CID: 01183830.

Electrical and Electronic Engineering

Year 4.

Module: Optimization ELEC97062 (EE4-29)

I confirm that the answers presented in the submitted pdf are my own work and that I have not been in contact with others during the exam period.

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#1 a)

$f(x) = x^3 - 2x + 2$

$f(0) = 2.$

in the interval $x \in [-2, -1]$

$-8 \leq x^3 \leq -1, \qquad -4 \leq x^3 - 2x \leq 1$

$-2 \leq x^3 - 2x + 2 \leq 3.$

$f(x = -1) = -1 + 2 + 2 = 3 > 0$

$f(x = -2) = -8 + 4 + 2 = -2 < 0.$

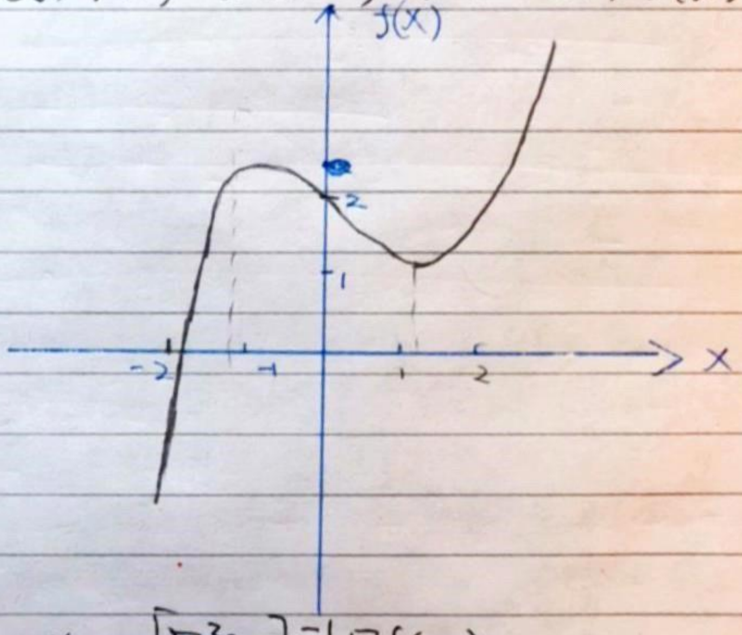
As the function is continuous and well defined for $x \in [-2, 1]$

$f(x)$ must have crossed the x-axis ($f = 0$), as $f(-1) > 0$

but $f(-2) < 0$.

$\frac{df}{dx} = 3x^2 - 2 \Rightarrow 0 \qquad x^2 = \frac{3}{2} \qquad x = \pm\sqrt{\frac{3}{2}}$ stationary point $\simeq \pm 1.22$

$x \to \infty,\ f(x) \to \infty \ ; \ x \to -\infty,\ f(x) \to -\infty$



#1 b)

$x_{k+1} = x_k - [\nabla^2 f(x_k)]^{-1} \nabla f(x_k)$

$\frac{df}{dx} = \nabla f = 3x^2 - 2.$

$\frac{d^2 f}{dx^2} = \nabla^2 f(x) = 6x$

$x_{k+1} = x_k - \frac{1}{6x_k}(3x_k^2 - 2) = \frac{6x_k^2 - 3x_k^2 + 2}{6x_k}$

$x_{k+1} = \frac{3x_k^2 + 2}{6x_k}$

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#1. ~~c) $x_0 = 0$. $x_1 = \frac{3\cdot 0+2}{0}$? not defined.~~

b) Newton's Iteration

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \qquad f'(x_k) = \frac{df}{dx}$$

$$f(x) = x^3 - 2x + 2, \quad f'(x) = 3x^2 - 2$$

$$x_{k+1} = x_k - \frac{x_k^3 - 2x_k + 2}{3x_k^2 - 2} = \frac{3x_k^3 - 2x_k - x_k^3 + 2x_k - 2}{3x_k^2 - 2}$$

$$x_{k+1} = \frac{2x_k^3 - 2}{3x_k^2 - 2}.$$

c) $x_0 = 0$, $x_1 = \frac{0-2}{0-2} = 1$, $x_2 = \frac{2\cdot 1 - 2}{3-2} = 0 \rightarrow$ repeat

assume $x_{2k} = 0$. $x_{2k+1} = \frac{0-2}{0-2} = 1$: this is not a function of x_{2k+1}

$\therefore$ all all numbers of x is 1. $x_{2k+1} = x_{2k+3} = 0$.

if $x_{2k} = 0$ and $x_{2k+2} = 0$.

assume $x_{2k+1} = 1$. $x_{2k+2} = \frac{2\cdot 1^3 - 2}{3\cdot 1^2 - 2} = 0$ not a function of x_{2k+1}

$\therefore$ if $x_{2k+1} = 1$, then $x_{2k+2} = 0$.

