Models for Longitudinal Data

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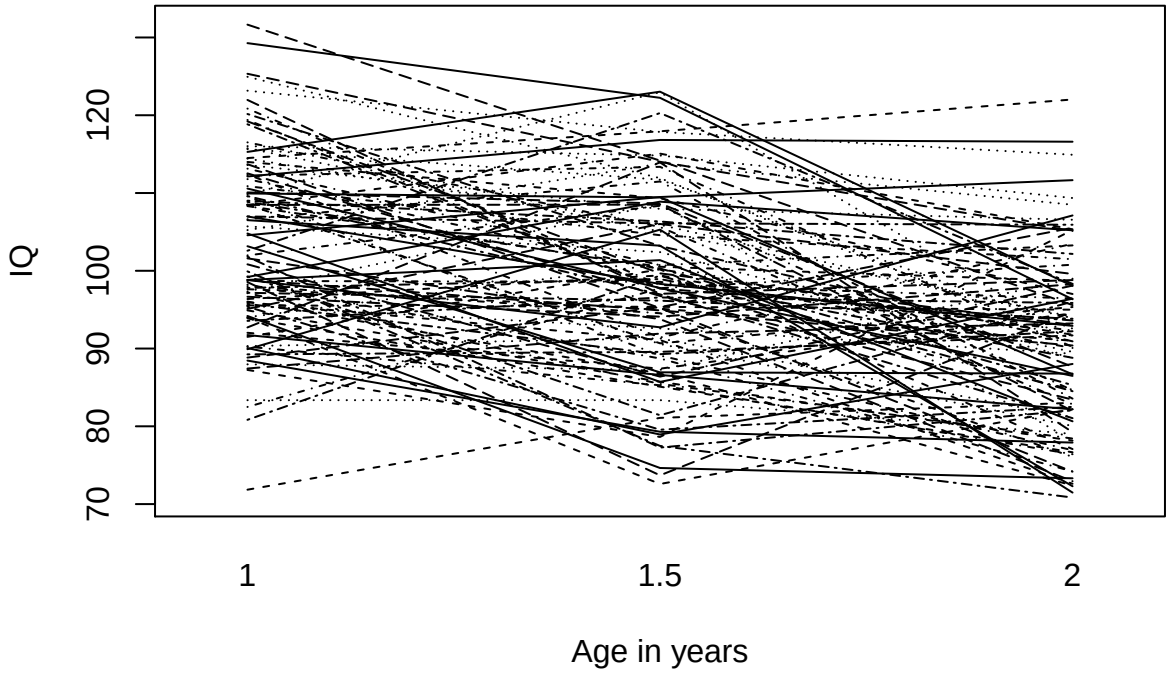
