Multiple Imputation

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Overview

- Why Impute?
- Amelia
- mice
- mi
- aregImpute



Listwise Deletion

- In almost all software, "listwise deletion" has been the default. If any variable is missing, a case is completely dropped from the analysis.
- Most people seem to agree that is bad—"biased parameter estimates"
- Until recently, practitioners ignored the problem.
- Research on this has been steadily accumulating since 1970s
- Statistical researchers are trying to find a more-or-less convenient,
 "idiot proof" procedure

Rubin proposed Multiple Imputation

- Jargon: MAR ("Missing at Random") means that the chance of a missing score is predictable using information in other variables (and not predictable by other unmeasured forces)
- Rubin's proposal
 - Use many variables, including the dependent variable and variables not planned for inclusion in the final model, to predict missings
 - Create several "Imputed" data sets.
 - Run Each analysis on Each Imputed Dataset
 - Combine the estimates, weight them to take uncertainty into account.

Do You Do It Yourself?

- Rubin suggested the imputations could be done at a data center when they supply the dataset. MI would be done "once and for all," and the imputed missings would be distributed as one collection.
- That approach was impractical for a number of reasons.
- Many routines to impute missing values have been proposed.
- This research area is still under active development
- Caution: I'm not an MI authority (just a guy demonstrating some R packages)

What are we looking for?

- What format does a routine expect from our data?
- Are the imputations returned in a manageable format?
- Is it difficult to conduct the analysis on each separate dataset?
- How to best pool the estimates together and summarize them?

Points of Concern

Why Impute?

- Calculation of "imputation averaged" results
 - Good theory/method exists for MLE of "slope coefficients".
- "Rubin's Rules" for slope & variance estimates
 - For slope estimates, average the imputed, $\hat{\beta} = \sum_{i=1}^{m} \hat{\beta}_{i}$
 - Variance estimate for $\hat{\beta}$ combines
 - **1** average of $\widehat{Var(\hat{\beta}_j)}$, $\sum_{i=1}^m = \widehat{Var(\hat{\beta}_j)}$, plus
 - ② a penalty for uncertainty between $\hat{\beta}_j$, $\frac{1}{1+m}\sum_{j=1}^{m}(\hat{\beta}_j-\hat{\beta}_j)^2$.
- ullet Less good theory/tools on other statistics (R^2 , deviance, etc.)
- Difficult choices about "openness" and "interoperability" with other R functions
- Caution about terminology: imputation sometimes means
 - The candidates to "fill in" for NAs
 - A completed data frame with the NAs are replaced by the candidates

Amelia

King, Gary, James Honaker, Anne Joseph, and Kenneth Scheve. 2001. "Analyzing Incomplete Political Science Data: An Alternative Algorithm for Multiple Imputation." The American Political Science Review 95(1): 49-69.

James Honaker, Gary King, Matthew Blackwell (2011).

Amelia II: A Program for Missing Data. Journal of
Statistical Software, 45(7), 1-47. URL

http://www.jstatsoft.org/v45/i07/.

Rough Sketch of Amelia

- Assume all variables are drawn from one Multivariate Normal Distribution, $MVN(\mu, \Sigma)$
- ullet Conduct series of complicated algorithms to estimate μ and Σ
- After estimating μ and Σ , then draw random samples from the MVN to fill in missing values
- Basic idea similar to "Norm" (J. Schafer), but algorithm may be faster (EM with "importance sampling")

```
amelia (x, m = 5, p2s = 1, frontend = FALSE, idvars = 1, frontend = 1, fronten
                         NULL, ts = NULL, cs = NULL, polytime = NULL,
                    splinetime = NULL, intercs = FALSE, lags = NULL
                    , leads = NULL, startvals = 0, tolerance = 0
                    .0001, logs = NULL, sqrts = NULL, lgstc = NULL,
                         noms = NULL, ords = NULL, incheck = TRUE,
                    collect = FALSE, arglist = NULL, empri = NULL,
                    priors = NULL, autopri = 0.05, emburn = c(0,0),
                          bounds = NULL, max.resample = 100, ...)
```

Note: amelia uses all of the supplied variables in imputations except vars declared as "idvars." To save memory, one should remove all extraneous variables first, rather than use the "idvars" feature to ask amelia to ignore them.

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Surprising, Possibly True

- Most people say "but my variables are not Normal." (gender, survey scales, etc)
- King (and others) argue the approximation is not harmful (various reasons)
- Amelia allows user to specify variables as "nominal" and "ordinal"
 - Nominal variables: The normal imputations are "rounded off" to values in the observed scale {0,1,2}
 - Ordinal variables: Optionally "rounded off" to integers, but instructions discourage that
 - They suggest a 7 point scale might meaningfully have imputed values in-between the integers

Grab Some Data, Impose Some Missings

Thanks to Chuck Cleland who suggested this example in r-help

```
options (digits =2)
if (!file.exists("examples")) dir.create("examples
if (!file.exists("examples/titanic.txt"))
 download.file ("http://lib.stat.cmu.edu/S/Harrell/
    data/ascii/titanic.txt", "examples/
    titanic.txt")
titanic <- read.table("examples/titanic.txt", sep
   = ',', header = TRUE)
titanic0 <- titanic
save(titanic0, file="examples/titanic0.rda")
set.seed (4321)
titanic$sex[sample(nrow(titanic), 10)] <- NA
titanic $pclass [sample(nrow(titanic), 10)] <- NA
titanic$survived[sample(nrow(titanic), 10)] <- NA
```

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The "Most Complete" Version of the Data Says ...

```
fullglm <- glm(survived ~ pclass + sex + age,
    family = binomial, data = titanic0)
library(xtable)
tout <- xtable(fullglm)
print(tout, type = "latex")</pre>
```

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	4.5222	0.4710	9.60	0.0000
pclass2nd	-1.4952	0.2820	-5.30	0.0000
pclass3rd	-2.8413	0.3389	-8.38	0.0000
sexmale	-3.0867	0.2411	-12.80	0.0000
age	-0.0493	0.0087	-5.65	0.0000

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After Imposing some more Missings, The ListWise Deletion Results

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	4.3706	0.4709	9.28	0.0000
pclass2nd	-1.4617	0.2847	-5.13	0.0000
pclass3rd	-2.7442	0.3403	-8.06	0.0000
sexmale	-3.0773	0.2414	-12.75	0.0000
age	-0.0464	0.0088	-5.30	0.0000

Use Amelia to create Imputed Data Sets

```
library(Amelia) # generate multiple imputations
titanic.amelia <- amelia(subset(titanic, select =
    c('survived', 'pclass', 'sex', 'age', '
    embarked')), m = 10, noms = c('survived', '
    pclass', 'sex', 'embarked'),
emburn = c(500, 500), p2s = F)</pre>
```

p2s=F turns off screen output that overflows the presentation software

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Note: Now use tools not from Amelia to Analyze the Data and Summarize it

- Lets try to use general purpose tools to estimate and summarize these models.
- The imputations are in an R list, so the general "lapply" function can be used to fit any kind of model that can accept a data frame as an argument.
- The R package mitools (Thomas Lumley) has tools to combine estimates of slopes and calcuate the Rubin-adjusted standard errors.

