

# Investigation of General Equilibrium Modelling for Domestic Sectoral Policy Analysis

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## Disclaimer

*The analysis and policy proposals presented in this paper are not government policy and only represents an academic investigation into this policy area.*

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## Abstract

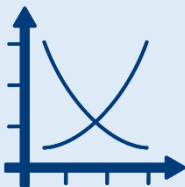
The UK, like many advanced economies subsidises advanced manufacturing sectors (Broadberry & Leunig, 2013), particularly if the technologies associated are considered 'Green'. For example, the USA passed the Inflation Reduction Act (IRA), (White House, 2022) and future EU investments will be made by The Green Deal Industrial Plan (GDIP), (European Commission, 2023). There are presumed to be multiple benefits to these decisions, however this is a market distortion to a true free market equilibrium. The neo-classical approach (Cordato, 1980) states that without market failures the government should not intervene in the market and that it may produce less optimum outcome, called Government Failure (Grand, 2009). The government must demonstrate the existence of market failures before intervening and that its decision would be an economic and social benefit to the UK. The former is covered in the Five Case Model, (HM Treasury, 2018). To confirm the latter, analysts must perform ex-ante modelling of the intervention's impacts (Munby, 2023).

This paper will investigate the potential introduction of General Equilibrium Modelling (GEM) to analyse interventions in advanced manufacturing sectors, with a specific focus on the automotive industry due to the author's area of work.

## Non-Technical Summary



Government policy of intervention into a sector (providing grants or tax relief) requires that economic modelling is completed. The current methodology used for advanced manufacturing sectors looks at the potential benefit of increased salary resulting from individuals moving to more productive jobs. It does not model what sectors would have reduced employment and the impacts of that on the wider economy. Nor does the current methodology model consider the impacts of trade.



This paper investigates the use of a Computable General Equilibrium Model (CGEM) to answer the question of impact on other sectors. CGEM attempts to model all the economy and the interaction between sectors based on the behaviours of economic actors: consumers, firms, and the government. CGEM can be expanded to include international trade.



This paper was unable to construct a satisfactory CGEM to be used in policy appraisal due to the weakness of the approach. The paper does address those weaknesses and proposes solutions. It also explains the benefits of continuing work in this area for policy appraisal and departmental collaboration.



## Context

### Government Departmental Changes

The Department for International Trade (DIT) was created on 14<sup>th</sup> July 2016 and became part of the Department of Business and Trade (DBT) as part of a Machinery of Government (MOG) change on 7<sup>th</sup> February 2023. DIT had a remit to complete trade policy analysis. To meet this remit the independent report: 'Trade modelling review expert panel', recommended development of Computable General Equilibrium Model (CGEM) capabilities, (Venables, 2022).

This remit for external policy focus has been combined with the Department for Business, Energy & Industrial Strategy (BEIS) domestic business policy focus. Therefore, allowing government to have a joined up domestic and international policy to strengthen UK industry. As DBT matures, this joined up policy direction will embed, and a requirement for shared analytical approaches will grow. Therefore, the domestic policy analysis approach of BEIS could be expanded by incorporating the approach previously used for external policy analysis in DIT.

### Collaboration

After discussing with ex-DIT trade analyst colleagues, the modelling approach investigated in this paper will be based on the work of (Böhringer, et al., 2003) and (Wing, 2004). These discussions formed the original motivation for the work.

### Policy Area - Market Failures

Government needs to conclude that there are market failures before it can intervene. The following section will illustrate there is a strong economic rationale for government intervention in the advanced manufacturing sector due to the presence of market failures. Some market failures affect the whole sector, while others are specific to a particular stage, such as R&D investment. Current His Majesty's Government (HMG) assessment of different market failures is categorised by the breakdown given in Figure 1. This paper will focus on the production side of the industry, specifically capital investments. The example of market failures below builds on the information provided in the Green Book 2022, Section 4.23<sup>1</sup>.

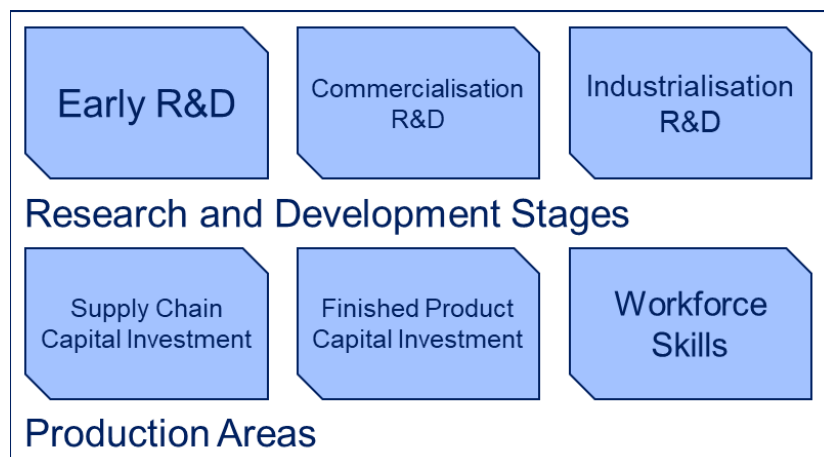


Figure 1 Industry Sections

### Positive Externality

The industry is currently transitioning to the production of Electric Vehicles (EV) from Internal Combustion Engine Vehicles (ICEV). EV has a positive externality of reduced CO<sub>2</sub>

<sup>1</sup> [https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment\\_data/file/1063330/Green\\_Book\\_2022.pdf](https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1063330/Green_Book_2022.pdf)



emissions when consumed compared to ICEV (USEPA, 2023), even when including the manufacturing process (FCAB, 2021).

As the Marginal Social Benefit (MSB) is greater than the Marginal Private Benefit (MPB) there will be an under consumption and production of the good (Pindyck & Rubinfeld, 2009). Therefore, the government can intervene to promote the consumption and production of that good.

### *Barriers to Entry and Market Power*

For the market to be considered a perfect competition it must contain many firms with none having high market power. This is enabled by lower barrier to entry and exit from the market. The automotive industry and its supply chain have high barriers to entry due to the very high initial fixed capital costs; for example, the new gigafactory<sup>2</sup> for Jaguar Land Rover (JLR) will require investment of £4bn (BBC News, 2023).

Certain elements of the electrified supply chain are already characterised by a monopolistic supply, for example over 70% of the natural graphite supply is controlled by China.<sup>3</sup>

This market power enables those Chinese firms to extract profits from European firms by charging above the equilibrium price. The well-established nature of these firms in Asia and the large amount of subsidisation they receive means that European new entrants to the market cannot compete.

This uncompetitive international behaviour results in EV supply chain projects typically having low margins<sup>4</sup>. Therefore, they attract low levels of investable finance.

### *Information Failures*

Although EV have been produced for years, this has not been at the scale required to transfer the entire new car market away from ICEV, a stated goal of the UK government (HMG, 2020) and the EU (Reid, 2023) by 2030 and 2035 respectively. There are multiple competing chemistry combinations that an EV cell can be, resulting in uncertainty in investment.

The performance and quality of an EV is driven by this chemistry and the quality of the production process. The knowledge to understand EV cells is complex and therefore the consumers of the good (motorists) will not be able to have accurate information and the market has failed (Pindyck & Rubinfeld, 2009). This is also true for Original Equipment Manufacturers (OEM) who use the cells as intermediate goods.

Due to the information failure, a firm can invest in one specific chemistry type, which does not develop as productively as others requiring them to retool and invest more. With the high risk of error, the industry will often keep with existing technologies and be too cautious, delaying investment until proven elsewhere.

This risk reduces the investment levels in the EV technologies further.

### **Policy Area – Intervention**

The above is a summary of the market failures which were used to justify the Automotive Transformation Fund (ATF), (Munby, 2023), a capital grant fund offering a share of the total budget, valued up to £1 billion, (HMG, 2022) to intervene in the EV supply chain. The total intervention into the sector is called the programme and individual cases are the projects.

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<sup>2</sup> Gigafactory is the term for a site making cells to go into the batteries of EVs.

<sup>3</sup> Redacted internal HMG source.

<sup>4</sup> Redacted internal HMG source.



### Impact/Benefit Justification

For any grant provided by HMG an ex-ante assessment is completed including an economic appraisal. This is done in line with 'Green Book' (GrB) principles. The most common approach is a Cost Benefit Analysis (CBA) appraisal using the Benefit Cost Ratio (BCR) as a metric. The GrB considers the social welfare benefit of bringing advanced manufacturing to the UK as the move to more productive jobs.

The GrB assumes that the economy is working at or near full employment and therefore only the wage above the next best alternative wage is a benefit. The next best alternative wage is assessed against a local alternative.

Advanced manufacturing, for example automotive, have higher average wages than alternative employment. Most advanced manufacturing occurs outside of London and the Southeast aligning policies with the Levelling Up<sup>5</sup> agenda.

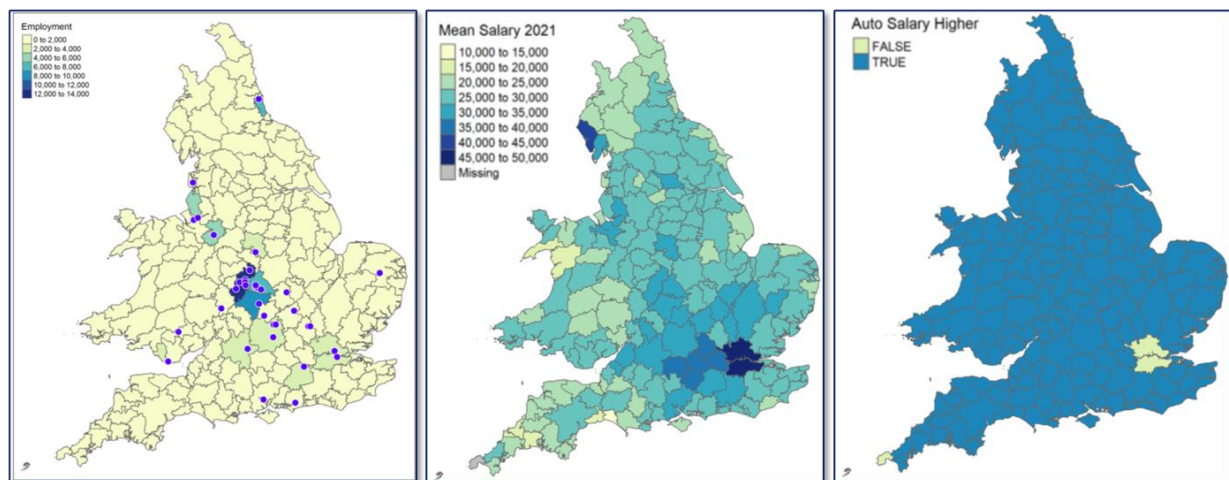


Figure 2 Automotive Employment Maps – ONS data

The maps in Figure 2 show the location of automotive employment, an example counterfactual salary, and the areas of the country in which automotive employment represents a move to a more productive employment. This is based on Office for National Statistics (ONS) data.

This requires a methodology to calculate the number of jobs which will shift into the sector based on the intervention. This paper presents the current methodology and a potential alternative for this calculation.

The above justification and benefit definition are used at the programme level intervention, but it has been used for individual cases. This paper will review the current and alternative methodologies when supporting the whole sector. **This is a live policy debate within HMG and contains commercially sensitive information. Therefore, where appropriate information has been redacted for a wider audience.**

<sup>5</sup> <https://www.gov.uk/government/publications/levelling-up-the-united-kingdom>



## Methodology

### Current Methodology

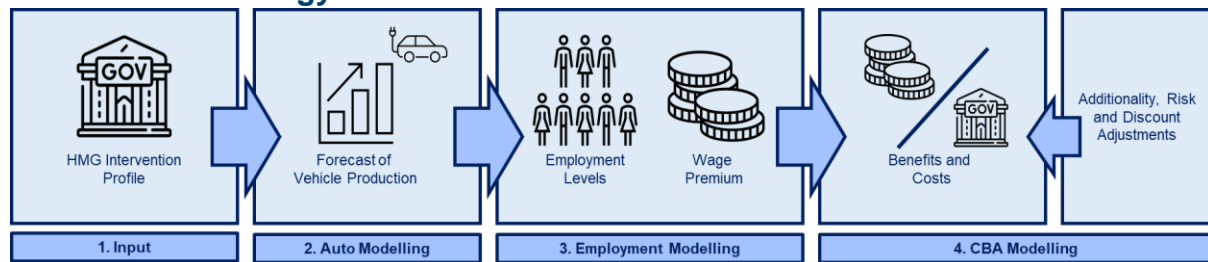


Figure 3 Current Methodology Overview

The current methodology builds a model of the automotive industry in isolation (Part 2 in Figure 3) to the rest of the economy. This methodology is currently in use in the appraisal of an active policy decision; **therefore, where appropriate information has been redacted for a wider audience, including the results of current modelling.** A version of the current methodology was originally derived by the author and was used to complete the Value for Money assessment presented on gov.uk by the then Permanent Secretary of BEIS, (Munby, 2023).

The model considers the capacity in the UK to produce vehicles, which is a capital-intensive production and therefore is limited by the number of factories. These factories are presented on the first map in Figure 2.

As a result of the transition from ICEV to EV the new supply chain will be a further limiting factor on the production capabilities in the UK<sup>6</sup>. Therefore, investing in either the OEM or EV supply chain will create new levels of production and employment.

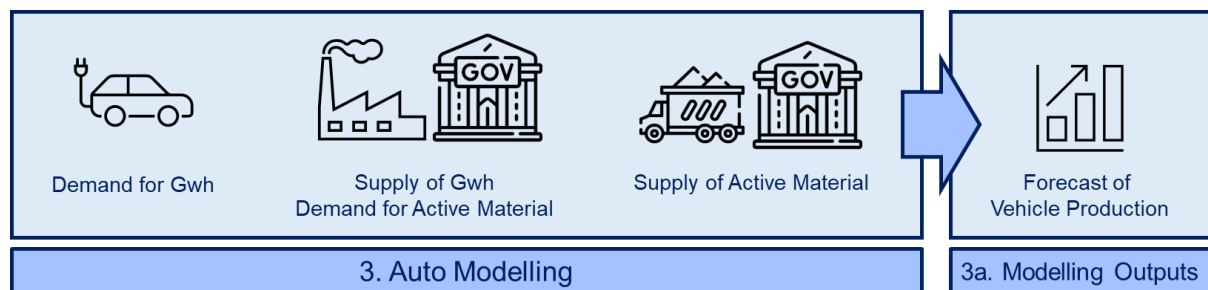


Figure 4 Automotive Modelling Diagram

The UK exports the majority of the vehicles produced, c80% annually. Demand for UK produced vehicles is exogenous in this methodology and cannot account for the external factors like IRA and GDIP.

The current methodology provides a quantitative assessment of the number of jobs in the sector with and without intervention. The difference in job numbers is multiplied by wage premium associated and is used as the benefit in the CBA, (Part 4 in Figure 3). It does not specifically model what the opportunity cost of the economic activity foregone is. There is no consideration given to: what sectors have a reduction in employment, potential changes in output and increases in prices and the wider social-economic impacts.

For this reason, the alternative methodology is proposed and will be investigated for usefulness in this paper. When incorporating the alternative methodology's results into the CBA modelling (Part 4 in Figure 3) the same GrB approaches to additionality, risk and

<sup>6</sup> Redacted internal HMG source and reasoning.





optimism bias would have to be used. There is a further discussion on this in Annex B:  
Technical: Additionality

### **Alternative Methodology**

The alternative methodology proposed is a Computable General Equilibrium Model (CGEM), which constructs a Walrasian general equilibrium in which supply and demand are equalised across all the interconnected markets in the economy, (Wing, 2004). It would be looking to replace Part 2 & 3 in Figure 3.

There is international precedent for the use of CGEM in policy formation, notably Norway and Australia, (Dixon & Jorgenson, 2013). They have been using single-country models since the 1960s and 1975 respectively. Since the 1990s other governments and Non-Governmental Organisations (NGO) have built and maintained CGEM to help policy assessment, including US International Trade Commission, the Organisation for Economic Cooperation and Development, and the World Bank, (Burfisher, 2011).

### **Uncertainties**

A key assumption for the current methodology is that advanced manufacturing sectors will continue to pay a wage premium compared to other sectors. This is exogenous to the current methodology but would be endogenous to CGEM. This wage premium is assumed to be a result of the productivity of those sectors. An investigation into this uncertainty for the current methodology can be found in Annex B: Technical: 8 - Wage Premium Uncertainty.

There is uncertainty that the data used in the CGEM will be detailed enough to account for the production change from ICEV to EV. There is a further discussion of this point in Generality of Production Function.

### **Project Aims**

To investigate the possibility and usefulness of developing a CGEM to analyse programme level intervention into advanced manufacturing sectors. The worked example will be the automotive sector and its transition from ICEV to EV production.

### **Project Objectives**

The project objectives for this paper are:

- Investigation into the structures presented in (Wing, 2004),
- Constructing a model based on this structure to establish the distribution of employment between different sectors,
  - In two cases, 'with' and 'without' policy intervention
- The comparative salary distribution of the two cases.

### **Motivation - Objectives**

The reasons for these objectives are:

- To produce a counterfactual which can be compared to the intervention state, determining the impact of the policy,
- To enable comparison of modelling results on the same primary economic benefit, wage premium.

The benefit of taking an CGEM approach would be to investigate the opportunity cost of the economic activity foregone. It would consider how government intervention will impact the different sectors of the economy. It will also make wage premium an endogenous variable.



## Economy structure - closed circular economy

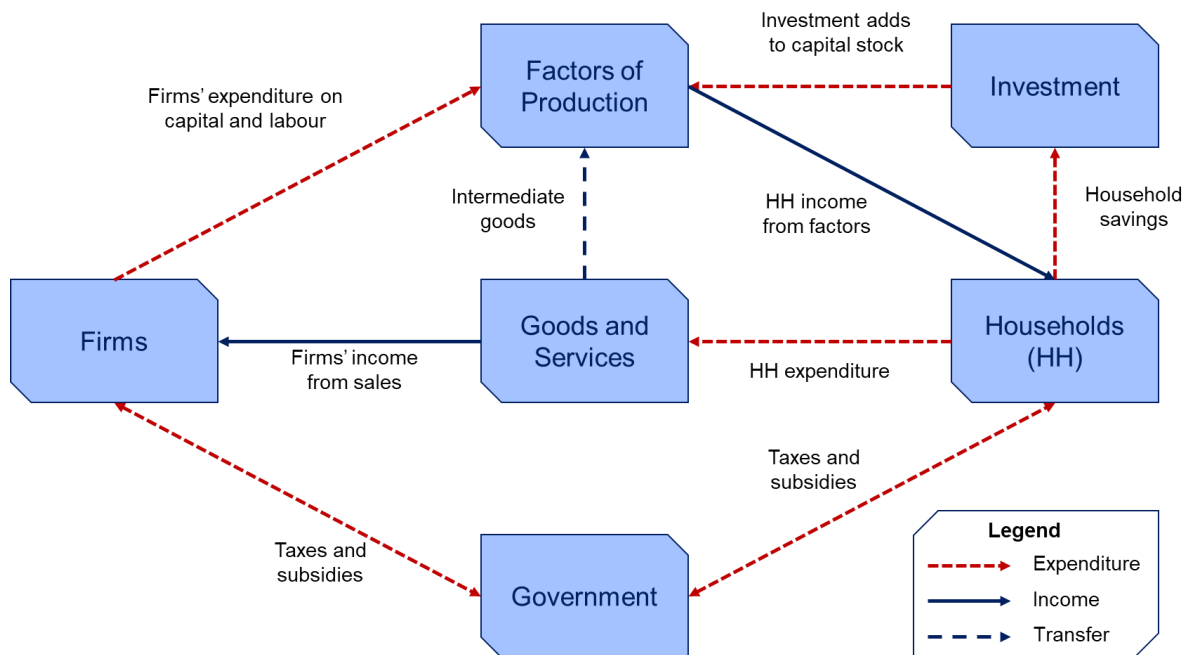


Figure 5 Close circular economy model

The underpinning idea in CGEM is a circular flow of commodities in a closed economy. The above model diagram is an autarky version of the model presented in the HMRC GEM paper, (HMRC, 2013) .

