DEPARTMENT OF COMPUTER SCIENCE

A Virtual Reality Learning Environment for Calculus

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Declaration

This dissertation is submitted to the University of Bristol in accordance with the requirements of the degree of BSc in the Faculty of Engineering. It has not been submitted for any other degree or diploma of any examining body. Except where specifically acknowledged, it is all the work of the Author.

James Daniel O'Reilly, Sunday 12th May, 2019

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Abstract

Virtual reality technology presents an exciting challenge for both educators and HCI researchers alike. As accessibility to this technology continues to increase, is is imperative that we research the effects it may have on learning experience and learning outcomes. The field of mathematics education is beginning to recognise the potential that immersive technologies may have to teach abstract concepts in mathematics.

This paper presents a comprehensive review of the relevant educational psychology for calculus, a review of previous technological aids in calculus education, and motivates the use of virtual reality technology as a teaching aid within this context. The development of a multi-user virtual reality application for teaching elementary calculus is then carefully detailed. The application was evaluated with secondary-level mathematics students to assess the effects on student engagement and collaboration. The results indicated that students were strongly engaged by the application, and pointed toward ways of improving the application to better facilitate and encourage productive, collaborative interactions.

Contents

Chapter 1

Introduction, Background and Literature Review

1.1 Terminology

The definitions of representation/inscription, visualisation, visualiser, and visual image are given below. The rationales for these terms and their respective definitions borrow heavily from the definitions outlined in Presmeg's review "Research on Visualisation Learning and Teaching Mathematics" in the 2006 Handbook of Research on the Psychology of Mathematics Education [1][2].

Visualisation: When a person conceives of or constructs some spatial arrangement, this formulation is guided by a visual image in the person's mind [3]. Visualisation is therefore defined as the set of processes used when constructing both visual mental imagery and any representations or inscriptions of a spatial nature involved in doing mathematics [4] [2].

Visual Image: A mental conception which depicts visual or spatial information [5].

Visualiser: A visualiser is someone who, given a choice of methods when problem solving, will prefer or tend to use visual methods.

Representation/Inscription: Roth (2004) [6] explains that graphical representations are known as inscriptions in the sociology of science, and that the term inscriptions is preferred to representations as the latter has various meanings and is inherently ambiguous. The term inscriptions will thus be used in this thesis. Any employment of the term representations will be accompanied by some descriptor such as numerical, symbolic, or graphical.

1.2 Basic Differential Calculus Concepts

The following is a brief review of the relevant theory. A full understanding of the concepts presented here is not necessary for a proper reading of this paper, but is useful when reading both Section 2 and Section 3.

Given some function $f(x)$, the derivative of $f(x)$ at a given point represents the rate of change of $f(x)$ at that point, which is equivalent to the slope of the line tangential to $f(x)$ at that point. In this paper, $f`(x)$ represents the first derivative and $f``(x)$ represents the second derivative. The relationship between a function and its derivative is illustrated in Figure 1.1.

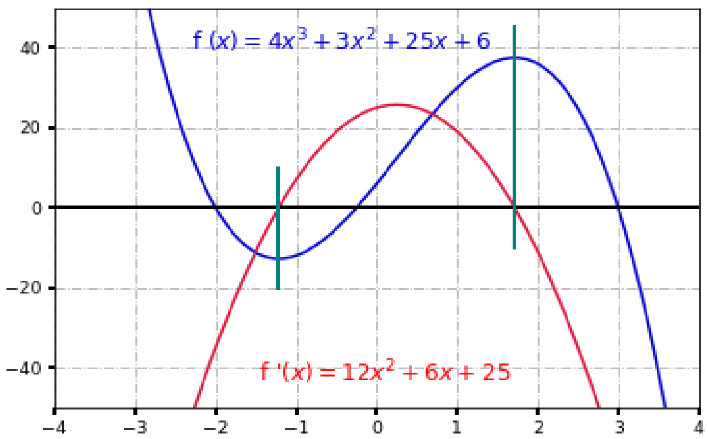


Figure 1.1: A figure showing how a function relates to its derivative.

Real world differential calculus problems usually fall under one of two categories:

1. Problems of optimisation
2. Problems of related rates of change

Problems of Optimisation

One common application of differential calculus is calculating the minimum or maximum value of a function. Problems of optimisation ask the student to find this minimum or maximum, subject to certain constraints. Given a function $f(x)$, any tangent to $f(x)$ at a maximum or minimum point will have a slope of zero and so $f'(x) = 0$.

Problems of optimisation can mostly be solved using these steps:

1. Calculate the derivative $f`(x)$.
2. Set the derivative $f`(x) = 0$ (As this is when the tangent will be horizontal).
3. Solve for x. This gives the x-coordinates of the maxima and minima.
4. Plug these x value(s) into the original equation $f(x)$ to calculate the y-coordinates of the maxima and minima.

Problems of Related Rates of Change

If a given quantity is changing over time, we know that the rate at which this quantity changes is given by the derivative. If two related quantities are changing over time, the rates at which these quantities change are related. For example, if a balloon is being filled with air, both the radius of the balloon and the volume of the balloon are increasing. In real world calculus problems, these related quantities are changing with respect to time. Developing further on the balloon example, the rate of change of volume $\boldsymbol{V}$, given by dV/dt is related to the rate of change of radius $\boldsymbol{r}$ given by dr/dt. In order to solve related rates of change problems, students are often required to use the Chain Rule. The *Chain Rule* defines an equation for how two such properties relate:

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \tag{1.1}$$

This rule can be illustrated using the balloon example:

We have that the volume of the balloon is given by $V = 4/3\pi r^3$ and so we can differentiate with respect to r to find $dV/dr = 4\pi r^2$. If the rate of change of radius $dr/dt = 2$, then the rate of change of volume can be found using the Chain Rule:

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} = 4\pi r^2 \cdot 2 = 8\pi r^2 \tag{1.2}$$

Problems of related rates can mostly be solved using the following steps:

1. Assign symbols to all variables involved in the problem.
2. State, in terms of the variables, the information that is given and the rate to be determined.
3. Find an equation relating the variables introduced in Step 1.
4. Using the chain rule, differentiate both sides of the equation found in step 3 with respect to the independent variable. This new equation will relate the derivatives.
5. Substitute all known values into the equation from step 4, then solve for the unknown rate of change.

1.3 The Psychology of Maths Education for Calculus

1.3.1 Student Difficulties in Calculus

A first course in calculus typically involves studying the theory of limits, derivatives, and integrals, followed by an application of this theory to real problems. David Tall's paper *"Students' Difficulties in Calculus"* [7] outlines the pedagogical issues encountered by the educator and the epistemological barriers faced by the typical calculus student.

Pedagogical Issues

The primary pedagogical issue is that students often learn to solve calculus problems on a case-by-case basis. This allows students who do not have a well-founded conceptual understanding to maintain varying and incompatible beliefs regarding certain central concepts. The student will then select from these different conceptions according to the context being considered, but will often be blind to any inconsistencies that arise. This issue is a result of students learning to pass a particular exam by focusing their attention on the procedural aspects of the calculus curriculum. The following is a excerpt from *"The Laboratory Approach to Teaching Calculus"* from the Mathematical Association of America:

> *"Much of what our students have actually learned ... – more precisely, what they have invented for themselves – is a set of "coping skills" for getting past the next assignment, the next quiz, the next exam. When their coping skills fail them, they invent new ones. The new ones don't have to be consistent with the old ones; the challenge is to guess right among the available options and not to get faked out by the teacher's tricky questions. ... We see some of the "best" students in the country; what makes them "best" is that their coping skills have worked better than most for getting them past the various testing barriers by which we sort students. We can assure you that that does not necessarily mean our students have any real advantage in terms of understanding mathematics."* [8]

Tall suggests that the primary coping mechanism employed by students when faced with an epistemological barrier, is not to attempt to grasp the concepts on a deeper level or to consolidate their inconsistent conceptions, but instead to focus on the procedural aspects of the calculus course in examinations. Teachers are aware of this and are therefore somewhat motivated to set procedural questions, understanding that questions which require a strong conceptual understanding are unlikely to be answered correctly. The result of this cycle of procedural learning and examination is that students are discouraged from attempting to formulate a deep and consistent understanding of central concepts in calculus.

Epistemological Barriers

There are three epistemological barriers that are particularly relevant to this thesis:

1. The inability or refusal of the student to use visual methods.
2. The inability of the student to interpret three dimensional representations.
3. Difficulty on the part of the student in selecting appropriate representations.

1. Given that functions are typically differentiated and integrated in their algebraic representation, it is understandable that this issue is often overlooked. However, when students encounter unfamiliar functions, the inability to construct an accurate graphical representation from the given algebraic representation causes problems [7].

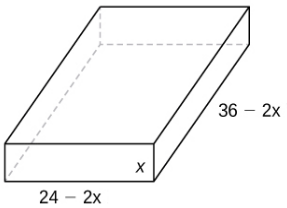
2. When given a real world problem (typically in three dimensions), students often struggle to formulate an accurate algebraic or symbolic representation from the given diagrammatical or verbal representation. Tall suggests this is due to the fact that "Many examinations for calculus focus on the symbolic manipulation rather than problem-solving" [7]. It is possible that Tall's explanation only partly accounts for the inability of the student to transition between representations, and that much of the problem also lies in students' inability to interpret two dimensional representations of three dimensional problems [9].

3. Finally, students often encounter difficulties in selecting and using the appropriate representations. Robert and Boschet reported that the students were most successful when they could reliably and flexibly elicit multiple approaches and switch between different representations (see Figure 1.2) [10]. An influential paper by Dreyfus and Eisenberg concluded that students of calculus are often reluctant to visualise [11]. When solving problems, students typically have a preference for algebraic and symbolic representations. The claims made by Dreyfus and Eisenberg are subject to some criticism which is discussed in Section 1.3.2.

A typical applied calculus question in which a graphical representation is required is with "Solids of Revolution". In calculating the volume of a solid, the student must visualise the three-dimensional solid with reference to a two-dimensional graphical representation. When studying the approach taken by students to such problems, Ferrini-Mundy discovered that incorrect solutions were rarely the result of solving the integral incorrectly, but instead the result of errors in setting up the integral [12][13]. This observation supports the claim that student difficulties with calculus are rarely the result of procedural errors, but are instead primarily related to their inability to construct or transition between different representations.

White and Mitchelmore [14] conducted a study identifying the difficulties faced by students when solving applied differential calculus problems. They discovered that students struggle most when attempting to generate a symbolic representation of the verbal problem situation. In these cases, interpreting or constructing a visual representation of the problem has been shown to help, as this allows the student to better recognise the relationships present and from there to construct an accurate symbolic representation of the problem.

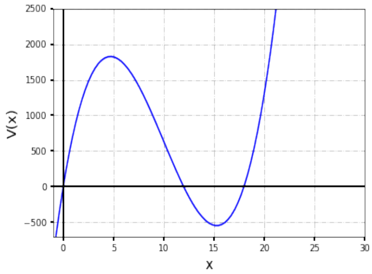
The general reluctance to visualise problems and the issues involved in mapping to and from graphical representations are critical epistemological barriers which must be overcome in order to improve the standard of calculus education. The sections that follow further emphasise the importance of visualisation training, discuss previous attempts at solving this problem, and will then motivate the use of technology as a teaching aid for calculus within this context.



(a) 3-Dimensional Representation

$$V(x) = 4x^3 - 120x^2 + 864x$$

(b) Algebraic Representation



(c) Graphical Representation

Figure 1.2: Different representations of the volume of a box.

1.3.2 On the Reluctance to Visualise

Having highlighted the problems that arise due to students' inability to use graphical representations, the following discusses Dreyfus' claim that there is a general reluctance on the part of students to visualise.

The point raised by Dreyfus and Eisenberg [11] at the Psychology of Mathematics Education Conference in 1991 was considered by Presmeg and Bergsten [15] and Healy and Hoyles [16]. Healy and Hoyles reported that students seldomly avail of the potential that visual methods offer and that, when presented with a symbolic representation of the problem, they are reluctant to engage visually or graphically. However, as the literature on the topic has progressed, the idea that students are "reluctant to visualise" has evolved since Dreyfus' 1991 paper.

Bremigan [17] examined the diagrams that mathematically capable high school students produced in solving calculus problems and investigated the relationship between the frequency of the use of diagrams and problem solving success. The results showed that mathematically capable high-school students frequently modified or constructed diagrams in solving applied calculus problems and that producing graphical representations of calculus problems was critical to the problem solving process.

