Wireless Communication Coursework 3

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1 System Model

1.1 Deployment

In this coursework, the downlink transmission from one base station (BS) to multiple users (UE) would be considered. To simplify the system model, some setting or numbers are fixed:

- There is one centre BS. And there are 6 interfering BSs uniformly located on a circle of radius 500m, centred around the centre BS.
- The centre BS serves K users. Users are randomly and uniformly located, with 35m < distance to BS < 250, in the centre cell.
- Each BS has 4 transmit antennas. The transmit power at BSs is fixed to 46dBm or 39.8W at each BS.
- The variance of noise at each UE is fixed to -174dBm or $4 \times 10^{-21} W$. This is extremely small and is close to thermal noise floor for 1 Hz bandwidth at room temperature.
- Path Loss [dB] is modeled by: $128.1 + 37.6 log_{10}(d[km])$; Shadowing is modeled by a log normal distribution with 8dB standard deviation.

1.2 Multi-User Multi-Cell MIMO: Downlink

The received signal vector at a user q in (centre) cell i at a time instance is:

$$\mathbf{y}_q = \Lambda_{q,i}^{-1/2}\mathbf{H}_{q,i}\mathbf{P}_{q,i}\mathbf{c}_{q,i} + \sum_{p \in \mathbf{K}_i, p \neq q} \Lambda_{q,i}^{-1/2}\mathbf{H}_{q,i}\mathbf{P}_{p,i}\mathbf{c}_{p,i} + \sum_{j \neq i}\sum_{l \in \mathbf{K}_j} \Lambda_{q,j}^{-1/2}\mathbf{H}_{q,j}\mathbf{P}_{l,j}\mathbf{c}_{l,j} + \mathbf{n}_q$$

- $\mathbf{y}_q \in \mathbb{C}^{n_{r,q}}$ is the received signal vector of user q.
- $\Lambda_{q,i}^{-1} \in \mathbb{R}$ refers to the path-loss and shadowing between BS i and UE q.
- $\mathbf{H}_{q,i} \in \mathbb{C}^{n_{r,q} \times n_{t,i}}$ represents the MIMO fading channel between BS i and UE q, and it would be further elaborated in the following sub-sections.
- $\mathbf{c}$ is the transmitted symbol vector. $\mathbf{P}$ is the precoding matrix.
- The 2nd term represents the multi-user interference and the 3rd term is the inter-cell interference. The 4th term $\mathbf{n}_q$ is the circular symmetric complex Gaussian noise at UE q.

At reciever, a suitable candidate of combiner is MMSE combiner, as it minimizes the total resulting noise by balancing mitigation of the effect of both noise and interference. The exact expression of MMSE combiner could be found in later sub-section. After applying a combiner $\mathbf{g}$ to stream l of user q in cell i:

$$z_{q,l} = \mathbf{g}_{q,l}\mathbf{y}_{q,l} = \Lambda_{q,i}^{-1/2}\mathbf{g}_{q,l}\mathbf{H}_{q,i}\mathbf{p}_{q,i,l}c_{q,i,l} + \sum_{m\neq l}\Lambda_{q,i}^{-1/2}\mathbf{g}_{q,l}\mathbf{H}_{q,i}\mathbf{p}_{q,i,m}c_{q,i,m}$$

$$+ \sum_{p\in\mathbf{K}_i, p\neq q}\Lambda_{q,i}^{-1/2}\mathbf{g}_{q,l}\mathbf{H}_{q,i}\mathbf{P}_{p,i}\mathbf{c}_{p,i} + \sum_{j\neq i}\sum_{l\in\mathbf{K}_j}\Lambda_{q,j}^{-1/2}\mathbf{g}_{q,l}\mathbf{H}_{q,j}\mathbf{P}_{l,j}\mathbf{c}_{l,j} + \mathbf{g}_{q,l}\mathbf{n}_q$$

The 2nd term is inter-stream interference. The 3rd is multi-user interference. And the 4th term is inter-cell interference. In this specific coursework, the transmission scheme is that, the centre BS only transmit towards one UE at any single time instance. Therefore the multi-user interference terms in the above two equations would equal to 0 in this case.

