UNIVERSITY OF
EXETER

Centre for Metamaterial
Research and Innovation

EPSRC Centre for
Doctoral Training
in Metamaterials

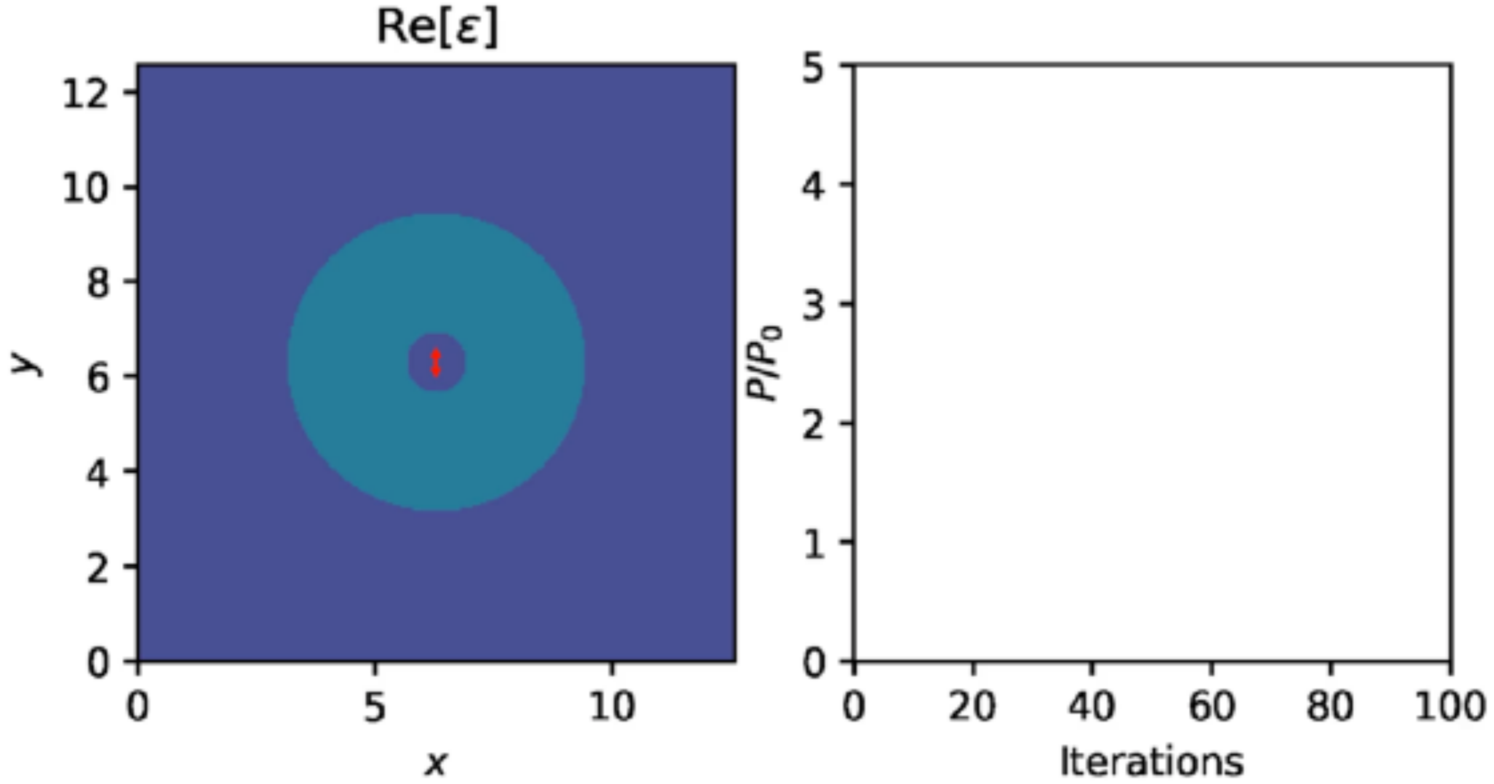
XM^2

UKRI

Engineering and
Physical Sciences
Research Council

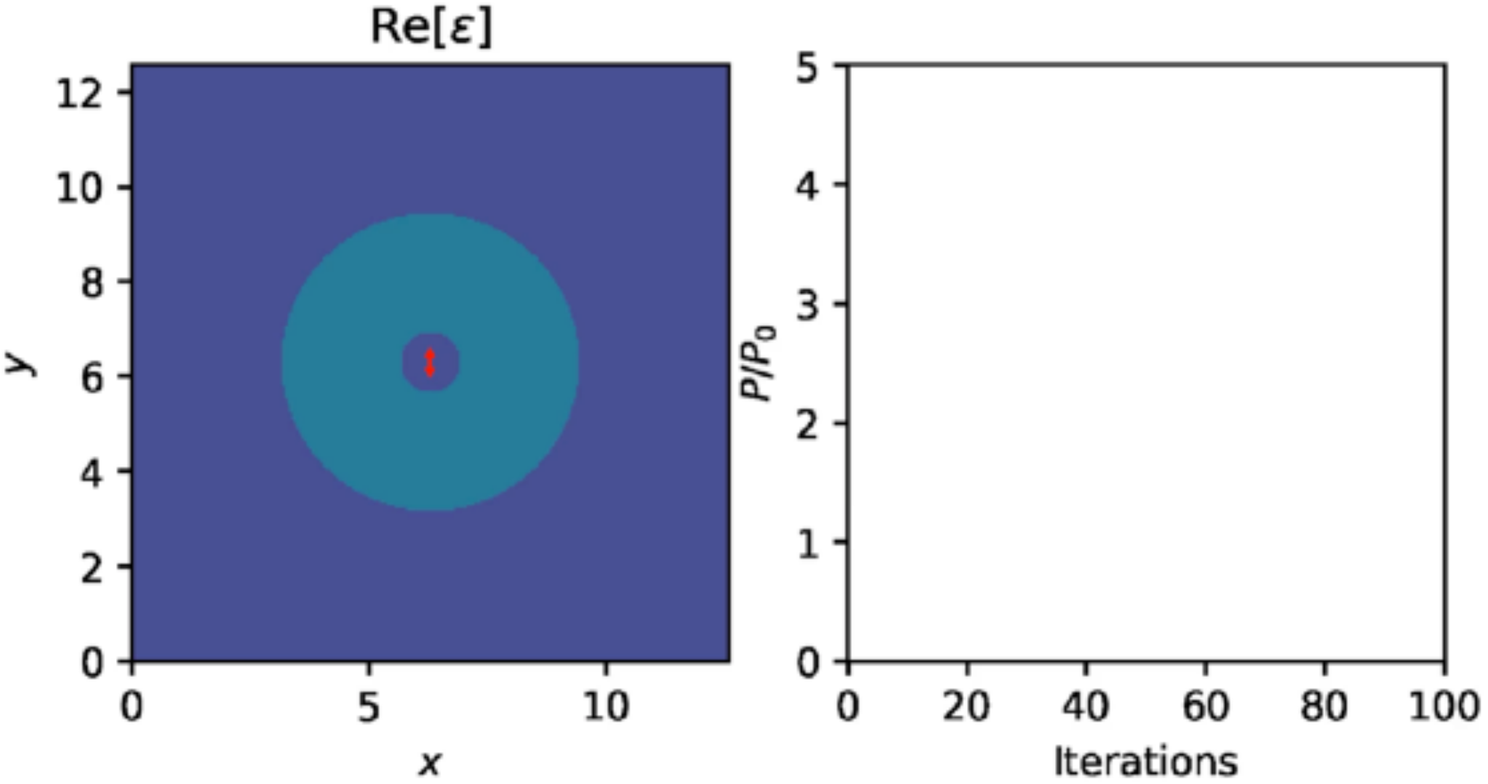
The Adjoint Method

- Sandro Mignuzzi, Stefano Vezzoli, Simon A. R. Horsley, William L. Barnes, Stefan A. Maier, and Riccardo Sapienza "Nanoscale Design of the Local Density of Optical States", Nano Lett. 19, 3, 1613–1617 (2019)
- Owen Miller, "Photonic Design: From Fundamental Solar Cell Physics to Computational Inverse Design", PhD thesis (2012)