This claim was buttressed by Stylianou [18], who conducted a study of the perceptions and use of visualisation by professional mathematicians and students alike, from which she drew the following conclusion:

> *"Both experts and novices perceive visual representations as a useful tool and frequently attempt to use them when solving problems. ... However, further analysis clearly showed that the changes may only be covering the surface; students may be willing to use visual representations but have very little training associated with this skill."*

Stylianou notes that students are willing to engage with visual modes of reasoning, but often do not possess the skills to do so effectively. The paper concludes by encouraging

> *"mathematics educators to make more explicit and informed decisions about visual representation use in curricular materials and instruction, providing opportunities for students to become more successful problem solvers."*

This sentiment is echoed by Dreyfus:

> *"It is therefore argued that the status of visualisation in mathematics education should and can be upgraded from that of a helpful learning aid to that of a fully recognised tool for learning and proof"*

The literature suggests that while gifted students and professional mathematicians regularly use visual representations as problem solving tools, typical mathematics students are unlikely to engage with visual modes of reasoning. This, however, is not a result of a general reluctance to visualise on the part of students, as Dreyfus claimed. More recent research indicates that students are willing to use visual methods, but few students possess the skills to do so. The body of research indicates that educators should encourage the use of visual methods in the mathematics classroom and provide adequate opportunities for students to develop their visual reasoning skills. Educational technologies such as graphics calculators, 2-D and 3-D graphics, augmented reality, and virtual reality each provide interesting and unique opportunities for encouraging and training visual modes of reasoning.

1.4 The Application of Educational Technology to Calculus

Many attempts have been made to integrate technology into the calculus classroom, some more successful than others. The motivation for the use of technological aids was twofold:

1. Students and educators were dissatisfied with the traditional, formalist approach to teaching calculus

2. New technologies were made available so why not use them?

In the past it would seem that many technological interventions were pragmatic, based on a superficial understanding of pedagogy and epistemology, and not based on empirical research. This lead to many failed attempts to integrate technology into the learning process. Furthermore, these technologies were often market driven and therefore pressure existed for these technologies to be adopted once developed, irrespective of their effectiveness as teaching and learning tools. This section reviews what has been learned from the previous use of technological aids in calculus education, followed by an analysis of the current uses of this technology.

1.4.1 A History of the Use of Educational Technology in Calculus

Graphic Visualisation

As part of the calculus reform movement in America which commenced in the late 1980's [19], educators began to appreciate the benefits that high-resolution graphics could bring to calculus education [20]. Heid(1998) [21] and Palmiter(1991) [22] studied the effects of this new technology on high-school calculus students and confirmed that the visual approach helped to develop a deeper conceptual understanding. One such example is the use of graphics calculators.[23]

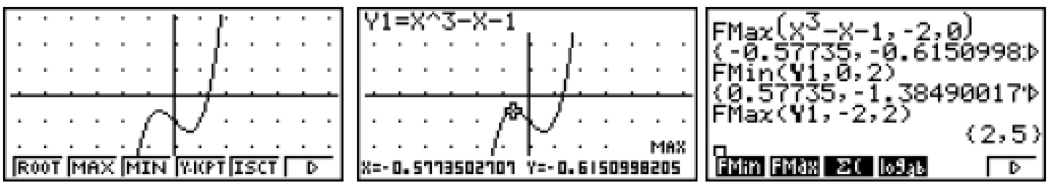


Figure 1.3: A graphics calculator obtaining maxima and minima from a graph or numerically [23].

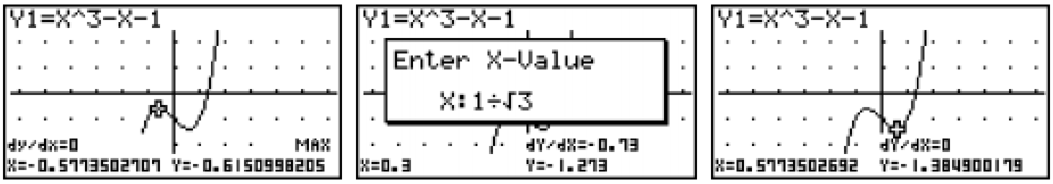


Figure 1.4: A graphics calculator exploring turning points [23].

Enactive Control

While the use of graphics for visualisation was a step forward, the process of looking at graphics is ultimately a passive exercise, where the student is a passive recipient of knowledge and does not otherwise engage or interact with the tool. The introduction of the mouse and keyboard as input devices for personal computers in the 1980's provided an interface through which students could interact with visual representations to explore mathematical concepts. This technology facilitated learning through active exploration.

Computer Algebra Systems

Computer algebra systems (CAS) were introduced in the 1990's. These CAS were capable of taking a symbolic representation of a problem and quickly producing a solution. This eliminated the focus on the procedural aspects of questions and allowed the student to instead focus on the underlying concepts. CAS whose designs have been informed by proper analysis and educational psychology have shown to produce noticeable improvements in students' conceptual understanding when implemented correctly, and can have no effect or adverse effects if implemented poorly [24].

1.4.2 The Current Use of Educational Technology in Calculus

The integration of these technologies into today's classroom has been somewhat slow and frustrating for those who are aware of the potential benefits. Computer Algebra Systems are rarely used in secondary education. Furthermore, many educators seem reluctant to use computer graphics as a teaching aid. The graphical tools that are available, however, are rarely interactive and adhere to the traditional model of the student as a passive recipient of knowledge. Perhaps there is a role for immersive technologies such as virtual reality that allow for visualisation of abstract mathematical concepts and facilitate learning through interaction and active engagement.

1.5 Immersive Technology in Education

The potential for VR technology in education was recognised long before the recent developments in the VR market involving companies such as Oculus, HTC, PlayStation, and Google. Significant research was conducted in the nineties evaluating the potential uses for this technology in education and defining researchable issues going forward. The following is an excerpt from Sandra Helsel's 1992 paper "Virtual Reality and Education":

> *"As a profession, education is responding powerfully to the notion of virtual reality curriculum. Educators seem to have an instant - and almost visceral - understanding of the learning potential that well-designed, virtual experiences could offer students."* [25]

However, it is necessary to include a note of caution. Ever since educators and business people recognised the potential of educational technologies, there has been significant pressure for their immediate deployment. This rapid general endorsement of emerging technologies does not allow adequate time for assessment of the effects - both positive and negative - that these technologies may have on current pedagogy, the classroom environment, the learning experience, and ultimately learning outcomes.

In my view, this approach is typified by the calculus reform movement in America. Commercial interests collaborated with educators to develop and implement new educational technologies, the effects of which took years to become apparent [26][27]. It is important that the motivation for these technologies is rooted in the desire to improve both learning experiences and learning outcomes, and care is needed to ensure that empirical evidence and research trump other commercial motives. The following is a consideration of the advantages of virtual reality technology in education.

1.5.1 Active Learning

Active learning refers to teaching and learning methods which put the student in charge of their own learning through meaningful activities. It helps the student to reflect upon concepts and the active application of these concepts. It is in direct contrast to the traditional approach, wherein the student is a passive recipient of knowledge. Intuitively this approach makes sense when teaching about three dimensional objects in space: interacting with a solid cube in the real world conveys far more information than observing a diagram of the same cube on a page or whiteboard. Joel Michael's paper "Where's the evidence that active learning works?" summarises research highlighting the benefits of active learning in science education [28].

Immersive technologies allow us to simulate life-like interactions with objects and structures for which it is impractical to give the student a physical copy. Further developing the cube example, what if we wanted the student to understand how the properties of the cube change when we vary certain parameters such as length, width or height? Or perhaps we wanted the student to understand how the three dimensional cube is formed from its net? This would be impractical with a physical model. Ultimately calculus is the mathematics of change, and using interactive animation within an immersive virtual environment allows us to represent the dynamic properties of three-dimensional objects.

1.5.2 Student Engagement

Research has shown that immersive learning environments can enhance student engagement. Hanson and Shelton's 2008 Paper "Design and Development of Virtual Reality: Analysis of Challenges face by educators", outlines how educators can (and should) take advantage of the immersive power of virtual reality as a means of stimulating engagement with learning activities [29]. Dickey (2003) showed that learners' engagement could be enhanced by virtual learning environments with photo-realistic computer graphics [30]. Chris Dede's "Immersive Interfaces for Engagement and Learning" evaluated student engagement and learning with an immersive technology curriculum. Dede's research indicated that "students were deeply engaged by this curriculum through actional and symbolic immersion and developed sophisticated problem solving skills" [31]. Furthermore, research into educational multi-user virtual environments has shown that students were more engaged in the immersive interface and learned as much or more when compared with a similar, paper-based curriculum [32].

Enhanced student engagement could be a result of the fact that immersive technology is inherently more engaging than traditional teaching methods, or simply because the change in instructional method helps students to maintain focus and prevents boredom. The VR learning environment developed during this project is intended to be used as a supplementary teaching tool, and not as a replacement for traditional teaching methods. Therefore it is not important exactly why student engagement is enhanced, but that it is enhanced nonetheless.

1.5.3 Collaborative Learning

There is not a consistent definition of Collaborative Learning with which all scholars agree. This paper will use the definition given by Dillenbourg in his 1999 paper "what do you mean by collaborative learning?" [33]: He defines collaborative learning as a situation in which two or more people learn or attempt to learn something together. More specifically, he defines the collaborative activity as

> *"...joint problem solving, and learning is expected to occur as a side effect of problem solving, measured by the elicitation of new knowledge or by the improvement of problem solving performance."* [8]

Dillenbourg then discusses the variety of scales on which collaborative learning can exist, including group size and time-scale. Naturally, a collaborative learning experience involving 30 students over the space of a year is not comparable to a collaborative learning experience involving two students during one lesson. Importantly, the educational application developed in this project was only evaluated with small groups of students on a single day. If applications such as the one presented in this paper were to be readily adopted into the classroom, theories about how culture develops in virtual learning environments would then be relevant and should be discussed.

In effect, collaborative learning describes environments or situations in which interactions between multiple agents *may* occur. These interactions are then expected to stimulate learning experiences. Importantly, these interactions are not guaranteed and so in order to maximise learning potential, we should be concerned with designing the learning environment in such a way that encourages these interactions. Dillenbourg suggests four ways to increase the probability of desirable interactions:

- **Set up the initial conditions.** Both the physical and social environment should be carefully constructed to maximise this probability. For example, how does one select the size of each group and also the members within each group? What criteria should be used for selection and how will these criteria impact social dynamics? Are some tasks inherently more collaborative than others?
- **Specify the 'collaboration' contract with a scenario based on roles.** This involves pre-defining the roles of each participant to maximise the probability of desirable interactions.
- **Scaffold productive interactions by encompassing interaction rules into the medium.**
- **Monitor and regulate the interactions.** This involves introducing a teacher into the space to help facilitate desirable interactions. The aim is not for the teacher to direct the students or to provide answers, but to perform "minimal pedagogical intervention in order to redirect the group's work in a productive direction or to monitor the social dynamics of the interaction." A proposed alternative is to provide the participants with the tools to self-regulate their interactions, for instance displaying the degree of asymmetry in action or the rate of acknowledgement in interaction.

The VR learning environment developed here seeks to use the methods suggested by Dillenbourg above. The application and applicability of these methods will be discussed in Section 2.

Dillenbourg also describes a useful framework for designing collaborative situations and learning environments. This framework has five elements:

- **Symmetry of Action:** Are the same actions available to each agent?

- **Symmetry of Status:** Do the agents have different status within the community?
- **Symmetry of Knowledge:** Do the agents have the same level of knowledge?
- **Shared Goals:** Do the agents share the same goals?
- **Division of Labour:** How are actions divided among the agents?

This framework allows the developer of a collaborative learning environment to understand those elements which contribute to collaboration, and how to engineer the environment to facilitate desirable interactions. Furthermore, this framework can be used to evaluate the degree of collaboration inherent in a situation or learning environment. Section 3.1.2 details a separate framework outlined by Dillenbourg which is used to evaluate collaboration in this application.

1.5.4 Distance Learning

Distance learning is the education of students who cannot always be physically present at school. Virtual reality learning environments can place students and educators from different locations in the same virtual room. This technology brings with it opportunities in remote tutoring and online courses. Particularly interesting is the capability to educate students who cannot attend school due to disabilities or illness, allowing them to learn and interact with their peers in a way previously impossible. The VR learning environment in this paper supports these capabilities, allowing participants in different locations to both hear, see and interact with each other in virtual space. Dickey has conducted extensive research into the affordances and constraints of virtual worlds for synchronous distance learning [30][34][35].

1.6 Constructivism and Virtual Reality

As a theory of learning, constructivism is a response to the problems faced by didactic approaches such as behaviorism and programmed instruction. It posits that learning is an active, constructive process in which students connect new information with previously acquired knowledge to construct new knowledge. Dewey (1916) argued that knowledge construction was the active process of being in an environment and that the educator's role was not to merely transmit knowledge, but to shape the learning environment based on an understanding of how environments can promote knowledge acquisition [36].

Constructivist theory also emphasises the ability of the learner to interact with and control learning processes while also coordinating this learning process with other learners. Any virtual learning environment designed with constructivist theory in mind should therefore be inherently interactive and collaborative.

Huang's paper "Investigating learners attitudes toward virtual reality learning environments: Based on a constructivist approach" outlines how an understanding of constructivist principles is necessary to develop effective virtual reality learning environments [37]. It presents the growing body of research that shows constructivism as fundamental and underlying our understanding of learning in a virtual reality learning environment [38][39][40].

