Wireless Communication Coursework 1

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Introduction

This coursework practiced the evaluation of the error rate performance at link level of basic SISO/SIMO/MISO transmission and reception schemes using MATLAB. The bit error rate (BER) is evaluated for Signal to Noise Ratio (SNR) ranging from 0dB to 20dB. In each experiment, a random bit stream of length 10^6 is generated. This is chosen because at 20dB, BER could be as low as around 10^{-5}. So a long bit stream of sufficient length would help measure the experimental BER more accurately. If it takes too long to run the code provided, lengh of bit stream could be reduced and SNR iteration could be changed from every 0.1dB to larger interval. Each MATLAB script is independent to run.

1 Uncoded QPSK transmission over a SISO Rayleigh fading channel

The channel model is $y = \sqrt{Es}hc + n$, where y is the received signal, h is the channel, assumed to be Rayleigh distributed, c is the symbol transmitted and n is an Additive White Gaussian Noise (AWGN). All variables are scalar and time index is omitted. Assume channel is estimated perfectly at receiver, using maximum likelihood detection: $z = yh^*$, where z is the output of detector and is the input into QPSK demodulator.

The bit error rate (BER) of SISO fading channel is given by equation: $\overline{P} = 1 - \sqrt{\rho/(1+\rho)}$, where ρ is the input SNR. In Figure 1, it is represented by the red line;the BER simulated from code is in blue line. It can be seen from figure that for SNR higher than 10dB, the experiment data line plotted (in blue) converges to the theoretical line. So the simulation confirms the thoery. On a log-log scale, BER decreases approximately linearly with increase in SNR, so BER in Rayleigh fading channel is much higher than that in a Additive White Gaussian Noise channel.

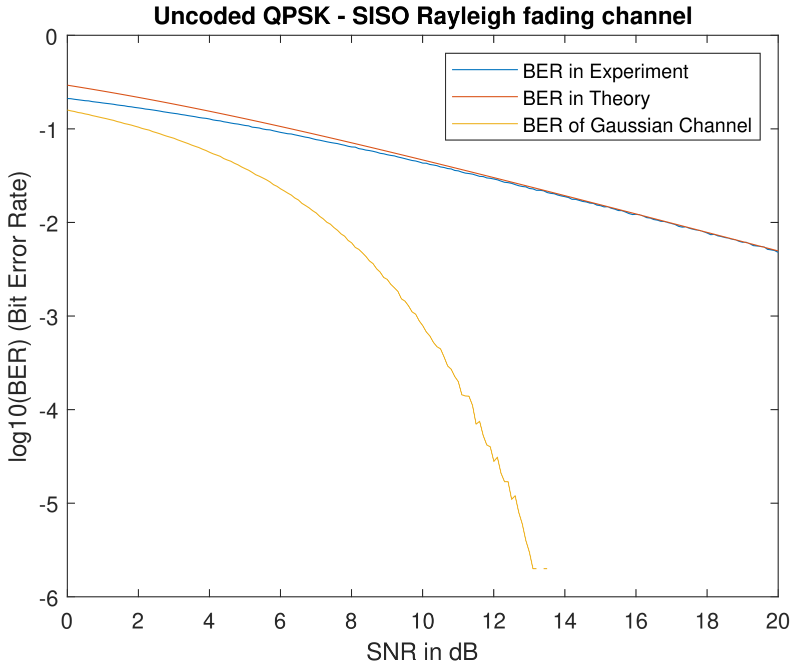
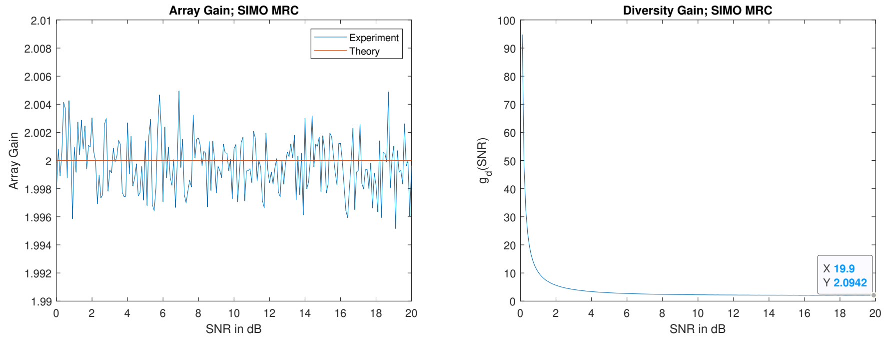


Figure 1: BER in SISO Rayleigh fading channel

2 Uncoded QPSK transmission over a SIMO i.i.d Rayleigh fading channel with MRC and two receive antennas

The channel model is $\mathbf{y} = \sqrt{Es}\mathbf{h}c + \mathbf{n}$, where $\mathbf{y}$ is the signal vector at receiver antennas, $\mathbf{h}$ is the channel vector, assumed to i.i.d be Rayleigh distributed, c is the symbol transmitted and n is an AWGN. Assume channel vecotr is perfectly known, using Maximal Ratio Combining: $g_n = h_n^*$ and $z = \mathbf{gy}$, where z is the output of combiner.