CGEM can be dynamic and track the evolution of the economy over time in response to a policy change. In the baseline, it is assumed the economy follows a steady-state growth path where all economic activities grow at a constant rate, (HMRC, 2013). This enables the model to forecast over the appraisal period of the policy intervention.

## Required Modelling Assumptions

The following are required modelling assumptions to build a working CGEM. Their justification and reasoning can be found in Annex B: Technical: 1 – Assumption Justification and Reasoning.

### Firms Assumptions

Firms are assumed:

1. To be profit maximisers,
2. To be producing homogenous products,
3. To have a homogenous production function within a single sector of the economy,
4. To be producing using labour, capital, and intermediate goods as input factors,
5. To be producing with a constant return to scale,
6. To have a Cobb-Douglas (CD) production function.

### Households Assumptions

Households are assumed:

7. To be utility maximisers, with a defined Utility Function (UF),
8. To gain income from selling labour as price takers, and collecting interest on invested capital at a fixed rate,
9. To save income at a fixed rate,



### Government Assumptions

The government is assumed:

10. To have a constant tax and spend policy throughout the model except for the policy being assessed.

All economic actors are assumed:

11. To be budget neutral in a single period; income = expenditure + savings.

### Modelling – CGEM

The data the CGEMs were calculated with is from the ONS. ONS produces Input-Output Analytical Tables which includes product-by-industry section derived from the annual Supply and Use Tables (SUTs) (ONS, 2023). From this the annual use of intermediate goods, labour expenditure and final output can be gathered. This data is disaggregated by SIC code as described in Annex B: Standard Industrial Classification (SIC). This classification of sectors has been implemented at two scales, 39 sectors and 5 sectors.

This data is the appropriate dataset for this work as it gives the depth of disaggregation (covers a range of sectors), breadth of data points (range of years) and is relatively consistent in presentation when matching across datasets.

### Model Derivation

Initial modelling derivation was built from the work presented in the MIT Joint Program on Science and Policy of Global Change report: Computable General Equilibrium Models and Their Use in Economy-Wide Policy Analysis, (Wing, 2004).

Set of commodities:  $i \in (1, N)$   
 Set of industry sectors:  $j \in (1, N)$   
 Set of primary factors:  $f \in (1, F)$   
 Set of final demands:  $d \in (1, D)$   
 For simplicity  $D = N$

In CGEM, the price and income elasticities of demand are crucial in determining the sectoral growth pattern and economic impact, (Hertel, 2012). For models in this paper the UF, is modelled as a CD, from this function the price elasticities of demand can be calculated. This is discussed further in the Utility Function section.

It assumes that the circular economy in the model conforms to:

- Cobb-Douglas Economy,
- Commodity market clearance,
- Factor market clearance,
- Full employment,
- Zero profit.

These assumptions can be formulised in the equations below:

$$y_i = \sum_{i=1}^N x_{ij} + \sum_{d=1}^D g_{id} = \sum_{i=1}^N x_{ij} + c_i + s_i \quad \text{Equation 1 Commodity Market Clearance}$$

$$V_f = \sum_{j=1}^N v_{fj} \quad \text{Equation 2 Factor Market Clearance}$$

$$y_j = \sum_{i=1}^N x_{ij} + \sum_{i=1}^F v_{fj} \quad \text{Equation 3 Full Employment}$$





$$m = \sum_{f=1}^F V_f = \sum_{i=1}^N \sum_{d=1}^D g_{id}$$

Equation 4 Zero Profit

Where:

Variable	Definition
$y_i$	Total output of commodity $i$
$x_{ij}$	Intermediate good $j$ required for commodity $i$
$g_{id}$	Final good use of commodity $i$ from factor rent $d$
$c_i$	Consumption of commodity $i$
$s_i$	Saving of commodity $i$
$V_f$	Total available primary factor $f$
$v_{fj}$	Primary factor $f$ used for sector $j$
$m$	Total rent from all primary factors employed

The households in this model are assumed to be utility maximisers (Assumption 7) and therefore the agent problem can be defined as:

$$\max_{c_i} U(c_1, \dots, c_N) \text{ subject to } m = \sum_{i=1}^N p_i(c_i + s_i)$$

Equation 5 Agent Utility Maximisation

Where the function  $U(\cdot)$  is defined as:

$$U = A_c c_1^{\alpha_1} c_2^{\alpha_2} \dots c_N^{\alpha_N} = A_c \prod_{i=1}^N c_i^{\alpha_i}$$

Equation 6 Cobb-Douglas Utility Function

This can be solved to produce the equations for  $c_i$  &  $\alpha_i$

$$c_i = \alpha_i \frac{(m - \sum_{i=1}^N p_i s_i)}{p_i}$$

Equation 7 Consumption of commodity  $i$

$$\alpha_i = \frac{c_i p_i}{(m - \sum_{i=1}^N p_i s_i)}$$

Equation 8 Alpha of commodity  $i$

The producers in this model are assumed to be profit maximisers (Assumption 1) and therefore the producer's problem can be defined as:

$$\max_{x_{ij}, v_{ij}} \pi_j = p_j y_j - \sum_{i=1}^N p_i x_{ij} - \sum_{f=1}^F w_f v_{fj} \text{ subject to } y_j = \vartheta_j(x_{1j}, \dots, x_{Nj}, v_1, \dots, v_{Fj})$$

Equation 9 Produces Profit Maximisation

Where  $\vartheta_j(\cdot)$  is specified as the production function. For this model it will be an extension of the CD investigated in Equation 22 to:

$$\vartheta_j(\cdot) = A_j (x_{1j}^{\beta_{1j}}, \dots, x_{Nj}^{\beta_{Nj}}) (v_{1j}^{\gamma_{1j}}, \dots, v_{Nj}^{\gamma_{Nj}}) = A_j \prod_{i=1}^N x_{ij}^{\beta_{ij}} \prod_{f=1}^F v_{fj}^{\gamma_{fj}}$$

Equation 10 Cobb-Douglas Production Function with Intermediate Goods



Where:  $(\beta_{1j} + \dots + \beta_{Nj} + \gamma_{1j} + \dots + \gamma_{Nj} = 1)$  *Equation 11 Constant Return to Scale*

From these equations  $\beta_{ij}$  and  $\gamma_{fj}$  can be derived as:

$$\beta_{ij} = \frac{p_i x_{ij}}{p_j y_j} \quad \& \quad \gamma_{fj} = \frac{w_{fj} v_{fj}}{p_j y_j}$$

The aim is to create a benchmark year, within which the price of each commodity  $p_i$  and wage  $w_{fj}$  are set to 1. This unity of prices and wages is a modelling assumption to enable algebraic simplification without the loss of generality in the modelling. There is no loss in generality of modelling as the prices and wages are relative to the base year and consumption/output is not modelled as unity. The following are the equations used to calculate the calibration parameters in the benchmark year:

$\alpha_i = \bar{g}_{ic} / \bar{G}_c$	$A_c = \frac{\bar{G}_c}{(\prod_{i=1}^N \bar{g}_{ic}^{\alpha_i})}$
$\beta_{ij} = \frac{\bar{x}_{ij}}{\bar{y}_j}$	$\gamma_{fj} = \frac{\bar{v}_{fj}}{\bar{y}_j}$
$A_j = \frac{\bar{y}_j}{(\prod_{i=1}^N \bar{x}_{ij}^{\beta_{ij}} \prod_{f=1}^F \bar{v}_{fj}^{\gamma_{fj}})}$	$s_i = \bar{g}_{is}$
$V_f = \bar{V}_f$	$\bar{m} = \sum_{f=1}^F \bar{V}_f$

*Equation 12 Calibration Equations*

With the above definitions for calibration the general equilibrium equations can be reformulated as:

$$\bar{y}_i = \sum_{j=1}^N \bar{x}_{ij} + \sum_{d=1}^D \bar{g}_{id} \quad \text{Equation 13 Benchmark Commodity Market Clearance}$$

$$\bar{V}_f = \sum_{j=1}^N \bar{v}_{fj} \quad \text{Equation 14 Benchmark Factor Market Clearance}$$

$$\bar{y}_j = \sum_{i=1}^N \bar{x}_{ij} + \sum_{f=1}^F \bar{v}_{fj} \quad \text{Equation 15 Benchmark Full Employment}$$

$$\bar{m} = \sum_{f=1}^F \bar{V}_f = \sum_{i=1}^N \sum_{d=1}^D \bar{g}_{id} \quad \text{Equation 16 Benchmark Zero Profit}$$

Rearranging Equation 13 to Equation 16 and inserting the calibration definitions from Equation 12, the following deltas can be derived:

$$\Delta_i^C = z \sum_{j=1}^N \beta_{ij} p_j y_j + \alpha_i \left( \sum_{f=1}^F w_f V_f - \sum_{j=1}^N p_j s_j \right) + p_i s_i - p_i y_i \quad \text{Equation 17 Delta Consumption}$$

$$\Delta_f^F = \sum_{j=1}^N \gamma_{fj} \frac{p_j y_j}{w_f} - V_j \quad \text{Equation 18 Delta Primary Factor}$$



$$\Delta_j^\pi = p_j - A_j \prod_{i=1}^N \left( \frac{p_i}{\beta_{ij}} \right)^{\beta_{ij}} \prod_{f=1}^F \left( \frac{w_f}{\gamma_{fj}} \right)^{\gamma_{fj}}$$

Equation 19 Delta Profit

$$\Delta_m = \sum_{f=1}^F w_f V_f - m$$

Equation 20 Delta Factor  
Clearance

These delta functions can be combined into a single function of functions:

$$\xi(\cdot) = [\Delta^c, \Delta^F, \Delta^\pi, \Delta^m]$$

Equation 21 Delta Function

For a given set  $\Psi(\alpha_i, A_c, \beta_{ij}, \gamma_{fj}, A_j, s_i, V_f, \bar{m})$  of calibration inputs a vector  $\mathbf{z} = [\mathbf{p}, \mathbf{w}, \mathbf{y}, m]$  is in equilibrium iff  $\xi(\mathbf{z}) = \mathbf{0}$

The difference of  $\xi(\mathbf{z})$  from  $\mathbf{0}$  can be interpreted as the amount the vector  $\mathbf{z}$  needs to change to move the model to equilibrium.

### Structural Choices

There is a trade-off between the computability of the model and the complexity, i.e., the number of features in the model. These options must be balanced when constructing the model. Examples of these trade-offs include the number of sectors in the model and the functions of economic actor behaviour.

### Model 1

This model is using the 5-sector disaggregation to be more computable. 2015 ONS Input-Output Table<sup>7</sup> data was used and a benchmark CGEM was constructed. This version of the model assumed no savings, capital, imports, exports, or government involvement.

However, these assumptions are not present in the data from ONS, which is presented in the current price year as opposed to at a unit production level. So, the outputs had to be calibrated to unity price and wages, which is presented in the code:

11\_Model\_1\_Calibration.R.

The ONS data provided a unit input-output for intermediate goods which could be used for the  $x_{ij}$  within the model. From this, a corresponding set of  $\beta_{ij}$  and  $\gamma_{ji}$  were calculated. The model constructed its own  $A_j$  (technology constant) to allow each sector to create enough output to cover both intermediate goods  $x_{ij}$  and final demand  $g_{id}$ .

Implementing this approach, the benchmark model was confirmed as a general equilibrium within the realms of computational tolerance, accounting for floating point errors.

This could be tested with the delta function in Equation 21.

### Model 2

For a more complex version of the model, capital will be introduced as an additional required primary factor. The construction began again with the ONS data, with an initial endowment of fixed capital levels in 2015 and employment figures.

The total output was scaled to match a salary cost of 1 and capital cost of 0.1 (to represent an economic wide interest rate of 10%). This model produced a set of outputs including intermediate goods required and final consumption levels for consumers which match the return on labour and capital,  $\Delta_j^F = \Delta^m = 0$ . However, as the initial input data is based on real world values, with the additional complexity of two primary factors the model could not be

<sup>7</sup> <https://www.ons.gov.uk/economy/nationalaccounts/supplyandusetables/datasets/ukinputoutputanalyticaltables-detailed>



engineered such that all the deltas were zero. The benchmark input was not a general equilibrium.

## Model 2 - Equilibrium

The model required a method to vary endogenous variables to reach an equilibrium, requiring the creation of an algorithm to search for the equilibrium point. The endogenous variables in Model 2 that could be altered are the vectors of price  $p$ , wages  $w$ , and consumption  $g$  (consumption alters total output  $y$ ), each of which contain 5 elements.

This gives 15 variables which could be scaled. Table 1 below presents the relationship between the individual deltas and an increase in a single variable. When the delta has a subscript  $j$  this means the change of that delta is for a different commodity. The equilibrium is found once all deltas equal zero.

Increase in Variable	$p_i$	$w_i$	$g_i$
$\Delta_i^c$	↓	↑	↓
$\Delta_j^c$	↑	↑	↑
$\Delta_i^F$	↑	↓	↑
$\Delta_i^\pi$	↑	↓	=
$\Delta_j^\pi$	↓	=	=
$\Delta^m$	=	↑	↓

Table 1 Delta changes due to variable change

The literature recommends an approach like a Newton-type steepest-descent optimisation algorithm (Kehoe, 1991). An attempt to construct this algorithm to find the equilibrium can be found in the R code: 12\_Model\_2\_Calibration.R. A flow diagram of the attempt is given in Figure 6. The function 'Update Values' contains the Newton-type steepest-descent function and updates values based on the directions derived in Table 1. However, this attempt was unsuccessful. The code could not converge on an equilibrium.

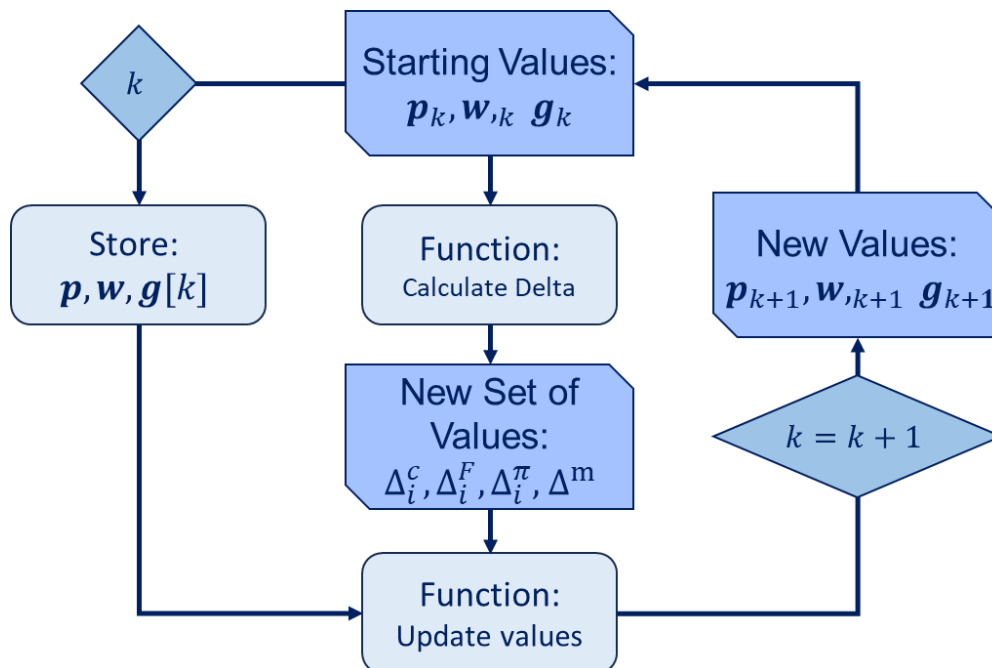


Figure 6 Flow Diagram for Equilibrium search



## Modelling Weaknesses

The CGEM produced could investigate the opportunity costs of the intervention better than the current methodology (if an equilibrium algorithm was derived), however the CGEM approach has multiple weaknesses. This section of the paper will discuss them and give some potential solutions where investigated.

### Weaknesses with the Benchmark Approach

The following explores the weaknesses with the benchmark aspect of the CGEM approach.

#### *Unity Prices and Wages*

The methodology reviewed in the documentation readily used in DBT (ex-DIT) assumes in the benchmark year prices and wages are set to 1. This enables the algebra to construct future years relative to the first year. However, this assumption negates the main benefit that a model would be trying to predict. Therefore, an attempt to construct a version that assumes wages which are not equal to 1 was created. The code can be found in 00\_Main\_Model\_v1.R. The code was unable to find the equilibrium point for this version. This version also attempted to model all 39 SIC commodity groups as described in Annex A: Dedication

This paper is dedicated to my wife for her love and support through the master's program. Without you, none of this would have been possible and sorry for the exam the week of our wedding.

Annex B: Technical: 2 – Standard Industrial Classification (SIC).

#### *Production Function*

The methodology presented by (Wing, 2004) and used in the modelling above calculates the CD coefficients for production from a single year of data. Depending on the sensitivity of the results to these coefficients this may lead to misleading results. A potential solution to this is to approximate the CD from many years of data.

The three factor Cobb-Douglas is given as:

$$Q = AG^{\alpha}L^{\beta}K^{\gamma}, \quad 0 < \alpha, \beta, \gamma < 1$$

Where:  $Q$  = Output,  $G$  = intermediate goods,  $K$  = capital used,  $L$  = labour used,  $A$  = technology constant.

#### *Equation 22 Factor CD Production Function*

Linear regression modelling and an author-derived algorithm were used to calculate the parameters of Equation 22. Full technical detail of the approach can be found in Annex B: Technical.

The benefit of this approach is to produce more robust coefficients.

### Weaknesses with the General CGEM Approach

The following explores CGEM approach weaknesses, additional to benchmarking weaknesses above.

