Forecasting Volatility for the S&P 500 Index Using GARCH Models

Report for the Computational Project

Managerial Forecasting and Decision Analysis (Fall 2023-EIND 468-588)

Max Taylor
Mechanical & Industrial Engineering Department
Montana State University
Bozeman, Montana, 59717
Email: Max.H.T@live.com

James Santarpio
Mechanical & Industrial Engineering Department
Montana State University
Bozeman, Montana, 59717
Email: Santarpioj1@gmail.com

Abstract

The purpose of this research is to forecast volatility using the best GARCH (General Autoregressive Conditional Heteroskedasticity) model to obtain the best forecasts possible. The focus of the research is based on the U.S. market index that is the S&P 500, using data from January 2010 to November 2023. The data was split into training data and testing data groups. The last 30 observations created the testing data set, so that the effectiveness of the forecast could be shown.

After investigating the data it appeared that a GARCH(1,1) model would be the most appropriate for our data. The model was able to forecast future values of S&P 500 volatility, but only to a certain extent. The model consistently overestimated the actual value of the data and for that reason undesired residuals were created that are not typical with a GARCH model.

Key words: GARCH, volatility, forecast.

Literature Review

GARCH models have been widely used for volatility forecasting since its development by Robert F. Engle in 1982. Volatility for financial time series is often conditional heteroskedastic - the variance is not only non constant over time but also conditional on the previous day's volatility. This conditional heteroskedasticity is not incorporated into other models such as ARIMA.

The key paper used in our analysis, Forecasting Volatility using Garch Models by Francisco Joao Matos Costa, involves forecasting volatility of an index using various different techniques. The paper also has an extensive literature review of the history behind volatility forecasting with GARCH models.

The paper evaluates and compares efficacy for different types of GARCH models. This is based on U.S. stock data, the NASDAQ-100. Our analysis plans to evaluate a GARCH model with a different financial time series data similar to Francisco Joao Matos Costa.

Problem Description

Increased volatility is a risk factor for all types of investors. For example, if an individual is nearing retirement, high volatility can lead to large losses at a point when this money may be necessary in the near future. Additionally it is important for companies to understand potential future volatilities in their investments. Money managers and banks need this critical information so they can plan for the future and keep their companies from sustaining large capital losses.

Being able to forecast volatility provides decreased future risk to a company. If volatility in an individual investment is likely to increase in the near future, it is of the investors interest to move this money to a less risky investment for the time being. Being able to forecast volatility is additionally important for the pricing of different assets as well as assessing credit risk for individuals/businesses. If volatility continues to increase, a company can become more likely to default.

There are some limitations to modeling volatility with GARCH. The first is that there is evidence stock prices are negatively correlated with changes in returns volatility. Volatility increases due to bad news and decreases due to good news. GARCH models do not account for this- they simply assume the magnitude of returns influences the conditional variance and not the positive or negative direction. Additionally, the non-negativity constraint of the parameters can create difficulties for the model.

Research has shown GARCH tends to outperform ARCH for forecasting volatility due to less overfitting, less variables, and not breaking the negativity constraint. We will be utilizing the GARCH model over the ARCH model in this analysis for these reasons.

Mathematical Modeling

Daily returns will be calculated using the following

$$r_t = \ln(P_t/P_{t-1})$$

Garch extends upon ARCH models. ARCH models assume returns is given by

$$r_t = \mu + \sigma_t * \varepsilon_t$$

Where ε is i.i.d. Random normally distributed error term with mean 0 and standard deviation 1. The residual term at time t, r_t - μ, can be determined by

$$a_t = \sigma_t * \varepsilon_t$$

In an arch model, volatility is calculated as

$$\sigma_t^2 = \alpha_0 + \alpha_1 * a_{t-1}^2$$

With α_0, α_1 > 1 to ensure positive variance. In this model, returns depend on all prior information up to t-1. There are lagged squared returns up to a parameter p.

In a GARCH(1,1) model, volatility is modeled as

$$\sigma_t^2 = \alpha_0 + \alpha_1 * a_{t-1}^2 + \beta_1 * \sigma_{t-1}^2$$

where $\alpha_0 > 0$, $\alpha_1 > 0$, $\beta_1 > 0$, and $\alpha_1 + \beta_1 < 1$. The GARCH model also includes lagged squared returns up to a certain parameter p and additionally includes a lagged conditional variance term up to a certain lag q.

Data

Our data contains the daily close price of the S&P 500 index from January 4, 2010, until November 28, 2023. The S&P 500 is the most popular stock index that is followed in the United States because it follows the 500 largest companies on the market. Being able to forecast the volatility of this index may provide some insight for the rest of the market. Close prices were used instead of open prices because it accounts for the market movements that happen throughout the market day and provides a more true direction of where the index is moving towards. The sample size was chosen to be around the last 13 years or so. This gave us a large sample size that encompasses all different types of market conditions that can help make the model more accurate to forecast for the future.

