# Implementing Join

## Join

1/87

DBMSs are engines to *store*, *combine* and *filter* information.

Filtering is achieved via selection and projection.

The *join* operation (⋈) is the primary means of *combining* information.

Because *join* is

- such an important operation in database applications/systems
- potentially very expensive to execute

many methods have been developed for its implementation.

(We use a running example to compare costs of the various join processing methods)

### ... Join

2/87

Types of join:

- simple equijoin (single-equality condition)

```
select * from R,S where R.i = S.j
```

- partial-match join (conjunction of equality conditions)

```
select * from R,S where R.a = S.b and R.c = S.d ...
```

- theta join (arbitrary expression as condition)

```
select * from R,S where R.a < S.b and R.c <> S.d ...
```

Focus on simple equijoin, since common in practice (`R.pk=S.fk`)

## Join Example

3/87

Consider a university database with the schema:

```
create table Student(
   id     integer primary key,
   name   text,  ...
);
create table Enrolled(
   stude  integer references Student(id),
   subj   text references Subject(code),  ...
);
create table Subject(
   code   text primary key,
   title  text,  ...
);
```

And the following request on this database:

*List names of students in all subjects, arranged by subject.*

### ... Join Example

4/87

The result of this request would look like:

```
Subj      Name
--------  -----------------
COMP1011  Chen Hwee Ling
```

```
COMP1011  John Smith
COMP1011  Ravi Shastri
...
COMP1021  David Jones
COMP1021  Stephen Mao
...
COMP3311  Dean Jones
COMP3311  Mark Taylor
COMP3311  Sashin Tendulkar
```

---

### ... Join Example

5/87

An SQL query to provide this information:

```
select E.subj, S.name
from   Student S, Enrolled E
where  S.id = E.stude
order  by E.subj, S.name;
```

And its relational algebra equivalent:

$Sort_{[subj]}$ ( $Project_{[subj,name]}$ ( $Join_{[id=stude]}(Student,Enrolled)$ ) )

The core of the query is the join $Join_{[id=stude]}(Student,Enrolled)$

To simplify writing of formulae, $S = Student$, $E = Enrolled$.

---

### ... Join Example

6/87

Some database statistics:

| Sym | Meaning | Value |
|---|---|---|
| $r_S$ | # student records | 20,000 |
| $r_E$ | # enrollment records | 80,000 |
| $C_S$ | `Student` records/page | 20 |
| $C_E$ | `Enrolled` records/page | 40 |
| $b_S$ | # data pages in `Student` | 1,000 |
| $b_E$ | # data pages in `Enrolled` | 2,000 |

Also, in cost analyses below, $N$ = number of memory buffers.

---

### ... Join Example

7/87

`Out` = $Student \bowtie Enrolled$ relation statistics:

| Sym | Meaning | Value |
|---|---|---|
| $r_{Out}$ | # tuples in result | 80,000 |
| $C_{Out}$ | result records/page | 80 |
| $b_{Out}$ | # data pages in result | 1,000 |

Notes:

- $r_{Out}$ ... one result tuple for each `Enrolled` tuple
- $C_{Out}$ ... result tuples have only `subj` and `name`

---

## Join via Cross-product

8/87

Join can be defined as a cross-product followed by selection:

*Join[Cond](R,S) = Select[Cond]( R × S )*

For the example query, could implement

*Join[id=stude](Student,Enrolled)*

as

*Select[id=stude](Student × Enrolled)*

Cross-product contains *20,000 × 80,000 = 1,600,000,000* tuples.

---

### ... Join via Cross-product

9/87

For `Temp` = *(Student × Enrolled)*

I/O costs:

- size of `Temp` relation $r = 16 \times 10^8$ records
- assuming $C_{Temp}=16$, then $b_{Temp} = 10^8$
- `Temp` is written once, then scanned once
- total I/O = $10^8.(T_w+T_r)$

Assuming $T_w=T_r=0.01s$, this will take around 500 hours!

---

### ... Join via Cross-product

10/87

Because

- cross-products are infrequent in practice (except to describe join)
- cross-products are large (typically **much** larger than the final join result)

DBMSs do **not** implement join via cross-product.

DBMSs implement only join and provide cross-product as:

*R × S = Join[true](R,S)*

or, in SQL

```
select * from R,S
```

---

# Nested-Loop Join

---

## Nested Loop Join

12/87

The simplest join algorithm:

- iteratively generates the cross-product
- checks join condition on each tuple

Algorithm to compute *Join[Cond](R,S)*:

```
for each tuple r in R {
    for each tuple s in S {
        if ((r,s) satisfies join condition) {
            add (r,s) to result
}   }   }
```

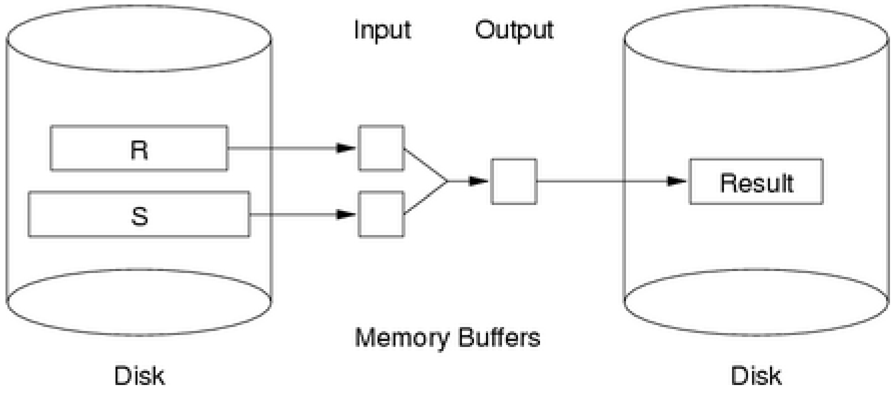
*R* is the *outer* relation; *S* is the *inner* relation.

---

### ... Nested Loop Join

13/87

Requires (at least) three memory buffers (2 input, 1 output).




