Avoiding the Abraham-Minkowksi Controversy in Light-Driven Deformation

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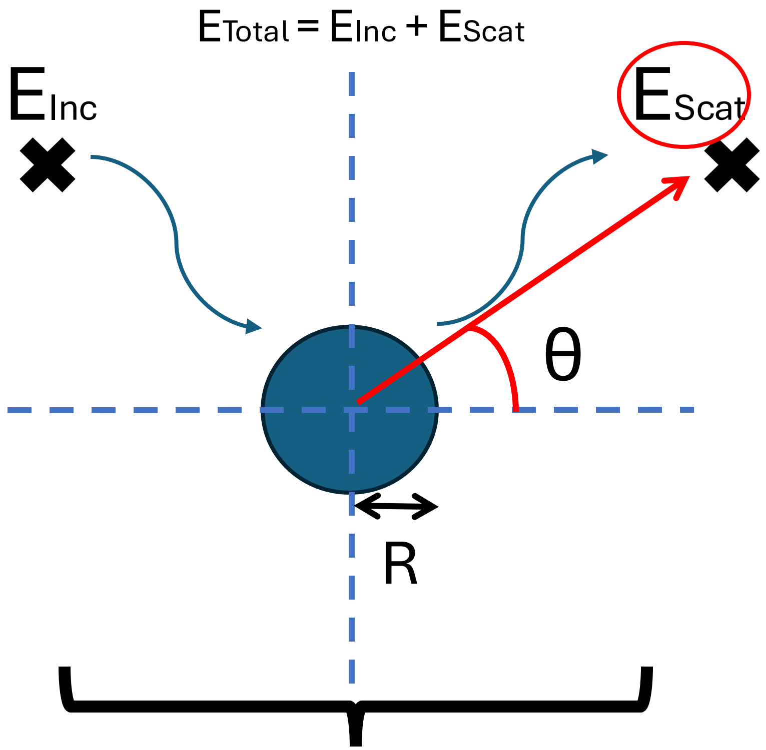
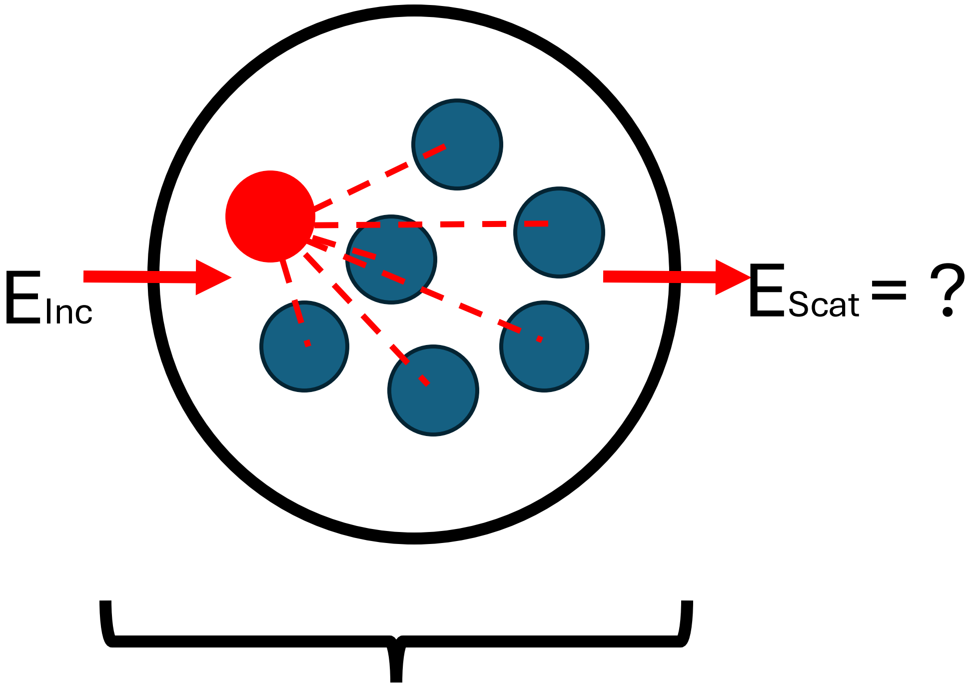
Goals of the project:

- Assess the validity of current simulation techniques in modelling flexible materials.
- Simulate the dynamic motion of light interacting with a flexible material.
- Account for the Abraham-Minkowski controversy when immersed in varying refractive media.
- Compare this model with real experimental systems (e.g. tensile forces on a red blood cells).