$\because$ ① $x_0 = 0$

② if $x_{2k} = 0$ (even index) $x_{2k+1} = 1$

or if previous even index makes $x_{2k} = 0$, then the next odd index of x makes $x_{2k+1} = 1$.

③ if $x_{2k+1} = 1$, then $x_{2k+2} = 0$

$x_{odd} = 1$ makes $x_{even} = 0$.

$\therefore$ $\{x_k\}$ sequence is a sequence $0, 1, 0, 1, \cdots$ that does not converge.

in particular: even index : $x_{2k} = x_{2k+2} = 0$

odd index : $x_{2k+1} = x_{2k+3} = 1$.

#1. d) $$x_{k+1} = \frac{2x_k^3 - 2}{3x_k^2 - 2}$$

$$x_{k+2} = \frac{2x_{k+1}^3 - 2}{3x_{k+1}^2 - 2} = \frac{2\cdot\left(\frac{2x_k^3-2}{3x_k^2-2}\right)^3 - 2}{3\cdot\left(\frac{2x_k^3-2}{3x_k^2-2}\right)^2 - 2}$$

$$F(0) = \frac{2\cdot\left(\frac{0-2}{0-2}\right)^3 - 2}{3\cdot\left(\frac{0-2}{0-2}\right)^2 - 2} = \frac{2\cdot 1 - 2}{3\cdot 1 - 2} = 0.$$

$$F(x) = \frac{2\left(\frac{2x^3-2}{3x^2-2}\right)^3 - 2}{3\cdot\left(\frac{2x^3-2}{3x^2-2}\right)^2 - 2}$$

$$F(1) = \frac{2\cdot\left(\frac{2-2}{3-2}\right)^3 - 2}{3\cdot\left(\frac{2-2}{3-2}\right)^2 - 2} = \frac{0-2}{0-2} = 1.$$

e) $f = x^2 - c = 0.$ $c > 0$

i) $\frac{df}{dx} = 2x,$ $\frac{d^2f}{dx^2} = 2.$

$x_{k+1} = \cancel{\ldots} = \cancel{x_k - x_k = 0}$

$$x_{k+1} = x_k - \frac{x_k^2 - c}{2x_k} = \frac{2x_k^2 - x_k^2 + c}{2x_k}$$

$$x_{k+1} = (x_k^2 + c)/(2x_k)$$

ii) $$x_{k+2} = \frac{x_{k+1}^2 + c}{2x_{k+1}} = \frac{\left(\frac{x_k^2+c}{2x_k}\right)^2 + c}{2\left(\frac{x_k^2+c}{2x_k}\right)}$$

$\times (2x_k)^2$ on both sides

$$G(x) = \frac{(x^2+c)^2 + 4x^2c}{4(x^2+c)x}$$

$$G(x) = \frac{x^4 + c^2 + 2x^2c + 4x^2c}{4(x^3 + cx)}$$

$$G(x) = \frac{x^4 + 6x^2c + c^2}{4(x^3 + cx)}$$

$3x^2$ $-c$
$1x^2$ $+c$

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#1. e) continue.

$$\bar{x} \cdot 4(\bar{x}^3 + c\bar{x}) = \bar{x}^4 + 6c\bar{x}^2 + c^2$$
$$4\bar{x}^4 + 4c\bar{x}^2 = \bar{x}^4 + 6c\bar{x}^2 + c^2.$$
$$3\bar{x}^4 - 2c\bar{x}^2 - c^2 = 0.$$
$$(3\bar{x}^2 + c)(\bar{x}^2 - c) = 0.$$

$\because c > 0$ AND consider real number only

$\therefore 3\bar{x}^2 + c \neq 0.$

$\therefore \bar{x}^2 - c = 0, \quad \bar{x} = \pm\sqrt{c}. \Rightarrow$ unique fixed points.

$$\bar{x}^2 - c = (\pm\sqrt{c})^2 - c = c - c = 0$$

$\therefore \bar{x} = \pm\sqrt{c}$ ~~is~~ are the solutions of the equation $x^2 - c = 0$.

if starting at $x_0 = 0$, the cubic equation has periodic behavior.

for the quadratic equation, if $x_0 = 0$, x_2 would be undefined as denominator would be 0. (see $G(x)$)

for quadratic equation, fixed points are unique: only at $\bar{x}$, $\bar{x} = G(\bar{x})$

the sequence would only be periodic if it starts at fixed points.

if $x_0 \neq \bar{x}$, that is not starting at fixed points.

