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## Nonstandard Finite Difference Time Domain Methodology for Harmonics Generation in Nonlinear Dielectrics

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**Abstract:** Nonstandard (NS) finite difference time domain (FDTD) methodologies have been developed to solve Maxwell's equations with high accuracy on a coarse grid [1]. In this paper we extend the NS-FDTD methodology to nonlinear optics. As an example we simulate second harmonic generation and compare with theory.

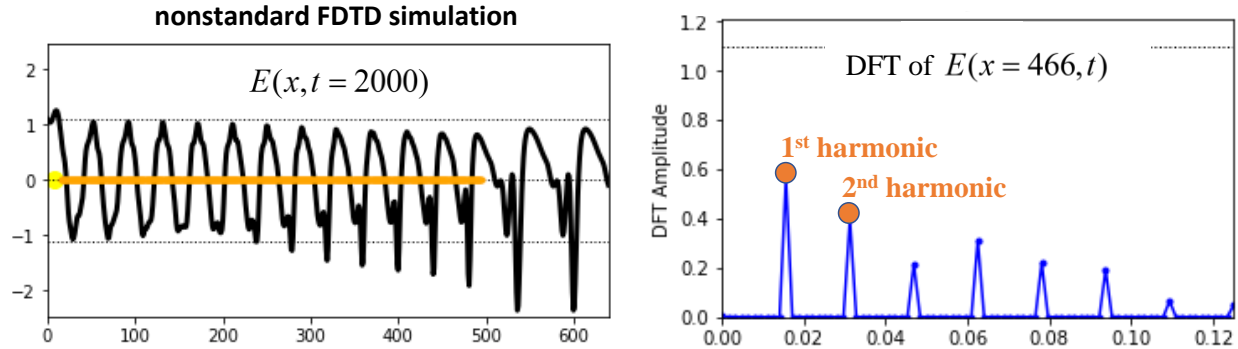
In units in which  $\varepsilon_0 = 1$ ,  $\varepsilon = 1 + \chi^{(1)}$ , then  $\mathbf{D} = \varepsilon \mathbf{E} + \mathbf{P}^{\text{NL}}$ ,  $\mathbf{P}^{\text{NL}}$  = nonlinear polarization.

Taking  $\mu = \text{constant} = \mu_0 = 1/c^2$  and  $\nabla \cdot \mathbf{E} = 0$ , Maxwell's equations reduce to

$$\left( \partial_t^2 - \frac{c^2}{\varepsilon} \nabla^2 \right) \mathbf{E} = -\frac{1}{\varepsilon} \partial_t^2 \mathbf{P}^{\text{NL}}, \quad \text{one dimension} \quad P^{\text{NL}} = \chi^{(2)} E^2 \rightarrow \left( \partial_t^2 - \frac{c^2}{\varepsilon} \partial_x^2 \right) E(x, t) = -\frac{\chi^{(2)}}{\varepsilon} \partial_t^2 E(x, t)^2 + s(t).$$

source turns on at  $t = 0$

The ordinary FDTD algorithm is numerically unstable. We have devised on based on a nonstandard finite difference model that is stable and accurate on a coarse numerical grid.



**(Left)** Source (yellow dot) generates an electromagnetic field in free space, which propagates into a nonlinear dielectric material (orange). The amplitude of the second harmonic and sum frequency amplitudes grow as the field propagates through the material. **(Right)** Discrete Fourier transform of the field.

We have extended this methodology to the 2- and 3-dimensions and to the full form of Maxwell's equations, and incorporated higher (than 2<sup>nd</sup>) order polarizations.

### Reference:

[1] J. B. Cole and S. Banerjee, Computing the Flow of Light. SPIE Press, 2017.  
<https://spie.org/Publications/Book/2250613?SSO=1>