Within the S&P 500 index price and the S&P 500 volatility data, there are no extreme or unusual trends. It is of interest to note that there is a steep drop off shortly after 2020 due to COVID-19, but the index made a very quick recovery that continued along the same trend that was occurring before that event. Also, the trend of the

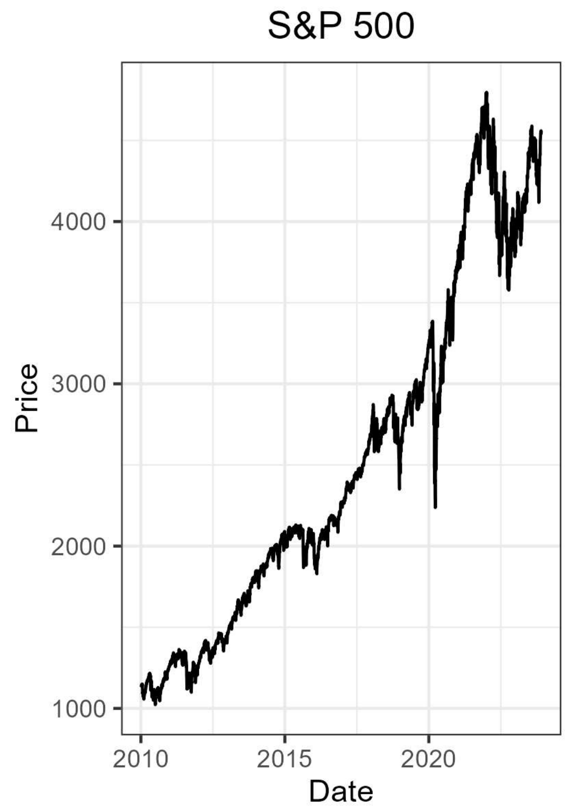

Figure 1: Time Series of S&P 500 Index Value

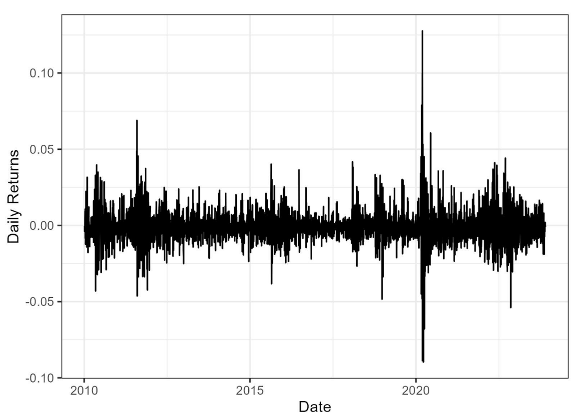

Figure 2: Time Series of the Daily Returns of the S&P 500 Index

index has been very unpredictable over roughly the last two years This can be seen in Figure 1. The returns showed signs of conditional heteroskedasticity, as expected for financial time series data. That characteristic is also required for the GARCH(1,1) model to be used effectively. Also, the large spike in the value of the returns can be seen shortly after the start of 2020 due to the increased volatility from COVID-19. This is shown in Figure 2.

Solution Method

To calculate the volatility of returns, we utilized the same procedure as Francisco Joao Matos Costa, which involved calculating the continuously compounded daily return as

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Our process began by analyzing raw data and observing any unusual trends. After this, we planned to fit and evaluate a GARCH(1,1) model as

$$\sigma_t^2 = \alpha_0 + \alpha_1 * a_{t-1}^2 + \beta_1 * \sigma_{t-1}^2$$

The reason for this is the strong evidence from the literature that this is one of the most effective volatility models available (Hansen & Lunde, 2004). We utilized the R statistical software as well as the rugarch package to fit the model. Within rugarch, the sGARCH specification was set to calculate a simple GARCH with parameters p = q = 1. The S&P 500 data was split into a training and testing set with the training data containing all data except the last 30 values.

Results and Conclusion

At an alpha of .05, the model was found to have significant values for all variables μ, α_1, and β_1, and α_0. We can conclude from this that both the lagged squared volatility values and the lagged squared residuals influence the current volatility and the intercept is not equal to zero. A plot of the standardized residuals qqline is shown in Figure 3. The residuals do not follow a normal distribution which is a violation of the GARCH(1,1) model. It appears that the distribution of the residuals have fat tails especially on the right side of the distribution. This right skew of the residuals hints at the ability of the model to overestimate the forecast values.