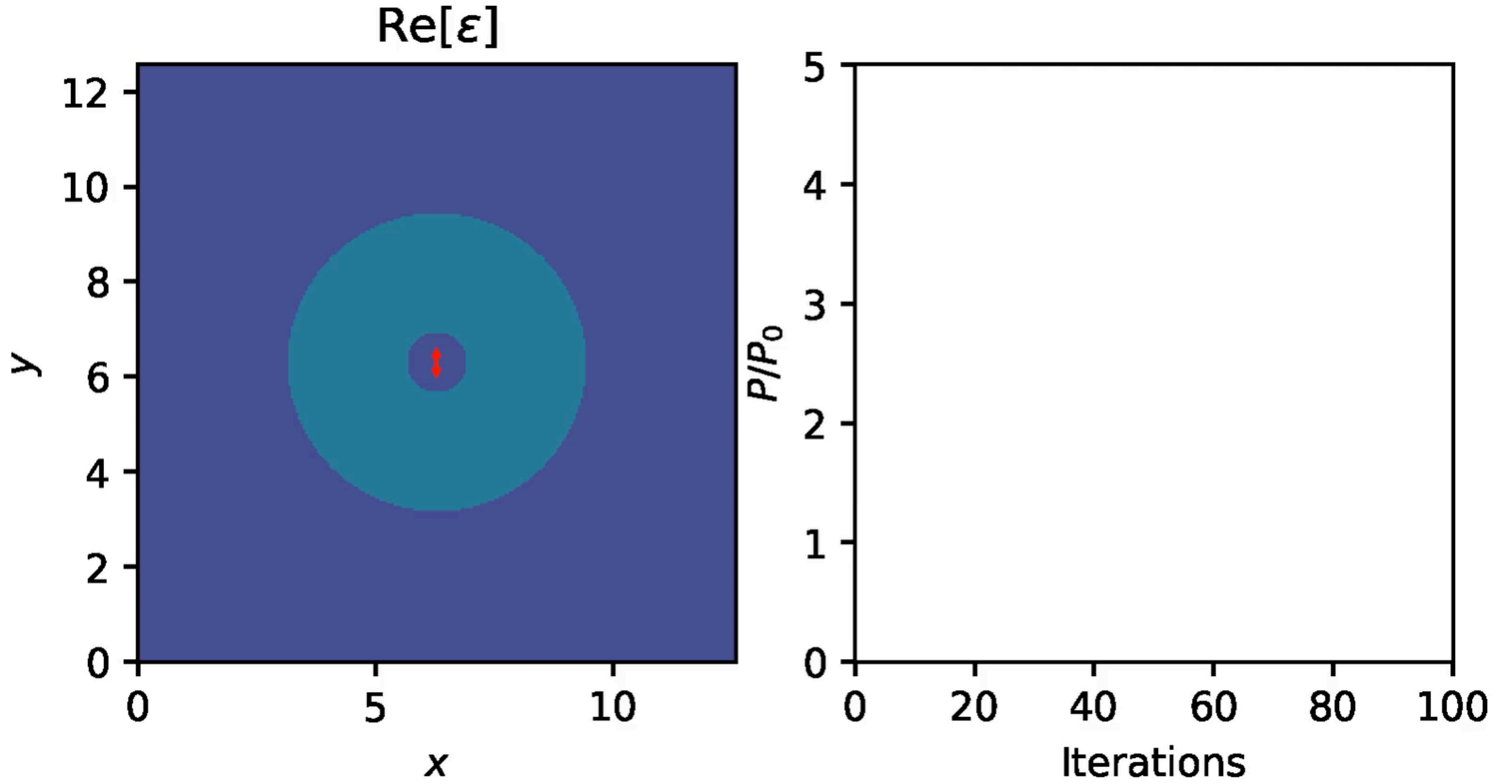


Re[ε]
y
x
0
5
10
0
2
4
6
8
10
12
P/P_0
Iterations
0
20
40
60
80
100
0
1
2
3
4
5

$$\delta P = \frac{1}{2} \mathrm{Im} \left[\boldsymbol{E}_{*}(\boldsymbol{r}) \cdot \boldsymbol{E}(\boldsymbol{r}) \right] \delta \varepsilon$$