lapply Conducts the glm for Each Imputed Set

Post Processing with "mitools"

```
options(digits=2)
library(mitools) # MIextract
betas <- Mlextract(allimplogreg, fun = coef)
vars <- Mlextract(allimplogreg, fun = vcov)
summary(Mlcombine(betas, vars))</pre>
```

```
Multiple imputation results:
      MIcombine.default(betas, vars)
                       se (lower upper)
            results
                                        missInfo
(Intercept) 4.03 0.532 2.926 5.128
                                            66 %
pclass2nd -1.44 \ 0.258 \ -1.951 \ -0.931
                                            27 %
pclass3rd -2.71 \ 0.308 \ -3.339 \ -2.081
                                            57 %
                                            18 %
sexmale
            -2.53 \ 0.175 \ -2.872 \ -2.184
              -0.05 \ 0.011 \ -0.073 \ -0.027
                                            73 %
age
```

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mi inference from mix offers effective df and fm

```
library (mix)
se.glm <- Mlextract(allimplogreg, fun = function(x){sqrt(diag(vcov(
    x)))})
as.data.frame(mi.inference(betas, se.glm))
```

```
est
                      std.err
                                 d f
                                      signif
                                                lower
                                                         upper
                                                                       fminf
(Intercept)
                4.03
                        0.532
                                 23 1.2e-07
                                                2.926
                                                         5 128
                                                                1 72
                                                                        0.66
pclass2nd
              -1.44
                        0.258
                                129 1.3e-07
                                               -1.951
                                                        -0.931
                                                                        0.27
pclass3rd
              -2.71
                        0.308
                                 31 \quad 6.3e - 10 \quad -3.339 \quad -2.081
                                                                        0.57
                                                                        0.18
sexmale
              -2.53
                        0.175
                                300 \ 0.0e + 00 \ -2.872 \ -2.184 \ 0.21
                                 18 \ 2.5e - 04 \ -0.073 \ -0.027 \ 2.39
                                                                        0.73
              -0.05
                        0.011
age
```

df: degrees of freedom associated with the t reference distribution used for interval estimates.

r: estimated relative increases in variance due to nonresponse.

fminf: estimated fractions of missing information.

Compare Side-by-Side: MI and LD results

```
df1 <- as.data.frame(mi.inference(betas, se.glm))</pre>
df2 \leftarrow cbind(df1[,1:2], ldbeta = coef(ldglm), ldse = sqrt(diag(vcov
     (ldglm))))
df2
```

```
est
                   std.err Idbeta
                                     ldse
(Intercept) 4.03
                     0.532
                           4.371
                                   0.4709
pclass2nd
            -1.44
                     0.258
                            -1.462 0.2847
pclass3rd
            -2.71
                     0.308
                           -2.744 0.3403
sexmale
                     0.175 - 3.077 0.2414
            -2.53
            -0.05
                     0.011 - 0.046 - 0.0088
age
```

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Make the Missings Worse!

```
\label{eq:set_seed} \begin{array}{l} \texttt{set.seed} \ (234234) \\ \texttt{titanic} \ <- \ \texttt{titanic0} \\ \texttt{titanic\$sex} \ [\texttt{sample} \ (\texttt{nrow} \ (\texttt{titanic}), \ 400)] \ <- \ \texttt{NA} \\ \texttt{titanic\$survived} \ [\texttt{sample} \ (\texttt{nrow} \ (\texttt{titanic}), \ 400)] \ <- \ \texttt{NA} \\ \texttt{titanic\$survived} \ [\texttt{sample} \ (\texttt{nrow} \ (\texttt{titanic}), \ 400)] \ <- \ \texttt{NA} \\ \end{array}
```

Then estimate

- new "ldglm" (listwise deletion estimate)
- 10 fresh imputed datasets and regressions for each

New Listwise Deletion Model IdgIm

	Estimate	Std. Error	z value	$\Pr(> z)$
(Intercept)	5.9001	0.9254	6.38	0.0000
pclass2nd	-2.1114	0.5233	-4.03	0.0001
pclass3rd	-4.4776	0.7632	-5.87	0.0000
sexmale	-3.6943	0.5480	-6.74	0.0000
age	-0.0657	0.0162	-4.07	0.0000

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MI summary estimates

summary (MIcombine (betas, vars))

```
Multiple imputation results:
      MIcombine.default(betas, vars)
            results
                         se (lower upper)
                                           missInfo
(Intercept)
              2.89 0.4254 2.011 3.770
                                               65 %
pclass2nd
              -0.91 0.2682 -1.453 -0.364
                                               53 %
pclass3rd
              -2.10 0.2799 -2.679 -1.526
                                               63 %
sexmale
              -1.73 0.2213 -2.184 -1.273
                                               63 %
              -0.04 \ 0.0083 \ -0.057 \ -0.023
                                               65 %
age
```

Compare Side-by-Side: MI and LD results

```
df2.1 <- as.data.frame(mi.inference(betas, se.glm))</pre>
df2.2 \leftarrow cbind("MI", df2.1[,1:2], "LD", Idbeta = coef(IdgIm), Idse =
      sqrt(diag(vcov(ldglm))), "full", est = coef(fullglm))
df2.2
```

```
"MI"
                      est
                          std.err
                                   "LD"
                                         Idheta
                                                  ldse
                                                        "full"
                                                                    est
(Intercept)
               MI
                    2.89
                           0.4254
                                     LD
                                          5.900
                                                 0.925
                                                           full
                                                                 4.522
pclass2nd
               MI - 0.91
                           0.2682
                                         -2.111
                                                 0.523
                                                           full
                                                                -1.495
pclass3rd
               MI - 2.10
                           0.2799
                                        -4.478 0.763
                                                           full
                                                               -2.841
sexmale
                           0.2213
                                      LD = 3.694 \ 0.548
               MI - 1.73
                                                           full
                                                               -3.087
               MI = 0.04
                           0.0083
                                      ID = 0.066 \cdot 0.016
                                                           full
                                                               -0.049
age
```

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Maybe confidence intervals help

```
"full"
                                                    2.5 % 97.5 %
                     lower
                             upper
                                              beta
               est
(Intercept)
              2.89
                     2.011
                            3.770
                                      full
                                            4.522
                                                    3.632
                                                            5.480
pclass2nd
             -0.91 -1.453
                           -0.364
                                      full
                                           -1.495
                                                   -2.059
                                                          -0.952
pclass3rd
             -2.10
                   -2.679
                           -1.526
                                      full
                                           -2.841
                                                  -3.527 - 2.196
                                                   -3.575
sexmale
             -1.73 - 2.184
                            -1.273
                                           -3.087
                                                           -2628
             -0.04 - 0.057
                           -0.023
                                           -0.049 -0.067 -0.033
age
                                      full
```

```
2.5 % 97.5 %
             LDbeta
(Intercept)
              5.900
                      4.218
                             7.866
pclass2nd
             -2.111
                     -3.192
                            -1.128
pclass3rd
             -4.478
                    -6.114 - 3.101
sexmale
             -3.694 - 4.879
                            -2.707
             -0.066 -0.099 -0.036
age
```

Multiple Imputation via Chained Equations

Stef van Buuren, Karin Groothuis-Oudshoorn (2011).

MICE: Multivariate Imputation by Chained Equations in
R. Journal of Statistical Software, 45(3): 1-67.

Stef van Buuren (2012). Flexible Imputation of Missing Data. Boca Raton, FL: Chapman & Hall/CRC Press.

