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Motion of atoms in a radiation trap

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The force exerted by optical-frequency radiation on neutral atoms can be quite substantial, particularly in the neighborhood of an atomic resonance line. In this paper we derive from quantum theory the optical force, its first-order velocity dependence, and its fluctuations for arbitrary light intensity, and apply the results to the problem of creating a stable optical trap for sodium atoms. New results include the position dependence of the velocity-dependent force, a complete expression for the momentum diffusion constant including the substantial contribution from fluctuations of the dipole force, and an estimate of trapping times in excess of 1 sec even in the absence of effective damping. The paper concludes with a discussion of the prospects and difficulties in providing sufficient damping to stabilize such a trap.

INTRODUCTION

The force exerted by optical-frequency radiation on neutral atoms can be quite substantial, particularly in the neighborhood of an atomic resonance line. This force can be viewed, equivalently, as the Lorentz force exerted by the field on the optical-frequency atomic dipoles, or as the consequence of momentum conservation in the absorption and reemission of light by the atoms. The total force has three distinguishable components, resulting, respectively, from absorption, spontaneous emission, and induced emission. The first two of these[1,2] have together been called the scattering force, and are the only forces present for plane-wave radiation. Spontaneous emission, by the symmetry of its angular distribution, contributes only to the fluctuations of this force component. The third force component[3-6] is dispersive in nature, and, below resonance, results in an attraction of the atom toward regions of higher field intensity. It has been called the dipole force, and is the near-resonance form of electrostriction in a gas.

The term resonance radiation pressure has been used, heuristically, to denote these forces. Among early proposals for their use were suggestions[3,2,7] for confining atoms to restricted regions of space. The concept of optically cooling atomic motions by resonance radiation forces[8] led to proposals[5,6,9] of spatial traps for atoms in which their temperature could be cooled to ~$(10^{-3}$–$10^{-4})$ K. Such a trap would render the atoms available for long times for spectroscopic studies or other novel experiments.

In this work we consider the motion of atoms situated in optical resonance radiation traps. We derive the radiation pressure forces, their velocity dependence, and their quantum fluctuations in a consistent way starting from fundamentals. The calculations yield new results on both the velocity dependence and quantum fluctuations of the force and lead to some new insights into the overall trapping problem. The average scattering force, by conservation of momentum, is in the direction of the incident wave vector. It has been detected in atomic-beam-deflection experiments[10,11] and by the generation of significant pressure differences in atomic vapors.[12] The strong velocity dependence on this force has led to the concept of optical cooling or damping.[8] If the frequency of the light is tuned below resonance, then any atomic motion either toward or away from the light beam results in a sizable Doppler shift leading to an incremental force opposing the motion. The idea of optical cooling was proposed independently within the context of ion trapping.[13] Experimental cooling of ions held in electromagnetic traps has recently been observed.[14,15] The average dipole force is directed along the gradient of the optical field intensity and is dispersive in character, being in the direction of the gradient when below resonance, and in the opposite direction when above resonance. Dipole forces have recently been observed experimentally in atomic beam focusing and defocusing experiments.[16] In a somewhat different context, nonresonant dipole forces have been invoked[17] to explain observations on the distortion of liquid surfaces by radiation pressure.[18] For the resonance radiation forces on atoms the effect of saturation and tuning must be considered[2,6] to obtain the greatest force for a given power.

Many uses have been suggested for radiative forces on atoms, such as velocity selection,[2] isotope separation[2,11,19,16] production of transversely cooled[8] or axially slowed and confined atomic beams,[16] and acceleration of neutral atoms to high velocity.[4] Transient dipole forces also exist for light tuned to exact resonance if the atom interacts with the field for times short compared

to the natural lifetime.[19-21] Atomic-beam deflection has recently been observed under these circumstances.[22] Analogous effects have been seen in the deflection of molecular beams by resonant microwave electric fields.[23] We shall, however, restrict our present discussion principally to the initially mentioned application of optical trapping and cooling.

Basically an optical trap consists of an optical field configuration[5,6,9] with a point of stable equilibrium such that any displacement of an atom from this point results in an average restoring force. The maximum kinetic energy (at the equilibrium point) of an atom that can be confined in a trap is defined as the well depth. An important aspect of such traps is the fact that the trapping forces have fluctuations due to the quantum nature of the interactions. These fluctuations constitute a source of heat. In the absence of cooling this will result in a finite retention time for atoms introduced into the trap. The fluctuations due to the scattering force can be readily appreciated on intuitive grounds.[24,5,9,14] Recently evidence on heating due to the scattering force was observed experimentally.[25] Fluctuations in the dipole force[26,9] are conceptually more difficult to understand and have not until now been adequately considered in the context of traps.

The heating of atoms by fluctuations must be counteracted by cooling if atoms are to be retained in the trap. Indeed the equilibrium kinetic energy of an atom in a trap results from a balance of the fluctuation heating and the degree of optical cooling that exists. The effectiveness of a trap in containing an atom is determined by the Boltzmann factor given by the ratio of the well depth to the equilibrium kinetic energy. In practice we seek a low average kinetic energy for trapped atoms and a large Boltzmann factor.

Letokhov *et al.*[5] proposed trapping atoms in large volumes on the standing wave maxima of 3 orthogonal pairs of standing waves tuned $\sim\Gamma/2$ below the atomic resonance, where Γ is the radiative decay rate of the atom, the reciprocal of the radiative lifetime τ. This tuning gives optimum damping and results in a minimum kinetic energy $\sim\hbar\Gamma$. Unfortunately this trap has a well depth which is also $\sim\hbar\Gamma$ giving a Boltzmann factor of about unity. Thus thermal excitation out of this trap is probable and the trap is very leaky. The traps proposed by Ashkin *et al.*[6,9] are based on strongly focused Gaussian beams tuned $\sim(10^2-10^3)\Gamma$ below resonance. In this way traps of depth $\sim(10^2-10^3)\hbar\Gamma$ are achieved but at a price of much reduced damping. In fact, the Boltzmann factor is again ~1. However, it has been proposed[9] to use additional optimally tuned damping beams with these focused beam traps with the intent of simultaneously obtaining trap depths of $(10^2-10^3)\hbar\Gamma$ and minimum temperatures $\sim\hbar\Gamma$ to give highly stable traps with Boltzmann factors of 100 or more.

