MATH-UA 329: Homework 3A

James Pagan, March 2024

Professor Güntürk

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1 Problem 1

Let $\mathbf{x}$ be the vector in X such that $\|\mathbf{x}\|_X = 1$ and and $\|\mathbf{STx}\|_Z = \|\mathbf{ST}\|_{X\to Z}$. The existence of this vector is ensured by Extreme Value Theorem, since $\|\mathbf{ST}\|_{X\to Z}$ is a supremum of the image of a compact set. Observing that $\|\mathbf{x}\|_X = 1$, we have that

$$\begin{aligned}
\|\mathbf{ST}\|_{X\to Z} &= \|\mathbf{STx}\|_Z \\
&\leq \|\mathbf{S}\|_{Y\to Z}\|\mathbf{Tx}\|_Y \\
&\leq \|\mathbf{S}\|_{Y\to Z}\|\mathbf{T}\|_{X\to Y}\|\mathbf{x}\|_X \\
&= \|\mathbf{S}\|_{Y\to Z}\|\mathbf{T}\|_{X\to Y}.
\end{aligned}$$

This completes the proof.