$G(x_k)$ would never equal to x_k; therefore never go into and trapped into fixed points.

Therefore quadratic equation would not have periodic behavior.

~~x_k will move towards fixed points, which as solutions,~~

if x_k converges to solutions, and solutions are those unique fixed points, then sequence $\{x_k\}$ converges ~~to solutions.~~ to one of the fixed point/solution

~~Also~~ $\sqrt{c} = G(\sqrt{c})$, $-\sqrt{c} = G(-\sqrt{c})$, there is no jump between fixed points $\Rightarrow$ not periodic. 4

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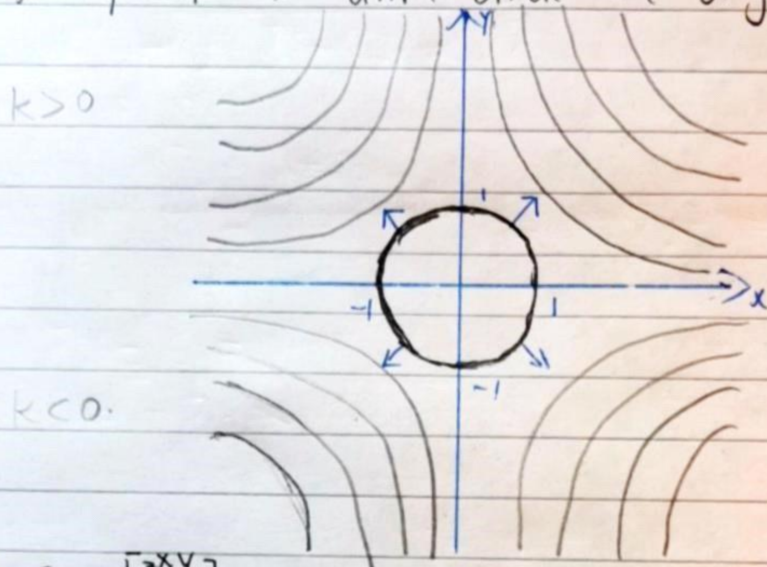
#2. $f(x,y) = x^2 y$, $g(x,y) = x^2+y^2=1$, $x \to$ symmetric $\to -x$

a). $x^2 y = k$. $y = \frac{k}{x^2}$

$x^2+y^2=1 \Rightarrow$ unit circle at origin. ~~=~~ admissible set.



- black: pencil: level lines.

- blue: axis and gradients.

$\nabla f = \begin{bmatrix} 2xy \\ y \end{bmatrix}$

$\nabla g = [2x, 2y]$

At any point in the admissible set, there is only one active constraint $g(x,y)$,

$\therefore$ there is only one gradient of active constraint: $\nabla g(x,y)$

it is linearyly independent with void ?

$\therefore$ all points are regular points.

b). $L = x^2 y + \lambda(x^2+y^2-1)$

① $\frac{\partial L}{\partial x} = 2xy + 2\lambda x = 0$

② $\frac{\partial L}{\partial y} = x^2 + 2\lambda y = 0$

③ $x^2+y^2-1=0$

} necessary conditions of optimality.

from ① $2x(y+\lambda)=0$.

if $x=0$, $x^2+y^2-1=0=0+y^2-1 \Rightarrow y^* = \pm 1$

$0+2\cdot\lambda y = 0$. $, y \neq 0 \Rightarrow \lambda = 0$.

$(x,y,\lambda) = (0, \pm 1, 0)$ $P_1: (0, 1, 0)$

5. $P_2: (0, -1, 0)$

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#2 b) continue

if $x \neq 0$, $y+\lambda=0$, $y=-\lambda$

$x^2+2\cdot\lambda(-\lambda)=0$, $x^2-2\lambda^2=0$, $x^2=2\lambda^2$

$x=\pm\sqrt{2}\lambda$

$2\lambda^2+\lambda^2-1=0$, $3\lambda^2=1$, $\lambda^2=\frac{1}{3}$. $\lambda=\pm\sqrt{\frac{1}{3}}$

$P3: (+\sqrt{\frac{2}{3}}, -\sqrt{\frac{1}{3}}, +\sqrt{\frac{1}{3}})$ (x, y, λ)