Currently the outstanding conceptual problems remaining for the understanding of optical traps for neutral atoms are the questions of the magnitude of the fluctuations of the dipole force and the viability of the concept of separate trapping and damping beams. In the earlier estimates[5,9] of minimum kinetic energy the contributions of the dipole force fluctuations were neglected. One of the principal results of this work is a quantum-mechanical calculation of the dipole fluctuations exact to arbitrary field strength. The results show that these fluctuations can often be large. In standing waves dipole force fluctuations make a contribution which in the absence of saturation effects is the same size as the scattering force fluctuations. Interestingly, their spatial variation is such that when added to the scattering fluctuations, there results a velocity diffusion constant which is independent of position in the standing wave. In Gaussian beam traps and at high powers dipole fluctuations can exceed the spontaneous scattering force fluctuations. Fortunately, however, conditions exist in the Gaussian trap of Refs. 6 and 9 where the dipole force fluctuation can be neglected relative to the scattering force fluctuations. This paper draws no new conclusions about the problem of using separate trapping and damping beams. However, we develop the whole subject of cooling and heating in a unified way and give some new insights. For example, a new calculation is given of the damping for a standing-wave field as a function of position, correct to all intensities, for low atomic velocities. An important aspect of this result is that the damping varies with position and is zero at the maximum of the standing-wave fields where atoms are expected to collect. This represents another complication for single-frequency standing-wave traps as proposed in Ref. 5. Although an atom can be viewed as a simple harmonic oscillator (SHO) only when saturation effects are absent, it is shown that a single beam trap is capable of stably trapping a SHO. This result at least shows that nature does not necessarily abhor an optical trap. Finally, our estimate of the magnitudes of the dipole and spontaneous force fluctuations point out a new experimental possibility. This involves an estimate of the retention lifetime of an atom put into a single beam trap in the absence of significant cooling. It is shown that by working at sufficiently low saturation and with low dipole fluctuations, lifetimes of many seconds are possible for deep traps using reasonable opti-

cal powers. This implies that an experimental demonstration of trapping is not contingent on cooling. Cooling can then be studied subsequently by the addition of separate damping beams, for example. Parenthetically, one further important use of traps may be as an experimental probe for the study of the fluctuations of the radiation pressure forces themselves.

The approach used in our calculation of the force, its velocity dependence, and the atom's momentum diffusion is to treat the interaction of the optical radiation field with the atom's momentum quantum mechanically. The quantum treatment of field and momentum is only necessary in finding the force fluctuations. That, however, being a needed result, the quantum treatment is used throughout. Two approximations are used. First, we treat the atom's position as a classical variable. In the case of sodium, or any comparably heavy atom, this approximation is justified by its small de Broglie wavelength, which amounts to only 0.03 μm for sodium atom whose kinetic energy is of the order of the natural width $\hbar\Gamma$ of its resonance level. For sodium, $\Gamma = (16.1\ \text{nsec})^{-1}$. Alternatively one can appeal to Heisenberg's uncertainty principle, which implies that if an atom's position is defined to $\sim\lambda/2\pi$, its momentum uncertainty is equal to that occasioned by the random scattering of one photon. As we are concerned here with the cumulative effects of the scattering of many photons, the neglect of an equivalent few more should have no major consequence. Second, we treat the velocity as small, retaining effects only to first order in v. In context, the reason for this is that atoms caught in an optical trap must be moving quite slowly. Again using numbers typical of the sodium resonance line at 590-nm wavelength, the trap depth might be of the order $10^3\hbar\Gamma$. A trapped atom with, say, one tenth of that energy would have an effective kinetic temperature of about 0.02 K, and would travel, at most, about 0.1 μm in one lifetime. This being a small fraction of an optical wavelength, it is appropriate, indeed desirable, to treat the atom as moving slowly. A symptom of such slowness is that the Doppler shift caused by the atom's motion is less than the natural width.

CALCULATION OF THE FORCES AND MOMENTUM DIFFUSION

In the electric-dipole approximation the force of radiation on a neutral, slowly moving atom is given by

$$f_i = \vec{\mu}\cdot\partial\vec{E}/\partial x_i\,, \tag{1}$$

where $\vec{\mu}$ is the atomic dipole moment, $\vec{E}$ is the electric field, and f_i is the ith Cartesian component of the force. Expression (1) includes both the force the electric field exerts on the dipole and that the magnetic field exerts of the associated current.

The force (1), and all the other quantities we need may be derived from the dipole Hamiltonian function,

$$\mathcal{H} = H_{\text{field}} + H_{\text{atom}} - \vec{\mu}\cdot\vec{E}(\vec{x})\,, \tag{2}$$

where

$$H_{\text{field}} = \frac{1}{8\pi}\int (E^2 + H^2)\,dV$$

and

$$H_{\text{atom}} = P^2/2M + \hbar\omega_0\sigma_{22}\,.$$

In (2) $\vec{\mu}$ is the atomic dipole moment, $\vec{E}$ is the electric field of the radiation, $\vec{H}$ the magnetic field, and σ_{22} is the projection operator for the upper atom level. Also $\vec{P}$ is the atom's momentum, $\vec{x}$ its position, and M its mass.

If we deal with sharply resonant two-level atoms, we can approximate the dipolar energy term by

$$\vec{\mu}\cdot\vec{E} \approx (\vec{\mu}_{12}\cdot\vec{E}^{\dagger})\sigma + \sigma^{\dagger}(\vec{\mu}_{21}\cdot\vec{E})\,. \tag{3}$$

Here the total field $\vec{E}$ has been expanded according to

$$\vec{E} = \vec{E}e^{-i\omega t} + \vec{E}^{\dagger}e^{i\omega t}\,,$$

where the new $\vec{E}$ contains the "positive"-frequency part of the field, and represents an energy-lowering operator, while $\vec{\mu}$ has been similarly expanded according to

$$\vec{\mu} = \vec{\mu}_{12}\sigma e^{-i\omega t} + \vec{\mu}_{21}\sigma^{\dagger}e^{i\omega t}\,,$$

where $\vec{\mu}_{12}$ is the dipole matrix element connecting the two pertinent atomic levels, and σ is the lowering operator for the atom. The operators $E^{\dagger}$ and $\sigma^{\dagger}$ are the Hermitian-conjugate energy-raising operators. In the dipolar energy (3), we have kept only the secular, or energy-conserving, terms, and have arranged them in normal order (lowering operators to the right). The explicit carrier frequency ω is added for later convenience. For a monochromatic applied field, ω will be chosen as the field frequency.

We now wish to find expressions for the mean force on the atom, the first-order velocity dependence of that force, and finally the two-time autocorrelation function of the force, which determines the momentum diffusion constant. The first two of these require only semiclassical theory, but the last requires that we stay with the quantum theory. We shall be working in the Heisenberg picture, where the operators are time dependent, satisfying the equations of motion

$$dO/dt = -(i/\hbar)[O,\mathcal{H}]\,. \tag{4}$$

Here the square bracket represents the commutator, and the (constant) quantum state of the system is prescribed at some appropriate initial time. Note that, e.g., $\sigma e^{-i\omega t}$ represents a Heisenberg operator. Using (2), (3), and (4) we can immediately write down the equation for the force on the atom, namely

$$\vec{f} \equiv d\vec{P}/dt = -\vec{\text{grad}}(\mathcal{H})$$
$$= \sigma^\dagger \vec{\text{grad}}[\vec{\mu}_{21}\cdot\vec{E}(x)] + \text{H.c.}\,, \tag{5}$$

where H.c. represents the Hermitian-conjugate operator. Note that (5) has the same form as the classical equation (1).

