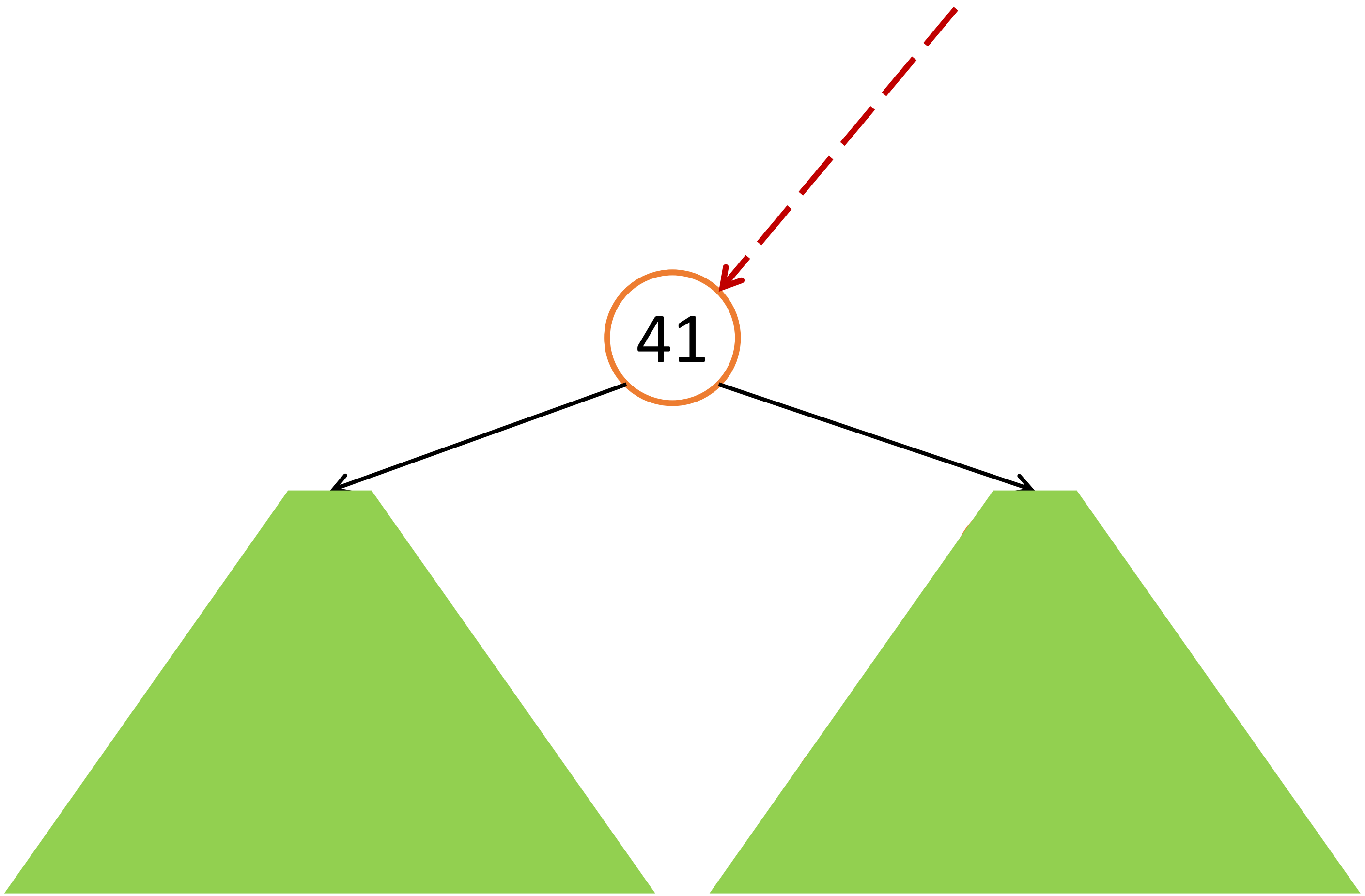
Search for a particular key

- Starting from the root
- Is “25” in the tree?



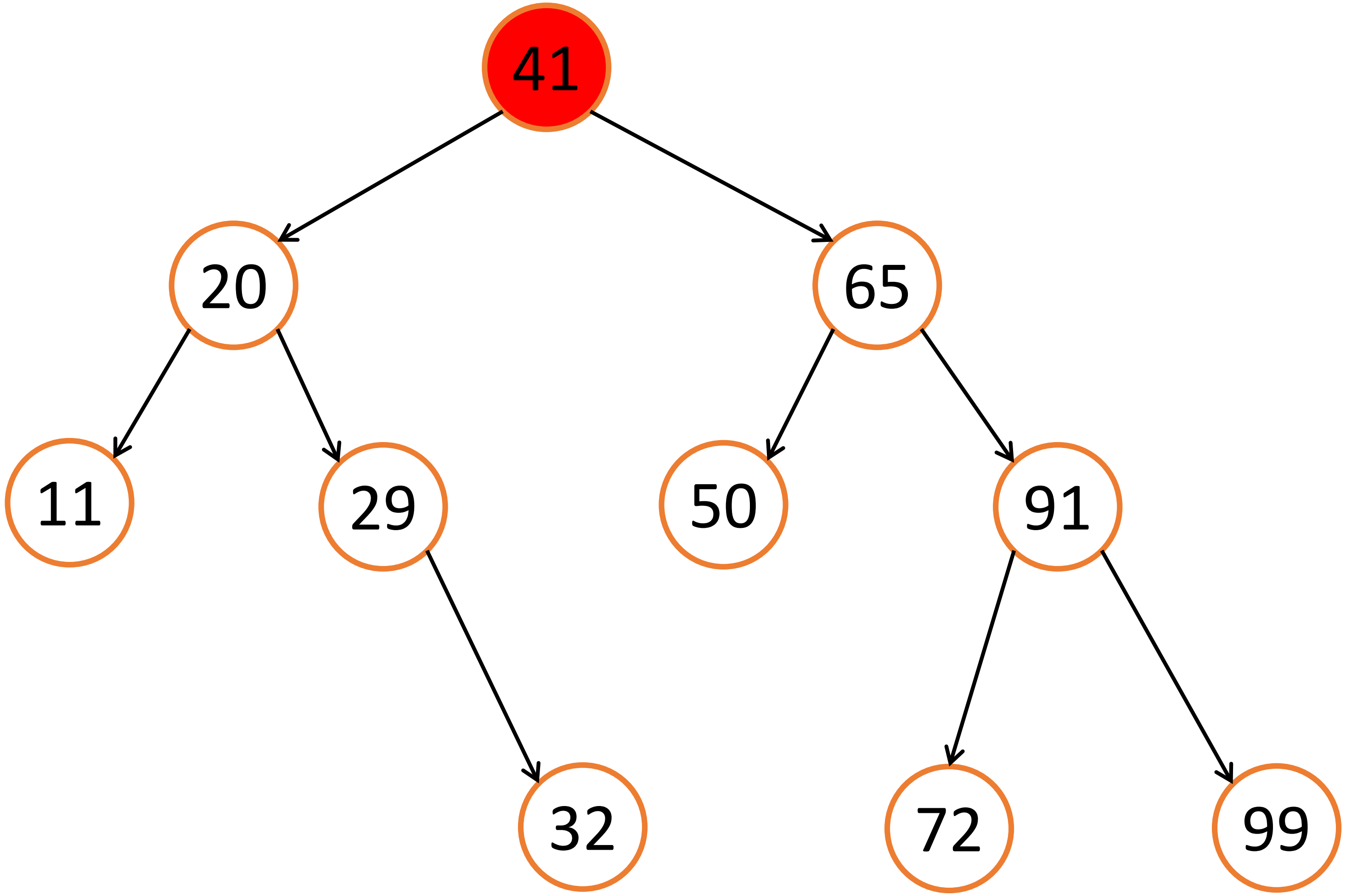
Go Left or Go Right?

- 25 < 41

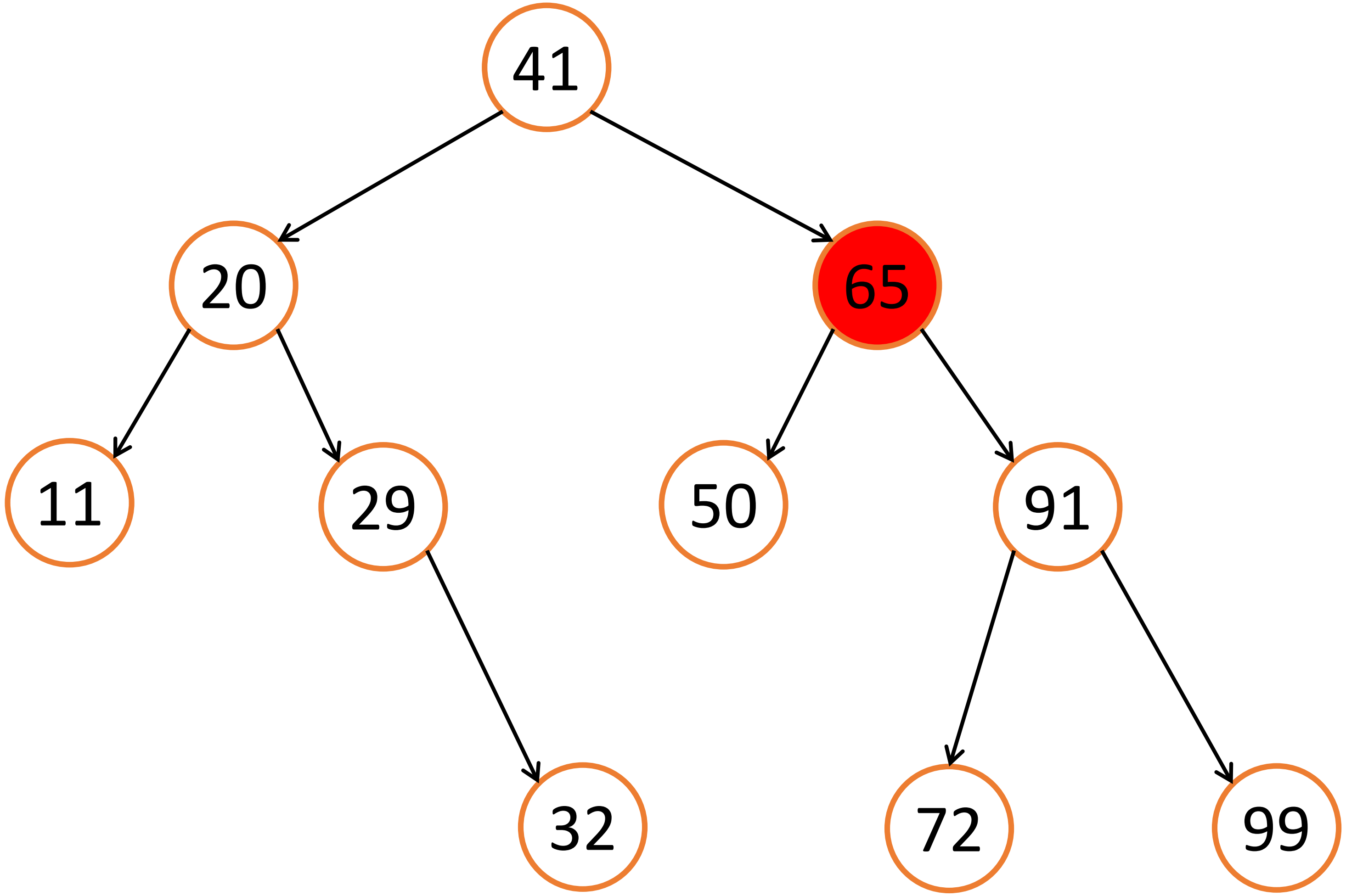


```cpp
template <class T>
bool BinarySearchTree<T>::exist(T x) {
    TreeNode<T>* current = _root;

    while (current) {
        if (current->_item == x)
            return true;
        else if (x > current->_item )
            current = current->_right;
        else
            current = current->_left;
    }
    return false;
}
```

Example: exist(72)



Example: exist(72)



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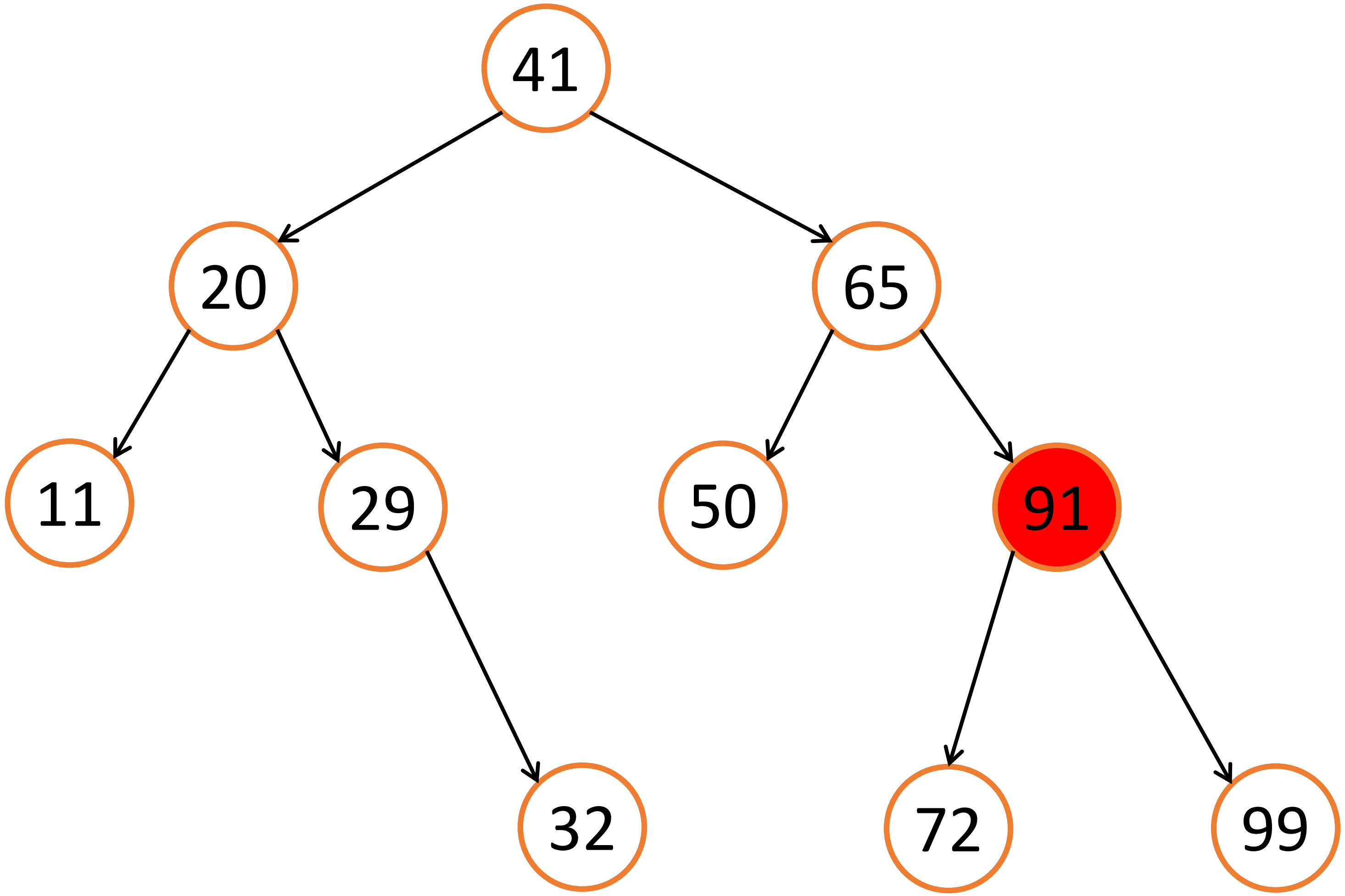

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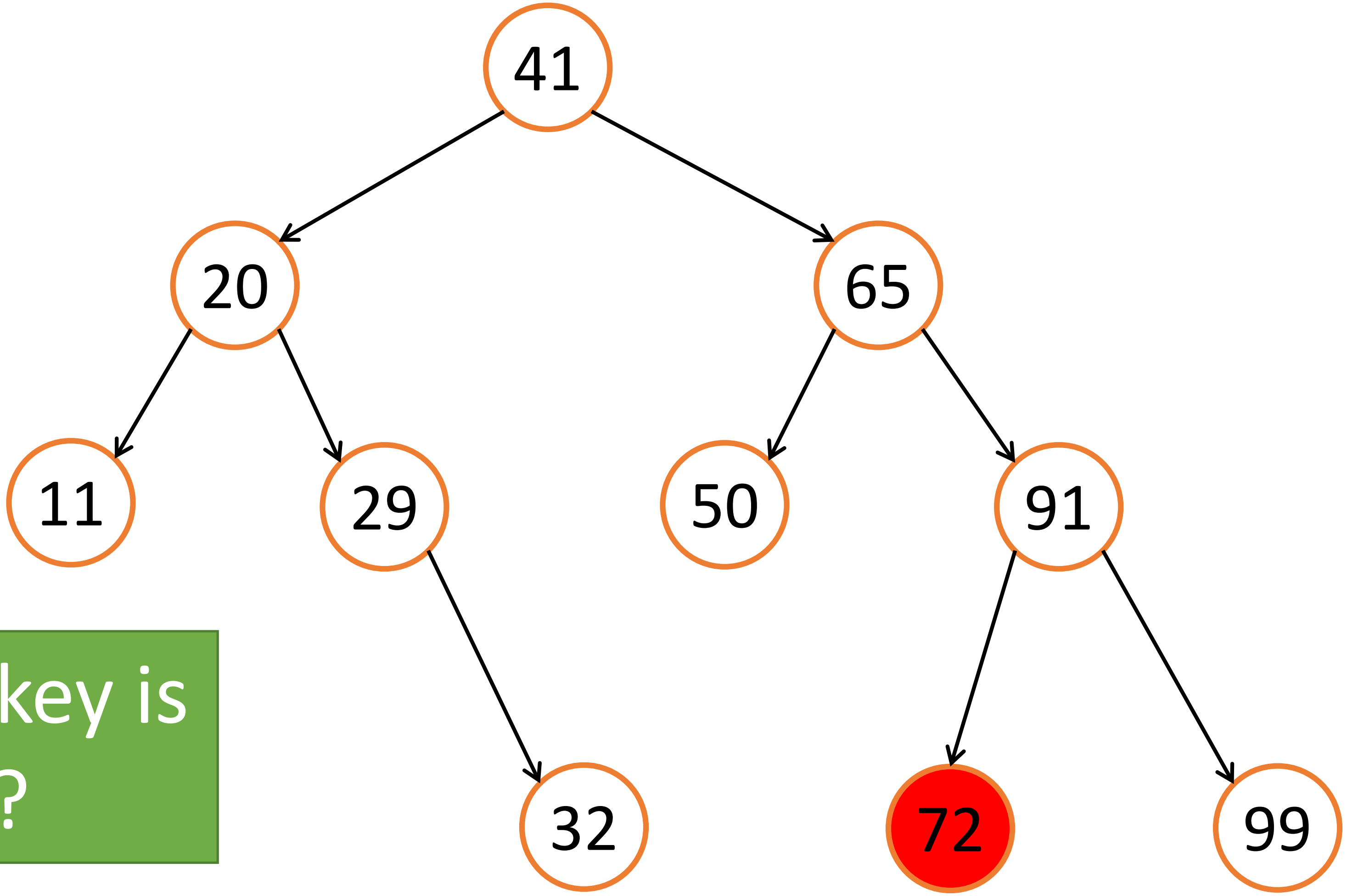
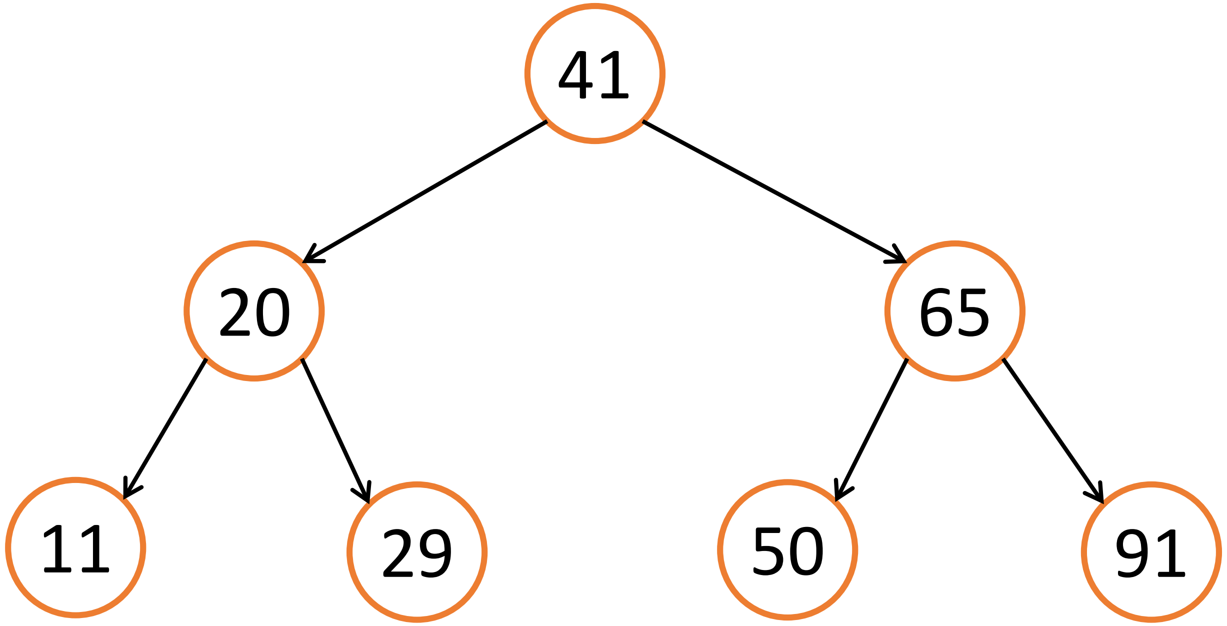


What happen if the key is NOT in the tree?

