Fuck

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1 Day 1

An operation $*$ on a set S is something that takes in two values of S and spits out one value of S. For instance,

1. Addition, subtraction, multiplication, and division are operations on rational numbers $\mathbb{Q}$.

2. Exponentiation is an operation on positive integers: let $a * b = a^b$.

3. Logic gates are an operation: they take in two numbers (each either 0 or 1) and spit out 0 or 1.

An operation $*$ on S is called **closed** if for all $a, b \in S$, the element $a * b \in S$ too. In other words, $*$ never "leaves" S. The operation $*$ is associative if for all $a, b, c \in S$, we have $a * (b * c) = (a * b) * c$. Furthermore, $*$ is **commutative** if $a * b = b * a$ for all $a, b \in S$. It's possible to have one of these without the other!

2 Day 2

We often denote S with the operation $*$ by the pair $(S, *)$. There are two other properties of operations we care about:

1. $(S, *)$ has an **identity element** if there exists e in S such that the operation "does nothing" with e — for all a in S, we have $a * e = e * a = a$.

2. $(S, *)$ has **inverses** if for *all* a in S, there exists some b in S that "cancels" a — namely $a * b = b * a = e$, where e is the identity.

We often write the inverse of a as a^{-1}, where this is *not necessarily* 1 *divided by* a. It's like how the inverse of a matrix $\mathbf{T}$ is written $\mathbf{T}^{-1}$. Notice that to have inverses, there must be an identity. Drill these five properties — closure, associativity, identity, inverses, commutativity — into your head!

1.

1. Find the identity of complex numbers $\mathbb{C}$ with respect to addition. Find the identity of *nonzero* complex numbers with respect to multiplication. 3. $\mathbb{Q}$ has an additive identity. Does each rational number $\mathbb{Q}$ under $+$ have an inverse?