### ... Nested Loop Join

14/87

Abstract algorithm for *Join[Cond](R,S)* (with 3 memory buffers):

```
for each page of relation R {
    read into buffer rBuf
    for each page of relation S {
        read into buffer sBuf
        for each record r in rBuf {
            for each record s in sBuf {
                if ((r,s) satisfies Cond) {
                    add combined(r,s) to OutBuf
                    write Outbuf when full
}   }   }   }   }
```

### ... Nested Loop Join

15/87

Detailed algorithm for *Join[Cond](R,S)* (with 3 memory buffers):

```
// rf: file for R, sf: file for S, of: output file
outp = 0; clearBuf(oBuf);
for (rp = 0; rp < nPages(rf); rp++) {
   readPage(rf, rp, rBuf);
   for (sp = 0; sp < nPages(sf); sp++) {
      readPage(sf, sp, sBuf);
      for (i = 0; i < nTuples(rBuf); i++) {
         rTup = getTuple(rBuf, i);
         for (j = 0; j < nTuples(sBuf); j++) {
            sTup = getTuple(sBuf, j);
            if (satisfies(rTup,sTup,Cond)) {
            rsTup = combine(rTup,sTup);
            addTuple(oBuf, rsTup);
            if (isFull(oBuf)) {
               writePage(of, outp++, oBuf);
               clearBuf(oBuf);
}   }   }   }    }   }
```

### ... Nested Loop Join

16/87

The three-memory-buffer nested loop join requires:

- read all $b_R$ pages of *R* once
- for each of page of *R*, read $b_S$ pages of *S*

Cost = $b_R + b_R\, b_S$

If we use *S* as the outer relation in the join

Cost = $b_S + b_S\, b_R$

It is (slightly) better to use smaller relation as outer relation.

## Nested Loop Join on Example



If `Student` is outer relation and `Enrolled` is inner:

$$\begin{aligned} \text{Cost} &= b_S + b_S\, b_E \\ &= 1{,}000 + 1{,}000 \times 2{,}000 = 2{,}001{,}000 \end{aligned}$$

If `Enrolled` is outer relation and `Student` is inner:

$$\begin{aligned} \text{Cost} &= b_E + b_E\, b_S \\ &= 2{,}000 + 2{,}000 \times 1{,}000 = 2{,}002{,}000 \end{aligned}$$

Cost of nested-loop join is too high (5 hours, if $T_r$=0.01 sec)

---

## Implementing Join Better



Aims of effective join computation:

- generate only relevant tuples from the cross-product
- generate these tuples with minimal disk I/O

Range of costs for *Join(R,S)*

- worst case cost = $b_R + b_R b_S$ (nested loop join)
- best case cost = $b_R + b_S$ (read each page once)

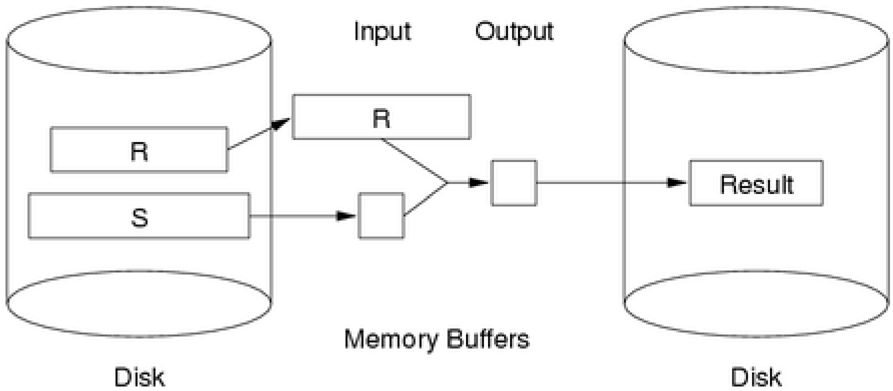
---

## Block Nested Loop Join



If at least $b_R$+2 memory buffers available:

- read the entire *R* relation into memory
- for each *S* page, check join condition on all `(r,s)` pairs

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### ... Block Nested Loop Join



Algorithm for nested loop join with $b_R$+2 memory buffers:

```
read all of R's pages into memory buffers
for each page of relation S {
    read page into S's input buffer
    for each tuple s in S's buffer {
        for each tuple r in R's memory buffers {
            if ((r,s) satisfies JoinCond)) {
                add (r,s) to output buffer
                write output buffer when full
}   }   }   }
```

Note that *R* effectively becomes the inner relation in this scheme.

### ... Block Nested Loop Join

21/87

This method requires:

- read $b_R$ pages of relation $R$ into buffers
- while $R$ is buffered, read $b_S$ pages of $S$

Cost = $b_R + b_S$

Notes:

- minimal I/O cost, but considers all *(r,s)* pairs
- thus, requires $r_R.r_S$ checks of the join condition

---

### ... Block Nested Loop Join

22/87

Further performance improvements:

- must reduce number of $R$ tuples matched against each $S$ tuple
- use access method to find small set of $R$ tuples matching $S$ tuple

Example:

- each $S$ joins with $k \ll r_R$ tuples of $R$
- $R$ tuples are stored in sorted array of memory buffers
- for each $S$ tuple, use binary search to find matching buffer
- scan around that buffer to find all matching *(R,S)* pairs
- requires approx $C_R.r_S$ checks of join condition

---

## Block Nested Loop Join on Example

23/87

If $\geq 1002$ memory buffers are available:

- read `Student` relation into memory
- scan `Enrolled` relation, computing join

$$\begin{aligned}\text{Cost} &= b_S + b_E \\ &= 1{,}000 + 2{,}000 = 3{,}000\end{aligned}$$

This is considerably better than $10^6$ (30 secs vs 5 hours).

But what if we have only $N$ memory buffers, where $N < b_R$ , $N < b_S$?

