OPTICS

Momentum in an uncertain light

Ulf Leonhardt

How much momentum does light transfer to a material through which it passes? This is a surprisingly opaque matter, contested for almost a century, that is still the object of theory and experimentation.

One of Rudolf Peierls' Surprises in Theoretical Physics1 is the difficulty of assessing the momentum of light in transparent materials such as glass or water. Despite its being a seemingly basic concept, rival theories23 predict values for the momentum that differ by considerable factors, and that have been hotly debated for many decades. Experimental tests 4.5 have been rare. Two papers now bring out the big guns to break the impasse: writing in Physics Letters A, Dereli, Gratus and Tucker6 resort to Einstein's general relativity for the requisite theory; and in Physical Review Letters, Campbell et al.7 use ultracold atoms in an experimental test.

Light's momentum describes the degree to which light sets other things in motion when they absorb or reflect it. The force of light is usually rather weak, but visual evidence for its existence can be seen, for example, in a comet's tail. Here, the action of sunlight transfers momentum onto the particles of the tail, pushing dust away from the comet (Fig. 1).

The debated issue is the ratio between the momentum, p, and the energy, E, of light in a medium. This ratio depends on the degree to which a material slows light from its speed in a vacuum, c. The

reduced speed is often written c/n, where n is the refractive index of the medium. In 1908, Hermann Minkowski2 proposed that the fundamental relationship between all these quantities is p = nE/c; a year later, Max Abraham³ postulated p = E/(nc) (Box 1, overleaf). These rivalling momenta differ by n^2 , which is a sizeable factor in most media: in water, n alone is 1.33, and in glass it is 1.46.

It might seem that this dispute could easily be settled by experiment: all it should take is some water and a laser beam illuminating its surface. The surface partially reflects the beam; the rest of the incident light enters the water. The water must take up any imbalance in momentum, so the surface should rise or fall, depending on the momentum of light in water compared with that in air. In Minkowski's case, the



Figure 1 | Light, momentum, action. The comet Hale-Bopp, seen here over the Joshua Tree National Park in southern California on the evening of 28 March 1997, has both a blue ion tail and a white dust tail. Whereas the ion tail is carried away by the 'solar wind' of charged particles from the Sun's atmosphere, the dust tail is pushed by the radiation pressure of the sunlight. The momentum transfer in this second case is weaker than that in the first, resulting in the splitting of the tails.

momentum in water is higher and the water should rise; in Abraham's case, the reverse is true and the water level should fall8. An experiment in the mid-1970s found that the water rises4. In a second experiment, the light pressure on a mirror suspended in water was measured, and the same result emerged. Thus, in both cases, Minkowski is the winner, Case closed.

Not so fast. Sometimes, one can directly calculate light forces without considering the momentum transfer. Careful calculation8 of the forces acting behind the scenes reveals that the momentum of light is in fact chameleonic: its precise value depends on the type of experiment performed. In fact, according to the calculations, the first of the experiments4 should have observed Abraham's momentum in action, whereas the second5 should indeed

see Minkowski's momentum. The probthe beam did not uniformly illuminate the surface. This imbalance created lateral light forces that curved and lifted the water to a greater extent than the momentum could push it, resulting in a 🙎 false interpretation of the result.

Campbell et al.⁷ applied for the first time the sophisticated tools of atom optics to measuring light's momentum 🖇 transfer. Like light, atoms are simultaneously particles and waves. In traditional optics, instruments made of matter act on light. In atom optics, the roles are reversed: light acts on atom waves. Campbell and colleagues performed their experiments on a few million atoms cooled to nearly absolute zero. At such low temperatures, atom waves oscillate in unison. The authors illuminated the atoms with two beams of light travelling in opposite directions, which together form a so-called standing wave. When the atoms absorb the photons, they begin to move in the two directions of the light, and the atom waves interfere with each other. The momentum transfer could be inferred from the interference fringes by fitting the profile of the fringes to the theory. To the surprise of the leader of the

experimental group7, Wolfgang Ketterle, it fitted Minkowski's prediction.

Theory can use equally sophisticated tools as experiment: for example, the general theory of relativity used by Dereli, Gratus and Tucker⁶. According to this theory, mass and momentum — including those of light — curve space and time. This effect is felt as the force of gravity. The authors first obtained Abraham's momentum in a similar way to that detailed in a long-forgotten paper from 1923 (ref. 9). This exploited an intriguing idea that originates from Fermat's 'principle of least time': that, because light rays choose their paths to minimize their travelling time, they tend to stay as long as possible in regions where the refractive index n is low and speed c/n high, and avoid regions of high n. In other words,

Box1|Theoretical routes to light's momentum

The reasoning behind both the Minkowski and the Abraham formulae for the momentum of light is best explained with a little support from two pillars of modern physics: wave-particle duality and Einstein's relativity.

