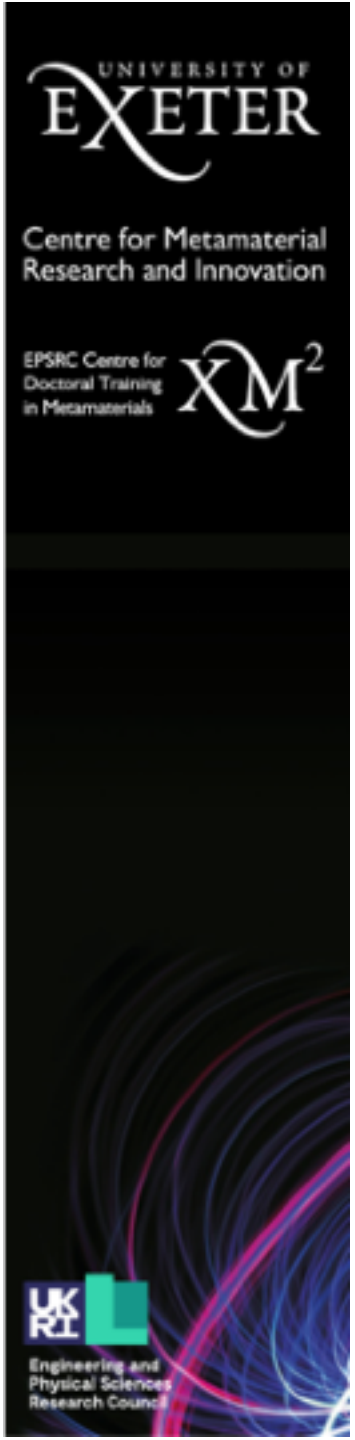
UNIVERSITY OF EXETER

Centre for Metamaterial Research and Innovation

EPSRC Centre for Doctoral Training in Metamaterials

XM²



UKRI Engineering and Physical Sciences Research Council

www.metamaterials.center

Continuous → Discrete

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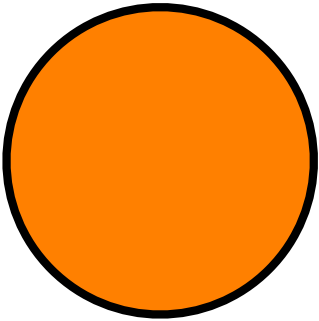
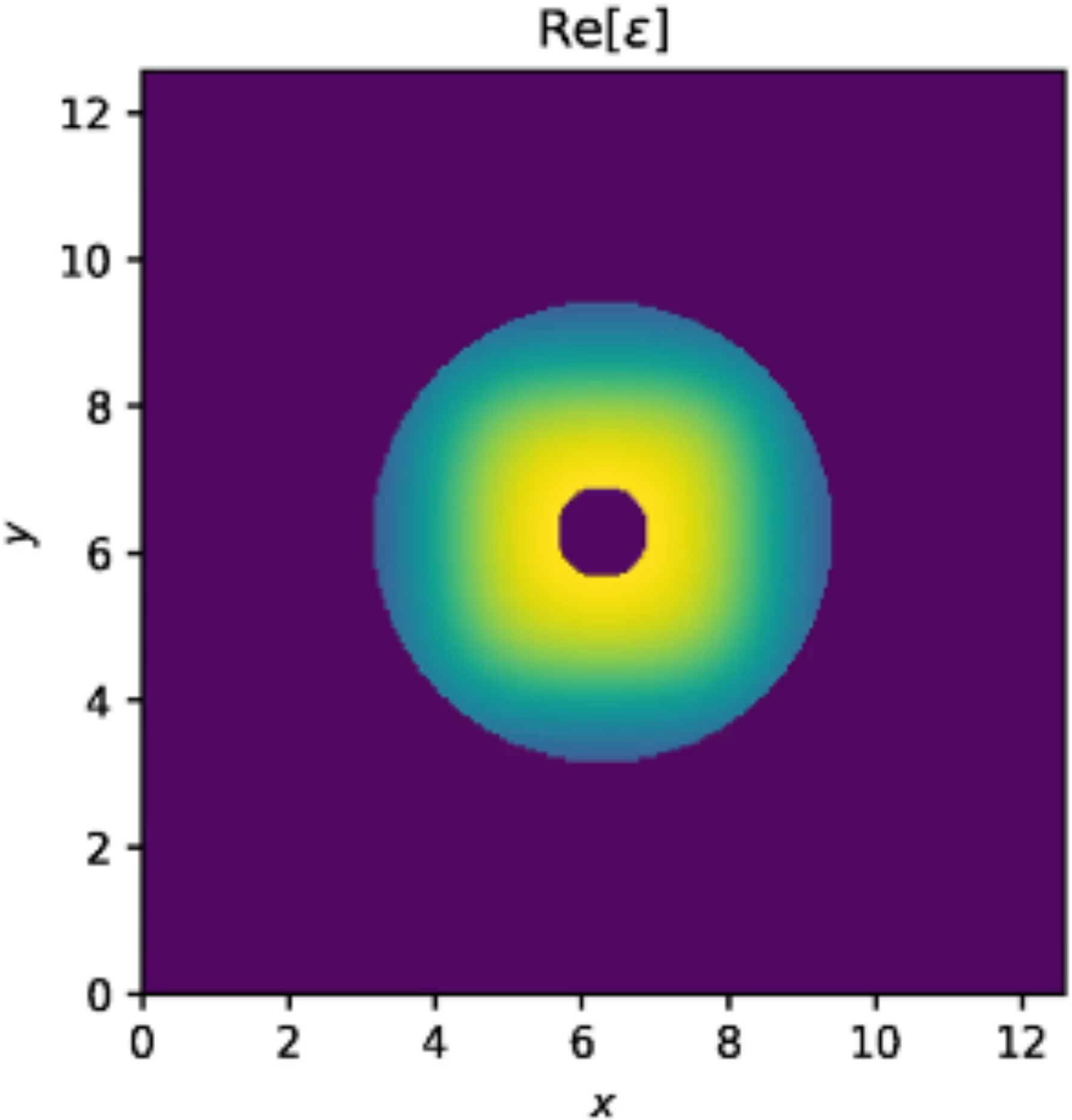