1.7 Merill's First Principles of Instruction

While constructivist principles provide a useful foundation when thinking about the design of VRLEs, they do not describe explicit steps that the designer can follow. In their paper *"Design and Development of Virtual Reality: Analysis of Challenges Faced by Educators"*, Shelton and Hansen advocate following *Merill's First Principles of Instruction* [41] which sets out explicit steps that the instructional designer can follow [29].

Merrill suggests that each instruction can be conceptualised through *Activation*, *Demonstration*, *Application*, and *Integration*. Both the developer and educator are responsible for appropriately designing the VR learning experience to incorporate each of these phases into the lesson.

1. **Activation:** This involves introducing the material to the students so that it is interesting and compelling. How does the presented material build upon previous knowledge?

2. **Demonstration:** Consider the role that the student plays in developing their own understanding. What interactions within the virtual reality environment will facilitate the construction of knowledge?

3. **Application:** According to Hanson, the application portion should define how learners are guided within the virtual reality environment, the order in which they interact with information within the environment, and how complexity is increased throughout the lesson.

4. **Integration:** Integration occurs when the students can use the knowledge they have acquired in the VRLE. This can be either inside or outside of the virtual environment. While it may occur outside of the virtual environment, it is important to consider if the educator wants to design a thorough and complete instructional exercise using VR technology.

Chapter 2

Designing the Virtual Reality Learning Environment

This section addresses each aspect of designing a virtual reality learning environment (VRLE) for teaching calculus. The idea has changed significantly since its original conception. Section 2.1 describes how the application was originally envisaged and how this concept has evolved over time. The rationale behind the key design elements is discussed, and also the technical challenges faced when developing a multiplayer VR application. Any texts presented in the virtual environment are included in Appendix A.

2.1 Initial Thoughts

The original intention was to build a generalised tool that educators could use to design and develop mathematical virtual reality learning environments. It was quickly realised however, that this was impossible given the time constraints and it was instead decided to design a VRLE that focused on the applications of differential calculus. Section 1.3.1 described the typical difficulties encountered by calculus students: visualisation and mapping between different representations. The aim of the project was to take applied differential calculus problems and model them in virtual reality. The students would be able to interact with these physical problems and see how their properties change in real time. Furthermore, these problems would be designed and modelled in such a way as to emphasise the relationship between different representations, while allowing students to visualise what these problems look like in three dimensions. This application is intended as a multi-user environment so that students could collaborate when working with problems, or to enable educators to share the space with students and use the environment as a direct teaching tool.

I also initially imagined that the entire application would be situated in the same virtual room. As development progressed, it became clear that splitting the application into different rooms would be beneficial as it more clearly delineates between the different stages of learning. The "Introduction Room" is where the students can become familiar with the basics of virtual reality and user input in the VRLE. The "Theory Room" explores the basic theories of differential calculus in a collaborative manner through interactive displays. Finally, the students progress to the application room, where they are exposed to three dimensional differential calculus problems which explore the applications of the theory. Top down diagrams for the layout of each room are included in Appendix A.

The sections that follow detail the development of the Introduction Room, Theory Room, and Application Room, respectively. Many functions and interactions are present in multiple rooms (interactive displays, for example). Rather than discussing these separately, they are addressed as they become relevant for each room, in the order that they would be presented to a student using the application.

2.2 Equipment and Software

This project required equipment that could offer a fully immersive experience and allow the participants to interact with both objects and the other participants in three dimensional space. There are two virtual reality head mounted displays which meet these requirements: the Oculus Rift and the HTC Vive. Fortunately, the application could be designed without a specific headset in mind and could then be adapted depending on which headset was available at the time of evaluation. Both the Oculus and the Vive systems consist of a headset, two hand-held controllers, and base stations that track the position of the user in three dimensional space. To run smoothly, VR applications also require a powerful computer with advanced graphics capabilities. The application was developed in Unity [42] and all scripts were written in C-Sharp. Any graphical representations used in the application were generated in Python.

2.3 Navigating the Virtual Learning Environment

Two options are available when deciding how the user will navigate the virtual environments: Standard movement with the joysticks (move the joystick in the direction that you want your avatar to move), or teleportation (point to a spot on the ground and teleport to that location). Teleportation was chosen for the following reasons:

- It can be somewhat disorientating for the user in virtual reality to move an avatar using a joystick when the user's physical body is still. It is important to minimise discomfort by removing anything that could detract from the learning experience.
- The space is large and navigating with the joystick would take time. Teleportation is much more efficient.
- Teleportation allows the developer to indirectly control the movement of the player in the space by limiting the areas to which the user can teleport. How the student navigates the space and the order in which the student visits certain areas can be controlled without explicitly telling the student where to go.

The teleportation mechanic from the *"SteamVR Interaction System* application was adopted for use here.

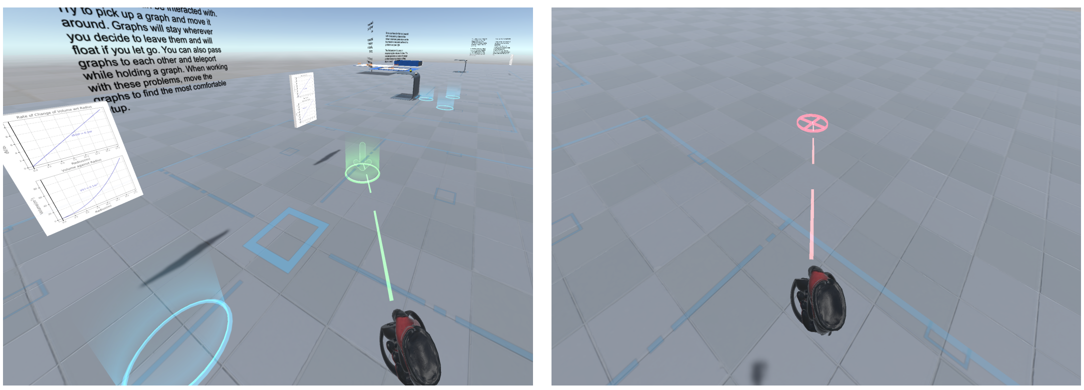


Figure 2.1: Teleportation

The target is green when the user points to an area that can be teleported to, and red otherwise. The teleportation function in the application has a limited range. Limiting the range is useful as again it allows the developer to implicitly control the movement of the player in the space. If, for example, the student is at station A, and should visit station B or station C before reaching station D. The environment can be designed with the teleportation function so that station D is only reachable from station B and C, thus forcing the student to first visit station B or station C without explicitly directing them on the desired path. This gives the student a sense of agency and control over their movement, while still ensuring they progress through the application as the developer intended. Furthermore, the use of teleportation points allows the developer more control over the specific position of the user in the application (see Figure 2.2).

As was previously mentioned, there are three rooms: An Introduction room, a Theory room and an Application room. A mechanic is required for switching efficiently and seamlessly between these rooms. In each room, there is a station containing two labelled virtual buttons which, when pressed, relocate the player to the chosen room instantly.

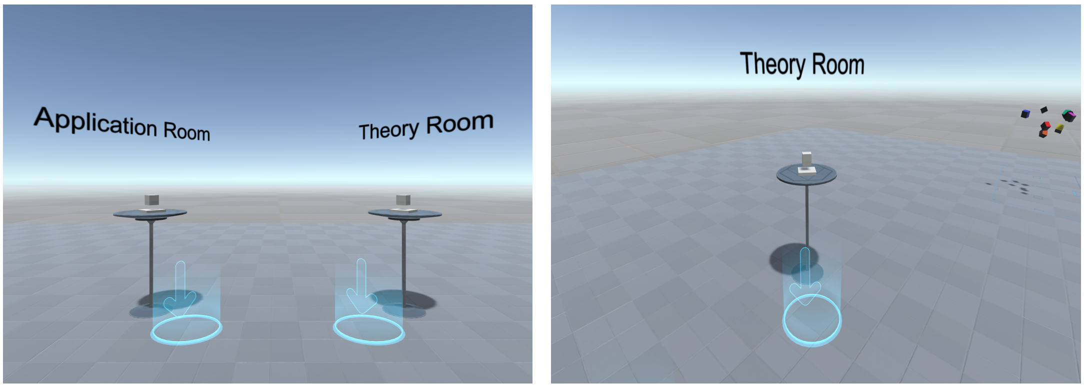
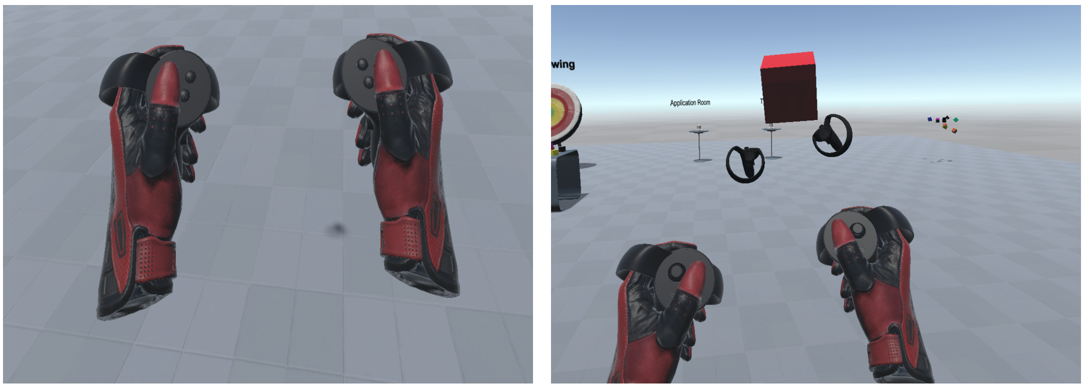


Figure 2.2: Room switching buttons

2.4 Designing Player Models

The player models are designed to be simplistic, with a coloured cube as a head. Each player can also see a model of their own hands which animate in response to controller input, and the position of their controllers. The first player loaded into the application has a red head, and the other a blue head. The players do not have bodies, and only the position of their head and controllers can be seen by the other player.



(a) Player hands (b) Player model head and hands

It was decided not to develop a full avatar: avatars require significant animation and this would also add to the difficulties in networking the application (see Section 2.8). Implementing a system with personalised avatars would add an interesting personal element to the application and allow for more expressive interactions between participants, but perhaps this could cause unwanted distractions in an educational setting. The simple and minimalist avatars shown above convey enough information about the other players in the space, but are not distracting. Players can still gesture to one another and communicate through body language. The only immediate improvement currently envisaged would be some way of showing the direction in which the head is facing (e.g. by using a face, hat or a simple nose).

2.5 Introduction Room Design

Upon entering the application, the student is first placed in the introduction room. The function of this room is to welcome the participants to the virtual environment and to allow them to familiarise themselves with the movement and methods of interaction used. A floor plan for the introduction room can be found in Appendix A.

The student first encounters a welcome text (see Appendix A). This text welcomes them to the VRLE and briefly explains how movement and interaction with objects work. The controls for teleportation and interaction are explained and the student is presented with a set of interactable, throwable cubes to teach her the basics of grabbing and taking ownership of objects in the environment. The students can pass or throw the cubes to one another and are also given a target for practise.

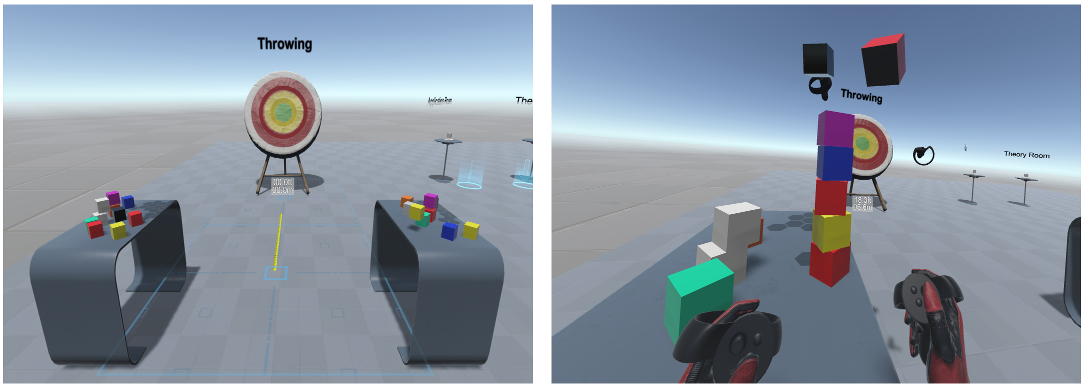


Figure 2.4: Interaction with cubes

2.6 Theory Room Design

The function of the theory room is to allow the students to cement their knowledge of basic differential calculus theory by experimenting with interactive graphics in a collaborative manner.

In its exploration of differential calculus, the Irish Leaving Certificate mathematics syllabus focuses heavily on the relationship between a function and its derivative, emphasising the behaviour of the derivative at the critical points of the original function. In order to interact with and understand the three-dimensional real world differential calculus problems presented in the application room, the students should have a strong understanding of this relationship. The students should already know this theory well, and so the aim of the theory room is not to teach new material, but to offer a brief recap of the theory before the students proceed to the Application Room where the theory is applied.