#### *Data Limitations – SIC Codes*

The SIC code 29 covers the production in the automotive industry, a breakdown is given in Figure 7. There is currently no split between the production of ICEV and EV which are both contained within SIC 29.



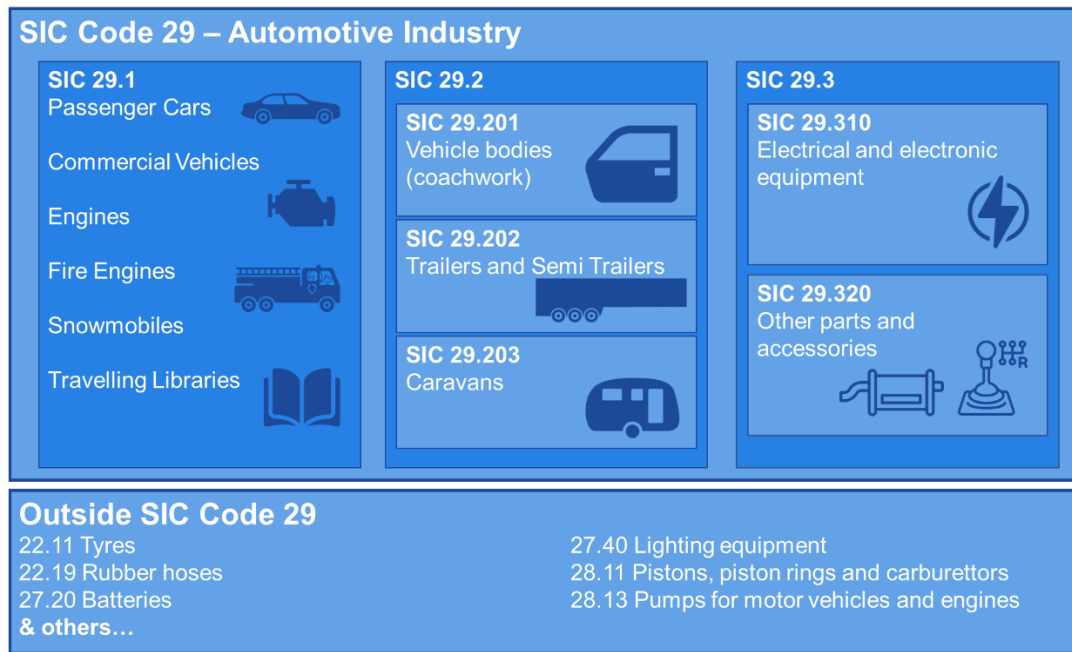


Figure 7 SIC 29 Breakdown

Any production function derived from SIC 29 will have capitalisation levels and intermediate goods for both ICEV and EV. The UK currently produces a small percentage of EV compared to ICEV. Therefore, it can be assumed that the production function based on the current sector will be valid for the next few years if EV production remains similar. However, as the industry transitions to produce more EV this assumption will fail.

### Generality of Production Function

The parameters of Equation 22 provide a more robust production function than the one used in Model 1 and 2. However, this approach still suffers from the data limitation described above.

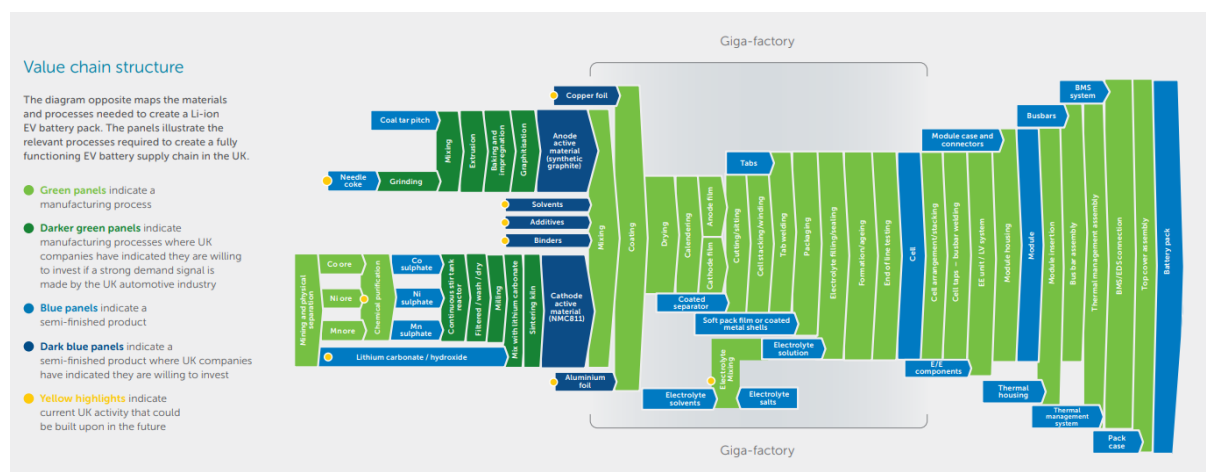


Figure 8 EV Battery Value Chain – Advance Propulsion Centre (APC)

The current methodology includes subject material expertise which can model the impacts of investing in the EV supply chain or in an OEM. Figure 8<sup>8</sup> is the EV battery value chain and

<sup>8</sup> Figure source: <https://www.apcuk.co.uk/wp-content/uploads/2021/09/Automotive-Batteries-Report-Summary-April-2019.pdf>





shows the level of subject matter expertise included in the current methodology and how expertise from external sources has been utilised, i.e., APC.

One way the difference between ICEV and EV can be incorporated into the CGEM would be for the commodities used to be disaggregated further. This is not possible with current data source (ONS, Input-Output Tables).

Below is a proposed method to mitigate the ONS data limitation when calculating the production function by the regression given in Annex B: Technical 4. Production Function 1.

The regression model will have to incorporate a term for the annual EV production percentage. This will try to determine how the intermediate goods and capitalisation level change because of the EV proportion changing. It will account for the increased inclusion of the value chain elements shown in Figure 8. This could allow for a model with an exogenous term of EV percentage which alters the production function over time.

Without the construction of the regression model to identify how EV production is changing the production function and mix of intermediate goods, an alternative methodology will not be as robust as the current methodology.

### *Multiplicity of Equilibrium*

It is extremely difficult to prove analytically that a given equilibrium solution is unique in real world CGEM, especially when there is the inclusion of taxes distortion. There have been examples of testing for multiplicity of equilibria in real-world CGE models. However, the complexity of doing so means they are uncommon. (Wing, 2004).

This does provide a doubt on the solution and subsequential policy advice provided by any CGE modelling.

### *Utility Function (UF)*

There are multiple implications for modelled consumer behaviour by using the CD UF which are discussed in this section by mathematical deriving elasticities. These include price elasticity of demand, cross-price elasticity of demand and income elasticity of demand.

### **Price elasticity of demand**

The price elasticity of demand is defined as  $\varepsilon \equiv \frac{p_i}{c_i} \frac{\partial c_i}{\partial p_i}$ , (Mac Lean, 2011) and can be calculated by differentiating Equation 7:  $\left[ c_i = \alpha_i \frac{(m - \sum_{i=1}^N p_i s_i)}{p_i} \right]$  with respect to  $p_i$ . In model 1 and 2 it was assumed there were no savings, which means the elasticity for those models is:

$$\varepsilon = \frac{p_i}{c_i} \frac{\partial c_i}{\partial p_i} = -\frac{\alpha_i m}{c_i p_i} \quad \text{Equation 23 Price elasticity for Model with no savings}$$

The derivative  $\frac{\partial c_i}{\partial p_i} < 0$ ,  $\forall p_i > 0$  as  $\alpha_i > 0$ , this implies that the demand will always decrease for increasing prices and vice versa. This means that Giffen goods cannot be included in the model.

The price elasticity of a model including savings is evaluated below, assuming  $s_i$  is not dependent on  $p_i$ .



$$\frac{\partial c_i}{\partial p_i} = \frac{\partial}{\partial p_i} \alpha_i \frac{(m - \sum_{i=1}^N p_i s_i)}{p_i} = \alpha_i \left( \frac{\partial}{\partial p_i} \frac{m - PS}{p_i} - \frac{p_i s_i}{p_i^2} \right), PS \equiv \sum_{j=1}^N p_j s_j \text{ \& } j \neq i$$

$$\therefore \frac{\partial c_i}{\partial p_i} = -\frac{\alpha_i(m - PS)}{p_i^2} \Rightarrow \varepsilon = -\frac{\alpha_i(m - PS)}{c_i p_i}$$

Equation 24 Price elasticity for Model with savings

The modelling assumes that the consumer budget is balanced within a single year (no borrowing), therefore  $m \geq PS \Rightarrow \frac{\partial c_i}{\partial p_i} < 0, \forall p_i > 0$ . Therefore, the price elasticity in a model has the same behaviour, i.e., no Giffen goods.

### Cross-price elasticity of demand

Cross-price elasticity of demand is defined as  $\varepsilon_c \equiv \frac{p_j}{c_i} \frac{\partial c_i}{\partial p_j}$  (Pindyck & Rubinfeld, 2009)

In the model with no saving there is no cross elasticity of demand as  $\frac{\partial c_i}{\partial p_j} = 0$ .

The cross elasticity of a model including savings is evaluated below, assuming  $s_i$  is not dependent on  $p_i$ .

$$\frac{\partial c_i}{\partial p_j} = \frac{\partial}{\partial p_j} \alpha_i \frac{(m - p_1 s_1 - \dots - p_N s_N)}{p_i} = \frac{\partial}{\partial p_j} \alpha_i \left( \frac{m}{p_i} - \frac{p_1 s_1}{p_i} - \dots - \frac{p_N s_N}{p_i} \right)$$

$$\frac{\partial}{\partial p_j} \alpha_i \frac{m}{p_i} = \frac{\partial}{\partial p_j} \alpha_i \frac{p_k s_k}{p_i} = 0$$

$$\Rightarrow \frac{\partial c_i}{\partial p_j} = -\frac{\alpha_i s_j}{p_i} \Rightarrow \varepsilon_c = -\frac{p_j \alpha_i s_j}{c_i p_i}$$

Equation 25 Cross Elasticity in Model with Savings

Price is defined as positive for all solutions to the model. Equation 25 implies that all cross elasticities are negative and therefore all goods are complementary.

### Income elasticity of demand

Income elasticity is defined as  $\varepsilon_I \equiv \frac{I}{c_i} \frac{\partial c_i}{\partial I}$ , where  $I$  is income, (Pindyck & Rubinfeld, 2009).

Income elasticity is the same in a model with constant saving rate (Assumption 9,  $\frac{\partial s_i}{\partial m} = 0$ ) or without savings:

$$\varepsilon_I = \frac{m}{c_i} \frac{\partial c_i}{\partial m} = \frac{m \alpha_i}{p_i c_i} > 0$$

Equation 26 Income Elasticity

This implies that an increase in income will always result in a linear increase in consumption of all goods. As an example of this model behaviour, Figure 9 has been created for a three good economy with constant return to scales in CD with:  $\alpha_1 = 0.15, \alpha_2 = 0.3, \alpha_3 = 0.55, \alpha_1 + \alpha_2 + \alpha_3 = 1$ . This assumes all other factors remain equal (e.g., price of commodities.)



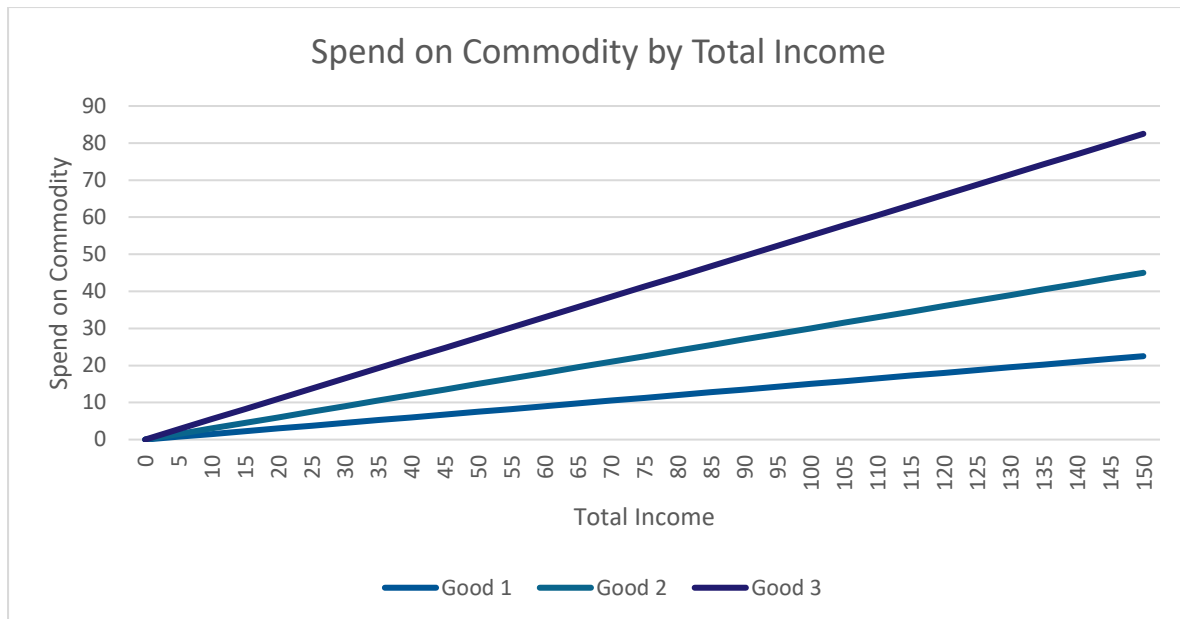


Figure 9 CD predicted spend as income changes.

This implication can be tested against real world data via the ONS 'Household expenditure by gross income decile group' dataset<sup>9</sup>. One method is comparing the total spend on each commodity group in each decile to the lower decile (i.e., Third decile group compared to Second decile group). From this you can infer the income elasticity. The assumption is that if individuals earning the salary in the second decile moved to the income of third decile, they would shift that spend on average to that of the third decile.

Reviewing the data from ONS infers that there is evidence to support a negative income elasticity at certain income levels for:

- Alcoholic drinks, tobacco & narcotics,
- Housing(net)<sup>10</sup>, fuel & power,
- Health,
- Education<sup>11</sup>

Table 2 shows the percentage of years that a negative elasticity is calculated when moving decile group for each commodity group. This is measuring absolute spend decreases not just proportional decreases.

<sup>9</sup> <https://www.ons.gov.uk/peoplepopulationandcommunity/personalandhouseholdfinances/expenditure/datasets/householdexpenditurebygrossincomedecilegroupuktablea4>

<sup>10</sup> Excluding mortgage interest payments, council tax and Northern Ireland rates

<sup>11</sup> Although the data in the Education group had gaps unlike others, so likely not significant as a result as other groups.



Commodity Group	Moving from Percentile X to Y								
	10 to 20	20 to 30	30 to 40	40 to 50	50 to 60	60 to 70	70 to 80	80 to 90	90 to 100
1	0%	0%	0%	0%	0%	0%	0%	0%	0%
2	33%	17%	8%	0%	0%	17%	8%	42%	0%
3	0%	0%	0%	0%	8%	8%	0%	0%	0%
4	0%	8%	0%	25%	17%	50%	50%	50%	0%
5	8%	0%	17%	17%	0%	25%	25%	8%	0%
6	0%	42%	8%	33%	33%	42%	33%	8%	8%
7	0%	0%	0%	0%	0%	8%	0%	0%	0%
8	0%	0%	0%	0%	8%	0%	0%	0%	0%
9	0%	0%	0%	0%	0%	8%	0%	0%	0%
10	67%	42%	17%	42%	75%	0%	42%	42%	0%
11	0%	0%	0%	0%	0%	0%	0%	0%	0%
12	0%	0%	0%	8%	0%	17%	0%	0%	0%

Table 2 Result of investigating commodity groups with negative income elasticities.

To examine the income elasticity with this data requires the assumption that firms within a commodity group are producing homogenous goods/services. This means that the prices paid for a unit of goods/service does not change. This removes the possibility that consumers will pay for a more luxury version of the same item as income increases, i.e., changing from Supermarket own brand to a premium brand food product. The inverse of this phenomenon has been seen in the post-COVID cost-of-living crisis, with 64% of consumers switching to cheaper brands (Grocery trader, 2022) according to a Shopmium survey<sup>12</sup>.

To compare the behaviour predicted by the CD UF to real world data further, regression modelling of UK consumption data has been completed. This is presented in the Regression Modelling of Consumption section.

### Autarky

The alternative methodology currently assumes the economy is an autarky. This was by design to enable an initial exploration of the model limiting complexity. For a more robust version of the methodology, it should consider international trade. In the last ten years, despite the reduction in vehicle production numbers from a high in 2016, the percentage of exports has remained around 80%. This is evidenced in data presented in Figure 10<sup>13</sup>.

<sup>12</sup> survey of 4,000 consumers by leading supermarket media and promotions business Shopmium.

<sup>13</sup> Source: <https://my.ihf.com/>



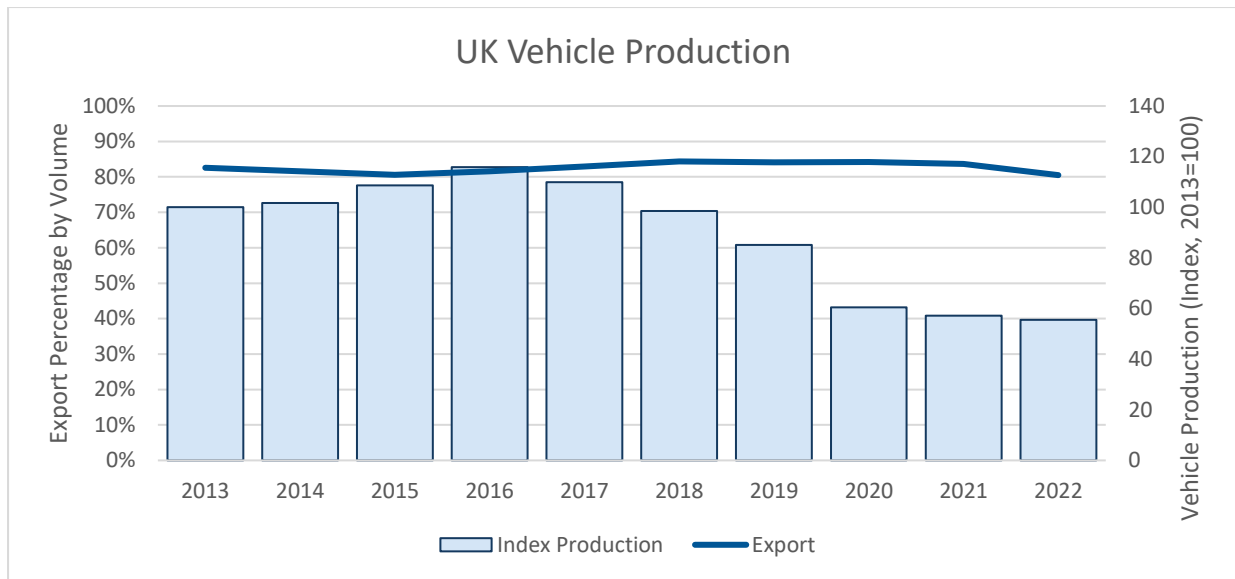


Figure 10 UK Vehicle Production Data

This is an area in which a future CGEM could be more insightful than the current approach. The current approach assumes that international demand for vehicles is exogenous and will absorb any increase in UK production.