$$\begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{m} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}_E & 0 \\ 0 & \boldsymbol{\alpha}_H \end{pmatrix} \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}$$

- Purcell, E. M. and Pennypacker, C. R., “Scattering and absorption of light by nonspherical dielectric grains” Astrophysical Journal 186 705 (1973)

$$\begin{pmatrix}\mathbf{E}(\mathbf{r})\\ \mathbf{H}(\mathbf{r})\end{pmatrix}=\begin{pmatrix}\mathbf{E}_s(\mathbf{r})\\ \mathbf{H}_s(\mathbf{r})\end{pmatrix}+\sum_n\begin{pmatrix}\xi^2\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_n)\alpha_E & i\xi\overleftrightarrow{\mathbf{G}}_{EH}(\mathbf{r},\mathbf{r}_n)\alpha_H\\ -i\xi\overleftrightarrow{\mathbf{G}}_{EH}(\mathbf{r},\mathbf{r}_n)\alpha_E & \xi^2\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_n)\alpha_H\end{pmatrix}\begin{pmatrix}\mathbf{E}(\mathbf{r}_n)\\ \mathbf{H}(\mathbf{r}_n)\end{pmatrix}$$


Re[ε]
12
10
8
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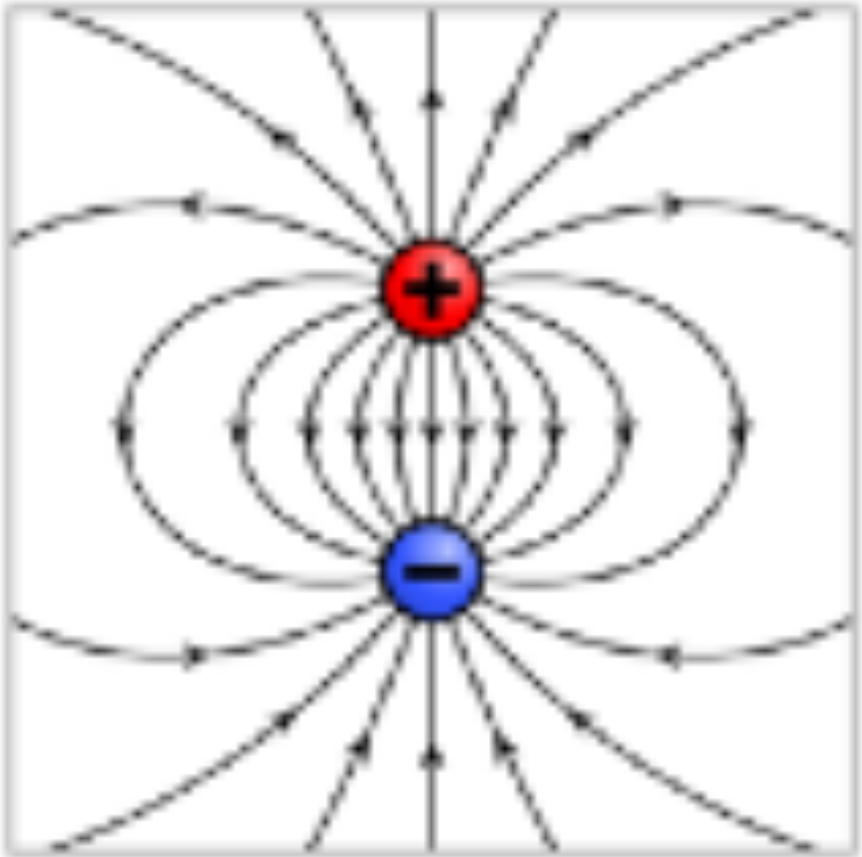
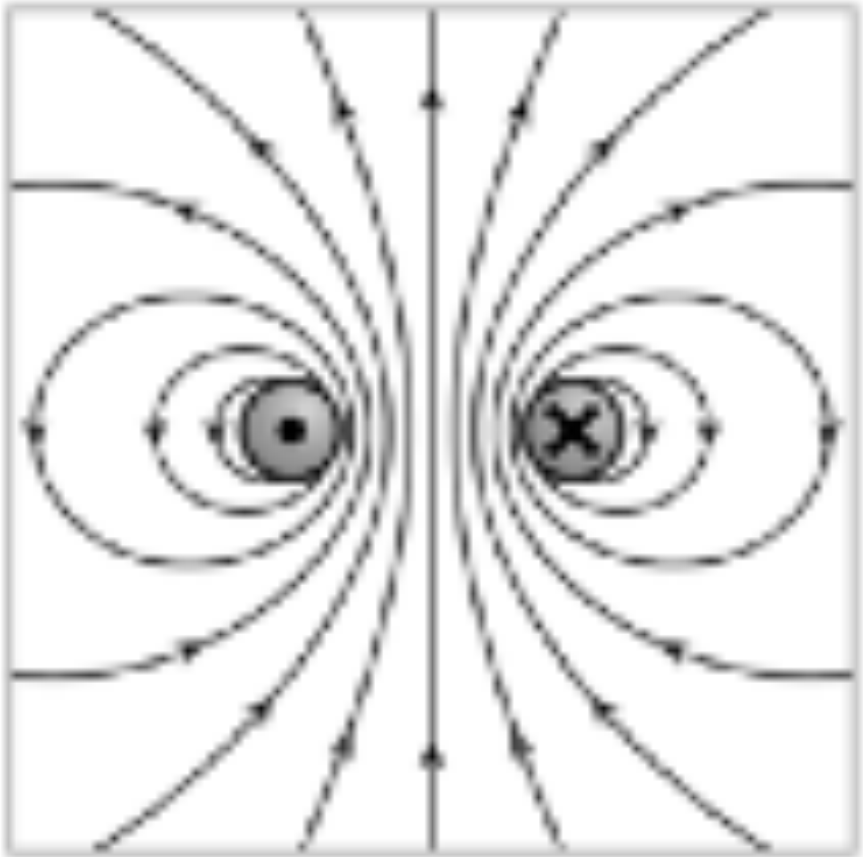
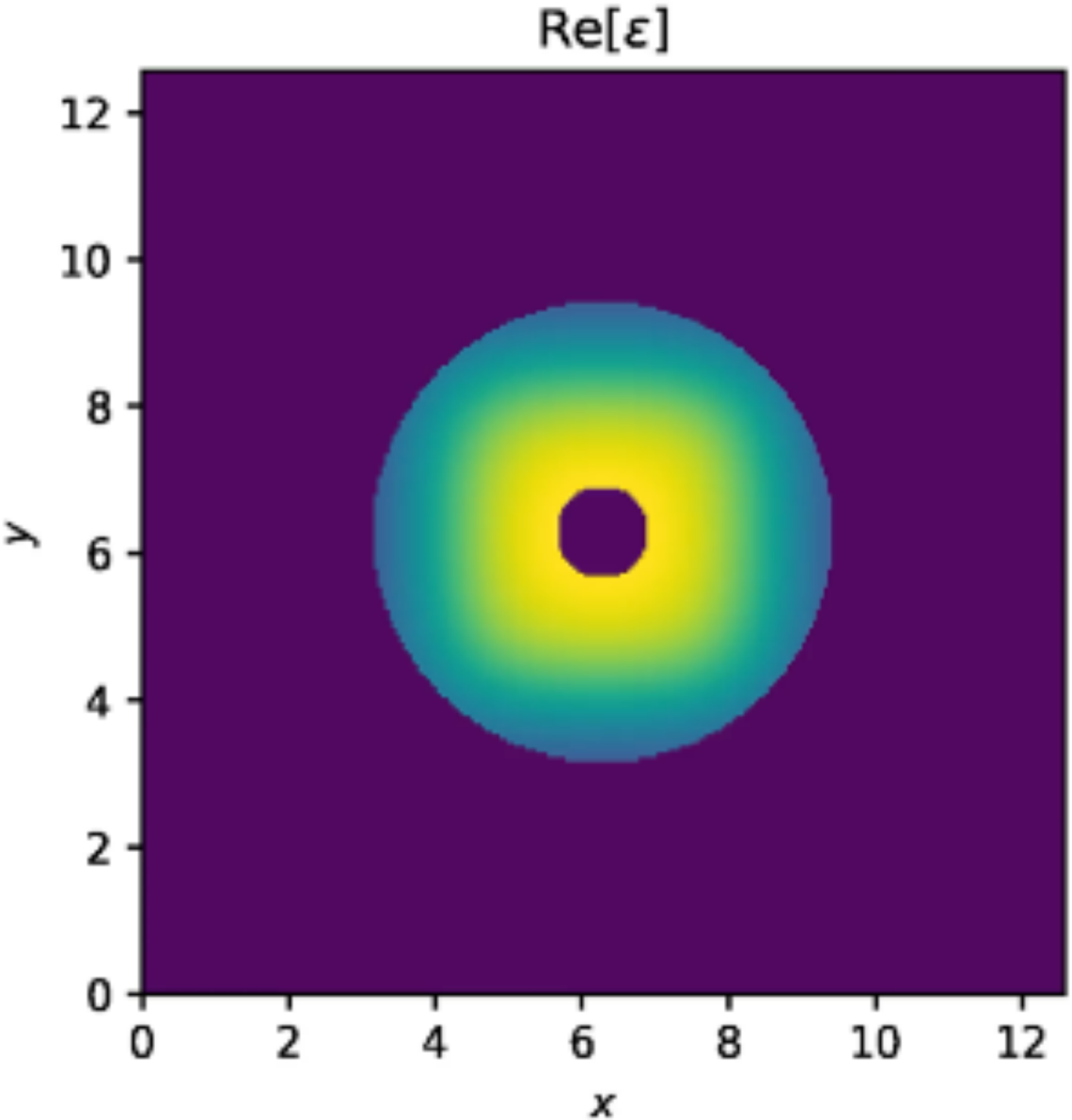
x