The GARCH(1,1) model was applied

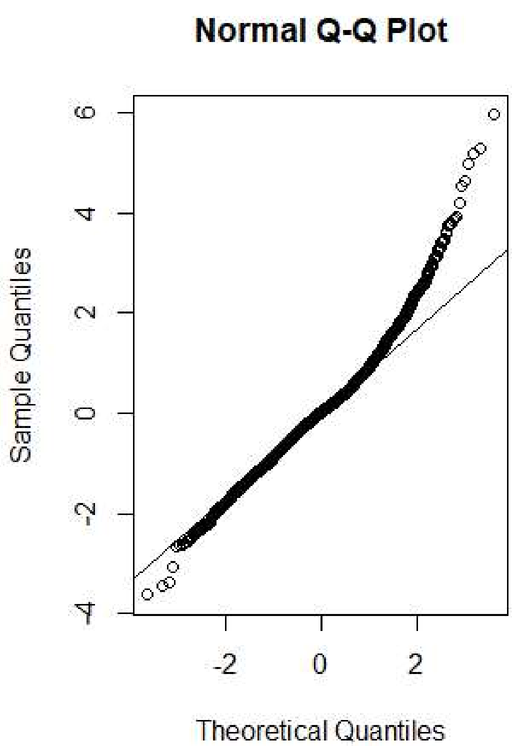

Figure 3: Normal Probability Plot of the Residuals

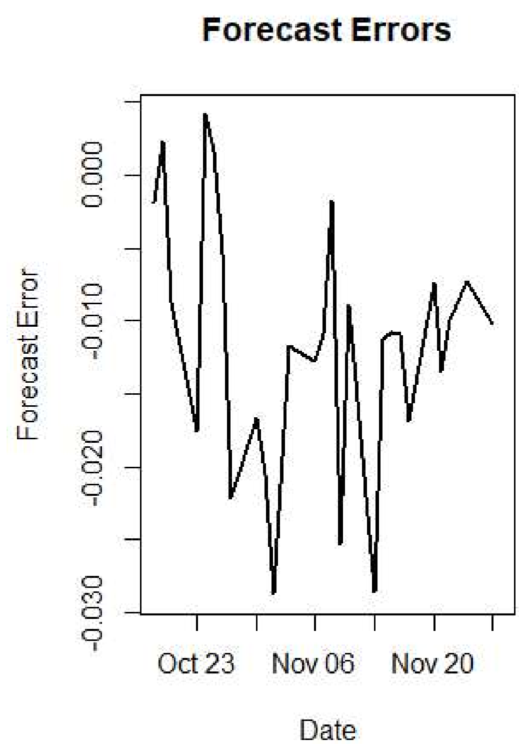

Figure 4: Plot of the Values of the Forecast Errors

to the testing data set to determine its ability to forecast accurately. It can be seen that the forecast errors are consistently negative values. This can be seen in Figure 4. When the forecast error value is negative it means that the model is overestimating and when the forecast error is positive it means that the model is underestimating. This is due to the forecast error being shown as:

$$e_t = P_t - F_t$$

Since there are such consistent error values less than zero this GARCH(1,1) forecast model has a bias to overestimate the true value of volatility. The real values plotted along with the forecast values can be seen in Figure 5. The previous three figures have

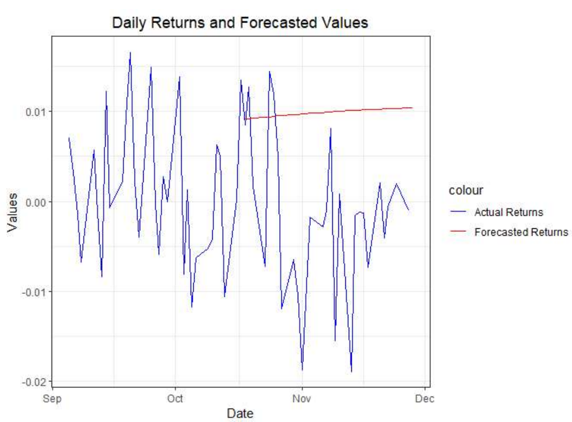

Figure 5: Actual Values Plotted with the Forecast Values

shown that the model does not produce errors that are inside a desired distribution or range. It is concluded that this model does not create a fit that effectively makes dependable forecasts.

After investigating the GARCH(1,1) model that was created, it has been realized that a different type of GARCH model might more accurately represent our data. The recommendation would be to use a NGARCH (Nonlinear General Autoregressive Conditional Heteroskedasticity) modeling technique. This is a more sophisticated model that takes more common factors that are associated with volatility into account. The correlation of previous returns with present returns is accounted for and it points out that volatility spikes are often clustered. NGARCH seems like a better fit for the data that was used in this study because of the volatility clustering and how the volatilities of previous days can be correlated to the current day.

References

Costa, Francisco J M. “Forecasting volatility using GARCH models.” (2017).

Hansen, Peter Reinhard and Lunde, Asger and Lunde, Asger, A Forecast Comparison of Volatility Models: Does Anything Beat a Garch(1,1)? (March 22, 2004). Brown Univ. Economics Working Paper No.

01-04, Available at SSRN:
https://ssrn.com/abstract=264571 or
http://dx.doi.org/10.2139/ssrn.264571