Rayleigh	$R < \lambda$
Mie	$R \sim \lambda$
Ray-tracing	$R > \lambda$

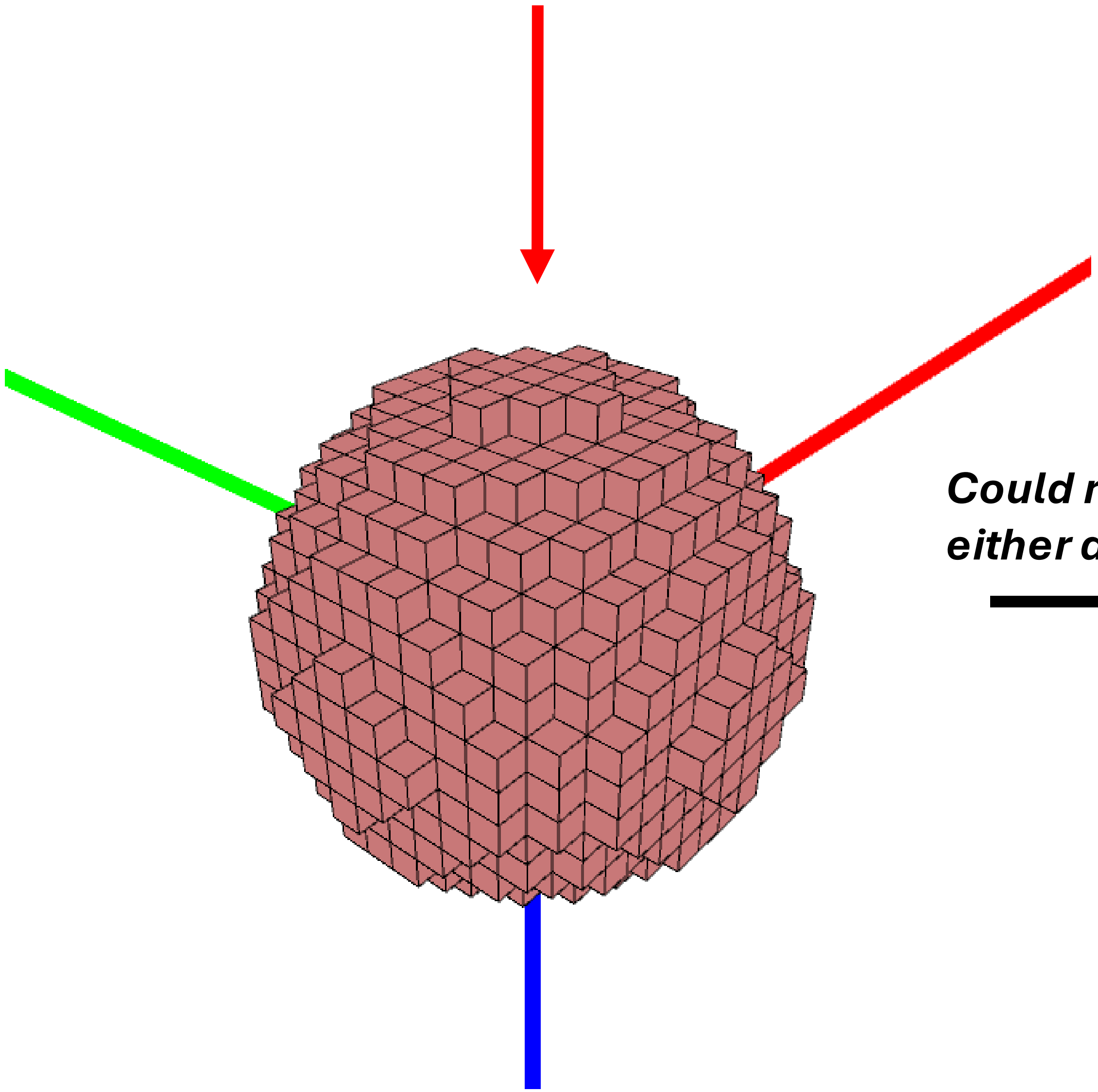


General overview

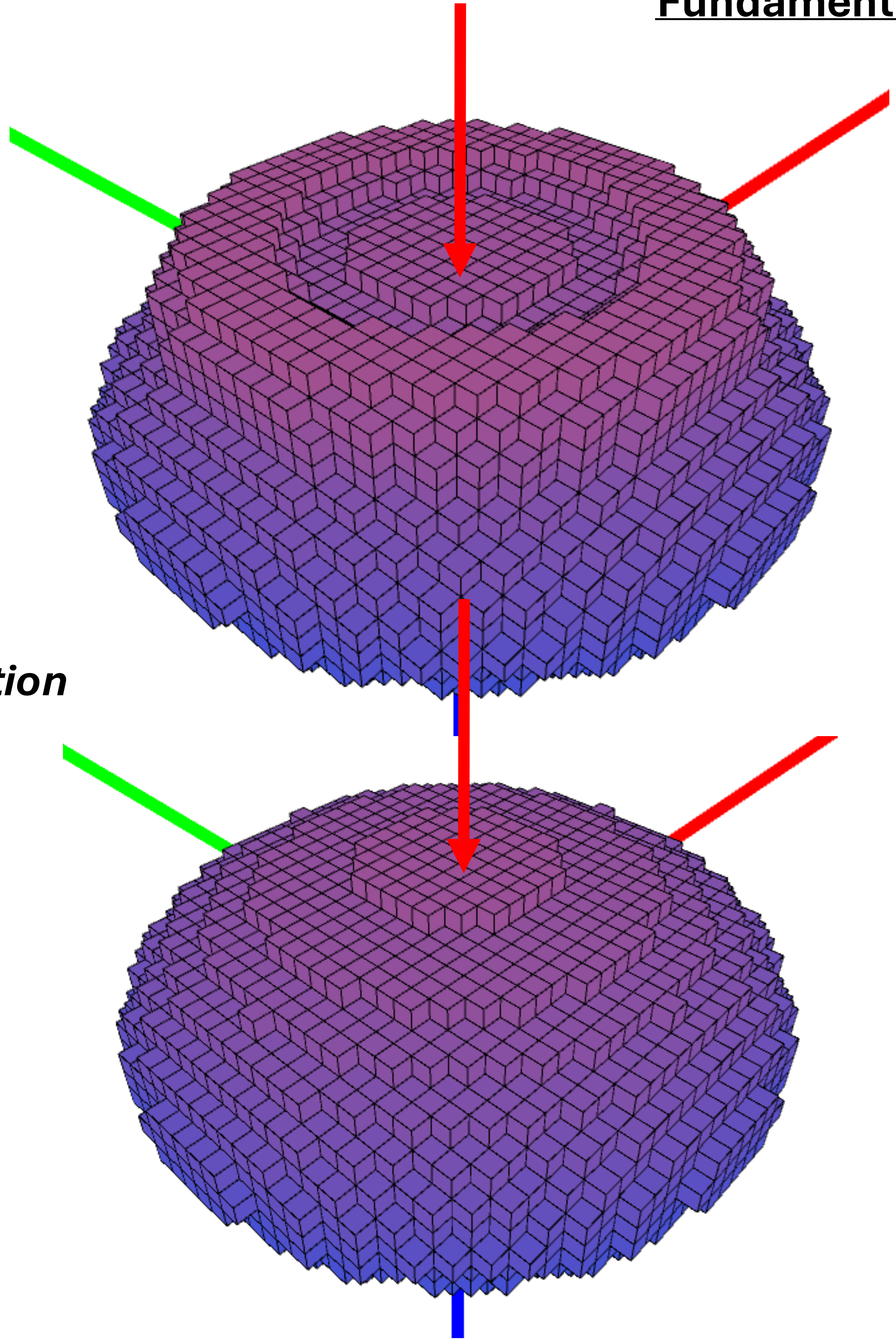


Complex interaction with many atoms

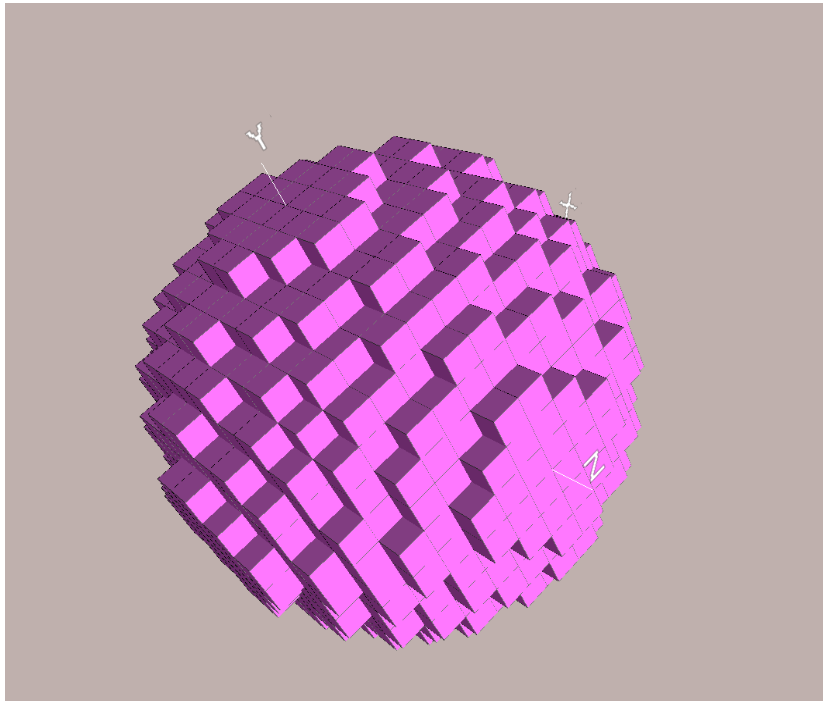
Fundamentals

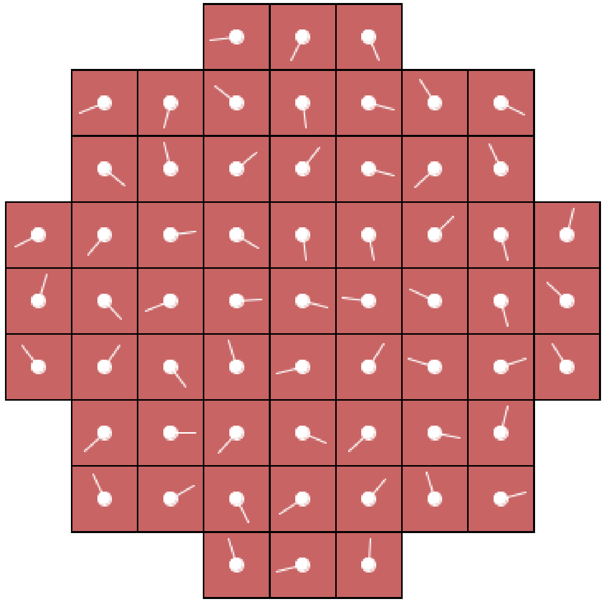


Could result in either deformation



DDA