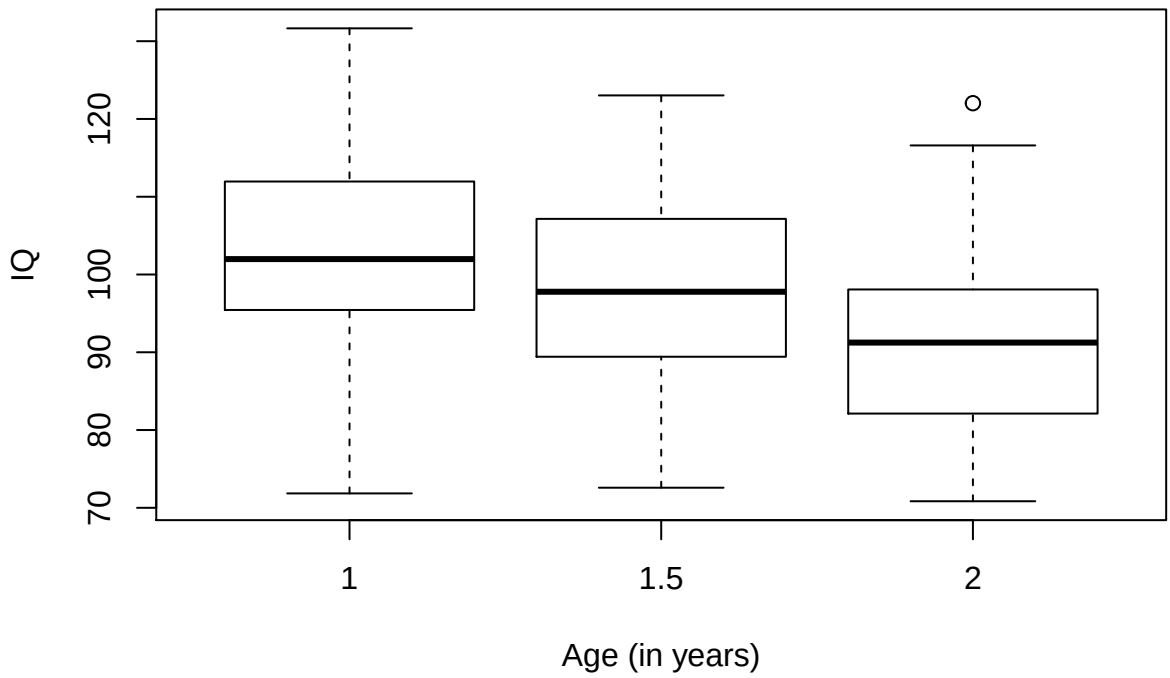
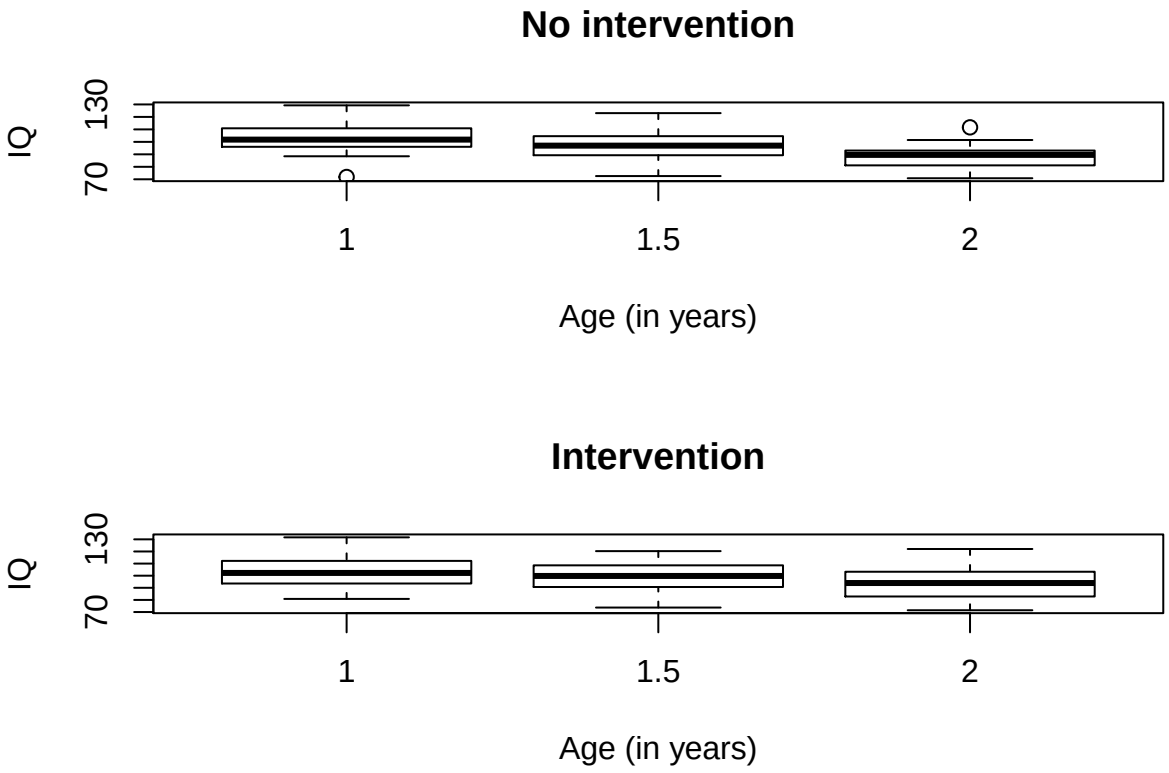


