ELEC97094/ELEC97095 Wireless Communications **Coursework 3**: System-level Performance Evaluation of LTE 4Tx MIMO Downlink

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1 Objective

Evaluate (using Matlab computer simulations) the performance of LTE 4Tx Single-User MIMO with quantized precoding using a simplified system level evaluation methodology.

2 Deployment

We consider a downlink transmission from one base station (BS) to multiple users (UE), as illustrated in Figure 1. The BS serves $K = 10$ UEs randomly and uniformly dropped (at a distance $> 35m$ and $< 250m$ from the BS) in a centre cell. The transmission is subject to interference from 6 neighboring base stations. The transmit power at each base station is fixed to 46dBm. The noise variance at the UE is fixed to -174dBm. The BSs are all equipped with $n_t = 4$ transmit antennas and the UEs are equipped with either $n_r = 1$ or 2 receive antennas. All deployments and channel model parameters are listed in Table 1.

Table 1: Deployment and channel model parameters

Parameter	**Explanation/Assumption**
Transmit power	46dBm
Noise variance	-174dBm
Number of users K dropped in the centre cell	$K = 10$ (baseline). Other values to be investigated to assess impact on performance.
Path Loss [dB]	$128.1 + 37.6 \log_{10}(d)$ with d the BS-user distance [km]
Shadowing model	Log-normal shadowing with 8dB standard deviation.
Shadowing correlation	0 for all links.
Antenna configurations $n_r \times n_t$	1×4 (baseline), 2×4

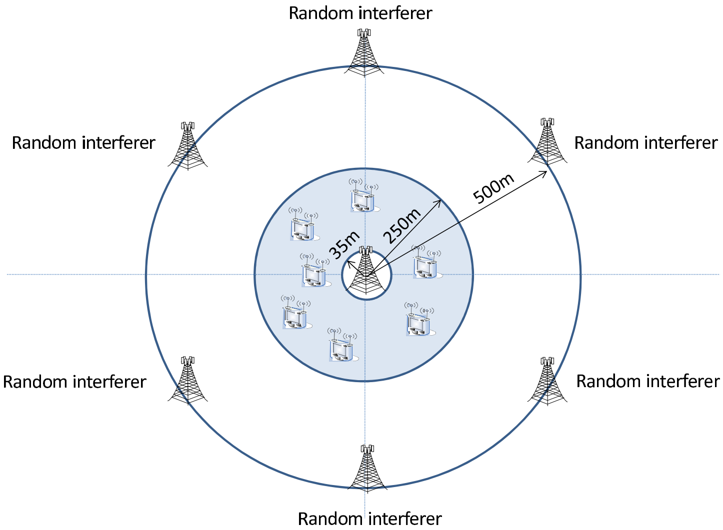

Figure 1: Deployment scenario: 1 centre BS/cell and 6 interfering BSs.

3 MIMO Channel Model

We assume for simplicity that the MIMO channel is flat fading and writes at time instant k between BS i and UE q as

$$\mathbf{H}_{k,q,i} = \tilde{\mathbf{H}}_{k,q,i}\mathbf{R}_{t,q,i}^{1/2} \tag{1}$$

where

- $\tilde{\mathbf{H}}_{k,q,i} \in \mathbb{C}^{n_r \times n_t}$ represents a spatially uncorrrelated Rayleigh flat fading channel matrix, whose entries are i.i.d. according to $\mathcal{CN}(0,1)$. The evolution of $\tilde{\mathbf{H}}_{k,q,i}$ is modeled by a first-order Gauss-Markov process

$$\tilde{\mathbf{H}}_{k,q,i} = \epsilon\tilde{\mathbf{H}}_{k-1,q,i} + \sqrt{1-\epsilon^2}\mathbf{N}_{k,q,i}. \tag{2}$$

  $\mathbf{N}_{k,q,i} \in \mathbb{C}^{n_r \times n_t}$ has i.i.d. entries with distribution $\sim \mathcal{CN}(0,1)$ and

$$\mathcal{E}\left\{\text{vec}\left(\tilde{\mathbf{H}}_{k-1,q,i}\right)\text{vec}\left(\mathbf{N}_{k,q,i}\right)^H\right\} = \mathbf{0}_{n_r n_t \times n_r n_t}. \tag{3}$$

  The time correlation ϵ represents the correlation between entries of $\tilde{\mathbf{H}}_{k,q,i}$ and $\tilde{\mathbf{H}}_{k-1,q,i}$.