Wave-particle duality is a fundamental tenet of quantum mechanics, and states that light, like everything else in the quantum world, simultaneously has characteristics of both a particle and a wave. Light acting as a wave oscillates

with period τ and wavelength During each period, the wave advances by an amount determined by its speed in the medium that contains it: thus, $\lambda = (c/n)\tau$, where c is the speed of light in vacuo and n is the refractive index of the medium. Light acting as particles consists of photons, each of which carries an amount of energy $E = h/\tau$, where his Planck's constant. Combining these two expressions, and after a little rearrangement, you obtain Minkowski's prediction² for

light's momentum, p = nE/c, if you require $p = h/\lambda$. This fundamental relationship between momentum and wavelength now bears the name of the de Broglie relation.

Alternatively, however, you can start from Einstein's energy-mass equivalence formula $E=mc^2$. This implies that light has mass, and mass multiplied by velocity c/n gives momentum. The expression that emerges in this case is p=E/(nc), Abraham's momentum³. U.L.

the refractive index defines an optimization measure in space and time: a material acts as a space-time geometry of its own?

Dereli, Gratus and Tucker⁶ also found a way to derive the Minkowski momentum from the principles of general relativity. The fact that they could deduce expressions for both momenta from the same starting point might be connected to another twist of the story. As atom optics has shown, the role of light and matter can be reversed. So, if a transparent material — the glass of a lens, for example acts on light as though to change the geometry of a situation, light should change the geometry of such materials as well. This leads to a theory of the momentum of light in ultracold atoms10: whenever the wave aspects of atoms dominate, as in Campbell and colleagues' interference experiment, the Minkowski momentum appears, but when the particle aspects are probed, the Abraham momentum is relevant.

How this might apply to the momentum of light in more ordinary materials such as glass or water remains unclear. When exactly is the Minkowski momentum applicable, and when the Abraham momentum? Whatever the final answer, one thing is clear: light continues to surprise.

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MALARIA

A protective paradox

Stephen L. Hoffman

The infectious form of the malaria parasite has thousands of proteins, making it tough to develop a vaccine for it. Narrowing down which proteins cause protective immune responses may help resolve the problem.

A vaccine against malaria would be the ideal means of preventing the hundreds of millions of cases of the disease that occur annually across the globe¹. But no malaria vaccine has yet been licensed, and there is little consensus on how to develop one. Initial success in inducing an immune response against the malaria parasite (*Plasmodium falciparum*) came more than 30 years ago²⁻⁴, with the discovery that human volunteers could be completely protected against malaria by exposure to radiation-weak-

ened sporozoites — the infectious parasite cells passed on by mosquitoes (Fig. 1). But until recently it has not been considered practicable to manufacture and administer such 'attenuated' sporozoites as part of a mass immunization strategy. — not least because it took more than 1,000 bites from mosquitoes infected with the irradiated cells to fully protect most of the volunteers against the disease.

Considerable effort has therefore gone into trying to define which of the thousands of proteins expressed by the parasite⁶⁷ are involved in the protective immunity elicited by the attenuated sporozoites, with the aim of eventually making vaccines against them. On page 937 of this issue, Kumar et al.⁸ report that one of these proteins, the circumsporozoite protein, might elicit much of the protective immunity induced by the attenuated sporozoites*. Puzzlingly, however, it seems that a protective response can occur in the absence of this protein too.

The first clinical trials using a purifiedprotein malaria vaccine began 20 years ago^{9,10}, but so far only volunteers immunized with vaccines based on the circumsporozoite protein11 — the major sporozoite surface protein have been reproducibly protected against P. falciparum sporozoite challenges. However, the protective immunity induced by the best circumsporozoite-protein vaccine is far lower12 than that induced by radiation-attenuated P. falciparum sporozoites4. One explanation for this dramatic difference is that the wholeparasite vaccine induces protective immune responses against tens, hundreds or even thousands of parasite proteins, whereas the singleprotein vaccine can elicit an immune response only against that protein13.

Accordingly, there are significant efforts to identify additional target proteins expressed in sporozoites, and in the parasite during the liver stages of infection, to construct a more effective vaccine. If one, or a few proteins, like the circumsporozoite protein, is responsible for the protection, efforts to identify other target proteins could be curtailed, and resources focused on optimizing immunization with these few proteins. If, however, strong immunity depends on immune responses against many proteins, it may be difficult, or even impossible, to construct an effective vaccine based on only parts of the parasite. So, resolution of whether protective immunity relies on one, a few or many proteins is crucial.

Kumar et al.⁸ used a mouse model of malaria where the animals are infected with the Plasmodium yoelii parasite, as P. falciparum does not produce malaria in mice. This mouse model is meaningful for human malaria because experiments using attenuated P. yoelii sporozoite vaccines have consistently paralleled or predicted results in humans exposed to attenuated P. falciparum sporozoites. To examine how important the circumsporozoite protein is in inducing immunity to attenuated sporozoites, the authors first genetically engineered mice that could not generate an immune response to the P. yoelii circumsporozoite protein (PyCSP).

Mammalian immunity is based on two broad types of immune reaction — antibody-based immunity and the cell-based immunity that involves T cells. Kumar et al. therefore had to disrupt both of these reactions to create PyCSP-tolerant mice. They first injected the gene

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