---

### ... Block Nested Loop Join on Example

24/87

In general case, read *outer* relation in runs of *N-2* pages

```
for each run of N-2 pages from R {
    read N-2 of R's pages into memory buffers
    for each page of relation S {
        read page into S's input buffer
        for each tuple s in S's buffer do
            for each tuple r in R's memory buffers {
                if ((r,s) satisfies JoinCond)) {
                    add (r,s) to output buffer
                    write output buffer when full
}   }   }   }   }
```

---

### ... Block Nested Loop Join on Example

25/87

Block nested loop join requires

- read $\lceil b_R/N\text{-}2 \rceil$ runs from $R$
- for each run, scan $b_S$ pages of $S$

Cost = $b_R + b_S \cdot \lceil b_R/N\text{-}2 \rceil$

Notes:

- the final run will typically be "short" (i.e. $< N\text{-}2$ pages)
- unless index/hash is used, we still do $r_R.r_S$ tuple comparisons

---

### ... Block Nested Loop Join on Example

26/87

Costs for various buffer pool sizes:

| *N* | Inner | Outer | #runs | Cost |
|---|---|---|---|---|
| 22 | `Student` | `Enrolled` | 50 | 101,000 |
| 52 | `Student` | `Enrolled` | 20 | 41,000 |
| 102 | `Student` | `Enrolled` | 10 | 21,000 |
| 1002 | `Student` | `Enrolled` | 1 | 3,000 |
| 22 | `Enrolled` | `Student` | 100 | 102,000 |
| 52 | `Enrolled` | `Student` | 40 | 42,000 |
| 102 | `Enrolled` | `Student` | 20 | 22,000 |
| 1002 | `Enrolled` | `Student` | 2 | 4,000 |

---

## Block Nested Loop Join in Practice

27/87

Why block nested loop join is very useful in practice ...

Many queries have the form

```
select * from R,S where r.i=s.j and r.x=k
```

This would typically be evaluated as

*Join [i=j] ((Sel[r.x=k](R)), S)*

If *|Sel[r.x=k](R)|* is small ⇒ may fit in memory (in small #buffers)

---

## Join Conditions and Methods

28/87

Nested loop join makes no assumptions about join conditions.

```
for each pair of tuples (r,s) {
    check join condition on (r,s)
    if satisfied, add to results
}
```

To improve join:

- reduce the number of tuple pairs considered
- but not easy to do for arbitrary join condition

As noted above, simple equijoin is a common join condition.

Thus, a range of other join algorithms has been developed specifically for equality join conditions.

---

## Index Nested Loop Join

29/87

Most joins considered so far have a common problem:

- repeated scans of *entire* inner relation $S$ are required

If there is an index on $S$, we can avoid such repeated scanning.

Consider $Join_{[R.i=S.j]}(R,S)$:

```
for each tuple r in relation R {
    use index to select tuples
        from S where s.j = r.i
    for each selected tuple s from S {
        add (r,s) to result
}   }
```

(For ordered indexes (e.g. Btree), this also assists join conditions like $R.i<S.j$)

---

### ... Index Nested Loop Join

30/87

This method requires:

- one scan of $R$ relation ($b_R$)
  - only one buffer needed, since we use $R$ tuple-at-a-time
- for each *tuple* in $R$ ($r_R$), one index lookup on $S$
  - cost depends on type of index and number of results
  - best case is when each $R.i$ matches few $S$ tuples

Cost = $b_R + r_R.Sel_S$ ($Sel_S$ is the cost of performing a select on $S$).

---

### ... Index Nested Loop Join

31/87

For index lookup:

- cost of locating first matching tuple
  - for B+ trees, typically 2-4 page reads
  - for hashing, typically 1-2 page reads
- cost of finding other matching tuples
  - if clustered, typically 1-2 page reads
  - if unclustered, up to $b_q$ page reads

Note: building an index "on the fly" to perform a join can be very cost-effective.

---

## Index Nested Loop Join on Example

32/87

Case 1: $Join_{[id=stude]}(Student,Enrolled)$

- `Student` is outer and `Enrolled` is inner
- `Enrolled` has a clustered B+ tree index on `stude` field
- B+ tree has depth 3 (root + internal + leaf)
- most of the time, the four matching records are in a single page

Cost = $b_S + r_S\ btree_E$

= $1,000 + 20,000 \times (3+1.01) = 80,000$

---

### ... Index Nested Loop Join on Example

33/87

Case 2: $Join_{[id=stude]}(Student,Enrolled)$

- `Student` is outer and `Enrolled` is inner
- `Enrolled` has an unclustered B+ tree index on `stude` field
- B+ tree has depth 3 (root + internal + leaf)
- assume worst case; matching records are all on different pages

Cost $= b_S + r_S\ btree_E$

$= 1{,}000 + 20{,}000 \times (3+4) = 150{,}000$

---

### ... Index Nested Loop Join on Example

34/87

Case 3: *Join[id=stude](Student,Enrolled)*

- `Enrolled` is outer and `Student` is inner
- `Student` is hashed on `id` field (e.g. linear hashing)
- there may be (short) overflow chains (e.g. 1.1 page reads/bucket)

Cost $= b_E + r_E\ hash_S$

$= 2{,}000 + 80{,}000 \times 1.1 = 90{,}000$

---

## Optimised Index Nested Loop Join

35/87

Consider the following scenario for *Join[R.i=S.j](R,S)*:

- `R.i` is not a primary key (so many tuples have same `R.i` value)
- `R` is sorted on `R.i` (or could be efficiently sorted on `R.i`)
- each `R.i` value does not match very many tuples

Could save repeated index scans with the same `R.i` value

- cache results of index scan for `R.i=`*k* in buffer
- if next `R` tuple also has `R.i=`*k*, re-use scan results

---

### ... Optimised Index Nested Loop Join

36/87

Abstract algorithm for optimised index nested loop join:

```
for each tuple r in relation R {
   if (prev == r.i)
      use selected tuples in buffer(s)
   else {
      use index to select tuples
         from S where s.j = r.i
      store selected tuples in buffer(s)
   }
   for each selected tuple s from S
      add (r,s) to result
   prev = r.i
}
```