To fully simulate reality, you would need 1 dipole per atom, however here a greatly reduced number is considered.

$$\sum_{k=1}^{N} \mathbf{A}_{jk}\mathbf{P}_k = \mathbf{E}_{inc,j}$$

$$\mathbf{P}_j = \alpha_j \mathbf{E}_j$$

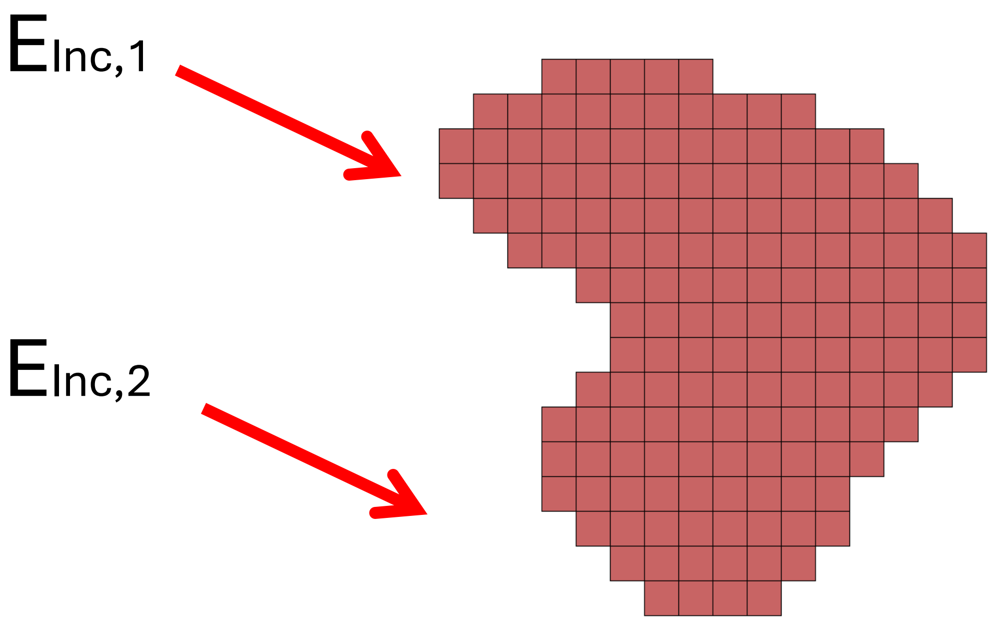
*α_j formulated using 'Clausius-Mossotti' relation

$$\mathbf{E}_{sca} = \frac{k^2 exp(ikr)}{r} \sum_{j=1}^{N} exp(-ik\hat{\mathbf{r}}.\mathbf{r}_j)(\hat{\mathbf{r}}\hat{\mathbf{r}} - I_3)\mathbf{P}_j$$