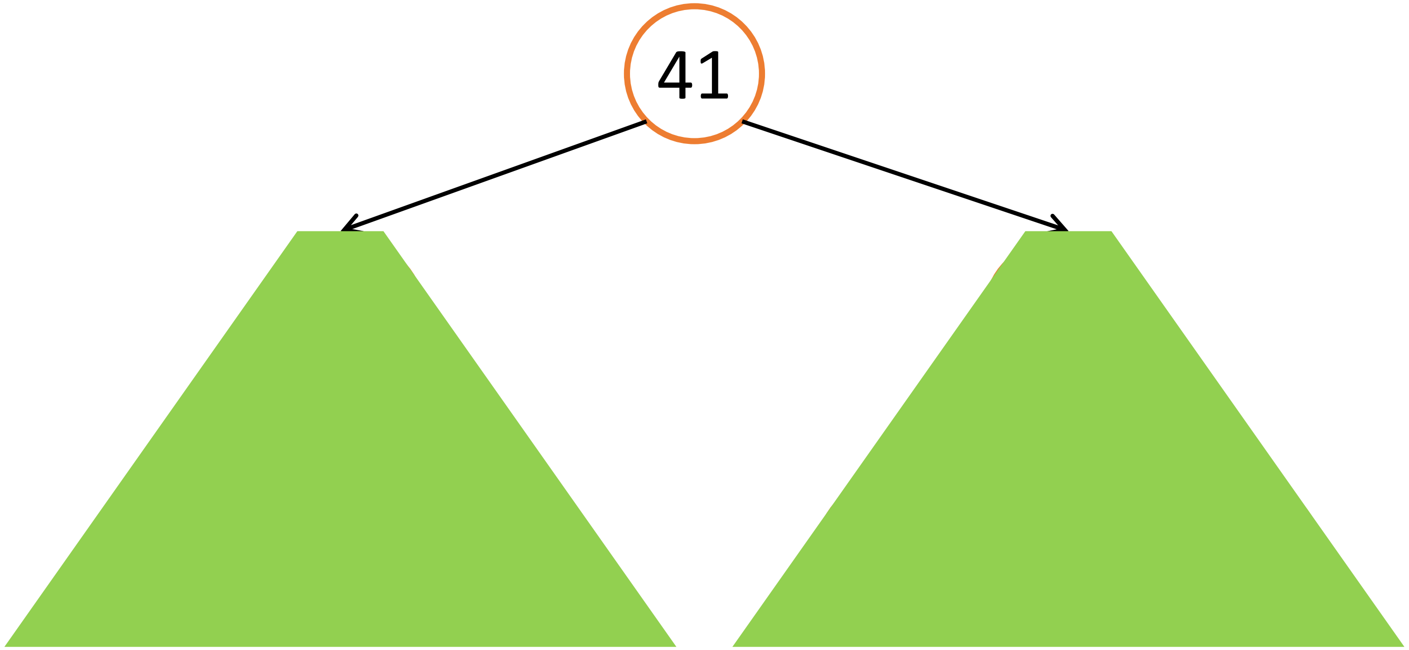
Inserting a Key

- Inserting 25



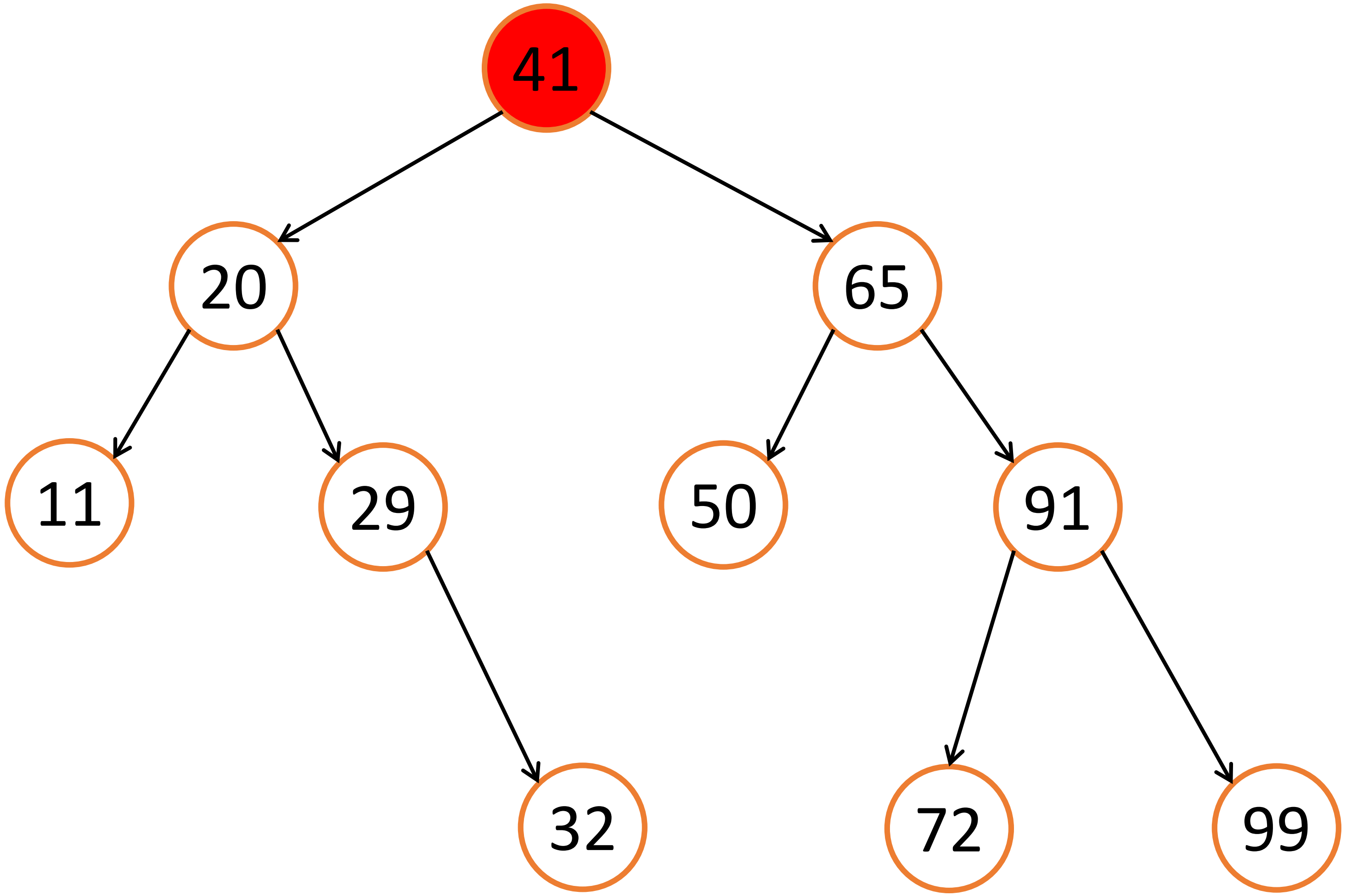
Inserting 25

- Go left or right?

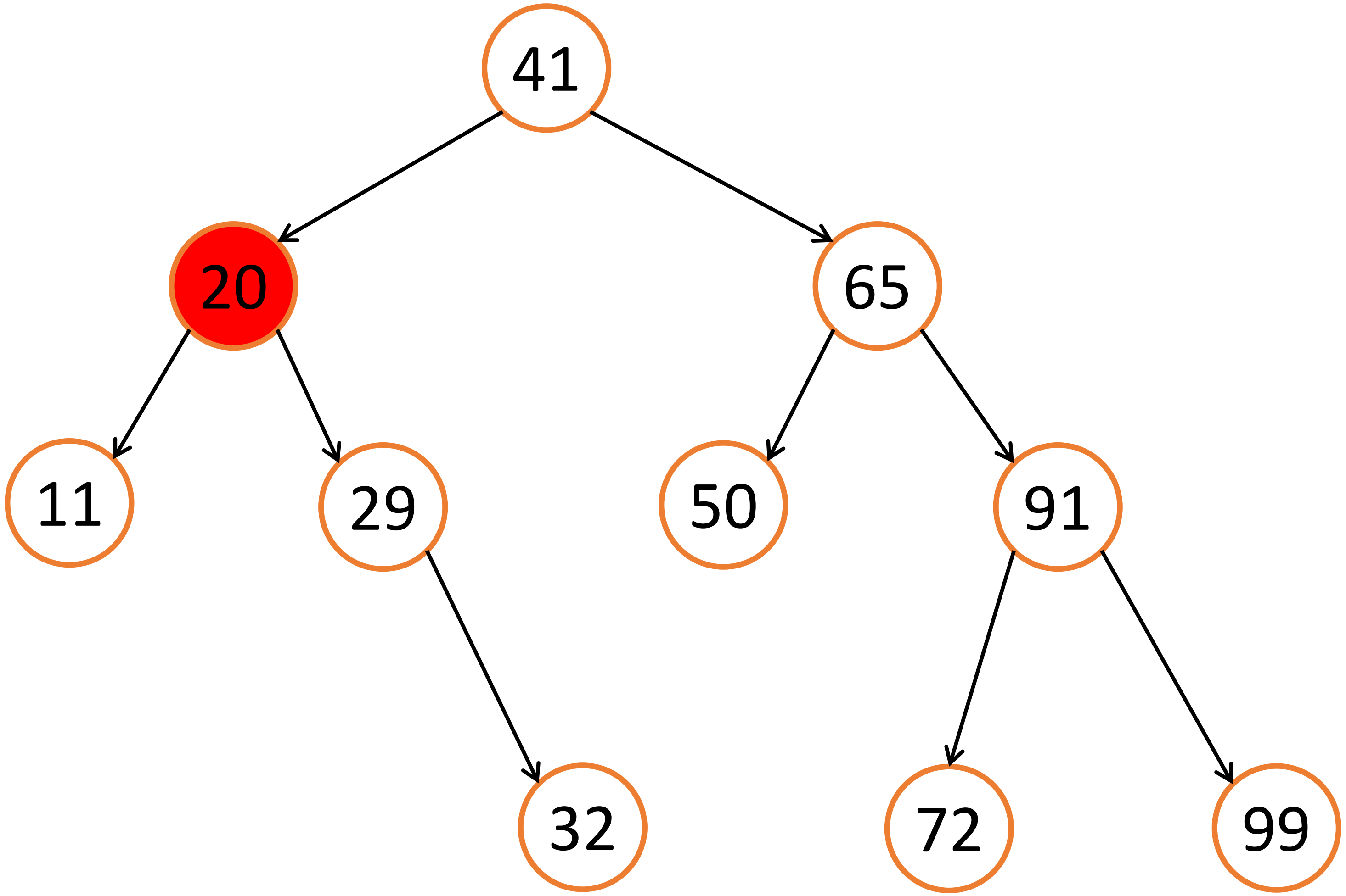


```cpp
template <class T>
TreeNode<T>* BinarySearchTree<T>::insert
                          (TreeNode<T>* current, T x)
{
  if (current->_item > x) {
    if (current->_left)
      current->_left = insert(current->_left, x);
    else
      current->_left = new TreeNode<T>(x);
  } else if (x > current->_item) {
    // try it yourself
  }
  else return current;
}
```

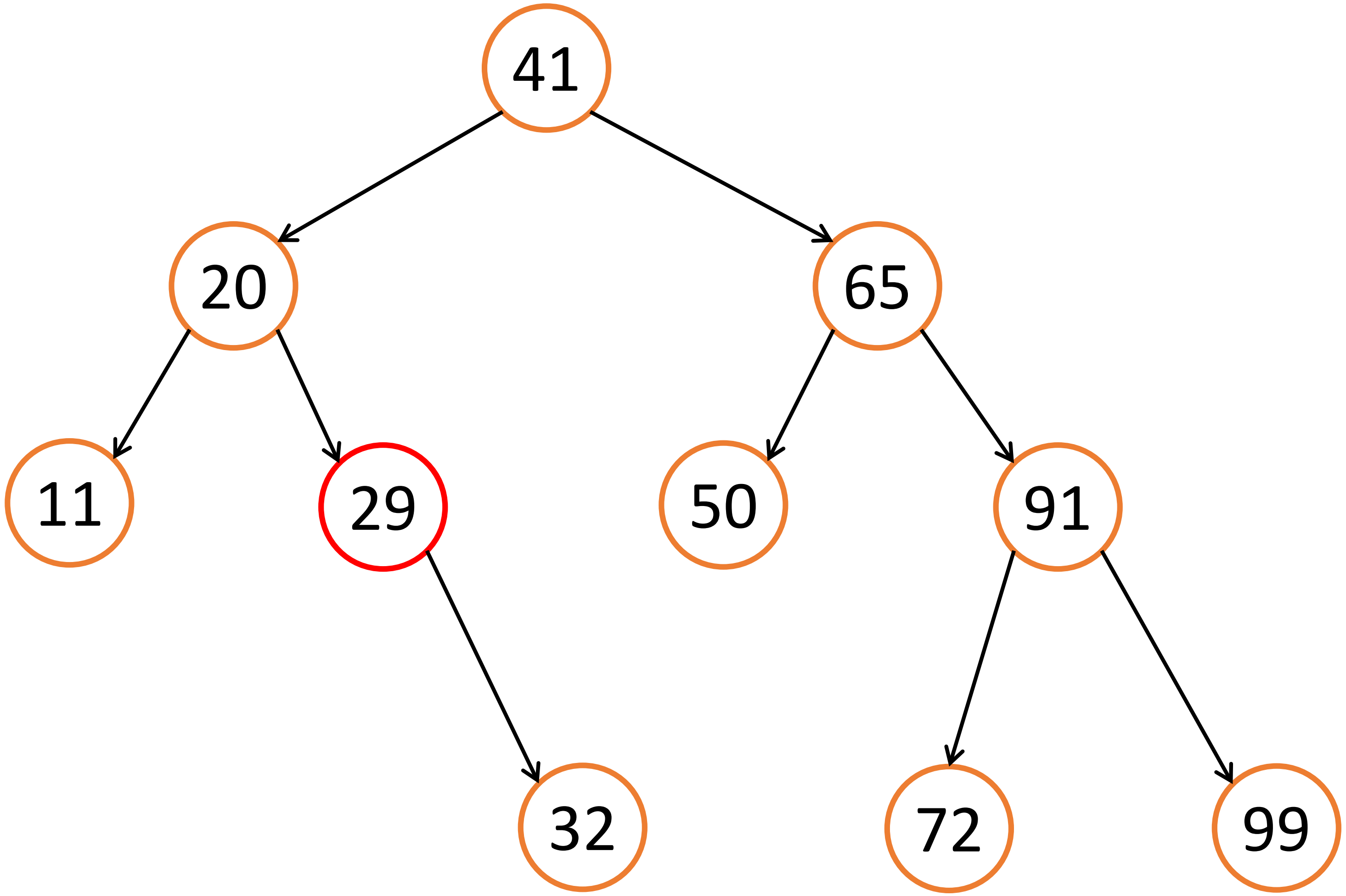
Insert(27)



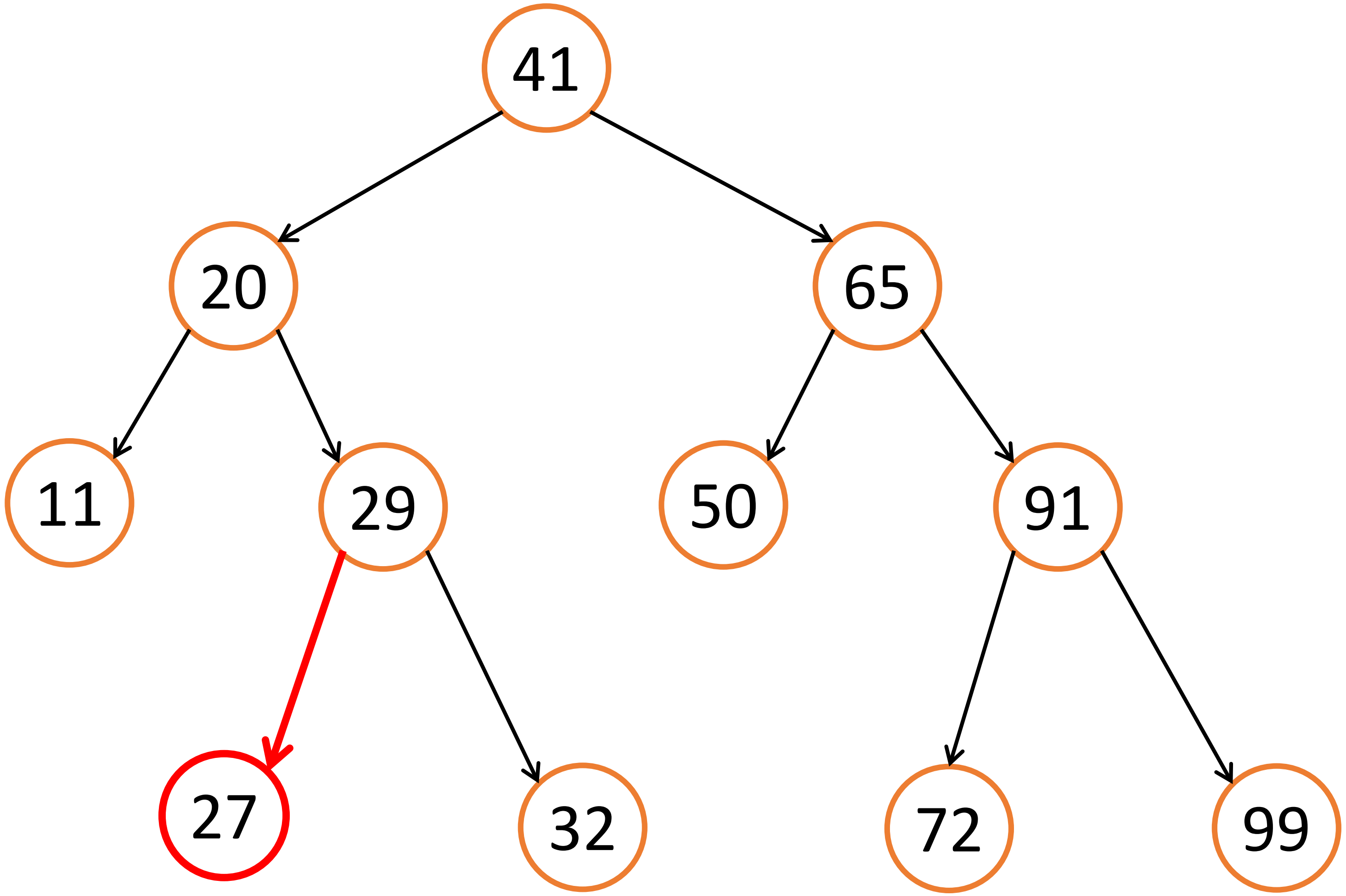
Insert(27)