Spaghetti plot.
Descriptives.

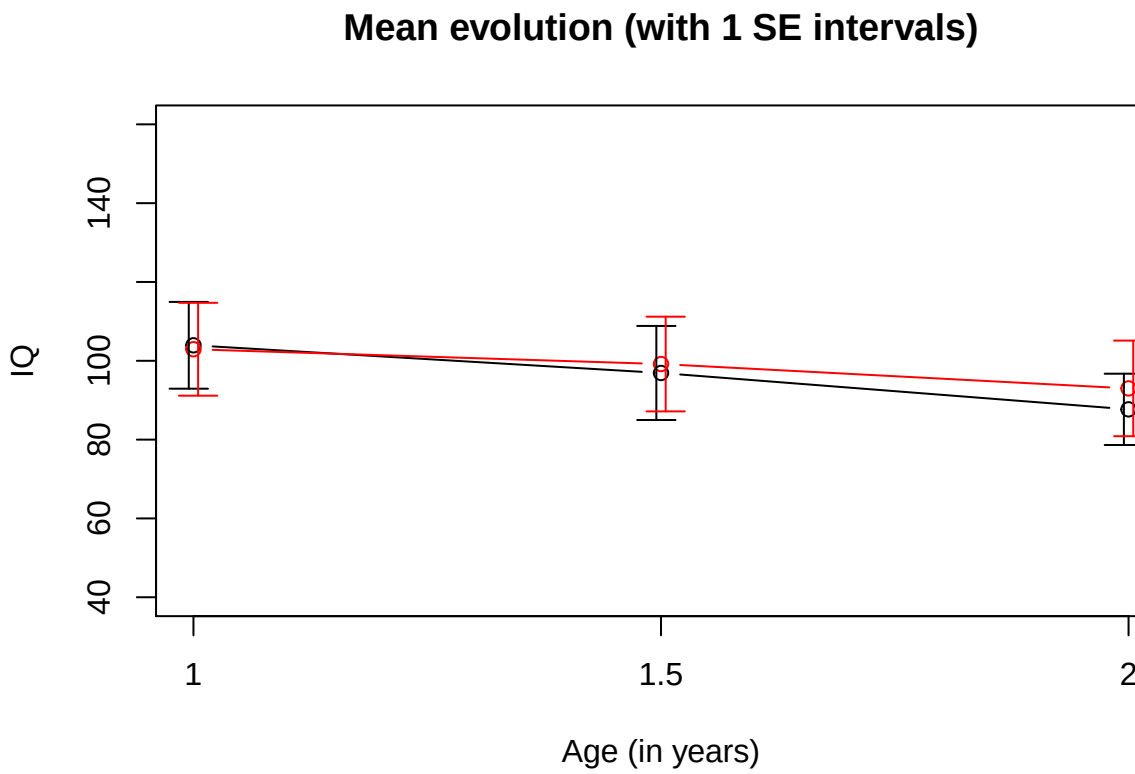


Boxplots.



Boxplots per program.

General function to plot error bars.



Plotting mean evolutions.

Correlation between IQ scores at different ages.

```
##       id program     cog.1   cog.1.5     cog.2
## 1      1       1 106.98289  98.31060  92.91342
## 2      2       1 108.86019 100.29307  85.29502
## 3      3       1 112.52438  96.76684  83.42649
## 4      4       1  90.24428  85.27380  76.41052
## 5      5       1 105.70738 102.39839  88.78872
## 6      6       1  93.88987  85.09601  76.66209
## 7      7       1 109.93899 109.43202  86.68573
## 8      8       1 106.98599 106.09735 105.92056
## 9      9       1 125.33166 114.01277 105.12163
## 10    10       1  82.49028  97.76043 100.77653
## 11    11       1 105.22740 113.77558 108.48978
## 12    12       1 114.49240 117.93460 122.02861
## 13    13       1  91.73435  86.60655  82.18117
## 14    14       1  95.37839  79.56730  82.49799
## 15    15       1 110.07345 108.65272  87.65045
## 16    16       1  98.01933  94.54154  76.07024
## 17    17       1  87.96931  87.25195  98.98177
## 18    18       1 108.31917 100.51574  98.70037
## 19    19       1  94.18916  74.64264  73.31470
## 20    20       1 120.16253 103.18718  90.92738
## 21    21       1 131.65178 114.16135  95.58954
## 22    22       1 120.85608  98.18288  96.44816
## 23    23       1 116.02689 114.82364 109.35596
```