1.3 Flat fading MIMO Channel Model

The channel matrix at time instant k between BS i and UE q could be written as $\mathbf{H}_{k,q,i} = \tilde{\mathbf{H}}_{k,q,i}\mathbf{R}_{t,q,i}^{1/2}$.

The spatially uncorrelated Rayleigh fading channel is represented by: $\tilde{\mathbf{H}}_{k,q,i} \in \mathbb{C}^{n_r\times n_t}$ and each entry in the matrix is a circular symmetric gausian random variable. A first-order Gauss-Markov process models its evolution: $\tilde{\mathbf{H}}_{k,q,i} = \epsilon\tilde{\mathbf{H}}_{k-1,q,i} + \sqrt{1-\epsilon^2}\mathbf{N}_{k,q,i}$, where $\mathbf{N}_{k,q,i} \sim CN(0,1)$ has i.i.d entries. The correlation in time domain is represented by ϵ.

$\mathbf{R}_{t,q,i}$ is the transmit correlation matrix of BS i and user q link. In this coursework, the exponential structure has been adopted to simplify the system:

$$\mathbf{R}_{t,q,i} = \begin{bmatrix} 1 & t_{q,i} & t_{q,i}^2 & t_{q,i}^3 \\ t_{q,i}^* & 1 & t_{q,i} & t_{q,i}^2 \\ t_{q,i}^{*2} & t_{q,i}^* & 1 & t_{q,i} \\ t_{q,i}^{*3} & t_{q,i}^{*2} & t_{q,i}^* & 1 \end{bmatrix}$$

For simplicity, the transmit correlation matrix between user in centre cell and interfering BS is identity matrix. For centre cell, $t_{q,0} = t_{space}e^{j\phi_q}$, where t_{space} is the magnitude of the spatial correlation coefficient, identical for all users. The phase of the correlation coefficient is user specific and is uniformly distributed between 2 and 2π.

1.4 Transmission Scheme

LTE 4Tx Single-User MIMO scenario has been considered. Script "Codebook.m" calculates and stores the precoders according to LTE specification. There is only a finite set of precoders, and the best precoder needs to be estimated by the receiver. For example, quantized precoding for spatial multiplexing with rank adaptation and rate maximization:

$$\mathbf{W}^* = \arg\max_{n_e}\max_{\mathbf{W}_i^{(n_e)}} R$$

n_e is the number of stream to transmit. Achievable rate R is explained in the next sub-section. Assuming receiver has perfect channel knowledge, the estimation is a function of the current channel matrix, and index (RI & PMI) is provided to BS. It has been assumed that this feedback is loss-less and instantaneous.

At BS, transmit symbols are linearly precoded: $\mathbf{c'}_i = \mathbf{P}_i\mathbf{c}_i = \mathbf{W}_i\mathbf{S}_i^{1/2}\mathbf{c}_i$. $\mathbf{P}_i \in \mathbb{C}^{n_{t,i}\times n_{e,i}}$ is the precoder matrix (or vector if only one stream), constrained by transmit power: $Trace(P^H P) = P_{tx}$. (My MATLAB code has some slight difference with Yang's note, on where to assign half the power to each stream when $ne = 2$, although the transmit power constraint has been checked to satisfy.) $\mathbf{W}_i \in \mathbb{C}^{n_{t,i}\times n_{e,i}}$ can be viewed as the beamforming direction. $\mathbf{S}_i \in \mathbb{R}^{n_{u,q}\times n_{u,q}}$ is the power allocation matrix and is a scaled identity matrix since it's uniform power allocation across stream.