Cost savings depend on repetition factor, #buffers, size of index scans

---

# Sort-Merge Join

---

## Sort-Merge Join

38/87

Basic approach:

- sort both relations on join attribute (reminder: *Join[R.i=S.j](R,S)*)
- scan together using merge to form result `(r,s)` tuples

Advantages:

- no need to deal with "entire" *S* relation for each *r* tuple
- deal with runs of matching *R* and *S* tuples

Disadvantages:

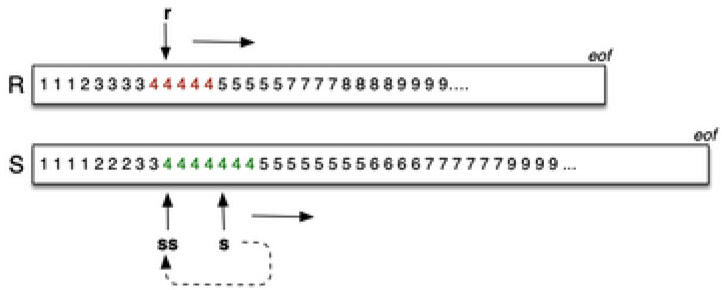
- cost of sorting both relations (relations may be sorted on join key?)
- some rescanning required when long runs of *S* tuples

---

### ... Sort-Merge Join 39/87

Method requires several cursors to scan sorted relations:

- `r` = current record in *R* relation
- `s` = start of current run in *S* relation
- `ss` = current record in current run in *S* relation




---

### ... Sort-Merge Join 40/87

Abstract algorithm for merge phase of *Join[R.i=S.j](R,S)*:

```
r = first tuple in R
s = first tuple in S
while (r != eof and s != eof) {
    // align cursors to start of next common run
    while (r != eof and r.i < s.j) { r = next tuple in R }
    while (s != eof and r.i > s.j) { s = next tuple in S }
    // scan common run, generating result tuples
    while (r != eof and r.i == s.j) {
        ss = s   // set to start of run
        while (ss != eof and ss.j == r.i) {
            add (r,s) to result
            ss = next tuple in S
        }
        r = next tuple in R
    }
    s = ss   // start search for next run
}
```

---

## Sidetrack: Iterators 41/87

Sort-merge join implementation is simplified by use of iterators.

- iterators give the appearance of tuple-at-a-time
- even when the underlying data is page-by-page
- and even in the pesence of auxiliary index structures

Typical usage of iterator:

```
Iterator iter; Tuple tup;
iter = startScan("Rel","i=5");
while ((tup = nextTuple(iter)) != NULL) {
    process(tuple);
}
endScan(iter);
```

---

### ... Sidetrack: Iterators 42/87

```
typedef struct {
    File   inf;  // input file
```

```
    Buffer buf;  // buffer holding current page
    int    curp; // current page during scan
    int    curr; // index of current record in page
} Iterator;

// simple linear scan; no condition
Iterator *startScan(char *relName) {
    Iterator *iter = malloc(sizeof(Iterator));
    iter->inf  = openFile(fileName(relName),READ);
    iter->curp = 0;
    iter->curr = -1;
    readPage(iter->inf, iter->curp, iter->buf);
}
```

---

### ... Sidetrack: Iterators

43/87

```
Tuple nextTuple(Iterator *iter) {
    // check if reached end of current page
    if (iter->curr == nTuples(iter->buf)-1) {
        // check if reached end of data file
        if (iter->curp == nPages(iter->inf)-1)
            return NULL;
        iter->curp++;
        iter->buf = readPage(iter->inf, iter->curp);
        iter->curr = -1;
    }
    iter->curr++;
    return getTuple(iter->buf, iter->curr);
}
// curp and curr hold indexes of most recently read page/record
```

---

### ... Sidetrack: Iterators

44/87

```
TupleID scanCurrent(Iterator *iter) {
    // form TupleID for current record
    return iter->curp + iter->curr;
}

void setScan(Iterator *iter, int page, int rec) {
    assert(page >= 0 && page < nPages(iter->inf));
    if (iter->curp != page) {
        iter->curp = page;
        readPage(iter->inf, iter->curp, iter->buf);
    }
    assert(rec >= 0 && rec < nTuples(iter->buf));
    iter->curr = rec;
}

void endScan(Iterator *iter) {
    closeFile(iter->buf);
    free(iter);
}
```

---

## Sort-Merge Join

45/87

Concrete algorithm using iterators:

```
Iterator *ri, *si;  Tuple rup, stup;

ri = startScan("SortedR");
si = startScan("SortedS");
while ((rtup = nextTuple(ri)) != NULL
       && (stup = nextTuple(si)) != NULL) {
    // align cursors to start of next common run
    while (rtup != NULL && rtup.i < stup.j)
           rtup = nextTuple(ri);
```

```
    if (rtup == NULL) break;
    while (stup != NULL && rtup.i > stup.j)
           stup = nextTuple(si);
    if (stup == NULL) break;
        // must have (r.i == s.j) here
...
```

---

### ... Sort-Merge Join

46/87

```
...
    // remember start of current run in S
    TupleID startRun = scanCurrent(si);
    // scan common run, generating result tuples
    while (rtup != NULL && rtup.i == stup.j) {
        while (stup != NULL and stup.j == rtup.i) {
            addTuple(outbuf, combine(rtup,stup));
            if (isFull(outbuf)) {
                writePage(outf, outp++, outbuf);
                clearBuf(outbuf);
            }
            stup = nextTuple(si);
        }
        rtup = nextTuple(ri);
        setScan(si, startRun);
    }
}
```

---

### ... Sort-Merge Join

47/87

Buffer requirements:

- for sort phase:
  - as many as possible (remembering that cost is $O(log_{\#Bufs})$ )
  - if insufficient buffers, sorting cost can dominate
- for merge phase:
  - one output buffer for result
  - one input buffer for relation *R*
  - (preferably) enough buffers for longest run in *S*

---

### ... Sort-Merge Join

48/87

Cost of sort-merge join.