Insert(27)

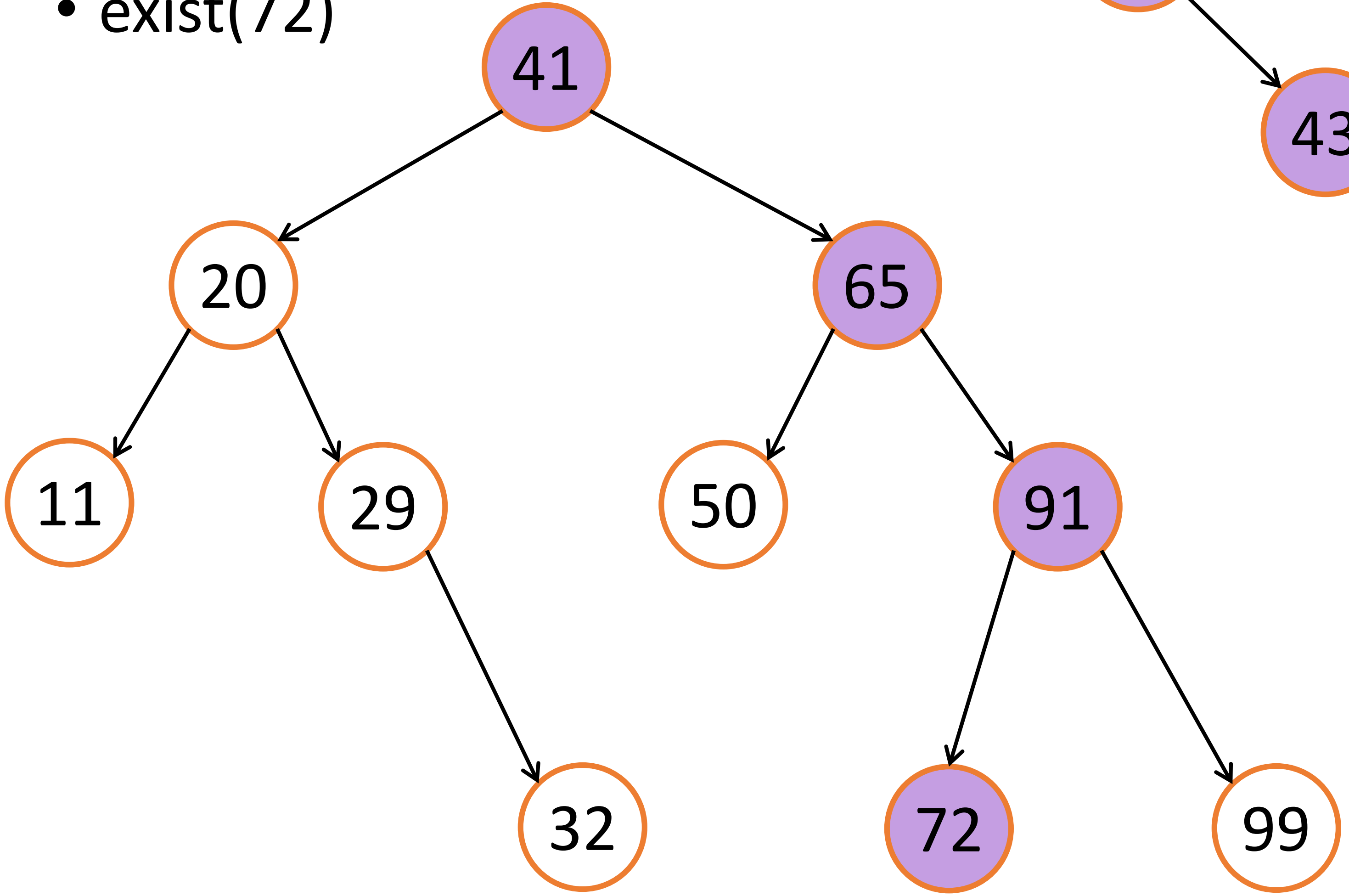


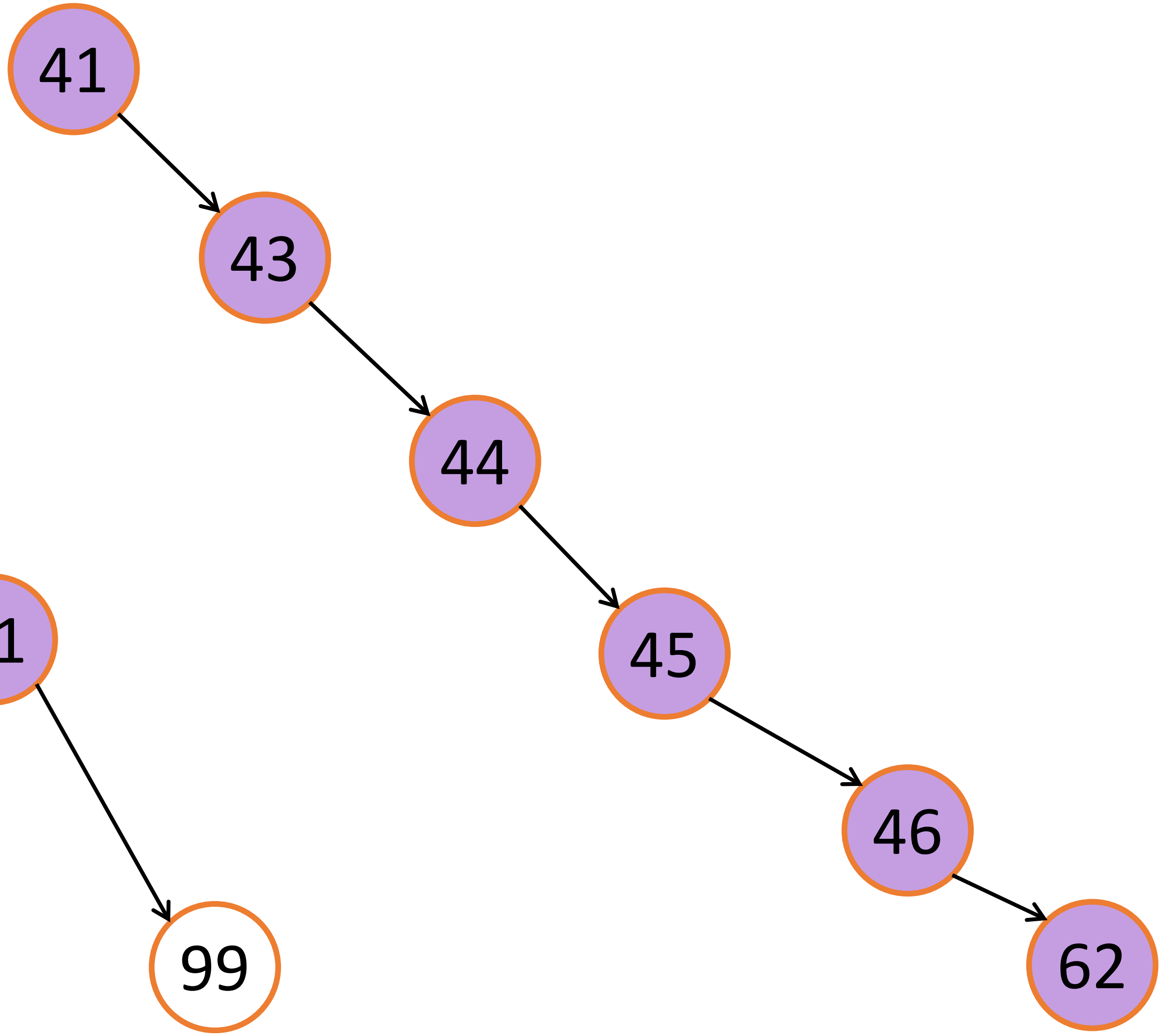
Insert(27)



What is the Time Complexity for Insert()/exist()?

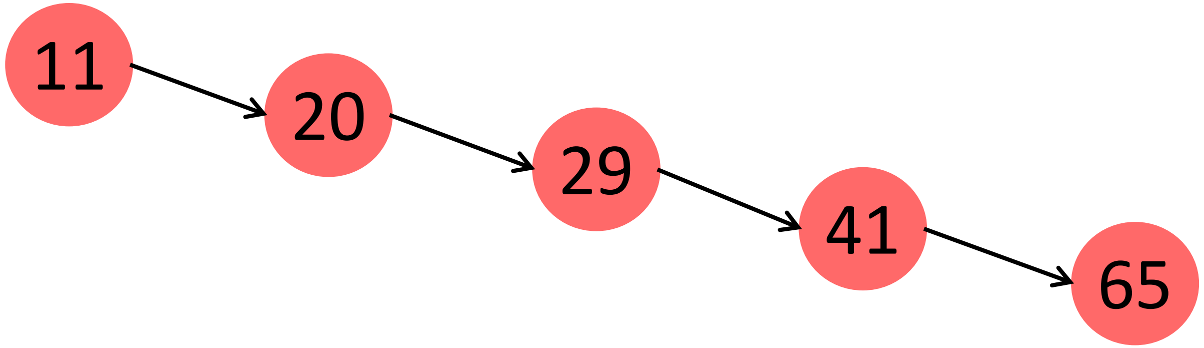
- exist(72)

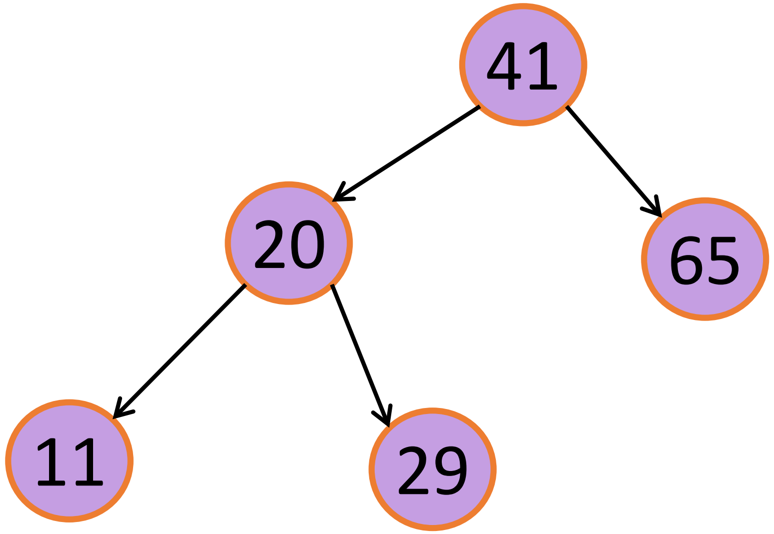


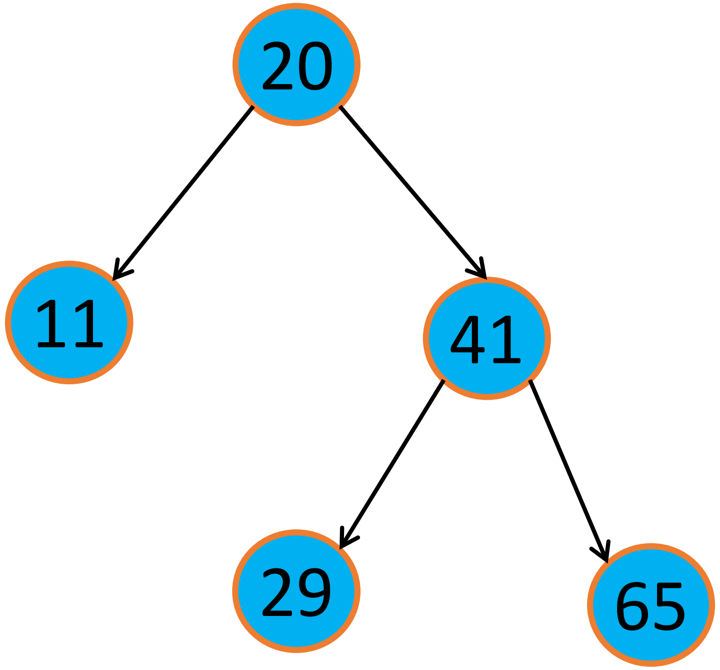


Tree Shapes

- Trees come in many shapes
  - same keys ≠ same shape
  - performance depends on shape
- What determines shape?
  - Order of insertions

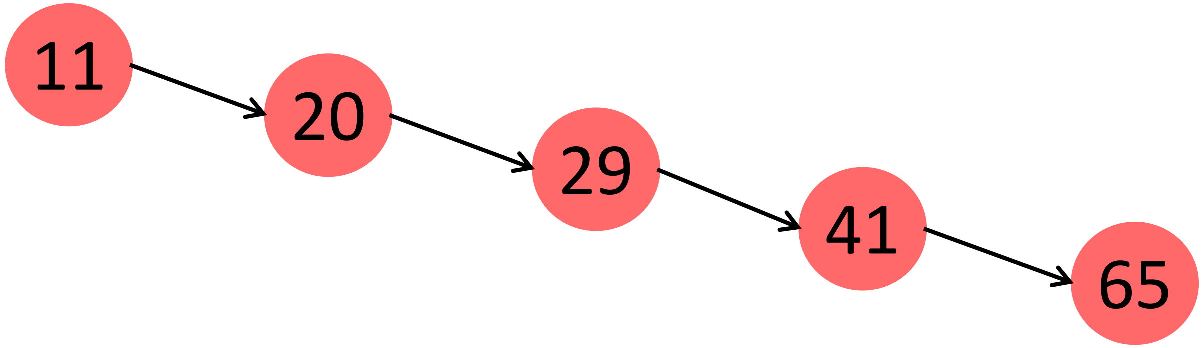


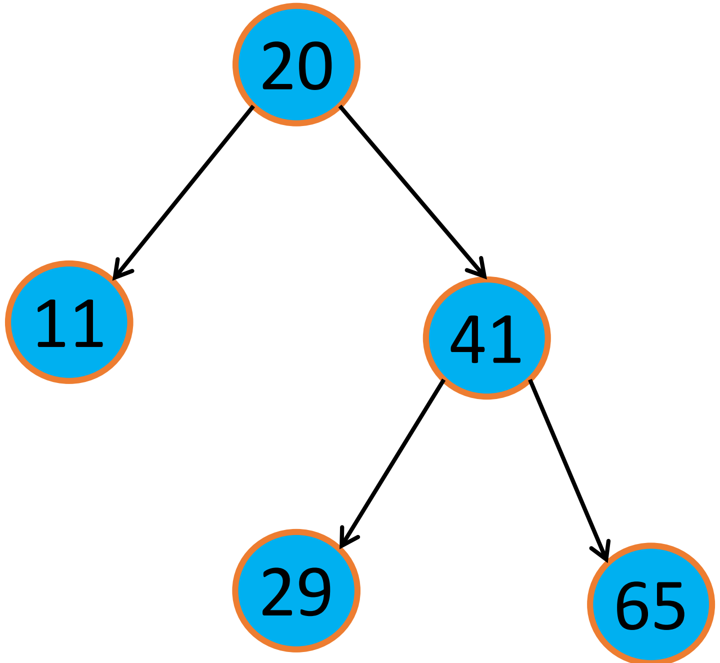




Time Complexity

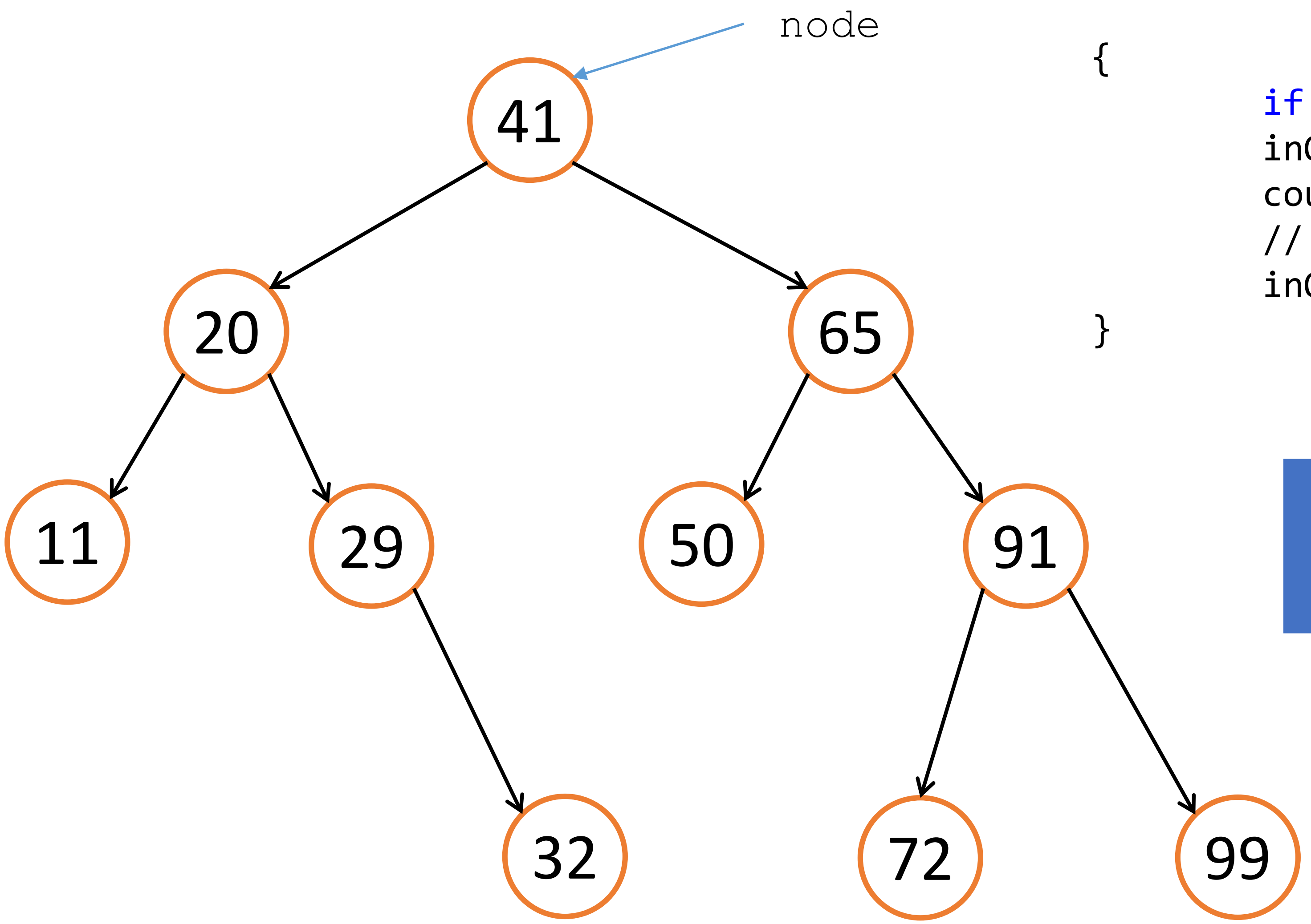
- Depending on the height of the tree
- A tree with n nodes
  - Height = O(n) or O(log n)
- Come back next week!