Rough Sketch

- Strategy quite different from Amelia and other MVN based theories
- MICE: separately process each column, predicting it from all the others
 - "The algorithm imputes an incomplete column (the target column) by generating 'plausible' synthetic values given other columns in the data."
- Cycle through columns over and over, until model converges (in MCMC sense), then draw samples to impute.

Recommends "predictive mean matching" to select imputed values

- When filling in missings, find cases with similar predicted values to the case in question
- From among those cases, collect their list of actual observed scores
- Draw imputations from that subset of actual scores
- "Automatically" solves the problem that imputations might have impossible values
 - Imputations for categorical variables always match the original scale (sex is always 0 or 1, never 0.64)
 - When a variable is badly skewed, the PMM always selects a realistic value.

The mice Interface

```
mice(data, m = 5, method = vector("character",
    length=ncol(data)), predictorMatrix = (1 -
    diag(1, ncol(data))), visitSequence = (1:ncol
    (data))[apply(is.na(data), 2, any)], post =
    vector("character", length = ncol(data)),
    defaultMethod = c("pmm", "logreg", "polyreg",
        "polr"), maxit = 5, diagnostics = TRUE,
    printFlag = TRUE, seed = NA, ...
)
```

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Special mice features

- "fine grained" management of imputation algorithms for different types of data
- Defaults:

data type	default	also available
numeric	pmm (predictive mean matching)	norm, 2level
binary	logreg (logistic regression)	lda
factor	polyreg (Bayesian polytomous regression)	
factor: ordinal	polr (prop. odds logistic (MASS))	

- Possible to
 - add user-defined predictive tools
 - control the sequence of column processing

Other Handy mice Features

- complete: function can
 - return any of the individual imputed data frames
 - return all data frames combined in the "long" format (rows stacked together)
 - return all frames combined in the "wide" format (columns side-by-side)
- pool: outputs many of Rubin's suggested diagnostic formulae (param, var, R^2)
- summary(pool()): distills parameter estimates

nhanes: small test data frame supplied with mice

```
library (mice)
nhanes
```

```
age
        bmi
              hyp chl
         NA
               NA
                    NA
          23
                   187
         NA
                   187
4
      3
         NA
               NA
                    NA
          20
                   113
6
      3
         NA
               NA 184
      1
          22
                  118
8
          30
                   187
          22
                   238
10
      2
         NA
               NA
                    NA
11
         NA
               NA
                    NA
12
         NA
               NA
                    NA
      3
13
          22
                   206
14
          29
                   204
15
          30
                    NA
         NA
16
      1
               NA
                    NA
17
      3
          27
                   284
18
          26
                  199
19
          35
                   218
20
          26
                    NA
21
         NA
               NA
                    NA
```

nhanes: small test data frame supplied with mice ...

```
33
                  229
23
         28
                  131
24
         25
                   NA
25
         27
                1 186
```

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Test That Out

```
imp <- mice(nhanes, printFlag = FALSE)</pre>
fit \leftarrow with (data = imp, exp = Im(bmi \sim hyp + chl))
fitpool <- pool(fit)</pre>
fitpool
```

```
Call: pool(object = fit)
Pooled coefficients:
(Intercept)
                    hyp
                                 chl
    20.592 -1.176
                               0.037
Fraction of information about the coefficients
   missing due to nonresponse:
(Intercept)
                                chl
                    hyp
       0.41
                   0.17
                                0.25
```

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What's all that?

Inside the outpute object from pool, there is a wealth of information that previous editions of mi did report automatically. That structure includes

```
fit <- list(call = call, call1 = object$call,
   call2 = object$call1, nmis = object$nmis, m =
   m, qhat = qhat, u = u, qbar = qbar, ubar = qbar
   ubar, b = b, t = t, r = r, dfcom = dfcom, df =
                fmi = fmi, lambda = lambda)
    df.
```

qhat: matrix of m complete data fits	b: within imputation variance
r: rel. incr var due to nonresponse	t: total variance of pooled estimates
qbar: pooled estimate	u: Variance matrices from m fits $(var \times var \times m)$
ubar: mean of variances across m fits	gamma: prop. variance explained by imputations
dfcom: df in complete analysis	df: df for pooled estimates
	fmi: fraction missing information

summary of pooled fit

round(summary(pool(fit))[,c(1:4,6:9)],2)

```
d f
                                   lo 95
                                          hi 95
                                                 nmis
                                                        fmi
               est
(Intercept)
             20.59
                    5.07
                           4.06
                                     9.38 31.81
                                                   NA 0.41
             -1.18 2.21
                         -0.53 18 -5.81
                                           3.46
hyp
                                                     8 0.17
chl
              0.04 0.02
                          1.50 \ 15 \ -0.02
                                           0.09
                                                   10 0.25
```

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It Gracefully Handles Factor Variables

```
Call: pool(object = fit2)

Pooled coefficients:
(Intercept) hyp chl
23.613 -1.486 0.027

Fraction of information about the coefficients missing due to
nonresponse:
(Intercept) hyp chl
0.42 0.25 0.34
```

Compare "real" and 2 imputed sets

cbind("F", nhanes, "imp1", complete(imp2,3), "imp2", complete(imp,3))

	"F"	age	bmi	hyp	chl	"imp1"	age	bmi	hyp	chl	"imp2"	age	bmi	hyp	chl
1	F	1	NA	ŇA	NA	imp1	1	35	1	187	imp2	1	30	1	199
2	F	2	23	1	187	imp1	2	23	1	187	imp2	2	23	1	187
3	F	1	NA	1	187	imp1	1	30	1	187	imp2	1	29	1	187
4	F	3	NA	NA	NA	imp1	3	25	1	284	imp2	3	26	1	206
5	F	1	20	1	113	imp1	1	20	1	113	imp2	1	20	1	113
6	F	3	NA	NA	184	imp1	3	20	2	184	imp2	3	26	2	184
7	F	1	22	1	118	imp1	1	22	1	118	imp2	1	22	1	118
8	F	1	30	1	187	imp1	1	30	1	187	imp2	1	30	1	187
9	F	2	22	1	238	imp1	2	22	1	238	imp2	2	22	1	238
10	F	2	NA	NA	NA	imp1	2	27	1	229	imp2	2	28	2	204
11	F	1	NA	NA	NA	imp1	1	30	1	131	imp2	1	33	1	206
12	F	2	NA	NA	NA	imp1	2	30	1	206	imp2	2	22	1	238
13	F	3	22	1	206	imp1	3	22	1	206	imp2	3	22	1	206
14	F	2	29	2	204	imp1	2	29	2	204	imp2	2	29	2	204
15	F	1	30	1	NA	imp1	1	30	1	238	imp2	1	30	1	199
16	F	1	NA	NA	NA	imp1	1	30	1	238	imp2	1	30	1	187
17	F	3	27	2	284	imp1	3	27	2	284	imp2	3	27	2	284
18	F	2	26	2	199	imp1	2	26	2	199	imp2	2	26	2	199
19	F	1	35	1	218	imp1	1	35	1	218	imp2	1	35	1	218
20	F	3	26	2	NA	imp1	3	26	2	218	imp2	3	26	2	206
21	F	1	NA	NA	NA	imp1	1	28	1	238	imp2	1	22	1	118
22	F	1	33	1	229	imp1	1	33	1	229	imp2	1	33	1	229

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Compare "real" and 2 imputed sets ...