At each time instant, BS decides which user to transmit to according to proportional fair scheduling: $q^* = \arg\max_{q\in\kappa} \frac{\gamma_q R_q}{\bar{R}_q}$, where γ_q is Quality of Service and is 1 for all users. $\bar{R}_q$ is the long-term average rate of user q and is calculated using an exponentially weighted low-pass filter:

$$\bar{R}_q(k+1) = (1 - 1/t_c)\bar{R}_q(k) + 1/t_c R_q(k), \text{if q scheduled at time k}$$

$$\bar{R}_q(k+1) = (1 - 1/t_c)\bar{R}_q(k),\ \text{if q not scheduled at time k}$$

1.5 Receive combiner and achievable rate

The noise-plus-interference auto-correlation of stream l of user q in cell i is given by (no inter-user interference term):

$$\mathbf{R}_{ni} = \sum_{m\neq l} \Lambda_{q,i}^{-1}\mathbf{H}_{q,i}\mathbf{p}_{q,i,m}(\mathbf{H}_{q,i}\mathbf{p}_{q,i,m})^H + \sum_{j\neq i} \Lambda_{q,j}^{-1}\mathbf{H}_{q,j}\mathbf{P}_j(\mathbf{H}_{q,j}\mathbf{p}_j)^H + \sigma_{n,q}^2\mathbf{I}_{n_r}$$

The MMSE receive combiner for stream l is: $\mathbf{g}_{q,l} = \Lambda_{q,i}^{-1/2}(\mathbf{H}_{q,i}\mathbf{p}_{q,i,l})^H\mathbf{R}_{ni}$.

The Signal to Interference plus noise Ratio (SINR) experied by stream l of user q in cell i is: $\rho_{q,l} = \frac{\Lambda_{q,i}^{-1}|\mathbf{g}_{q,l}\mathbf{H}_{q,i}\mathbf{p}_{q,i,l}|^2}{\mathbf{g}_{q,l}\mathbf{R}_{ni}\mathbf{g}_{q,l}^H}$. The maximum achievalbe instantaenous rate achievable by user q in cell i is $R_{q,i} = \sum_{l=1}^{n_{u,q}} \log_2(1+\rho_{q,l})$.

It should be noticed that the above all equations apply only to a particular precoder. The receiver needs to calculate this for all the precoders in the codebook (2x16) at each estimation instant. The maximum achievable instantaneous rate is fed back to transmitter as Channel Quality Indicator (CQI) and is used in calculation of long term average rate in proportional fair scheduling. Optimal number of streams is fed back as Rank Indicator (RI) thus combined with Precoding Matrix Indicator(PMI), it tells transmitter which precoder to use.

2 CDF of user long term SINR

In this section, cumulative distribution(CDF) of user long term signal to interference plus noise ratio(SINR) is the focus. In the long term, fluctuation of MIMO fading channel would be ignored, therefore path loss plus shadowing would be the only multiplicative factor for signal (and interference). For user q in cell i, the long term SINR is given by:

$$\rho_{LT,q} = \frac{\Lambda_{q,i}^{-1} E_{s,i}}{\sigma_{n,q}^2 + \sum_{j \neq i} \Lambda_{q,j}^{-1} E_{s,j}}$$

By dropping a large number of i.i.d users, the simulated result is given by Figure 1. The long term SINR for the majority of the users is between -10dB to 30dB. Roughly 75% of the users would have long term SINR higher than 0dB. This can be viewed as a rough network performance estimation.

The conjecture from the shape of CDF is confirmed by MATLAB Kolmogorov-Smirnov test, which confirms that the data points in simulation is normally distributed, with mean equal to 8.50dB and standard deviation of 12.96dB. On after-thought, this is perfectly reasonable: each user and its long term SINR should be independent and identically distributed (i.i.d). From Law of Large Numbers, many i.i.d instances of this random variable would follow Gaussian distribution, regardless of individual user probability model of long term SINR value.