The next step, solving for the field at the atom, is plagued by the usual difficulties of quantum electrodynamics. However, if we can approximate that field by the sum of the external field and the radiation reaction field, assuming that the effect of the divergent local dipole field can be suitably renormalized into the excitation energy of the atom, our purpose is served. Thus we assume that $\vec{E}(\vec{x})$ can be developed as

$$\vec{E}(\vec{x}) = \vec{E}^0(\vec{x}) + i\tfrac{2}{3}k^3\vec{\mu}_{12}\sigma\,, \tag{6}$$

where $E^0(\vec{x})$ is the free external incident field, and the other term is the local reaction field, with $k = \omega/c$, where ω is the frequency of the atomic dipole. Recalling that the reaction field has no gradient at the atom's position, we note that it does not contribute to the force (5). Hence we can write

$$\vec{f} = -i\sigma^\dagger \vec{\text{grad}}G + \text{H.c.}, \tag{7}$$

where

$$G \equiv i\vec{\mu}_{21}\cdot\vec{E}^0(\vec{x},t)/\hbar\,.$$

We now turn to the equations of motion of the atomic operators. Let σ_{11} be the projection operator for the lower atomic level, and D be the population difference

$$D \equiv \sigma_{11} - \sigma_{22}\,.$$

(Note $\sigma_{11} + \sigma_{22} = 1$.) Using the operator relations

$$\sigma_{jj}\sigma_{kl} = \sigma_{jl}\delta(j,k)\,, \tag{8}$$

where $\sigma_{12} \equiv \sigma$, $\sigma_{21} \equiv \sigma^\dagger$, and $\delta(j,k)$ is the Kronecker delta function, we gain from (4) the usual equations of motion

$$\dot{\sigma} - i\Omega\sigma = (i/\hbar)D(\vec{\mu}_{21}\cdot\vec{E})\,,$$

and

$$\dot{D} = (2i/\hbar)[(\vec{\mu}_{12}\cdot E^\dagger)\sigma - \sigma^\dagger(\vec{\mu}_{21}\cdot E)]\,,$$

where $\Omega \equiv \omega - \omega_0$ is the detuning of the atomic resonance frequency ω_0 from the chosen ω. Next we make the reaction field approximation (6), to obtain [using (8)]

$$\dot{\sigma} + (\Gamma/2 - i\Omega)\sigma = DG$$

and

$$\dot{D} + \Gamma D = \Gamma - 2(G^\dagger\sigma + \sigma^\dagger G)\,, \tag{9}$$

where $\Gamma = (4/3\hbar)k^3|\mu_{12}|^2$ is the usual expression for the natural radiative decay rate of the atom. Note that the operator order in Eqs. (9) has become important, for while $\vec{E}(\vec{x})$ commutes with $\sigma^\dagger$ at the same time, for example, as they represent different physical entities, it is evident from (6) that the external field $\vec{E}^0(\vec{x})$ (and hence G) does not. The reason for choosing normal ordering will surface in the next paragraph.

Let us now consider the quantum expectation value of (7) and (9). We may represent the initial state of the system by a Dirac ket $|S\rangle$, or sometimes more simply just by $\rangle$. The quantum expectations of (7) and (9) are then just those equations surrounded by angular brackets, as in $abc \to \langle abc\rangle$. If the initial field is the coherent state $|E'(\vec{r})\rangle$ we can apply the well-known result that

$$\vec{E}^0(\vec{r},t)\,|\vec{E}'(r)\rangle = |\vec{E}'(\vec{r})\rangle\vec{E}'(\vec{r},t)\,, \tag{10}$$

where $\vec{E}'(\vec{r},t)e^{-i\omega t}$ represents the classical field satisfying the vacuum Maxwell equations and having $\vec{E}'(\vec{r})$ as its initial value. We use $\vec{r}$ to represent any point in space, and $\vec{x}$ to represent the atom's position. Thus, because of the normal ordering of Eqs. (7) and (9), if we assume the above initial coherent field state, we achieve the following equations

$$\langle\vec{f}\rangle = -i\hbar[\langle\sigma\rangle^*\vec{\text{grad}}g - \langle\sigma\rangle\vec{\text{grad}}g^*]\,,$$
$$\langle\dot{\sigma}\rangle + (\Gamma/2 - i\Omega)\langle\sigma\rangle = \langle D\rangle g\,, \tag{11}$$
$$\langle\dot{D}\rangle + \Gamma\langle D\rangle = \Gamma - 2(g^*\langle\sigma\rangle + g\langle\sigma\rangle^*)\,,$$

where

$$g = i\vec{\mu}_{21}\cdot\vec{E}'(\vec{x},t)/\hbar = \langle G\rangle\,.$$

Here the complex conjugate (*) has replaced the Hermitian conjugate (†) of (7) and (9), and we have with a minimum of complexity obtained the appropriate semiclassical equations with damping.

If we represent $\vec{\text{grad}}g$ by

$$\vec{\text{grad}}g = (\vec{\alpha} + i\vec{\beta})g\,, \tag{12}$$

where $\vec{\alpha}$ and $\vec{\beta}$ are real (Note: if $g = ue^{i\phi}$ with u and ϕ real, then $\vec{\alpha} = \vec{\text{grad}}\ln u$ and $\vec{\beta} = \vec{\text{grad}}\phi$) then the force equation expands to

$$\langle\vec{f}\rangle = \vec{\alpha}[i\hbar(g^*\langle\sigma\rangle - g\langle\sigma\rangle^*)]$$
$$+ \hbar\vec{\beta}(g^*\langle\sigma\rangle + g\langle\sigma\rangle^*)\,. \tag{13}$$

In (13), the coefficient of $\vec{\alpha}$ is the negative of the

expectation value of the dipolar interaction energy; i.e., $\langle \vec{\mu} \cdot E^0 \rangle$ if we use (3). This is the dipole force familiar also for dc fields. The coefficient of $\hbar\vec{\beta}$ is the absorption rate, as may be seen from the third of Eqs. (11). This is the force component usually called the scattering force.

If the external field $E'(\vec{x}, t)e^{-i\omega t}$ is monochromatic, of frequency ω, and the atom is motionless, then $\vec{E}'$ and hence g is independent of time, and the stationary solution of (11) is

$$\langle \sigma \rangle = g/[\gamma(1+p)], \qquad \langle D \rangle = (1+p)^{-1}, \tag{14}$$

where

$$\gamma \equiv \Gamma/2 - i\Omega, \quad p \equiv 2|g|^2/|\gamma|^2,$$

and the force, from (13), is

$$\langle \vec{f} \rangle = \hbar p(1+p)^{-1}(-\Omega\vec{\alpha} + \Gamma\vec{\beta}/2). \tag{15}$$

The quantity p is called the saturation parameter.