In-order-traversal(v)

```
template <class T>
void BinarySearchTree<T>::inOrder
                              (TreeNode<T>* node)
{
    if (!node) return;
    inOrder(node->_left);
    cout << node->_item << " ";
    // or do something about it
    inOrder(node->_right);
}
```

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{
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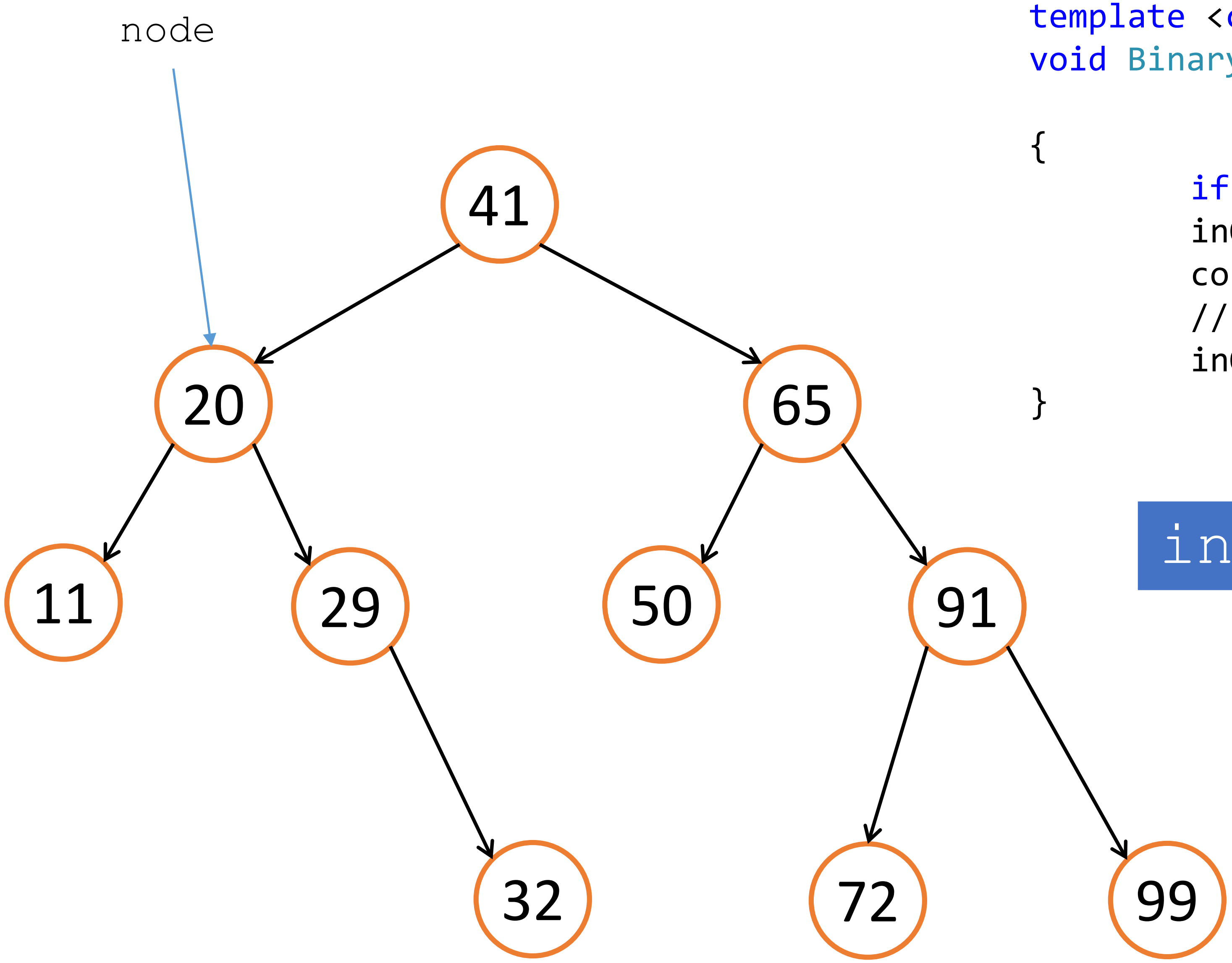
        // or do something about it
        inOrder(node->_right);
}
```

Start by calling `inOrder(_root)`

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        // or do something about it
        inOrder(node->_right);
}
```
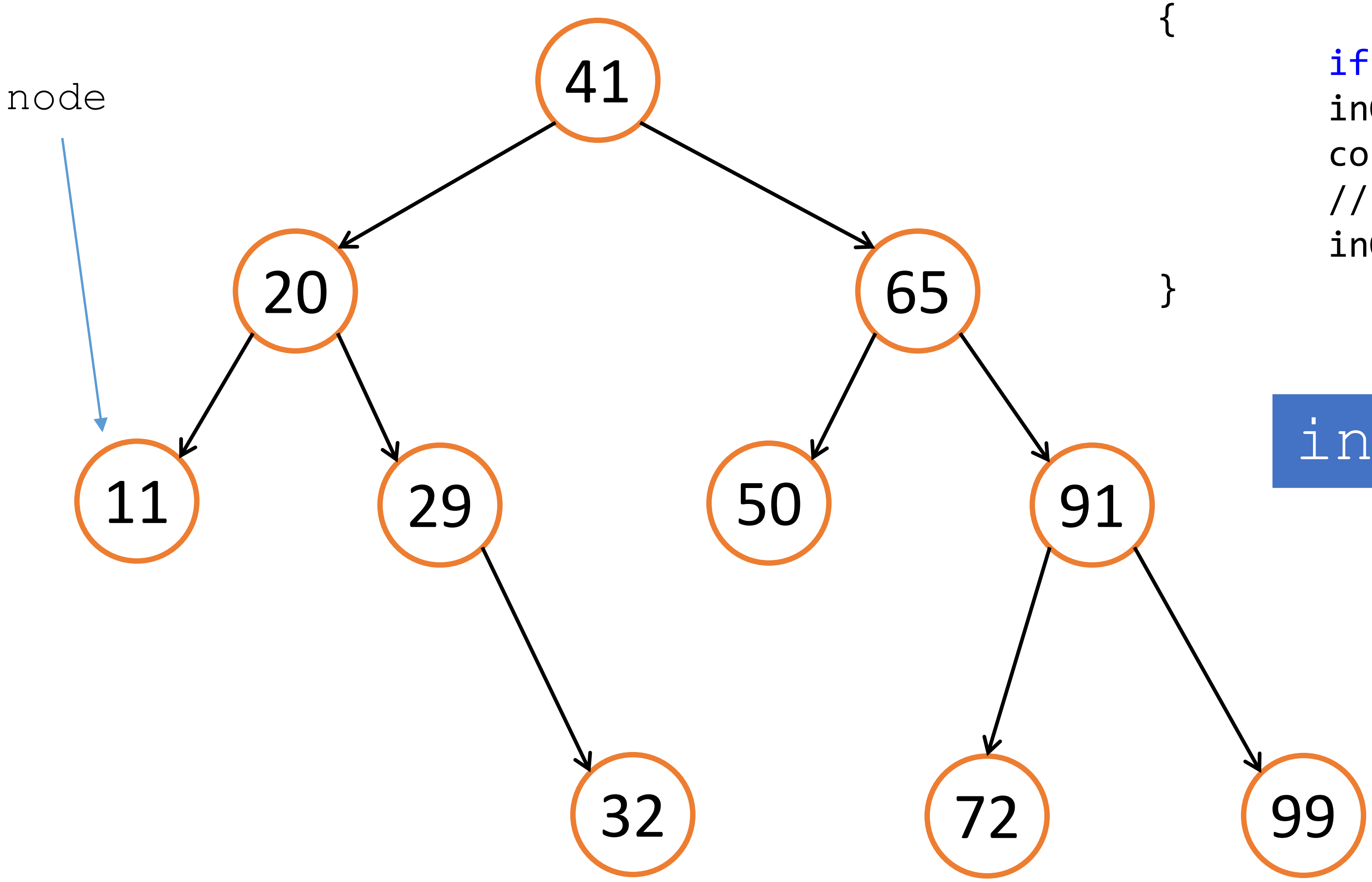

inOrder(node->left)

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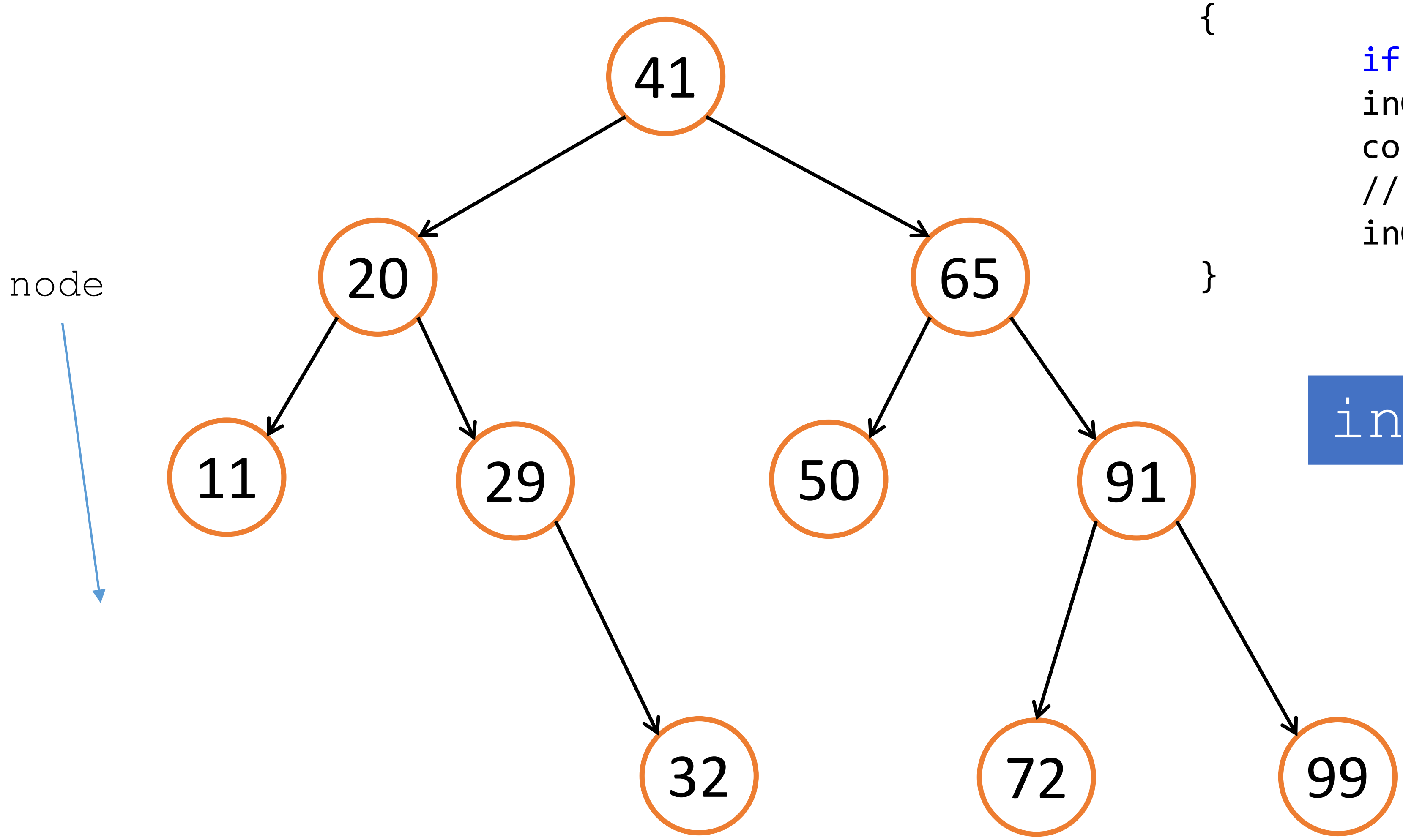


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inOrder(node->left)

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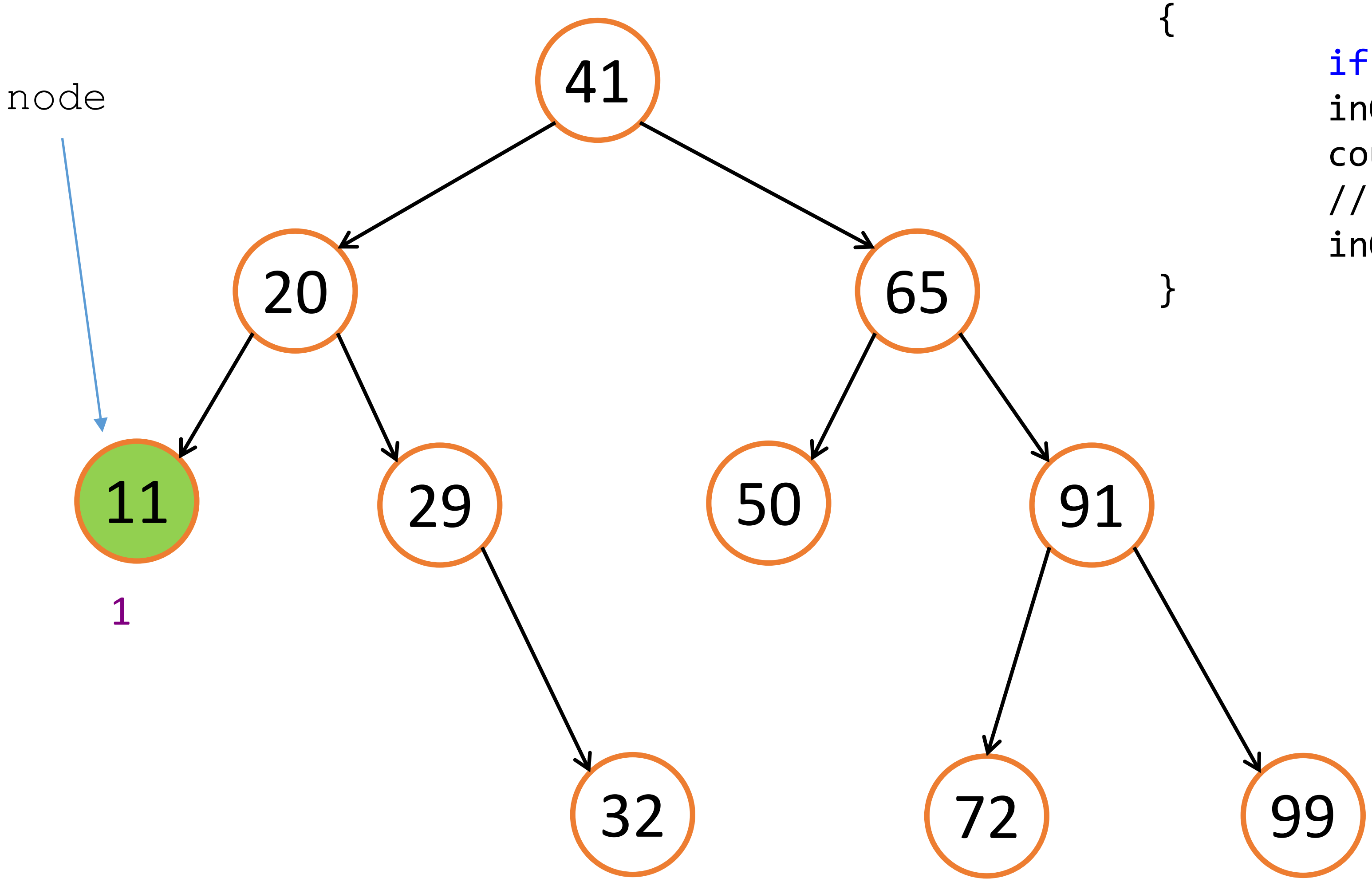
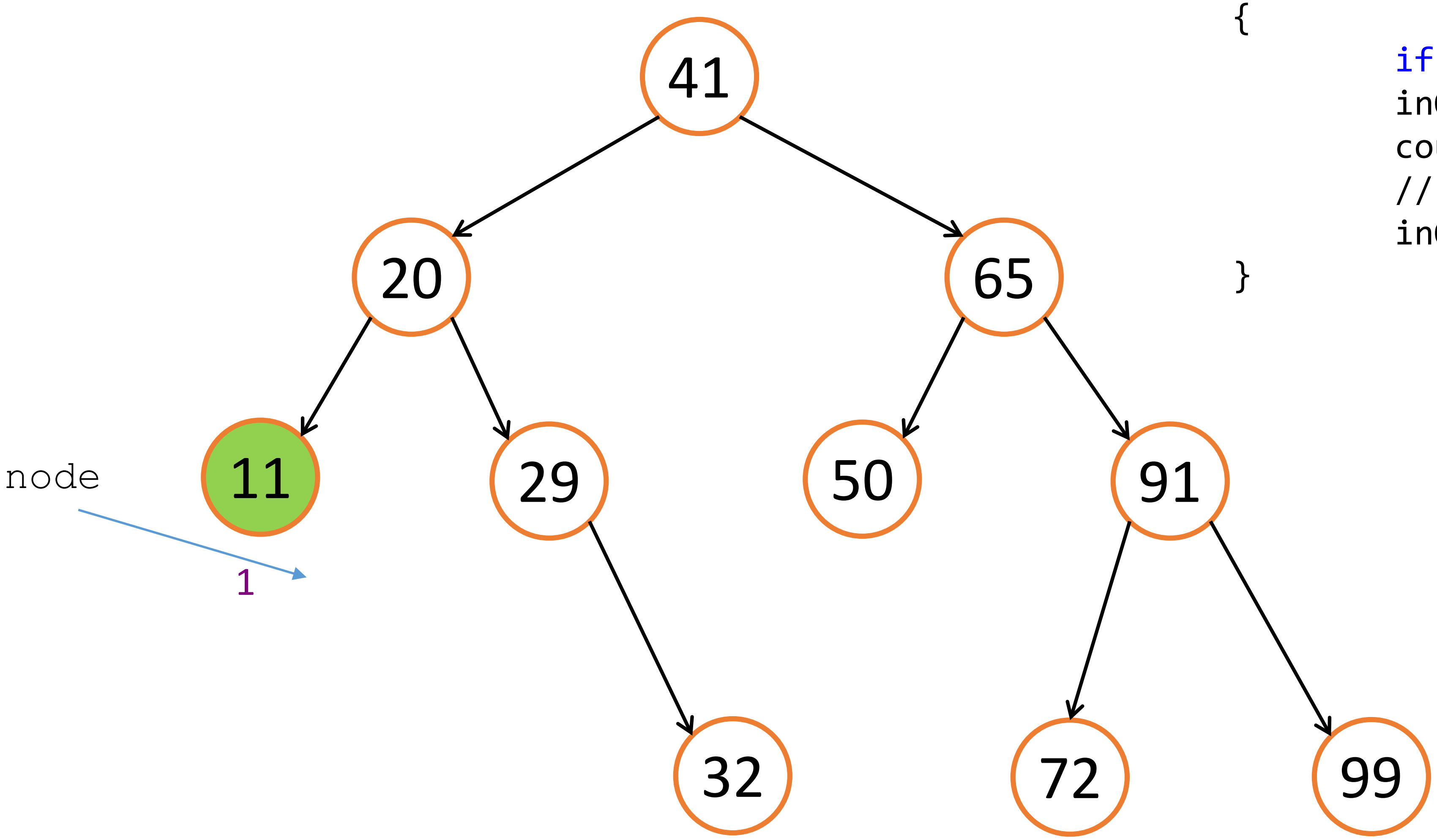
        // or do something about it
        inOrder(node->_right);
}
```