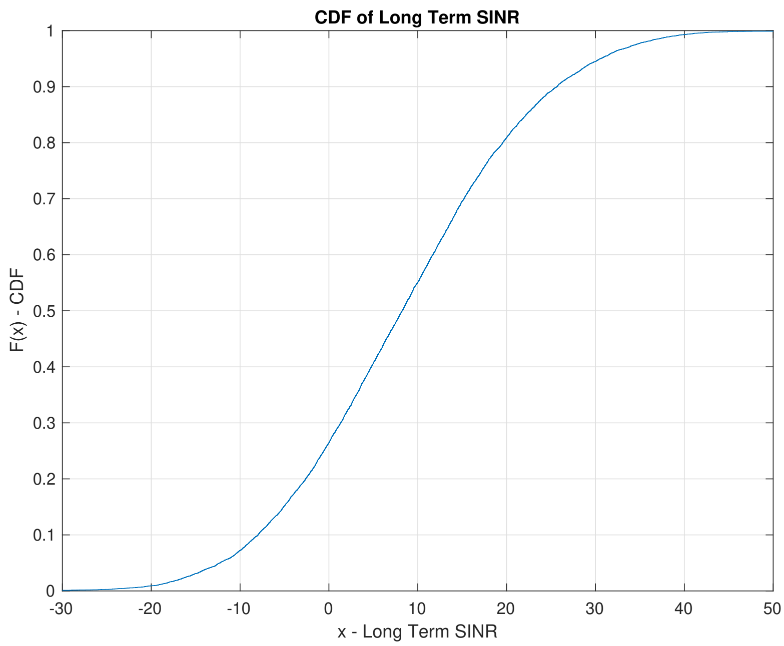


Figure 1: CDF of long term SINR

3 Influence of number of antennas

This section exploits how the number of receive antennas impact the performance in terms of CDF of user average rate. If each UE has only 1 receive antenna, the channel effectively becomes a multi-user multiple input single output (MISO) channel. In a MISO channel, benefits like array gain and diversity gain are possible to exploit, but no spatial multiplexing gain could be enjoyed. In contrast, MIMO channel offers multiplexing gain, which could be interpreted as the ability to transmit in parallel several independent streams. $R \approx g_s \log 2(\rho)$, the pre-log factor of the rate at high SNR is the multiplexing gain. $g_s \approx n = \min(n_r, n_t)$. In this coursework, $n_t = 4$ is fixed. If $n_r = 1$, g_s becomes unity thus no benefit. If $n_r = 2$, there could be a multiplexing gain of 2. This is reflected on Figure 2: at any CDF value, user average rate of 2 antennas is always higher than that with only 1 antenna.

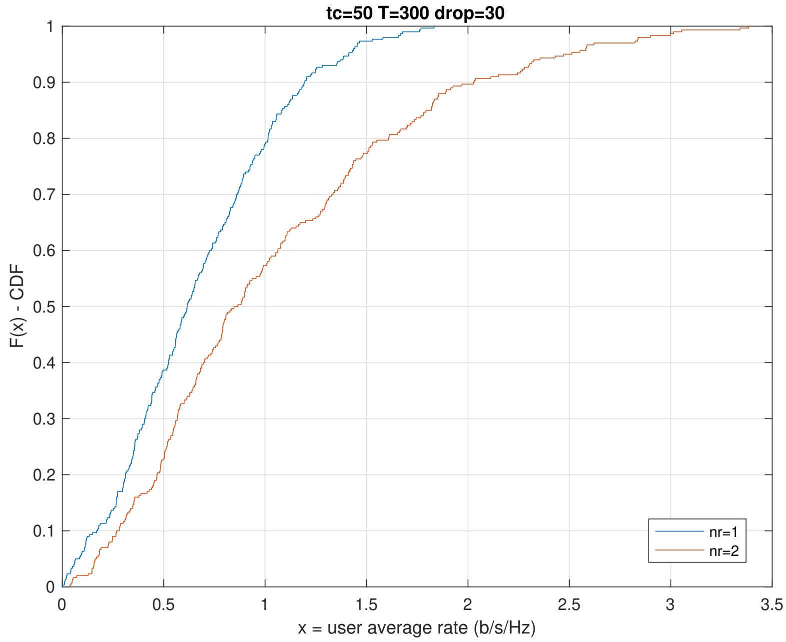


Figure 2: CDF of user average rate: influence of number of receive antennas

Although kstest shows that user average rate follows Gaussian distribution for both single and double receive antennas, histogram (experimental probability distribution function) shows that both have a obvious tail on the right, especially for two receive antennas. (See Figure 3). Approximately 15% of data points for $n_r = 2$ is higher than the maximum data point of $n_r = 1$. Qualitatively speaking, with multiple antenna, the linear precoder has the flexibility to decide whether to transmit single symbol stream or multiple symbol stream, thus increases the likelihood of higher capacity.