23	F	1	28	1 131	imp1	1	28	1 131	imp2	1	28	1 131
24	F	3	25	1 NA	imp1	3	25	1 284	imp2	3	25	1 206
25	F	2	27	1 186	imp1	2	27	1 186	imp2	2	27	1 186

How About the Titanic Data?

```
load("/home/pauljohn/SVN/SVN-guides/Rcourse/DataSets/titanic0.rda")
set.seed(234234)
titanic <- titanic0
titanic$sex[sample(nrow(titanic), 400)] <- NA
titanic$sex[sample(nrow(titanic), 400)] <- NA
titanic$pclass[sample(nrow(titanic), 400)] <- NA
miceTitanic <- mice( subset( titanic, select = c('survived', 'pclass', 'sex', 'age', 'embarked')), m = 10, maxit = 10,
    printFlag=FALSE)
miceFitTitanic <- with(data = miceTitanic, exp = glm(survived ~
    pclass + sex + age, family = binomial))
pool(miceFitTitanic)</pre>
```

```
Call: pool(object = miceFitTitanic)
Pooled coefficients:
(Intercept) pclass2 pclass3 sex2
                                                  age
            -1.24
                       -2.81
                                     -2.31
      3.97
                                                -0.05
Fraction of information about the coefficients missing due to
    nonresponse:
(Intercept) pclass2
                         pclass3
                                    sex2
                                                  age
                0.75
                           0.77
                                      0.81
      0.92
                                                 0.93
```

Here's the error you see if you forget to subset the variables with select

How About the Titanic Data? ...

```
Error: chunk 7 (label=mice70) 
Error in nnet.default(X, Y, w, mask = mask, size = 0, skip = TRUE, softmax = TRUE, too many (3210) weights Execution halted
```

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summary of pooled fit

round(summary(pool(miceFitTitanic)), 2)

	est	se	t	d f	Pr(> t)	lo 95	hi 95	nmis	fmi	
		lamb	da							
(Intercept)	3.97	1.03	3.9	10.1	0.00	1.7	6.3	NA	0.92	0
.90										
pclass2	-1.24	0.40	-3.1	16.7	0.01	-2.1	-0.4	NA	0.75	0
.72										
pclass3	-2.81	0.41	-6.9	15.8	0.00	-3.7	-1.9	NA	0.77	0
.74										
sex2	-2.31	0.35	-6.5	13.8	0.00	-3.1	-1.6	NA	0.81	0
.79										
age	-0.05	0.02	-2.5	9.8	0.03	-0.1	0.0	680	0.93	0
.92										

Descriptive 42/96 K.U.

The mi Package (Gelman's Columbia U Team)

Yu-Sung Su, Andrew Gelman, Jennifer Hill, Masanao Yajima. 2011. "Multiple Imputation with Diagnostics (mi) in R: Opening Windows into the Black Box". *Journal of Statistical Software*. 45(2) Kobi Abayomi, Andrew Gelman and Marc Levy. (2008). "Diagnostics for multivariate imputations". *Applied Statistics* 57, Part 3: 273-291.

"Generate a multiply imputed matrix applying the elementary functions iteratively to the variables with missingness in the data randomly imputing each variable and looping through until approximate convergence."

Rough Sketch

- Strategy similar to mice and aregImpute: proceed one-variable-at-a-time
- Predict each variable from each of the others
 - Start with median/mode for NAs
 - Conduct "n.iter" iterations, or until convergence
 - Provides a large set of mi.XXX functions to impute variables of different types
- Draw bootstrap sample to create imputed values
- Allows limited "preprocessing" of numeric variables (similar idea to aregImpute's use of avas)
- As "Opening Windows into the Black Box" implies, this is intended to be less "mysterious," more "informative", and easier to diagnose MI processes.



Type-Dependent Imputation Methods

type	mi name	model						
binary	mi.binary	logistic						
unordered	mi.categorical	multinomial						
ordinal	mi.polr	continuation logistic						
continuous	mi.continous	regression						
count	mi.count	Bayesian Poisson (w overdispersion)						

Descriptive 45/96 K.U.

Interface

Descriptive 46/96 K.U.

Steps to Use mi

- Create an "information table"
- Use mi to create imputations
 - Runs 1 separate "chain" for each desired imputation
- pooling methods implemented for common R estimators like "Im.mi","glm.mi","Imer.mi"
 - These cycle through all imputed data frames
 - · create estimates for each
- display or other extractor methods can present results

Information Table for nhanes data with factor variables in it

```
inf <- mi.info(nhanes2)
inf</pre>
```

```
names include order number.mis all.mis
                        type collinear
            Yes
                    NA
                                         No
    age
   unordered—categorical
                                   No
           Yes
    bmi
                                         Nο
   positive-continuous
                                 Nο
3
    hyp
          Yes
                                         Nο
                    binary
                                   No
4
            Yes
    chl
                                10
                                         Nο
   positive - continuous
                                 No
```

Can customize variable types at this stage



Descriptive 48/96 K.U.

Run mi

Caution: 2013-05-21 runtime errors were observed with n.imp = 10, only 8 will succeed.

```
\label{eq:milmputeNhanes2} \begin{array}{ll} \mbox{milmputeNhanes2} < - \mbox{mi(nhanes2} \; , \; \mbox{info} \; = \; \mbox{info} \; = \; \mbox{n.imp} \; = \; \mbox{8} \; , \; \; \mbox{n.iter} \; = \; \\ \mbox{500} \; , \; \mbox{add.noise} \; = \; \mbox{FALSE}) \end{array}
```

Descriptive 49/96 K.U.

```
M1 < - \text{Im.mi(bmi} \sim \text{hyp} + \text{chl}, \text{ milmputeNhanes2)} display(M1)
```

```
Separate Estimates for each Imputation
** Chain 1 **
lm(formula = formula, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 21.17 4.42
hypyes 0.09 2.64
chl
          0.03
                    0.02
n = 25, k = 3
residual sd = 4.72, R-Squared = 0.07
** Chain 2 **
lm(formula = formula, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 18.71 3.84
hypyes -1.16 3.15
chl
          0.05
                    0.02
n = 25. k = 3
```

 Descriptive
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 K.U.

```
residual sd = 5.18. R-Squared = 0.24
** Chain 3 **
lm(formula = formula, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 21.79 4.41
hypyes -2.43 2.54
chl
         0.03 0.02
n = 25. k = 3
residual sd = 4.79, R-Squared = 0.08
** Chain 4 **
lm(formula = formula, data = mi.data[[i]])
           coef est coef se
(Intercept) 19.37 3.88
hypyes -0.05 2.41
chl
          0.03
                     0.02
n = 25. k = 3
residual sd = 4.26, R-Squared = 0.12
** Chain 5 **
Im(formula = formula, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 23.78
                     4 09
```

Descriptive 51/96 K.U.

```
hypyes
         -0.70
                     2.78
chl
           0.02
                     0.02
n = 25. k = 3
residual sd = 4.40, R-Squared = 0.02
** Chain 6 **
Im(formula = formula, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 16.29 4.66
hypyes -4.31 2.41
chl
          0.06
                     0.02
n = 25, k = 3
residual sd = 4.96, R-Squared = 0.24
** Chain 7 **
lm(formula = formula, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 26.90 3.74
      1.38 2.05
hypyes
chl
          0.00
                     0.02
n = 25, k = 3
residual sd = 4.09, R-Squared = 0.02
```