In a similar manner to the introduction room, the students are presented with a text upon relocating to the theory room. This text introduces the concept of a linear driver, which forms the key means of interacting with objects in the environment. The students are also given a practise demonstration showing how the linear driver operates.

2.6.1 Linear Drivers

A mechanical analogue is necessary to allow the students and interact and engage with the mathematical objects and graphs in the VRLE. According to constructivist theory, it is exactly these interactions that facilitate knowledge construction in the VRLE. While the application would still be educational and engaging without interaction, it would also fail to utilise one of the most beneficial affordances of VR.

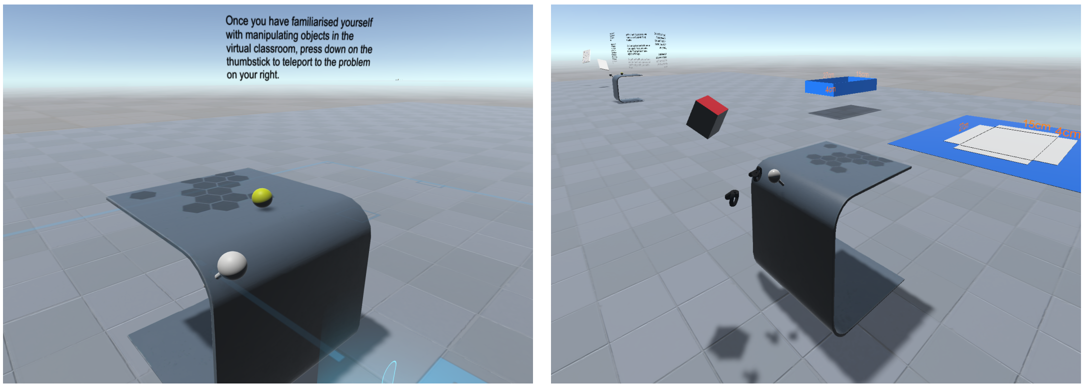


Figure 2.5: Images showing the linear driver demonstration and the linear driver used in the optimisation problem.

The nature and necessity of a linear driver is best illustrated with an example. If a student seeks to understand how the volume of a cylinder changes as the radius of the cylinder varies, then the relationship between these properties can be presented algebraically, graphically, or diagramatically. To enable these representations to be dynamic and interactive, the student could be allowed to vary the radius, and could then see how the volume changes as the radius is varied. For the graphical representation, this could be a vertical line at the current radius value, which moves along the graph. For the diagrammatic representation, the diagram changes accordingly.

How should the student vary this radius value? One obvious consideration is via manual input: a virtual number pad and an enter button would suffice. This input would seem "clunky" and perhaps create a disconnect between the student and the representations. Furthermore, using manual input would effectively discretise the possible inputs for the radius, rather than allowing for a continuous range of inputs. A better mechanic would be to implement some form of slider or lever that the student could move between a start and end point (see Figure 2.5). The set of possible slider positions between these points is then linearly mapped to the value of the radius, so that moving the slider changes the value of the radius. This allows for a direct connection to the representation and gives a continuous rather than discrete set of possible inputs.

To implement this logic in the application, first an animation is created. This animation could either be the movement of the x-value line along the graph in the graphical case, or animation of the cylinder changing in the diagrammatic case. Each of these animations are defined over a certain time frame. The animation is then connected via a script to the linear driver so that the animation begins when the linear driver is at the start point, and the animation completes when the linear driver is at the end point. The script then automatically maps all values between the start and end points.

Figure 2.5 illustrates how the concept of linear driver is introduced to the students. This demonstrative example connects the position of that particular handle to an animation for the yellow ball. The text presented to the students is included in Appendix A. To summarise, the linear driver is an object that the student slides to interact with the different representations.

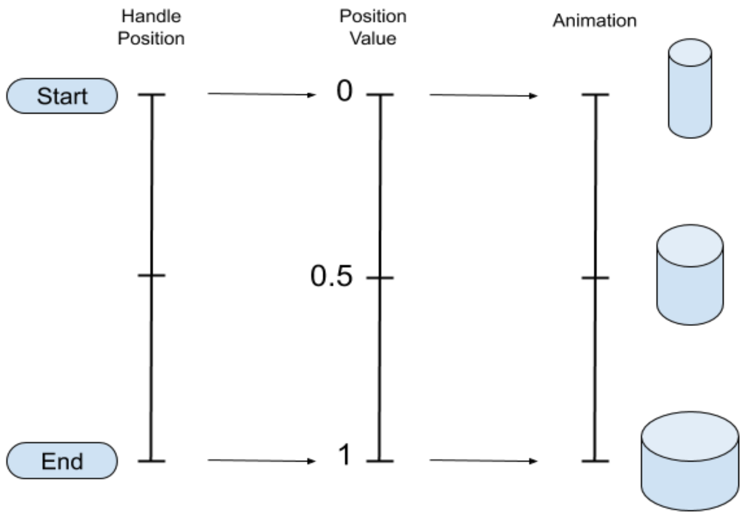


Figure 2.6: Linear mapping from the position of the handle to the animation.

The mapping of the position of the linear driver to the animation states is called a linear mapping.

2.6.2 Modelling the Theory Room

To remind the students of the relevant theory, a virtual blackboard with a set of three interactive graphs is modelled. A cubic function and its first and second derivatives are displayed:

$$f(x) = 4x^3 + 3x^2 + 25x + 6 \tag{2.1}$$

$$f‘(x) = 12x^2 + 6x + 25 \tag{2.2}$$

$$f“(x) = 24x + 6 \tag{2.3}$$

There are two linear drivers in front of the interactive graphs. The first linear driver is mapped to an animation which moves a vertical line along the graphs corresponding to the value of x. When this value is at a maximum or minimum point, the colour of the line changes, indicating that the x-value is at a critical point (see Figure 2.8). The first linear driver is also mapped to a display which gives the value of x, $f(x)$, $f‘(x)$, and $f“(x)$ (see Figure 2.8). The second driver is mapped to an animation which displays the tangent at every point on the graph.

Without some sort of direction, it is unclear how exactly the participants are expected to interact with these displays. An introductory text is included which aims to scaffold the interaction in such a way as to promote active and collaborative learning, without giving explicit directions to the students. Pieces of this text are included below.

> *"In front of you are a set of graphs. The graphs show a cubic function, along with its first and second derivatives. ... Together with your partner, use the handles to interact with the graphs. Try to relate the behaviour of these graphs to what you have already learned about differential calculus. ... What does the derivative represent? ... Where is the point of inflection and how does it relate to the other graphs? How do the tangents behave at maxima and minima? ... This is about learning maths in a collaborative way, working through and discussing ideas with your partner."*

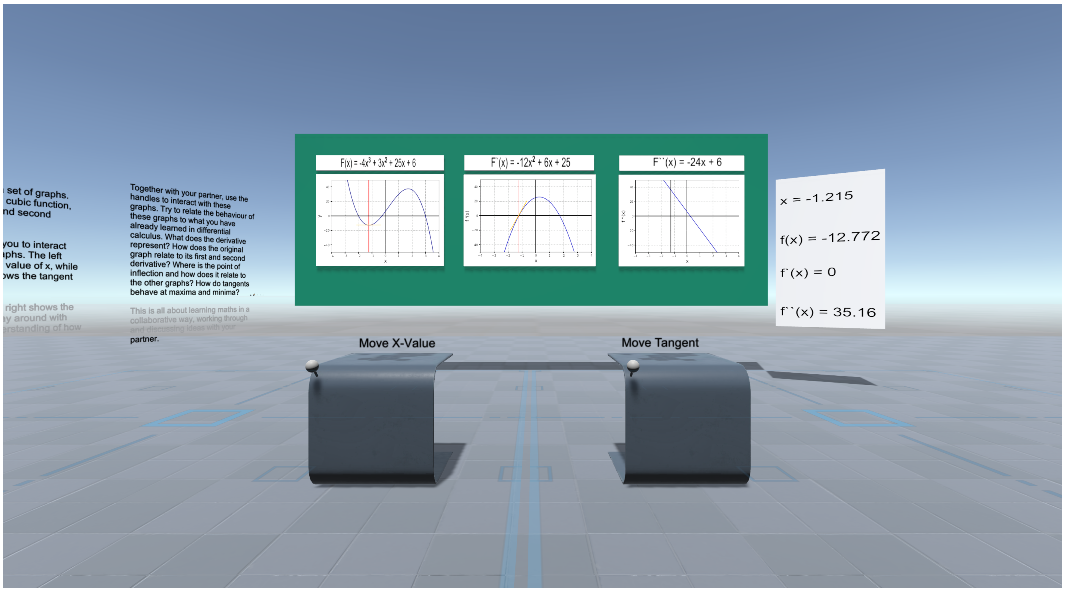


Figure 2.7: A full view of the problem presented in the Theory Room. The linear drivers, graphic displays, and value display can be seen.

It is hoped that the students will use the interactive graphs together to remind themselves of the theory they have previously learned. With one student controlling the x-value and another student controlling the position of tangents, the relationship of the function to its derivatives at critical points can be seen. Furthermore, this relationship is represented numerically on the value display (see Figures 2.7 and 2.8).

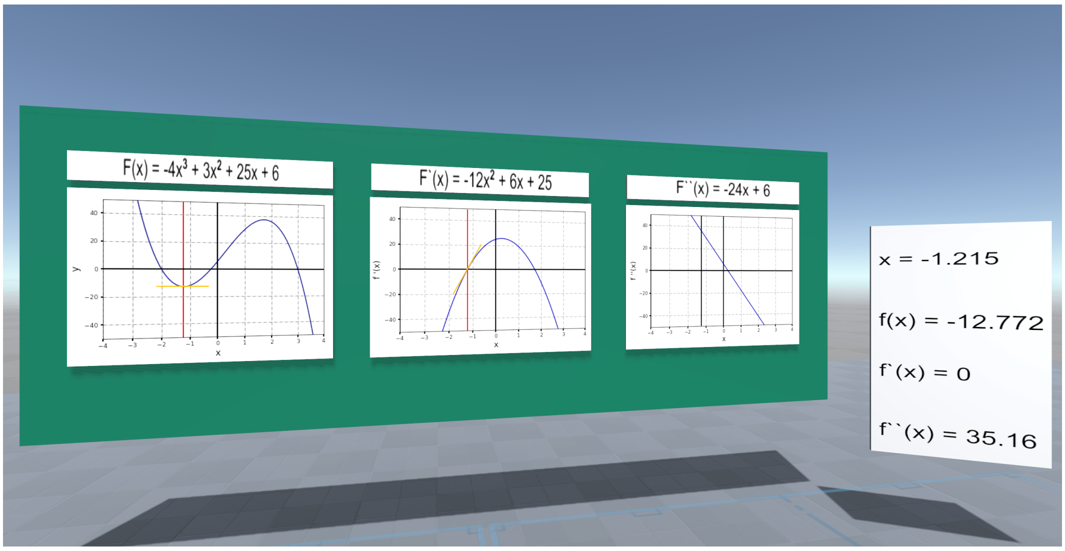


Figure 2.8: An image showing the graphic displays at a critical point. Note that $f`(x) = 0$ on the value display on the right.

2.7 Application Room Design

The function of the application room is to present the students with real world calculus problems to which they can apply the theory that was recapped in Theory Room. It is the largest room in the application and builds on every interaction and mechanic used thus far. The students are presented with dynamic problems with which they can directly interact. Importantly, the students will have been exposed to these problems on paper before entering the application, satisfying the *activation* component of Merrill's First Principles. This section discusses the rationale behind the problems chosen, how exactly the students are expected to interact with these problems, and the challenges faced when modelling the problems in a multi-user virtual reality environment.

Displays

In the Application Room the students are presented with three-dimensional representations of calculus problems. As they interact with these representations, it is important to illustrate how this relates to the graphical representation. It would be possible to implement a virtual blackboard, comparable to the display in the Theory Room. However such a display is static and would not allow the student to interact with the display directly as they explore the space. The problems in the Application Room are three dimensional and so the students are rewarded for walking around to view the problem from different perspectives. To facilitate this, displays were designed to be grabbed, held, and moved were designed.

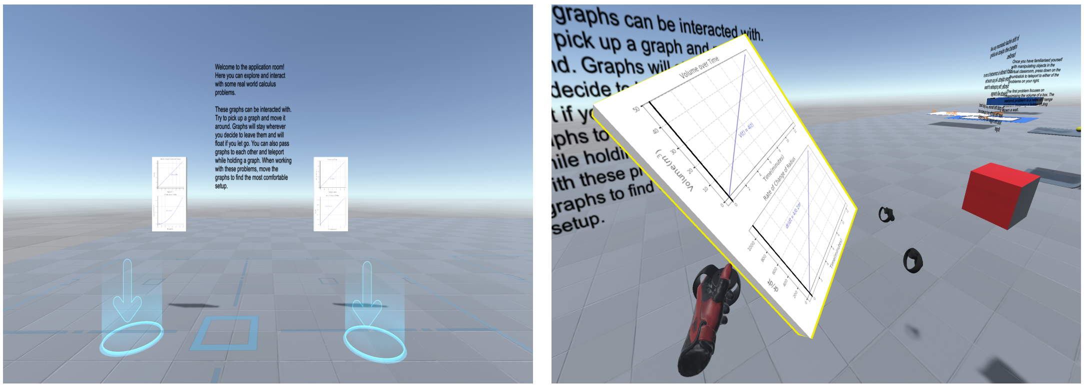


Figure 2.9: Images showing the introduction to interactable graphic displays and users passing displays to each other.

