MTH210 — Discrete Mathematics II

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Content by Week

1 Sequences and Series

A **sequence** is an ordered set of numbers.

$2, 4, 6, 8, 10\dots$ is an example of a sequence of positive numbers.

$a_1, a_2, a_3, \dots, a_n$ denotes an infinite sequence.

- A sequence is defined **analytically** if each term a_i is defined by some function $f(i) = a_i$
- A sequence is defined **recursively** if the first k terms are given **explicitly** and the rest are given through a recursive function $a_n = f(a_{n-1}, a_{n-2}, \dots, a_{n-k})$ for $n > k$.
- Even if two sequences are equal for small indexes, does not indicate that they don't diverge at some further point.

A **series** is the sum of all the terms in a sequence.

If m and n are integers and $m \leq n$, the symbol $\sum\limits_{k=m}^{n} a_k$ is the summation from k, defined as:

$$\sum_{k=m}^{n} a_k = a_m + a_{m+1} + a_{m+2} + \cdots + a_n$$

- We call k the **index** of the summation.
- m is the **lower limit** of the summation.
- n is the **upper limit** of the summation.

1.1 Sums and Products

If m and n are integers and $m \leq n$, the symbol $\prod\limits_{k=m}^{n} a_k$ is read as product from k equals m to n of a sub k, it can be written as:

$$\prod_{k=m}^{n} a_k = a_m \cdot a_{m+1} \cdot a_{m+2} \times \cdots \times a_n$$

Theorem 1.1. *The following properties hold for any integer $n \geq m$, given $a_m, \dots$ and $b_m, \dots$ sequences of real numbers.*

- $\sum\limits_{k=m}^{n} a_k + \sum\limits_{k=m}^{n} b_k = \sum\limits_{k=m}^{n} (a_k + b_k)$
- $c \cdot \sum\limits_{k=m}^{n} a_k = \sum\limits_{k=m}^{n} (c \cdot a_k)$, *given some constant c*
- $(\prod\limits_{k=m}^{n} a_k) \cdot (\prod\limits_{k=m}^{n} b_k) = \prod\limits_{k=m}^{n} (a_k \cdot b_k)$

Theorem 1.2. *The binomial theorem, also called n choose r is computed by using the following formula for $0 \leq r \leq n$:*

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

2 Placeholder

3 Placeholder

4 Placeholder

5 Placeholder

6 Placeholder

7 Placeholder

8 Placeholder