Descriptive 52/96 K.U.

```
** Chain 8 **
lm(formula = formula, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 28.36
                     4.70
hypyes
       -0.10 3.78
chl
         -0.01
                     0.03
n = 25, k = 3
residual sd = 6.24, R-Squared = 0.00
Pooled Estimates
lm.mi(formula = bmi \sim hyp + chl, mi.object = milmputeNhanes2)
           coef.est coef.se
                     6.08
(Intercept) 22.04
hypyes -0.91 3.33
chl
           0.02
                     0.03
```

```
cbind(b=coef(M1), se=se.coef(M1), t=coef(M1)/se.coef(M1))
```

Descriptive 53 / 96

```
b se t
(Intercept) 22.045 6.082 3.62
hypyes -0.910 3.334 -0.27
chl 0.025 0.033 0.76
```



Descriptive 54/96 K.U.

titanic\$survived[sample(nrow(titanic), 400)] <- NA

```
\label{load} $$ [ oad("/home/pauljohn/SVN/SVN-guides/Rcourse/DataSets/titanic0.rda") $$ set.seed(234234) $$ titanic <- titanic0 $$ titanic0 (titanic), 400) $$ [ sample(nrow(titanic), 400) $$ [ <- NA titanic$pclass[sample(nrow(titanic), 400) $$] <- NA $$ [ NA titanic$pclass[sample(nrow(titanic), 400) $] <- NA $$ [ NA titanic$pclass[
```

Grab Subset, then compute min.info (prepare for imputation)

Subset required to avoid use of extraneous variables by imputer. Can customize inf to change variable types, if desired.

Note, mi did not converge with "embarked" included as predictor, so it was omitted here

```
names include order number mis all mis
                                                                     type
           collinear
  survived
                 Yes
                                    400
                                              No
                                                                   binary
            No
                 Yes
                                    400
    pclass
                                              No unordered—categorical
            No
3
                 Yes
                                    400
                                              No
                                                                   binary
        sex
            No
4
        age
                 Yes
                          4
                                    680
                                              No
                                                    positive - continuous
            No
```

Descriptive 56/96 K.U.

Create 10 Titanics

```
\begin{array}{ll} \mbox{milmpTitanic} < - \mbox{ mi(ss, info} = \mbox{inf, n.imp} = 10, & \mbox{n.iter} = 400, \\ \mbox{add.noise} = \mbox{FALSE}) \end{array}
```

n.iter set higher, convergence can take more than 100 iterations



Descriptive 57/96 K.U.

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Use "glm.mi" from mi on the List of Imputed Datasets

```
M2 <- glm.mi( survived \sim pclass + sex + age, milmpTitanic, family = binomial(link = "logit")) display(M2)
```

```
Separate Estimates for each Imputation
** Chain 1 **
glm(formula = formula, family = family, data = mi.data[[i]])
           coef est coef se
(Intercept) 3.70
                     0.32
pclass2nd -0.64
                     0.21
pclass3rd -4.12 0.24
sexmale -1.61
                    0.17
age
           -0.04
                     0.01
 n = 1313, k = 5
  residual deviance = 1053.3, null deviance = 1740.9 (difference =
      687.6)
** Chain 2 **
glm(formula = formula, family = family, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 4.73
                     0.37
```

Descriptive 58/96 K.U.

```
pclass2nd
          -1.34
                     0.23
pclass3rd
          -4.03
                     0.27
sexmale
           -2.93
                     0.20
           -0.07
                     0.01
age
 n = 1313. k = 5
  residual deviance = 929.3, null deviance = 1559.7 (difference =
      630.4)
** Chain 3 **
glm(formula = formula, family = family, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 4.85
                     0.35
pclass2nd -1.33
                     0.22
pclass3rd -2.86 0.23
sexmale -2.62
                     0.17
age
           -0.09
                     0.01
 n = 1313, k = 5
  residual deviance = 1067.9, null deviance = 1628.9 (difference =
      561.0)
** Chain 4 **
glm(formula = formula, family = family, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 5.30
                     0.38
```

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Descriptive 59/96 K.U.

```
pclass2nd
          -1.67
                     0.24
pclass3rd
          -3.39
                     0.26
sexmale
         -3.56
                     0.20
           -0.06
                     0.01
age
 n = 1313. k = 5
  residual deviance = 967.6, null deviance = 1694.5 (difference =
      726.9)
** Chain 5 **
glm(formula = formula, family = family, data = mi.data[[i]])
           coef est coef se
(Intercept) 5.52
                     0.39
nclass2nd -2.12
                     0.23
pclass3rd -4.84 0.28
sexmale -1.73
                     0.17
age
          -0.08
                     0.01
 n = 1313, k = 5
  residual deviance = 1027.7, null deviance = 1698.3 (difference =
      670.6)
** Chain 6 **
glm(formula = formula, family = family, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 5.43
                     0 39
```

Descriptive 60 / 96 K.U.

```
pclass2nd
          -1.40
                     0.24
pclass3rd
          -3.46
                     0.26
sexmale
         -3.74
                     0.21
           -0.07
                     0.01
age
 n = 1313. k = 5
  residual deviance = 937.2, null deviance = 1699.5 (difference =
      762.3)
** Chain 7 **
glm(formula = formula, family = family, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 4.55
                     0.34
pclass2nd -1.58
                     0.21
pclass3rd -4.32 0.26
sexmale -1.74
                     0.16
age
          -0.06
                     0.01
 n = 1313, k = 5
  residual deviance = 1080.9, null deviance = 1678.8 (difference =
      597.9)
** Chain 8 **
glm(formula = formula, family = family, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 4.30
                     0.32
```

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Descriptive 61/96 K.U.

```
pclass2nd
          -0.88
                     0.21
pclass3rd
          -3.51
                     0.22
sexmale
         -2.30
                     0.17
           -0.04
                     0.01
age
 n = 1313. k = 5
  residual deviance = 1185.1, null deviance = 1796.8 (difference =
      611.8)
** Chain 9 **
glm(formula = formula, family = family, data = mi.data[[i]])
           coef est coef se
(Intercept) 5.00
                     0.38
nclass2nd -1.54
                     0.25
pclass3rd -3.26 0.26
sexmale -3.90 0.21
age
          -0.05
                     0.01
 n = 1313, k = 5
  residual deviance = 918.0, null deviance = 1691.9 (difference =
      773.9)
** Chain 10 **
glm(formula = formula, family = family, data = mi.data[[i]])
           coef.est coef.se
(Intercept) 4.74
                     0.35
```

Descriptive 62 / 96 K.U.

```
pclass2nd
           -1.48
                      0.22
pclass3rd
            -4.09
                      0.25
sexmale
            -1.48
                      0.16
age
            -0.08
                      0.01
 n = 1313, k = 5
  residual deviance = 1083.3, null deviance = 1654.8 (difference =
      571.5)
Pooled Estimates
glm.mi(formula = survived \sim pclass + sex + age, mi.object =
    milmpTitanic,
    family = binomial(link = "logit"))
            coef.est coef.se
(Intercept) 4.81
                      0.68
pclass2nd
                      0.48
         -1.40
pclass3rd -3.79
                      0.67
sexmale
          -2.56 1.00
age
            -0.06
                      0.02
```

Descriptive 63/96 K.U.

```
\label{eq:mititanicResult} \begin{tabular}{ll} $(miTitanicResult <- cbind(b = coef(M2), se = se.coef(M2), t = coef(M2)/se.coef(M2))) \end{tabular}
```

```
b se t
(Intercept) 4.812 0.681 7.1
pclass2nd -1.399 0.483 -2.9
pclass3rd -3.789 0.668 -5.7
sexmale -2.560 0.996 -2.6
age -0.064 0.019 -3.3
```

Descriptive 64/96 K.U.