When the user lets go of a display, it floats in place, allowing the student to place the displays to achieve a desired setup. The displays contain graphs which show the properties of the three dimensional problem, such as the relationship between radius and volume. These graphs are interacted with via the same linear driver that is linked to the three dimensional representation, showing the relationship between representations and highlighting when the graph or its differential is at a maximum or a minimum. The displays are introduced in a similar manner to the linear driver, with a separate station that instructs the students about their use.

Designing the Problem Questions The problem questions must be designed according to strict criteria. Each problem question must be appropriate for the Leaving Certificate syllabus and should build on the theory presented in the Theory Room. Furthermore, the problems should be designed to best make use of the affordances of VR when modelled. This means that they should be three dimensional and have a well-defined method of interaction. This section gives a formal declaration of each problem question along with model solutions. This is followed by a discussion of how Merrill's First Principles apply, and an outline of how the students are expected to interact with these problems in an immersive virtual learning environment.

2.7.1 Designing an Optimisation Problem

Original sketches of the optimisation problem are included in Appendix A.

An open topped box is made from a 220cm x *155cm piece of cardboard by removing a square from each corner of the box and folding up the flaps on each side. What size square should be* *cut out of each corner in order to maximise the volume of the box?*

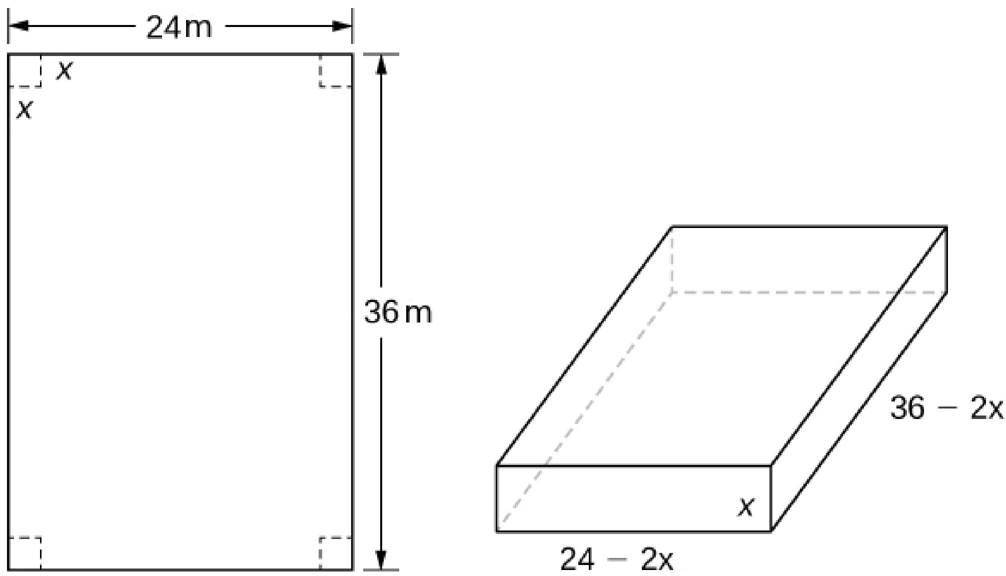
Model Solution

1. Create a diagram from the verbal description of the problem.
2. Assign a variable to the side of the square removed from each corner of the box. Say x, for example.
3. Calculate the length of the sides of the box in terms of x.
4. Calculate the possible range of x, given that the length of a side cannot be negative.
5. Use these dimensions, as well as the formula for the volume of a cuboid, to derive a formula for the volume of the box in terms of x.
6. Differentiate this formula and set the result equal to zero.
7. Solve for x. One of the two values of x will be within the possible range of x. This is the correct solution.

Solution: A diagram should first be created to represent the problem question. In this diagram a variable should be assigned to the side of the square removed from the corner of the box and then represent the sides of the box in terms of this variable.

Letting x be the height of the box, then the length and width of the box are given by:

$Length = 360 - 2x$ and $Width = 240 - 2x$



The student should observe that the box is a valid box only if x is within a certain range. The length of each edge of the box is required to be greater than or equal to zero.

$$x \geq 0 \tag{2.4}$$

$$24 - 2x \geq 0 \tag{2.5}$$

$$36 - 2x \geq 0 \tag{2.6}$$

Solving each of these equations for x:

$$0 \leq x \leq 12 \tag{2.7}$$

The student should now represent the volume of the box in terms of x. The formula for volume is given by:

$$Volume = Length \cdot Width \cdot Height \tag{2.8}$$

The volume V of the box is therefore given by:

$$V(x) = (36 - 2x) \cdot (24 - 2x) \cdot xV(x) = 4x^3 - 120x^2 + 864x \tag{2.9}$$

In order to find the value of x that maximises this volume, the student should differentiate with respect to x and then set the answer to 0.

$$V'(x) = 12x^2 - 240x + 864 \tag{2.10}$$

$$V'(x) = x^2 - 20x + 72 \tag{2.11}$$

Setting $V'(x)$ equal to zero and then using the quadratic formula (or otherwise):

$$x = 10 + 2\sqrt{7} or x = 10 - 2\sqrt{7} \tag{2.12}$$

As $10 + 2\sqrt{7}$ is not within the range for x defined earlier, the volume is maximised when

$$x = 10 - 2\sqrt{7} \tag{2.13}$$

The maximum volume is given by:

$$V(10 - 2\sqrt{7}) = 640 + 448\sqrt{7} \approx 1825cm^3 \tag{2.14}$$

Comments: Any difficulties in solving this question will most likely result from an inability to move from the verbal representation to a graphical representation, and then from that graphical representation to a symbolic or algebraic representation. Once the student has derived the correct formula for the volume of the box, it is unlikely any errors will be made in the procedures that follow [12][13]. Importantly however, even students who can successfully move between representations and derive the correct solution are unlikely to be picturing what the optimised box actually looks like in three dimensions. Furthermore, students who complete the question are not necessarily aware of how the volume of the box changes with respect to x, they have knowledge only about the single point at which the volume is maximised, and are not encouraged to consider the behaviour about that point.

2.7.2 Modelling the Optimisation Problem in Virtual Reality

When modelling this problem, the aim is to present the student with graphics and animations that elucidate a more tangible relationship between the different representations, allowing the student to more easily see how to derive this relationship themselves.

The box was the first element of the problem to be modelled. A model of the net and the complete box are included in the virtual environment, as seen in Figure 2.10. The box and net are modelled to scale. Both the box and the net are animated to match the exact behaviour of the box in the problem question. These animations are linked to a single linear driver. Along each side of the box is a text which displays the exact length of the given side. This text changes with the animation.

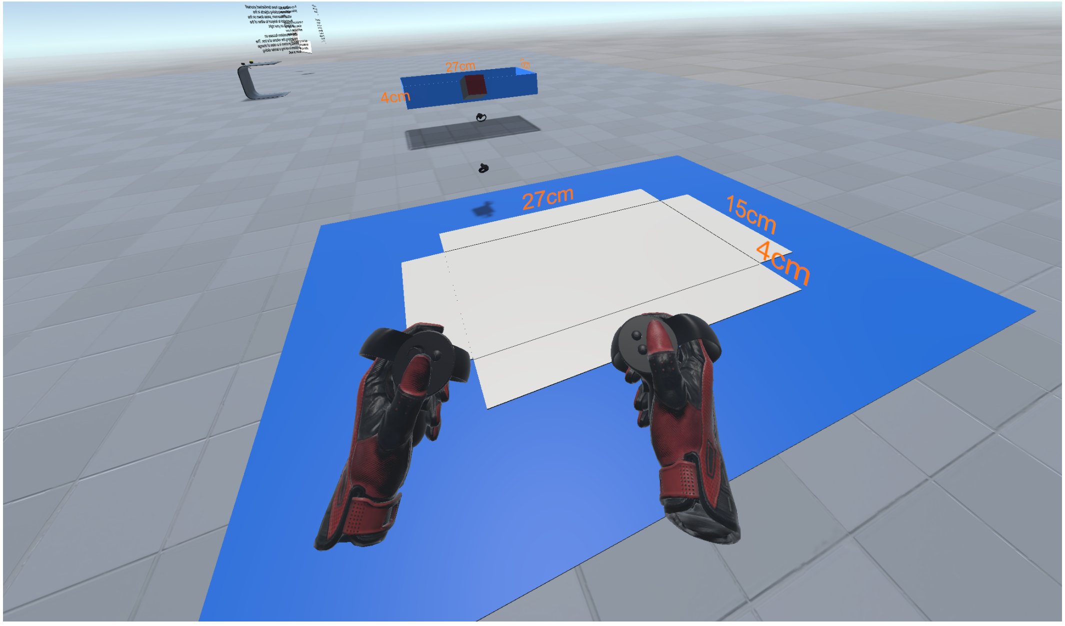


Figure 2.10: an image of the net in the foreground, and the three-dimensional box in the background. Both models have dimensions displayed in orange.

The student is also be presented with a group of interactable graphic displays as described in Section 2.7. These will display graphs showing:

- The volume of the box $V(x)$ with respect to x.
- The rate of change of volume of the box $V`(x)$ with respect to x.
- The length of the box $L(x)$ with respect to x.
- The width of the box $W(x)$ with respect to x.

On each of these displays, a vertical line is animated at the current x-value. This animation is linked to the same linear driver. Therefore as the student moves the handle on the linear driver, they can see the box and net changing in real time, with text displaying the dimensions of the box. At the same time, the graphic displays shown in Figure 2.11 also show how this x-value changes, and highlight when it has reached a critical value, indicating that the box is at its maximum value.

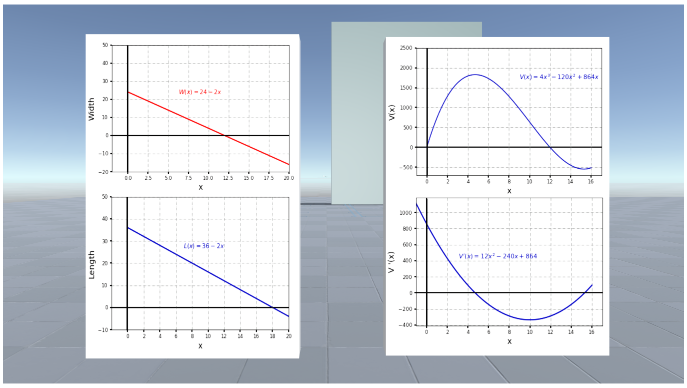


Figure 2.11: Graphic displays for the optimisation problem. Length and width are displayed on the left. Volume and rate of change of volume are displayed on the right.

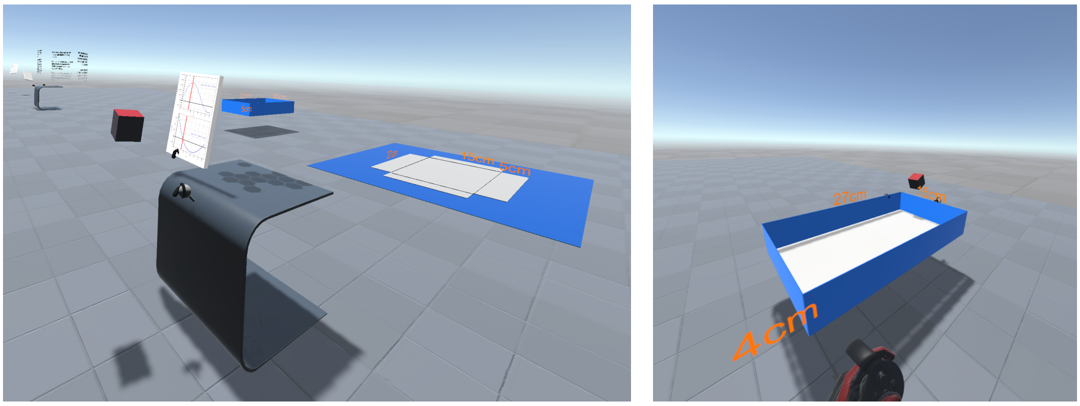


Figure 2.12: Interacting with the Optimisation Problem

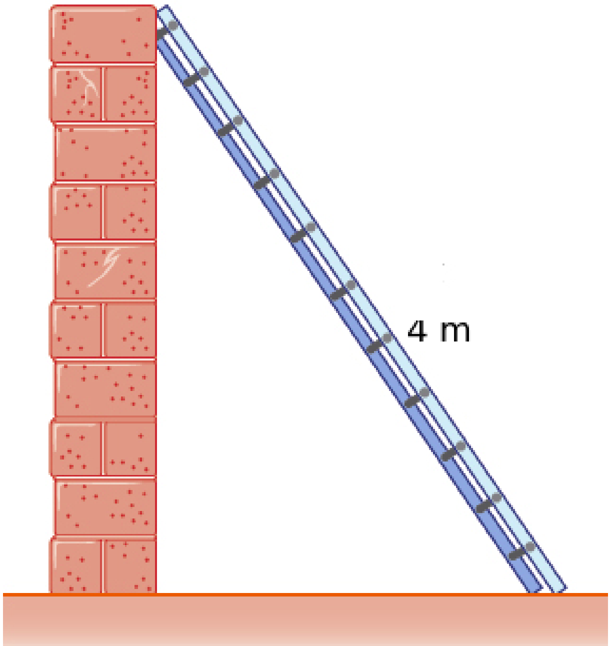
2.7.3 Designing a Related Rates of Change Problem

A 4-meter ladder is leaning against a wall. If the top of the ladder is sliding down the wall at a rate of 0.6 meters per second, how fast is the bottom of the ladder moving along the ground when the bottom of the ladder is 3 meters from the wall?