$P4: (-\sqrt{\frac{2}{3}}, -\sqrt{\frac{1}{3}}, +\sqrt{\frac{1}{3}})$

$P5: (+\sqrt{\frac{2}{3}}, +\sqrt{\frac{1}{3}}, -\sqrt{\frac{1}{3}})$

$P6: (-\sqrt{\frac{2}{3}}, +\sqrt{\frac{1}{3}}, -\sqrt{\frac{1}{3}})$

c) $\frac{\partial^2 L}{\partial x^2} = 2y+2\lambda$ $\frac{\partial^2 L}{\partial y^2} = 2\lambda$, $\frac{\partial^2 L}{\partial x \partial y} = 2x$

$$\nabla^2 L = \begin{bmatrix} 2y+2\lambda & 2x \\ 2x & 2\lambda \end{bmatrix}$$

$\left(\frac{\partial g}{\partial x}\right) s = [2x, 2y]\, s = 0$

P_1: $[0, 2]\, s = 0$, $s = \begin{bmatrix} a \\ 0 \end{bmatrix}$ $a>0$

$\nabla^2 L(P_1) = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} a \\ 0 \end{bmatrix}' \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix}' \begin{bmatrix} 2a \\ 0 \end{bmatrix} = 2a^2 > 0$

$\therefore P_1$ is a local minimum. (strict)

P_2: $[0, -2]\, s = 0$, $s = \begin{bmatrix} a \\ 0 \end{bmatrix}$ $a>0$

$\nabla^2 L(P_2) = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}$ $s'\nabla^2 L s = \begin{bmatrix} a \\ 0 \end{bmatrix}' \begin{bmatrix} -2a \\ 0 \end{bmatrix} = -2a^2 < 0$

$\therefore P_2$ is a local maximizer. (strict)

$P3$: $[2\sqrt{\frac{2}{3}}, -2\sqrt{\frac{1}{3}}]\, s = 0$. $s = \begin{bmatrix} a \\ \sqrt{2}a \end{bmatrix}$ $y+\lambda=0$ for P_3 to P_6.

$$\nabla^2 L(P3) = \begin{bmatrix} 0 & 2\sqrt{\frac{2}{3}} \\ 2\sqrt{\frac{2}{3}} & 2\sqrt{\frac{1}{3}} \end{bmatrix}$$

$s'\nabla^2 L s = \begin{bmatrix} a \\ \sqrt{2}a \end{bmatrix}' \begin{bmatrix} \frac{4}{\sqrt{3}}a \\ \frac{4\sqrt{2}}{3}a \end{bmatrix} = \frac{4}{\sqrt{3}}a^2 + \frac{8\sqrt{3}}{3}a^2 > 0$

P_3 is a strict local minimum.

#2 $P_4: \left[-2\sqrt{\tfrac{2}{3}},\ -2\sqrt{\tfrac{1}{3}}\right] s = 0, \quad s = \begin{bmatrix} a \\ -\sqrt{2}a \end{bmatrix}$

c.) $\nabla^2 L(P_4) = \begin{bmatrix} 0 & -2\sqrt{\tfrac{2}{3}} \\ -2\sqrt{\tfrac{2}{3}} & 2\sqrt{\tfrac{2}{3}} \end{bmatrix}$

$$\begin{bmatrix} a \\ -\sqrt{2}a \end{bmatrix}' \begin{bmatrix} 0 & -2\sqrt{\tfrac{2}{3}} \\ -2\sqrt{\tfrac{2}{3}} & 2\sqrt{\tfrac{2}{3}} \end{bmatrix} \begin{bmatrix} a \\ -\sqrt{2}a \end{bmatrix} = \begin{bmatrix} a \\ -\sqrt{2}a \end{bmatrix}' \begin{bmatrix} \frac{4}{\sqrt{3}} a \\ -\frac{2\sqrt{6}+4\sqrt{3}}{3} a \end{bmatrix} =$$

$$= \frac{4\sqrt{3}}{3} a^2 + \frac{4\sqrt{6}+4\sqrt{3}}{3} a^2 > 0.$$

P_4 is a local minimum (strict)

$P_5: \left[+2\sqrt{\tfrac{2}{3}},\ +2\sqrt{\tfrac{1}{3}}\right] s = 0, \quad s = \begin{bmatrix} a \\ -\sqrt{2}a \end{bmatrix}$

$\nabla^2 L(P_5) = \begin{bmatrix} 0 & +2\sqrt{\tfrac{2}{3}} \\ +2\sqrt{\tfrac{2}{3}} & -2\sqrt{\tfrac{1}{3}} \end{bmatrix} = -\nabla^2 L(P_4)$

with the same s as used in P_4.

$s' \nabla^2 L(P_5)\, s < 0$

P_5 is a strict local maximum.