The Hmisc & rms Packages

Harrell, Frank E. 2010. Regression Modeling Strategies: With Applications to Linear Models, Logistic Regression, and Survival Analysis. Springer.

Frank E Harrell Jr (2013). rms: Regression Modeling Strategies. R package version 3.6-3.

'aregImpute' takes all aspects of uncertainty in the imputations into account by using the bootstrap to approximate the process of drawing predicted values from a full Bayesian predictive distribution. Different bootstrap resamples are used for each of the multiple imputations, i.e., for the 'i'th imputation of a sometimes missing variable, 'i=1,2,... n.impute', a flexible additive model is fitted on a sample with replacement from the original data and this model is used to predict all of the original missing and non-missing values for the target variable."

Rough Sketch

Predict each variable from each of the others

Start with random selections for NAs

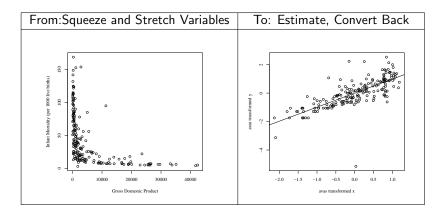
Do this "burnin"+"n.impute" times

- Draw bootstrap sample, fit a "flexible" model to it, predict outcomes for all cases from that model.
- Default uses predictive mean matching to select an imputed value for each NA. (regression extrapolation is alternative)

Note: Special emphasis on nonlinearity in imputation fitting (similar to avas)

areglmpute

What's Avas?



Interface

```
aregImpute(formula, data, subset, n.impute = 5,
   group = NULL, nk = 3, tlinear = TRUE, type = c(
   'pmm', 'regression'), match = c('weighted', '
   closest'), fweighted = 0.2, curtail = TRUE,
   boot.method = c('simple', 'approximate bayesian)
   '), burnin = 3, x = FALSE, pr = TRUE, plotTrans
    = FALSE, tolerance = NULL, B = 75)
```

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Important Detail about "rms"

- Prof. Harrell is a long-standing, highly distinguished programmer and statistician (SAS PROC Logistic in mid 1980s)
- He has developed his own set of fitting functions in S over 2 decades and they are not perfectly interchangeable with R functions
 - rms "ols" is not exactly same as R's "lm"
 - rms "Irm" is not exactly same as R's glm(y~x, family=binomial(link="logit"))
 - aregImpute and other rms functions are tailored for rms "fitting" routines", but tolerate some R routines (with warnings).
- Summary and Plotting functions for "rms" objects are usually expecting different options than functions in base R

Syntax Example

- [1] "f \leftarrow aregimpute (\sim age + bmi + hyp + chl, data= nhanesf, nk=0)"
 - Can't actually run that, though, because of this error (which I have not solved)
- Error in aregImpute (\sim age + bmi + hyp + chl, data = nhanesf, nk = 0): a bootstrap resample had too few unique values of the following variables: age Execution halted
- Warning in aregImpute (\sim age + bmi + hyp + chl, data = nhanesf, nk = 1): hyp has the following levels with < 5 observations: yes Consider using the group parameter to balance bootstrap samples

↓□▶ ↓□▶ ↓□▶ ↓□▶ □ ♥Q♠

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If that did work, we would run fit.mult.impute

```
fmi <- fit.mult.impute(bmi \sim hyp + chl, lm, f, data = nhanesf)
```

```
sqrt(diag(vcov(fmi)))
```

 Even if aregImpute did create imputations, it would be accompanied by this warning

```
Warning in fit.mult.impute(bmi ~ hyp + chl, lm, f, data = nhanesf): Not using a Design fitting function; summary(fit) will use standard errors, t, P from last imputation only. Use vcov(fit) to get the correct covariance matrix, sqrt(diag(vcov(fit))) to get s.e.
```

Caused by my use of R's "Im", rather than rms's own function "ols"

4□▶ 4□▶ 4□▶ 4□▶ 4□▶ 4□ = 900

Descriptive 71/96 K.U.

The Titanic Rises Again

```
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
Iteration 10
Iteration 11
Iteration 12
Iteration 13
```

Descriptive 72/96 K.U.

Use glm in fit.mult.impute

 $\begin{array}{lll} {\sf rmsFmiTitanic} < & {\sf fit.mult.impute(survived \sim pclass + sex + age, \\ {\sf glm, family=binomial(link=logit), rmsImputeTitanic, data=titanic, fit.reps=TRUE)} \end{array}$

```
Variance Inflation Factors Due to Imputation:
(Intercept) pclass2nd
                        pclass3rd sexmale
                                                     age
                              4.5
                                         1.9
                                                     2.5
Rate of Missing Information:
(Intercept) pclass2nd
                        pclass3rd sexmale
                                                    age
      0.65
                 0.53
                             0.78
                                         0 48
                                                    0 59
d.f. for t-distribution for Tests of Single Coefficients:
                        pclass3rd sexmale
(Intercept) pclass2nd
                                                     age
                               15
                                           39
                                                      26
The following fit components were averaged over the 10 model fits:
  fitted.values linear.predictors
```

summary (rmsFmiTitanic)



Descriptive 73/96 K.U.

Use glm in fit.mult.impute ...

```
Call:
fitter (formula = formula, family = ..1, data = completed.data)
Deviance Residuals:
   Min
           10 Median
                           3Q
                                  Max
-2.143 \quad -0.735 \quad -0.386 \quad 0.680
                                2.901
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
                       0.30409 13.96 < 2e-16 ***
(Intercept) 4.24518
pclass2nd -1.37042
                       0.20975 -6.53 -6.4e -11 ***
pclass3rd -2.85903
                       0.19864
                                -14.39 < 2e-16 ***
         -2.37998
sexmale
                       0.15689 - 15.17 < 2e-16 ***
                       0.00569 - 9.71 < 2e-16 ***
age
           -0.05526
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 1680.1 on 1312 degrees of freedom
Residual deviance: 1148 6
                         on 1308
                                   degrees of freedom
AIC: 1159
Number of Fisher Scoring iterations: 5
```

Recall the Cautionary Warning about Fitting Functions?