$$\delta \mathbf{E}(\mathbf{r}) = -\left[\xi^2 \mathbf{G}(\mathbf{r}, \mathbf{r}_n)\alpha_E \nabla \mathbf{E}(\mathbf{r}_n) + i\xi \nabla \times \mathbf{G}(\mathbf{r}, \mathbf{r}_n)\alpha_H \nabla \mathbf{H}(\mathbf{r}_n)\right] \delta \boldsymbol{r}_n$$


Re[ε]
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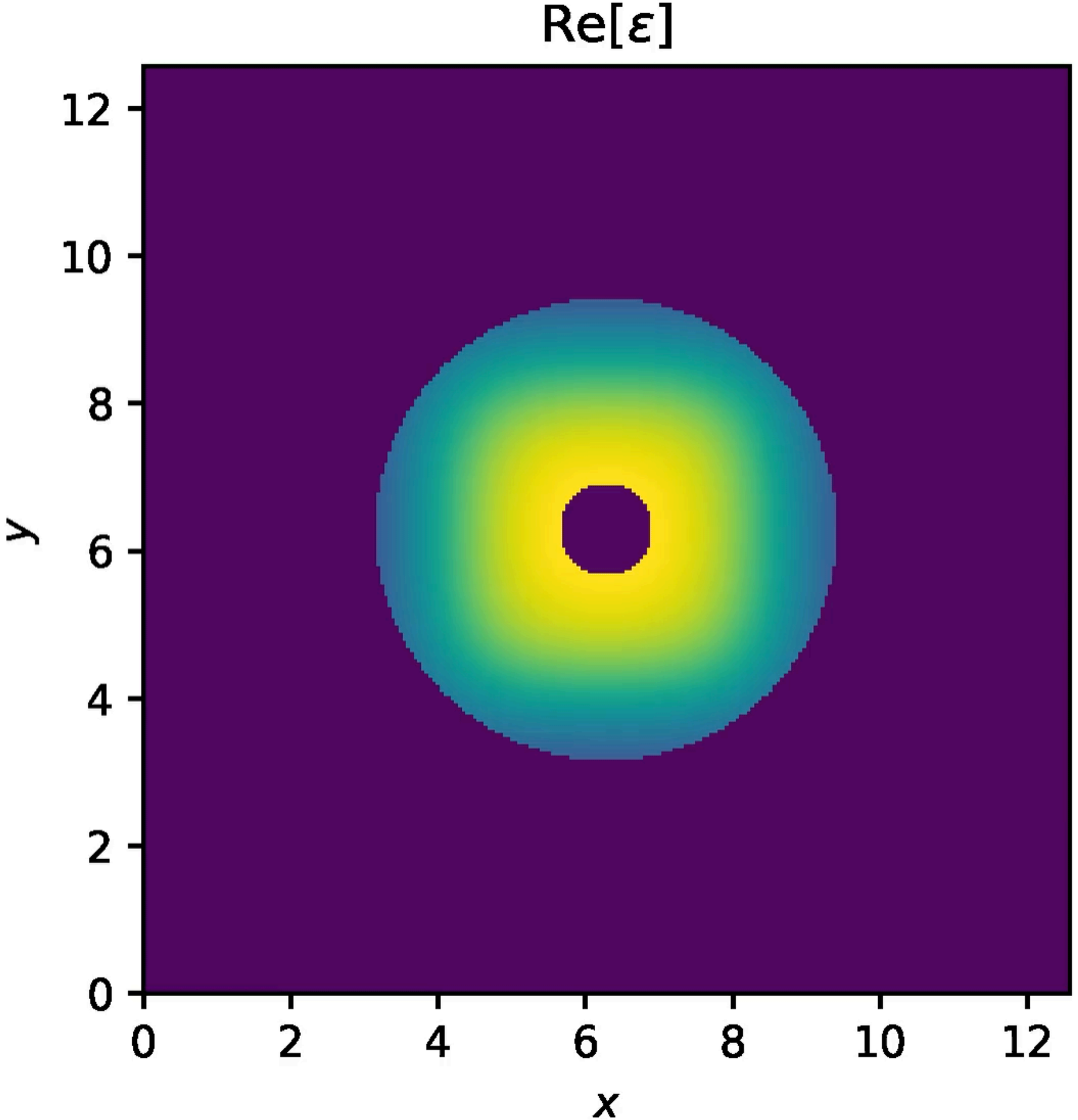
Re[ε]
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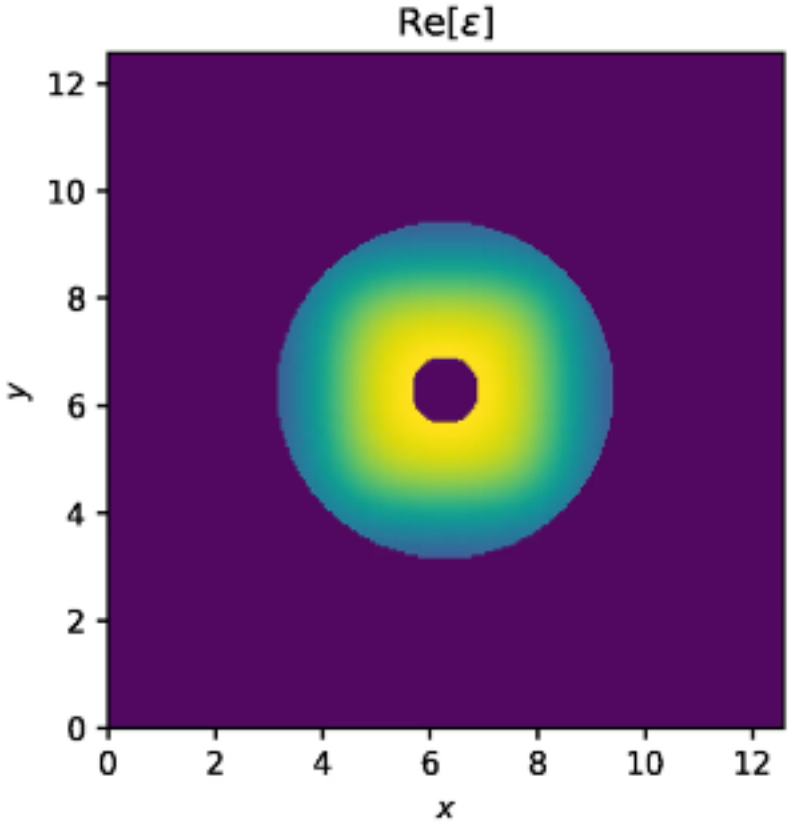
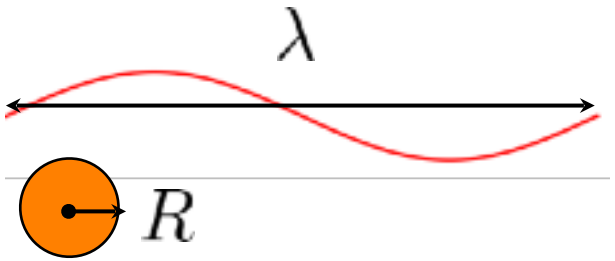
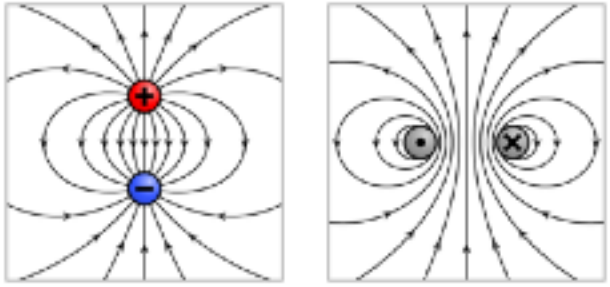
Continuous → Discrete