- $\mathbf{R}_{t,q,i}$ is the transmit correlation matrix of BS i-user q link. Denoting by $i=0$ the centre BS and by $i=1,\ldots,6$ the interfering BS, a simplified model is given by the exponential structure

$$\mathbf{R}_{t,q,i} = \begin{bmatrix} 1 & t_{q,i} & t_{q,i}^2 & t_{q,i}^3 \\ t_{q,i}^* & 1 & t_{q,i} & t_{q,i}^2 \\ t_{q,i}^{*2} & t_{q,i}^* & 1 & t_{q,i} \\ t_{q,i}^{*3} & t_{q,i}^{*2} & t_{q,i}^* & 1 \end{bmatrix}. \tag{4}$$

  Assume for simplicity that $t_{q,i} = 0\ \forall q$ for $i = 1,\ldots,6$ and $t_{q,0} = te^{j\phi_q}\ \forall q$ where t is the magnitude of the correlation coefficient (same for all users) and ϕ_q is a user-specific phase of the correlation coefficient, randomly distributed between 0 and 2π.

Further details on the MIMO channel assumption are provided in Table 2.

Table 2: Deployment and channel model parameters

Parameter	**Explanation/Assumption**
Time correlation ϵ	$\epsilon = 0.85$ (baseline). Other values to be investigated to assess the impact of time correlation on performance.
Spatial correlation t	$t = 0.5$ (baseline). Other values to be investigated to assess the impact of spatial correlation on performance.

4 Transmission Scheme

We aim at investigating the performance of LTE Single-User MIMO, consisting in Spatial Multiplexing with quantized precoding (as defined by LTE specifications). In the centre cell, the BS schedules one UE at a time (using Spatial Multiplexing with quantized precoding) so as to maximize a proportional fairness metric. To do so, the UEs report a Precoding Matrix Indicator (PMI), a Channel Quality Indicator (CQI) and a Rank Indicator (RI). It is assumed that there is no delay between the feedback and the transmission and no feedback errors on the uplink. The precoder is chosen in a codebook of precoders defined for RI = 1 to 2. LTE 4Tx codebook is provided in the appendix [1] (yellow text). The RI and PMI together determines the index of the preferred precoder in the codebook. RI refers to the number of streams (denoted as Number of Layers in LTE specifications) that are transmitted to the UE and PMI is the index of the preferred precoder in the codebook corresponding to RI. The UE selects the RI and PMI that maximizes its rate. The CQI refers to the rate achievable by the selected RI and PMI. MMSE is considered as the baseline receiver. It could be designed differently depending on the assumptions on the knowledge of the inter-cell interference characteristics. Further details on the system-level assumptions are provided in Table 3.

The performance is measured in terms of the cumulative distribution function (CDF) of the user average rate ($bits/s/Hz$). To do so, you will have to generate X drops of K users. Each drop will last for T time instants (where T should be sufficiently long such that the scheduler rate converges). At each time instant, the instantaneous rate achieved by each user is recorded (PS: note that if a user is not scheduled at a given time instant, its instantaneous rate is 0). At the end of the drop, the average rate achieved by each user is computed (as an average over all instantaneous rates). After X drops, XK values of user average rate are available and the CDF can be computed.

5 Tasks

The following tasks should be performed:

1. Write the system model. Detail the expressions of the achievable rate and receiver combiner.

2. Show the cumulative distribution function (CDF) of the user long term SINR. The long term SINR is calcluated by ignoring the MIMO fading channel and only accounts for path loss and shadowing. For user q in cell i, the long-term SINR is given by

$$SINR_{LT,q} = \frac{\Lambda_{q,i}^{-1} E_{s,i}}{\sigma_{n,q}^2 + \sum_{j \neq i} \Lambda_{q,j}^{-1} E_{s,j}}.$$

Table 3: System-level assumptions

Parameter	Explanation/Assumption
Interference modeling	Random precoding, i.e. the precoder is randomly generated at each interfering BS and is selected from the LTE codebook
Scheduling	Proportional fair in time domain
Transmission mode	Single-user MIMO
Codebook of precoder	4Tx LTE codebook (see Appendix)
Feedback information	RI (1-bit), PMI (4-bit), CQI (unquantized)
Feedback delay/error	no delay, no error
Receiver	MMSE
Rank adaptation	the UE dynamically selects the preferred tranmission rank (RI)
Power allocation among streams	uniform power allocation among streams for a given transmission rank
Coding and modulation, link adaptation/abstraction	Shannon capacity expression, i.e. it is assumed that the rate achievable with the transmission of a stream is given by $\log_2(1 + SINR)$ where $SINR$ is the instantaneous signal to interference plus noise ratio.