**This term is for far-field scattering, another term for near-field scattering can also be included in the summation*

*Equations from the ***'ADDA manual'***, Maxim A. Yurkin, and ***'Discrete-dipole approximation for scattering calculations'*** descriptions*

Note that $\boldsymbol{A}_{jk}$ gives $\boldsymbol{E}$ field at $\boldsymbol{r}_j$ from dipole at $\boldsymbol{r}_k$, where $\boldsymbol{A}_{jk}$ elements are each 3x3 matrices



***Must repeat entire calculation for different incident beams, despite same geometry**

+Easily applicable to asymmetric shapes

-Slow to calculate for many dipoles

(1) Characterising Beams (Incident & Scattered)

Spherical Harmonic Terms Vectorised → Vector Spherical Wavefunctions (VSWF)

$$\mathbf{B}_{nm}(\theta,\phi) = \mathbf{r}\nabla Y_n^m(\theta,\phi)$$

$$\mathbf{C}_{nm}(\theta,\phi) = \nabla \times (\mathbf{r} Y_n^m(\theta,\phi))$$

$$\mathbf{P}_{nm}(\theta,\phi) = \hat{\mathbf{r}} Y_n^m(\theta,\phi)$$

Standard spherical harmonic,
Seen solving Laplacian on unit sphere

$$\mathbf{E}_{\text{inc}}(r) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} a_{nm}^{(2)} \mathbf{M}_{nm}^{(2)}(kr) + b_{nm}^{(2)} \mathbf{N}_{nm}^{(2)}(kr)$$

$M_{nm} = M_{nm}(B_{nm}, C_{nm}, P_{nm})$

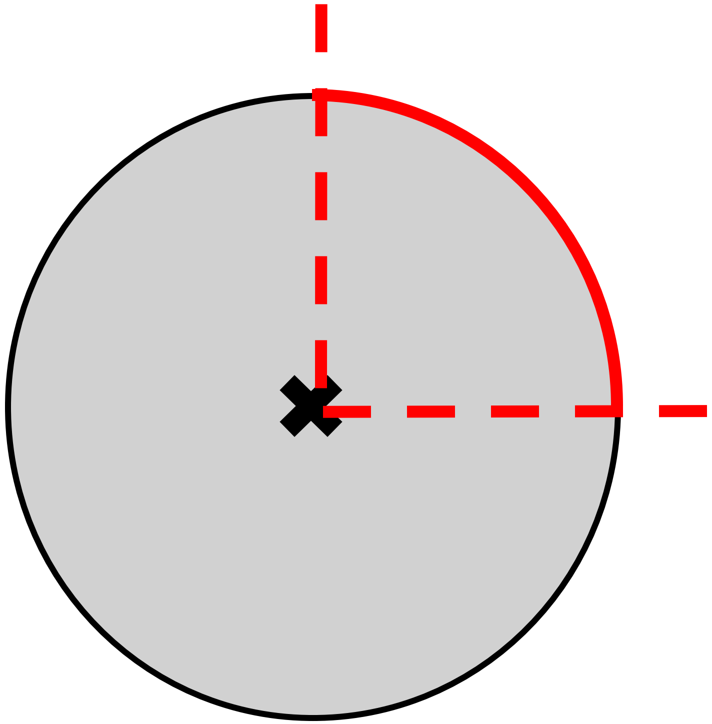
$N_{nm} = N_{nm}(B_{nm}, C_{nm}, P_{nm})$

P = TA

- **P**=(p_{11},q_{11},...) : Scattered coefficients
- **A**=(a_{11},b_{11},...) : Incident coefficients
- **T**=Linear matrix (T-matrix)

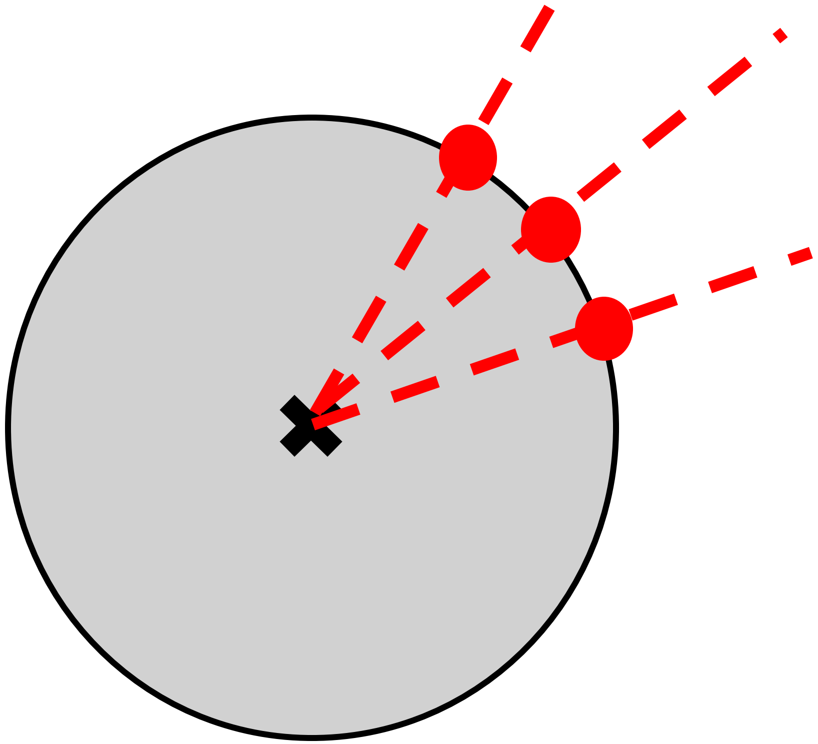
Equations from **'Calculation of the T-matrix: general considerations and application of the point-matching method',* T.A. Nieminen, 2002

(2) Characterising T-Matrix



Extended Boundary Condition Method (EBCM)