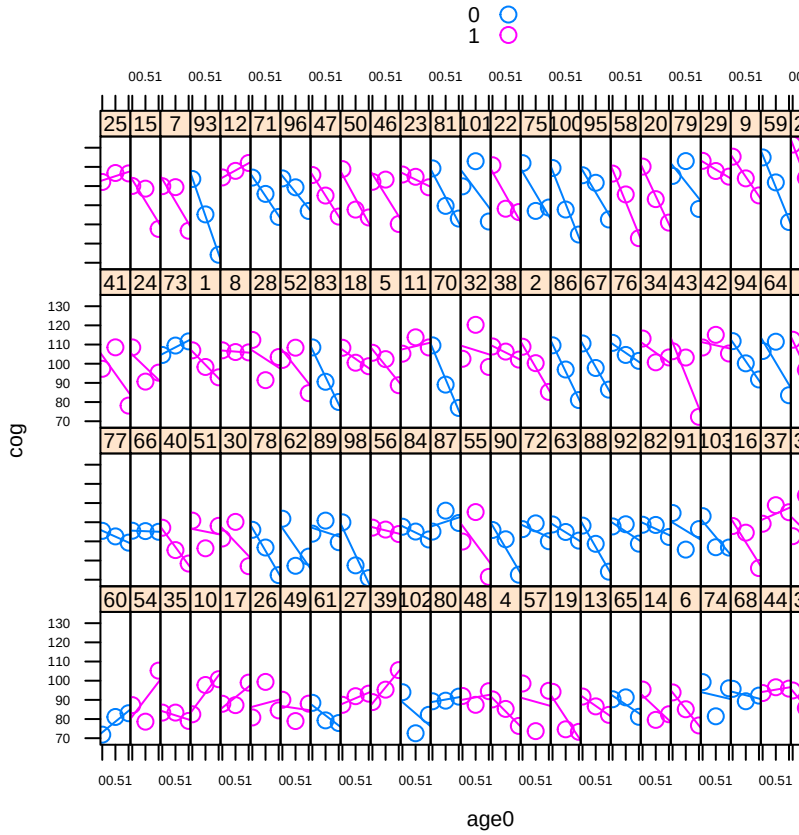
```
## 24   24       1 108.50925  90.67410  95.33989
## 25   25       1 112.12633 116.82108 116.59443
## 26   26       1  80.83364  99.37240  84.50895
## 27   27       1  87.42831  91.92213  93.23212
## 28   28       1 112.40877  91.41400 103.47415
## 29   29       1 123.16009 117.85460 114.91998
## 30   30       1  91.50417 100.18023  77.07159
## 31   31       1  98.81854  92.73035 107.11404
## 32   32       1 102.57830 120.27905  98.31582
## 33   33       1  92.71612 113.93789  79.24296
## 34   34       1 113.13320 100.72077 103.26529
## 35   35       1  83.36287  83.36361  79.02352
## 36   36       1  94.91031  85.72218  78.16092
## 37   37       1  99.20004 108.88909 105.20391
## 38   38       1 109.00945 106.33822 102.16497
## 39   39       1  88.83725  95.32658 105.52882
## 40   40       1  97.11882  85.45274  78.42828
## 41   41       1  97.34675 108.54913  78.15538
## 42   42       1 108.46227 115.06198 105.33598
## 43   43       1 106.51196 103.23088  72.29365
## 44   44       1  93.55798  96.57147  95.85989
## 45   45       1  96.66016  94.09513  98.27780
## 46   46       1 112.22206 113.36683  90.21796
## 47   47       1 115.75380 105.04866  94.08408
## 48   48       1  92.02678  87.41049  94.53681
## 49   49       1  90.05997  78.94745  88.01800
## 50   50       1 118.84066  97.71249  93.61212
## 51   51       1 100.83136  86.38944  98.09643
## 52   52       1 101.98162 108.36219  84.64359
## 53   53       1 113.60019 107.96398 106.56405
## 54   54       1  87.24872  78.63402 105.24226
## 55   55       1  89.91285 105.28723  71.47761
## 56   56       1  97.17166  96.15962  93.83076
## 57   57       1  98.54182  73.69470  94.72776
## 58   58       1 116.51412 105.63534  82.82452
## 59   59       0 124.94527 111.91136  91.24352
## 60   60       0  71.84974  81.08563  83.02724
## 61   61       0  88.45453  79.30242  77.91012
## 62   62       0 101.81687  77.26865  82.05273
## 63   63       0  98.83434  94.90388  90.39131
## 64   64       0 106.62630 111.48140  83.56083
## 65   65       0  90.31671  91.30650  81.15678
## 66   66       0  95.55040  95.36279  94.99702
## 67   67       0 110.57535  97.77947  86.48163
## 68   68       0  95.73969  89.27407  92.22706
## 69   69       0 108.46676 108.50177  81.96456
## 70   70       0 109.57738  89.10167  76.92818
## 71   71       0 114.42217 105.91386  93.90563
## 72   72       0  96.37015  99.40169  90.07637
## 73   73       0 104.56600 109.48258 111.65059
## 74   74       0  99.27624  81.42511  96.03642
## 75   75       0 121.96798  97.03722  98.80731
## 76   76       0 110.88300 104.57459 101.48922
## 77   77       0  95.49981  92.58608  89.33631
```

```
## 78   78       0  96.03821  86.79500  72.39639
## 79   79       0 115.28714 123.02920  98.05151
## 80   80       0  89.25232  89.57222  91.68889
## 81   81       0 119.21283  99.67131  93.03702
## 82   82       0  98.68876  98.57612  92.32044
## 83   83       0 108.50237  90.58188  80.04558
## 84   84       0  97.63610  95.12270  90.99789
## 85   85       0  98.82146 101.36631  80.64234
## 86   86       0 109.60582  96.96958  81.04709
## 87   87       0  95.09582 106.02006  99.58778
## 88   88       0  98.22010  88.72318  74.19584
## 89   89       0  94.03053 100.83250  89.57231
## 90   90       0  95.94445  91.09095  72.61159
## 91   91       0 104.79640  85.71041  96.40972
## 92   92       0  97.79446  98.93178  88.80250
## 93   93       0 113.70390  95.20994  74.17274
## 94   94       0 111.78680 100.19471  91.77763
## 95   95       0 115.63780 111.71134  92.56601
## 96   96       0 113.99454 109.34016  97.01341
## 97   97       0 129.27171 122.23883  96.33804
## 98   98       0  99.92436  77.48019  70.84332
## 99   99       0 101.59745  93.78219  72.94243
## 100 100       0 119.40051  97.75317  84.70497
## 101 101       0 109.84539 122.96455  91.49998
## 102 102       0  93.98954  72.58405  82.19806
## 103 103       0 103.16078  86.94306  86.77639
```

```
##             cog.1   cog.1.5     cog.2
## cog.1   1.0000000 0.5816070 0.3263912
## cog.1.5 0.5816070 1.0000000 0.4371109
## cog.2   0.3263912 0.4371109 1.0000000
```