Step 1: sort each relation (if not already sorted):

- Cost = $2.b_R\ (1 + log_{N-1}(b_R\ /N))\ +\ 2.b_S\ (1 + log_{N-1}(b_S\ /N))$
  (where $N$ = number of memory buffers)

Step 2: merge sorted relations:

- if every run of values in *S* fits completely in buffers,
  merge requires single scan, Cost = $b_R + b_S$
- if some runs in of values in *S* are larger than buffers,
  need to re-scan run for each corresponding value from *R*

---

## Sort-Merge Join on Example

49/87

Case 1: $Join_{[id=stude]}(Student,Enrolled)$

- *Student* and *Enrolled* already sorted on *id#*
- memory buffers *N=4*; all runs are of length $< 2$

Cost = $b_S + b_E\ =\ 3,000$ (i.e. minimal cost)

---

### ... Sort-Merge Join on Example

50/87

Case 2: $Join_{[id=stude]}(Student,Enrolled)$

- relations are not sorted on *id#*
- memory buffers $N=32$; all runs are of length $< 30$

$$
\begin{aligned}
\text{Cost} &= sort(S) + sort(E) + b_S + b_E \\
&= b_S \lceil \log_{30} b_S \rceil + b_E \lceil \log_{30} b_E \rceil + b_S + b_E \\
&= 1{,}000 \times 3 + 2{,}000 \times 3 + 1{,}000 + 2{,}000 \\
&= 12{,}000
\end{aligned}
$$

---

### ... Sort-Merge Join on Example

51/87

Case 3: $Join_{[id=stude]}(Student,Enrolled)$

- *Student* and *Enrolled* already sorted on *id#*
- memory buffers $N=3$ ($S$ input, $E$ input, output)
- one-quarter of the "runs" in $E$ span two pages
- there are no "runs" in $S$, since *id#* is a primary key

Cost depends on which relation is outer and which is inner.

---

### ... Sort-Merge Join on Example

52/87

Case 3 (continued) ...

If $E$ is outer relation:

- Cost = $b_E + b_S = 3{,}000$

If $S$ is outer relation:

- one-quarter of $E$ runs require two page reads
- each $E$ run is processed once for matching *S.id* value
- Cost = $b_S + b_E + r_S/4 = 8{,}000$

---

## Sidetrack 2: More on Iterators

53/87

Above description of iterators:

- involved simple scan of a single table
- with no condition to select tuples

In the general case, an iterator involves:

- one (selection) or two (join) tables
- with a condition to determine relevant tuples

A typical SQL query involves many iterators

- one for each relational operator in query plan
- connected in a demand-driven network of query nodes

---

### ... Sidetrack 2: More on Iterators

54/87

Requires a more general definition of execution state:

```
typedef struct {
    Oper   op;     // operation (sel,sort,join,...)
```

```
    Reln   r1;     // first relation
    Reln   r2;     // second relation (if any)
    Buffer *bufs; // buffers used by operation
    int    curp1; // index of current page for r1
    int    curr1; // index of current record in page
    int    curp2; // index of current page for r2
    int    curr2; // index of current record in page
    Cond   cond;  // condition for choosing tuple(s)
} Iterator;
```

For PostgreSQL details, see **`include/nodes/execnodes.h`**

---

# Hash Join

---

## Hash Join

56/87

Basic idea:

- use hashing as a technique to partition relations
- to avoid having to consider all pairs of tuples

Requires sufficent memory buffers

- to hold substantial portions of partitions
- (preferably) to hold largest partition of outer relation

Other issues:

- works only for equijoin `R.i=S.j` (but this is a common case)
- susceptible to data skew (or poor hash function)

Variations: *simple*, *grace*, *hybrid*.

---

## Simple Hash Join

57/87

Basic approach:

- hash part of the outer relation *R* into memory buffers (build)
- scan the inner relation *S*, using hash to search (probe)
    - if R.i=S.j, then h(R.i)=h(S.j) (hash to same buffer)
    - only need to check one memory buffer for each *S* tuple

Makes the assumption: whole of *S* hashes into memory

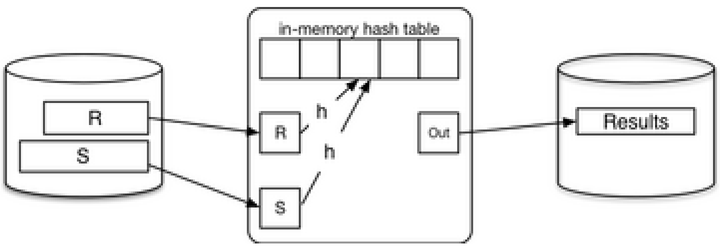
- requires *R* to be smaller than memory buffers
- requires a uniform hash function (no overflows)

---

### ... Simple Hash Join

58/87

Data flow:




---

### ... Simple Hash Join

59/87

Algorithm for ideal simple hash join *Join[R.i=S.j](R,S)*:

```
for each tuple r in relation R
   { insert r into buffer[h(R.i)] }
for each tuple s in relation S {
   for each tuple r in buffer[h(S.j)] {
      if ((r,s) satisfies join condition) {
         add (r,s) to result
      }
   }
}
```

Cost = $b_R + b_S$ (minimum possible cost)

---

### ... Simple Hash Join

60/87

Consider that we have *N* buffers available (2 input, 1 output, N-3 hash)

If $b_R \le N\text{-}3$ buffers, no need to hash (use nested loop).

In practice, size of hash table $b_{hR} > b_R$ (e.g. data skew)
⇒ hash table for *R* is even less likely to fit in memory

Can be handled by a variation on above algorithm:

- scan *R*, making hash table with *N-3* buffers
- once hash table built, scan *S* (standard probe phase)
- if more *R* tuples, build new table and repeat

---

### ... Simple Hash Join

61/87

Algorithm for realistic simple hash join *Join[R.i=S.j](R,S)*:

```
for each tuple r in relation R {
   if (buffer[h(R.i)] is full) {
      for each tuple s in relation S {
         for each tuple rr in buffer[h(S.j)] {
            if ((rr,s) satisfies join condition) {
               add (rr,s) to result
            }
         }
      }
      clear all hash table buffers
   }
   insert r into buffer[h(R.i)]
}
```

Note: requires multiple passes over the *S* relation.