To do so, drop a large number of users and evaluate $SINR_{LT,q}$ for all of them. Plot the CDF of the long term SINR.

3. Show and discuss the influence of the number of receive antennas (1×4 vs. 2×4) on the performance (CDF of user average rate).

4. Show and discuss the influence of the scheduling time scale t_c of the proportional fair scheduler on the performance (CDF of user average rate).

5. Show and discuss the influence of velocity/time correlation ϵ and the number of users K on the performance (CDF of user average rate).

6. Show and discuss the influence of the spatial correlation t on the performance (CDF of user average rate).

6 Recommendation

The project requires that you combine several fundamental concepts learned throughout the course. Mastering the following concepts/problems will help you to address the tasks requested in the project:

- Generate random variables using matlab.
- Derive the expression of the MMSE combiner and the SINR at the output of the combiner.
- Generate using matlab spatially and temporally correlated MIMO channels as defined above.
- Generate the average rate vs SNR of MIMO transmission strategies.

- Derive the MMSE combiner with SM transmission, compute the SINR for each stream and the achievable rate.
- Be familiar with the operation of quantized precoding and be able to derive the MMSE combiner with a SM transmission based on quantized precoding.
- Be familiar with PF scheduler.
- Write a multi-cell system model, compute the long-term SINR, instantaneous SINR and achievable rate.

When you run simulations, you should make sure the drops are long enough such that the scheduler converges.

7 Deliverables

The project is conducted **individually** using Matlab. All Matlab files must have been written by yourself. Each student is requested to submit (on Blackboard)

1. A pdf **report** detailing the results. Format: Font size 10 pt, maximum 10 pages, single-spacing. In your report, you should clarify whether your simulation results are inline with the theory and explain the rationale behind the observations.

2. **All Matlab files** with comments. The files should be self-explanatory and the examiner should be able to run the code and get the same results as those provided in the report. Explain how to run the code.

Deadline for report submission on Blackboard: **17 March 2020, 23:59:00** (London time). Oral interview (about 15min per person): **19 and 20 March 2020**. Schedule to come.

A Appendix: LTE 4Tx codebook [1]

Table 6.3.4.2.2-1: Large-delay cyclic delay diversity.

Number of layers υ	U	$D(i)$
2	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & e^{-j2\pi/2}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & e^{-j2\pi i/2}\end{bmatrix}$
3	$\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & e^{-j2\pi/3} & e^{-j4\pi/3}\\ 1 & e^{-j4\pi/3} & e^{-j8\pi/3}\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 0\\ 0 & e^{-j2\pi i/3} & 0\\ 0 & 0 & e^{-j4\pi i/3}\end{bmatrix}$
4	$\frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & e^{-j2\pi/4} & e^{-j4\pi/4} & e^{-j6\pi/4}\\ 1 & e^{-j4\pi/4} & e^{-j8\pi/4} & e^{-j12\pi/4}\\ 1 & e^{-j6\pi/4} & e^{-j12\pi/4} & e^{-j18\pi/4}\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & e^{-j2\pi i/4} & 0 & 0\\ 0 & 0 & e^{-j4\pi i/4} & 0\\ 0 & 0 & 0 & e^{-j6\pi i/4}\end{bmatrix}$

6.3.4.2.3 Codebook for precoding and CSI reporting

For transmission on two antenna ports, $p \in \{0,1\}$, and for the purpose of CSI reporting based on two antenna ports $p \in \{0,1\}$ or $p \in \{15,16\}$, the precoding matrix $W(i)$ shall be selected from Table 6.3.4.2.3-1 or a subset thereof. For the closed-loop spatial multiplexing transmission mode defined in [4], the codebook index 0 is not used when the number of layers is $\upsilon = 2$.