From (15) we can demonstrate that the scattering force is associated with spontaneous emission, the dipole force with a coherent redistribution of the incident field due to stimulated emission. The stationary upper-state probability $\langle \sigma_{22} \rangle = p/[2(1+p)]$. Hence one may express the scattering force component as

$$\langle \vec{f} \rangle_{\text{scat}} = \Gamma \langle \sigma_{22} \rangle \hbar\vec{\beta}.$$

This evidently may be regarded as the result of removing quanta of average momentum $\hbar\vec{\beta}$ from the incident external field at the same rate that the atom is undergoing spontaneous decay. The dipole force by contrast depends on the detuning Ω as well as on the excitation of the atom. It must therefore depend on the phase relationship between the mean dipole $\vec{\mu}_{12}\langle \sigma \rangle$ and the external field $\vec{E}'$. It may be regarded as the result of the redistribution of field momentum caused by coherent interference between the emitted field of the dipole and the outgoing waves of the incident field. For example, an atom below resonance (negative Ω) sitting in a focused Gaussian beam is pulled toward the beam focus by the dipole force because the atom acts rather like a weak positive lens. The "light scattering" involved in generating this force is the intrabeam coherent forward scattering.

Now we examine the first-order velocity dependence of the force. A moving atom experiences a modified field since

$$\frac{d\vec{E}'(\vec{x}, t)}{dt} = \frac{\partial \vec{E}'(\vec{x}, t)}{\partial t} + (\vec{v} \cdot \vec{\text{grad}})\vec{E}'(\vec{x}, t).$$

For the same monochromatic field as above, we have, using (12),

$$\dot{g} = \vec{v} \cdot (\vec{\alpha} + i\vec{\beta})g. \tag{16}$$

This modifies the solution of (11). We can obtain an expression for the force accurate to first order in the velocity by taking the time derivative of the zero-order solutions (14), using (16), and then using these first-order results for $\langle \dot{\sigma} \rangle$ and $\langle \dot{D} \rangle$ to re-solve Eqs. (11) to first order in $\vec{v}$. We find thus

$$\langle \dot{D} \rangle \cong -\frac{2p}{1+p}(\vec{v} \cdot \vec{\alpha})\langle D \rangle,$$

$$\langle \dot{\sigma} \rangle \cong \left((\vec{v} \cdot \vec{\sigma})\frac{1-p}{1+p} + i(\vec{v} \cdot \vec{\beta})\right)\langle \sigma \rangle,$$

and it is then straightforward to solve Eqs. (11) again to find the modified force. In the simple case of a plane wave of wave vector $\vec{k}$, we have $\vec{\alpha} = 0$, $\vec{\beta} = \vec{k}$, so that in this case the only change in Eq. (15) is the Doppler shift $\Omega \to \Omega - \vec{v} \cdot \vec{k}$. This change modifies p. Then to first order we find

$$\langle \vec{f} \rangle = \hbar p(1+p)^{-1}(\Gamma/2)\vec{k}\left(1 + \frac{2\Omega(\vec{v} \cdot \vec{k})}{|\gamma|^2(1+p)}\right). \tag{17}$$

For negative Ω, this velocity dependence damps the motion of the atom along $\vec{k}$, a necessity for a stable trap. Note that the damping force is maximized for $p = 1$ and $\Omega = -\Gamma/2$. Another relatively simple case that has been examined in the literature is the case of a pure standing wave. For this case we have in (12), $g = 2g_0 \cos(\vec{k} \cdot \vec{x})$; whence $\vec{\beta} = 0$, $\alpha = -\vec{k}\tan(\vec{k} \cdot \vec{x})$, and we obtain after some algebra the result

$$\langle \vec{f} \rangle = \frac{\hbar p}{1+p}[\Omega\vec{k}\tan(\vec{k} \cdot \vec{x})] \times \left(1 + \frac{\Gamma^2(1-p) - 2p^2|\gamma|^2}{\Gamma|\gamma|^2(1+p)^2}(\vec{v} \cdot \vec{k})\tan(\vec{k} \cdot \vec{x})\right). \tag{18}$$

Here p represents the local value of the saturation parameter, or $p = 4p_0 \cos^2(\vec{k} \cdot \vec{x})$, where p_0 is the saturation parameter corresponding to one of the two oppositely directed traveling waves that comprise the standing wave. For small values of p_0, the force reduces to

$$\langle \vec{f} \rangle = 2\hbar\vec{k}p_0\Omega\{\sin(2\vec{k} \cdot \vec{x}) + (\Gamma/|\gamma|^2)\vec{v} \cdot \vec{k}[1 - \cos(2\vec{k} \cdot \vec{x})]\}.$$

Our result (18) is consistent with a derivation[27] of the first few spatial Fourier components of the force, which were derived for all velocities. In particular, we checked that the spatial averages were the same to first order in the velocity at all intensities. Two things are noteworthy about (18). First, the velocity-dependent force vanishes at the standing-wave maxima (e.g., at $\vec{k} \cdot \vec{x} = 0$), exactly where one might expect trapped atoms to accumulate. In addition, there is a sign reversal

of this force when $p^2/(1-p) = \Gamma^2/2|\gamma|^2$, which, for large detuning $|\Omega| \gg \Gamma$, can have a quite small value. Thus, for negative Ω, which yields traps at the standing-wave maxima (a desirable feature since this also makes a trap for the directions perpendicular to $\vec{k}$), the damping which exists at low intensities in the neighborhood of the maxima reverses sign and transforms into heating at rather small values of the saturation parameter p. These features complicate the conception of trapping atoms in a standing wave, as in Ref. 5.

We turn now to the investigation of the force fluctuations, which ultimately determine how long an atom will stay in the trap. We assume a monochromatic field, negligible velocity, and quasistationary conditions. We seek the value of the momentum diffusion constant $2D_p$ due to the quantum fluctuations of the force. It is given by

$$\begin{aligned} 2D_p &= (d/dt)(\langle \vec{P}\cdot\vec{P}\rangle - \langle\vec{P}\rangle\cdot\langle\vec{P}\rangle) \\ &= 2\,\mathrm{Re}(\langle \vec{P}\cdot\vec{f}\rangle - \langle\vec{P}\rangle\cdot\langle\vec{f}\rangle) \\ &= 2\,\mathrm{Re}\int_{-\infty}^{0} dt\,[\langle \vec{f}(t)\cdot\vec{f}(0)\rangle - \langle\vec{f}(t)\rangle\cdot\langle\vec{f}(0)\rangle], \end{aligned} \tag{19}$$

where the second line results from $\vec{f} = d\vec{P}/dt$, and the third from expressing $\vec{P}$ as the time integral of the force. Note that the product of two Hermitian operators is Hermitian only if they commute, which $\vec{P}$ and $\vec{f}$ do not. The time zero is arbitrary, and the time minus infinity is an exaggeration, since the autocorrelation of the force lasts only for the order of the atomic lifetime Γ^{-1}. For quasistationary conditions, we can advance the time arguments of the integrand by $|t|$ obtaining equivalently,

$$D_p = \mathrm{Re}\int_0^{\infty} dt\,[\langle \vec{f}(0)\cdot\vec{f}(t)\rangle - \langle f\rangle^2]. \tag{20}$$