Model Solution:

1. First create a diagram from the verbal representation of the problem question.
2. Assign variables to
   (a) The distance of the bottom of the ladder from the wall. Let's call this b.
   (b) The distance of the top of the ladder from the ground. Let's call this a.
3. Understand what the question is asking the student to find. The rate of change of the distance of the ladder from the wall, which is db/dt.
4. Represent db/dt in terms of other rates of change using the chain rule: $db/dt = db/da \cdot da/dt$.
5. Find da/db and da/dt.
6. Multiply to get db/dt
7. find db/dt when $b = 3$

Solution
A diagram should first be constructed to represent the problem question. In this diagram a variable should be assigned to the height of the top of the ladder and assign another variable to the distance of the bottom of the ladder from the wall.



The student should discern what the question is asking, which is to find the rate of change of distance between the ladder and the bottom of the wall. From the diagram above this is db/dt. In order to get an equation for db/dt, one must use the chain rule:

$$\frac{db}{dt} = \frac{da}{dt} \cdot \frac{db}{da} \tag{2.15}$$

It is given in the question that $da/dt = -0.6$. db/da must then be found, which first requires an equation in terms of a and b. Using Pythagoras' Theorem, we have that:

$$a^2 + b^2 = 16 \tag{2.16}$$

$$b = (16 - a^2)^{\frac{1}{2}} \tag{2.17}$$

$$\frac{db}{da} = \frac{1}{2} \cdot (16 - a^2)^{\frac{-1}{2}} \cdot (-2a) \tag{2.18}$$

$$\frac{db}{da} = \frac{-a}{\sqrt{16 - a^2}} \tag{2.19}$$

Notice that db/da is written in terms of a and not b. Therefore need to find the value of a when $b = 3$:

$$a = \sqrt{(16 - 3^2)} = \sqrt{7} \tag{2.20}$$

Using that,

$$\frac{db}{dt} = \frac{da}{dt} \cdot \frac{db}{da} \tag{2.21}$$

This gives,

$$\frac{db}{dt} = \frac{-\sqrt{7}}{\sqrt{16 - \sqrt{7}^2}} \cdot (-0.6) \tag{2.22}$$

$$\frac{db}{dt} = \frac{0.6\sqrt{7}}{3} = \frac{\sqrt{7}}{5} m/s \tag{2.23}$$

Comments
Any difficulties in solving this question will most likely again be a result of an inability on the part of the student to move from the verbal representation to a graphical representation, and then from that graphical representation to a symbolic or algebraic representation. Once the student recognises the need to use the chain rule, and therefore requires an equation describing the relationship between b and a in order to find da/db or db/da, it is unlikely any errors will be made in the procedures that follow [12][13]. Importantly however, even students who can successfully move between representations and derive the correct solution may not be viewing exactly what this represents in terms of the position or speed of the ladder. Furthermore, students may not be aware of how db/dt behaves about the specific point in question, as they are not encouraged to consider the behaviour about that point.

2.7.4 Modelling the Related Rates Problem in Virtual Reality

When modelling this problem in virtual reality, the aim is to present the student with graphics and animations that elucidate a more tangible relationship between the different representations, allowing the student to more easily see how to derive this relationship themselves.

A model of a realistic ladder against a wall was first included in the virtual environment. Both the ladder and the wall are to scale and match the dimensions of the presented problem include images. The rate at which the animated ladder slides down the wall also matches the question provided. This animation is linked to a singular linear driver. The distance of the top of the ladder from the ground, the distance of the bottom of the ladder from the wall, and the length of the ladder are each displayed. These values change according to the animation.

The student is also presented with a group of interactable graphic displays. These will display graphs showing:

- A graph of b against a.
- A graph of both b against time.
- A graph of a against time.
- The rate of change of the distance of the top of the ladder from the ground, with respect to the distance of the bottom of the ladder from the wall: da/db.

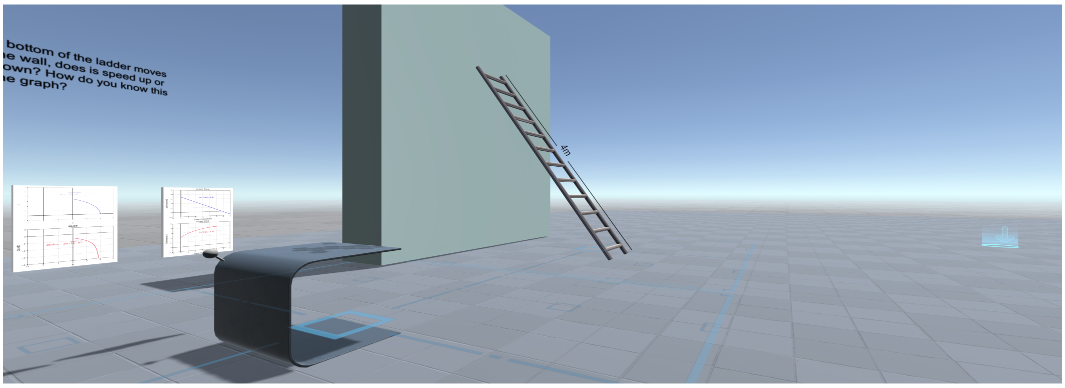


Figure 2.13: Model for the related rates of change problem.

On each of these displays, a vertical line is animated at the current point of the animation. This animation is again linked to the same linear driver. As the student moves the handle on the linear driver, they can see the ladder slide down the wall in real time, with text displaying the relevant distances. At the same time, the graphic displays mentioned above also show how the current position of the ladder relates to the relevant rates of change.

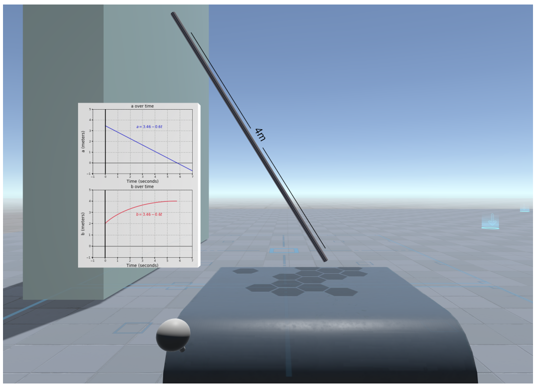


Figure 2.14: Linear driver and graphic display for the related rates of change problem.

2.7.5 Interaction and Collaboration

Interaction

When these problems were originally conceptualised, it was imagined that the student would interact with the objects directly (see Appendix A). To see how the properties of the box changed over time, the student could effectively "pull" on the side of the box to make it longer. Similarly, it was imagined that the student would interact with the ladder directly, pulling or pushing it up or down the wall. However modelling this method of interaction in virtual reality was difficult, and networking the application was much simpler when using the linear driver as the primary method of interaction (see Section 2.6.1).

Although the students would have been already exposed to the problems before entering the virtual environment, a brief description of the problems is included in the virtual environment (see Appendix A). This text guides them in their interactions and suggests some questions to consider as they explore. The intent is to scaffold the interactions rather than give a set of explicit directions. If there is an instructor sharing the virtual environment with the students, the instructor would instead be responsible for this guidance.

Collaboration

Dillenbourg sets out methods which can be used to increase the probability of collaborative interactions in learning environments (see Section 1.5.3). The social and physical environment of the application is carefully constructed to maximise this probability:

- There should be symmetry in knowledge and status between the two students using the application, as they are learning the same curriculum and are at the same stage in their mathematics education. This reduces the probability of one student taking a dominant role in the interaction.
- The accompanying texts attempt to regulate the content of their interactions by suggesting questions and encouraging the students solve the questions provided in a collaborative manner.
- The students have shared goals. Dillenbourg outlines this as a key element for promoting collaborative interactions.
- It is not possible to operate the linear driver while inspecting the three dimensional representations closely. This can be seen in the layout for the Application Room in Appendix A. Moving from the linear driver allows the students to view the problem from different angles and gain a more intuitive understanding. One student must operate the linear driver while the other can either explore the problem space or interact with the graphs. The students must communicate with one another to achieve their desired setup and must work together make use of the tools provided. The student in possession of the display must communicate synchronously with the student operating the linear driver in order to direct them toward the critical points.

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2.7.6 Applying Merrill's First Principles of Instruction

This section outlines how these problems follow *Merrill's First Principles of Instruction* (see Section 1.7).

Activation

The students are exposed to the verbal representation of the problem before entering the virtual learning environment. A quick lesson shows how the box can be constructed and how the equation for the volume can be derived. The students are then prompted to suggest how one could find the maximum volume of this box using the methods they have learned in differential calculus. The students are given a similar lesson for the related rates of change problem. Once the students understand the basic concepts of the question and what it is they are trying to find, the activation portion of the instruction is complete and they can enter the virtual learning environment.

Demonstration

Which interactions within the virtual environment facilitate construction of knowledge? The interactions with both the graphs and the three dimensional representation of the box should elucidate to a more tangible relationship between the graphical and physical representations, and lead to a deeper conceptual understanding of the problem, rather than a mere procedural understanding.

Application

How are learners guided in the virtual environment? They are guided via the text displayed, or via an instructor if one is present in the virtual environment with the students. In a more indirect sense, the students are guided by the fact that they can only teleport to certain areas of the room. The optimisation problem can only be reached by first passing through the tutorial for using displays and the tutorial for using the linear driver. The ladder problem can only be reached from the optimisation problem. While the students follow a linear progression, the experience does not feel as though it is explicitly guided. How is complexity increased throughout the course of the lesson? Complexity isn't necessarily increased, more so the students move from learning theory within the virtual environment to applying this theory to real world problems which are modelled in the environment.

2.8 Networking

To enable players in remote locations to share the same virtual environment, the gamestates of multiple clients need to be synchronised over a network. This network includes the clients and a server. Each client needs to both send data to and receive data from the server.

This is illustrated with a simple example: synchronising the position and rotation of player head models. The position of an object in the space is represented by a vector storing the x, y and z coordinates.

```
position = (position.x, position.y, position.z)
```

Similarly, the rotation of an object is represented by a vector storing the clockwise rotation in degrees about the x, y and z axes.

```
rotation = (rotation.x, rotation.y, rotation.z)
```

When player one moves her head, the headset moves, which updates the position and rotation vectors of the head model locally on player one's client. Every frame (60 times a second), the these vectors are sent to the server. The server then sends the position and rotation vectors to the second client, which is reading from the server on every frame update. These position and rotation vectors are then handled locally on player two's client to update the position and rotation of player one's head model.

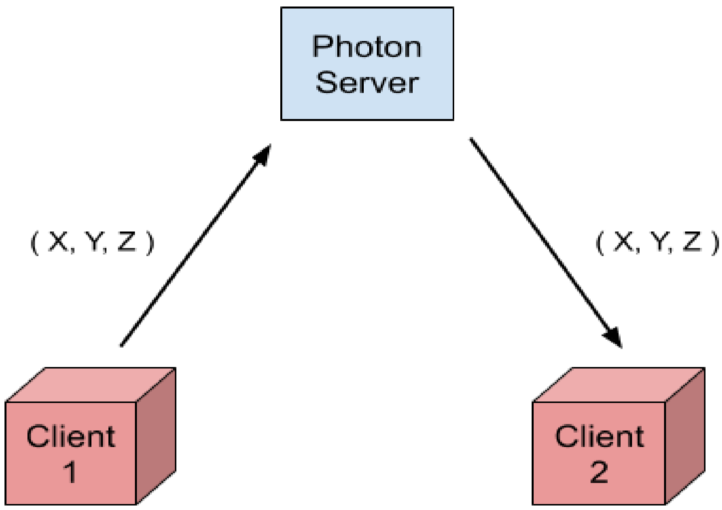


Figure 2.15: Synchronising the player position of the network.

```
void Update() {
    if(photonView.isMine) {
        switch (index) {
            case (1):
                transform.position =
                    OculusManager.Instance.head.transform.position;
                transform.rotation =
                    OculusManager.Instance.head.transform.rotation;
            break;
            case (2):
                transform.position =
                    OculusManager.Instance.leftHand.transform.position;
                transform.rotation =
                    OculusManager.Instance.leftHand.transform.rotation;
            break;
            case (3):
                transform.position =
                    OculusManager.Instance.rightHand.transform.position;
                transform.rotation =
                    OculusManager.Instance.rightHand.transform.rotation;
            break;
        }
    }
}
```

Listing 2.1: Client-side logic for networking the playermodel positions

2.8.1 Development Challenges

The bulk of virtual reality applications are single player and therefore do not implement networking. There is limited online content detailing how to network VR applications. As a result of this, implementing networking was extremely difficult and time-consuming. The majority of time spent developing was devoted to understanding and implementing networking for virtual reality applications. The technicalities and difficulties faced networking the application are discussed below. I have also included a list of useful resources for readers interested developing their own collaborative virtual reality learning environments (see Appendix B).