- fit.mult.impute warns the user that when a fitting routine is not from rms, then the standard errors and significance tests are based only on the last fitted model
- One should instead extract the standard errors from the covariance matrix
- Which I do on the next slide

Create a Homemade Summary for the Fit.Mult.Impute Output

sqrt (diag (vcov (rmsFmiTitanic)))

```
(Intercept)
               pclass2nd
                             pclass3rd
                                             sexmale
                                                               age
     0.3041
                  0.2097
                                0.1986
                                             0.1569
                                                           0.0057
```

(rmsTitanicResult <- cbind(b=rmsFmiTitanic\$coefficients, se=diag(vcov(rmsFmiTitanic))))

```
se
(Intercept) 4.245
                    9.2e - 02
pclass2nd
             -1.370 4.4e-02
pclass3rd
             -2.859 3.9e-02
sexmale
             -2.380 2.5e-02
             -0.055 3 2e - 05
age
```

mitools

Here's a Problem: I don't believe the se result. Compare

```
require (mitools)
rmsbetas <- Mlextract(rmsFmiTitanic$fits, fun=coef)</pre>
rmsvars <- Mlextract(rmsFmiTitanic$fits, fun=vcov)</pre>
rmsTitanicMItools <- summary(MIcombine(rmsbetas.rmsvars))
```

```
Multiple imputation results:
      MIcombine.default (rmsbetas, rmsvars)
                        se (lower upper) missInfo
            results
(Intercept)
            4.245 0.5458
                           3.112
                                    5.378
                                              68 %
pclass2nd
             -1.370 0.3132 -2.008 -0.733
                                              55 %
pclass3rd
             -2.859 0.4361 -3.789 -1.929
                                              80 %
sexmale
             -2.380 0.2273 -2.840 -1.920
                                              51 %
             -0.055 0.0093 -0.074 -0.036
                                              62 %
age
```

for(i in 1:length(rmsFmiTitanic\$fits)) print(summary(rmsFmiTitanic\$ fits [[i]]))

```
Call:
fitter(formula = formula, family = ..1, data = completed.data)
Deviance Residuals:
  Min
           10 Median
                          3Q
                                 Max
-2.318 \quad -0.722 \quad -0.375 \quad 0.624
                               3.065
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
(Intercept) 4.27977
                       0.33643 12.72 < 2e-16 ***
pclass2nd -1.30687
                       0.21447 -6.09 1.1e-09 ***
pclass3rd -2.77282
                       0.20172
                                -13.75 < 2e-16 ***
         -2.34844
                                -14.46 < 2e-16 ***
sexmale
                       0.16246
                       0.00612 -9.38 < 2e-16 ***
           -0.05740
age
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 1680.1 on 1312 degrees of freedom
```

4 D > 4 A > 4 B > 4 B > Descriptive 78 / 96

```
Residual deviance: 1148.6 on 1308 degrees of freedom
AIC: 1159
Number of Fisher Scoring iterations: 5
Call:
fitter (formula = formula, family = ..1, data = completed.data)
Deviance Residuals:
   Min
           10 Median
                          3Q
                                 Max
-2488 - 0657 - 0350
                       0.629
                               2 976
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
(Intercept) 4.70439
                       0.32223 14.60 < 2e-16 ***
                       0.21563 -7.87 3.5e - 15 ***
pclass2nd -1.69747
pclass3rd -2.86445
                       0.20043
                               -14.29 < 2e-16 ***
         -2.11856
sexmale
                       0.16525
                               -12.82 < 2e-16 ***
age
           -0.06367
                       0.00564
                               -11.29 < 2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
```

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```
Null deviance: 1698.3 on 1312 degrees of freedom
Residual deviance: 1103.0 on 1308
                                   degrees of freedom
AIC · 1113
Number of Fisher Scoring iterations: 5
Call:
fitter (formula = formula, family = ..1, data = completed.data)
Deviance Residuals:
  Min
           10 Median
                           30
                                  Max
-2522 - 0584 - 0326
                       0 549
                                2 669
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
(Intercept) 5.15318
                       0.36868 13.98 < 2e-16 ***
pclass2nd -1.68279
                       0.23263 \quad -7.23 \quad 4.7e - 13 ***
                                -15.64 < 2e-16 ***
pclass3rd -3.72679
                       0.23826
sexmale -2.52675
                       0.17290
                                -14.61 < 2e-16 ***
           -0.06950
                       0.00684
                                -10.16 < 2e-16 ***
age
                       0.001 '** 0.01 '* 0.05 '. 0.1 ' 1
Signif. codes:
```

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```
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 1691.9 on 1312 degrees of freedom
Residual deviance: 1050.4 on 1308
                                    degrees of freedom
AIC · 1060
Number of Fisher Scoring iterations: 5
Call:
fitter(formula = formula, family = ..1, data = completed.data)
Deviance Residuals:
   Min
           10 Median
                        3Q
                                  Max
-2.573 \quad -0.648 \quad -0.351 \quad 0.585
                                2.765
Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) 4.26266
                       0.31396 13.58 < 2e-16 ***
pclass2nd -1.43351
                       0.21260 -6.74 \cdot 1.6e - 11 ***
pclass3rd -3.00432
                       0.21207
                                -14.17 < 2e-16 ***
sexmale -2.47417 0.16366 -15.12 < 2e-16 ***
            -0.05497
                       0.00572
                                -9.61 < 2e-16 ***
age
```

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 Descriptive
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```
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 1689.4 on 1312 degrees of freedom
Residual deviance: 1108.5 on 1308 degrees of freedom
AIC: 1119
Number of Fisher Scoring iterations: 5
Call:
fitter(formula = formula, family = ..1, data = completed.data)
Deviance Residuals:
           10 Median
                        3Q
                                  Max
-2.523 \quad -0.672 \quad -0.375 \quad 0.578
                                2.588
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
(Intercept) 4.08354
                       0.31901 12.80 <2e-16 ***
pclass2nd -1.46783 0.21463 -6.84 8e-12 ***
pclass3rd -2.91258
                       0.20744
                                -14.04
                                        <2e-16 ***
```

```
sexmale
           -2.50116
                       0.16286 -15.36 <2e-16 ***
            -0.04957
                       0.00576 -8.61 <2e-16 ***
age
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 1678.8 on 1312 degrees of freedom
Residual deviance: 1119.7 on 1308 degrees of freedom
AIC: 1130
Number of Fisher Scoring iterations: 5
Call:
fitter (formula = formula, family = ..1, data = completed.data)
Deviance Residuals:
   Min
           1Q Median
                        3Q
                                 Max
-2.637 \quad -0.750 \quad -0.404 \quad 0.616
                                2 819
Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) 4.16399 0.33749
                                12.34 < 2e-16 ***
```

```
pclass2nd
          -1.38169
                       0.21875
                               -6.32 2.7e-10 ***
pclass3rd
        -2.82804
                       0.20494
                               -13.80 < 2e-16 ***
         -2.30791
                               -14.40 < 2e-16 ***
sexmale
                       0.16029
age
           -0.05521
                       0.00624 -8.85 < 2e-16 ***
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 1686.8 on 1312 degrees of freedom
Residual deviance: 1172.5 on 1308 degrees of freedom
AIC · 1182
Number of Fisher Scoring iterations: 5
Call:
fitter(formula = formula, family = ..1, data = completed.data)
Deviance Residuals:
  Min
           1Q Median
                       30
                                  Max
-2.810 \quad -0.747 \quad -0.186 \quad 0.629
                                2 685
Coefficients:
```

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```
Estimate Std. Error z value Pr(>|z|)
(Intercept) 4.0348
                      0.2995 13.47 < 2e-16 ***
                      0.2085 -6.01 1.8e-09 ***
pclass2nd -1.2541
pclass3rd -2.4904 0.1952 -12.76 < 2e-16 ***
sexmale -2.2320 0.1633
                               -13.66 < 2e-16 ***
            -0.0525 0.0056 -9.37 < 2e-16 ***
age
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 1694.5 on 1312 degrees of freedom
Residual deviance: 1100.1 on 1308 degrees of freedom
AIC · 1110
Number of Fisher Scoring iterations: 5
Call:
fitter (formula = formula, family = ..1, data = completed.data)
Deviance Residuals:
  Min
        10 Median
                          3Q
                                 Max
-2.497 \quad -0.678 \quad -0.323
                      0.529
                               2.812
```