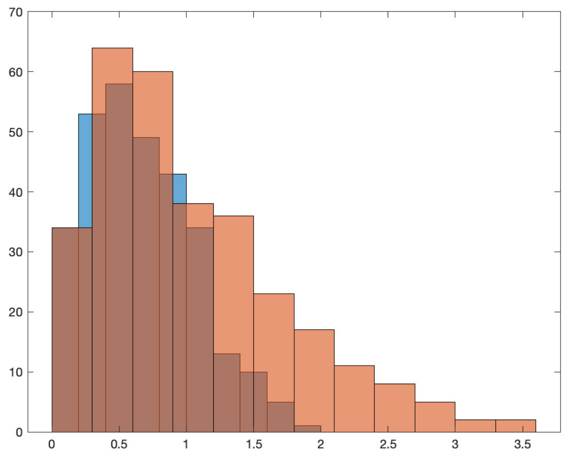
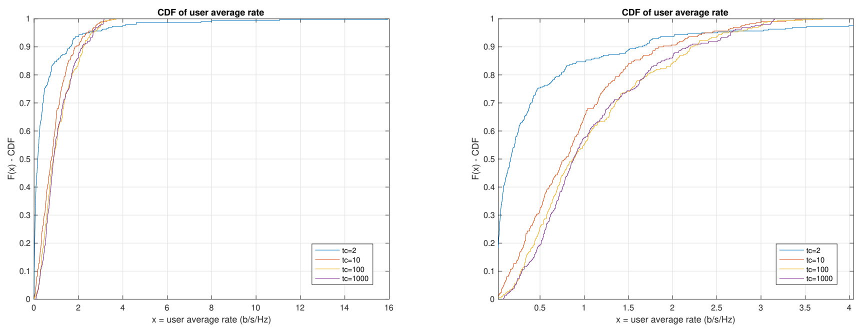


Figure 3: Histogram of user average rate: ($n_r = 1$: blue, n_r=2: orange)

4 Influence of scheduling time scale

This section exploits how the scheduling time scale t_c of the proportional fair scheduler impacts the performance in terms of CDF of user average rate. The bigger the value of of t_c, the longer the drop needs to be. Necessary value for user average rate to converge has been tested in 'Testground4.m'. Figure 4 shows that when t_c increases, the proportional of user which can have relatively higher average rate, for example more than 1 bits/s/Hz, increases. Although the shift is small, one possible explanation is that when $1-1/t_c$ becomes smaller as t_c increases, so user average rate would decay in a slower speed when the user is not scheduled.



(a) Overview

(b) Zoom

Figure 4: Influence of scheduling time scale on user average rate

When t_c is small, for example when $t_c = 2$: first, if user q is not scheduled, then $\bar{R}_q(k+1) = \frac{1}{2}\bar{R}_q(k)$. The user average rate would decay in an exponential speed. If t_c is just slightly greater then 1, then decay would be even faster. Therefore most of the user average rate in simulation snapshot has a relatively low value. Second, if user q is scheduled, then $\bar{R}_q(k+1) = 1/2\bar{R}_q(k) + \frac{1}{2}CQI$, where CQI is the instantaneous user achievable rate. This means $\bar{R}_q(k+1)$ would have a large upward jump in value. This explains the large variance of data points for $t_c = 2$ on the figure, and explains the possibility to observe very high rate in simulation snapshot.