An international trade CGEM will also be able to contextualise UK government policy intervention against USA IRA and EU GDIP policies. However, this development option will require a considerable larger dataset and calibration. There is a discussion in Annex B: Technical: 9 - Trade Locations, of how to focus the modelling on key trade partners.

### Black Box

Simulation models like CGEM are criticized for being 'Black Boxes', (Burfisher, 2011), whose results are difficult to explain by clear causal chains, (Dixon & Jorgenson, 2013). This is especially true for proprietary commercial modelling, which is the reason R, an open-source coding language, was used when creating a new CGEM.

One of the key approaches to overcome this weakness is in the presentation of the methodology and results. The recommendation would be:

- Narrowing the scope of the results to key areas of interest to the audience,
- Focus on communicating how policy intervention impacts key metrics (employment, trade ratios).



## Regression Modelling of Consumption

It should be noted the following estimators from regression models are not the elasticities. The sign of the estimators (- or +) does indicate the sign of the elasticities. To calculate whether the elasticities are inelastic or elastic the price level and quantity consumed would have to be incorporated into the equations.

The data for this analysis is the ONS 'Household expenditure by gross income decile group' dataset. The manipulation of the data prior to regression modelling is discussed in Annex B: Technical: 11 – Data Manipulation – Regression Modelling of Consumption.

### Income Elasticity of Demand

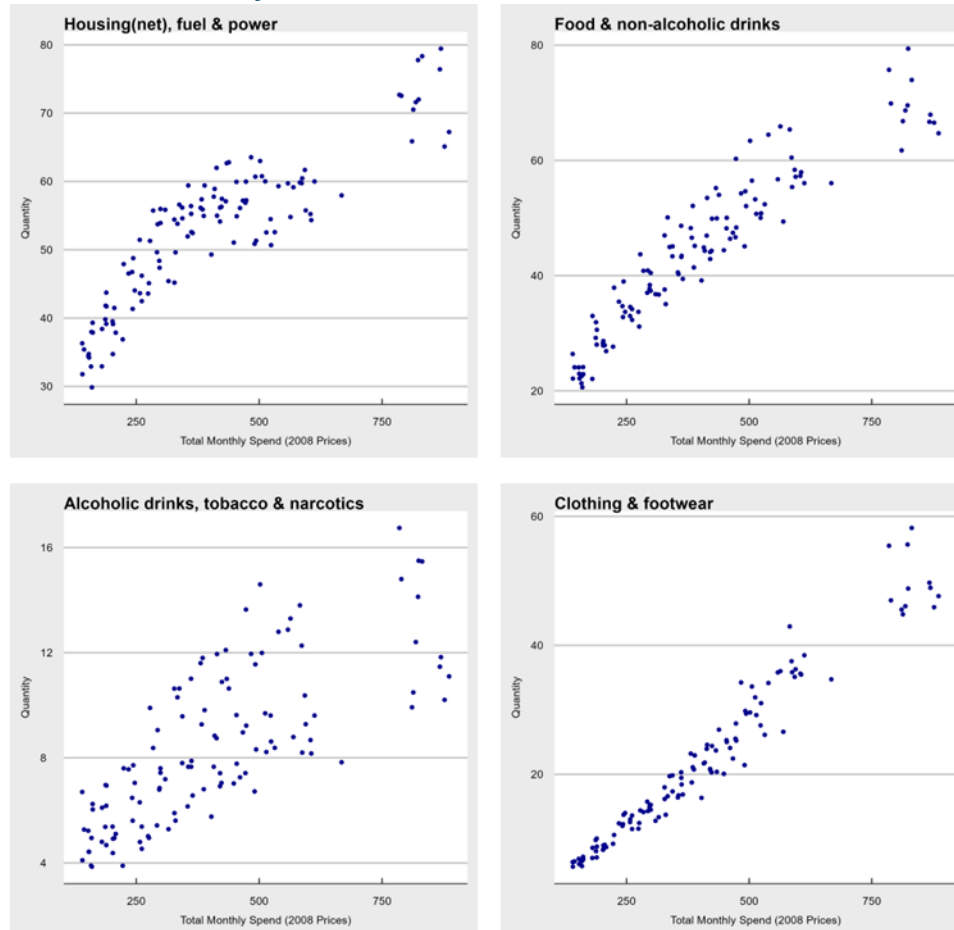


Figure 11 Plot of monthly quantity consumed of commodities by income level from 2008-2019

Regression analysis can be run on this ONS data to test the implication of the CD of linear relationship between total income and commodity quantity. As it is an area of specific interest in this paper, Group 7 – Transport has been split into 7.1 Automotive purchases and 7 – other (all other consumption in Group 7), results shown in Figure 12.





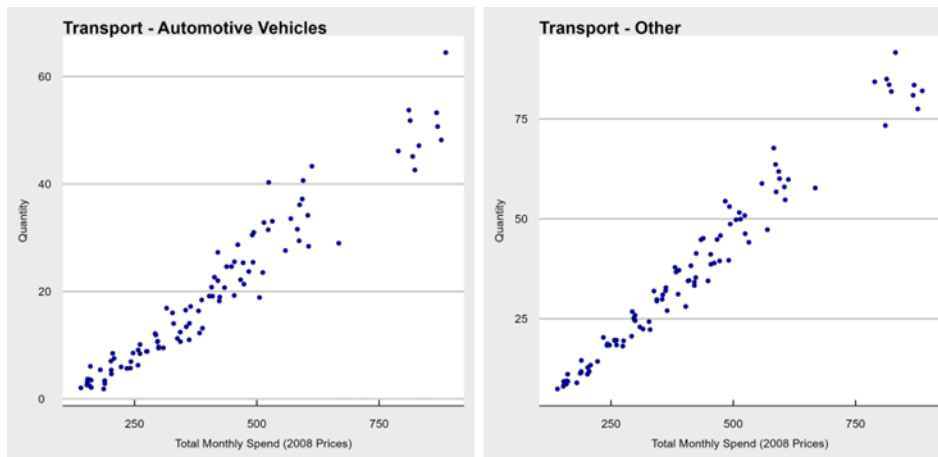


Figure 12 Plot of monthly quantity consumed by income level for disaggregated Transport group.

### Simple Linear Regression

$$\text{Quantity} = \alpha \cdot \text{Income} + \text{Intercept} + \varepsilon, \quad \text{where } \varepsilon = \text{error}$$

Equation 27 1<sup>st</sup> Income Regression - Simple Linear Regression

Commodity	Intercept	$Pr(>  t )$	$\alpha$	$Pr(>  t )$	$r^2$	Adjusted $r^2$
Commodity 1	17.8	4.18 E-50	0.066	5.37 E-76	0.878	0.877
Commodity 2	3.80	2.36 E-39	0.011	4.16 E-57	0.566	0.562
Commodity 3	-3.98	1.11 E-12	0.064	5.94 E-88	0.954	0.953
Commodity 4	33.0	8.55 E-58	0.049	5.51 E-43	0.789	0.787
Commodity 5	-2.88	7.67 E-05	0.074	5.42 E-74	0.932	0.932
Commodity 6	0.179	0.587	0.012	1.46 E-47	0.818	0.817
Commodity 7.1	-9.52	6.33 E-20	0.072	6.62 E-62	0.938	0.938
Commodity 7 - Other	-8.53	5.72 E-29	0.110	3.12 E-96	0.968	0.967
Commodity 8	3.89	3.93 E-22	0.019	6.02 E-55	0.924	0.923
Commodity 9	-11.2	9.90 E-30	0.169	1.20 E-116	0.98	0.98
Commodity 10	-5.97	3.18 E-09	0.028	1.35 E-25	0.628	0.625
Commodity 11	-10.5	1.79 E-35	0.109	3.73 E-105	0.944	0.943
Commodity 12	-4.06	1.22 E-13	0.091	1.97 E-105	0.964	0.963

Table 3 1<sup>st</sup> Regression Results

Modelling the relationship between quantity and income as a linear relationship yields good results for certain commodities. This is inferred by the very low  $Pr(> |t|)$  on all the Intercepts and  $\alpha$  except commodity 6's Intercept. The  $r^2$  and adjusted  $r^2$  are also very high for all commodities (except 4 and 10).

This model could be improved by including the assumption that the intercept is zero. This is a reasonable assumption as it implies without an income that an individual does not consume any commodities.

### Linear Regression with zero intercept

$$\text{Quantity} = \hat{\alpha} \cdot \text{Income} + \varepsilon, \quad \text{where } \varepsilon = \text{error}$$

Equation 28 2<sup>nd</sup> Income Regression – Linear regression with zero intercept



Commodity	$\hat{\alpha}$	$Pr(>  t )$	$r^2$	Adjusted $r^2$
Commodity 1	0.102	4.5 E-92	0.962	0.962
Commodity 2	0.019	4.8 E-85	0.919	0.918
Commodity 3	0.056	2.0 E-113	0.984	0.984
Commodity 4	0.115	8.7 E-69	0.921	0.921
Commodity 5	0.068	1.0 E-108	0.983	0.983
Commodity 6	0.012	1.9 E-88	0.962	0.962
Commodity 7.1	0.053	5.8 E-67	0.951	0.951
Commodity 7 - Other	0.093	1.6 E-98	0.984	0.984
Commodity 8	0.027	5.0 E-92	0.974	0.973
Commodity 9	0.147	4.0 E-125	0.990	0.990
Commodity 10	0.017	1.8 E-30	0.699	0.696
Commodity 11	0.088	7.0 E-104	0.971	0.970
Commodity 12	0.083	3.0 E-132	0.989	0.989

Table 4 2<sup>nd</sup> Regression Results

All the  $r^2$  and adjusted  $r^2$  values have improved, showing a potentially better model. The statistical confidence of the  $\hat{\alpha}$  values is very high as all the  $Pr(> |t|)$  are very low (significantly less than 0.001). This implies this model is a good fit. To confirm that the 2<sup>nd</sup> regression is statistically better an Analysis of Variance (ANOVA) test was completed with the 1<sup>st</sup> regression set as the Null hypothesis. If the resulting  $Pr(> F)$  is less than 0.001 this is considered a significant result to reject the Null hypothesis, i.e., the Linear regression with zero intercept is a statistically significant better model.

Commodity	$Pr(> F)$
Commodity 1	2.27 E-34
Commodity 2	1.35 E-15
Commodity 3	5.48 E-10
Commodity 4	1.64 E-59
Commodity 5	0.00067
Commodity 6	0.450
Commodity 7.1	2.54 E-19
Commodity 7 - Other	4.70 E-15
Commodity 8	1.70 E-33
Commodity 9	3.01 E-20
Commodity 10	2.92 E-09
Commodity 11	2.92 E-16
Commodity 12	2.02 E-07

Table 5 ANOVA Results – 1<sup>st</sup> vs 2<sup>nd</sup> Regressions

Therefore, for all (except for Commodity 6) the ANOVA test implies that there is statistically significant evidence to reject the Null hypothesis and use the second regression.

There is still evidence to support rejecting the Null hypothesis for commodity 6. This lower significance can be explained by the 1<sup>st</sup> regression model producing an intercept close to zero (0.179). Therefore, it is hard to test the small change in the intercept between regression models. When considering it is still a positive result from the ANOVA and there is improvement to  $Pr(> |t|)$  of  $\alpha$ ,  $r^2$  and adjusted  $r^2$  the Null hypothesis should be rejected.



This regression evidence so far supports the modelled behaviour of the CD UF. It should be noted that the sum of  $\hat{\alpha}$  in the linear regression model with zero intercept is 0.88. This potentially can be explained by an increasing marginal rate of savings as income increases.

Visual inspection of the charts in Figure 11 infers that a logarithmic regression should be tried. This would imply that for some of the commodities as income increases the proportion of income spent on them decreases. Implying the marginal utility of consumption of those commodities decreases, relative to the marginal utility of consumption of other commodities, as income increases.

### Logarithmic Regression

$$\text{Spend} = \alpha_L \cdot \text{Log}(\text{Income}) + \text{Intercept} + \varepsilon, \quad \text{where } \varepsilon = \text{error}$$

Equation 29 3<sup>rd</sup> Income Regression – Logarithmic

Commodity	Intercept	Pr(>  t )	$\alpha_L$	Pr(>  t )	$r^2$	Adjusted $r^2$
Commodity 1	-112.6	2.0 E-76	26.7	1.1 E-92	0.905	0.904
Commodity 2	-18.6	1.7 E-45	4.58	2.9 E-61	0.584	0.580
Commodity 3	-123	6.9 E-52	24.7	1.8 E-59	0.884	0.883
Commodity 4	-64.8	4.3 E-28	20.0	4.6 E-50	0.840	0.839
Commodity 5	-136.5	1.4 E-40	27.8	3.5 E-48	0.830	0.829
Commodity 6	-22.8	7.1 E-36	4.74	6.7 E-44	0.793	0.791
Commodity 7.1	-143.7	6.4 E-40	27.7	9.3 E-45	0.866	0.864
Commodity 7 - Other	-215.7	9.8 E-52	42.7	6.3 E-58	0.911	0.910
Commodity 8	-34.4	2.5 E-50	7.82	2.0 E-63	0.965	0.965
Commodity 9	-322	6.2 E-55	64.4	1.5 E-62	0.900	0.899
Commodity 10	-52.3	1.2 E-14	9.83	6.6 E-17	0.463	0.459
Commodity 11	-208.4	6.3 E-51	41.1	1.1 E-57	0.855	0.854
Commodity 12	-172.3	1.6 E-54	34.8	1.3 E-62	0.890	0.889

Table 6 3<sup>rd</sup> Regression Results

It is not expected that this model will perform better than the 2<sup>nd</sup> regression for all commodities. The ANOVA test was used again to confirm which commodities are likely to have logarithmic relationships. If an ANOVA test results in a Non-Applicable (NA) error there is no evidence to reject the Null hypothesis (linear regression with zero intercept).

Commodity	Pr (> F)
Commodity 1	8E-67
Commodity 2	1.6E-43
Commodity 3	NA
Commodity 4	7.2E-65
Commodity 5	NA
Commodity 6	NA
Commodity 7.1	0.0255
Commodity 7 - Other	NA
Commodity 8	1.2E-30
Commodity 9	NA
Commodity 10	NA
Commodity 11	NA
Commodity 12	NA

Table 7 ANOVA Results – 2<sup>nd</sup> vs 3<sup>rd</sup> Regression



For commodity 1,2,4 and 8 there is significant statistical evidence to reject the Null hypothesis. For Commodity 7.1 the evidence is not significant enough to reject and the linear regression with zero intercept has better  $\Pr(> |t|)$ ,  $r^2$  and adjusted  $r^2$  results. Therefore, for all other commodities (3, 5, 6, 7.1, 7 – Other, 9, 10, 11 and 12) the Null hypothesis is not rejected. The final relationships are given in Table 8 and example charts of the regression and data are given in Figure 13, with all charts in Annex F: Charts and Results.

Code	Group (Description)	Relationship
1	Food & non-alcoholic drinks	Logarithmic
2	Alcoholic drinks, tobacco & narcotics	Logarithmic
3	Clothing & footwear	Linear
4	Housing(net) <sup>14</sup> , fuel & power	Logarithmic
5	Household goods & services	Linear
6	Health	Linear
7.1	Transport – Automotive	Linear
7 - Other	Transport – Other	Linear
8	Communication	Logarithmic
9	Recreation & culture	Linear
10	Education	Linear
11	Restaurants & hotels	Linear
12	Miscellaneous goods & services	Linear

Table 8 Income Regression Modelling Results

Logarithmic relationships between income and quantity implies a non-constant elasticity. These relationships are not possible with the CD UF which is a weakness when comparing to the available data. The weakness is exacerbated by the intervention being appraised changing the income levels in the economy.

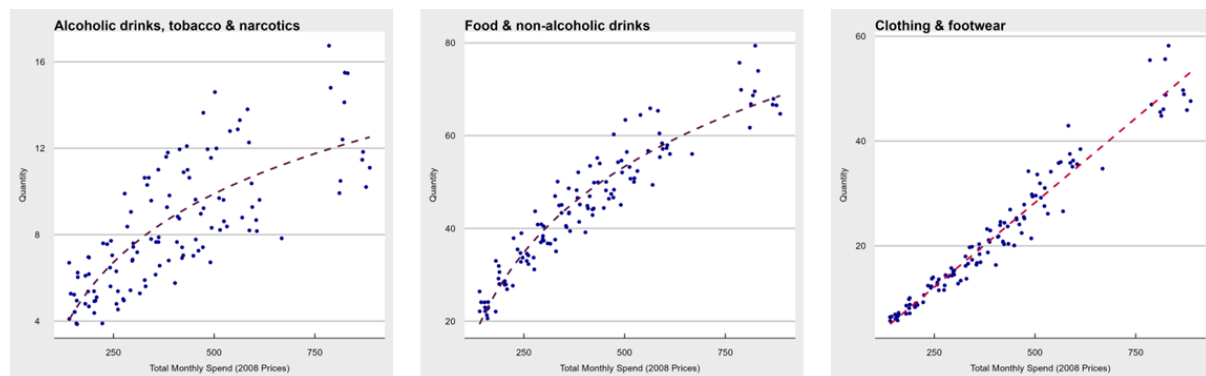


Figure 13 Example of Income Regression Results and Data

A guide to find all charts produced for this paper can be found in Annex C: Code Blocks.

<sup>14</sup> Excluding mortgage interest payments, council tax and Northern Ireland rates



## Price Elasticity of Demand

Following the same approach as income elasticity of demand a simple regression model of price vs quantity was constructed.

$$Quantity = \beta \cdot Price + Intercept + \varepsilon, \quad \text{where } \varepsilon = \text{error}$$

Equation 30 1<sup>st</sup> Price Regression - Linear.