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Notice the "fit.reps=T" option? It allows inspection of each fitted model ...

```
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
                      0.34465 12.30 < 2e-16 ***
(Intercept) 4.23938
nclass2nd -1.28286
                      0.22106 -5.80 6.5e-09 ***
pclass3rd -2.95459
                      0.21243
                               -13.91 < 2e-16 ***
sexmale
       -2.62029
                      0.16949
                               -15.46 < 2e-16 ***
          -0.05305
                      0.00634 - 8.36 < 2e-16 ***
age
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 1669.2 on 1312 degrees of freedom
Residual deviance: 1075.9 on 1308
                                  degrees of freedom
AIC: 1086
Number of Fisher Scoring iterations: 5
Call:
fitter(formula = formula, family = ..1, data = completed.data)
Deviance Residuals:
```

```
Min
           10 Median
                           3Q
                                  Max
-2.417
       -0.693 \quad -0.330
                        0.607
                                2.893
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
(Intercept) 3.75758
                       0.29209 12.86 < 2e-16 ***
pclass2nd -0.97306
                       0.20516 -4.74 2.1e-06 ***
pclass3rd -2.36932
                       0.19073
                               -12.42 < 2e-16 ***
         -2.34050
sexmale
                       0.15930
                               -14.69 < 2e-16 ***
           -0.04948
                       0.00538 -9.19 < 2e-16 ***
age
Signif. codes: 0 '*** 0.001 '** 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 1680.1 on 1312 degrees of freedom
Residual deviance: 1133.4 on 1308
                                   degrees of freedom
AIC: 1143
Number of Fisher Scoring iterations: 5
Call:
fitter (formula = formula, family = ..1, data = completed.data)
```

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```
Deviance Residuals:
  Min
           10 Median
                          3Q
                                  Max
-2.458 \quad -0.696 \quad -0.376 \quad 0.618
                                2.838
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
(Intercept) 3.77253
                       0.30409 12.41 < 2e-16 ***
pclass2nd -1.22397
                       0.20975 -5.84 5.4e -09 ***
pclass3rd -2.66704
                       0.19864
                                -13.43 < 2e-16 ***
sexmale
         -2.33004
                       0.15689
                                -14.85 < 2e-16 ***
           -0.04719
                       0.00569 - 8.30 < 2e-16 ***
age
Signif. codes:
               0 '*** 0.001 '** 0.01 '* 0.05 '. ' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 1680.1 on 1312 degrees of freedom
Residual deviance: 1148.6 on 1308
                                   degrees of freedom
AIC: 1159
Number of Fisher Scoring iterations: 5
```

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Using "Irm" from rms package (instead of glm from R)

 $\begin{array}{lll} \text{fmi2} & < - \text{ fit.mult.impute (survived } \sim \text{ pclass } + \text{ sex } + \text{ age} \,, \, \, \text{lrm} \,, \\ \text{rmsImputeTitanic , data= titanic , fit.reps=TRUE)} \end{array}$

```
Variance Inflation Factors Due to Imputation:
Intercept pclass=2nd pclass=3rd sex=male
                                                  age
                 2.1
                            4.5
                                                  2.5
      2.8
                                      1.9
Rate of Missing Information:
 Intercept pclass=2nd pclass=3rd sex=male
                                                  age
     0.65
                0.53 0.78
                                  0.48
                                                 0.59
d.f. for t-distribution for Tests of Single Coefficients:
 Intercept pclass=2nd pclass=3rd sex=male
                                                  age
                  32
                             15
                                        39
                                                   26
The following fit components were averaged over the 10 model fits:
 stats linear.predictors
```

fmi2

Using "Irm" from rms package (instead of glm from R) ...

Logistic R	egression	n Model										
fitter(formula = formula, data = completed.data)												
		Model Likelihood Discrim.			Discrin	nination	Rank					
			Ratio Test Indexes			lexes						
Obs	1313 0.862	LR c		568.84	R2	0.486	С					
0	869 .724	d . f .		4	g	2.123	Dxy					
1 0.7	444 26	Pr(>	chi2)	<0.0001	gr	8.441	gamma					
max deriv 0.32		gp	0.328	tau—a								
					Brier	0.134						
1				Pr(> Z)								
Intercept pclass=2nd												
pclass=3rd												
sex=male age				<0.0001 <0.0001								

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Using "Irm" from rms package (instead of glm from R) ...

 Please note: the standard errors in the output based on Irm match the std.errors estimated by MItools. Thus I conclude sqrt(diag(cov(fit.mult.impute.object) did not give correct results.

First Try at summary fails:

summary (fmi2)

Error in summary.rms(fmi2): adjustment values not defined here or with datadist for pclass sex age Execution halted Error: Cannot convert file

rms fitters require a "datadist" object must be defined

```
d <- datadist(titanic)</pre>
options (datadist="d")
summary (fmi2)
```

Effec		Response : survived					
Factor	Low	High	Diff.	Effect	S.E.	Lower 0.9	5 Upper 0.95
age	21	41	20	-1.11	0.19	-1.47	-0.74
Odds Ratio	21	41	20	0.33	NA	0.23	0.48
pclass — 1st:3rd	3	1	NA	2.86	0.44	2.00	3.71
Odds Ratio	3	1	NA	17.44	NA	7.42	41.01
pclass — 2nd:3rd	3	2	NA	1.49	0.32	0.87	2.11
Odds Ratio	3	2	NA	4.43	NA	2.38	8.26
sex — female:male	2	1	NA	2.38	0.23	1.93	2.83
Odds Ratio	2	1	NA	10.80	NA	6.92	16.87

Descriptive

What Should You Do Now?

- Can't ignore "missing data problem" any more, but
- No "lead pipe cinch" solution exists at this time
- I wish there were decisive results comparing these algorithms to find out "which one is best."
- Until then, I expect researchers will use whatever tools are prevalent in their fields

Encouraging Titanic News: 4 Results are Mostly the Same

Amelia, aregImpute

mi . mice

```
cbind ("Amelia"="Amelia", df1 [ , 1:2 ], "rms"="rms", rmsTitanicMItools [,1:2])
```

```
| Amelia est std.err rms results se | (Intercept) Amelia 4.03 0.532 rms 4.245 0.5458 | pclass2nd Amelia -1.44 0.258 rms -1.370 0.3132 | pclass3rd Amelia -2.71 0.308 rms -2.859 0.4361 | sexmale Amelia -2.53 0.175 rms -2.380 0.2273 | age Amelia -0.05 0.011 rms -0.055 0.0093 |
```

```
mi b se t mice est se t [
(Intercept) "mi" "4.81" "0.68" "7.07" "mice" "3.97" "1.03" "3.87" pclass2nd "mi" "-1.4" "0.48" "-2.89" "mice" "-1.24" "0.4" "-3.12 [
pclass3rd "mi" "-3.79" "0.67" "-5.67" "mice" "-2.81" "0.41" "-6.92 [
sexmale "mi" "-2.56" "1" "-2.57" "mice" "-2.31" "0.35" "-6.55 [
age "mi" "-0.06" "0.02" "-3.31" "mice" "-0.05" "0.02" "-2.48 [
]
```

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