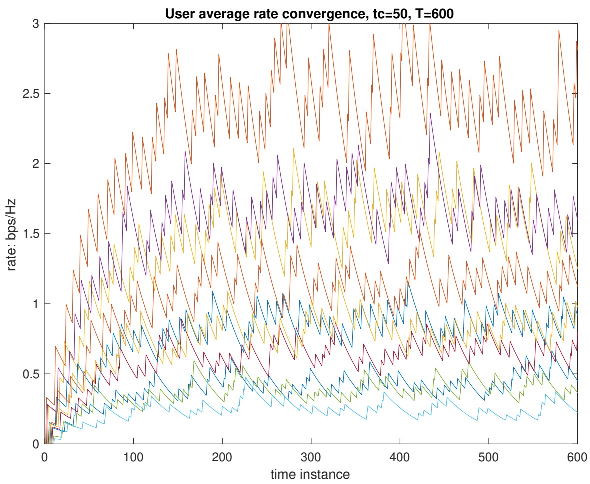


Figure 5: Average Rate convergence simulation

When t_c has medium value, e.g. $t_c = 100$, simulation show typical behaviour of proportional fair scheduling. Using Testground4.m and adjusting the constants (recommend $t_c = 50, T = 600$), convergence of user average rate could be clearly observed. By detail observation of Figure 5, we can see after convergence, the average rate of each user would decrease not for long before it gets boosted. That's when the user got schedule. Also difference user would have difference equilibrium average rate (height of lines with different colours), all the users are 'frequently' transmitted. This is exactly what Proportional Fair scheduling wants to achieve.

At very large scheduling time scale, $t_c = 10^n, n > 4$, the PF scheduler becomes effectively a rate maximization scheduler. If user is scheduled, its average rate would not change noticeably, because the additional term, $\frac{1}{t_c}R_q(k)$, would be very small. If the user is not scheduled, its average rate still does not change much, as $1 - \frac{1}{t_c} \approx= 1$. Assuming each user has the same Quality of Service, little change in user average rate means little change in weight of each user ($w_q = \frac{QoS}{\bar{R}_q}$). Therefore in stability, the BS will always transmit to the user with best achievable rate until channel condition changes. Unfortunately simulating large value like $t_c = 10000$ mean at least $T = 30000$ is needed, this takes too long to complete on a personal computer and is not presented in any figure in this report.

5 Influence of channel time correlation and number of user

This section exploits how the channel time correlation ϵ and number of users in the cell impact the performance in terms of CDF of user average rate.

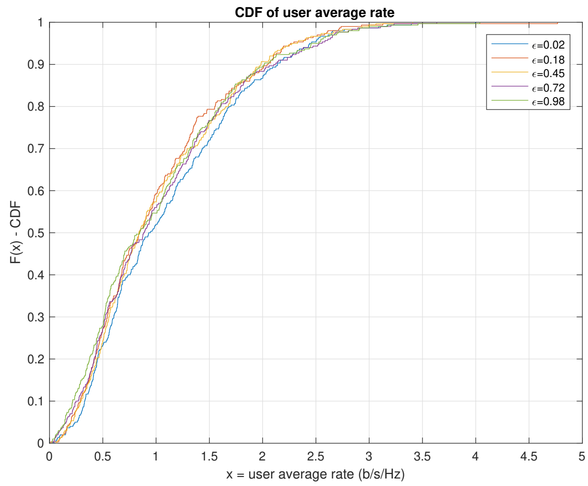
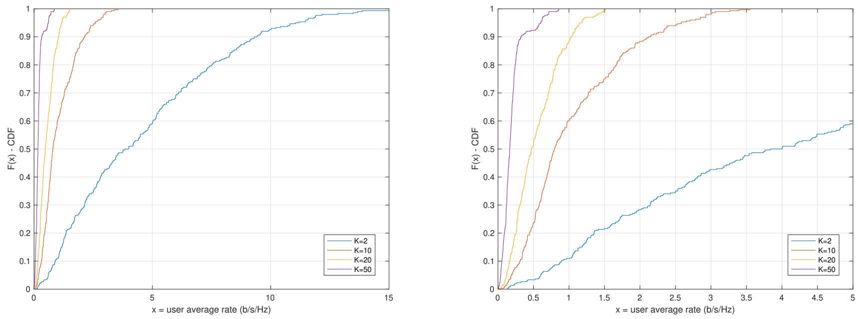


Figure 6: Influence of time correlation)

Figure 6 shows that the time correlation value ϵ doesn't really change the CDF of user average rate.