Table 6.3.4.2.3-1: Codebook for transmission on antenna ports $\{0,1\}$ and for CSI reporting based on antenna ports $\{0,1\}$ or $\{15,16\}$.

Codebook index	Number of layers υ	
	1	2
0	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
1	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$
2	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ j\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1 & 1\\ j & -j\end{bmatrix}$
3	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -j\end{bmatrix}$	-

For transmission on four antenna ports, $p \in \{0,1,2,3\}$, and for the purpose of CSI reporting based on four antenna ports $p \in \{0,1,2,3\}$ or $p \in \{15,16,17,18\}$, the precoding matrix W shall be selected from Table 6.3.4.2.3-2 or a subset thereof. The quantity $W_n^{\{s\}}$ denotes the matrix defined by the columns given by the set $\{s\}$ from the expression $W_n = I - 2u_n u_n^H / u_n^H u_n$ where I is the 4×4 identity matrix and the vector u_n is given by Table 6.3.4.2.3-2.

Table 6.3.4.2.3-2: Codebook for transmission on antenna ports $\{0,1,2,3\}$ and for CSI reporting based on antenna ports $\{0,1,2,3\}$ or $\{15,16,17,18\}$.

Codebook index	u_n	Number of layers υ			
		1	2	3	4
0	$u_0=\begin{bmatrix}1 & -1 & -1 & -1\end{bmatrix}^T$	$W_0^{\{1\}}$	$W_0^{\{14\}}/\sqrt{2}$	$W_0^{\{124\}}/\sqrt{3}$	$W_0^{\{1234\}}/2$
1	$u_1=\begin{bmatrix}1 & -j & 1 & j\end{bmatrix}^T$	$W_1^{\{1\}}$	$W_1^{\{12\}}/\sqrt{2}$	$W_1^{\{123\}}/\sqrt{3}$	$W_1^{\{1234\}}/2$
2	$u_2=\begin{bmatrix}1 & 1 & -1 & 1\end{bmatrix}^T$	$W_2^{\{1\}}$	$W_2^{\{12\}}/\sqrt{2}$	$W_2^{\{123\}}/\sqrt{3}$	$W_2^{\{3214\}}/2$
3	$u_3=\begin{bmatrix}1 & j & 1 & -j\end{bmatrix}^T$	$W_3^{\{1\}}$	$W_3^{\{12\}}/\sqrt{2}$	$W_3^{\{123\}}/\sqrt{3}$	$W_3^{\{3214\}}/2$
4	$u_4=\begin{bmatrix}1 & (-1-j)/\sqrt{2} & -j & (1-j)/\sqrt{2}\end{bmatrix}^T$	$W_4^{\{1\}}$	$W_4^{\{14\}}/\sqrt{2}$	$W_4^{\{124\}}/\sqrt{3}$	$W_4^{\{1234\}}/2$
5	$u_5=\begin{bmatrix}1 & (1-j)/\sqrt{2} & j & (-1-j)/\sqrt{2}\end{bmatrix}^T$	$W_5^{\{1\}}$	$W_5^{\{14\}}/\sqrt{2}$	$W_5^{\{124\}}/\sqrt{3}$	$W_5^{\{1234\}}/2$
6	$u_6=\begin{bmatrix}1 & (1+j)/\sqrt{2} & -j & (-1+j)/\sqrt{2}\end{bmatrix}^T$	$W_6^{\{1\}}$	$W_6^{\{13\}}/\sqrt{2}$	$W_6^{\{134\}}/\sqrt{3}$	$W_6^{\{1324\}}/2$
7	$u_7=\begin{bmatrix}1 & (-1+j)/\sqrt{2} & j & (1+j)/\sqrt{2}\end{bmatrix}^T$	$W_7^{\{1\}}$	$W_7^{\{13\}}/\sqrt{2}$	$W_7^{\{134\}}/\sqrt{3}$	$W_7^{\{1324\}}/2$
8	$u_8=\begin{bmatrix}1 & -1 & 1 & 1\end{bmatrix}^T$	$W_8^{\{1\}}$	$W_8^{\{12\}}/\sqrt{2}$	$W_8^{\{124\}}/\sqrt{3}$	$W_8^{\{1234\}}/2$
9	$u_9=\begin{bmatrix}1 & -j & -1 & -j\end{bmatrix}^T$	$W_9^{\{1\}}$	$W_9^{\{14\}}/\sqrt{2}$	$W_9^{\{134\}}/\sqrt{3}$	$W_9^{\{1234\}}/2$
10	$u_{10}=\begin{bmatrix}1 & 1 & 1 & -1\end{bmatrix}^T$	$W_{10}^{\{1\}}$	$W_{10}^{\{13\}}/\sqrt{2}$	$W_{10}^{\{123\}}/\sqrt{3}$	$W_{10}^{\{1324\}}/2$
11	$u_{11}=\begin{bmatrix}1 & j & -1 & j\end{bmatrix}^T$	$W_{11}^{\{1\}}$	$W_{11}^{\{13\}}/\sqrt{2}$	$W_{11}^{\{134\}}/\sqrt{3}$	$W_{11}^{\{1324\}}/2$
12	$u_{12}=\begin{bmatrix}1 & -1 & -1 & 1\end{bmatrix}^T$	$W_{12}^{\{1\}}$	$W_{12}^{\{12\}}/\sqrt{2}$	$W_{12}^{\{123\}}/\sqrt{3}$	$W_{12}^{\{1234\}}/2$
13	$u_{13}=\begin{bmatrix}1 & -1 & 1 & -1\end{bmatrix}^T$	$W_{13}^{\{1\}}$	$W_{13}^{\{13\}}/\sqrt{2}$	$W_{13}^{\{123\}}/\sqrt{3}$	$W_{13}^{\{1324\}}/2$
14	$u_{14}=\begin{bmatrix}1 & 1 & -1 & -1\end{bmatrix}^T$	$W_{14}^{\{1\}}$	$W_{14}^{\{13\}}/\sqrt{2}$	$W_{14}^{\{123\}}/\sqrt{3}$	$W_{14}^{\{3214\}}/2$
15	$u_{15}=\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}^T$	$W_{15}^{\{1\}}$	$W_{15}^{\{12\}}/\sqrt{2}$	$W_{15}^{\{123\}}/\sqrt{3}$	$W_{15}^{\{1234\}}/2$