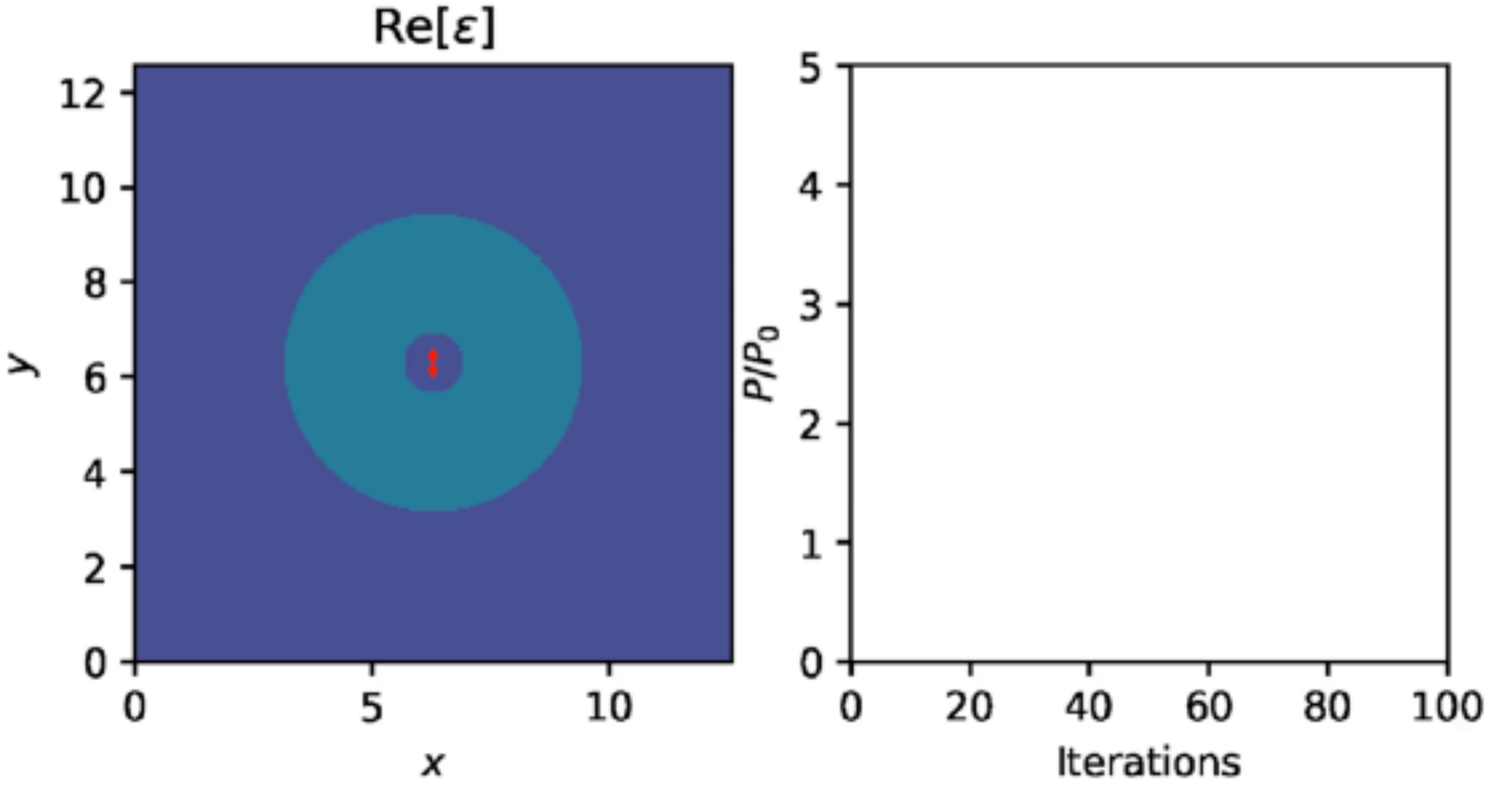
Re[ε]
y
x
0
2
4
6
8
10
12
0
5
10
P/P0
Iterations
0
1
2
3
4
5
0
20
40
60
80
100



Re[ε]
12
10
8
6
4
2
0
y
0
5
10
x
5
4
3
2
1
0
P/P0
0
20
40
60
80
100
Iterations

The Adjoint Method

$$\delta P = \frac{1}{2}\mathrm{Im}\left[\boldsymbol{E}_*(\boldsymbol{r}) \cdot \boldsymbol{E}(\boldsymbol{r})\right]\delta\varepsilon$$