---

### ... Simple Hash Join

62/87

Cost depends on *N* and on properties of data/hash.

Worst case:

- $h(i)=k$ so read only $C_R$ tuples before hash table "full"
- each hash table for *R* occupies one buffer with $C_R$ tuples
- degenerates to nested-loop-with-3-buffers case ⇒ $b_R + b_R b_S$

Best case:

- perfect uniform distribution of hash values
- each hash table of *R* holds $(N\text{-}3)C_R$ tuples from *N-3* pages
- number of hash tables built = $n_{hR} = \lceil b_R / (N\text{-}3) \rceil$
- read all of *S* for each hash table ⇒ $b_R + n_{hR}.b_S$

---

## Grace Hash Join



Basic approach:

- partition both relations on join attribute using hashing
- scan through corresponding pairs of partitions to form results

Similar approach to sort-merge join, except:

- sort-merge: partitioning achieved by sorting (runs)
- hash: partitioning achieved by hashing

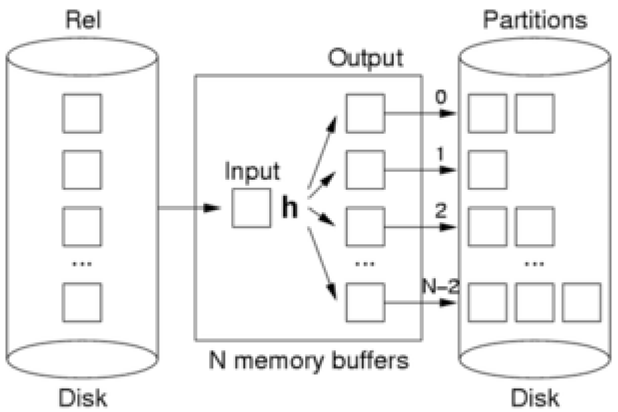
Requires enough buffer space to hold largest partition of inner relation.

---

### ... Grace Hash Join



Partition phase:



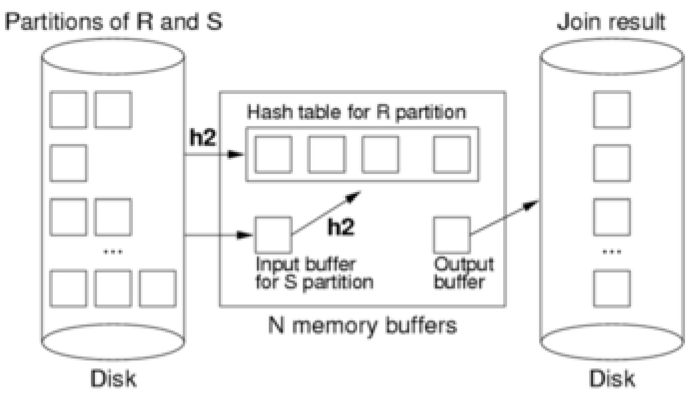

This is applied to each relation *R* and *S*.

---

### ... Grace Hash Join



Probe/join phase:




The second hash function (`h2`) simply speeds up the matching process.
Without it, would need to scan entire *R* partition for each record in *S* partition.

---

### ... Grace Hash Join



Abstract algorithm for *Join[R.i=S.j](R,S)*:

```
// assume h(val) generates [0..N-2]
// assume h2(val) generates [0..N-3]

// Partition phase (each relation -> N-1 partitions)
// 1 input buffer, N-1 output buffers

for each tuple r in relation R
    add r to partition h(r.i) in output file R'
```

```
for each tuple s in relation S
    add s to partition h(s.j) in output file S'
...
```

---

### ... Grace Hash Join

67/87

Abstract algorithm for *Join[R.i=S.j](R,S)* (cont.)

```
// Probe/join phase
// 1 input buffer for S, 1 output buffer
// N-2 buffers to build hash table for R partition

for each partition p = 0 .. N-2 {
    // Build in-memory hash table for partition p of R'
    for each tuple r in partition p of R'
        insert r into buffer h2(r.i)

    // Scan partition p of S', probing for matching tuples
    for each tuple s in partition p of S' {
        b = h2(s.j)
        for all matching tuples r in buffer b
            add (r,s) to result
}   }
```

---

### ... Grace Hash Join

68/87

Concrete algorithm for partitioning:

```
Buffer iBuf, oBuf[N-1];
File inf, outf[N-1]; char rel[100];
int i, r, h, ip, op[N-1]; Tuple tup;
for (i = 0; i < N-1; i++) {
    clearBuf(oBuf[i]);   op[i] = 0;
    rel = sprintf("%s%d","Rel",i);
    outf[i] = openFile(fileName(rel),WRITE));
}
inf = openFile(fileName("Rel"),READ);
for (ip = 0; ip < nPages(inf); ip++) {
    iBuf = readPage(inf, ip);
    for (r = 0; r < nTuples(iBuf); r++) {
        tup = getTuple(iBuf, r);
        h = hash(tup.i, N-1);
        addTuple(oBuf[h], tup);
        if (isFull(oBuf[h])) {
            writePage(outf[h], op[h]++, oBuf[h]);
            clearBuf(oBuf[h]);
}   }   }
```

---

### ... Grace Hash Join

69/87

Cost of grace hash join:

- #pages in all partition files of $Rel \cong b_{Rel}$ (maybe slightly more)
- partition relation *R* ... Cost = $b_R.T_r + b_R.T_w = 2b_R$
- partition relation *S* ... Cost = $b_S.T_r + b_S.T_w = 2b_S$
- probe/join requires one scan of each (partitioned) relation
  Cost = $b_R + b_S$
- all hashing and comparison occurs in memory $\Rightarrow$ $\cong 0$ cost

