MATH-UA 349: Honors Algebra II

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1 Problem 1

1.1 Part (a)

Performing reduction, it is easy to verify that

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 7 & 2 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 2 & 1 & 17 \\ -1 & 0 & -10 \\ 0 & 0 & 1 \end{bmatrix}$$

Observe that $(x, y, z) \in \mathbb{Z}^3$ is mapped to zero by the left-hand side if and only if $x = y = 0$. Therefore, the solutions are

$$\begin{bmatrix} 2 & 1 & 17 \\ -1 & 0 & -10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix} = \boxed{\begin{bmatrix} 17z \\ -10z \\ z \end{bmatrix}}$$

across all $z \in \mathbb{Z}$.