- Sandro Mignuzzi, Stefano Vezzoli, Simon A. R. Horsley, William L. Barnes, Stefan A. Maier, and Riccardo Sapienza “Nanoscale Design of the Local Density of Optical States”, Nano Lett. 19, 3, 1613–1617 (2019)
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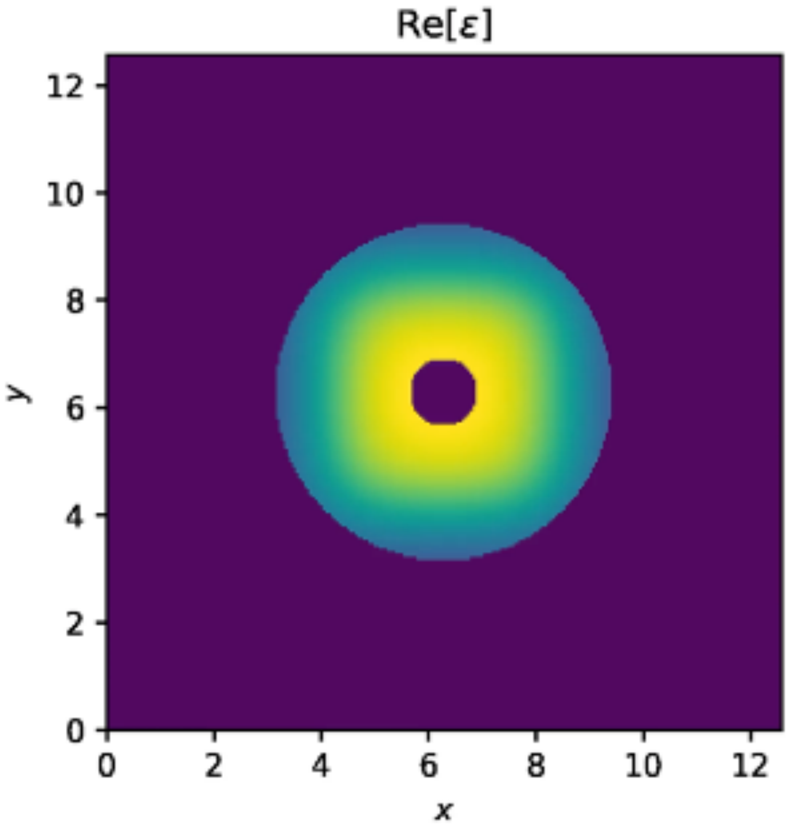
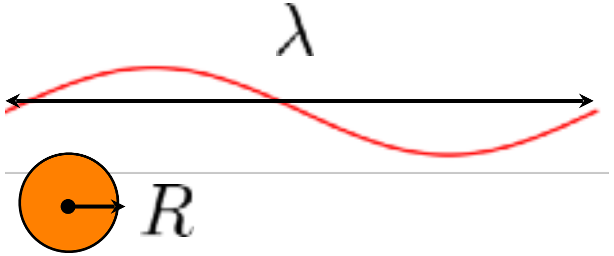
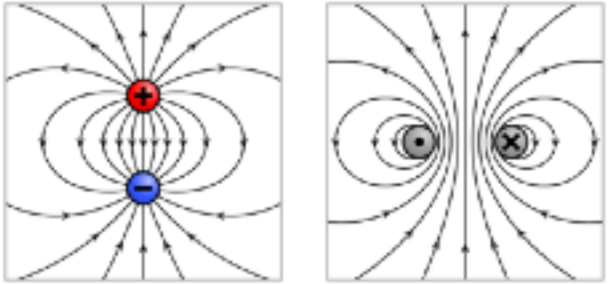
Continuous → Discrete





$$kR \leq 1$$



$$\begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{m} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}_E & 0 \\ 0 & \boldsymbol{\alpha}_H \end{pmatrix} \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{E}(\mathbf{r}) \\ \mathbf{H}(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \mathbf{E}_s(\mathbf{r}) \\ \mathbf{H}_s(\mathbf{r}) \end{pmatrix} + \sum_n \begin{pmatrix} \xi^2 \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_n)\alpha_E & i\xi \overleftrightarrow{\mathbf{G}}_{EH}(\mathbf{r},\mathbf{r}_n)\alpha_H \\ -i\xi \overleftrightarrow{\mathbf{G}}_{EH}(\mathbf{r},\mathbf{r}_n)\alpha_E & \xi^2 \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_n)\alpha_H \end{pmatrix} \begin{pmatrix} \mathbf{E}(\mathbf{r}_n) \\ \mathbf{H}(\mathbf{r}_n) \end{pmatrix}$$

$$\delta\mathbf{E}(\mathbf{r}) = -\left[\xi^2 \mathbf{G}(\mathbf{r},\mathbf{r}_n)\alpha_E \nabla \mathbf{E}(\mathbf{r}_n) + i\xi \nabla \times \mathbf{G}(\mathbf{r},\mathbf{r}_n)\alpha_H \nabla \mathbf{H}(\mathbf{r}_n)\right] \delta r_n$$

- Purcell, E. M. and Pennypacker, C. R., "Scattering and absorption of light by nonspherical dielectric grains" Astrophysical Journal 186 705 (1973)