Commodity	Intercept	Pr(>  t )	$\alpha_L$	Pr(>  t )	$r^2$	Adjusted $r^2$
Commodity 1	85.3	0.058	-0.419	0.73034	0.035	0.026
Commodity 2	18.3	4.2E-09	-0.106	0.4576	0.267	0.261
Commodity 3	48.6	0.543	-0.265	0.92517	0.003	-0.005
Commodity 4	70.3	0.107	-0.185	0.00171	0.022	0.014
Commodity 5	29.9	0.953	-0.028	0.27006	0.000	-0.008
Commodity 6	5.81	0.762	-0.008	0.11753	0.001	-0.008
Commodity 7.1	32.1	0.906	-0.12	0.91639	0.000	-0.01
Commodity 7 - Other	62.6	0.534	-0.256	0.79101	0.004	-0.006
Commodity 8	9.28	0.507	0.025	0.00058	0.003	-0.005
Commodity 9	56.9	0.994	0.005	0.53417	0.000	-0.008
Commodity 10	7.02	0.840	-0.015	0.05318	0.002	-0.007
Commodity 11	51.8	0.418	-0.188	0.43696	0.006	-0.002
Commodity 12	53.6	0.615	-0.21	0.73758	0.002	-0.006

Table 9 1<sup>st</sup> Price Regression Results

The price regression model did not yield statistically significant results and the chart of price vs quantity data indicates why. There are significant fixed effects in the model, namely the income effect on quantity, therefore a Fixed Effect model had to be implemented.

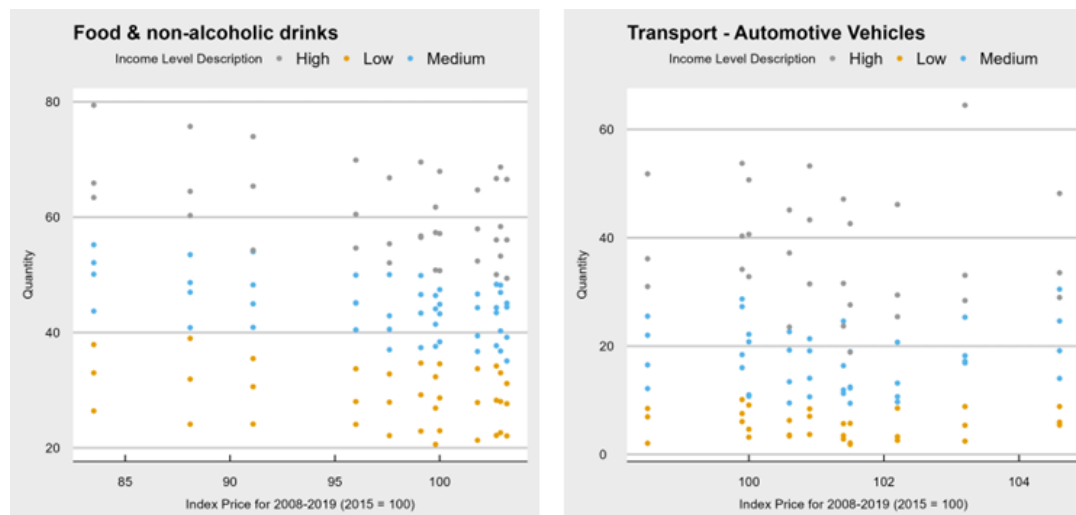


Figure 14 Price vs Spend Data

## Fixed Effects Regression

$$Quantity = \hat{\beta} \cdot Price + Intercept + \varepsilon \mid Income, \quad \text{where } \varepsilon = \text{error}$$

Equation 31 Price vs Quantity Fixed Effect regression.

The income level is fixed within the model by fixing the income decile group across years.



<i>Commodity</i>	$\hat{\beta}$	$Pr(>  t )$	$r^2$	<i>Adjusted <math>r^2</math></i>
<i>Commodity 1</i>	-0.419	1.9 E-30	0.985	0.985
<i>Commodity 2</i>	-0.106	1.9 E-41	0.938	0.938
<i>Commodity 3</i>	-0.265	7.9 E-05	0.979	0.979
<i>Commodity 4</i>	-0.185	6.8 E-08	0.929	0.929
<i>Commodity 5</i>	-0.028	0.649	0.939	0.939
<i>Commodity 6</i>	-0.008	0.526	0.838	0.838
<i>Commodity 7.1</i>	-0.120	0.634	0.926	0.926
<i>Commodity 7 - Other</i>	-0.256	4.4 E-09	0.991	0.991
<i>Commodity 8</i>	0.025	0.002	0.967	0.967
<i>Commodity 9</i>	0.005	0.961	0.988	0.988
<i>Commodity 10</i>	-0.021	0.203	0.736	0.736
<i>Commodity 11</i>	-0.188	6.2 E-08	0.980	0.980
<i>Commodity 12</i>	-0.210	5.8 E-05	0.985	0.985

Table 10 2<sup>nd</sup> Price Regression Results

This model produces more statistically significant results. It has also shown an expected result, that all the  $\hat{\beta}$  in the model are negative. This implies that commodities have this property, as price increases the quantity consumed decreases. Not all the  $\hat{\beta}$  are statistically significant, which could be explained by too few data points or that the consumption behaviour of that commodity is more dependent on the cross-price elasticity. These commodities include 5, 6, 7.1, 9 and 10.

### LASSO and Ridge Regression

To investigate the cross-price elasticity a multi regression method could be used. One of the assumptions of this model type is 'No Perfect Multicollinearity' (Stock & Watson, 2007). It is very unlikely that the price changes in the data are perfectly multicollinear however it is likely that the predictors would be imperfectly multicollinear. This is due to the commodities being made from similar intermediate goods and therefore having similar inflationary pressures controlling their prices.

Imperfect multicollinearity does not mean that the Ordinary Least Squares (OLS) regression is not possible however it will mean that at least one regressor will be imprecisely estimated, (Stock & Watson, 2007). To manage the data's imperfect multicollinearity, Penalised Regression Methods (PRM) were implemented:

- Ridge Regression (RR),
- Least Absolute Shrinkage and Selection Operator (LASSO).

RR is a means of estimating regression coefficients when data are high-dimensional and/or contain correlated variables (Hoerl & Kennard, 1970). LASSO regression is a shrinkage and variable selection method for regression models, which is well suited to overcoming overfitting issues, in terms of the number of variables ultimately included in the model, (Ranstam & Cook, 2018). This overestimation of how well the model performs in terms of using the included variables to explain the observed variability is known as optimism bias.

The regression model to be solved was:

$$Quantity = \alpha I + \sum_{i=1}^N \beta_i p_i + \varepsilon, \quad I = income$$

Equation 32 Cross-price and Income Regression





The approaches were implemented using the GLMNET package in R<sup>15</sup>. The difference between RR and LASSO in this package is changing the elastic net mixing parameter,  $\theta$  (Hastie, et al., 2023), which has a range  $\theta \in [0,1]$  where  $\theta = 0$  is RR and  $\theta = 1$  is LASSO. The results can be compared by the  $r^2$  figures produced by GLMNET, presented in Table 11. This shows that the LASSO has outperformed RR.

COMMODITY GROUP	LASSO $r^2$	RR $r^2$
COMMODITY 1	0.952	0.932
COMMODITY 2	0.952	0.932
COMMODITY 3	0.952	0.932
COMMODITY 4	0.952	0.932
COMMODITY 5	0.952	0.932
COMMODITY 6	0.952	0.932
COMMODITY 7.1	0.952	0.932
COMMODITY 7 - OTHER	0.952	0.932
COMMODITY 8	0.952	0.932
COMMODITY 9	0.952	0.932
COMMODITY 10	0.952	0.932
COMMODITY 11	0.952	0.932
COMMODITY 12	0.952	0.932

Table 11 LASSO vs RR  $r^2$

However further refinement of the model is required, as the results are counterintuitive and more likely modelling anomalies than insights. The full table of estimators can be found in Annex F: Charts and Results, Table 21 and Table 22.

PRM model interpretations:

- Majority of commodities have a zero-price elasticity of demand.
- All commodities are supplementary goods for commodity 1 (Food & non-alcoholic drinks) as the estimator is positive. However, the estimator of commodity 1 for commodity 1 is also positive, implying all things being equal, a price increase in commodity 1 increases quantity consumed of all commodities, which is not possible.
- A price change in commodity 12 has no impact on the quantity of any commodity consumed including that commodity, which is unlikely and contradicts the previous regression models.

Although this attempt at calculating the cross-price elasticities via PRM has not been successful this work has still been included in the paper. The reason is, that further work on this method will yield insights into consumer behaviours that should be incorporated when constructing the UF for a CGEM.

## Regression Results

The consumer behaviour that regression modelling has highlighted that should be included in a UF include:

- Logarithmic relationship between income and quantity in commodities 1, 2, 4 & 8,
- Although the PRM work was inconclusive it did infer that not all cross-price elasticities will be negative.

<sup>15</sup> <https://cran.r-project.org/web/packages/glmnet/index.html>



## Conclusion

This paper was an investigation into the use of CGEM as a policy analysis tool for domestic intervention into an advanced manufacturing sector. Due to the weakness discussed above a satisfactory model to appraise the policy decision has not been able to be completed. The first conclusion must be for the ongoing policy decision to be analysed using the previous framework.

However, this does not mean that this approach could not be utilised with additional work. It would require collaboration with analysts with additional skills in programming, data science and specific trade-data knowledge.

The results of this paper conclude that a future methodology should:

- Make use of the multi-year Cobb-Douglas production function parameters,
- Include intermediate goods values from ONS,
- Model an exogenous constant saving rate,
- Establish the labour - primary factor costs (salary) based on ONS data,
- Incorporating a tax system, ideally the simplest, either a Poll Tax or flat rate of income tax and corporation tax policy,
  - The justification of a simple tax system is that it increases the likelihood that the model's equilibrium can be shown to be unique,
- Model government expenditure as an increased consumption of commodities based on ONS Input-Output table data,
- Complete a regression model to determine future production function and intermediate good balance as the automotive sector transition to EV production as described in Generality of Production Function,
- If feasible, include economic data of the UK's key automotive trade partners (list of 18 given in Table 16),
- Have a presentation style appropriate for wider audiences to mitigate the risk that the model is perceived as a 'black box',
- Include further refinement of the PRM for the cross-price elasticity of demand which should be incorporated to a new UF, alongside the logarithmic income elasticity of demand,
- Adjust any results to account for additionality, aligned with the results of the RGF evaluation as discussed in Annex B: Technical: 10 - CBA Modelling Considerations.

This paper also recommends for the current methodology that further regression analysis is completed on automotive production against vehicle production figures, as discussed in Annex B: Technical: 8 - Wage Premium Uncertainty.

The impact of continuing CGEM modelling is not only on the potential insights for policy decision making but an opportunity to bring ex-DIT and ex-BEIS analytical teams in DBT together. This paper has highlighted the challenges with completing a successful CGEM which could be overcome by utilising the skills and knowledge within the ex-DIT team.



## Annex A: Dedication

This paper is dedicated to my wife for her love and support through the master's program. Without you, none of this would have been possible and sorry for the exam the week of our wedding.

## Annex B: Technical

### 1. Assumption Justification and Reasoning

The reason for some of the assumptions in the Required Modelling Assumptions section is that CGEM is modelling a perfectly competitive market; specifically, 1, 2, 3, 7, & 8. They are justifiable as assumptions in a model as assuming that the UK economy is a perfectly competitive market. The scope of this paper does not discuss whether the UK is a perfectly competitive market. The reasoning and justification for the other assumptions are given in Table 12 below.

Assumption	Reasoning	Justification
4	The inclusion of these three factors allows for a model which explores the topics of interest for the policy area. Labour is required to understand wage premium impacts, capital to understand how government intervention could be achieved and the intermediate goods are included to understand interaction between the different sectors of the economy.	The production function is limited to 3 factors and excludes entrepreneurship and land as other factors would increase complexity without providing insight for the policy question.
5	This helps with analytical simplification in model derivation and also allows the regression modelling of the CD production function.	This is a widely used assumption in analysis of production, (Lim & Shumway, 1992) however if not valid can lead to mistakes in output combinations. One example is between risk-neutral optimising firms and those which are risk averse. However, these considerations are outside the scope of this paper therefore the paper will assume the simplifying assumption.
6	Ensure that the production function is algebraically manageable for the modelling, while enabling it to contain the necessary factors of production.	There would be limited increased benefit to answering the policy question for incorporating this increase to complexity.
9	Analytical simplification to make this saving rate exogenous to the model.	There would be limited increased benefit to answering the policy question for incorporating this increase to complexity.
10	Analytical simplification and would allow the model to be computable.	There would be limited increased benefit to answering the policy question for incorporating this increase to complexity.
11	Analytical simplification to reduce the modelling requirements.	There would be limited increased benefit to answering the policy question for incorporating this increase to complexity.

Table 12 Modelling Assumptions - Reasoning and Justifications

### 2. Standard Industrial Classification (SIC)

SIC was first introduced into the UK in 1948 for classifying business establishments, (ONS, 2009), and can be used across multiple ONS datasets as a method of managing data disaggregated by production sector.



The classifications are in a hierarchy structure, section (letter), division (2-digit), group (3-digit), class (4-digit) and subclass (5-digit). There are 39 classifications used in parts of this report modelling, which are:

<b>Code</b>	<b>Description</b>	<b>5-Sector Code</b>
A01	01: Crop and animal production, hunting and related service activities	A
A02	02: Forestry and logging	A
A03	03: Fishing and aquaculture	A
B	B: MINING AND QUARRYING	A
C10T12	10-12: Manufacture of food, drink, and tobacco	B
C13T15	13-15: Manufacture of textiles, wearing apparel, leather, and leather products	C
C16	16: Manufacture of wood and of products of wood and cork	C
C17	17: Manufacture of paper and paper products	C
C18	18: Printing and reproduction of recorded media	C
C19	19: Manufacture of coke and refined petroleum products	C
C20	20: Manufacture of chemicals and chemical products	C
C21	21: Manufacture of basic pharmaceutical products and pharmaceutical preparations	C
C22	22: Manufacture of rubber and plastic products	C
C23	23: Manufacture of other non-metallic mineral products	C
C24	24: Manufacture of basic metals	C
C25	25: Manufacture of fabricated metal products, except machinery and equipment	C
C26	26: Manufacture of computer, electronic and optical products	C
C27	27: Manufacture of electrical equipment	C
C28	28: Manufacture of machinery and equipment	C
C29	29: Manufacture of motor vehicles, trailers, and semi-trailers	D
C30	30: Manufacture of other transport equipment	C
C31_32	31-32: Manufacture of furniture and other manufacturing	C
C33	33: Repair and installation of machinery and equipment	A
D	D: ELECTRICITY, GAS, STEAM AND AIR CONDITIONING SUPPLY	E
E	E: Water supply; sewerage, waste management and remediation activities	E
F	F: Construction	E
G	G: Wholesale and retail trade; repair of motor vehicles and motorcycles	E
H	H: Transportation and storage	E
I	I: Accommodation and food service activities	E
J	J: Information and communication	E
K	K: Financial and insurance activities	E
L	L: Real estate activities	E
M	M: Professional, scientific, and technical activities	E
N	N: Administrative and support service activities	E
O	O: Public administration and defence; compulsory social security	E
P	P: Education	E
Q	Q: Human health and social work activities	E
R	R: Arts, entertainment, and recreation	E
S	S: Other service activities	E



### 3. Further Reduction

When completing CGEM the following groupings of industry sectors have been used.

5-Sector Code	Definition
A	Raw Materials, Manufacturing Input
B	Food Goods
C	Manufactured Non-Food Goods
D	Automotive Production
E	Services and Other Goods

### 4. Production Function 1

The first attempt at a regression model of a production function was on the two-factor CD which for SIC 29 give the result:

$$Q_1 = 2.601 L^{0.435} K^{0.565}$$

*Equation 33 Two Factor CD model for SIC based on linear regression.*

$K$  is the capital used,  $L$  is the Labour used and 2.601 is the technology constant.

This section gives the mathematical derivation of the linear regression model used to calculate the parameters of the CD production function, given in Equation 10:

$$\vartheta_j(\cdot) = A_j \left( x_{1j}^{\beta_{1j}}, \dots, x_{Nj}^{\beta_{Nj}} \right) \left( v_{1j}^{\gamma_{1j}}, \dots, v_{Nj}^{\gamma_{Nj}} \right) = A_j \prod_{i=1}^N x_{ij}^{\beta_{ij}} \prod_{f=1}^F v_{fj}^{\gamma_{fj}}$$

For this regression we will model all the intermediate goods as a single input,  $G$ , which yields the three factor CD:

$$Q = AG^{\alpha} L^{\beta} K^{\gamma}$$

*Equation 34 Three Factor CD Production Function*

If you assume constant return to scale, then  $\gamma = 1 - \alpha - \beta$ .

$$\frac{Q}{K} = AG^{\alpha} L^{\beta} K^{-\alpha-\beta}$$

$$\ln \frac{Q}{K} = \ln A + \alpha \ln \frac{G}{K} + \beta \ln \frac{L}{K}$$

$$\hat{Q} = \hat{A} + \alpha \hat{G} + \beta \hat{L}, \quad \text{Where: } \hat{Q} \equiv \ln \frac{Q}{K}, \quad \hat{A} \equiv \ln A, \quad \hat{G} \equiv \ln \frac{G}{K}, \quad \hat{L} \equiv \ln \frac{L}{K}$$

*Equation 35 Multivariable Linear Regression Model of CD*

The constants in this equation can be estimated by multi-variable linear regression which was complete in R. The simple linear regression used to calculate the parameters can return answers in the whole range  $\mathbb{R}$ . Therefore, for some of the sectors there may be better estimators of the function outside the bounds of the parameters,  $0 < \alpha, \beta, \gamma < 1$ .



### Linear Regression Results

Sector	$\hat{A}$	Alpha	Beta	Gamma
A01	1.364	0.414	0.111	0.475
A02	1.602	0.188	0.353	0.458
B	1.113	0.666	0.118	0.215
C18	1.058	0.587	0.259	0.154
C19	1.282	0.698	0.072	0.229
C22	0.941	0.512	0.445	0.043
C23	1.261	0.396	0.470	0.135
C25	1.677	0.110	0.538	0.353
C28	1.036	0.466	0.459	0.075
C31_32	0.843	0.754	0.220	0.026
D	0.873	0.752	0.087	0.162
F	1.758	0.571	0.099	0.331
H	1.111	0.314	0.565	0.121
N	0.952	0.597	0.384	0.020
O	1.094	0.138	0.630	0.232
P	1.248	0.319	0.358	0.323
Q	1.904	0.293	0.244	0.462

Table 13 Linear Regression Modelling Results

## 5. Production Function 2

To find the parameters for the sectors that linear regression failed, other techniques were required. The aim was to find the local minimum Mean Squared Error (MSE) within the parameter range. Given the level of accuracy any eventual modelling would use, the first approach was to use a brute force search method. Calculating the MSE of all possible combination of  $[0.01 < \alpha, \beta, \gamma < 0.97, A \in (0.84, 1.90)]^{16}$ , at 2-digits (i.e., 0.23, 0.76, etc).

$$MSE = \sum (Q - \bar{Q})^2, \quad \bar{Q} = \bar{A} \bar{G}^{\bar{\alpha}} \bar{L}^{\bar{\beta}} \bar{K}^{\bar{\gamma}}$$

This approach would give a local minimum within the discrete search space, that would be a false local minimum of the continuous search space. i.e., it is the minimum MSE for the finite discrete set of parameters searched over.