A quantity written without a specific time argument is assumed to have its stationary equilibrium value; and $\langle f\rangle^2 \equiv \langle\vec{f}\rangle\cdot\langle\vec{f}\rangle$. Inserting (7) for the forces, and using (10) we obtain the result

$$\begin{aligned} \langle \vec{f}(0)\cdot\vec{f}(t)\rangle = \hbar^2\{&\langle \sigma^\dagger(0)\sigma(t) + \sigma(0)\sigma^\dagger(t)\rangle\,|(\mathrm{grad}\,g)^2| \\ &- \langle\sigma^\dagger(0)\sigma^\dagger(t)\rangle(\mathrm{grad}\,g)^2 \\ &- \langle\sigma(0)\sigma(t)\rangle(\mathrm{grad}\,g^*)^2 \\ &+ \langle\sigma^\dagger(0)\mathrm{com}(0,t)\sigma(t)\rangle\}, \end{aligned} \tag{21}$$

where

$$\begin{aligned} \mathrm{com}(0,t) \equiv{}& \vec{\mathrm{grad}}G(0)\cdot\vec{\mathrm{grad}}G^\dagger(t) \\ &- \vec{\mathrm{grad}}G^\dagger(t)\cdot\vec{\mathrm{grad}}G(0). \end{aligned}$$

The last term of (21) arises upon rearranging the field operators into normal order so that (10) may be used. One can show that the field *gradient* operators commute with the atomic operators at all times.

Consider first the final term of (21). It is the only term which depends on the quantum fluctuations of the field (i.e., on a field commutator). One would thus expect it to yield the effects of spontaneous emission. The quantity $\mathrm{com}(0,t)$ is a free field commutator; its value, casting out a possible high-frequency divergence as usual, is

$$\mathrm{com}(0,t) = k^2\Gamma\delta(t).$$

The last term of (21) thus reduces to

$$(\hbar k)^2\Gamma\langle\sigma_{22}\rangle\,\delta(t), \tag{22}$$

and indeed this result can be modeled by the random instantaneous emission of quanta of momentum $\hbar k$ at an average rate $\Gamma\langle\sigma_{22}\rangle$, as one might expect.

The remaining terms of (21) encompass the effects of the external field gradient interacting with the atomic dipole fluctuations, and constitute the same result one would obtain from a semiclassical theory. The autocorrelation times involved are, as we have mentioned above, of the order of the atomic lifetime, and their modeling in terms of the emission and absorption of quanta is not at all obvious. To proceed with the evaluation, we see that we need quantities such as

$$u \equiv \int_0^{\infty} dt\,[\langle\sigma^\dagger(0)\sigma(t)\rangle - \langle\sigma^\dagger\rangle\langle\sigma\rangle]. \tag{23}$$

Suppose we have solved the set of first-moment equations in (11) for *arbitrary* initial conditions to yield a result of the form

$$\begin{aligned} \langle\sigma(t)\rangle = {}& a_0(t) + a_1(t)\langle\sigma(0)\rangle \\ &+ a_2(t)\langle\sigma^\dagger(0)\rangle + a_3(t)\langle D(0)\rangle. \end{aligned} \tag{24}$$

One can show by study of its equation of motion[28] that the quantity $\langle\sigma^\dagger(0)\sigma(t)\rangle$ has the similar solution

$$\begin{aligned} \langle\sigma^\dagger(0)\sigma(t)\rangle &= a_0(t)\langle\sigma^\dagger(0)\rangle + a_1(t)\langle\sigma^\dagger(0)\sigma(0)\rangle \\ &\quad + a_2(t)\langle\sigma^\dagger(0)\sigma^\dagger(0)\rangle + a_3(t)\langle\sigma^\dagger(0)D(0)\rangle \\ &= [a_0(t) + a_3(t)]\langle\sigma^\dagger(0)\rangle + a_1(t)\langle\sigma_{22}(0)\rangle. \end{aligned} \tag{25}$$

The solution for u that we seek follows from insertion of (25) into (23), yielding

$$u = (A_0 + A_3)\langle\sigma^\dagger\rangle + A_1\langle\sigma_{22}\rangle, \tag{26}$$

where

$$A_i \equiv \int_0^{\infty} dt\,[a_i(t) - \langle\sigma\rangle\,\delta(i,0)], \quad i = 0\text{–}3. \tag{27}$$

The other terms of (20) may be treated similarly;

we obtain thus

$$2D_p = 2\hbar^2 \mathrm{Re}[\,(2A_0\langle\sigma^\dagger\rangle + A_1)\,|(\mathrm{grad}g)^2| + (2A_0\langle\sigma\rangle + A_2)(\mathrm{grad}g^*)^2]] + (\hbar k)^2\Gamma\langle\sigma_{22}\rangle\,. \tag{28}$$

The quantities A_i may be conveniently found by taking and solving the Laplace transform of Eqs. (11). One finds thus

$$A_0 = \frac{g}{|\gamma|^2(1+p)^2}\left[\frac{\gamma}{\Gamma} - \frac{\Gamma}{\gamma} - i2p\left(\frac{\Omega}{\Gamma}\right)\right],$$
$$A_1 = \frac{\Gamma + p\gamma}{\gamma\Gamma(1+p)}, \quad A_2 = -\frac{2g^2}{|\gamma|^2\Gamma(1+p)}, \tag{29}$$
$$A_3 = \frac{g}{\gamma\Gamma(1+p)}\,.$$

Finally, using these results, along with the stationary values (14) of the atomic variables, and using (12) for grad$g$, we obtain the final result

$$2D_p = \hbar^2\alpha^2\Gamma\frac{p}{2(1+p)^3}\left[1 + \left(\frac{\Gamma^2}{|\gamma|} - 1\right)p + 3p^2 + \frac{4|\gamma|^2}{\Gamma^2}p^3\right] + \hbar^2\beta^2\Gamma\frac{p}{2(1+p)^3}\left[1 + \left(3 - \frac{\Gamma^2}{|\gamma|^2}\right)p + p^2\right]$$
$$+ 2\hbar^2(\vec{\alpha}\cdot\vec{\beta})\Omega\frac{p^2}{(1+p)^3}\left[\frac{\Gamma^2}{|\gamma|^2} + p\right] + (\hbar k)^2\Gamma\frac{p}{2(1+p)}\,. \tag{30}$$

We now have in hand the quantities necessary for discussion of trap stability.