In theory, at high SNR, BER becomes $\overline{P} = (4\rho)^{n_r}\binom{2n_r-1}{n_r}$; so the diversity gain should be $g_d = n_r = 2$. From 2b, we can see that at high SNR, diversity gain calculated from experiment data converges to 2, so it meets the expectation from theory. At low SNR, experimental diversity gain could be much higher, suggesting MRC has more benefit in low SNR region. The theoretical output SNR of combiner is $\overline{\rho}_{out} = \rho n_r$, so the array gain in theory is $g_a = n_r = 2$. From 2a, the experimental array gain oscillates around 2, so confirms the theory.



(a) Array Gain (b) Diversity Gain

Figure 2: SIMO i.i.d Rayleigh fading channel with MRC: BER and Diversity Gain

3 Uncoded QPSK transmission over a MISO i.i.d Rayleigh fading channel with MRT with two transmit antennas

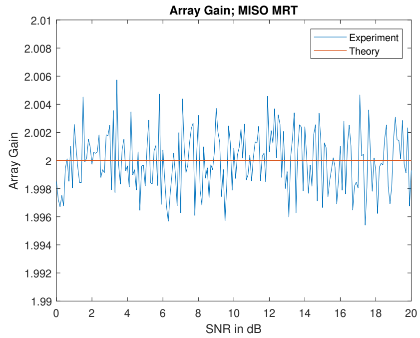
The channel model is $y = \sqrt{Es}\mathbf{hc'} + n = \sqrt{Es}\mathbf{hw}c + n$, where y is the received signal, $\mathbf{h}$ is a horizontal channel vector, assumed to i.i.d be Rayleigh distributed, $\mathbf{w}$ is a weight vector or precoder or beamformer, c is the symbol transmitted and n is an AWGN. Assume channel vector is perfectly known at transmitter, Matched Beamforming or Maximal Ratio Transmission is used: $\mathbf{w} = \mathbf{h}^H/||\mathbf{h}||$.

The symbol error rate is upper-bounded at high SNR by $\overline{P} \leq \overline{N}_e(\rho d_{min}^2/4)^2$, so theoretical diversity gain equal to $n_t = 2$. From Figure 3b, it is clear that diversity gain is initially larger but converges to 2 at high SNR as theory expected. The theoretical output SNR of combiner is $\overline{\rho}_{out} = \rho n_t$, so the array gain in theory is $g_a = n_t = 2$. From 3a, the experimental array gain oscillates around 2, so confirms the theory. Comparing MRT to MRC of SIMO in the previous section, they have the same performance.

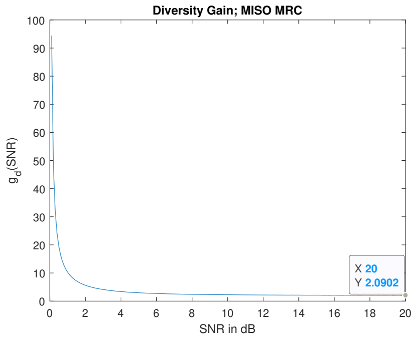
4 Alamouti Scheme with QPSK over a MISO i.i.d Rayleigh fading channel and two transmit antennas

Alamouti Scheme doesn't require channel knowledge at transmitter, but utilizes Space Time Coding over two symbol periods:

$\mathbf{y} = [y_1, y_2*]^T = \sqrt{Es}\mathbf{H}_{eff}\mathbf{c} + \mathbf{n}$. $\mathbf{H}_{eff} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$. $\mathbf{c} = \begin{bmatrix} c_1/\sqrt{2} \\ c_2/\sqrt{2} \end{bmatrix}$. $[z_1, z_2]^T = \mathbf{H}_{eff}^H\mathbf{y} = \sqrt{Es}(|h_1|^2 + |h_2|^2)\mathbf{I}_2\mathbf{c} + \mathbf{H}_{eff}^H\mathbf{n}$.



