MTH108 — Linear Algebra

James Li — 501022159

Professor: K. Q. Lan

Email: klan@torontomu.ca

Content by Week

1 Euclidean Spaces

A Euclidean Space is a mathematical space in which points and lines can be represented by a set of coordinates in the respective dimension of the space, and every point can be represented in a defined set. For example:

$$\mathbb{R}^3 = (x, y, z); x, y, z \in \mathbb{R}$$

is three-dimensional space represented using coordinates in terms of (x, y, z), where $x, y, z \in \mathbb{R}$.

Theorem 1.1. *Given two vectors $\vec{a}, \vec{b}$ and some constant k, $\vec{a}$ and $\vec{b}$ are called **parallel** if:*

$$\vec{a} = k\vec{b} \Leftrightarrow \vec{a} // \vec{b}$$

1.1 Products of Vectors with Constants

Theorem 1.2. *Given a constant k in $\mathbb{R}$ and some vector $\vec{a}$ in $\mathbb{R}^2$, the product of $k\vec{a}$ is:*

$$k\vec{a} = k(x_1, y_1) = (k \cdot x_1, k \cdot y_1), x, y \in \mathbb{R}$$

To represent a vector in Linear Algebra, we can use the following notation (*using the previously mentioned vector $\vec{a}$ as an example*):

$$\vec{a} = (x_1, y_1) = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

1.1.1 Examples:

Take $\vec{a} = \left(\begin{smallmatrix} 2 \\ -1 \end{smallmatrix} \right)$ and $\vec{b} = \left(\begin{smallmatrix} 4 \\ -1 \end{smallmatrix} \right)$, compute $-2\vec{a} + 3\vec{b}$:

$$\begin{aligned} -2\vec{a} + 3\vec{b} &= 2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 3 \begin{pmatrix} 4 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} 12 \\ -3 \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ -1 \end{pmatrix} \end{aligned} \tag{1}$$

1.2 Products of two Vectors

Theorem 1.3. *Given two vectors in $\mathbb{R}^n, \vec{a} = \left(\begin{smallmatrix} x_1 \\ \vdots \\ x_n \end{smallmatrix} \right), \vec{b} = \left(\begin{smallmatrix} y_1 \\ \vdots \\ y_n \end{smallmatrix} \right)$, the product of $\vec{a} \cdot \vec{b}$ is:*

$$\begin{aligned} \vec{a} \cdot \vec{b} = (x1, \ldots, x_n) \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \\ = x_1y_1 + x_2y_2 + \cdots + x_ny_n \end{aligned}$$

*This is known as the **Dot Product**.*

1.2.1 Examples:

Take $\vec{A} = \left(\begin{smallmatrix} 1 \\ -1 \\ 2 \\ 3 \end{smallmatrix} \right)$ and $\vec{b} = \left(\begin{smallmatrix} 2 \\ 1 \\ -1 \\ 1 \end{smallmatrix} \right)$, Find the dot product of $\vec{a} \cdot \vec{b}$:

$$\begin{aligned} \vec{a} \cdot \vec{b} &= (2) + (-1) + (-2) + (3) \\ &= 2 \end{aligned} \tag{2}$$

2 Placeholder

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