DISCUSSION OF THE DIFFUSION CONSTANT

The above result for the diffusion constant has some properties which are at first sight somewhat surprising; hence it merits some discussion. For small excitation of the atom (to first order in the saturation parameter p) we find

$$2D_p \approx \tfrac{1}{2}\hbar^2\Gamma p(k^2 + \alpha^2 + \beta^2)$$
$$\approx \hbar^2\Gamma\langle\sigma_{22}\rangle\,(k^2 + \alpha^2 + \beta^2)\,. \tag{31}$$

We see here three terms, each associated with one of the elementary processes, absorption (β^2), induced emission (α^2), and spontaneous emission (k^2). The spontaneous-emission term can be alternatively associated with the interaction of the semiclassical dipole (note that for small p, $\langle\sigma_{22}\rangle \approx |\langle\sigma\rangle|^2$) with the zero-point field fluctuation, while the other two terms may be similarly associated with interaction of the semiclassical field gradient (recall that $\alpha^2 + \beta^2 = |g^{-1}\mathrm{grad}g|^2$) with the zero-point dipole fluctuation. In this regard we note that

$$\langle\vec{\mu}\cdot\vec{\mu}\rangle = |\mu_{12}|^2\langle\sigma^\dagger\sigma + \sigma\sigma^\dagger\rangle = |\mu_{12}|^2,$$

independent of the state of the atom. Thus an atom even in its ground state has a substantial random dipole moment which gives rise to a random force in interaction with the external classical field gradient. Another view may be had by noting that a weakly excited two-level system is indistinguishable from a one-dimensional harmonic oscillator, whose zero-point fluctuation can interact with the external field gradient. In passing, we remark that calculation of the diffusion constant for a one-dimensional harmonic oscillator yields exactly (31), with $\langle\sigma_{22}\rangle$ replaced by $\langle n\rangle$, the mean excitation number of the oscillator.

An interesting and perhaps somewhat unexpected aspect of (31) occurs when the atom is in the presence of several beams of radiation of the same frequency but different directions. In particular, consider the standing-wave case

$$g = 2g_0\cos(\vec{k}\cdot\vec{x})\,.$$

Then

$$\vec{\beta} = 0, \quad \vec{\alpha} = -\vec{k}\tan(\vec{k}\cdot\vec{x}),$$
$$D_p = (\hbar k)^2\Gamma p_0\,,$$

where $p_0 = 2|g_0|^2/|\gamma|^2$ represents the saturation parameter corresponding to a single one of the two associated traveling waves. We see that the diffusion constant is independent of the atom's position in the standing wave, even though the field strenth, excitation of the atom, and mean force are strongly position dependent. The explanation of this curious behavior we have noted above; that is, in the field minima the diffusion results from the interaction of the zero-point dipole fluctuation with the large gradient of the external field amplitude. Further, we see that in this approximation, the diffusion depends only on p_0; hence if we hold p_0 fixed while increasing $|\Omega|$, we can increase the depth of the sinusoidal potential while the diffusion constant remains unchanged. This is encouraging with regard to trapping atoms, but fortunately does not ensure a stable trap, because the damping is reduced as $|\Omega|$ increases.

The other particularly interesting feature of (30) is the term proportional to $\alpha^2 p^4(1+p)^{-3}$, which becomes dominant at large p. It is the only term which does not saturate or decrease for large p. Thus, if $p \gg 1$, and $\vec{\alpha} \neq 0$ we find,

$$2D_p \to 2\hbar^2\alpha^2 p|\gamma|^2/\Gamma = 4\hbar^2\alpha^2|g|^2/\Gamma\,. \tag{32}$$

It is noteworthy that this term can contribute significantly to the diffusion constant even for small p when the detuning Ω is large. Thus it merits careful consideration in constructing a trap. Its proportionality to α^2 shows it to be associated with the dipole force.

At large p we can understand (32) by a not-too-complex argument. In this limit, the atom is strongly coupled to the externally excited field mode, and it is appropriate to consider the atom and that one field mode as a single quantum system, according to the picture of the "dressed" atom.[29] This system has two quasistationary states, in each of which the atom has an equal mixture of upper and lower levels, with its dipole $\vec{\mu}_{12}\langle\sigma\rangle$, respectively, in phase with and in opposition to the field, giving them equal and opposite interaction energies $\pm|\vec{\mu}_{12}\cdot\vec{E}'|$ and thus equal and opposite dipole forces. Each time this system spontaneously emits a photon into one of the unexcited field modes, the atom of necessity finds itself in its lower state, and thus in an equal mixture of the above two quasistationary states. Thus, after each spontaneous emission event the atom is forced randomly in either direction by the field gradient, the same steady force persisting until the next decay. Finally, the spontaneous decay rate of each quasistationary state is $\Gamma/2$, since in each such state the upper atomic level probability is $\frac{1}{2}$ and the spontaneous emission rate is always proportional to the upper level probability. On this basis one can calculate the resulting diffusion constant. The interaction energies are

$$\pm\hbar|g|,$$

hence the forces $\vec{f}$ are $\pm\hbar\,\vec{\text{grad}}|g| = \pm\hbar\vec{\alpha}|g|$. The diffusion constant from (19) is

$$2D_p = 2\int_{-\infty}^{0} dt\langle\vec{f}(t)\cdot\vec{f}(0)\rangle,$$

where we have used the fact that the forces are real. To make the required average we observe that the force is constant between decays and takes a new random direction at each decay. Thus $2D_p$ reduces to

$$2D_p = 2f^2\langle t_d\rangle,$$

where $-t_d$ is the time of the last spontaneous decay prior to time zero. Since the spontaneous decay rate is $\Gamma/2$, we have $\langle t_d\rangle = 2/\Gamma$, and hence

$$2D_p = 4f^2/\Gamma = 4\hbar^2\alpha^2|g|^2/\Gamma,$$

in exact agreement with (32).

In conclusion of this section, we will risk some remarks concerning the photon concept. It is most precise, and often useful, to think of the momentum exchange between atom and field as occurring in quantum units $\hbar\vec{k}$. In the present case one can nicely understand the scattering force and its associated fluctuations in such terms. However, the dipole force and its associated fluctuations cannot be simply understood on this basis; in particular, our heuristic picture of the fluctuations in the high saturation limit invokes a *steady* force giving many $\hbar\vec{k}$ of momentum to the atom, *interrupted* by the spontaneous-emission events. The photon concept does *not* seem particularly helpful in understanding this part of the force on the atom.

IMPLICATIONS FOR TRAPPING

We are now prepared to consider the problem of trapping. First we note that the mean dipole force [from Eq. (15)] may be written as the negative gradient of a potential U.[6] Since $\vec{\text{grad}}\,p = 2\vec{\alpha}p$, we have

$$\langle\vec{f}\rangle_{\text{dip}} = -\vec{\text{grad}}[(\hbar\Omega/2)\ln(1+p)], \tag{33}$$

so that

$$U = (\hbar\Omega/2)\ln(1+p).$$

This is called the trap potential. The mean scattering force

$$\langle\vec{f}\rangle_{\text{scat}} = \hbar\Gamma p(1+p)^{-1}\vec{\beta}/2 \tag{34}$$

is nonconservative and must be offset by the dipole force. As we shall see, the trap parameters may be chosen so that the mean scattering force is negligibly small. Thus for negative detuning Ω (i.e., below resonance) atoms might be expected to collect at the positions of the field amplitude maxima.