```
cout << node->_item << " ";
```

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}
```
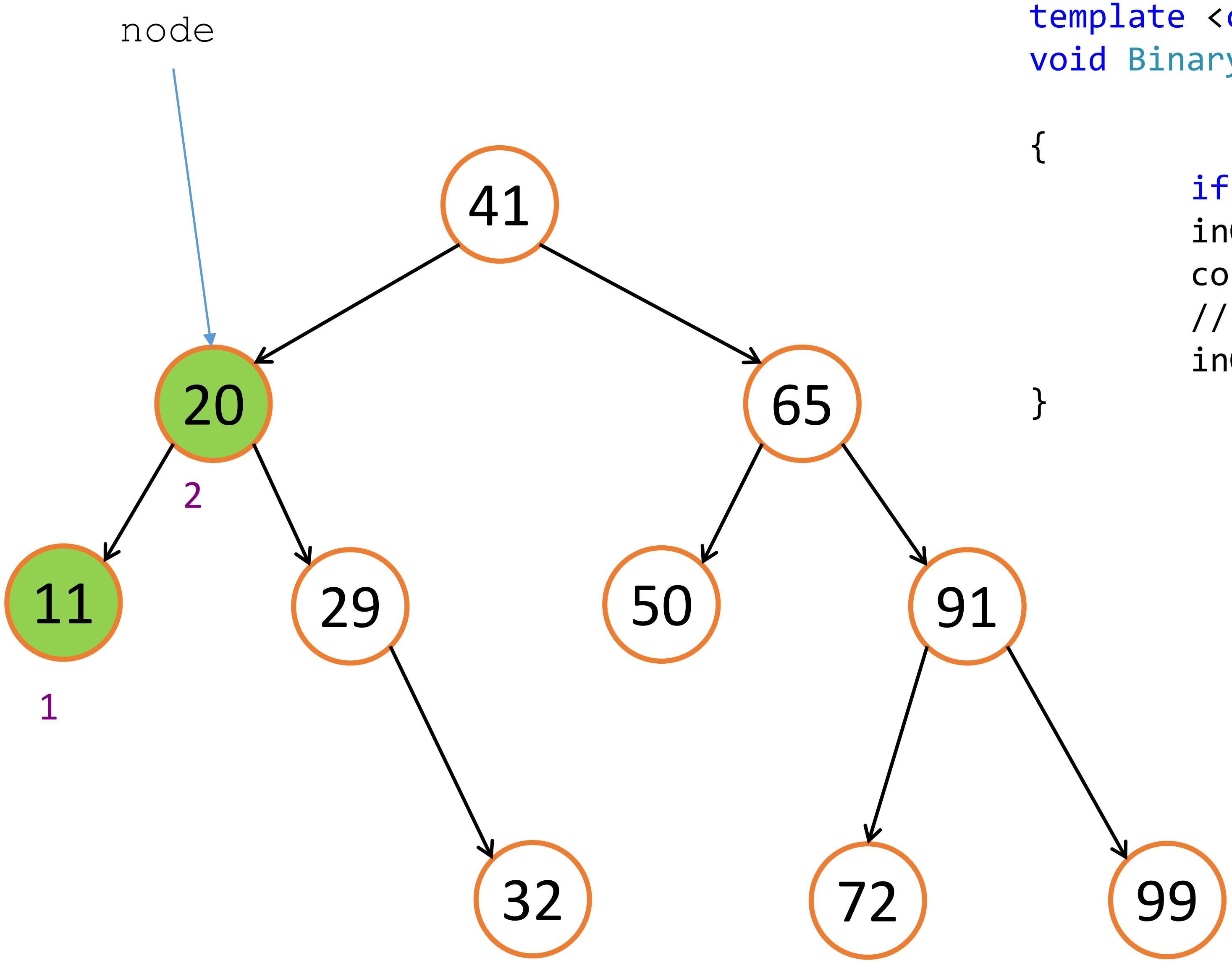

inOrder(node->_right)

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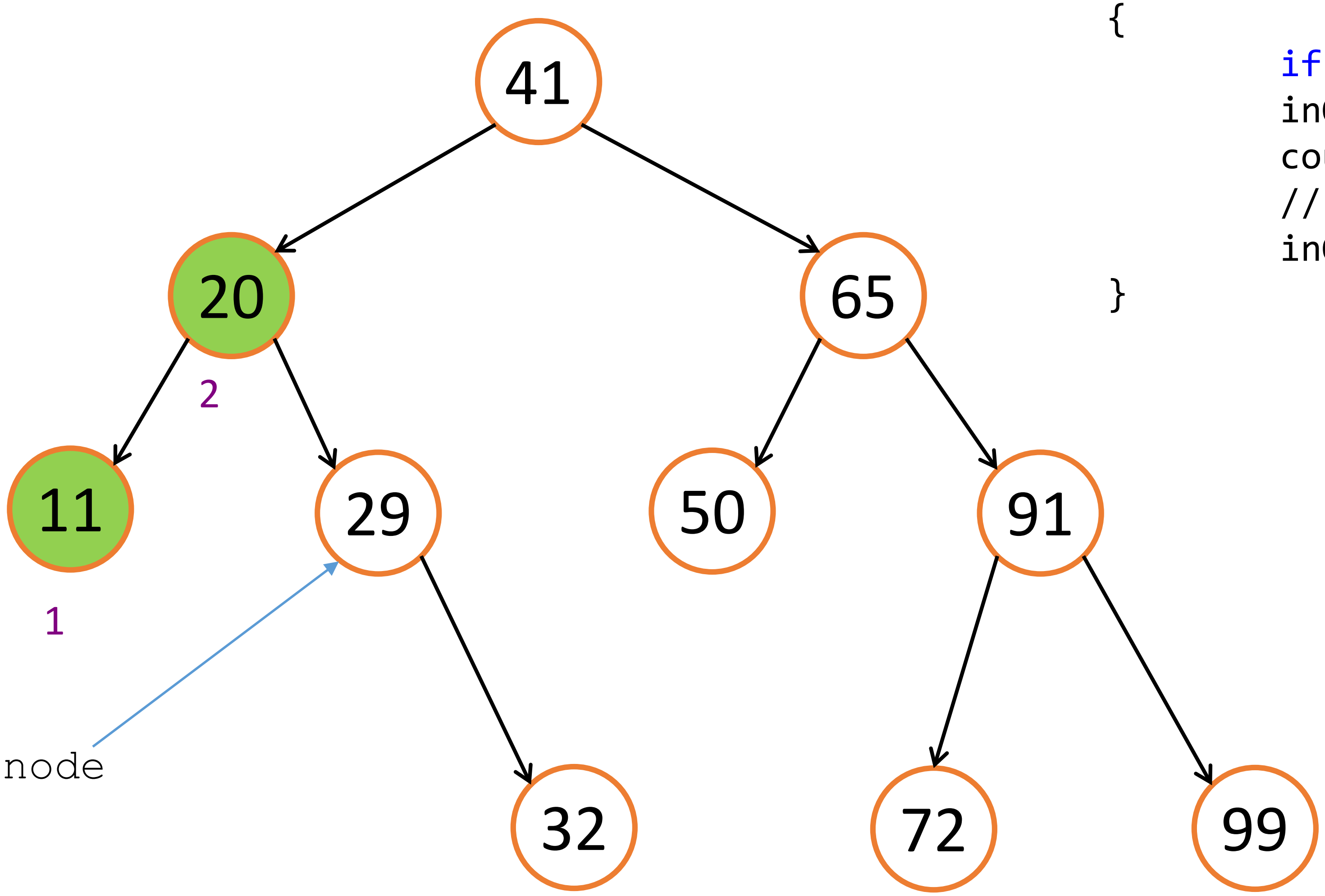
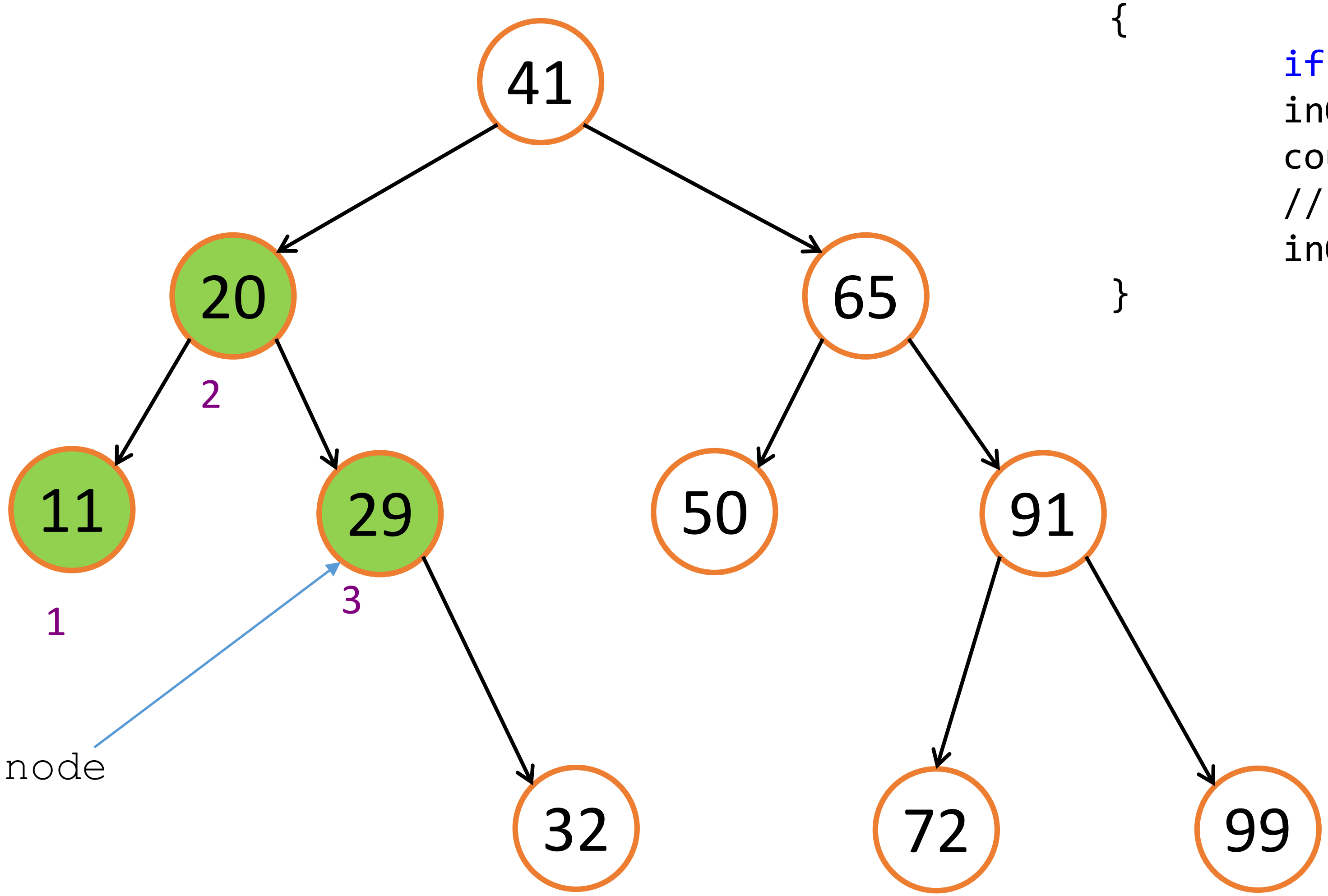


inOrder(node->_right)

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{
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        cout << node->_item << " ";
        // or do something about it
        inOrder(node->_right);
}
```

```
inOrder(node->_left) (null)
cout << node->_item << " ";
```

```
template <class T>
void BinarySearchTree<T>::inOrder
                          (TreeNode<T>* node)
{
        if (!node) return;
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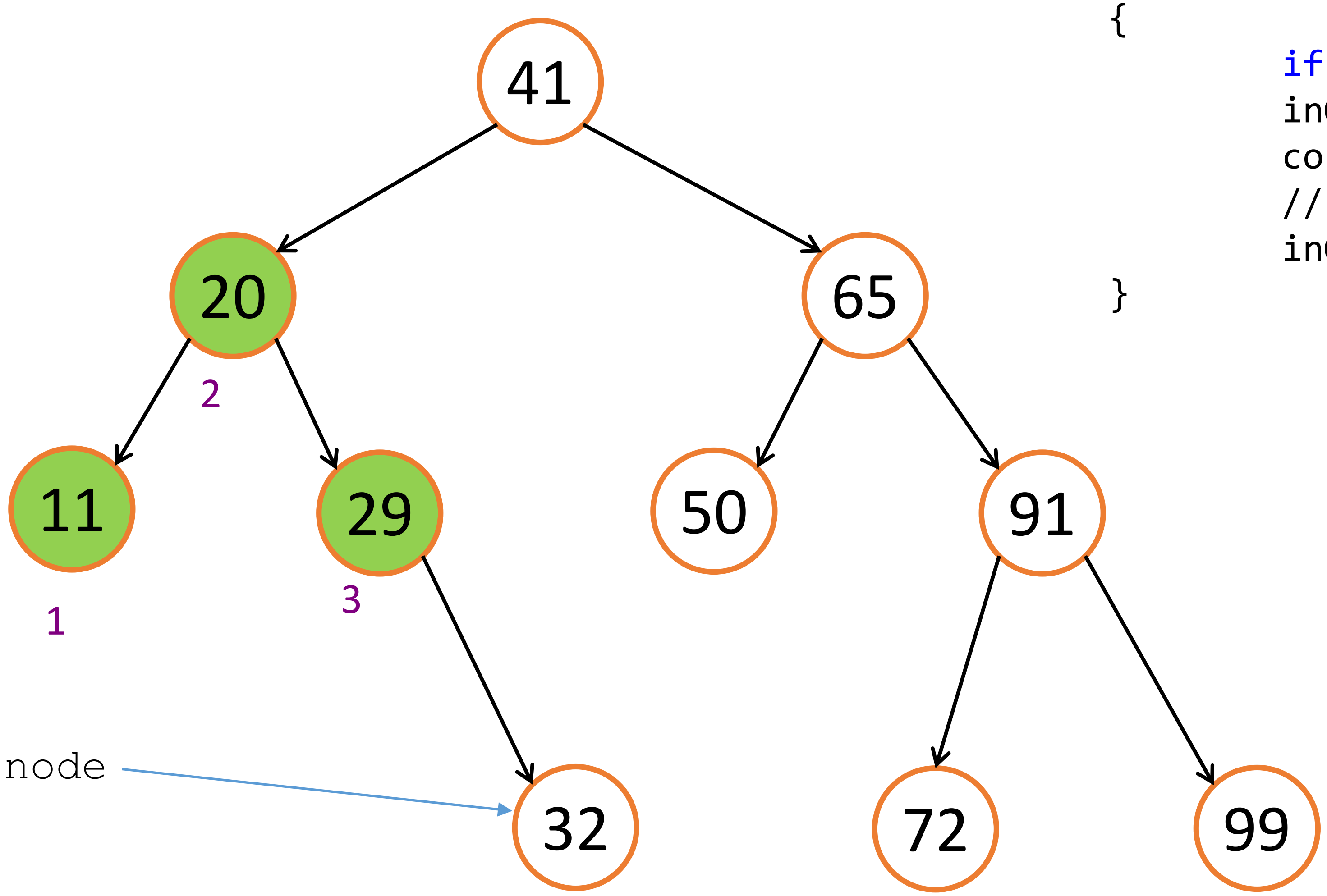

inOrder(node->_right)

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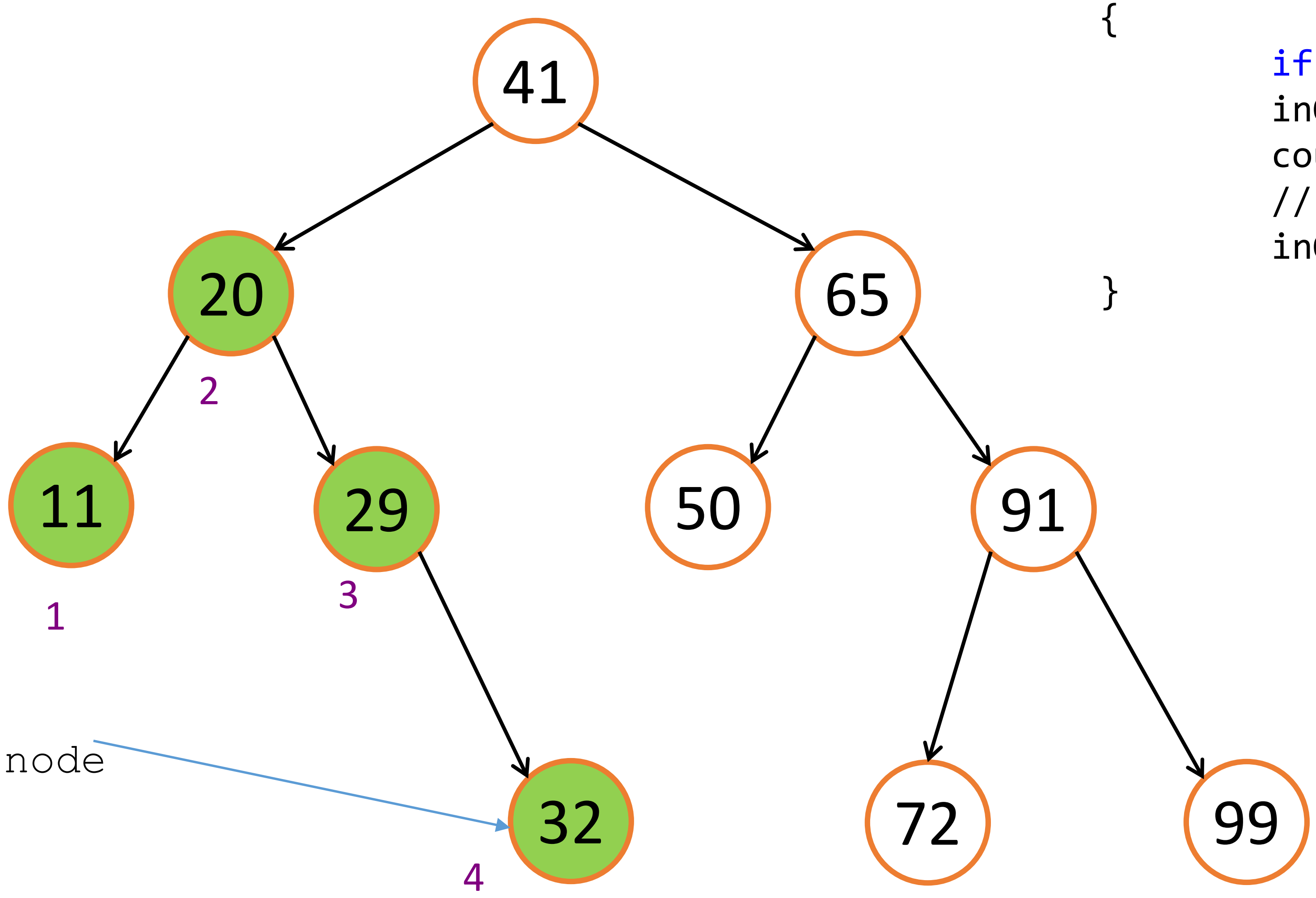
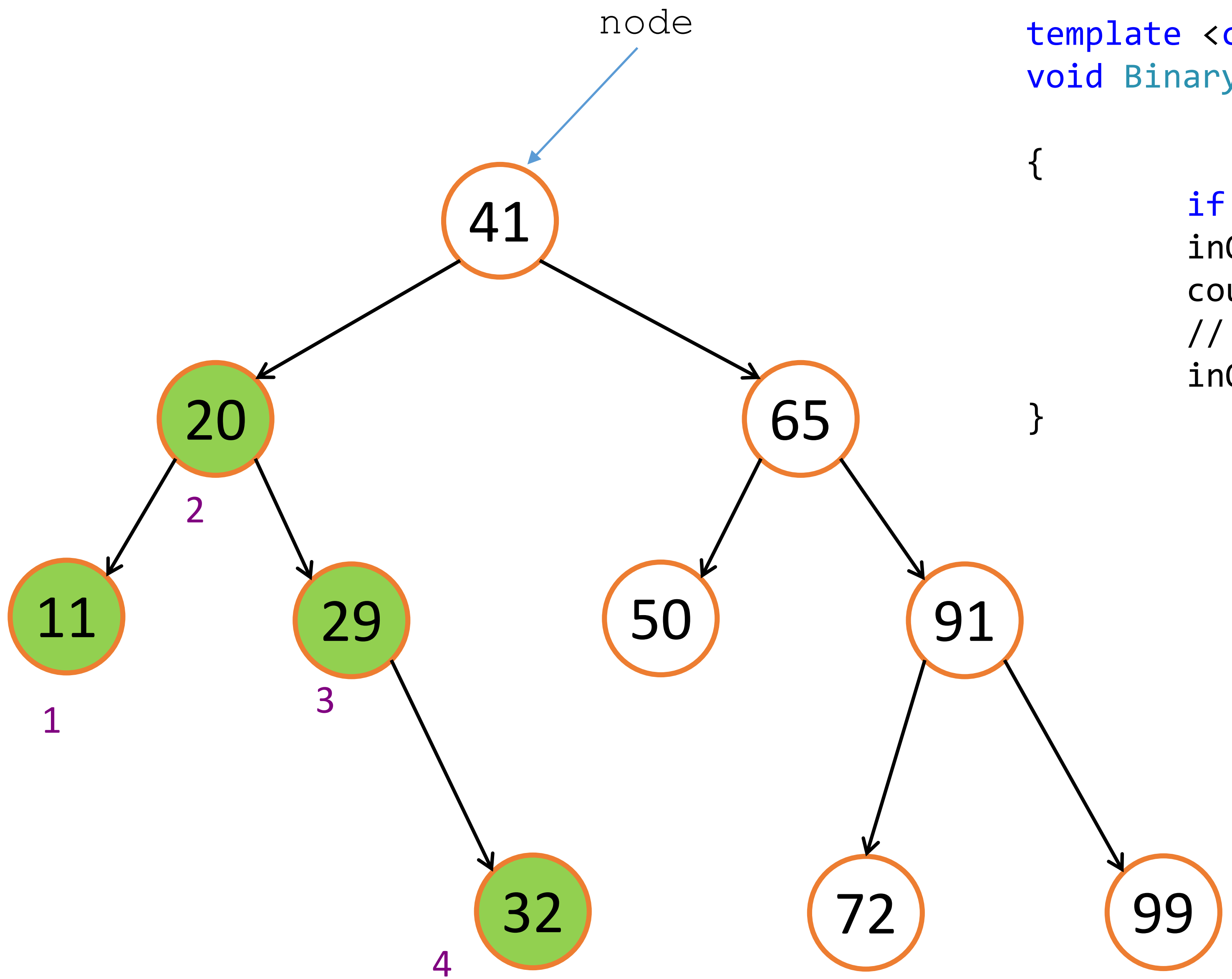



```
inOrder(node->_left) (null)
cout << node->_item << " ";
inOrder(node->_right) (null)
```