Where the bar denotes the found estimators satisfying the equation:

$$\alpha_1, \beta_1, A_1 \text{ s.t. } \min_{MSE} SE = \sum (Q - \bar{Q})^2, \quad \bar{Q} = \bar{A} \bar{G}^{\bar{\alpha}} \bar{L}^{\bar{\beta}} \bar{K}^{\bar{\gamma}} [0.01 < \alpha, \beta, \gamma < 0.97, A \in (0.84, 1.90)],$$

However, this was too computationally intensive and only one result was found after 18 hours of run time. There were 22 sectors that needed finding (c16 day runtime).

Sector	A	Alpha	Beta	Gamma
A03	0.95	0.8	0.01	0.199

## 6. Production Function 3

The next search method to find the local minimums was an algorithmic search based on shrinking spheres. The starting point for the algorithm is the central point,  $\alpha = 0.33$ ,  $\beta = 0.33$ ,  $A = 1.24^{17}$ .

•

<sup>16</sup> This is the range of A that were found in the other linear regressions.

<sup>17</sup> This based on the mean A result from Linear Regression





### Algorithm Steps:

- Step 1 is to create new set of points which are selected at random on the surface of the sphere centred at the start point,
- The radius of the sphere decreases with each iteration of the algorithm,
- The MSE of each element of the set is calculated,
- The element with the smallest MSE is the new start point, return to step 1,

The algorithm is illustrated in Figure 15. The code can be found in LM Regression 7.R. The name of the functions in Figure 15 correspond to those in the code.

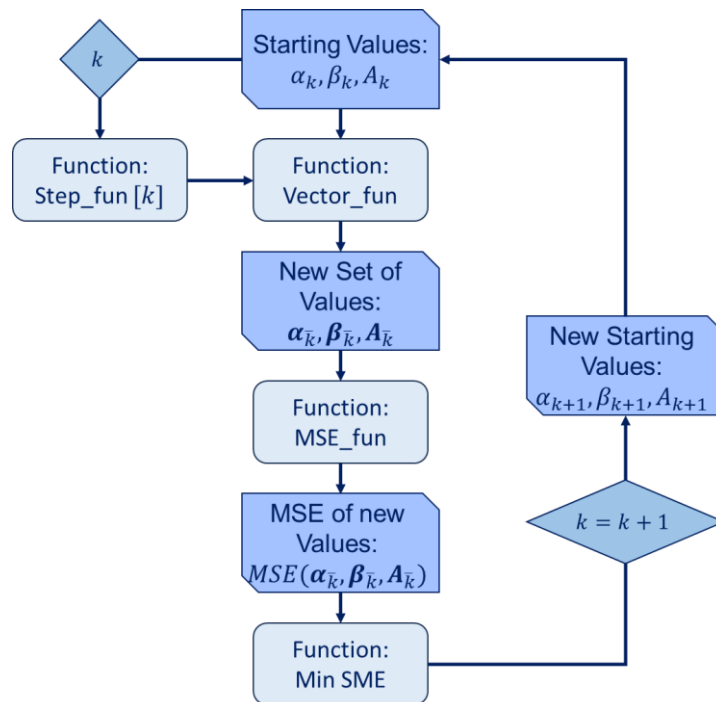


Figure 15 Flow Diagram of Search Algorithm



### *Production Function Results*

<b>Sector</b>	<b>A</b>	<b>Alpha</b>	<b>Beta</b>	<b>Gamma</b>
A01	3.91	0.41	0.11	0.48
A02	4.96	0.19	0.35	0.46
A03	29.92	0.41	0.01	0.58
B	3.04	0.67	0.12	0.22
C10T12	7.23	0.70	0.03	0.27
C13T15	8.06	0.75	0.18	0.06
C16	3.30	0.88	0.07	0.05
C17	141.98	0.29	0.57	0.13
C18	2.88	0.59	0.26	0.15
C19	3.60	0.70	0.07	0.23
C20	4.82	0.81	0.13	0.06
C21	84.22	0.41	0.49	0.09
C22	2.56	0.51	0.44	0.04
C23	3.53	0.40	0.47	0.13
C24	3.00	0.87	0.07	0.06
C25	5.35	0.11	0.54	0.35
C26	18.37	0.62	0.31	0.08
C27	3.57	0.87	0.08	0.06
C28	2.82	0.47	0.46	0.07
C29	8.08	0.73	0.20	0.06
C30	28.15	0.58	0.40	0.02
C31_32	2.32	0.75	0.22	0.03
C33	67.14	0.40	0.43	0.16
D	2.39	0.75	0.09	0.16
E	6.48	0.80	0.14	0.06
F	5.80	0.57	0.10	0.33
G	37.54	0.37	0.01	0.63
H	3.04	0.31	0.56	0.12
I	5.76	0.81	0.13	0.06
J	16.51	0.66	0.28	0.06
K	162.35	0.32	0.62	0.07
L	289.52	0.15	0.48	0.37
M	60.66	0.43	0.48	0.09
N	2.59	0.60	0.38	0.02
O	2.99	0.14	0.63	0.23
P	3.48	0.32	0.36	0.32
Q	6.71	0.29	0.24	0.46
R	17.81	0.61	0.31	0.07
S	62.86	0.41	0.48	0.12

*Table 14 Linear Regression and Search Algorithm Results*

These coefficients could be used in any future CGEM modelling without the bias of having a single benchmark year. Please note any combination of  $\alpha + \beta + \gamma \neq 1$  is due to rounding errors.



## 7. Household expenditure data

The groupings of commodity and services in the ONS 'Household expenditure by gross income decile group' are:

Code	Group (Description)
1	Food & non-alcoholic drinks
2	Alcoholic drinks, tobacco & narcotics
3	Clothing & footwear
4	Housing(net) <sup>18</sup> , fuel & power
5	Household goods & services
6	Health
7	Transport
8	Communication
9	Recreation & culture
10	Education
11	Restaurants & hotels
12	Miscellaneous goods & services

Table 15 ONS Household Expenditure Commodity groups

## 8. Wage Premium Uncertainty

Wage premium is calculated by taking the counterfactual salary away from the likely salary post intervention in the automotive sector. It is an exogenous assumption in the current methodology and is a key uncertainty. The automotive salary cannot be compared directly to the national mean as it does not account for the regional distribution of automotive salaries. The current methodology does not make assumptions about what sectors the new economic activity has displaced. Therefore, the counterfactual salary is the average local salary for the area of the new job.

The location in the following analysis will be disaggregated by Travel to Work Area (TTWA). The concept of TTWA is to create self-contained labour market areas. However, as commuting patterns are too diffuse, completely self-contained units are not possible. A statistical definition is used instead:

- At least 75% of the area's resident workforce work in the area and,
- At least 75% of the people who work in the area also live there.

(ONS, 2016)

Data on salary for different sectors and location is provided by the Annual Survey of Hours and Earnings (ASHE)<sup>19</sup> from the ONS. ASHE data is disaggregated by TTWA or by industry but not by both simultaneously. Therefore, it is not possible to directly know the automotive wage premium in each TTWA from the dataset. Therefore, a counterfactual salary must be constructed by the weight average across different TTWA.

Not all automotive jobs are the same with different skills, experience and Non-Vocational Qualifications (NVQ) levels required. A higher skilled automotive job will receive a higher salary but the counterfactual salary will also be higher. The ASHE data is disaggregated by percentiles therefore for the weighted average each percentile of automotive data is compared to each percentile of the local salaries.

<sup>18</sup> Excluding mortgage interest payments, council tax and Northern Ireland rates

<sup>19</sup> <https://www.ons.gov.uk/surveys/informationforbusinesses/businesssurveys/annualsurveyofhoursandearningsashe>



The level of automotive employment in a TTWA is taken from the ONS Business Register and Employment Survey (BRES)<sup>20</sup> and the available data is from 2015 – 2022.

This is an updated version of the wage premium calculation previously presented in this master programme (Lambert, et al., 2021).

$$WP_{auto} = \frac{\sum_{i=1}^N (x_i \sum_{j=1}^M (y_j^{auto} - y_{ji}^{all}))}{M \sum_{i=1}^N x_i}$$

Equation 36 Auto Wage Premium Calculation

Where:

$x_i$  = is the number of employees in TTWA  $i$

$y_{ji}^{all}$  = is the salary of all employees in TTWA  $i$  for percentile  $j$

$y_j^{auto}$  = is the auto salary in percentile  $j$

$N$  = the number of TTWAs

$M$  = the number of percentiles

This has been calculated for SIC code 29100 which covers production of automotive vehicles, from 2015 – 2022. The data is recorded annually in the price year of that data. Therefore, the Gross Domestic Product (GDP) deflator has been used to make the premiums comparable.

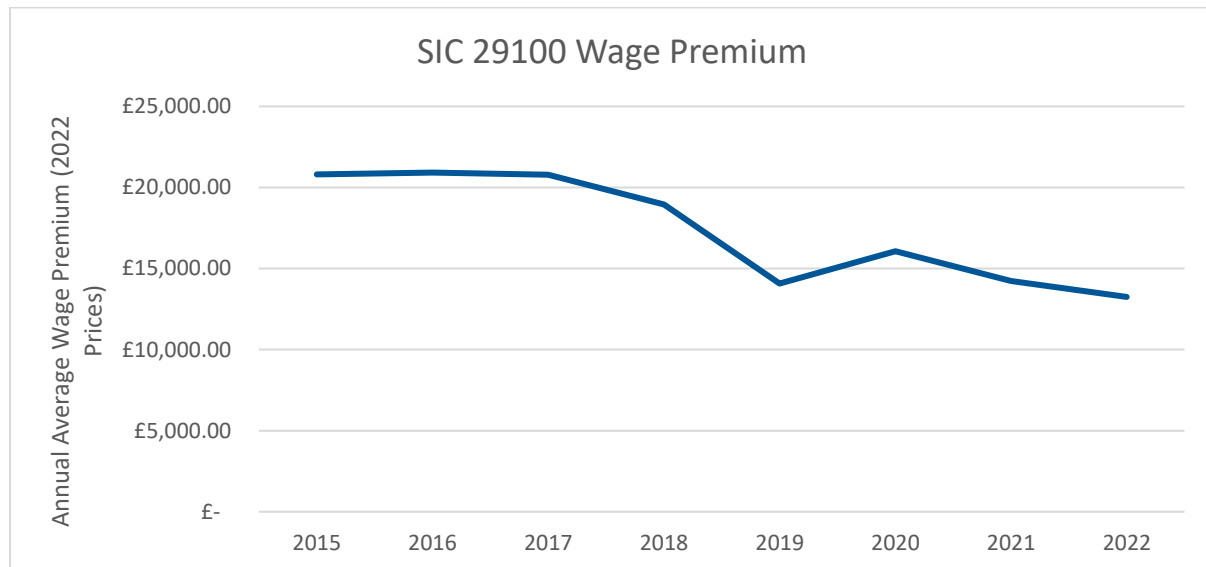


Figure 16 Average wage premium of automotive production

The automotive industry has been hit by external shocks recently. These include the Global financial crisis (2009), the trade uncertainty resulting from Brexit (2016) and COVID (2020). These dates have been highlighted in Figure 17 which is IHS data<sup>21</sup>.

<sup>20</sup> <https://www.nomisweb.co.uk/>

<sup>21</sup> This data is from a license provided to DBT and therefore the actual figures have been altered randomly but the inference taken from the chart remains true.



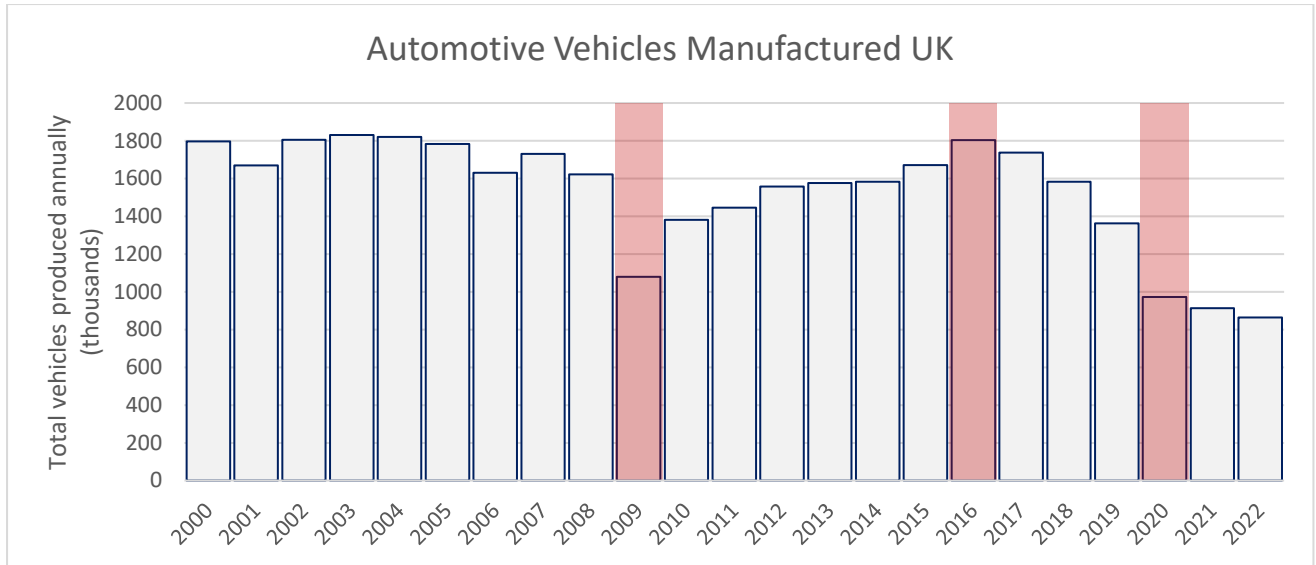


Figure 17 Total Annual Vehicle Production

This reduction in production since 2016 while maintaining similar capital capacity could have resulted in the depression of the automotive salaries. It would hold therefore that as production recovers (as it has historically), that the wage premium will follow. To test this hypothesis thoroughly it is recommended that a wage premium vs production regression be produced on all the data from 2000 to present.

#### Data Limitations – Wage Premium

There are current data limitations for the ONS BRES data as pre 2015 is not disaggregated by TTWA. Therefore, an alternate location division would be required for the above recommendation to be actioned.

### 9. Trade Locations

The UK has exported vehicles to 74 different countries and territories<sup>22</sup>. To include all these as individually modelled economies in a CGEM would vastly increase the complexity. The UK does not trade equally with all countries, 53% of exports have gone to EU countries between 2013 and 2022 (average calculated on volume of trade not value).

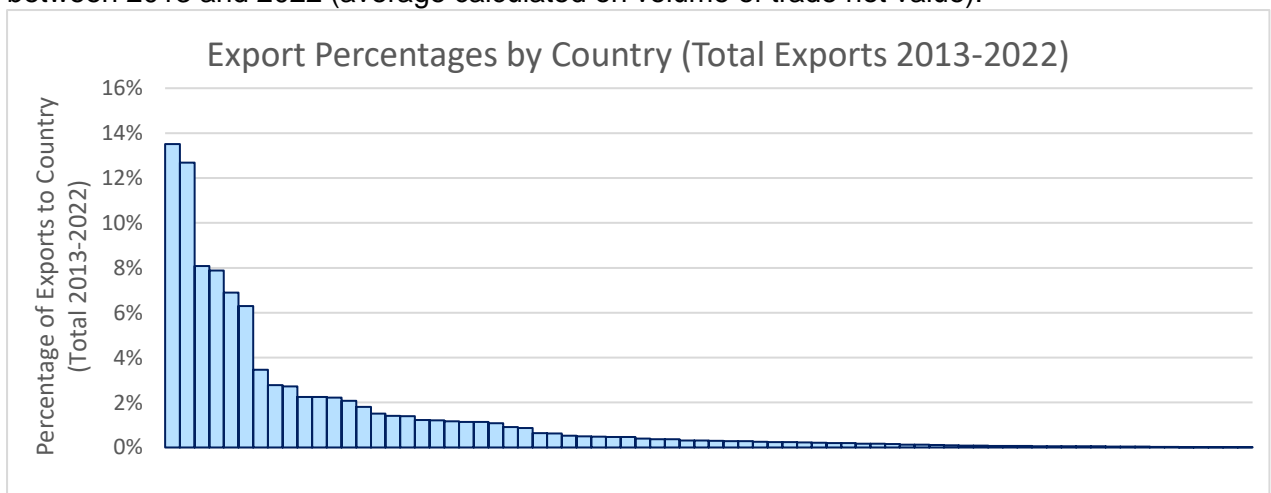


Figure 18 Export Percentage by Country

<sup>22</sup> Based on IHS data from <https://my.ihs.com/>



Figure 18 shows the percentage of exports to individual countries or territories by volume between 2013 to 2022. It shows that export volumes are focused on specific countries. Figure 19 shows the percentage of trade data included in a model if you add the above countries in order of percentage of exports of vehicles received, (blue line – Trade Data)

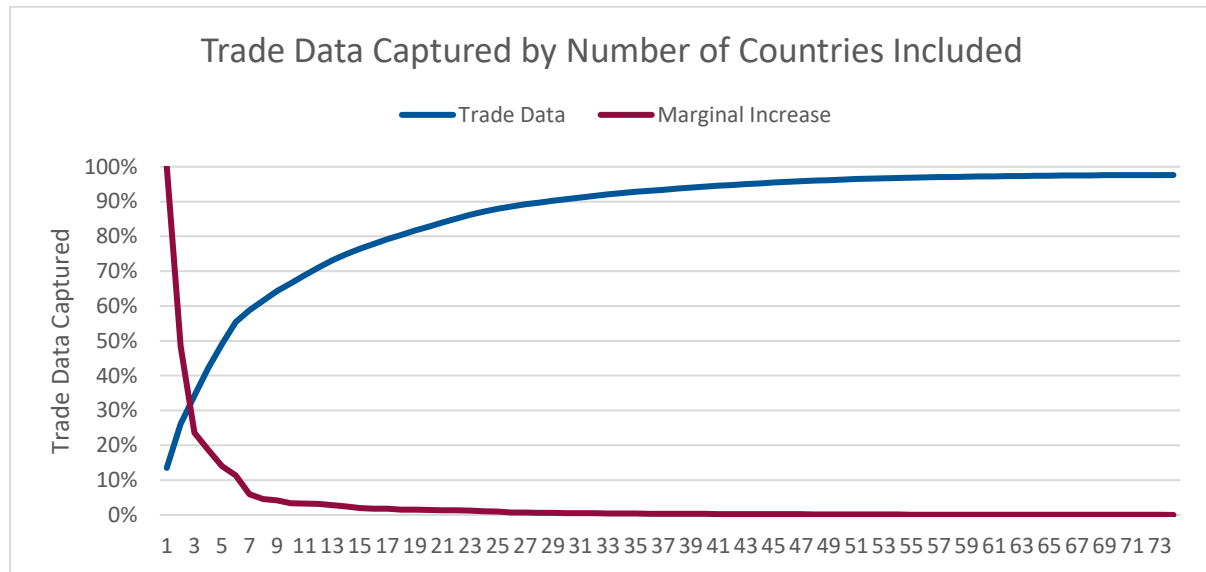


Figure 19 Trade Data Captured by Number of Countries in a Model

The marginal increase is defined as the percentage increase in trade data captured if the next country is included in a model and shown in Figure 19 as the red line – Marginal Increase. Therefore, most economic activity (80%) would be included by modelling the first 18 countries in the list. The marginal benefit of an additional country after that point would be an increase of 2% or less. Therefore, the recommendation for a CGEM which includes international trade should include the following 18 countries:

Country	Export Percentage
United States	14%
Germany	13%
Mainland China	8%
France	8%
Italy	7%
Spain	6%
Russia	3%
Belgium	3%
Poland	3%
Australia	2%
Netherlands	2%
Japan	2%
Turkey	2%
Sweden	2%
Canada	2%
Switzerland	1%
South Korea	1%
Portugal	1%

Table 16 - 18 largest automotive export countries



## 10. CBA Modelling Considerations

### *Additionality*

The Green Book defines additionality as a real increase in social value that would not have occurred in the absence of the intervention being appraised. It is a concept that enables analysts to understand and gauge how much of an intervention is additional, (HM Treasury, 2022). An intervention with an additionality rate of 75%, is stating that 75% of the intervention results in additional benefits and 25% of benefits would have occurred anyway. It is helpful in ensuring that appraisal is not biased to be overly optimistic.

