*Dirichlet-Multinomial distribution*

We here use a Dirichlet-Multinomial distribution:

where is the proportion at age in the available data such that , *N* is the total number of samples in the available data (which is restricted to any positive real number), is the estimated proportion at age such that and is the estimated variance inflation coefficient. The first term does not depend upon the parameters, but ensures that as , :

i.e., that the Dirichlet-multinomial reduces to the multinomial likelihood in this circumstance.

*Effective sample size*:

We define the effective sample size of the Dirichlet-Multinomial distribution as the sample size of a multinomial distribution that has the same variance. We use a Dirichlet distribution:

where , is the true proportion at age, and is the effective sample size of the Dirichlet distribution:

Similarly, the variance of a single element from a multinomial distribution:

where *N* is the sample size, is:

Defining observed proportion , we see that:

Therefore the variance of the observed proportion at age for a Dirichlet-multinomial distribution is approximately:

We therefore define the estimated effective sample size *Neff* as the sample size of a multinomial sample with identical variance:

i.e., that the effective sample size is the harmonic sum of *N* and . Preliminary results indicate that this approximation is very similar to estimates using Monte Carlo sampling to numerically generate samples from a Dirichlet-multinomial distribution, and then this Monte Carlo variance to the variance of a multinomial distribution.

*Two alternative parameterizations*

[Explain why I re-define ]