2 Linear Regression Per Person

Creating the time variable.

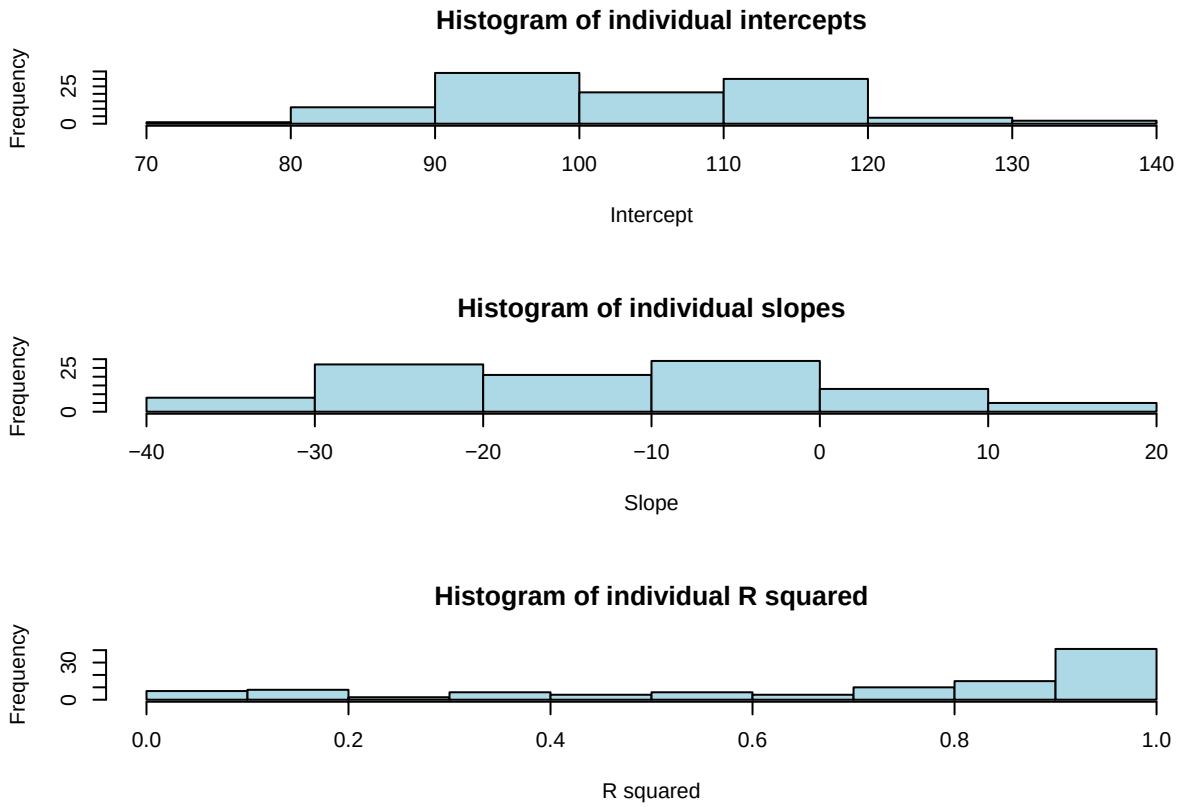


Displaying the linear regression per person.