$P_6.\ \left[-2\sqrt{\tfrac{2}{3}},\ +2\sqrt{\tfrac{1}{3}}\right] s = 0. \quad s = \begin{bmatrix} a \\ \sqrt{2}a \end{bmatrix}$

$\nabla^2 L(P_6) = \begin{bmatrix} 0 & -2\sqrt{\tfrac{2}{3}} \\ -2\sqrt{\tfrac{2}{3}} & -2\sqrt{\tfrac{1}{3}} \end{bmatrix} = -\nabla^2 L(P_3)$

with the same s used in P_3.

$s' \nabla^2 L(P_6)\, s < 0$

P_6 is a strict local maximum

#2
d) $f(P_1) = 0^2 \cdot 1 = 0$ local min
$f(P_2) = 0^2 \cdot (-1) = 0$ local max. } they are the same.

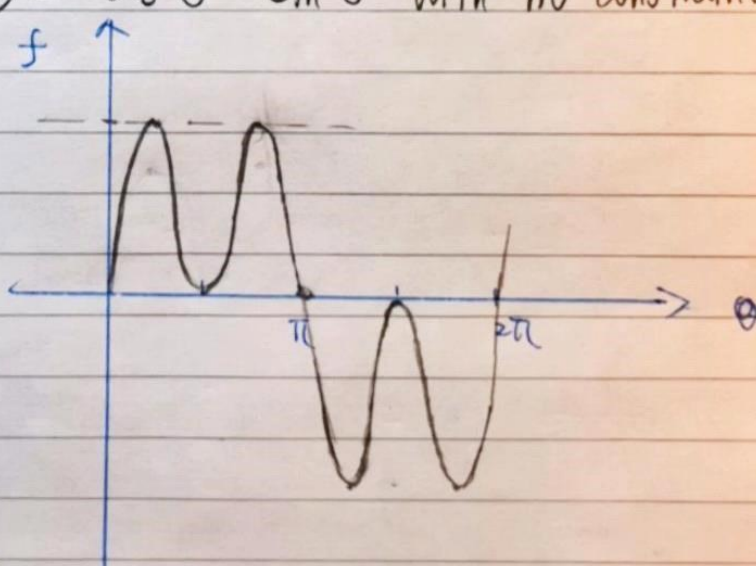
$f(P_3) = \frac{2}{3} \cdot (-\sqrt{\frac{1}{3}}) = -\frac{2\sqrt{3}}{9}$ local min
$f(P_4) = f(P_3) = -\frac{2\sqrt{3}}{9}$ local min as well

$f(P_5) = \frac{2}{3} \cdot \sqrt{\frac{1}{3}} = \frac{2\sqrt{3}}{9}$ local max
$f(P_6) = f(P_5) = \frac{2\sqrt{3}}{9}$ local max.

global maximizer: P5 and P6. $f = \frac{2\sqrt{3}}{9}$
P5, $(x, y, x) = (\cdot\sqrt{\frac{2}{3}}, \sqrt{\frac{1}{3}}, -\sqrt{\frac{1}{3}})$
P6, $= (-\sqrt{\frac{2}{3}}, \sqrt{\frac{1}{3}}, -\sqrt{\frac{1}{3}})$

global minimizer P3 and P4. $f = -\frac{2\sqrt{3}}{9}$.
P3 $(\cdot\sqrt{\frac{2}{3}}, -\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}})$
P4 $(-\sqrt{\frac{2}{3}}, -\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}})$

e) $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$ by trignomatry
so the ~~con~~ original constraint is naturally satisfied.
$f(\theta) = \cos^2\theta \cdot \sin\theta$ with no constraint.



the plot $f(\theta)$ verifies that there are 2 global maximum and 2 global minimum. P_1 and P_2 $(x, y) = (0, \pm 1)$ corresponds to $\theta = \frac{1}{2}\pi$ and $\frac{3}{2}\pi$ θ8. which are local min and max.

01183830 E.E4-29.

#3. turn max problem into a min problem.

a) $L = -\ln(r_1) - 2\ln(r_2) - 3\ln(r_3)$
$+ \rho_1(r_1 + r_2 + r_3 - c) + \rho_2(-r_1) + \rho_3(-r_2) + \rho_4(-r_3)$

$\frac{\partial L}{\partial r_1} = -\frac{1}{r_1} + \rho_1 - \rho_2 = 0.$
$\frac{\partial L}{\partial r_2} = -\frac{2}{r_2} + \rho_1 - \rho_3 = 0.$
$\frac{\partial L}{\partial r_3} = -\frac{3}{r_3} + \rho_1 - \rho_4 = 0.$

$r_1 - r_2 + r_3 - c \leq 0$, $\rho_1 \geq 0, \rho_2 \geq 0, \rho_3 \geq 0, \rho_4 \geq 0.$

disregard positivity constraints.

complementarity conditions.