**This method requires a homogeneous & isotropic particle.*



Point-Matching Method (PMM)

$$\hat{\mathbf{n}} \times (\mathbf{E}_{\text{inc}}(r) + \mathbf{E}_{\text{scat}}(r)) = \hat{\mathbf{n}} \times \mathbf{E}_{\text{int}}(r),$$

$$\hat{\mathbf{n}} \times (\mathbf{H}_{\text{inc}}(r) + \mathbf{H}_{\text{scat}}(r)) = \hat{\mathbf{n}} \times \mathbf{H}_{\text{int}}(r),$$

+Faster and more accurate calculation

-Not applicable to random shapes, need high symmetry

**This system of equations (coefficients for VSWF) can be solved for each coefficient, which can then give the T matrix as the combination of these coefficients.*

Equations from* *'Calculation of the T-matrix: general considerations and application of the point-matching method'***, T.A. Nieminen, 2002

Abraham-Minkowski

Minkowski:

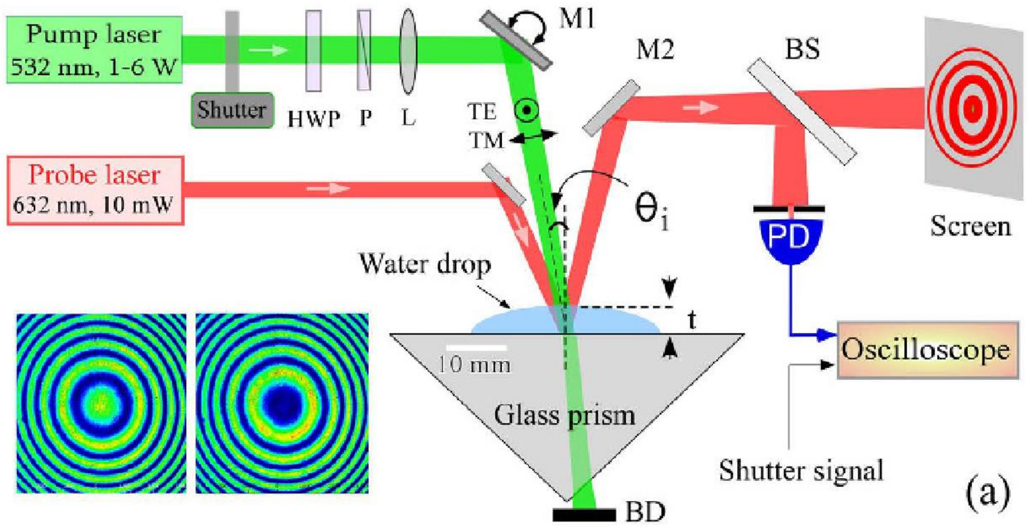
$$p = \frac{h}{\lambda}$$

$$\lambda = \left(\frac{c}{n}\right)T$$

$$E = \frac{h}{T}$$

$$p_M = p = \left(\frac{E}{c}\right)n = p_0 n$$

This system behaves differently to what may be expected with standard radiation pressure



Abraham:

$$E = mc^2$$

$$p = mv$$

$$v = \frac{c}{n}$$

$$p_A = p = \frac{E}{c}\frac{1}{n} = p_0\frac{1}{n}$$

These are therefore equivalent in a vacuum (n=1)