2.1 Linear regression of cog on age per participant.

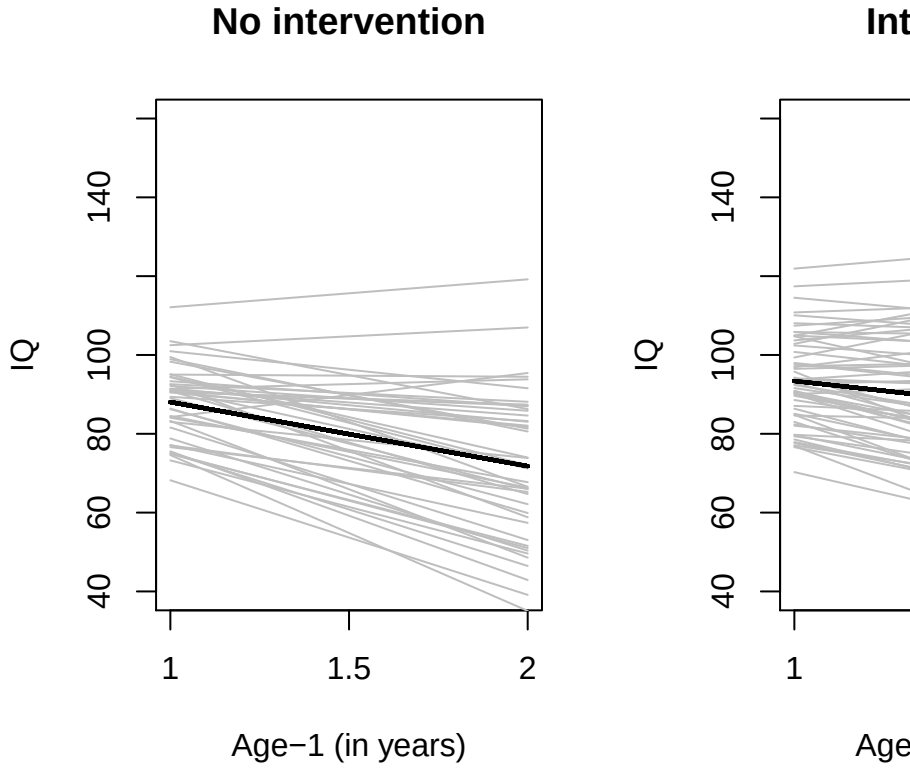
Coefficients.

R-Squared.


Histogram of individual intercepts
Frequency
0
25
70
80
90
100
110
120
130
140
Intercept
Histogram of individual slopes
Frequency
0
25
−40
−30
−20
−10
0
10
20
Slope
Histogram of individual R squared
Frequency
0
30
0.0
0.2
0.4
0.6
0.8
1.0
R squared

Histograms.

3 Linear regression per person and group



Plotting individual regression lines per group.

4 Fitting the Model

4.1 Installing the Packages

Creating the time variable.

Fitting the model with maximum likelihood.

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: cog ~ 1 + age0 * program + (1 + age0 | id)
##    Data: early.int1
##
##      AIC      BIC   logLik deviance df.resid
##   2332.5   2362.4  -1158.3   2316.5      301
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.25362 -0.59088  0.02131  0.56850  2.29366
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  id       (Intercept) 84.02    9.166
```

```
##           age0        39.44    6.281    -0.55
##  Residual             60.31    7.766
## Number of obs: 309, groups:  id, 103
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  104.3007     1.7274  60.380
## age0         -16.2555     1.8860  -8.619
## program       -0.9646     2.3020  -0.419
## age0:program   6.3187     2.5133   2.514
##
## Correlation of Fixed Effects:
##             (Intr) age0   progrm
## age0        -0.629
## program     -0.750  0.472
## age0:progrm  0.472 -0.750 -0.629
```

4.2 Estimating the fixed effects via bootstrap

```
##
## PARAMETRIC BOOTSTRAP
##
##
## Call:
## bootMer(x = early.lmer1, FUN = fixef, nsim = 250, use.u = TRUE)
##
##
## Bootstrap Statistics :
##        original      bias    std. error
## t1* 104.3007437  0.03075438    1.025165
## t2* -16.2554565 -0.09608901    1.535844
## t3*  -0.9646326 -0.01093838    1.433200
## t4*   6.3187112  0.07253470    2.098457

##
## Number of bootstrap replications R = 250
##               original  bootBias bootSE   bootMed
## (Intercept)  104.30074  0.030754 1.0252 104.37044
## age0         -16.25546 -0.096089 1.5358 -16.26872
## program       -0.96463 -0.010938 1.4332  -0.95388
## age0:program   6.31871  0.072535 2.0985   6.17482
```

4.3 Calculating confidence intervals for the fixed effects via Wald, bootstrap and profile likelihood