$\rho_1(r_1 + r_2 + r_3 - c) = 0 \qquad \rho_2 r_1 = 0, \rho_3 r_2 = 0, \rho_4 r_3 = 0.$

b). ~~if $r_1 + r_2 + \rho_3 - c = 0, \rho_1$~~

$\rho_1 = \frac{1}{r_1} + \rho_2 = \frac{2}{r_2} + \rho_3 = \frac{3}{r_3} + \rho_4.$

$\frac{1}{r_1} = \rho_1 - \rho_2 \Rightarrow r_1 = \frac{1}{\rho_1 - \rho_2}$
$\frac{2}{r_2} = \rho_1 - \rho_3 \Rightarrow r_2 = \frac{2}{\rho_1 - \rho_3}$
$\frac{3}{r_3} = \rho_1 - \rho_4 \rightarrow r_3 = \frac{3}{\rho_1 - \rho_4}$

if $\rho_2 = \rho_3 = \rho_4 = 0$. and $\rho_1 \neq 0$.

$r_1 = \frac{1}{\rho_1}, \ r_2 = \frac{2}{\rho_1}, \ r_3 = \frac{3}{\rho_1}$

$r_1 + r_2 + r_3 - c = 0.$

$\frac{1}{\rho_1}(1+2+3) - c = \frac{6}{\rho_1} - c = 0.$

$\rho_1 = \frac{6}{c}, \ r_1 = \frac{c}{6}, \ r_2 = \frac{c}{3}, \ r_3 = \frac{c}{2}$

This is a candidate optimal solution. ↓

$\left(\rho_1 = \frac{6}{c}, \rho_2 = \rho_3 = \rho_4 = 0, r_1 = \frac{c}{6}, r_2 = \frac{c}{3}, r_3 = \frac{c}{2}\right)$

$r_1 + r_2 + r_3 = c \cdot \left(\frac{1}{6} + \frac{1}{3} + \frac{1}{2}\right) = c$

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#3. continue

c)

$f(r) = -\ln(r_1) - 2\ln(r_2) - 3\ln(r_3)$

$g(r) = r_1 + r_2 + r_3 - c.$

$\frac{\partial g(r)}{\partial r} = [1, 1, 1]$

~~$\nabla_r f$~~

$L = -\ln(r_1) - 2\ln(r_2) - 3\ln(r_3) + \lambda(r_1 + r_2 + r_3 - c)$

$$\nabla_r L = \begin{bmatrix} -\frac{1}{r_1} + \lambda \\ -\frac{2}{r_2} + \lambda \\ -\frac{3}{r_3} + \lambda \end{bmatrix}$$

$\frac{d}{dr}\left(-\frac{1}{r}\right) = -(-1)\frac{1}{r^2}$

$\frac{\partial g(r)}{\partial r} \nabla_r L(r, \lambda) = -\frac{1}{r_1} - \frac{2}{r_2} - \frac{3}{r_3} + 3\lambda$

$$S(r, \lambda) = -\ln(r_1) - 2\ln(r_2) - 3\ln(r_3) + \lambda(r_1 + r_2 + r_3 - c) + \frac{1}{\varepsilon}(r_1 + r_2 + r_3 - c)^2 + \eta\left(-\frac{1}{r_1} - \frac{2}{r_2} - \frac{3}{r_3} + 3\lambda\right)^2$$

$\frac{\partial S}{\partial r_1} = -\frac{1}{r_1} + \lambda + \frac{2}{\varepsilon}(r_1 + r_2 + r_3 - c) + 2\eta\left(-\frac{1}{r_1} - \frac{2}{r_2} - \frac{3}{r_3} + 3\lambda\right) \cdot \left(\frac{1}{r_1^2}\right)$

$\frac{\partial S}{\partial r_2} = -\frac{2}{r_2} + \lambda + \frac{2}{\varepsilon}(r_1 + r_2 + r_3 - c) + 2\eta\left(-\frac{1}{r_1} - \frac{2}{r_2} - \frac{3}{r_3} + 3\lambda\right)\left(\frac{2}{r_2^2}\right)$

$\frac{\partial S}{\partial r_3} = -\frac{3}{r_3} + \lambda + \frac{2}{\varepsilon}(r_1 + r_2 + r_3 - c) + 2\eta\left(-\frac{1}{r_1} - \frac{2}{r_2} - \frac{3}{r_3} + 3\lambda\right)\left(\frac{3}{r_3^2}\right)$

$\frac{\partial S}{\partial \lambda} = (r_1 + r_2 + r_3 - c) + 2\eta\left(-\frac{1}{r_1} - \frac{2}{r_2} - \frac{3}{r_3} + 3\lambda\right) \cdot 3$