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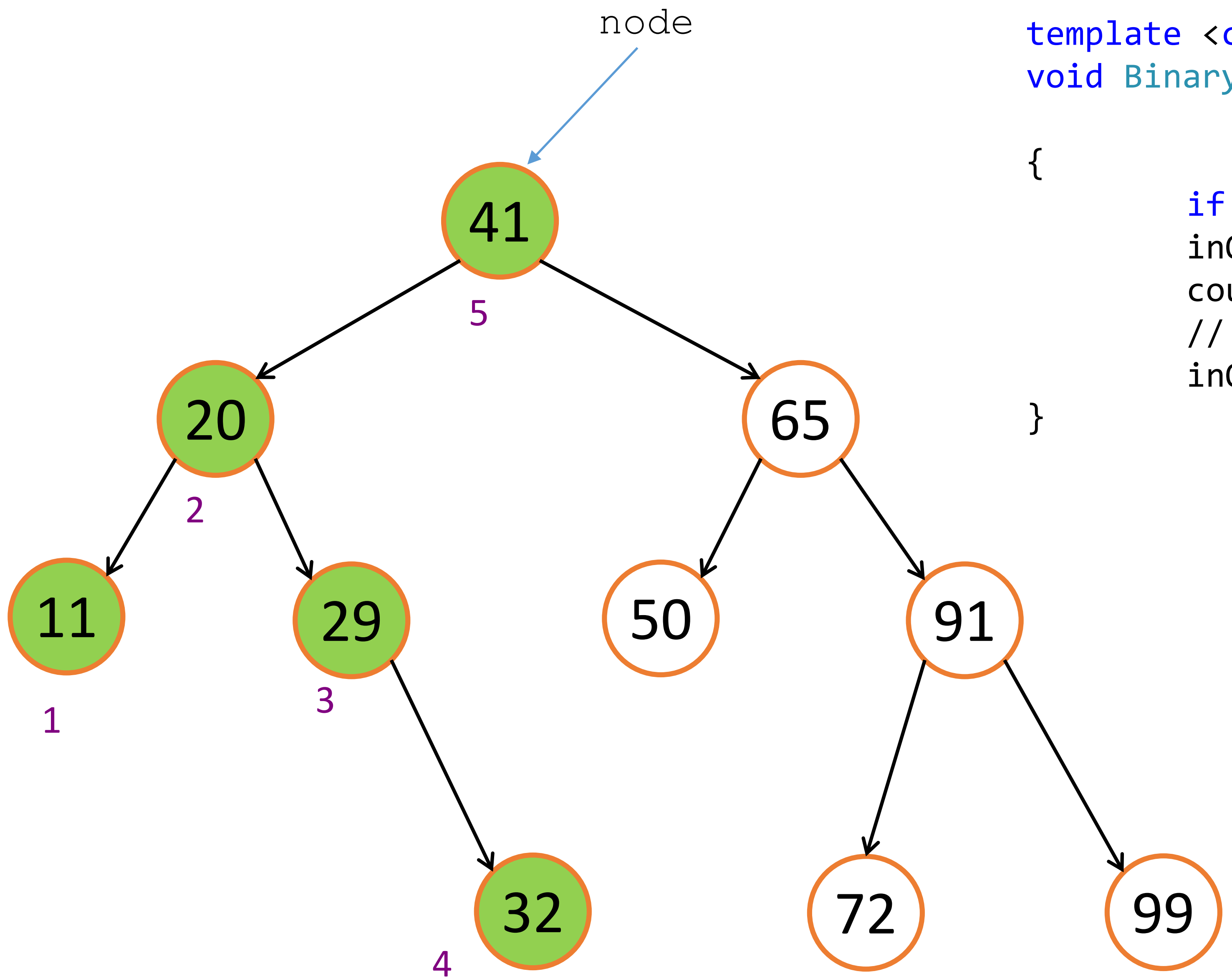
```

return
return
return

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```
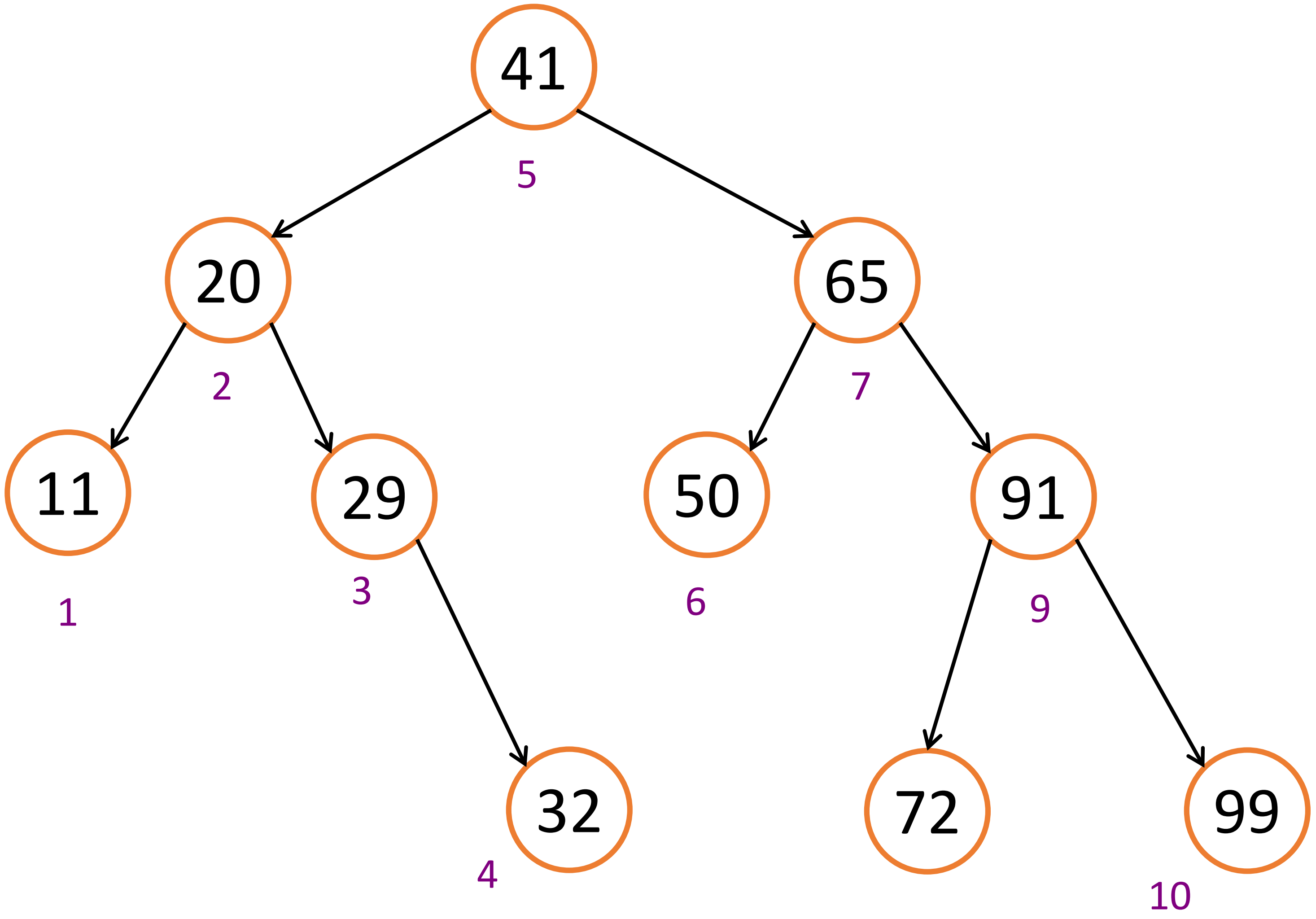

```
cout << node->_item << " ";
```

In-order



11 20 29 32 41 50 65 72 91 99

- Notice something?

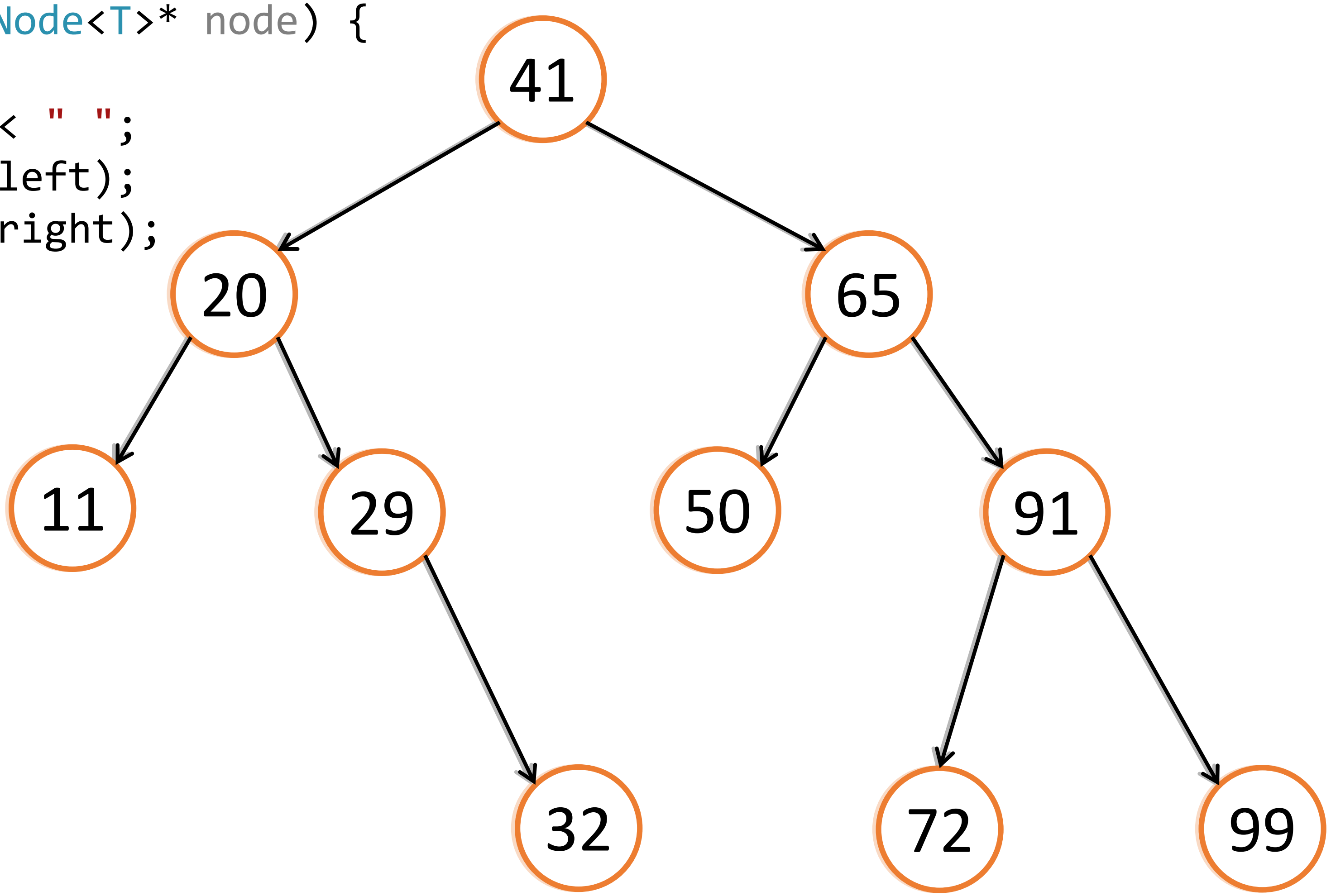
In-order-traversal(v): O(n)

```
template <class T>
void BinarySearchTree<T>::inOrder
                              (TreeNode<T>* node)
{
    if (!node) return;
    inOrder(node->_left);
    cout << node->_item << " ";
    inOrder(node->_right);
}
```

Pre-order-traversal(v): O(n)

```
template <class T>
void BinarySearchTree<T>::
            preOrderPrint(TreeNode<T>* node) {
    if (!node) return;
    cout << node->_item << " ";
    preOrderPrint(node->_left);
    preOrderPrint(node->_right);
}
```