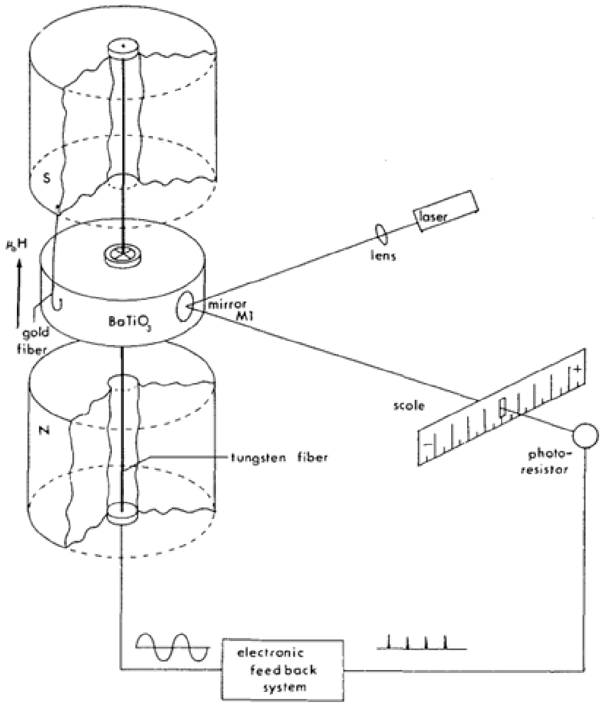


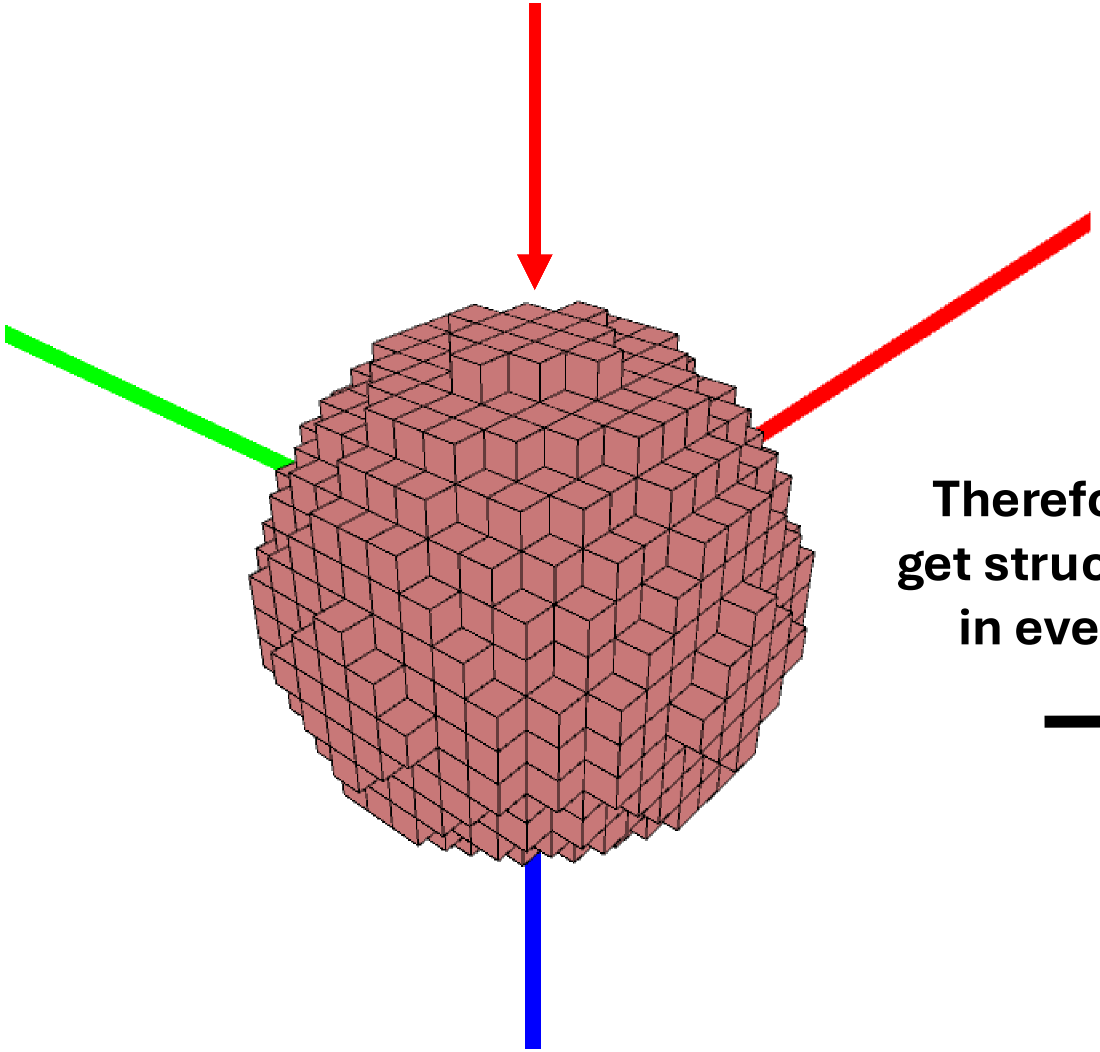
FIG. 1. Diagram of the experimental apparatus.

**Proof for Abraham and Minkowski momentum from 'Momentum in an uncertain light', by Ulf Leonhardt*

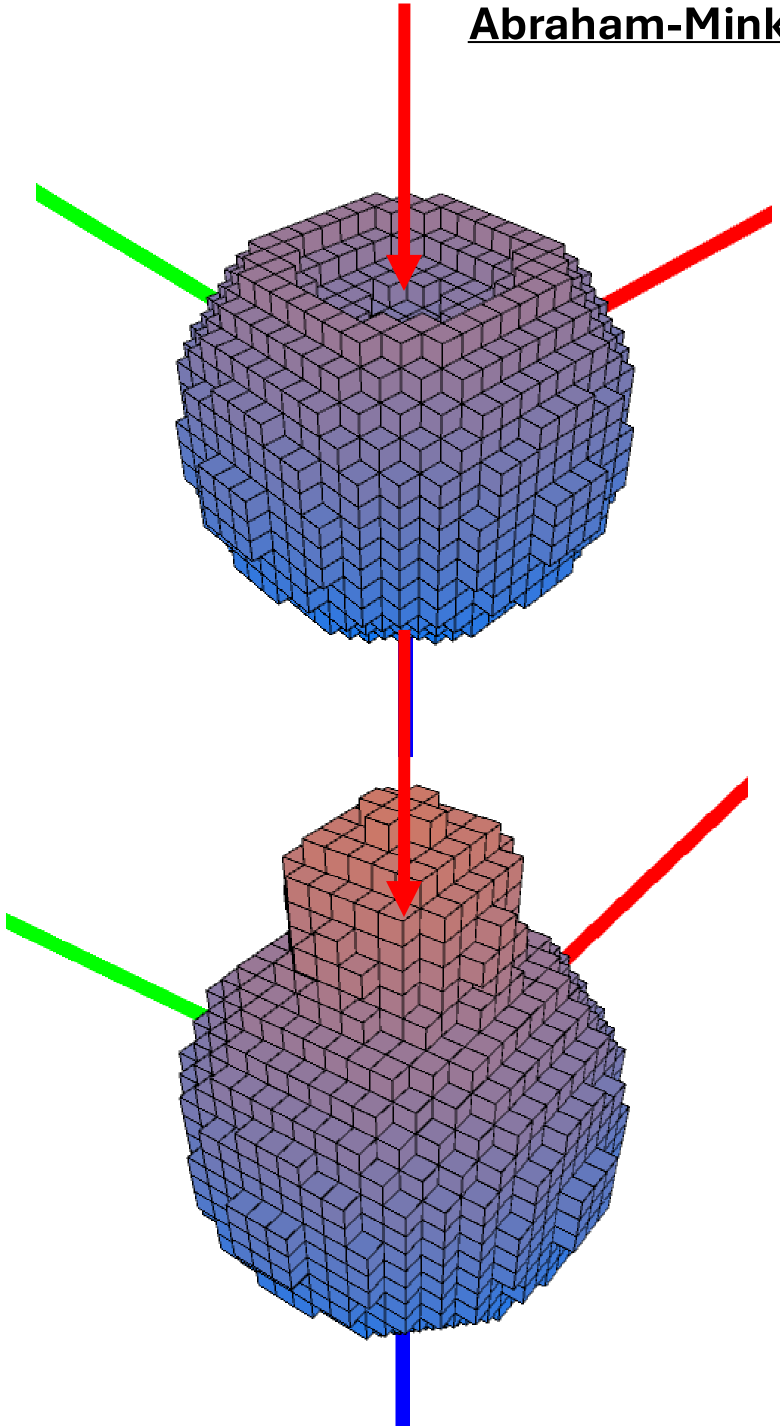
**'Nanomechanical effects of light unveil photons momentum in medium', Gopal Verma, 2017, upper figure*