The important question is then how long a trapped atom, subject to the force fluctuations, will remain in the trap. If sufficient damping could be obtained, a stable trap would ensue, but as we shall see, any simple single-frequency trap is unstable for a two-level atom. We shall return to the discussion of damping below. Neglecting the effects of damping altogether, it turns out that it should be possible to keep an atom in the trap for a considerable time, of the order of seconds. Define the trap depth U_0 as the maximum of $|U|$, i.e.,

$$U_0 = |U|_{\max}.$$

Also, let W be the energy of the atom relative to the bottom of the trap; i.e.,

$$W = U_0 + U + P^2/2M.$$

A trapped atom gains energy due to the force fluctuations, so that, in the absence of damping,

$$dW/dt = D_p/M.$$

The residence time T of an atom in the trap is thus of the order of

$$T \approx U_0(dW/dt)^{-1} \approx MU_0/(D_p)_{\max}. \tag{35}$$

The use of $(D_p)_{\max}$ here, rather than some more accurate average, makes this a conservative estimate. It turns out that the optimum value of p is always very much smaller than unity, and the optimum detuning is always very much larger than Γ. If we define a normalized detuning parameter by

$$q \equiv -\Omega/\Gamma, \tag{36}$$

then we can here approximate $p \ll 1$ and $q \gg 1$. In the small p approximation, we find for the trap depth,

$$U_0/\hbar\Gamma = qp_{\max}/2. \tag{37}$$

We want this quantity to be reasonably large compared to unity, which shows immediately that for small p, the detuning q must be large. We now need a suitable approximation for D_p. For small p, we need keep from (30) only the terms linear in p, and the potentially troublesome $\alpha^2 p^4$ term. Thus, when $q \gg 1$, $p \ll 1$,

$$2D_p \approx (\hbar^2\Gamma p/2)(k^2 + \alpha^2 + \beta^2 + 4\alpha^2 q^2 p^3). \tag{38}$$

It is helpful to note how the residence time depends on the parameters describing the trap, particularly the light intensity, trap depth, and trap dimension. If we define intensity by

$$I \equiv (c/4\pi)\bar{E}^2 = (c/2\pi)\left|E'\right|^2,$$

which is the same as the Poynting vector for a plane wave, then one can demonstrate for a free atom the relation

$$p(1 + 4q^2) = I/I_s, \tag{39}$$

where

$$I_s = \hbar\omega\Gamma k^2/12\pi = 6.29 \text{ mW/cm}^2,$$

where the evaluation[30] is for the case of sodium. For $q \gg 1$ we have

$$4pq^2 = I/I_s. \tag{40}$$

Using expressions (37) and (40), we can express $p_{\max}$ and q in terms of the trap depth and the light intensity $I_{\max}$ at the bottom of the trap. In particular,

$$q = (I_{\max}/4I_s)/(2U_0/\hbar\Gamma),$$
$$p_{\max} = (2U_0/\hbar\Gamma)^2/(I_{\max}/4I_s),$$

and (41)

$$q^2 p_{\max}^4 = (2U_0/\hbar\Gamma)^6/(I_{\max}/4I_s)^2.$$

One can observe, then, from (35), (38), and (41), that so long as the $\alpha^2 q^2 p^4$ term in the diffusion constant remains small, the trap lifetime is proportional to $(I_{\max}/U_0)$, whereas if the $\alpha^2 q^2 p^4$ term becomes dominant, the proportionality changes to $(I_{\max}^2/U_0^5)$ and the making of a deeper trap becomes very costly. Recalling that $\vec{\text{grad}}\, p = 2\vec{\alpha}p$, we observe that α is inversely proportional to the trap dimension; hence smaller traps are limited to smaller depths.

To get an idea of the numbers involved, we will look at two types of traps that have been proposed, namely a standing-wave trap and a traveling-wave Gaussian beam trap. For the former, we consider only longitudinal trapping in the standing-wave maxima. Recalling that here the atom field coupling has the form

$$g = 2g_0 \cos kx,$$

where g_0 is the magnitude of the coupling constant corresponding to a single traveling plane wave, we have

$$p = 4p_0 \cos^2 kx, \quad \vec{\alpha} = -kx \tan kx, \quad \beta = 0,$$

where $\hat{x}$ is the x directed unit vector, so that (38) becomes

$$2D_p = 2(\hbar k)^2 \Gamma p_0 (1 + 256 q^2 p_0^3 \sin^2 kx \cos^6 kx). \tag{42}$$

The maximum value of $\sin^2\theta\cos^6\theta$ is $\frac{27}{256}$, occurring when $\cos^2\theta = \frac{3}{4}$. Thus the p_0^4 term will begin to be in evidence when [using $p_{\max} = 4p_0$, $I_{\max} = 4I_0$, in (41)]

$$27q^2 p_0^3 = 27(U_0/2\hbar\Gamma)^4/(I_0/I_s) \gtrsim 1,$$

or (43)

$$\tfrac{27}{16}(U_0/\hbar\Gamma)^4 \gtrsim I_0/I_s,$$

where I_0 is now the intensity of one of the traveling waves that comprise the standing wave. Thus for $(U_0/\hbar\Gamma) = 100$, we need $I_0 \gtrsim 1$ MW/cm^2 for sodium to avoid the effects of the troublesome dipole force fluctuation term, but we then have [using (36), (41), and (42)]

$$q \approx 8 \times 10^5, \quad p_0 \approx 6 \times 10^{-5},$$
$$T \approx 2q(M/\hbar k^2) \approx 5 \text{ sec}. \tag{44}$$

The detuning required is about 270 cm^{-1}, or about 1.6% of the $3p$ state energy. For this trap there is no average scattering force, so its negligibility is ensured.

The other pertinent example is the traveling-wave Gaussian beam trap. Assuming the beam is focused at the origin, and travels in the x direction, we have now[31]

$$g = g_0(b/is)\exp[ik(x + r^2/2s)], \tag{45}$$

where

$$s = x - ib, \quad r^2 = y^2 + z^2,$$

and b is the confocal length. Then, one has

$$p = p_0[b^2/(b^2+x^2)] \exp[-kbr^2/(x^2+b^2)], \quad (46)$$

and if $kb \gg 1$, then to good approximation,

$$\vec{\beta} \approx k\hat{x}, \quad \vec{\alpha} \approx -(x\hat{x} + kbr\hat{r})/(x^2+b^2). \quad (47)$$

In evaluating the diffusion coefficient (38) for this case we can ignore $\alpha^2 p$ with respect to the other terms, leaving

$$2D_p \approx (\hbar k)^2 \Gamma p(1 + 2\alpha^2 k^{-2} q^2 p^3). \quad (48)$$

The quantity $2\alpha^2 k^{-2} q^2 p^3$ of (48) maximizes at $x = 0$, $r^2 = b/3k$, where it equals $2q^2 p_0^3/3ekb$. Thus corresponding to (43), the p^4 term will in this case be negligible if

$$(2/3ekb)(2U_0/\hbar\Gamma)^4 \lesssim I_0/4I_s,$$

or, approximately

$$(16/kb)(U_0/\hbar\Gamma)^4 \lesssim I_0/I_s. \quad (49)$$

From this expression it is evident that somewhat deeper traps may be obtained if the trap is less sharply focused. Recalling from (43) that the $1/e$ beam radius w_0 at the beam focus is related to b by

$$kb = (kw_0)^2.$$

we see that we can gain trap depth in proportion to the square root of the beam radius, but of course only at the expense of a less localized trap. For comparable light intensity and fairly tight focus, one sees that the trap depth and residence time are comparable to the standing-wave case. In the present case

$$T \approx (m/\hbar k^2)q, \quad (50)$$

a factor of two less than for the standing-wave case for the same detuning.