One possible reason is that, the deployment and transmission scheme in this coursework exploits a form of selection diversity, as it only transmit to one user at a time. If the channel is fast-varying, that is $0 \approx \epsilon << 1$ (see Section 1.3), peaks of user channel strength of users can be utilized in high frequency. Scheduler may switch to a user with better channel condition in each shorter average transition cycle (although still balanced by fairness). If the channel is slowly-varying, that is $0 << \epsilon \approx 1$, the channel condition for each user changes slowly. Scheduler could choose to transmit to a user for a longer time as long as the channel condition is still "good". Since channel varies slowly, the good channel could sustain longer. The multi-user diversity, that is be able to choose from pool of users with diverse channel condition, makes the impact of channel time correlation negligible under this coursework setting.

Another interpretation of this constant ϵ is the velocity of the user: the higher the velocity, the smaller the time correlation. However, this may not apply in this coursework setting. In this coursework, users are dropped randomly. But i.i.d path-loss plus shadowing of each user is fixed in simulation when time index evolves forward. This means the location of each user in simulation is stationary during after each drop. In this perspective the user velocity is always zero in this coursework.



(a) Overview

(b) Large number of users

Figure 7: Influence of number of users K on CDF of user average rate

It is clear from Figure 7a and 7b, that when the number of users in the centre cell increases, the CDF of user average rate shifts leftward (or rotate anti-clockwise). When the cell is under-loaded, e.g. K=2, more than 50% of the user could have rate greater than 3.5bits/s/Hz, while almost none of the user in other more crowed group could have this rate. For the over-crowded cell, for example K=50, almost no user can have rate higher than 1bits/s/Hz, which could be considered as pretty low. One simple explanation is that when K increases, each user would on average be scheduled less often by proportional fair scheduler. Each user needs to on average wait longer before they get transmitted for one scheduling period, therefore the user average rate is lower by definition.

6 Influence of spatial correlation

This section exploits how the magnitude of channel spatial correlation t (t_space in code) impacts the performance in terms of CDF of user average rate. (See expressions in Section 1.3)

By observing Figure 8, it is clear that when magnitude of spatial correlation (t_space) increases, the user average rate would decrease. This is more obvious for $0.7 < CDF < 1$ part. In previous tasks, the value of t_space is fixed, so $\mathbf{R}$ also has a fixed norm. In this task, changing spatial correlation from 0 to 1 means norm of $\mathbf{R}$ = would vary from 1 to 4. (See Testground5.m) Therefore each $\mathbf{R}$ generated during user dropping has been normalized before multiplied with the spatially uncorrelated Rayleigh fading channel matrix. This has made the lines in simulation output less entangled.

Spatial correlation in channel matrix could be influenced by a lot of factors. It might be simplified to the Kronecker Product of transmitter and receiver antenna correlation. Whether scattering is isotropic or highly directional will affect t_space. Also, antenna spacing and antenna angle spread are two very important factors.

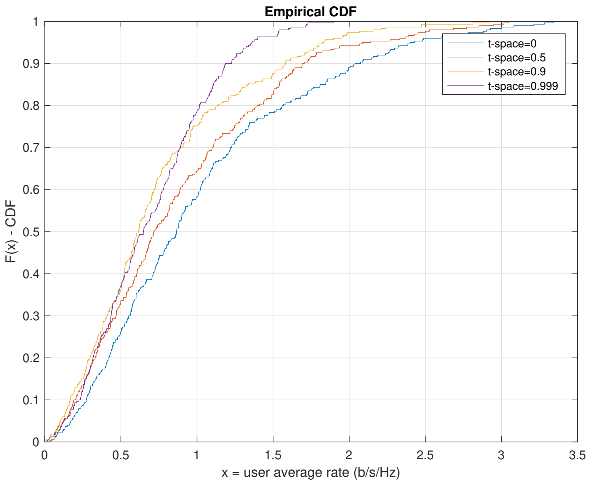


Figure 8: Influence of spatial correlation)

The eigenvalues of **R** are only function of the magnitude t. As t approaches 1, one eigenvalue would approach 0. This would make **R** and channel matrix more singular or badly conditioned. This in turn affects the 'separability' of virtual data pipes, (or strength of the weaker/smaller data pipe) thus reduces the effective multiplexing capability. Therefore higher spatial correlation reduces the user average rate.