```
##                   2.5 %     97.5 %
## (Intercept)  100.915097 107.686391
## age0         -19.951912 -12.559002
## program       -5.476396   3.547131
## age0:program   1.392761  11.244662

##                               2.5 %       97.5 %
```

```
## sd_(Intercept)|id         6.8981635  11.19026344
## cor_age0.(Intercept)|id  -1.0000000   0.06097266
## sd_age0|id                0.6633188   9.60636071
## sigma                     6.7861169   8.69849960
## (Intercept)             100.8424628 107.71536998
## age0                    -20.2665438 -12.32832435
## program                  -5.2967370   3.63354473
## age0:program              1.0641531  11.30936156

##                              2.5 %     97.5 %
## sd_(Intercept)|id         7.005366  11.406182
## cor_age0.(Intercept)|id  -1.000000   1.000000
## sd_age0|id                0.000000   9.975354
## sigma                     6.814978   8.953288
## (Intercept)             100.883287 107.718200
## age0                    -19.986640 -12.524273
## program                  -5.518786   3.589521
## age0:program              1.346481  11.290942
```

4.4 Get the KR-approximated degrees of freedom

4.5 Likelihood ratio tests

```
## Data: early.int1
## Models:
## early.lmer1.noprog: cog ~ 1 + age0 + (1 + age0 | id)
## early.lmer1.intprog: cog ~ 1 + age0 + program + (1 + age0 | id)
## early.lmer1: cog ~ 1 + age0 * program + (1 + age0 | id)
##                      npar    AIC    BIC  logLik deviance  Chisq Df Pr(>Chisq)
## early.lmer1.noprog      6 2336.8 2359.2 -1162.4   2324.8
## early.lmer1.intprog     7 2336.7 2362.8 -1161.3   2322.7 2.0840  1    0.14885
## early.lmer1             8 2332.5 2362.4 -1158.3   2316.5 6.1345  1    0.01326 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

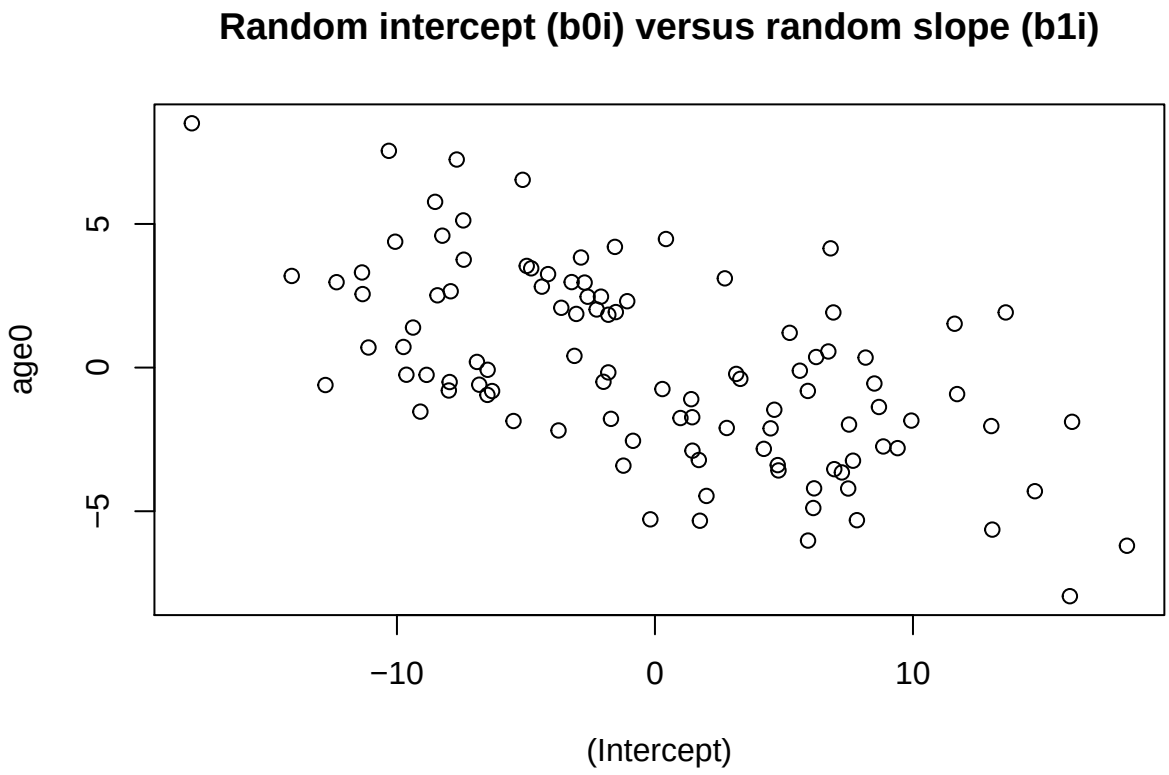
4.6 Random effects covariance matrix

```
##             (Intercept)      age0
## (Intercept)    84.01946 -31.89378
## age0          -31.89378  39.44496
## attr(,"stddev")
## (Intercept)        age0
##    9.166213    6.280522
## attr(,"correlation")
##             (Intercept)       age0
## (Intercept)   1.0000000 -0.5540135
## age0         -0.5540135  1.0000000
```