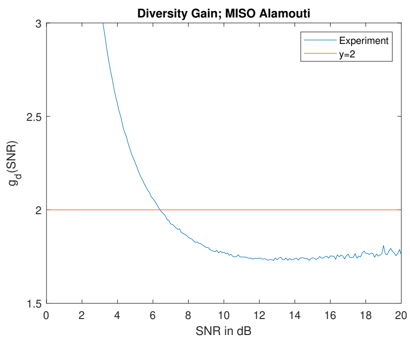
(a) Array Gain



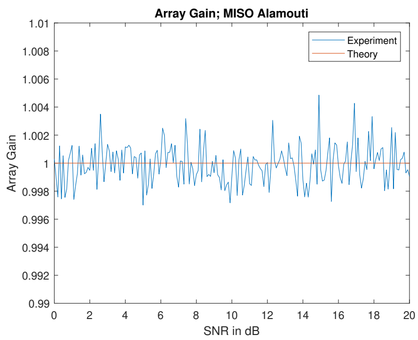
(b) Diversity Gain

Figure 3: MISO i.i.d Rayleigh fading channel with MRT: SNBout and Array Gain

The symbol error rate is upper-bounded at high SNR by $\overline{P} \leq \overline{N}_e(\rho d_{min}^2/8)^2$, so diversity gain should equal to $n_t = 2$. However, as observed from figure, diversity gain measured from experiement data is still below 2 at 20dB. Using Alamouti_playground.m, it can be confirmed that diversity gain in simulation would converge towards 2 at higher SNR. This is probably because Alamouti scheme doesn't have CSIT, diversity is more effectivly utilized at higher SNR. As no energy is beamformed towards receiver, there should be no array gain. And simulation confirms this: Figure 4b.



(a) Diversity Gain



(b) Array Gain

Figure 4: MISO i.i.d Rayleigh fading channel with Alamouti Scheme

5 Comparisons

Due to the page limit, the bit error rate and output SNR of the 4 scenarios are combined to one graph, in order to save space and make comparisons. The .mat data files in the folder serve this purpose. The figure 5 below compares the Bit Error Rate performance of all the above scenarious. It is clear that without the diversity gain of multiple antennas, SISO has the most gentle gradient at high SNR. MISO MRT and SIMO MRC has the same performace, because they are just maximal ratio combining at different places. Compare to these two, Alamouti scheme has the same gradient but is shifted upwards on graph because it has no array gain. The figure 6 below compares the output SNR performance of the 3 multiple-0antenna scenarious. MISO MRT and SIMO MRC has the same performace, although the blue line is almost covered by green line so not very visible. It is clear that on these two line, at each point Y is 3dB larger than X, confirming the array gain of $g_a = No.Antennas = 2$. The red line below confirms the theoretical expectation that Alamouti scheme does not have array gain.

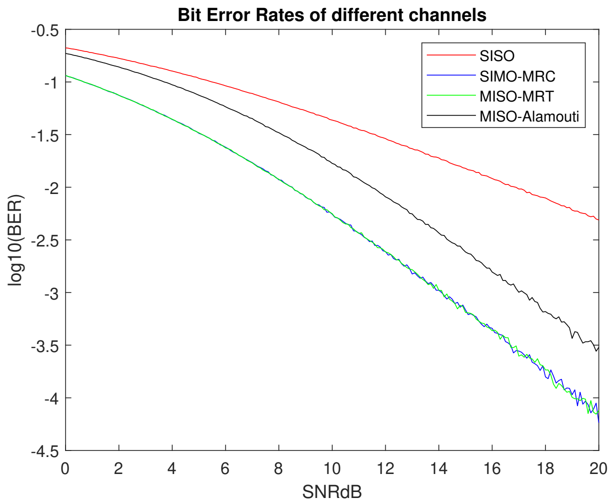


Figure 5: BER in different scenarious

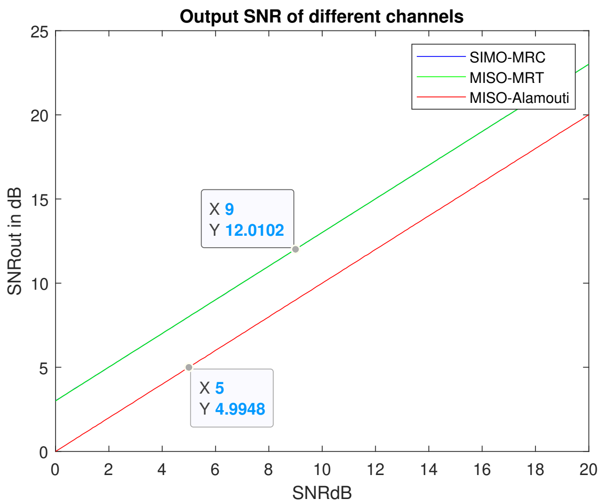


Figure 6: Output SNR in different scenarious