The best evidence for additionality is the evaluation of previously completed similar programmes. For the intervention into advanced manufacturing sectors the evaluation report of the Regional Growth Fund (RGF) has been used.<sup>23</sup>

The RGF was launched in October 2010 and ran over six rounds, with options to apply for exceptional funding outside of the main bidding rounds. There were five schemes within the RGF, details in Table 17. The description in the table were taken from the RGF evaluation report (BEIS, 2022).

Scheme	Description	Characteristics
Scheme 1: <b>Regional Projects</b>	Funding to individual businesses or consortia more than £1m to support investments, R&D, and upgrades to plants.	258 projects supported from 2011/12 to 2016/17, across 219 businesses.
Scheme 2: <b>Place-Based Programmes</b>	Investments in transport, public realm, and infrastructure to unlock commercial developments.	38 spatial interventions in 31 Local Authorities between 2013 and 2015.
Scheme 3: <b>National Programmes</b>	Funding to intermediaries, such as banks, to then support Small to Medium size Enterprises (SME) with loans primarily, but also grants, advice and other support.	31 programmes involving over 17,000 businesses from 2012/13 to 2015/16.
Scheme 4: <b>Regional Programmes</b>	Funding to local intermediaries to then support SMEs with grants, advice, and other support.	104 programmes with over 15,000 business beneficiaries from 2012/13 to 2015/16.
Scheme 5: <b>AMSCI</b>	Funding to support R&D, skills, and investment in supply chains with projects often involving collaborations between primes and suppliers.	Three funded interventions which have supported 685 incidences of support, involving 473 businesses many of which collaborated, from 2012/13 to 2015/16.

Table 17 Details of RGF

For the purposes of capital intervention into advanced manufacturing sectors the most relevant category is the Regional Projects (Scheme 1). Regional Projects tended to be grants received by large businesses, with firms and business consortia receiving funding directly from BEIS (now DBT). Such projects typically involved capital investment by a business (for example the upgrade / expansion of premises or the installation of new plant and machinery). This would be the same for the policy being appraised by the current and alternative methodologies.

Growth in employment amongst beneficiary businesses is observed in the evaluation, but a key question here is whether this would have occurred without support. The study used different control groups to estimate this, using counterfactuals derived from combinations of

<sup>23</sup> <https://www.gov.uk/government/publications/regional-growth-fund-evaluation>





different match pools and model specifications. The difference is the average treatment effect on treated (ATT), (BEIS, 2022).

Using this approach, and adjusting for employment growth seen in comparable businesses, up to 49,417 additional job years were estimated to have been created/safeguarded out of the total 57,653 job years created/safeguarded during 2011-2014 by the Regional Projects (Scheme 1).

This is equivalent to an 86% additionality rating; this percentage should be incorporated into the CBA modelling completed on the results of the current and alternative methodologies.

### 11. Data Manipulation – Regression Modelling of Consumption

The data for this analysis is the ONS 'Household expenditure by gross income decile group' dataset. It provides multiple years of data which can be compared together when accounting for the different price years.

The income in each year becomes comparable once the GDP deflator<sup>24</sup> has been used to bring all data points to a single price year.

A comparable price of commodities is calculated using Consumer Price Index (CPI), as shown in Equation 37.

The relative quantity change was calculated by assuming a unit price of 1 in 2008. Quantity consumed of commodity was given by:

$$Q_{it} = \frac{spend_i}{price_{it}}, \quad price_{it} = \frac{price_{i,2008}}{\frac{CPI_{i,t}}{CPI_{i,2008}}} \quad \text{Equation 37 Relative Quantity Consumed}$$

$CPI_{i,t}$  was taken from ONS data<sup>25</sup>, this data covers 2008-2019.

<sup>24</sup> <https://www.gov.uk/government/collections/gdp-deflators-at-market-prices-and-money-gdp>

<sup>25</sup> <https://www.ons.gov.uk/economy/inflationandpriceindices/datasets/consumerpriceindices>



## Annex C: Code Blocks



All the R code used for this project can be found at:

<https://github.com/James-Lambertcoding/Dissertation>

Alternatively please scan the QR code for access.

This GITHUB also contains all the data used and charts produced in this paper.

The code is split into two sections:

1. R Model – CGEM
2. R Model – Regressions

The code in section 1 covers the CGEM.

CGEM Model	Associated Code
Model 1	11_Model_1_Calibration.R
Model 2	12_Model_2_Calibration.R
Non-Unity Wage and 39 Commodity Model	00_Main_Model_v1.R (which runs) <ul style="list-style-type: none"> <li>• 01_Packages.R</li> <li>• 02_read_data_v2.R</li> <li>• 02a_Start_point_data.R</li> <li>• 03_Calibration_v3.R</li> <li>• 04_CGE_Functions_v_2.R</li> </ul>

Table 18 GCEM Code Mapping

The 'Non-Unity Wage and 39 Commodity Model' was more complex and therefore the code blocks were sub-divided to better handle version control and enable easier quality assurance of the code.

The code in section 2 covers regression analysis of UFs via investigating household expenditure and regression of the production function.

Regression Model	Associated Code
Price and Income Models for Utility Function	Income_Price_Elasticity_v2.R
LASSO and Ridge Regression Functions	Income_Price_Elasticity_v2.R
Production Function Regression	LM Regression 7.R
Production Function Search Function	LM Regression 7.R
Fixed Effect Regression	Fixed_Effects_Regression_v1.R

Price and Income Models for Utility Function Charts are in Output\_3 and split into:

Folder	Associated Code
Plots Linear Income	Commodity Spend vs Income with Linear Regression Line
Plots Log Income	Commodity Spend vs Income with Logarithmic Regression Line
Plots No Regression Lines	Commodity Spend vs Income with No Regression Line
Plots Price	Commodity Spend vs Price with No Regression Line



### 1. Risk Register

The risk register is based upon the methodology used in project management within DBT and specifically in the programme management of the ATF. The risks are measured against likelihood and impact on a 5-level RAG (Red, Amber, Green) scale. These metrics are combined to give an overall RAG rating by the risk rating matrix. Any rating Amber or above requires a mitigation in place.

Risk Rating Matrix						
		Likelihood				
		Green	Green/Amber	Amber	Amber/Red	Red
Impact	Green	Green	Green	Green/Amber	Green/Amber	Amber
	Green/Amber	Green	Green/Amber	Green/Amber	Amber	Amber/Red
	Amber	Green/Amber	Green/Amber	Amber	Amber/Red	Amber/Red
	Amber/Red	Green/Amber	Amber	Amber/Red	Amber/Red	Red
	Red	Amber	Amber/Red	Amber/Red	Red	Red

Figure 20 Risk Rating Matrix



Risk	Description	Impact	Likelihood	Rating	Mitigation
Unavailability of data	Although the data was going to be sourced from the ONS, there is no guarantee that the disaggregation available would be suitable	Amber/Red	Green/Amber	Amber	There are private data providers that DBT has contracts with which could be used to procure additional data sources.
Data loss	Due to corruption of hard drives or mishandling of version control there could be a loss of work either completely or subsections.	Amber/Red	Green/Amber	Amber	The dissertation and code have been backed up across multiple devices and cloud platforms including, personal computer, DBT servers and Google cloud. Version Control has been implemented using GITHUB to ensure previous states of the code are retrievable. This adds additional redundancy to stop complete data loss. Comments are used exhaustively throughout to ensure clarity in understanding parts of the code.
Policy change	There could be a change in strategic direction for the policy area, meaning that the policy question being answered was no longer relevant.	Amber	Green	Green/Amber	N/A (Rating below Amber) This work provides additional benefits besides implications for a policy decision.
Modelling not completable	There is no guarantee that a complete CGEM would be codable within the time of the project.	Amber /Red	Amber	Amber/Red	The policy intervention will be assessed by the current methodology within the department. This ensures that the ability to provide robust analysis for policy decision is not solely dependent on this project.
Subject matter difficulties	Barriers to completing the work because of difficulties in using the CGEM approach.	Red	Amber	Amber/Red	The author has ensured good working relationships with subject matter experts within the department to provide guidance and support.
Time constraints	Balancing work on project with work and life. There could be other commitments stopping the completion of the project.	Amber/Red	Amber/Green	Amber	A project timeline was created to ensure that time was divided across all the aspects of the project. This would ensure that blockers in one aspect of the project do not hold back reaching a suitable conclusion. Ensure that there is flexibility to include more time/days on the project as necessary.

Table 19 Project Risk Register



			Proposal										Exam										Draft Deadline										Final Deadline			
			20-Mar	27-Mar	03-Apr	10-Apr	17-Apr	24-Apr	01-May	08-May	15-May	22-May	29-May	05-Jun	12-Jun	19-Jun	26-Jun	03-Jul	10-Jul	17-Jul	24-Jul	31-Jul	07-Aug	14-Aug	21-Aug	28-Aug	04-Sep									
			Time Available	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	7	1	1	3	1	4	3									
			Contingency Available	0	0	0	0	0	2	0	0	0	0	0	0	2	2	2	0	2		0	2		2	0										
Task	Sub-task	Line Item																																		
Research	Reading	Background																																		
		Modelling																																		
Outreach	Trade Analysts																																			
Modelling	Policy																																			
	Data	Research																																		
		Cleaning																																		
		Model 1																																		
		Model 2																																		
		Equilibrium Search																																		
		Full Sectors																																		
		Production Function																																		
		Utility Function																																		
		Context																																		
		Methodology - Current																																		
	CGEM	Methodology - Alternative																																		
		Modelling																																		
		Weakness																																		
		Technical Annex																																		
		Conclusion and Recommendations																																		
		Non Technical Summary																																		
		Narrative																																		
		Diagrams																																		
		Referencing																																		
		Spelling and Grammar																																		

Figure 21 Project plan

The project plan was created to ensure that enough time remained to complete tasks and evaluate when contingency days were required to complete steps. One of the main uses was to know when a certain activity had to be stopped (particularly in the modelling) to ensure time to complete the other stages. This enabled understanding the trade-off between continuing coding/modelling and focusing on completing the write-up.



Table 20 below is taken from (Institute for Apprenticeships, 2019) as presented by Queen Mary University of London.

High Level	individual KSBs	Pass/Distinction	Mapping
Applied economic analysis	<p><b>S1</b> Apply micro-economic and macro-economic theories and modelling, including econometric, to inform a range of business and policy decisions.</p> <p><b>S2</b> Convert the policy or other question into a tractable appraisal, evaluation or other analysis drawing on the most appropriate analytical method, including non-market valuation methods. Analysis considers, inter alia: the counterfactual, opportunity cost, risk and uncertainty and how to estimate discount rates and costs of capital.</p> <p><b>S3</b> Critically assess available information sources and judge validity and usefulness for the issue at hand; clean and manipulate data; be aware of data limitations and explain them; clearly describe and present data using data visualisation techniques; and draw out and explain policy and business implications to clients</p>	<p>Apply their economic knowledge to inform a range of business and/or policy decisions.</p>	<p><a href="#">Impact/Benefit justification</a> shows that economic knowledge has been applied to a business area and policy decision.</p> <p>There is a discussion of how evaluation results from previous policies can be incorporated into current economic appraisal in the <a href="#">Additionality</a> section and how this is best practise.</p>
		<p>Assess and argue what an appropriate method and data sources are and identifies any limitations in the policy/business situation.</p>	<p>The ONS data was identified as the appropriate data source for the project because of the multiple years of data and the reliability. The limitations to their datasets which are discussed in <a href="#">Data Limitations – SIC Codes</a> and <a href="#">Data Limitations – Wage Premium</a>.</p>
		<p>Give a clear explanation of the assumptions made in their analysis and argue effectively for why these are appropriate.</p>	<p>The desired modelling assumption for the CGEM are presented in <a href="#">Desired Modelling Assumptions</a> with further explanation given to explain reason and justification for them in <a href="#">Annex B: Technical: 1 – Assumption Justification and Reasoning</a>. To ensure they were presented clearly a table format was used in <a href="#">Table 12</a></p>
		<p>Choose a presentation style and data visualisation tools that effectively describe the analysis and draw out recommendations for policy/business decisions.</p>	<p>Multiple plots and maps have been used through the project to illustrate the analysis: particularly:  <a href="#">Figure 2</a>  <a href="#">Figure 10</a>  <a href="#">Figure 11</a></p> <p>There are diagrams used to help explain the modelling approach at a high level:  <a href="#">Figure 1</a>  <a href="#">Figure 3</a>  <a href="#">Figure 4</a></p> <p>At a detailed level flow diagrams for depicting the logic of algorithms:</p>



			<a href="#">Figure 6</a> <a href="#">Figure 15</a>
		Identifies data relevant to the issue and the limitations of that data when judging validity and usefulness	The relevant data for the project is the Input Output Tables whose weakness are discussed in <a href="#">Data Limitations – SIC Codes</a> . Within this section there is a discussion of the data usefulness and why in the CGEM approach it should continue to be used. In the conclusion the limitations of the data are considered.
		Evaluate how limitations in the method/data selected could be improved upon and judge what the risks are for the project's conclusions.	The paper contains a complete section on the modelling weakness, which inform the conclusion of the paper. There is an in-depth mathematical derivation of the weakness to the CD UF used in the initial modelling. Specifically, the inability for the CD UF to handle income elasticity of demand and cross-price elasticity of demand, covered in <a href="#">Utility Function (UF)</a> .
		Explore a range of ways to improve their assumptions through new analysis and create convincing arguments to support their judgements.	The weakness of the initial UF analysis and production function assumption are explored, and alternate versions are provided. <a href="#">Production Function</a> , <a href="#">Generality of Production Function</a> and <a href="#">Utility Function (UF)</a> .
		Make predictions of the likely impact of their recommendations on the business/policy situation	The paper provides a prediction on the impact of this project on the analytical working environment between ex-DBT and ex-BEIS analysts in the <a href="#">Conclusion</a> .
<b>Project management and planning</b>	<b>S5</b> Scope areas of work identifying objectives, analytical methods, resources required and potential delivery risks. Able to recognise when complementary expertise is required e.g., scientists, other social scientists, and data specialists.	Set out a clear project scope and had the correct resources to deliver the requirements of the project	This is evidenced by the inclusion of <a href="#">Project Aims</a> and <a href="#">Project Objectives</a> . The resources were in place to work on the project, as best as possible given time constraints. Additional support was gained by reaching out to trade analysts within Ex-DIT
		Understood the risks to the project and set out evidence of how these were mitigated during the project.	Evidenced by the Risk Register: <a href="#">Table 19 Project Risk Register</a>





		Have, where required, drawn on other sources of expertise and opinion to inform their results and ensure maximum impact.	The initial stages of developing this work required collaborating with trade analyst in ex-DIT who have the sources and the expertise to inform the work in the project.
		Demonstrate how the learning they have generated during the project could be used to inform future projects and/or the wider workplace.	This is evidenced in the recommendations made in the <a href="#">Conclusion</a> . The work can be used to inform wider workplace analytical priorities. The recommendation examines the impact of continuing this work on the joining of ex-DIT and ex-BEIS into DBT.
		Devised processes for interdisciplinary working or tools to improve the effectiveness of interdisciplinary working.	The join-up and buy-in from trade analysts has improved interdisciplinary working and enabled a smoother transition to a joint DBT analytical working environment.
Effective communication	S7 Clearly communicate economic principles and concepts to non-economists; present trade-offs and uncertainties and articulate these clearly; frame advice, drawing on knowledge of stakeholders' positions, for maximum impact	Communicate complex economic ideas to a non-economist audience.	This is evidenced by the <a href="#">Non-Technical Summary</a> and the <a href="#">Conclusion</a> .
		Explore options and trade-offs.	Exploration of options and trade-offs is given in <a href="#">Structural Choices</a> and by the paper covering multiple models and different functions that could be used in the approach.
		Set out key uncertainties.	The two key uncertainties in the modelling approach are set out in <a href="#">Uncertainties</a> .
		Frames advice showing awareness of how stakeholders will react to analysis or recommendations.	The advice in the conclusion is to continue working with the ex-DIT analysts and it shows how to best land the advice generated by the project by anticipating stakeholder reactions.
		Demonstrates that they can tailor their communication approach to the needs of different audiences.	This is evidenced by the <a href="#">Non-Technical Summary</a> and the language in the <a href="#">Conclusion</a> being tailored to the non-analytical audience. There is the inclusion of maps and charts to help communicate to those who benefit from visual explanations. Evidenced by <a href="#">Figure 2</a> <a href="#">Figure 10</a> <a href="#">Figure 15</a>