2.8.2 Photon Networking Service

Photon [43] is a service that provides a server and server side logic for networking multiplayer applications. Photon also provides a supported Unity plugin with documentation. This Photon plugin has a unique identification number which is linked to the server. Each client running the application will share this number and communicate with the server. This is how the server knows which clients to synchronise over the network. The developer is then responsible for implementing any client side logic (as in Listing 2.1) using Photon's C-sharp documentation.

2.8.3 Networked Objects in the Application

The process for networking the player models has already been discussed. The same logic for synchronising position and rotation applies to the controller models and also the graphic displays. The Photon voice plugin [44] also allows the clients to communicate over voice chat and was simple to implement. This facilitates synchronous communication in the virtual environment. The following sections discuss the technical aspects of networking the animations and also implementing the logic for transferring ownership of objects.

2.8.4 Networking Animations

When player one interacts with the linear driver, the animation linked to that linear driver changes on player one's client. The challenge is to network this animation so that player two sees this change in real time on their local client. Any animation is effectively a combination of changes in position, rotation and scale of the different sub-objects which form the object being animated. An obvious solution would be to apply the same logic applied above to each of the sub-objects, synchronising each one over the network separately. While this is a valid solution, it results in lots of data being streamed over the network and is inefficient.

The nature of the linear driver presents an elegant solution. Recall from Section 2.6.1 that there is a linear mapping from the position of the handle to the animation state. The position of the handle is given by a singular value between zero and one. Instead of networking each object in the animation, it is instead possible to just synchronise this singular linear mapping value and then allow the clients to update the animation locally using this value. This significantly reduces the amount of data being streamed over the network, as only one float is being sent and received on every frame.

Linear Interpolation

Using this method however, the animation would appear to jump from position to position for the client who was not controlling the linear driver, rather than moving smoothly from one point in the animation to the next. To understand why this was the case and how this issue was resolved, it is necessary to speak about "inbetweening" and linear interpolation. In animation, inbetweening is the process of generating intermediate frames between two key frames. The problem faced in this application was that the animation would jump from one frame to another and would not display the frames in between. As the animation is linearly mapped to a single value, it means that this value was jumping between values rather than transitioning smoothly (for example jumping from 0.1 to 0.2 and not the values in between). Linear interpolation was used to solve this issue. Linear interpolation is a method of curve fitting using linear polynomials to construct new data points within the range of a discrete set of known data points. When a client receives this value from the server, it linearly interpolates between this value and the previous value received and then moves to this interpolated value rather than the value originally received, resulting in a smoother animation.

A simple linear interpolation script was written to interpolate between the current local handle position and the incoming handle position being streamed over the network. Rather than instantaneously updating to the incoming value, the local hand position is instead set to the value some fraction between the current value and the incoming value, depending on how smooth the developer wants the animation to be. This new position then becomes the current local handle position on the next frame, and the process repeats every frame. This results in a smoother animation, with the only drawback being that the animation is a fraction of a second behind the animation on the other client's screen. The logic is displayed in Listings 2.2.

```
private float Interp(float current, float target, float divisor) {
        if (target != current) {
            return current + (target - current)/divisor;
        } else {
            return target;
        }
    }

    // Update is called once per frame
    void Update()
    {
        if (isHandAttached) {
            driverFloat =
                gameObject.GetComponent<LinearDrive>().linearMapping.value;
            currentFloat =
                gameObject.GetComponent<LinearDrive>().linearMapping.value;
        } else {
            currentFloat = Interp(currentFloat, driverFloat, 10f);
            gameObject.GetComponent<LinearDrive>().linearMapping.value =
                currentFloat;
```

```
        }
    }

    void IPunObservable.OnPhotonSerializeView(PhotonStream stream,
        PhotonMessageInfo info) {
        Debug.Log("SerializeView running");
        if (stream.isWriting) {                // Send data
            stream.SendNext(driverFloat);
        } else if (stream.isReading) {         // Recieve data
            driverFloat = (float) stream.ReceiveNext();
        }
    }
```

Listing 2.2: Linear Interpolation Logic

2.8.5 Transferring Ownership

What if one player is in control of an object (e.g a cube, graphic display, or linear driver handle) and the other player attempts to take control of that object? If this ownership transfer is not handled correctly, the gamestates on each client will no longer be synchronised. When a player first grabs an object, they then have ownership over that object. If another player attempts to grab that object, a request is sent to the server to take ownership. The majority of applications require the current owner to accept this request before ownership is transferred to prevent players from stealing objects from other players without their permission. This would be problematic in a typical game, but for this particular application requests are automatically accepted as it allows for seemless transfer of objects from one student to another.

```
[PunRPC]
        void RPCDetachOthers() {
            Hand hand = GetComponent<InteractableNetworked>().hand;
            hand.DetachObject(gameObject);
        }
        public void TakeOwnership()
        {
            Debug.Log("Taking ownership");
            base.photonView.RequestOwnership();
            this.photonView.RPC("RPCDetachOthers", PhotonTargets.Others);
        }
```

Listing 2.3: Implementing Ownership Transfer

Chapter 3

Evaluation

The application was evaluated with six Leaving Certificate mathematics students from Colaiste Choilm in Cork, Ireland. While the literature reviewed suggests VRLE's could be beneficial to students of mathematics at primary, secondary, and post-secondary level, this application was developed with late secondary level students in mind. These students are familiar with the fundamental concepts of calculus but are not necessarily gifted at maths.

The evaluation took place in an otherwise empty classroom to minimise distraction. The technology was evaluated over a ninety minute time period during the school day. Ten minutes were allocated at the beginning to expose the students to the problems. During this ten minutes the students worked together to find a solution to the problems. These solutions were then discussed among the class. A brief, five minute demonstration was then given to the students how on using the virtual reality headset and controllers. The students were then split into pairs: one pair of boys, one pair of girls, and one mixed pair. Each pair spent twenty minutes in the application together. Notes were taken documenting their interactions. To prevent the students from learning by watching their peers, evaluations were conducted privately.

Following the principles of iterative development, it would be appropriate to evaluate the application with secondary-level students at each stage of development. This would allow a better understanding of how the target demographic would respond to the application. However, this was impractical given the constraints and scope of the project. Instead, the application was evaluated and play-tested regularly with other developers and university students at the Bristol VR Lab. While this feedback was useful, using experienced game developers and other students as play-testers throughout the development was non-optimal as they are already familiar with VR applications and many of the functionalities of this particular application.

3.1 The Purpose of this Evaluation

Educational technologies are typically evaluated with regards to their effects on both learning outcomes and the learning experience.

3.1.1 Learning Outcomes

Learning outcomes are statements that describe significant and essential learning that learners should achieve, that can be reliably demonstrated at the end of a course or program.

Statements about the effect of educational activities on learning outcomes should be supported with quantitative data. For educational technologies this data is typically gathered over the course of an entire curriculum with a large group of pupils and a control group. Examination results from before and after the technology was implemented could be used to analyse the effect on learning outcomes. As it was only possible to evaluate this technology on a single day, it was not possible to obtain data about learning outcomes. Furthermore, as the application was evaluated with a small group (n = 6) and without a control group, it would be unreasonable to make definitive statements about the effects of the technology on learning outcomes.

Examination results are not the only relevant learning outcome. This technology was partly motivated by the scientific literature which indicated that such technologies could be used to increase the use of visual modes of reasoning in the mathematics classroom. However, it is difficult to make definitive statements about the construction of mental imagery. The level of analysis required to study how students construct mental imagery is not feasible for a project of this scale. It may even be impossible to understand this given the current limitations of neuroscience.

Given all of the above, evaluating the impact of this technology on learning outcomes was not possible due to the limitations of the project.

3.1.2 Learning Experience

A learning experience refers to any interaction, course, or program in which learning takes place, including virtual experiences. Two key elements of the learning experience which this technology sought to improve were student engagement and collaborative learning. Both student engagement and collaboration could be effectively evaluated within the constraints of this project.

Evaluating Student Engagement

According to O'Brien (2016), user engagement (UE) is a quality of user experience characterised by the depth of an actor's cognitive, temporal, affective and behavioural investment when interacting with a digital system [45]. The measurement of user engagement has long been an active area of research in the field of human computer interaction (HCI). One tool developed to measure UE is the User Engagement Scale (UES) [46]. The UES is a 31-part questionnaire which seeks to measure six dimensions of engagement: aesthetic appeal, focused attention, novelty, perceived usability, felt involvement, and endurability. Recently, there have been questions raised about the validity of the six factors. Others have raised concerns about the length of the questionnaire and the need for a brief version and documentation guiding its use. A recent paper by O'Brien, Cairns, and Hall addresses these concerns and provides a revised UES, along with recommendations for its use by HCI researchers [47].

Research conducted using principal components analysis to analyse the UES suggests that it actually has four factors, rather than the six factors originally proposed. Of the six original UES subscales, only three remain: focused attention, aesthetic appeal, and perceived usability. Novelty, felt involvement and endurability have instead been combined into a fourth factor: reward factor. O'Brien (2018) confirmed this fourth factor and proposes a revised user engagement scale which grouped the questionnaire items differently [47]. This four factor model will be used for the purposes of evaluating the application presented in this paper. It is possible to measure only specific subscales of the UES, depending on which scales are most relevant to the technology being evaluated. While both focused attention (FU) and perceived usability (PU) are most relevant, it was decided in this evaluation to instead measure engagement as a holistic construct and so the entire UES questionnaire was employed. The questionnaire used is presented in Appendix B.

The 31 item questionnaire is structured as follows:

- 7 items measure focused attention.
- 8 items measure perceived usability.
- 5 items measure aesthetic appeal.
- 10 items measure reward factor.

Scoring the Questionnaire: The questionnaire uses a five point scale, ranging from "Strongly Disagree" to "Strongly Agree".

1. First, any items which phrase a statement in the negative are reverse coded. For example, the second perceived usability item states: "I found this application confusing to use". Any responses to this question on the five point scale are reverse coded, so that a confusing application receives a low score.
2. Scale scores are calculated by summing the scores for the items in each of the four subscales and dividing by the number of items in that subscale
3. An overall engagement score is calculated by adding the average of each subscale and dividing by four.

Evaluating Collaborative learning

The ability of the application to facilitate and encourage collaborative learning was measured using the framework set out by Dillenbourg.[33] He delineates between the different meanings for 'collaboration'. Each of *Situations*, *interactions*, and *mechanisms* can be described as collaborative.

When evaluating how well this application promotes collaboration between students, both collaborative situations and collaborative interactions can be considered. When evaluating collaborative situations, one can observe the symmetries of action, knowledge, and status, as well the sharing of goals and the division of labour. The virtual learning environment developed here was designed according to this framework and therefore mostly satisfies the characteristics of collaborative situations. Naturally, collaborative situations promote collaborative interactions. This evaluation therefore assesses whether the *interactions* in the virtual learning environment were particularly collaborative and seeks to understand which elements of the environment facilitated or hindered collaboration.

Dillenbourg outlines three criteria for collaborative interactions: interactivity, synchronicity, and negotiability.

- **Interactivity:** According to Dillenbourg, a measure of interactivity among peers is not defined by the number or frequency of interactions, but by the extent to which these interactions influence the peers' cognitive processes.
- **Synchronicity:** Synchronous communication is required for multiple agents to perform a task together in real time. Asynchronous communication is categorised by a time delay in sending, receiving, and acknowledging information. Interactions which involve asynchronous communication are usually described as cooperative, whereas collaborative interactions require synchronous communication.
- **Negotiability:** Collaborative interactions are negotiable. One agent's view does not hold more value on the sole basis of authority. In this regard, the negotiability of a collaborative interaction is strongly related to the symmetry of status between the agents partaking in

> the interaction. For example, tutoring dynamics tend to be less collaborative as there is less room for negotiation between the agents. In interactions among peers, there is the expectation that the agents must justify their standpoint, and therefore the structure of the dialogue is more complex than in hierarchical, vertical interactions where status is asymmetrical.

Collaborative interactions cannot be measured in the same manner as engagement, due to the inherent complexity of interactions among agents. When evaluating the degree of collaboration in the VRLE here, written observations of the interactions taking place were made. These observations were noted on a laptop. Care was taken to note when, where and why collaborative interactions occurred and when collaborative interactions did not occur at times or places where they were expected. The analysis is presented in Section 4.2.

