MTH310 Calculus & Computational Methods II

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Content by Week

1 Integration Practice and Theory/Application

1.1 FToC I

Recall that we can solve a definite integral using the following definition:

$$\int_a^b f(x)dx = F(b) - F(a)$$

1.2 Change of Variables / U-Substitution

Suppose we need take the antiderivative of $\int 2x\cos(x^2)dx$, let us suppose that $g(x) = x^2$, then we know $g'(x) = 2x$. We also know that $\int \cos(x)dx = \sin(x) + C$. If we combine these, we can derive the answer as:

$$\sin(x^2) + C = \int 2x\cos(x^2)dx$$

Theorem 1.1. *Let us take $u = g'(x) \to \frac{du}{dx} = g'(x)$ and $du = g'(x)dx$. We can then derive the following:*

$$\begin{aligned} f(g(x)) &= \int (f \cdot g)'x \\ &= \int f'(g(x))g'(x)dx \\ &= \int f'(u)du \end{aligned} \tag{1}$$

1.3 Examples:

Consider the following substitution $u = 3x \to du = 3x \to \frac{1}{3}du = dx$, we can then solve:

$$\begin{aligned} \int \cos(3x)dx &= \frac{1}{3}\int \cos(u)du \\ &= \frac{1}{3}\sin(u) + C \\ &= \frac{1}{3}\sin(3x) + C \end{aligned} \tag{2}$$

Consider the following substitution $u = 2x^2 + 1 \to du = 4x\ dx$, we can then solve:

$$\begin{aligned} \int \frac{x}{2x^2+1}dx &= \frac{1}{4}\int \frac{du}{u} \\ &= \frac{1}{4}ln(u) + C \\ &= \frac{1}{4}ln(2x^2+1) + C \end{aligned} \tag{3}$$

Consider the following substitution $u = 1 + x \to du = dx \to u - 1 = x$

$$\begin{aligned} \int x\sqrt{1+x}\ dx &= \int (u-1)u^{\frac{1}{2}} \\ &= \int u^{\frac{3}{2}} - u^{\frac{1}{2}}du \\ &= \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + C \\ &= \frac{2}{5}(1+x)^{\frac{5}{2}} - \frac{2}{3}(1+x)^{\frac{3}{2}} + C \end{aligned} \tag{4}$$

2 Placeholder

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