For the purpose of CSI reporting for eight CSI reference signals the codebooks are given in section 7.2.4 of [4].

6.3.4.3 Precoding for transmit diversity

Precoding for transmit diversity is only used in combination with layer mapping for transmit diversity as described in Section 6.3.3.3. The precoding operation for transmit diversity is defined for two and four antenna ports.

For transmission on two antenna ports, $p \in \{0,1\}$, the output $y(i)=\begin{bmatrix}y^{(0)}(i) & y^{(1)}(i)\end{bmatrix}^T$, $i=0,1,...,M_{\text{symb}}^{\text{ap}}-1$ of the precoding operation is defined by

$$\begin{bmatrix} y^{(0)}(2i) \\ y^{(1)}(2i) \\ y^{(0)}(2i+1) \\ y^{(1)}(2i+1) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & j & 0 \\ 0 & -1 & 0 & j \\ 0 & 1 & 0 & j \\ 1 & 0 & -j & 0 \end{bmatrix} \begin{bmatrix} \mathrm{Re}\left(x^{(0)}(i)\right) \\ \mathrm{Re}\left(x^{(1)}(i)\right) \\ \mathrm{Im}\left(x^{(0)}(i)\right) \\ \mathrm{Im}\left(x^{(1)}(i)\right) \end{bmatrix}$$

for $i=0,1,...,M_{\text{symb}}^{\text{layer}}-1$ with $M_{\text{symb}}^{\text{ap}}=2M_{\text{symb}}^{\text{layer}}$.

For transmission on four antenna ports, $p \in \{0,1,2,3\}$, the output $y(i)=\begin{bmatrix}y^{(0)}(i) & y^{(1)}(i) & y^{(2)}(i) & y^{(3)}(i)\end{bmatrix}^T$, $i=0,1,...,M_{\text{symb}}^{\text{ap}}-1$ of the precoding operation is defined by

References

[1] 3rd Generation Partnership Project (3GPP), "3GPP TS 36.211, Technical Specification Group Radio Access Network, Evolved Universal Terrestrial Radio Access (E-UTRA), Physical Channels and Modulation (Release 11)," Dec. 2012.