			<p>There are diagrams used to help explain the modelling approach at a high level:</p> <p><a href="#">Figure 1</a></p> <p><a href="#">Figure 3</a></p> <p><a href="#">Figure 4</a></p>
Horizon scanning	<p><b>S6</b> Use horizon scanning methodologies to anticipate new trends, opportunities and challenges that may influence outcomes of interest to client.</p>	<p>Show how trends, future opportunities and future challenges will affect their analysis.</p>	<p>This is evidenced by the inclusion of the description of <a href="#">Government Departmental Changes</a> which has helped motivate this project.</p> <p>In <a href="#">Wage Premium Uncertainty</a>, there is a discussion of how future changes to the automotive industry will impact the analysis.</p>
		<p>Demonstrate how their conclusions are resilient to future events or represent least regret solutions.</p>	<p>In <a href="#">Data Limitations – SIC Codes</a> there is a discussion of how future changes to the automotive industry will impact the modelling. It presents a method to build resilience into the modelling for these future events.</p>
Maintaining quality standards	<p><b>S8</b> Design Quality Assurance processes and implement these, following organisational best practices, and drawing on sources of external expertise; critically assess economic analysis and improve it.</p> <p><b>B1</b> Ethical conduct: analyst attributes sources and ideas to their originator; provides honest advice on all relevant aspects to an issue; avoids bias.</p> <p><b>B4</b> Rigour: demonstrates a commitment to detail.</p>	<p>Devise a robust quality assurance process for their work to ensure that analysis and written outputs are accurate and error free.</p>	<p>When creating the code for the models in this project, steps have been taken to allow robust quality assurance. This includes:</p> <ul style="list-style-type: none"> <li>• Proper version control by using GITHUB,</li> <li>• Completing thorough comments to enable others to follow,</li> <li>• For complex modelling the code blocks have been broken out, evidenced by <a href="#">Annex C: Code Blocks</a>.</li> </ul>
		<p>Clearly set out and reference all the sources they use, including both data sources and the source of ideas.</p>	<p>A referencing system has been used and a <a href="#">Bibliography</a> provided to set out clearly the sources of ideas. The sources of data are provided with links to original data sets where possible. The latter is evidenced by footnotes <a href="#">1</a>, <a href="#">7</a>, <a href="#">9</a>, and <a href="#">13</a> (non-exhaustive list). There has been necessary redaction of sensitive sources.</p>
		<p>Selected the appropriate level of detail necessary to achieve the required output.</p>	<p>The paper is presented at different levels, <a href="#">Non-Technical Summary</a> and <a href="#">Conclusion</a> for policy focussed readers as well as a detailed mathematical derivation for analytically focussed readers, <a href="#">Model Derivation</a>. This will enable the recommendation of the paper to be understood by all readers.</p>



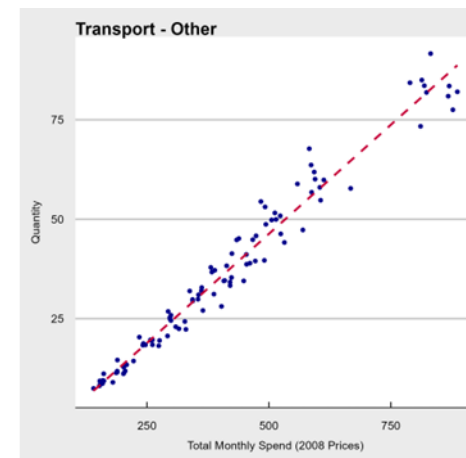
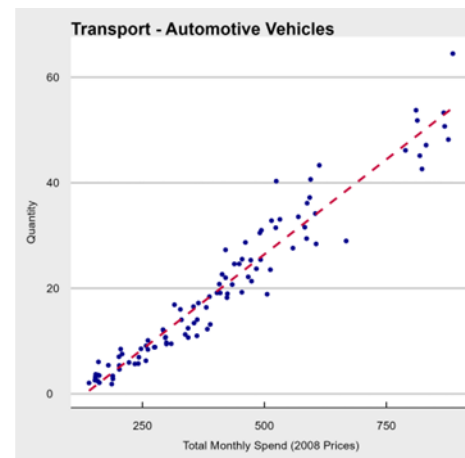
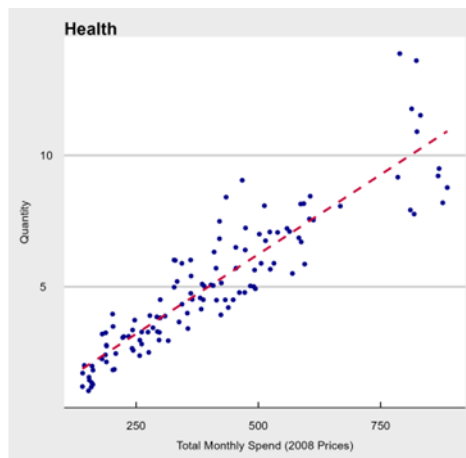
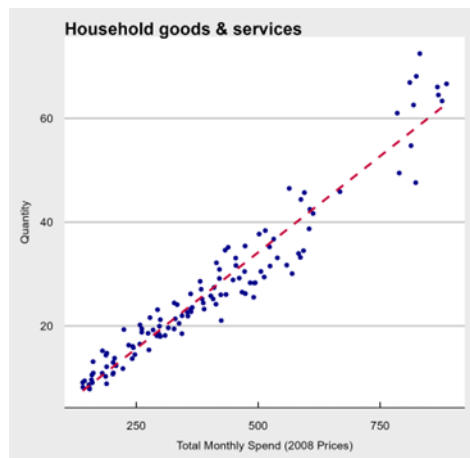
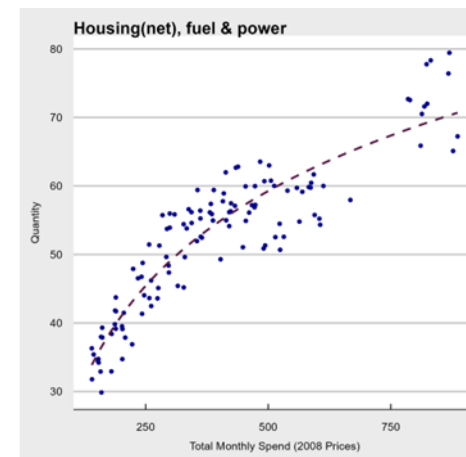
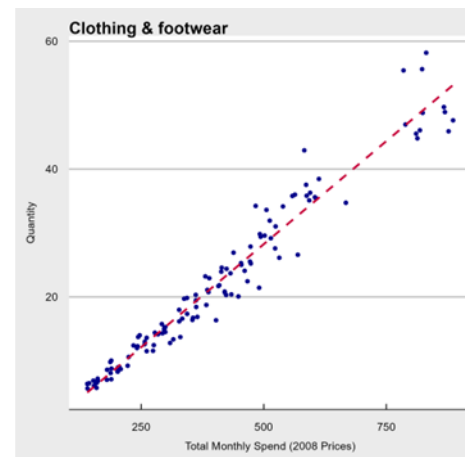
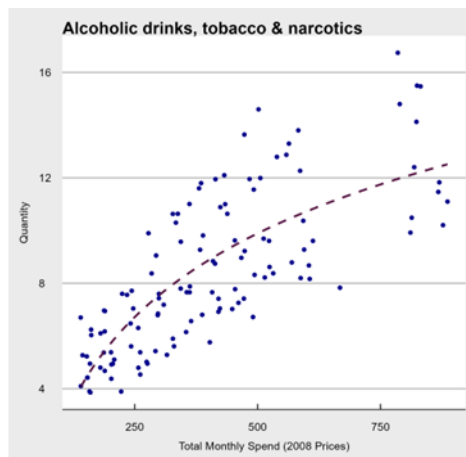
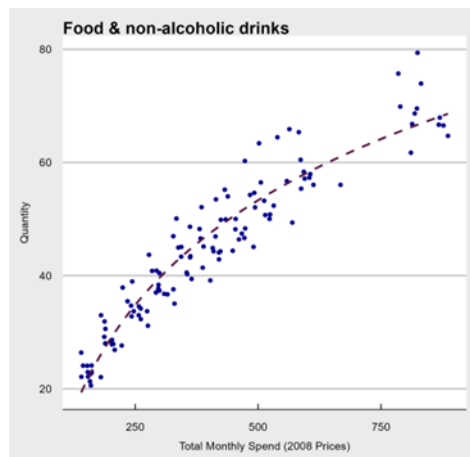
<ul style="list-style-type: none"> <li>• Compare their results with results from other methods or studies to check validity of results and conclusions.</li> </ul>	<p>The results from the equilibrium model were checked against the delta function to test their validity. The project is looking at an alternative methodology for a policy analysis. The conclusions of the that analysis can be checked against the current methodology.</p>
<ul style="list-style-type: none"> <li>• Describe the steps taken to ensure that their analysis is free from bias.</li> </ul>	<p>The analysis has been based on national statistics which are produced at the highest standard and should have limited bias in them.</p> <p>There is a discussion on how to incorporate the GrB definition of additionality to counteract optimism bias when appraising the results of any methodology used, found in <a href="#">Additionality</a> section.</p>
<ul style="list-style-type: none"> <li>• Ensure that inconvenient facts or analyses that do not fit their argument are addressed.</li> </ul>	<p>This paper included a thorough exploration of the weakness of the approach and provides full details of the limitation in the author's ability to complete a full complex CGEM. Therefore, this shows that all facts and analysis has been included.</p>
<ul style="list-style-type: none"> <li>• Draw on new sources of expertise that are external to their organisation which substantially improve the robustness of, or insight from, findings.</li> </ul>	<p>The current methodology as presented in this paper draws on subject matter expertise, particular the APC. This is evidenced by the information presented in <a href="#">Figure 8</a>.</p>
<ul style="list-style-type: none"> <li>• Challenge conventional wisdom and/or existing approaches in a sensitive and effective way.</li> </ul>	<p>This paper challenges the conventional wisdom of the current methodology to sector intervention analysis. It highlights the potential weaknesses and posits potential solutions. However, it balances a full recommendation of the alternative approach by reviewing the weakness of CGEM. There is also a discussion about communicating the analysis and recommendations in the <a href="#">Black Box</a> section.</p>

Table 20 QMUL KSBs



## Annex F: Charts and Results

Income and spend regression chart results:



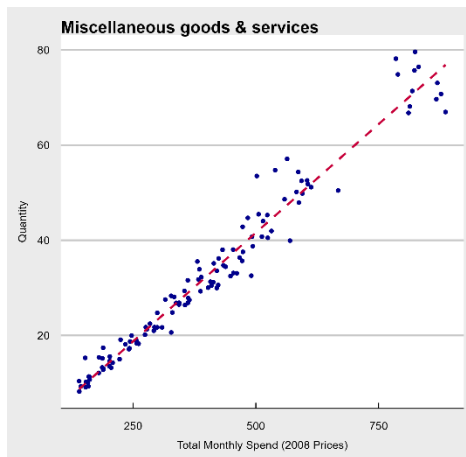
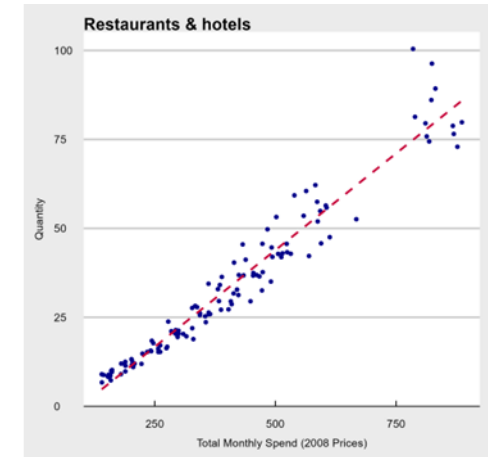
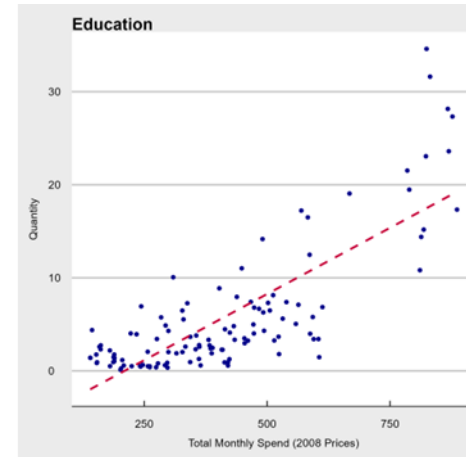
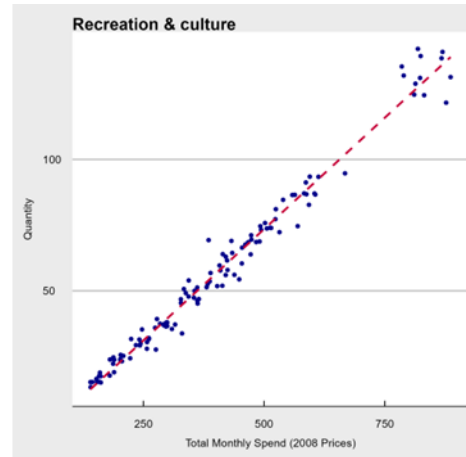
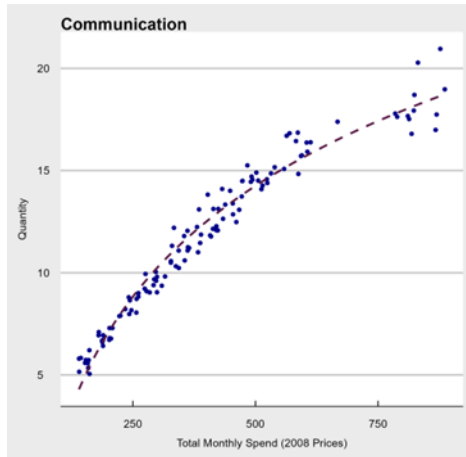


Figure 22 Quantity vs Income Charts with recommended relationship plotted (Linear or Logarithmic).



## 1. LASSO Modelling Results

LASSO	Regressors													
Commodity Groups	Income	Price 1	Price 2	Price 3	Price 4	Price 5	Price 6	Price 7.1	Price 7 Other	Price 8	Price 9	Price 10	Price 11	Price 12
Commodity 1	82.709	0.067	-0.16	0	0	0	0	0	-0.063	0	-0.149	-0.09	-0.016	0
Commodity 2	16.41	0.012	0	0	0	-0.059	0	0	0	0	-0.039	0	-0.039	0
Commodity 3	53.105	0.065	-0.013	0	-0.275	-0.046	0	0	0	0	0	-0.241	0	0
Commodity 4	81.737	0.049	0.164	0	-0.266	0	0	-0.107	0	0	0	-0.224	-0.065	0
Commodity 5	34.398	0.074	0	0	0.125	0	-0.401	0	0	-0.105	0	0	0.112	0
Commodity 6	7.745	0.012	0	0	0	0	0	0	0	0	0	-0.075	0	0
Commodity 7.1	21.002	0.072	0.006	0	1.235	0	-0.433	0	-1.107	0	0	0.63	0.019	0
Commodity 7 - Other	43.179	0.11	0	-0.062	0	0	0	-0.107	0	-0.19	-0.018	-0.061	0	0
Commodity 8	-0.857	0.019	-0.066	0	0	0	0	0	0.047	0	0	0.065	0	0
Commodity 9	65.39	0.17	-0.037	0	0.211	0	0	0	-0.121	0	0	-0.82	0	0
Commodity 10	-173.829	0.029	0.26	0.012	0.028	0.59	-1.402	0	1.399	-1.058	1.746	0.596	-0.46	0
Commodity 11	31.517	0.109	-0.238	-0.137	0	0	0	0	0	0	0	-0.062	0	0
Commodity 12	20.819	0.091	-0.074	0	0	0	0	0	0	0	-0.075	-0.044	-0.04	0

Table 21 LASSO Model Regressor Results

The cells highlighted in blue represent the model's estimation of linear quantity responsiveness to self-price changes.



## 2. Ridge Regression Modelling Results

LASSO	Regressors													
Commodity Groups	Income	Price 1	Price 2	Price 3	Price 4	Price 5	Price 6	Price 7.1	Price 7 Other	Price 8	Price 9	Price 10	Price 11	Price 12
Commodity 1	63.852	0.061	0	-0.009	0	0	-0.191	0	0	0	-0.016	0	0	0
Commodity 2	15.075	0.011	0	0	0	-0.053	0	0	0	0	-0.03	0	-0.037	0
Commodity 3	1.748	0.058	0	0	0	0	0	0	0	0	0	-0.008	0	-0.024
Commodity 4	62.764	0.045	0	0	-0.171	0	0	0	0	0	0	-0.063	-0.055	0
Commodity 5	0.12	0.067	0	0	0	0	0	0	0	0	0	0	0	0
Commodity 6	2.182	0.011	0	0	0	0	0	0	0	0	0	-0.015	0	0
Commodity 7.1	-6.815	0.065	0	0	0	0	0	0	0	0	0	0	0.003	0
Commodity 7 - Other	9.553	0.099	0	0	0	0	-0.091	0	0	0	0	0	0	-0.047
Commodity 8	4.675	0.017	0	0	0	0	0	0	0	0	0	0	0	0
Commodity 9	-4.299	0.152	0	0	0	0	0	0	0	0	0	0	0	0
Commodity 10	-5.749	0.026	0	0	0	-0.045	0	0	0.054	0	0	0	-0.004	0
Commodity 11	10.027	0.098	-0.034	0	0	-0.137	0	0	0	0	0	0	0	0
Commodity 12	3.935	0.082	0	0	0	0	0	0	0	0	-0.045	0	0	0

Table 22 Ridge Regression Model Regressor Results

The cells highlighted in blue represent the model's estimation of linear quantity responsiveness to self-price changes.





Abb. and Acronyms	Full Name
<b>ASHE</b>	Annual Survey of Earnings and Hours
<b>APC</b>	Advanced Propulsion Centre
<b>ATF</b>	Automotive Transformation Fund
<b>ATT</b>	Average Treatment effect on Treated
<b>BCR</b>	Benefit Cost Ratio
<b>BEIS</b>	Department for Business, Energy, and Industrial Strategy
<b>BRES</b>	Business Register and Employment Survey
<b>CBA</b>	Cost Benefit Analysis
<b>CD</b>	Cobb-Douglas
<b>CGEM</b>	Computable General Equilibrium Model
<b>CO2</b>	Carbon Dioxide
<b>CPI</b>	Consumer Price Index
<b>DBT</b>	Department for Business and Trade
<b>DIT</b>	Department for International Trade
<b>EV</b>	Electric Vehicles
<b>GDP</b>	Gross Domestic Product
<b>GEM</b>	General Equilibrium Model
<b>GrB</b>	Green Book
<b>HH</b>	Households
<b>HMG</b>	His Majesty's Government
<b>ICEV</b>	Internal Combustion Engine Vehicles
<b>JLR</b>	Jaguar Land Rover
<b>LASSO</b>	Least Absolute Shrinkage and Selection Operator
<b>MOG</b>	Machinery of Government Change
<b>MPB</b>	Marginal Private Benefit
<b>MSB</b>	Marginal Social Benefit
<b>NA</b>	Non-Applicable
<b>NGO</b>	Non-Governmental Organisations
<b>NVQ</b>	Non-Vocational Qualifications
<b>OEM</b>	Original Equipment Manufacturers
<b>OLS</b>	Ordinary Least Square
<b>ONS</b>	Office for National Statistics
<b>PRM</b>	Penalised Regression Model
<b>RAG</b>	Red, Amber, Green
<b>RGF</b>	Regional Growth Fund
<b>RR</b>	Ridge Regression
<b>SIC</b>	Standard Industrial Classifications
<b>SME</b>	Small to Medium sized Enterprises
<b>SUTs</b>	Supply and Use Tables
<b>TTWA</b>	Travel To Work Area
<b>UF</b>	Utility Function
<b>VfM</b>	Value for Money

Table 23 Abbreviations and Acronyms used in the paper.



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