Eng Maths One
: 18.5 / 20 (92.5%)

Question	Mark / Max
1	3 / 4
2	2 / 2
3	4 / 4
4	2 / 2
5	6 / 6
6	1 / 1
7	0.5 / 1

2212304

1.) a.

$$J(\varepsilon) = \frac{16J_0}{\varepsilon^2 - 6\varepsilon + 25}$$

Quotient rule

$J'(\varepsilon) =$

+2

$u = 16J_0 \quad du = 0$

$v = \varepsilon^2 - 6\varepsilon + 25 \quad dv = 2\varepsilon - 6$

$$\frac{du \cdot v - dv \cdot u}{v^2}$$

$$= \frac{0(\varepsilon^2 - 6\varepsilon + 25) - 16J_0(2\varepsilon - 6)}{(\varepsilon^2 - 6\varepsilon + 25)^2}$$

$$J'(\varepsilon) = \frac{96J_0 - 32J_0\varepsilon}{(\varepsilon^2 - 6\varepsilon + 25)^2}$$

+2 correct use of ratio rule

Find maximum

+1

$$0 = \frac{96J_0 - 32J_0\varepsilon}{(\varepsilon^2 - 6\varepsilon + 25)^2} = 0 = 96J_0 - 32J_0\varepsilon$$

$32J_0\varepsilon = 96J_0$

$\varepsilon = 3$

as $\varepsilon = \frac{E}{E_0}$

$3 = \frac{E}{E_0} \quad 3E_0 = E$

22/11/20

+0 No attempt to prove there is a maximum

+1 correct method for finding maximum

2212304

1b.) Completing the Square on original

+1

$\varepsilon^2 - 6\varepsilon + 25$

$$(\varepsilon - 3)^2 - 9 + 25 \Rightarrow \frac{16J_0}{(\varepsilon - 3)^2 + 16} \Rightarrow$$

+1

$$\frac{J_0}{\frac{\varepsilon - 3^2 + 16}{16}} \Rightarrow \frac{J_0}{\frac{(\varepsilon - 3)^2}{16} + 1} \Rightarrow$$

$$\frac{J_0}{\left(\frac{\varepsilon - 3}{4}\right)^2 + 1}$$

22/11/20

+1 correct completion of square

+1 correct rearrangement

2212304

1c.

$$I = \int_3^7 \frac{J_0}{1 + \left(\frac{\varepsilon - 3}{4}\right)^2} d\varepsilon$$

+1

$\tan u = \left(\frac{\varepsilon - 3}{4}\right)$

$du \sec^2 u = \left(\frac{d\varepsilon}{4}\right) \quad : d\varepsilon = 4\sec^2 u \, du$

$1 + \tan^2 u = \sec^2 u$

$1 + \left(\frac{\varepsilon - 3}{4}\right)^2 = \sec^2 u$

+1 Find boundaries: $\varepsilon = 7 \quad \varepsilon = 3$

$\tan u = 1 \quad \tan u = 0$

$u = \frac{1}{4}\pi \quad u = 0$

$$= \int_0^{\frac{\pi}{4}} \frac{J_0}{\sec^2 u} 4\sec^2 u \, du$$

+1

$$\therefore = 4J_0 \int_0^{\frac{\pi}{4}} \frac{\sec^2 u}{\sec^2 u} du = 4J_0 \int_0^{\frac{\pi}{4}} 1 \, du$$

+1

$$= 4J_0 [u]_0^{\frac{\pi}{4}} = 4J_0 \cdot \frac{\pi}{4} \quad \therefore$$

$$\int_3^7 \frac{J_0}{1 + \left(\frac{\varepsilon - 3}{4}\right)^2} d\varepsilon = \frac{4\pi}{4} J_0$$

22/11/2020

+1 correct substitution

+1 correct evaluation of arctan

+1 correct integral

+1 correct evaluation of limits

2212304

+1

$$Adj(a) = \begin{pmatrix} -2b & 2 & 1-b \\ 1+2a & -3 & a-1 \\ -b & 1 & b-a \end{pmatrix}$$

2c.

$$\frac{1}{11}\begin{pmatrix} 4 & 2 & 3 \\ 5 & -3 & 1 \\ 2 & 1 & -4 \end{pmatrix}$$

+1

C_{11}: $-2b = 4$, $b = -2$

C_{21}: $1 + 2a = 5$, $a = 2$

C_{13}: $1 - b = 3$, $b = -2$

C_{23}: $1 = a - 1$, $a = 2$

C_{33}: $b - a = -4$

$-2 - a = -4$

$a = 2$

$\therefore \quad a = 2$

$b = -2$

2d.

+0.5

$1 + 2a - 3b$

$= 1 + 2 \times 2 - 3 \times -2 = 11.$

22/11/20

+1 correct transpose

+1 values of a and b consistent in all cases

+0.5 Not identified determinant from Inverse, only calculated using a and b

2212304

2a.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ a & b & 1 \\ 1 & 0 & 2 \end{pmatrix}$$

+1

$\det(A) = 1(1-b) - 2(b-a)$

+1

$= 1 + 2a - 3b$

b.

$Adj(A) =$

+2

$A_{11} = -2b \quad A_{21} = 2 \quad A_{31} = 1 - b$

$A_{12} = 2a + 7 \quad A_{22} = -3 \quad A_{32} = a - 1$

$A_{13} = -b \quad A_{23} = 1 \quad A_{33} = b - a$

+2 +1

$$= \begin{pmatrix} -2b & (2a+1) & -b \\ 2 & -3 & 1 \\ (1-b) & (a-1) & (b-a) \end{pmatrix}$$

22/11/20

+1 correct expansion of determinant

+1 correct simplification

+2 correct elements in the minor matrices

+2 correct calculation of final answer in all elements

+1 correct signs