Total Cost = $3\ (b_R + b_S)$

---

### ... Grace Hash Join

70/87

The above cost analysis assumes:

- every partition of *R* fits in memory buffers at once

We achieve this situation if:

- data has uniform distribution
- hash function gives uniform distribution
  (all partitions are similar size)
- we have $N\text{-}1 \geq \lceil \sqrt{b} \rceil$ memory buffers
  (giving *N-1* partitions, each with $\cong b_R/(N\text{-}1)$ pages)

---

### ... Grace Hash Join

71/87

Possibilities for dealing with "over-long" partitions of *R*

- handle each over-long partition via scanning
  - requires over-long partitions to be scanned multiple times
  - essentially, such partitions are treated via nested loop join
- apply hash join recursively to over-long partitions
  - increases i/o by needing to partition parts of file multiple times
- use a different hash function with better distribution properties
  - but difficult to find such hash functions "on the fly"
- use the relation with the best partitioning as the "outer" relation

---

## Grace Hash Join on Example

72/87

For the example $Join_{[id=stude]}(Student,Enrolled)$:

- assume that we have a good hash function and $N = \sqrt{1000} = 32$

$$\text{Cost} = 3\,(b_S + b_E)$$

$$= 3\,(1{,}000 + 2{,}000) = 9{,}000$$

---

## Hybrid Hash Join

73/87

An optimisation if we have $\sqrt{b_R} < N < b_R{+}2$

- create *k* partitions using *N* buffers where $k \ll N$
- with grace join, would use *k* output buffers (one per partition)
- what to do with *N-k* remaining buffers? (ignore input buffer)
- use them to hold *m* partitions of *R* in memory (no disk writes)
- other partitions are handled as before (using *k-m* output buffers)

When we come to scan and partition *S* relation

- any tuple with hash in range *0..m-1* can be resolved
- other tuples are written to one of *k-m* partition files for *S*

Final phase is same as grace join, but with only *k-m* partitions.

---

### ... Hybrid Hash Join

74/87

Some observations:

- for *k* partitions, each partition has expected size $ceil(b_R/k)$
- holding *m* partitions in memory needs $m{\times}ceil(b_R/k)$ buffers
- since we have *k-m* output buffers, we must have $mb_R/k + (k\text{-}m) \leq N$
- for every partition/block held in memory, we save on disk i/o
- saving is $m/k \times 2(b_R + b_S)$

Other notes:

- if $N = b_R{+}2$, using block nested loop join is simpler

- cost depends on $N$ (but less than grace hash join)

---

### ... Hybrid Hash Join

75/87

Need to choose appropriate $m$ and $k$ to minimise cost

- base cost: $3 \times (b_R + b_S)$ (grace join)
- i/o saving: $m/k \times 2(b_R + b_S)$
- constraint: $mb_R/k + (k-m) \leq N$

Approach to maximise saving:

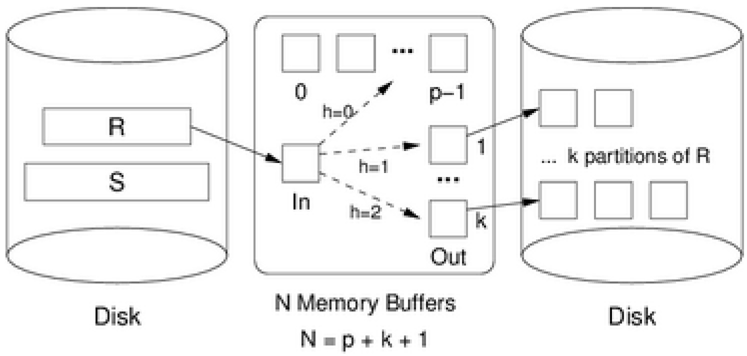
- have one large in-memory partition ($m = 1$)
- use as many as possible of $N$ buffers for partition
- use as few output buffers as possible (minimise $k$)

---

### ... Hybrid Hash Join

76/87

Data flow for hybrid hash join (partitioning $R$):

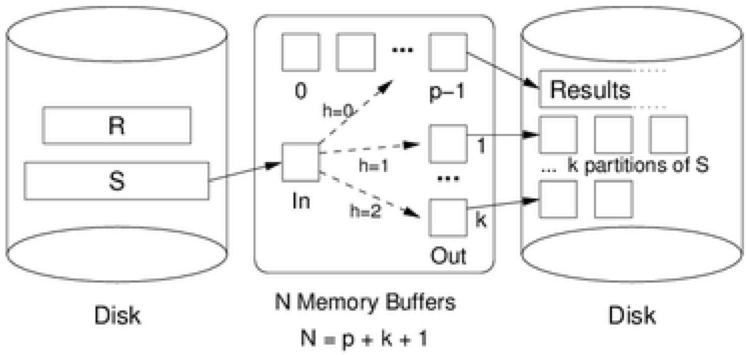



---

### ... Hybrid Hash Join

77/87

Data flow for hybrid hash join (partitioning $S$):




After this, proceed as for grace hash join.

---

### ... Hybrid Hash Join

78/87

Cost of hybrid hash join:

- assume: large $N$ total buffers, $m$ partitions in memory, $k$ partitions on disk
- read both tables: $b_R + b_S$
- total partitions for each table: $m+k$
- assuming uniform hashing, #pages in each R partition $P_R = ceil(b_R/(m+k))$
- assuming uniform hashing, #pages in each S partition $P_S = ceil(b_S/(m+k))$
- in Pass 1, $k*P_R + k*P_S$ pages written to disk partitions
- all joining of $m$ in-memory partitions is handled in memory
- in Pass2, $k*P_R + k*P_S$ pages read back from disk partitions

Cost = $b_R + b_S + k*P_R + k*P_S + k*P_R + k*P_S$
= $b_R + b_S + 2 * k * (P_R + P_S)$
= $b_R + b_S + 2 * k * (ceil(b_R/(m+k)) + ceil(b_S/(m+k)) )$