$r_b = \left(\frac{c}{6}, \frac{c}{3}, \frac{c}{2}\right) \qquad r_1 + r_2 + r_3 = c.$

$\frac{\partial S}{\partial r_1}\Big|_{r_b} = -\frac{6}{c} + \lambda + 2\eta \cdot \left(\frac{36}{c^2}\right)\left(-\frac{6}{c} - \frac{6}{c} - \frac{6}{c} + 3\lambda\right)$

$= -\frac{6}{c} + \lambda + \frac{72\eta}{c^2}\left(-\frac{18}{c} + 3\lambda\right) \Rightarrow 0$

$\frac{\partial S}{\partial r_2}\Big|_{r_b} = -\frac{6}{c} + \lambda + 2\eta \cdot \left(2 \cdot \frac{9}{c^2}\right)\left(-\frac{18}{c} + 3\lambda\right) \Rightarrow 0$

$= -\frac{6}{c} + \lambda + \frac{36\eta}{c^2}\left(-\frac{18}{c} + 3\lambda\right) \Rightarrow 0.$

the two equation can coexist if $-\frac{18}{c} + 3\lambda = 0$

$3\lambda = \frac{18}{c}, \quad \lambda = \frac{6}{c}.$

verify $-\frac{6}{c} + \lambda + 0 = 0, \quad -\frac{6}{c} + \frac{6}{c} = 0$ ✓

~~simi~~ $\frac{\partial S}{\partial r_3} = -\frac{6}{c} + \lambda + 0 + 0 = 0$ ✓

$\frac{\partial S}{\partial \lambda} = 0 + 2\eta \cdot 0 = 0$ ✓

therefor $(r, \lambda) = \left(\frac{c}{6}, \frac{c}{3}, \frac{c}{2}, \frac{6}{c}\right)$ is a stationary point of $S(r, \lambda)$

EE4-29.
~~EE-429~~

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#3 d).

$\rho_1 \geq 0, \quad \rho_1 \left[\left(\sum_{i=1}^{n} r_i \right) - c \right] = 0.$

if $\rho_1 > 0$, $\sum_{i=1}^{n} r_i = c$.

$L = \sum -\ln(r_i) \cdot \alpha_i + \rho_1 \left(\sum_{i=1}^{n} r_i - c \right) + \sum_{i=2}^{n+1} \rho_i (-r_{i-1})$

$\frac{\partial L}{\partial r_i} = -\frac{\alpha_i}{r_i} + \rho_1 - \rho_{i+1} \Rightarrow 0.$

assume $\rho_1 > 0$, $\rho_i = 0$ for all i from 2 to n+1

$\frac{\partial L}{\partial r_i} = -\frac{\alpha_i}{r_i} + \rho_1 = 0, \quad r_i = \frac{\alpha_i}{\rho_1}$

$\sum_{i=1}^{n} r_i = \sum_{i=1}^{n} \frac{\alpha_i}{\rho_1} = \frac{1}{\rho_1} \left(\sum \alpha_i \right) = c.$

$\rho_1 = \frac{\sum \alpha_i}{c}. \quad r_i = \frac{\alpha_i}{\rho_1} = \frac{\alpha_i}{\sum \alpha_i} c. = \left(\frac{\alpha_i}{\sum_{i=1}^{n} \alpha_i} \right) \cdot c$

A candidate optimal solution:

$r_i = \frac{\alpha_i c}{\left(\sum_{i=1}^{n} \alpha_i \right)}$, $\rho_2 = \rho_3 = \cdots = \rho_{n+1} = 0$

$\rho_1 = \left(\sum_{i=1}^{n} \alpha_i \right) / c$

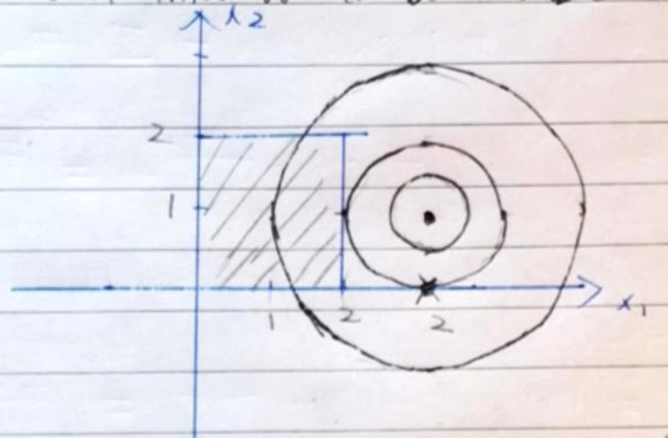
01183830. EE4-24

\# 4. $0 \le x_1 \le 2, \quad 0 \le x_2 \le 2.$

a). Admissible set is a square (inside)

$f = (x_1 - 3)^2 + (x_2 - 1)^2$

Level lines would be ~~[illegible]~~ circles around (3,1)



shaded: admissible set

circles in black: leve lines.