**'Measurement of the Abraham Force in Specimen of Barium Titanate Specimen', G. B. Walker, 1975, lower figure*

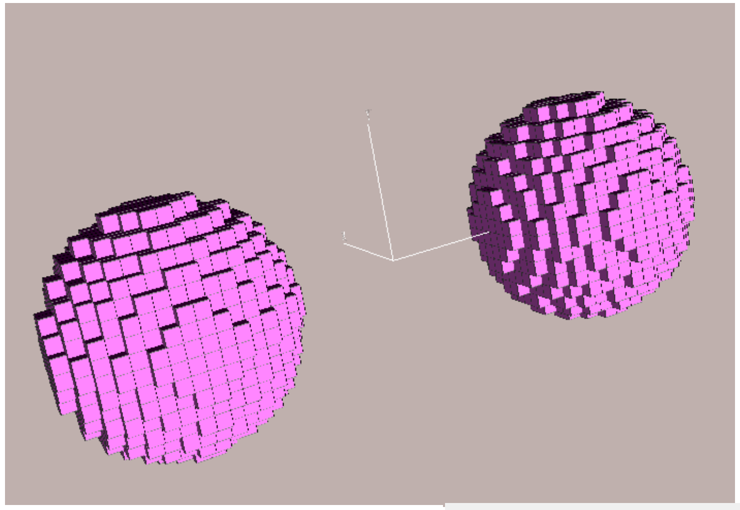
Abraham-Minkowski

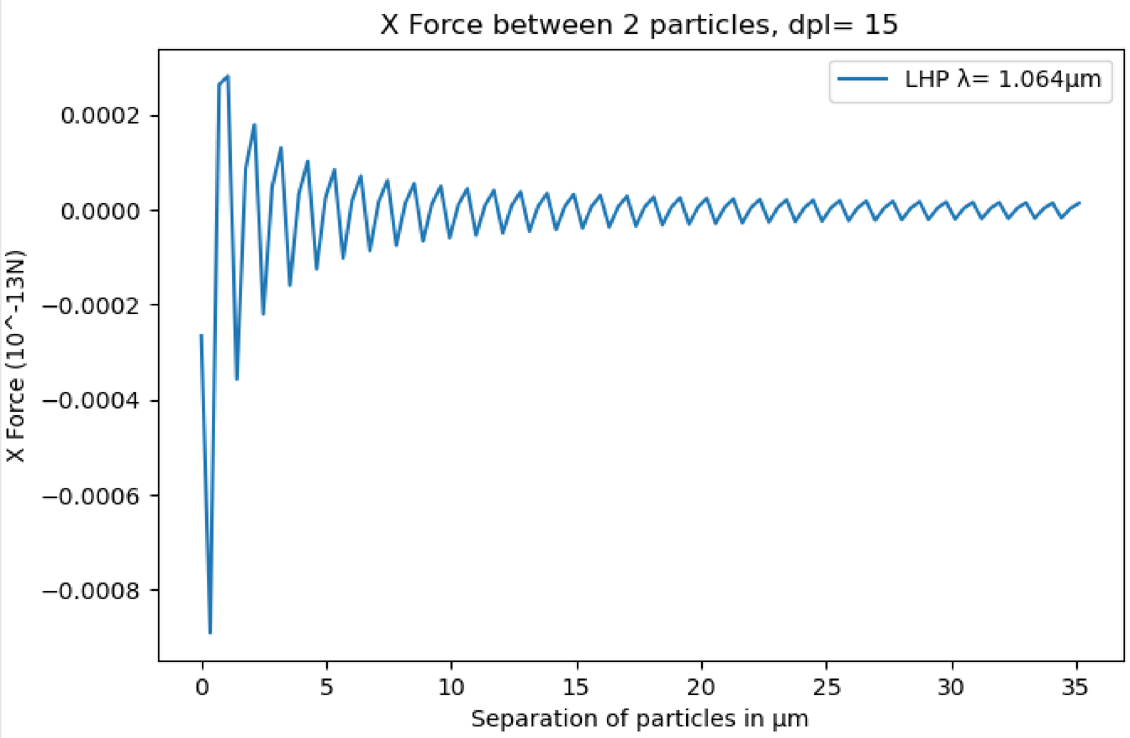


Therefore possible to get structures like these in even very simple beams

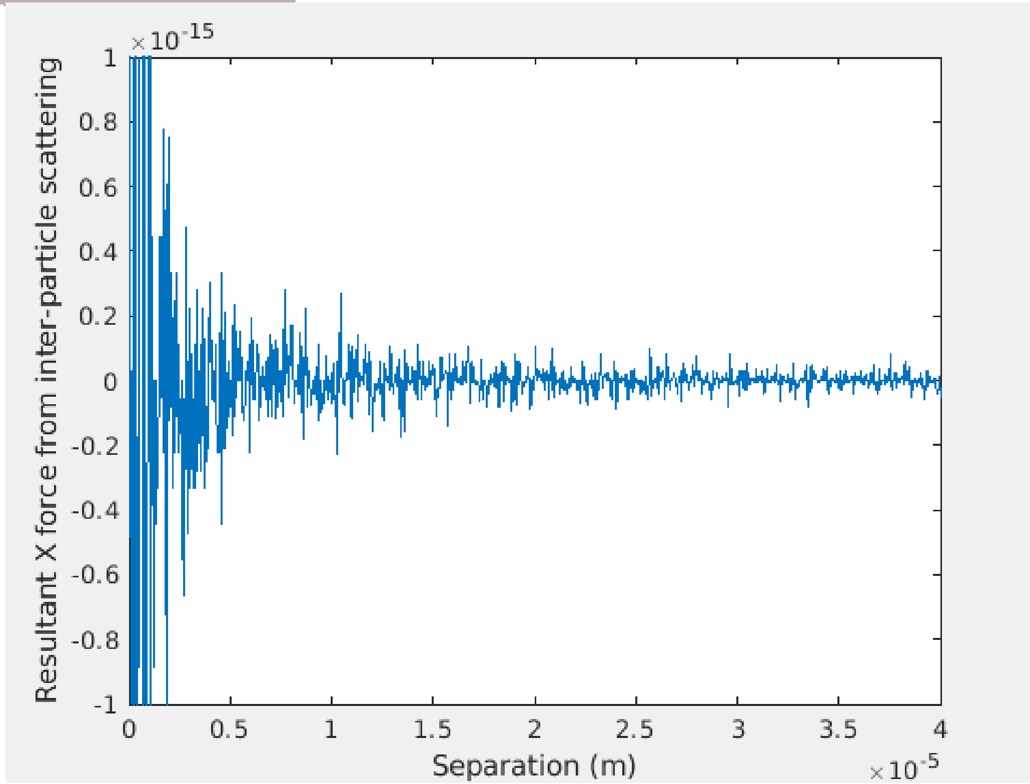


**This adds another level complexity to the situation that needs to be