$$kR \leq 1$$



$$\begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{m} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}_E & 0 \\ 0 & \boldsymbol{\alpha}_H \end{pmatrix} \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{E}(\mathbf{r}) \\ \mathbf{H}(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \mathbf{E}_s(\mathbf{r}) \\ \mathbf{H}_s(\mathbf{r}) \end{pmatrix} + \sum_n \begin{pmatrix} \xi^2 \overleftrightarrow{\mathbf{G}}(\mathbf{r}, \mathbf{r}_n)\alpha_E & i\xi \overleftrightarrow{\mathbf{G}}_{EH}(\mathbf{r}, \mathbf{r}_n)\alpha_H \\ -i\xi \overleftrightarrow{\mathbf{G}}_{EH}(\mathbf{r}, \mathbf{r}_n)\alpha_E & \xi^2 \overleftrightarrow{\mathbf{G}}(\mathbf{r}, \mathbf{r}_n)\alpha_H \end{pmatrix} \begin{pmatrix} \mathbf{E}(\mathbf{r}_n) \\ \mathbf{H}(\mathbf{r}_n) \end{pmatrix}$$

$$\delta\mathbf{E}(\mathbf{r}) = -\left[\xi^2 \mathbf{G}(\mathbf{r}, \mathbf{r}_n)\alpha_E \nabla\mathbf{E}(\mathbf{r}_n) + i\xi\nabla \times \mathbf{G}(\mathbf{r}, \mathbf{r}_n)\alpha_H \nabla\mathbf{H}(\mathbf{r}_n)\right] \delta\boldsymbol{r}_n$$

- Purcell, E. M. and Pennypacker, C. R., “Scattering and absorption of light by nonspherical dielectric grains” Astrophysical Journal 186 705 (1973)

Continuous → Discrete

$$\delta P = \mathrm{Im}\left[\mathbf{E}_*(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r})\right] \delta\varepsilon$$

$$\delta P = \mathrm{Im}\left\{\mathbf{p}^* \cdot \left[\xi^2 \overleftrightarrow{\mathbf{G}}(\mathbf{r}, \mathbf{r}_n)\alpha_E \nabla \mathbf{E}(\mathbf{r}_n) + i\xi \nabla \times \overleftrightarrow{\mathbf{G}}(\mathbf{r}, \mathbf{r}_n)\alpha_H \nabla \mathbf{H}(\mathbf{r}_n)\right]\right\} \delta \boldsymbol{r}_n$$