The optimal solution would be as close to (3,1) as possible. as $f(x)$ is a sum of squares. $f(x)$ increases always, when moving away from (3,1)

The optimal solution would be a level line making tangent with boundry of admissible set.

From plot, it can be seen that $(x_1, x_2) = (2,1)$ is the optimal solution.

b). $\nabla f = \begin{bmatrix} 2(x_1 - 3) \\ 2(x_2 - 1) \end{bmatrix}$ $\quad \nabla f(x_0) = \begin{bmatrix} 2(0-3) \\ 2(0-1) \end{bmatrix} = \begin{bmatrix} -6 \\ -2 \end{bmatrix}$

i) $\min_x x^T \cdot \begin{bmatrix} -6 \\ -2 \end{bmatrix} = -6x_1 - 2x_2$ $\quad 0 \le x_1 \le 2$

$0 \le x_2 \le 2.$

· minimum would be found for $(-6x_1 - 2x_2)$ when x_1 and x_2 take their positive maximum. $\xi = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$= -6 \cdot 2 - 2 \cdot 2 = -12 - 4 = -16.$

ii) ~~$[2,2]\begin{bmatrix} -6 \\ -2 \end{bmatrix} = -12 - 4 = -16 < 0.$~~

~~$x_1 = x_0 + t_0 \left(\begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} -6 \\ -2 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + t_0 [$~~

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#4. b) continue ii)

$\xi - x^0 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$[2, 2] \begin{bmatrix} -6 \\ -2 \end{bmatrix} = -16 < 0.$

$x_1 = x_0 + t_0 \begin{bmatrix} 2 \\ 2 \end{bmatrix} = t_0 \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad (x_0 = 0)$

$f(x_1(t_0)) = (2t_0 - 3)^2 + (2t_0 - 1)^2 = f_0$

$\frac{df_0}{dt_0} = \cancel{4(2t_0-3)}$

$= 4(2t_0 - 3) + 4(2t_0 - 1) = 0$

$8t_0 - 12 + 8t_0 - 4 = 0$

$t_0 = 1.$

$x_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

c). $\nabla f(x_1) = \begin{bmatrix} 2(2-3) \\ 2(2-1) \end{bmatrix} = \begin{bmatrix} 2 \cdot (-1) \\ 2 \cdot 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

Problem: $\min_x \; x^T \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \min_x \; -2x_1 + 2x_2$

$0 \leq x_1 \leq 2, \quad 0 \leq x_2 \leq 2.$

$(-2x_1 + 2x_2)$ decreases when x_1 increase and x_2 decrease.

$\therefore \xi = \begin{bmatrix} \max(x_1) \\ \min(x_2) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$(\xi - x^1) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}, \quad \nabla f(x^1) = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$[0, -2] \cdot \begin{bmatrix} -2 \\ 2 \end{bmatrix} = 0 - 4 = -4.$

$|-4| < |-16| \Rightarrow$ therefore optimality gap has been reduced.

$x_2 = x_1 + t_1 \begin{bmatrix} 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2t_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2-2t_1 \end{bmatrix}$

$f_1 = f(x_2(t_1)) = (2-3)^2 + (2-2t_1-1)^2 = 1 + (1-2t_1)^2$

$f_1 = 1 + 1 + 4t_1^2 - 4t_1 = 4t_1^2 - 4t_1 + 2.$

$\frac{df_1}{dt_1} = 8t_1 - 4 = 0 \Rightarrow t_1 = \frac{1}{2}.$

$x_2 = \begin{bmatrix} 2 \\ 2-2t_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 - 2 \cdot \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

\# 4 d)

$x_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\xi = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\nabla f(x_2) = \begin{bmatrix} 2(2-3) \\ 2(\cancel{1}-1) \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

$(\xi - x_2) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \underline{0}.$

$(\xi - x_2)^T \nabla f(x_2) = 0 \cdot -2 + 0 \cdot 0 = 0.$

↳ optimality gap is zero

↳ $(x_1, x_2) = (2, 1)$ is the optimal solution.

original problem from a), $(x_1^*, x_2^*) = (2, 1)$

Therefore the result in d) is consistent ~~wint~~ with result of part a).

Therefore $x^* = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$