```
template <class T>
void BinarySearchTree<T>::
          preOrderPrint(TreeNode<T>* node) {
       if (!node) return;
       cout << node->_item << " ";
       preOrderPrint(node->_left);
       preOrderPrint(node->_right);
}
```

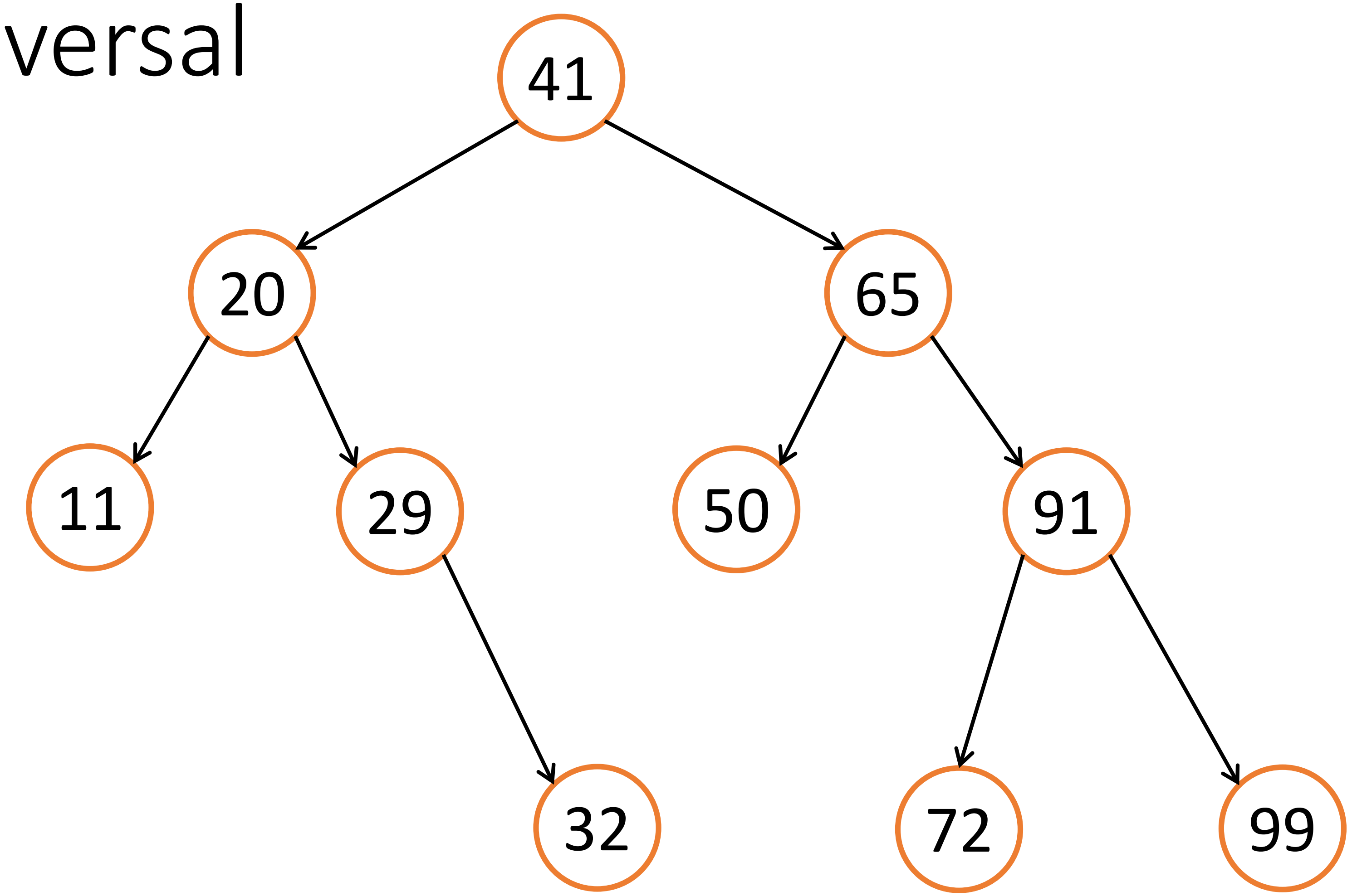


41 20 11 29 32 65 50 91 72 99

Post-order-traversal(v): O(n)

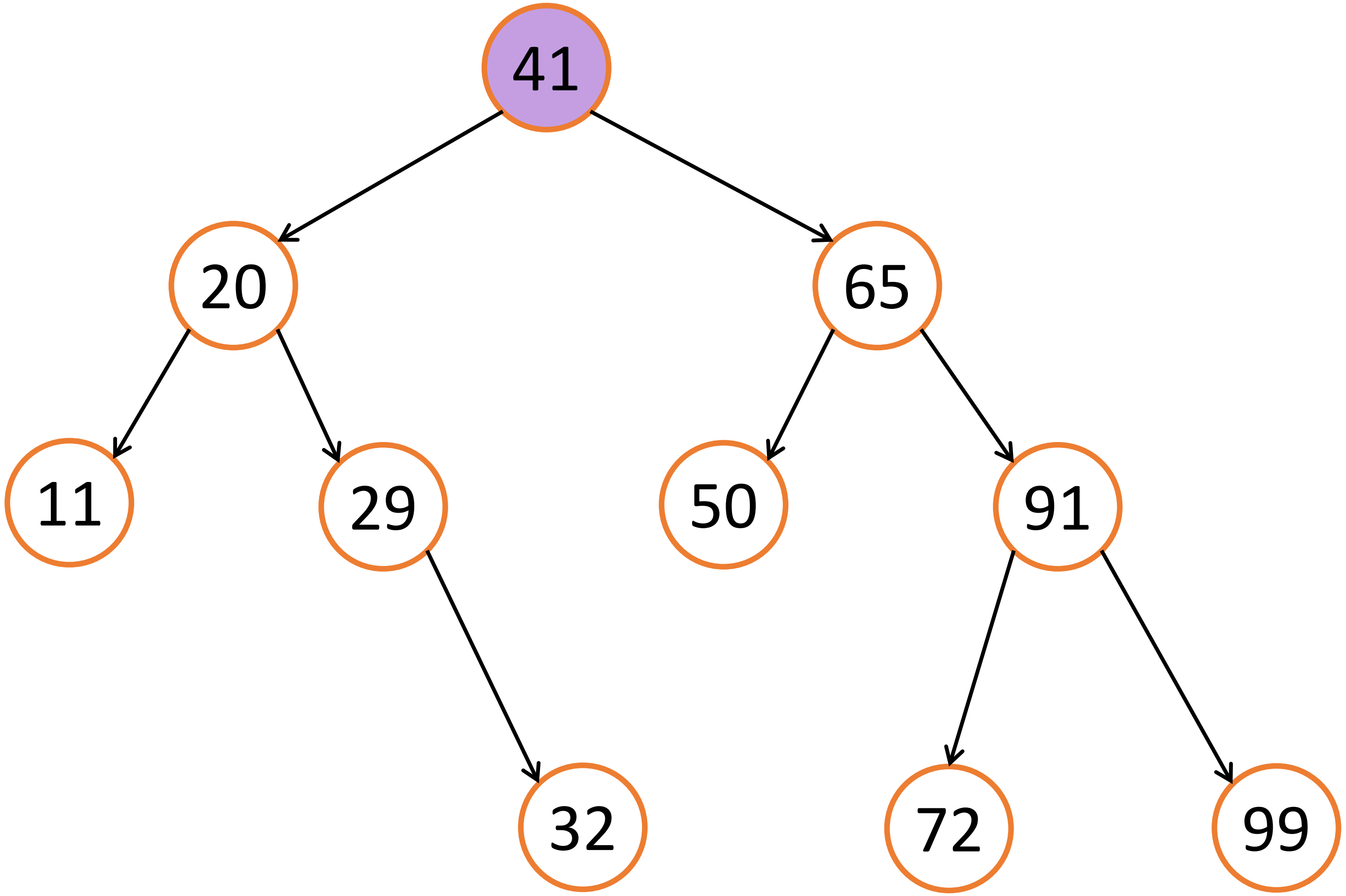
```
template <class T>
void BinarySearchTree<T>::
            postOrderPrint(TreeNode<T>* node) {
    if (!node) return;
    postOrderPrint(node->_left);
    postOrderPrint(node->_right);
    cout << node->_item << " ";
}
```

Post-order Traversal



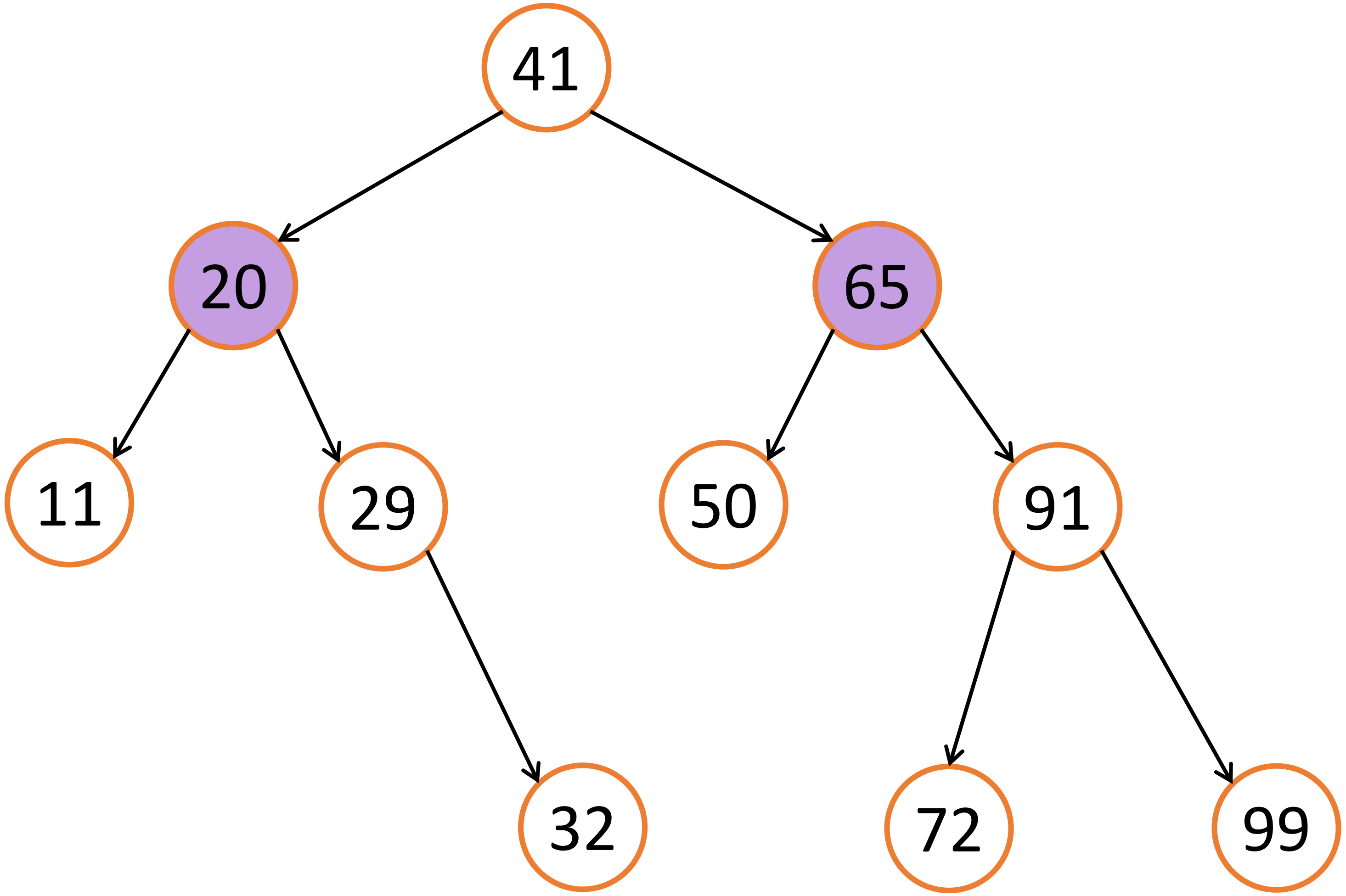
11 32 29 20 50 72 99 91 65 41

Level-order Traversal



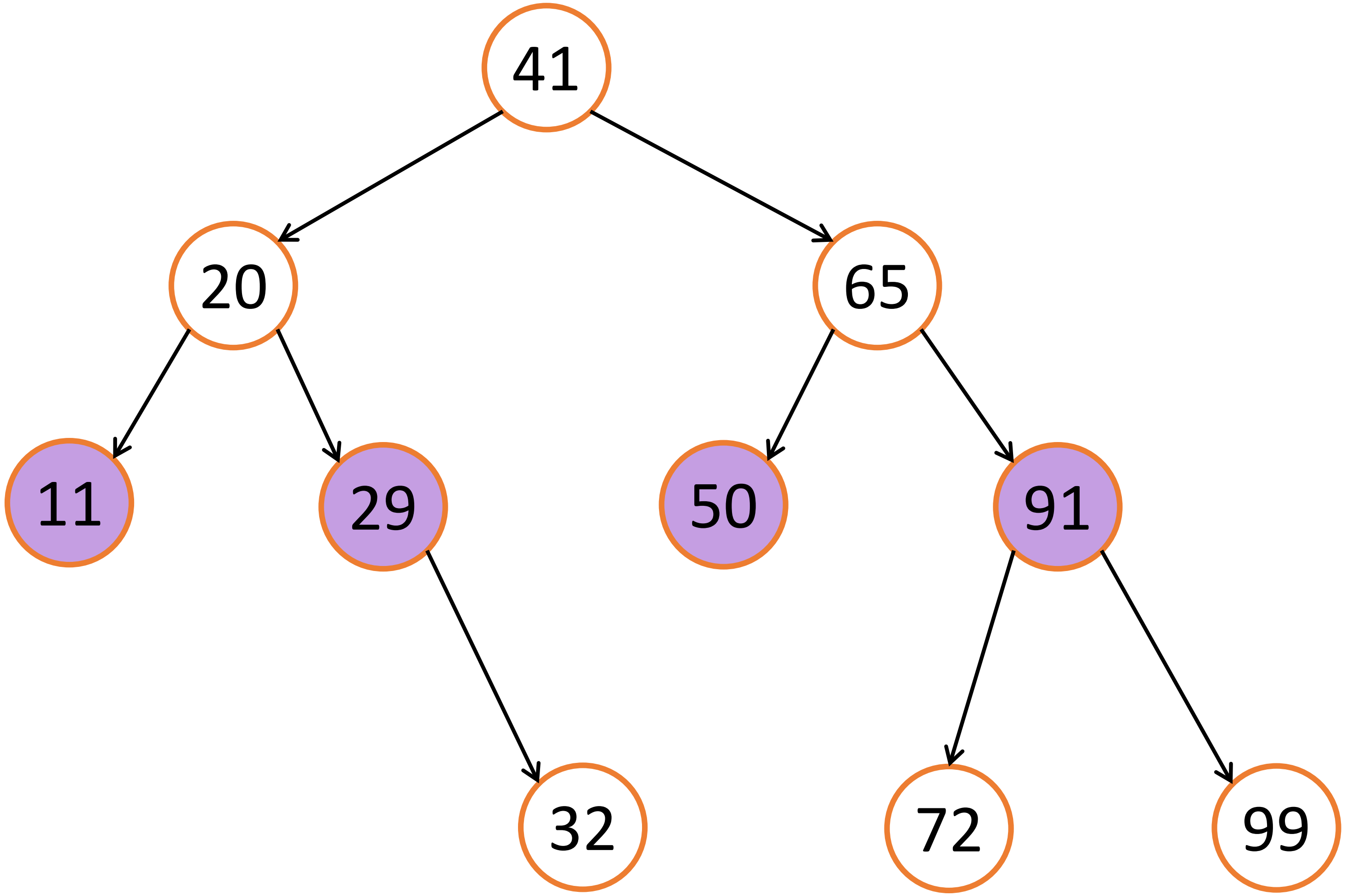
41

Level-order Traversal



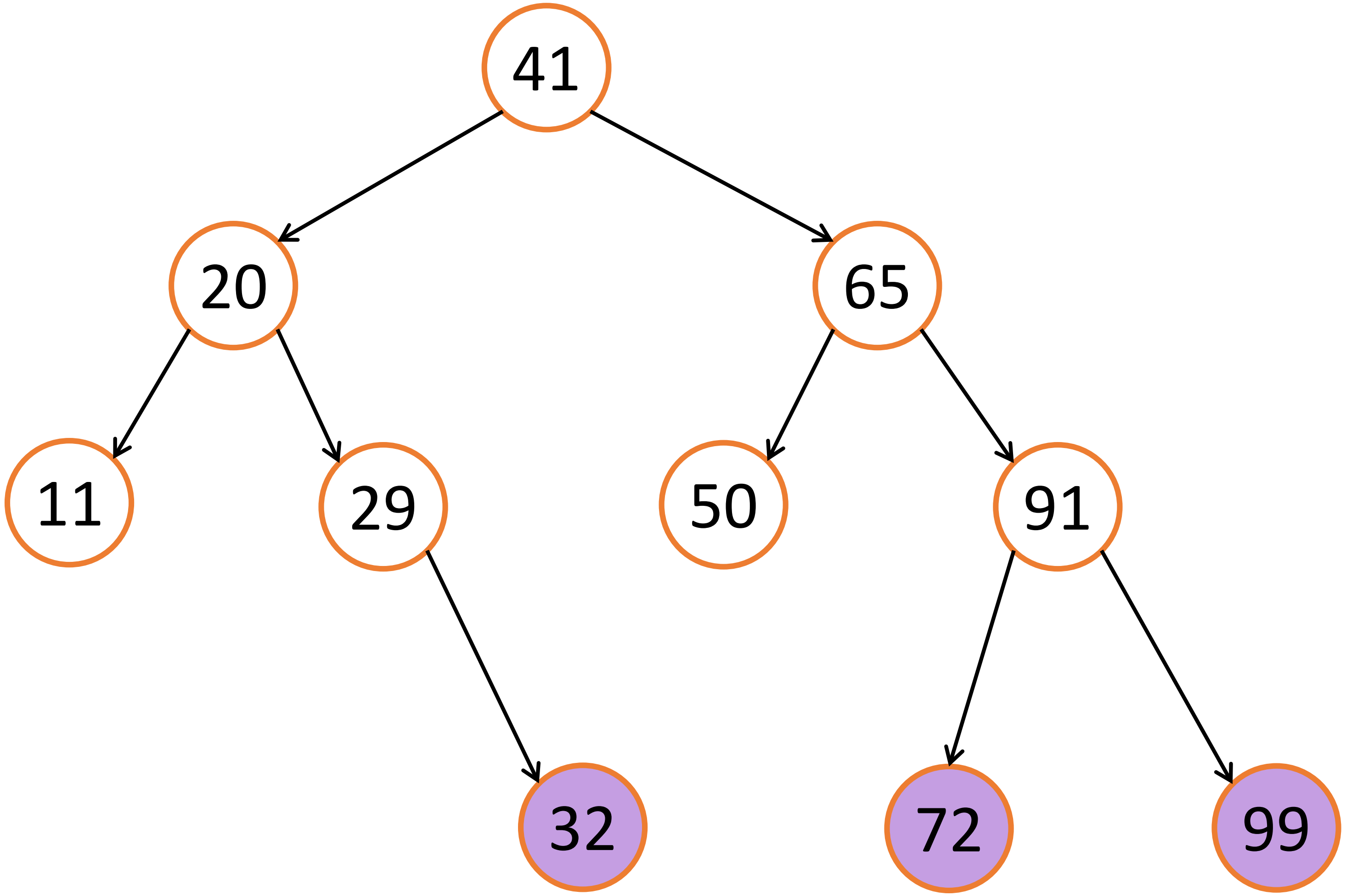
41 20 65

Level-order Traversal



41 20 65 11 29 50 91

Level-order Traversal



41 20 65 11 29 50 91 32 72 99

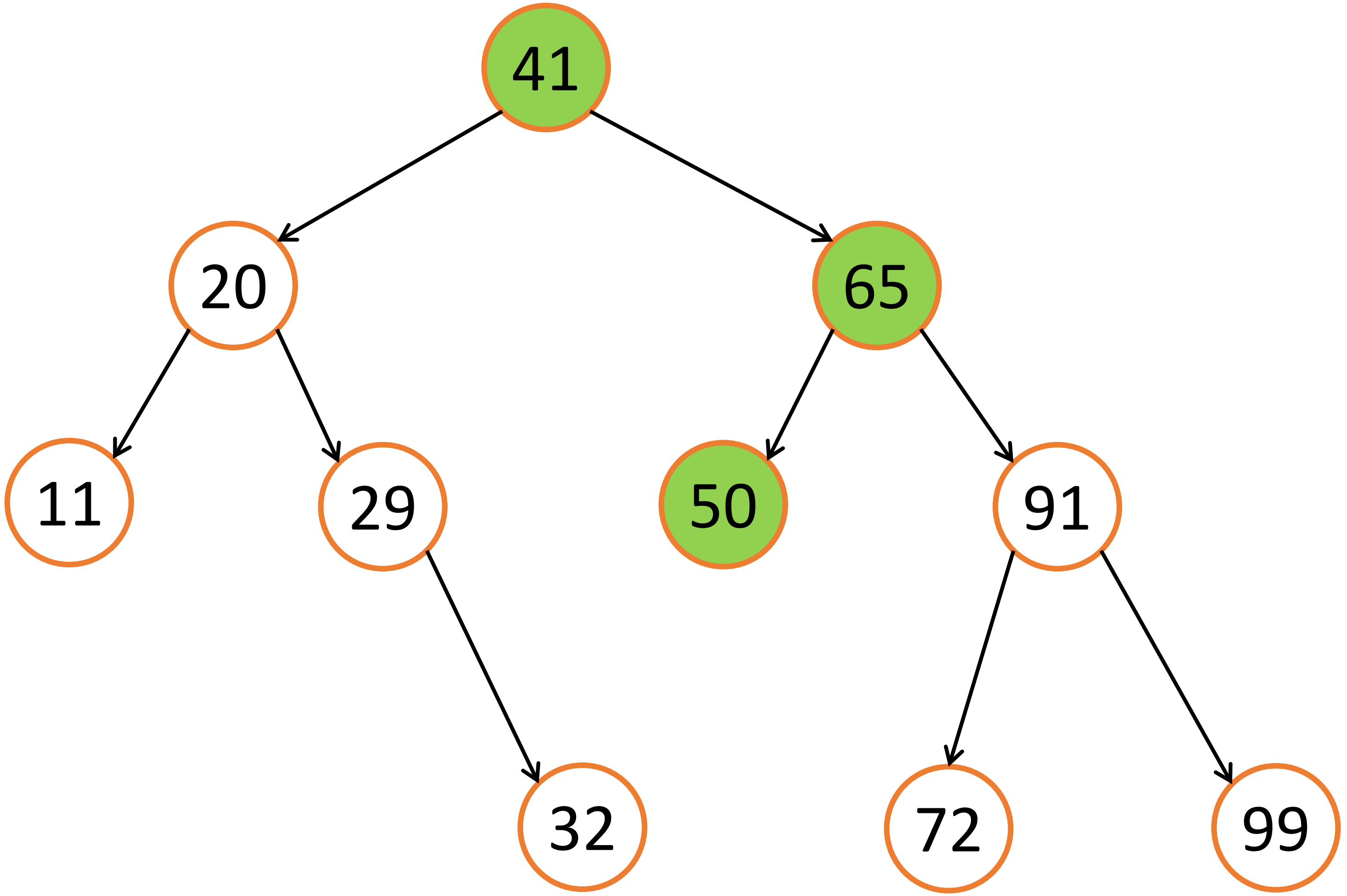
Application: Airport Scheduling

6:35	**7:00**	**7:19**	**8:21**	**12:21**	**14:23**	**14:42**			

- What is the next plane after 8:30?