We can check that the mean scattering force is negligible. From (33), (34), and (47), one finds the ratio of the mean x-directed dipole force to the mean scattering force to be

$$\langle f_{\rm dip}\rangle_x / \langle f_{\rm scat}\rangle_x = -2qx/k(x^2+b^2).$$

This ratio maximizes at $x = b$, where it is $-2q/kb$. If we are thinking of values of q near 10^6, then so long as $kb \lesssim 10^4$, the mean scattering force will not be important.

The interesting point here is that the traps we have considered, with light intensities of 1 MW/cm^2 or greater and with depths of $100\hbar\Gamma$ or greater, can hold atoms for periods of several seconds even in the absence of effective damping.

Finally, consider the question of damping. The component of the mean scattering force which is proportional to velocity damps the atom's motion. If the components of the damping force may be expressed in terms of a damping tensor γ_{ij} by

$$f_i = -M \sum \gamma_{ij} v_j,$$

then the equation of motion of the energy W may be expanded from its from (35) to read now

$$dW/dt \approx U_0/T - M \sum \gamma_{ij} v_i v_j.$$

If the trap can be so structured that the velocity distribution remains nearly isotropic, then we may average over directions to obtain

$$dW/dt \approx U_0/T - 2\bar{\gamma}(E_k), \quad (51)$$

where $\bar{\gamma} = \sum \gamma_{ii}/3$, and (E_k) is the kinetic energy. If we further assume that $(E_k) \approx W/2$, as it is for a harmonic potential well, then we find the relation

$$dW/dt \approx U_0/T - \bar{\gamma}W. \quad (52)$$

If steady-state conditions come about, then

$$W/U_0 = (\bar{\gamma}T)^{-1} = D_p/M\bar{\gamma}U_0. \quad (53)$$

Thus, the trap will be stable if $(\bar{\gamma}T) > 1$, and unstable otherwise.

It is quickly evident that a single beam trap is unstable. Consider the Gaussian beam trap. The damping is very nearly that for a plane wave; hence, for an x-directed beam, picking out the velocity-dependent term from (17), we find (for $p \ll 1$)

$$\gamma_{xx} = \frac{\hbar k^2}{M} \frac{pq}{q^2 + \frac{1}{4}}.$$

Using (37) and (48), there results

$$\bar{\gamma}T = \tfrac{1}{3} p(1 + 1/4q^2)^{-1} \ll 1, \quad p \ll 1.$$

Hence in traps with $p \ll 1$, as in the example discussed above, the damping is ineffectual. An attempt to increase $\bar{\gamma}T$ by increasing p (with $U_0 \propto pq =$ constant) fails, for while γ_{xx} then varies as p^2, so does D_p (and hence T^{-1}) after its q^2p^4 term becomes dominant. Similar arguments pertain to the standing-wave trap. Thus while single-beam (really single-frequency) traps can contain an atom for a sufficiently long time to envisage experiments, they are essentially unstable.

We have proposed[9] the idea of stabilizing the trap by using one or more additional light beams, tuned closer to resonance, whose sole purpose is to damp the atomic motion. It turns out that nature has put an obstacle in the way of this solution, namely the dynamic Stark shift of the damping beam's resonance as the atom moves around in the trap.

To examine this idea, we suppose that in addition to a Gaussian trapping beam, whose param-

eters will now be labeled by the subscript t, there is also a damping beam (subscript d) tuned closer to resonance. If both saturation parameters are small, and the damping beam is not strongly focused, then the damping due to the trapping beam and the trapping due to the damping beam can both be safely neglected. The only significant interaction between the two beams is the dynamic Stark shift.

The important parameters of the two-beam problem are thus

$$U_0 = \hbar\Gamma q_t p_t/2\,,$$

$$\bar{\gamma} = \hbar k^2 p_d q_d/3M(q_d^2+\tfrac{1}{4})\,,$$

$$2D_p = (\hbar k)^2\Gamma(p_t+p_d)\,,$$

where we must add the two beams' contributions to the momentum diffusion, and the detuning parameter q_d of the damping beam is subject to the Stark shift. With these values, we find for the equilibrium energy [see (53)]

$$W/\hbar\Gamma = D_p/M\bar{\gamma}\hbar\Gamma = 3(p_t+p_d)(q_d^2+\tfrac{1}{4})/2p_d q_d\,.$$

If $p_d \gg p_t$, this expression reduces to

$$W/\hbar\Gamma = \tfrac{3}{2}\left(q_d + \frac{1}{4q_d}\right). \tag{54}$$

Without the effects of Stark shift we could quickly minimize this by setting $q_d = \frac{1}{2}$, thus maximizing the damping and obtaining an equilibrium energy of the order of $\hbar\Gamma$. The Stark shift, however, has the following effect. As an atom of energy W (with respect to the bottom of the trap), moves in the trap, it encounters changes in the potential equal to W. Now for small p_t, the Stark shift of the resonance as seen by the damping beam is just twice the potential U; that is

$$q_d = q_{d0} - 2U/\hbar\Gamma\,,$$

where q_{d0} is the unperturbed detuning parameter. Thus for a change in potential equal to W, the change in q_d is

$$\delta q_d = -2W/\hbar\Gamma = -3\left(q_d + \frac{1}{4q_d}\right),$$

where we have used (54). One sees that the change in q_d occasioned by the motion of the atom in the trap is larger than q_d itself, and hence the effect of the damping beam is not at all simple. We have shown that if only one dimension is considered, the trap is still stable, because the damping is most effective at the bottom of the trap, where the atom's momentum is largest. For three dimensions, that conclusion may not hold.

Lest one think that nature somehow will not allow an optical trap to be stable, we remark that a simple-harmonic-oscillator dipole (SHO) can be trapped even by a single beam. For the case of the linear SHO one takes the small p limit of the theory, and then replaces

$$p \to 2\langle n\rangle\,,$$

where $\langle n\rangle$ is the mean excitation number for the SHO. The results so derived are valid for any $\langle n\rangle$. For the Gaussian beam trap, we then have, for $q \gg 1$,

$$\bar{\gamma}T = \tfrac{2}{3}\langle n\rangle\,,$$

and the trap is stable for large $\langle n\rangle$. For a real atom, however, we don't have the privilege of large $\langle n\rangle$, and some clever methods are called for. If the two-beam trap also turns out to be unstable in three dimensions, then there are several possible ways to proceed. One can reduce the Stark shift by a factor of 2 by using different upper levels for the trapping and damping resonances, so that only the lower level of the damping resonance is shifted. In addition, or alternatively, one might use a third beam tuned near a resonance of the upper damping level to cancel the Stark shift of the damping resonance.

Thus it would seem in principle possible to form a stable trap for atoms. Obtaining the optimum form of damping may take some experimentation, but since the traps can contain slow atoms for long times anyway, such experimentation would seem feasible and worthwhile.

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