considered with materials immersed in different refractive indices*





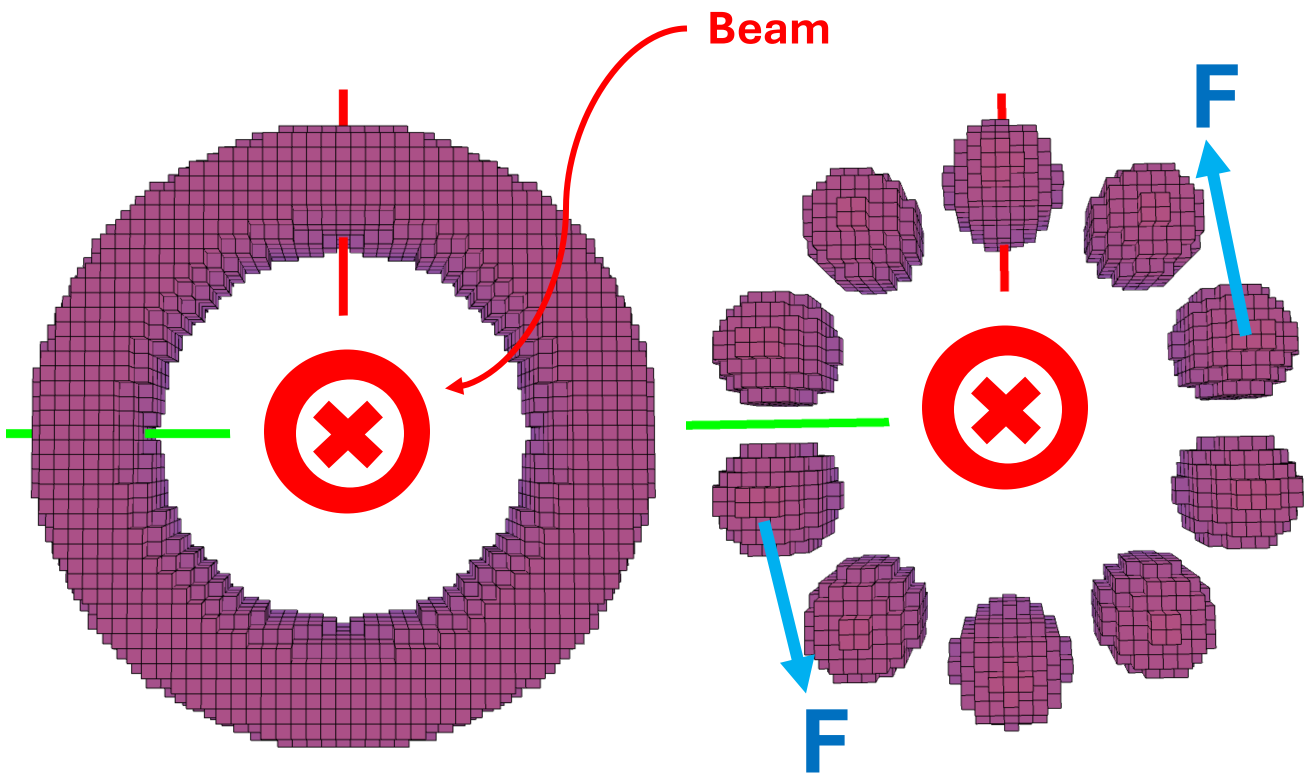
ADDA ***(DDA)***



OTT ***(T-Matrix)***

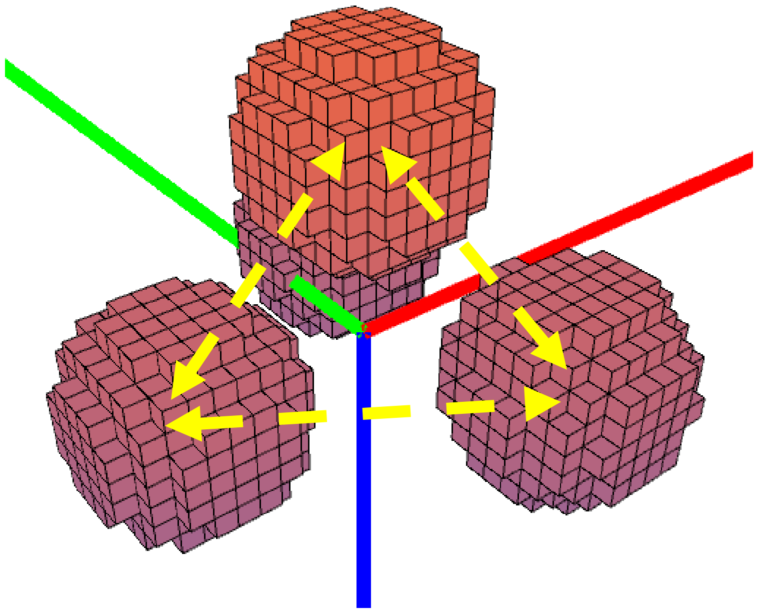
Future Research

Continuous Limit of Particles



Considering discrepancy in forces

Spring Connected Particles



Joined with springs between adjacent particles, in addition to other scattering forces experienced