1 Group Cosets

You can prove most of these in your head; it's just writing all these down is pretty useful.

Basic Properties of Cosets: If $H \subseteq G$ and $a, b \in G$ with $h \in H$,

- $H(ab) = (Ha)b$ and $(ab)H = a(bH)$
- $|Ha| = |aH| = H$.
- $Ha = Hb$ or $Ha \cap Hb = \varnothing$; $aH = bH$ or $aH \cap bH = \varnothing$;

Cosets and Subgroups: If $N \trianglelefteq G$ and $H \subseteq G$ with $a, b \in G$,

- $aNa^{-1} = N$ or $aNa^{-1} \subseteq N$.
- $aN = Na$.
- $Ha \subseteq Ga$ and $aH \subseteq aG$.

Homomorphisms: If $\varphi : G \to H$ is a homomorphism,

- $\varphi(a)\varphi(H) = \varphi(aH)$ and $\varphi(H)\varphi(a) = \varphi(Ha)$

2 Ring Cosets

For a ring R with an ideal I and an element $r \in R$, we define the coset of rings as follows: $r + I = \{r + i \mid i \in I\}$.

Basic Properties of Cosets: If $I \trianglelefteq R$ and $r, j \in R$ with $i \in I$,

- $I + (r + j) = (I + r) + j$ and $(r + j) + I = r + (j + I)$
- $r + I = I + r$.
- $I + r = I + j$ or $I + r \cap I + j = \varnothing$.
- $I + r = I + j \iff r - j \in I$.

Homomorphisms: If $\varphi : G \to H$ is a homomorphism,

- $\varphi(r + I) = \varphi(r) + \varphi(I) = \varphi(I) + \varphi(r) = \varphi(I + r)$.