4.7 Predicted random effects

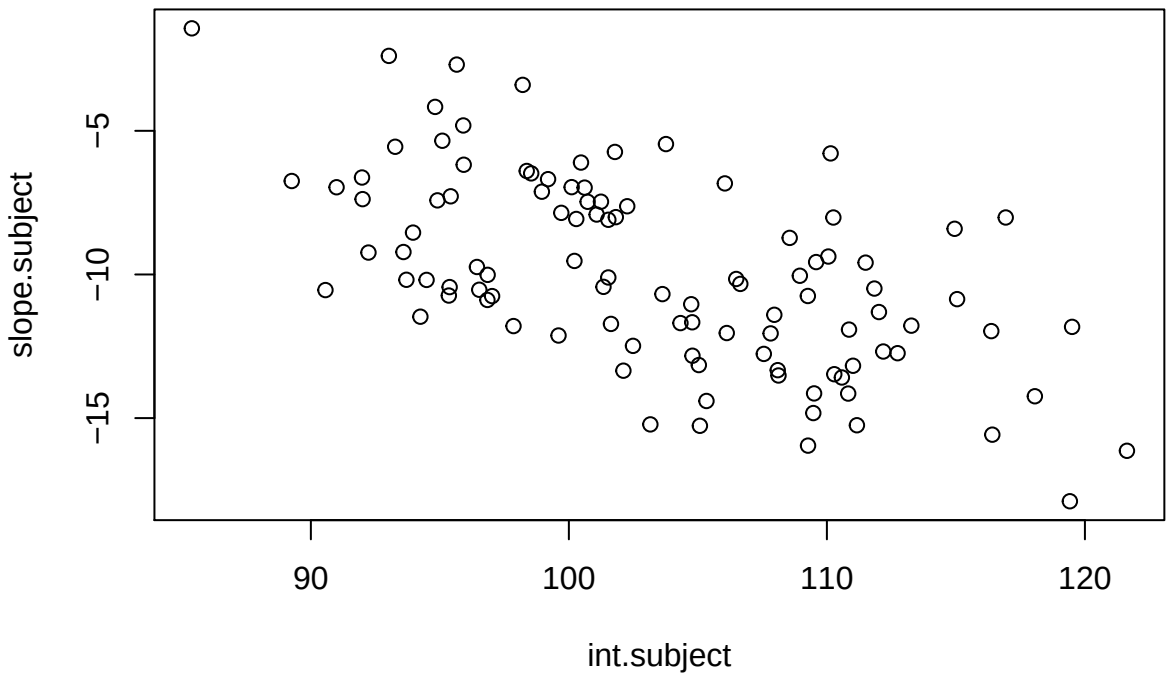
```
##    (Intercept)       age0
```

```
## 1     1.406085 -1.0998501
## 2     1.700819 -3.2167576
## 3     1.996405 -4.4674896
## 4   -11.103349  0.6994181
## 5     1.444739 -1.7280748
## 6    -9.633038 -0.2479249
## 7     4.787570 -3.5798370
## 8     5.221722  1.2096924
## 9    14.723774 -4.3028253
## 10   -7.682855  7.2428573
```



OLS vs LM Estimates ## Creating the subject specific intercepts and slopes

```
##    (Intercept)      age0    program age0:program
## 1    105.70683 -17.35531 -0.9646326     6.318711
## 2    106.00156 -19.47221 -0.9646326     6.318711
## 3    106.29715 -20.72295 -0.9646326     6.318711
## 4     93.19739 -15.55604 -0.9646326     6.318711
## 5    105.74548 -17.98353 -0.9646326     6.318711
## 6     94.66771 -16.50338 -0.9646326     6.318711
```


Random intercept versus random slope (Including the fixed effects
slope.subject
−5
−10
−15
90
100
110
120
int.subject