*How to determine k:*

- *set m=1 and so size of partition ≅ N ⇒ k ≅ $b_R$/N*
- *need to ensure that $\lceil b_R/k \rceil + k \le N$* *(allowing for input buffer)*
- *choose k close to $b_R$/N but satisfying constraint*

---

## Hybrid Hash Join on Example

79/87

*Case 1: N = 100 buffers, $b_R$ = 1000*

- *k = 10 ⇒ 1000/10 + 10 = 110 buffers; not less than 100*
- *k = 12 ⇒ 1000/12 + 12 = 96 buffers*
- *Cost = (3-2/12).(1000+2000) = 8500*

*Case 2: N = 200 buffers, $b_R$ = 1000*

- *k = 5 ⇒ 1000/5 + 5 = 205 buffers; not less than 200*
- *k = 6 ⇒ 1000/6 + 6 = 173 buffers*
- *Cost = (3-2/6).(1000+2000) = 8000*

*Case 3: N = 502 buffers, $b_R$ = 1000*

- *k = 2 ⇒ 1000/2 + 2 = 502 buffers*
- *Cost = (3-2/2).(1000+2000) = 6000*

---

## Pointer-based Join

80/87

*Conventional join algorithms set up R ↔ S connections via attribute values.*

*Join could be performed faster if direct connections already existed.*

- *in OODBMSs, they generally already exist in the form of object references (oids)*
- *in RDBMSs, they could be introduced via extra rid attributes*

*Such a modification to conventional RDBMS structure would be worthwhile:*

- *if we know in advance what kind of joins will be required*
- *adding the extra rid attributes into tuples is feasible*

---

### ... Pointer-based Join

81/87

*The basic idea for pointer-based join is:*

```
for each tuple r in relation R {
    for each rid associated with r {
        fetch tuple s from S via rid
        add (r,s) to result relation
    }
}
```

*Often, each R tuple is associated with only one rid, so the inner loop is not needed.*

---

### ... Pointer-based Join

82/87

*The advantage over value-based joins:*

- *rather than find S tuples via value-based lookup* *(e.g. hashing, index)*

- *we find S tuples by direct fetch with rid* *(much faster per tuple)*
- *requires no assumption about sorted-ness of relations*
- *does not require large numbers of buffers*

*The (potential) disadvantages:*

- *every `fetch` goes to a different page of S*
  *(this essentially returns us to the worst-case scenario for nested-loop join)*
- *the join only works in "one direction" (from R to S)*
- *requires additional data for each different join type*
- *requires tuples to be larger ⇒ $b_R$ is larger*

---

## General Join Conditions

83/87

*Above examples all used simple equijoin e.g. $Join_{[i=j]}(R,S)$.*

*For theta-join e.g $Join_{[i<j]}(R,S)$:*

- *index nested loop join: need B+ tree index on inner relation*
- *sort-merge join can be adapted, but is not very effective*
- *hash join is inapplicable*
- *other methods are essentially unchanged*

---

### ... General Join Conditions

84/87

*For multi-equality (pmr) join e.g. $Join_{[i=j \wedge k=l]}(R,S)$*

- *index nested loop join:*
  - *build index on all join fields of inner relation*
  - *e.g. if S is inner, build index on (S.j,S.l)*
- *sort-merge join:*
  - *sort both relations on combined join fields*
  - *e.g. sort R on (R.i,R.k), sort S on (S.j,S.l)*
- *hash-join:*
  - *use multi-attribute hashing on combined join fields*
- *other methods are essentially unchanged*

---

## Join Summary

85/87

*No single join algorithm is superior in some overall sense.*

*Which algorithm is best for a given query depends on:*

- *sizes of relations being joined, size of buffer pool*
- *any indexing on relations, whether relations are sorted*
- *which attributes and operations are used in the query*
- *number of tuples in S matching each tuple in R*

*Choosing the "best" join algorithm is critical because the cost difference between best and worst case can be very large.*

*E.g. $Join_{[id=stude]}$(Student,Enrolled): 3,000 ... 2,000,000*

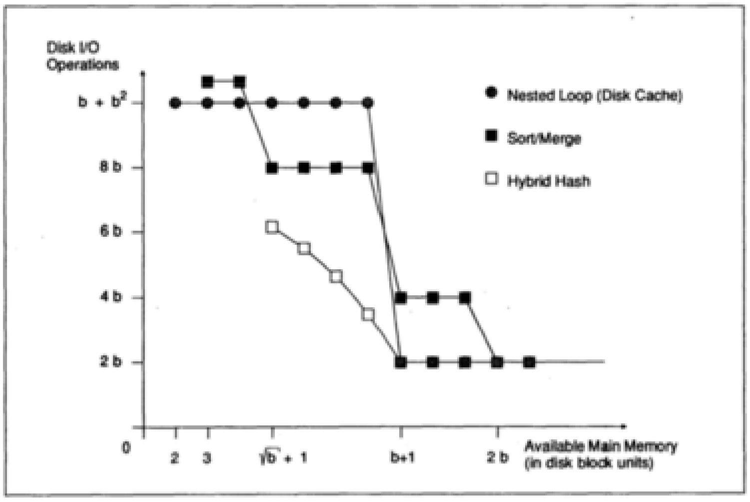
*In some cases, it may be worth modifying access methods "on the fly" (e.g. add index) to enable an efficient join algorithm.*

---

### ... Join Summary

86/87

*Comparison of join costs* *(from Zeller/Gray VLDB90, assumes $b_R = b_S = b$)*




---

## Join in PostgreSQL

87/87

*Join implementations are under:* **`src/backend/executor`**

*PostgreSQL suports three kinds of join:*

- *nested loop join (**`nodeNestloop.c`**)*
- *sort-merge join (**`nodeMergejoin.c`**)*
- *hash join (**`nodeHashjoin.c`**) (hybrid hash join)*

*Query optimiser chooses appropriate join, by considering*

- *physical characteristics of tables being joined*
- *estimated selectivity (likely number of result tuples)*

---

*Produced: 30 Apr 2020*