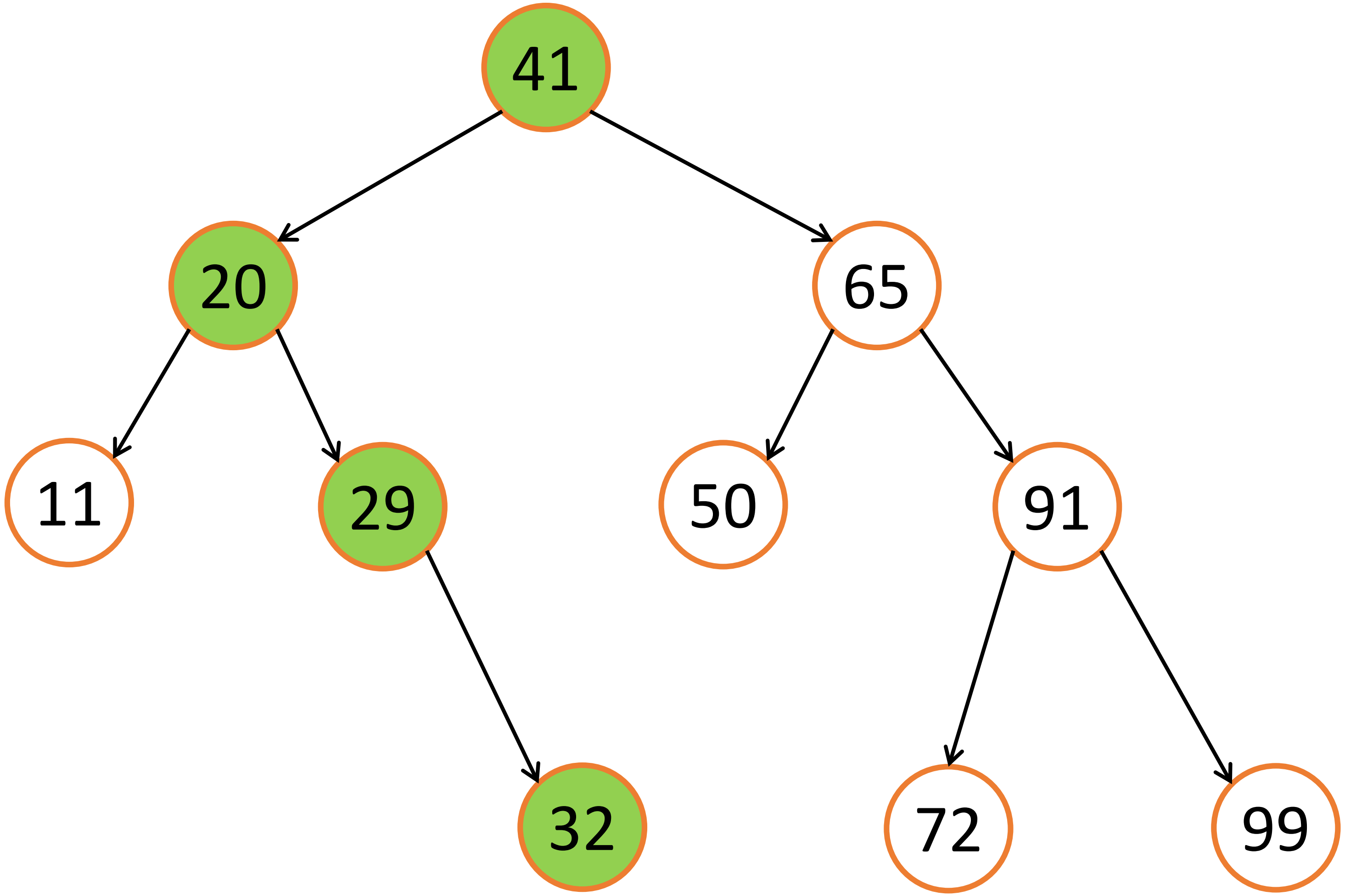
Successor Queries

- successor(42)
- When 42 is NOT in the tree



Successor Queries

- successor(33)
- When 33 is NOT in the tree

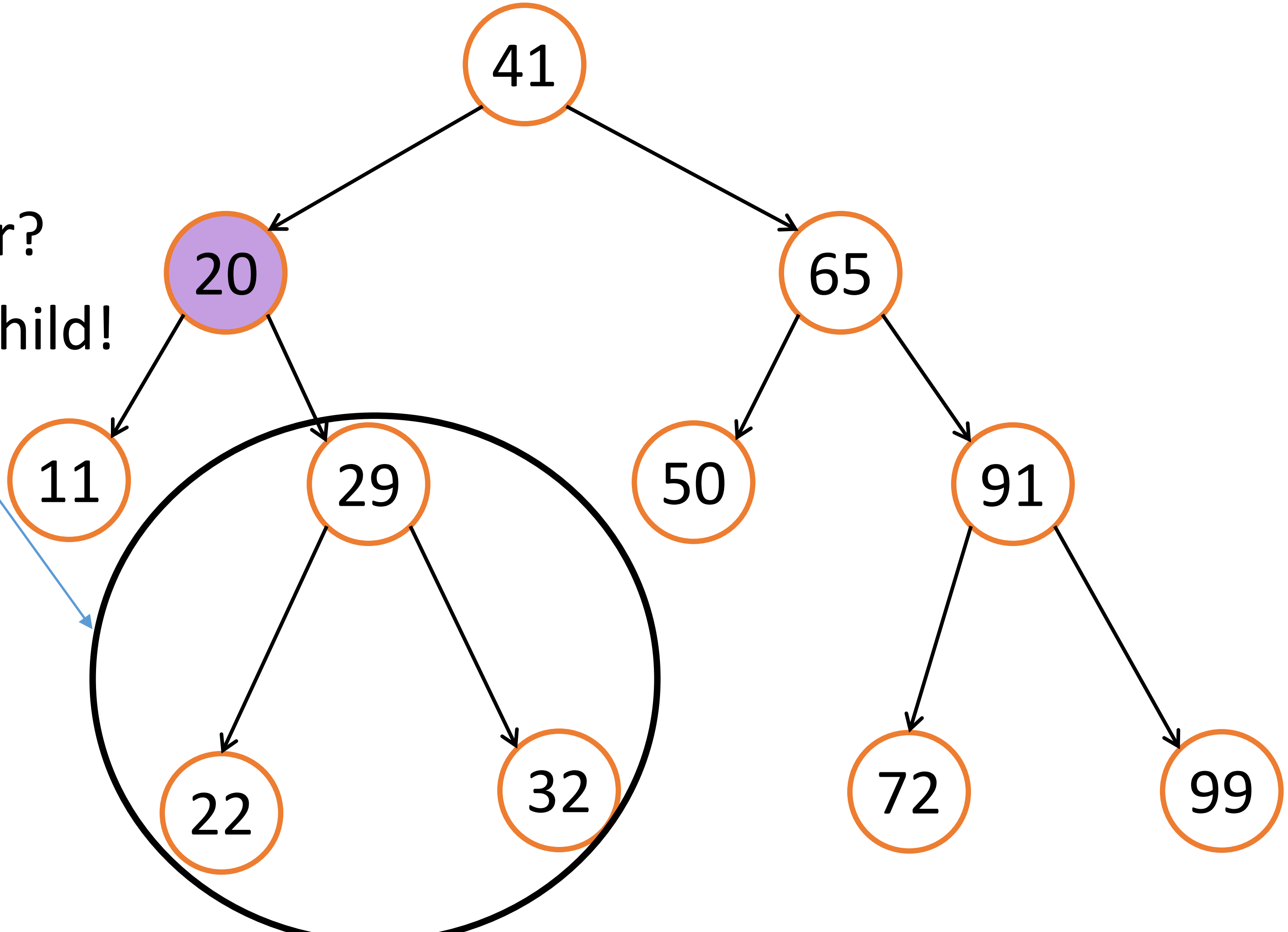


Basic strategy: Successor(key)

1. Search for key in the tree.
2. If (result > key), then return result.
3. If (result <= key), then ….?

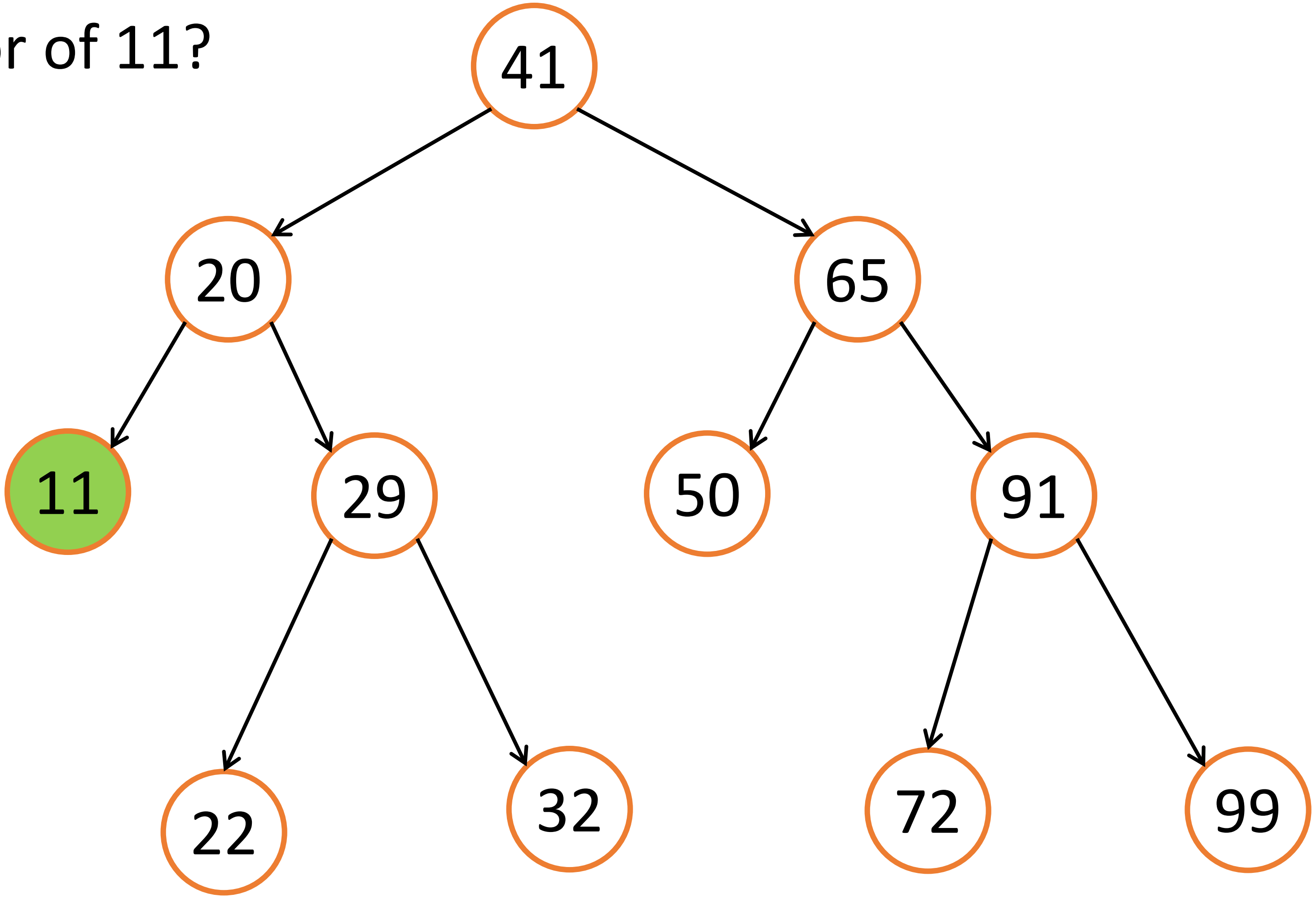
Successor Queries

- If the key is IN the tree
- s.g. successor(20)
- Which one is the successor?
- The minimum of its right child!
- Why?



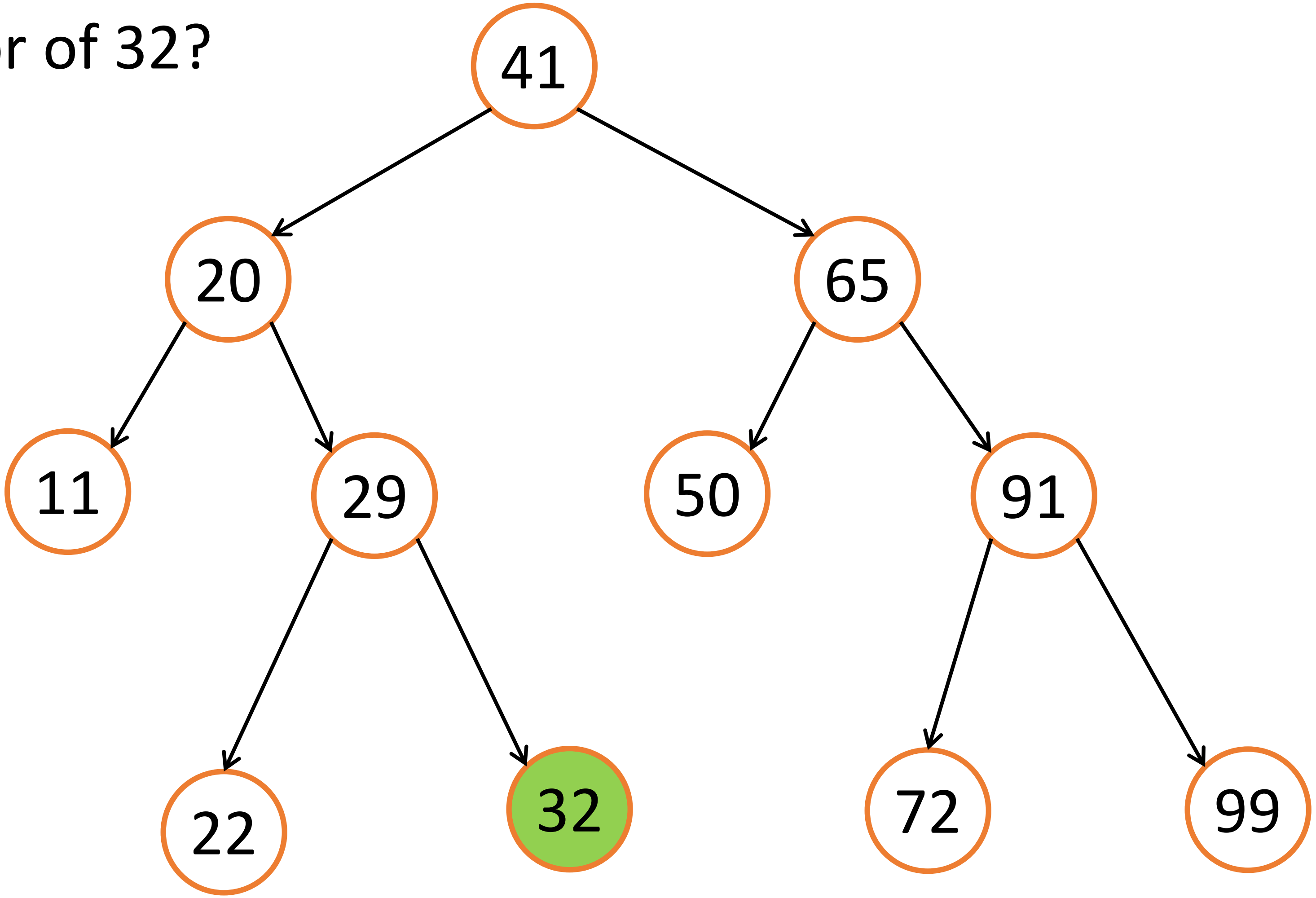
Successor Queries: No Right Child?

- Which one is the successor of 11?

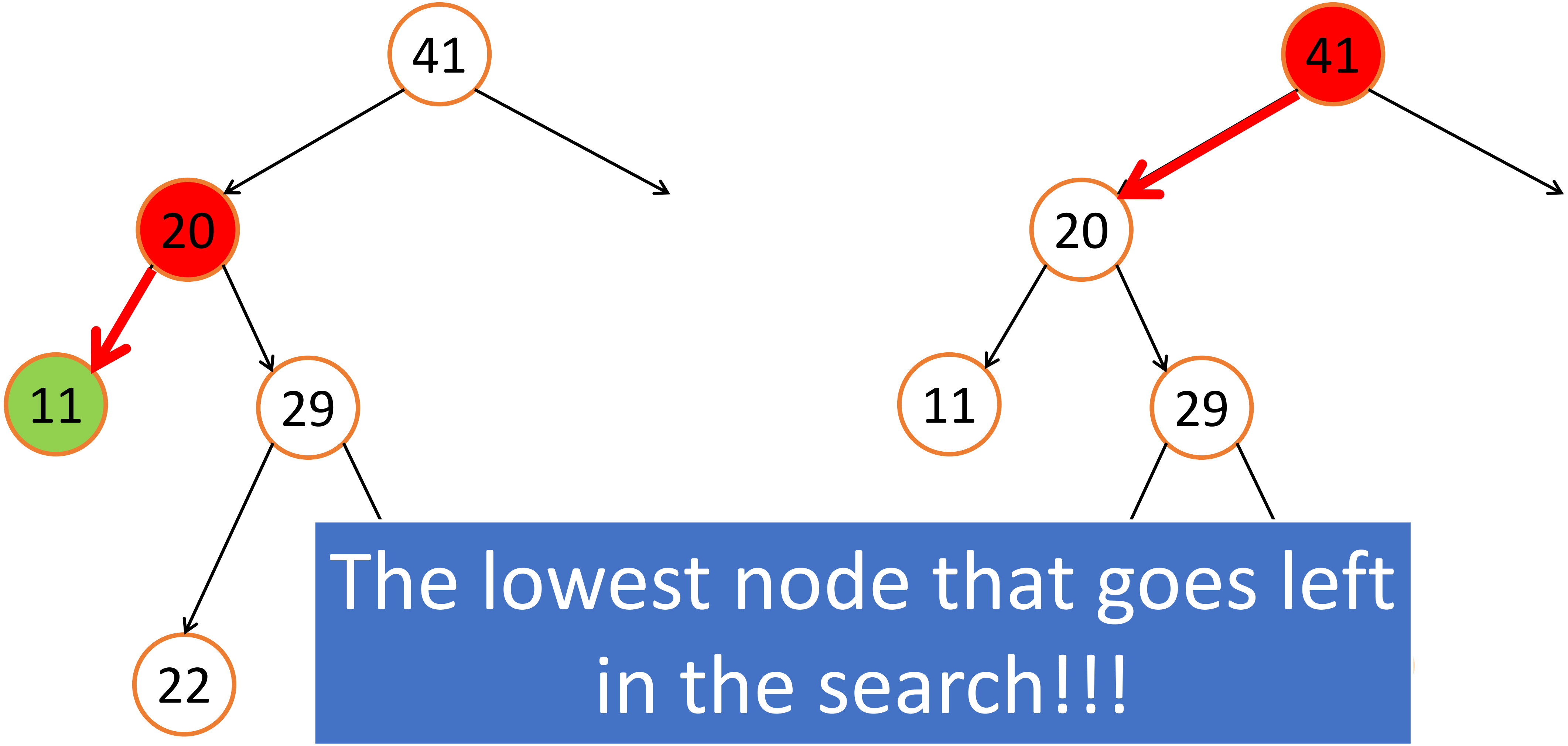


Successor Queries: No Right Child?

- Which one is the successor of 32?



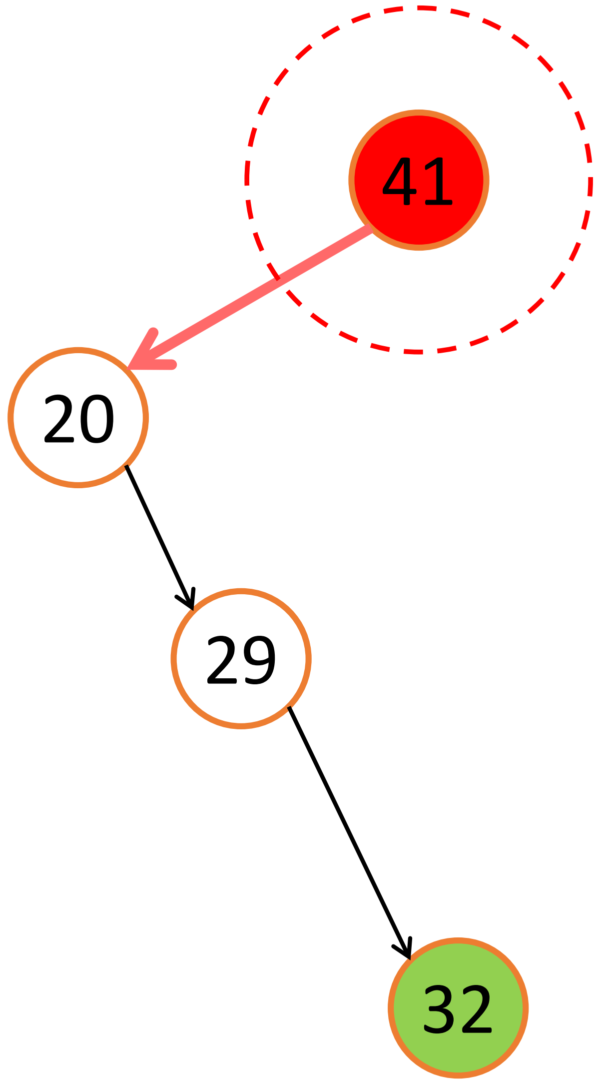
What is the common pattern?



The lowest node that goes left in the search!!!

Successor Queries

- Follow the search to find the node for x
- If the node has a right child
  - return the max of its right subtree
- Follow the search path from the root, find the last parent that goes left to find x



Works for The key NOT in the Tree

- successor(33)