Chapter 4

Results and Discussion

4.1 User Engagement

Each item from the 31-item questionnaire has six scores; one from each student. The scores for each question were then averaged. The results for each question are displayed below, according to the subscale to which the question belongs: Focused Attention (FA), Perceived Usability (PU), Aesthetic Appeal (AE), or Reward Factor (RW).

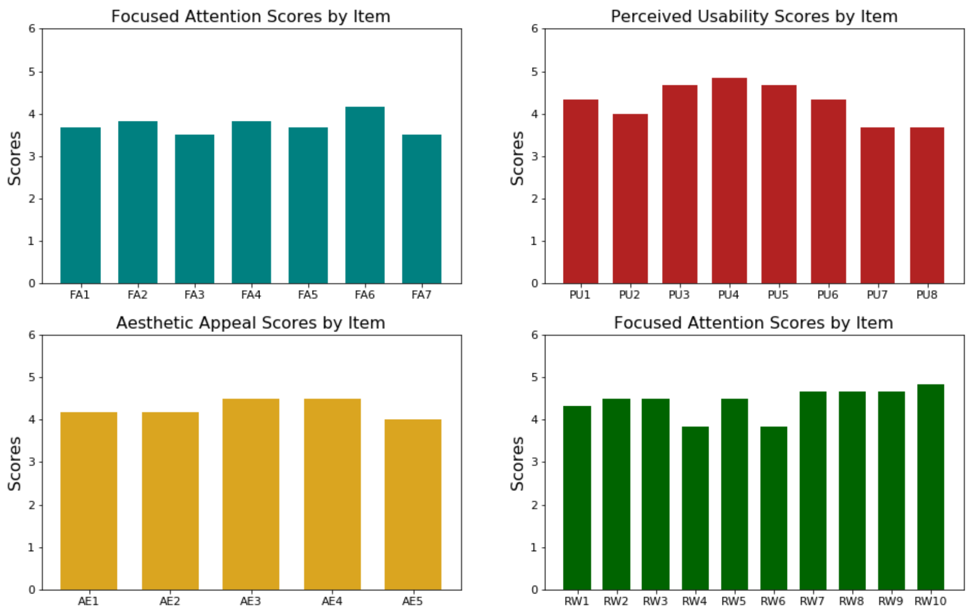


Figure 4.1: Histograms showing the average score for the items in each subscale

The score for the items within each subscale were then averaged to give the final score for each subscale. These scores are presented in Figure 4.2. The black lines on each bar represent the standard deviation of the result for each subscale.

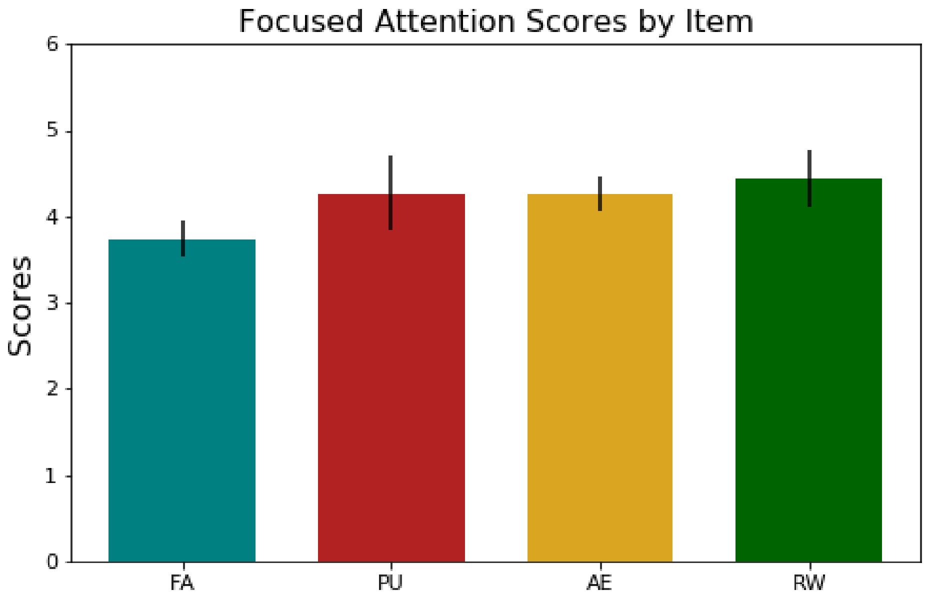


Figure 4.2: A histogram showing the average score for each subscale

4.1.1 User Engagement Results

Focused Attention

Focused attention had the lowest average score of 3.74, with a standard deviation of 0.215. Looking at the first graph in figure 5.1, it is clear that focused learning items consistently scored between three and four on average. Notably, FA6 - "I was absorbed in this experience." Scored highest with 4.166 and FA7 - "During this experience I let myself go." scored lowest with 3.5. The FA scores were negatively skewed by the results from one student who gave FA3 and FA4 a score of 2, and gave a score of 3 to the other questions. Without this student the FA average increases to 3.89. However, given the fact that this data was collected from a small group of six students, it is would be incorrect to consider this student an "outlier".

The relatively poor scores for the focused attention items, while seeming at first to reflect poorly on the immersive power of the application, should be considered in the context of a functional classroom environment. It is not desirable to have students completely unaware of their real world environment when in a classroom, and we want their sense of time to remain intact.

Perceived Usability

Perceived usability received an overall score of 4.27 with an standard deviation of 0.42. The scores for this subscale had the highest variance. PU4 - "I felt discouraged during this application" received a score of 4.83. Generally, 4.27 is a very high score for usability, and implies that the students did not struggle in their interactions with the VR equipment, the user interface, or the in game controls.

It was noted during the evaluation that the students who did not play video games struggled to familiarise themselves with the controls, whereas the students who had experience with similar input devices were immediately comfortable and confident using the Oculus controllers. The former consistently gave lower scores for perceived usability items and scored PU1 - "I felt frustrated during this experience" poorly. Two such students struggled to connect the pressing down of the joystick to the teleportation function. One of these students audibly expressed their frustration while in the environment, complaining that they could not understand how to teleport.

It is not intuitive to push a joystick down, however this is a universal function on gaming consoles. I postulate that some students' ability to quickly familiarise themselves with such controls is likely related to more experience with similar gaming consoles. If this technology were to be introduced into the classroom environment as part of a mathematics curriculum, structured lessons about the controls should be given to the students to ensure that they have equal opportunity to learn and interact comfortably with the virtual environment.

This lesson could be incorporated into the application as part of the introduction room. Rather than briefly explaining the controls and then asking the students to attempt to teleport around the space, an animated graphic could be presented to the students showing exactly how to press the thumbstick down. This could then be followed by a quick teleportation challenge which gives the students a variety of targets to teleport to. The students would not be able to proceed with the application until they have completed the challenge.

Aesthetic Appeal

Aesthetic appeal received an overall score of 4.26 with an standard deviation of 0.2. The AE subscale results had significantly less variance than other subscales which scored equally well. No positive nor negative comments were received from the students about the aesthetic qualities of the VRLE.

Reward Factor

Reward factor scored highest with a score of 4.43 and a standard deviation of 0.326. RW10 - "This experience was fun" received a score of 4.833. I posit that this result and other results in the reward section may be positively skewed by the fact that this was the first fully immersive virtual reality experience for the students, and so it is naturally going to be both rewarding and fun. Discounting for this fact, and even taking a liberal estimate of the effect this had on the RW scores, the results still show that the students enjoyed and were motivated by this educational experience. It supports the claims that this technology could be used as a supplementary tool in the mathematics classroom to promote student engagement.

RW6 - "I continued to use this application out of my curiosity" received mixed scores from the students. This may reflect the fact that some students felt obliged to continue using the application as they were taking part in an evaluation, and were not otherwise motivated by their curiosity. Other students were strongly motivated by their curiosity; so much so they decided to highlight this fact by heavily emphasising their positive response to RW6 with extra ticks marks and circles on the questionnaire.

While there is not enough evidence to buttress this claim, I suspect that the application will be most rewarding for students who already possess an interest in mathematics. It is possible that this application is therefore best suited for students who are already interested in pursuing mathematics or STEM related subjects after secondary education, providing them with an immersive environment in which they can explore mathematical concepts and satisfy their curiosity.

Overall Result

Averaging the results for the subscales gives an overall user engagement score of 4.175 out of 5 for the application.

4.1.2 Final Remarks on User Engagement

When user engagement was evaluated as a holistic construct, the results indicated that students were strongly engaged by the application. It is important to note that the results presented here are likely positively skewed due to the novelty factor that virtual reality technology brings. If technology of this kind were to be integrated into a mathematics curriculum, this novelty would naturally dissipate over time. Nevertheless, the ability of this application to incite curiosity and engage students still stand.

Looking at the different subscales more closely, it is clear that the application did not adequately satisfy the Focused Attention dimension of user engagement. This is perhaps one of the most important user engagement dimensions to consider when evaluating engagement with educational technologies. Each of the items in the FA subscale scored similarly, and so it is difficult to discern exactly which elements of the application negatively contributed to the FA dimension. Furthermore, the size of the subject group makes it unreasonable to definitively claim which specific dimensions of focused attention should be addressed. Ultimately, a similar study with a larger test group would need to be performed and clinical interviews with the students conducted to more accurately determine how to improve focused attention in this virtual learning environment.

The perceived usability dimension scored particularly well which implies that the students found the controls and user interface easy to navigate and understand. It would be helpful to include an in depth tutorial on the teleportation function as mentioned above. I do not see any other immediate improvements that could be made in this regard. However, I would add that educators should be careful to ensure their students are familiar with the controls and to be cognisant of the impact that levels of previous exposure to similar interfaces may have on the usability of the technology.

4.2 Collaboration

During the evaluation, the interactions between students in the VRLE were monitored. There were a total of three pairs of students. The following sections present these observations and discuss their inplications. Note that the observations presented below do not allow the evaluator to state the degree to which the environment is collaborative, but instead allow for a discussion of the elements of the VRLE which could be changed to increase the probability of collaborative interactions occurring.

4.2.1 Interactivity and Negotiability

A measure of interactivity among peers is not defined by the number or frequency of interactions, but by the extent to which these interactions influence the peers' cognitive processes. Collaborative interactions are also negotiable: One agent's view does not hold more value on the sole basis of authority. Interactions, both negotiable and influential, were expected to occur at each of the problems modelled in the virtual environment. This section analyses why these interactions arose and, in the cases where these interactions failed to develop, proposes adjustments which could be made to the learning environment in response.

The extent to which the interactions influenced the students' cognitive processes was seemingly dependant on whether there was a symmetry of knowledge between the students. In two of the pairs, one student had a distinctly superior understanding of differential calculus. Rather than using this understanding to help the other to understand the problem as was expected, the student instead dominated the interaction and stated the answers to each question while the other student remained silent. This was then followed by a declaration that the problem had been solved and a suggestion of moving onto the next problem. As such, the interaction was unproductive as neither of the students' cognitive processes were influenced by the interaction.

There was limited negotiability as the passive student did not ask questions in an attempt to improve their understanding of the problem.

The final pair of students had a commensurate understanding of calculus concepts and consequently worked together to answer the questions put forward in the theory problem as well as the optimisation problem. When one student expressed a lack of understanding about the point of inflection in the theory problem, the other student used the linear driver to explain the concept with reference to the graphs displayed in Figure 2.7 and Figure 2.8. The same student also explained the relationship between the volume graph and the rate of change graph presented in the optimisation problem. This was followed by an acknowledgement of understanding by the other student. Each of these interactions involved negotiation between the students, where the students each put forward possible answers which were responded to and improved upon. This dynamic was present throughout the application: one student would state a lack of understanding and the students would collaborate to achieve a shared understanding of the problem.

The results indicate that in order to maximise negotiability, interactivity, and ultimately collaborative learning, the students should be selected carefully to ensure that knowledge is symmetrical. Another possible solution would be to introduce an instructor to the environment to monitor the dynamic and ensure that neither student entirely dictates the interaction. It is essential that both students are comfortable expressing a lack of understanding. For this reason it may also be advisable to select students who are comfortable with one another and have an established dynamic.

4.2.2 Synchronicity

Synchronous communication is required for multiple agents to perform a task together in real time. This VRLE facilitates synchronous communication as the Photon voice implementation does not have a time-delay. However, there were periods during the evaluation where students communicated asynchronously or did not communicate at all.

Each of the three pairs communicated synchronously when attempting to understand the problems in the Theory Room and the Application Room. For example, this was evident when one student was operating the linear driver and the other student was holding or viewing the display to which that linear driver was linked. The student holding the display gave explicit directions to the student operating the driver, telling them the direction in which to move the driver (left or right) and the magnitude of the movement (more or less). This concluded with an instruction to stop moving the driver when the display indicated that a critical point was reached. Students also communicated synchronously when working together to synchronise the x-value and tangent positions for the problem presented in the Theory Room. It was noted that students would face each other and use hand gestures when communicating, utilising body language as they would in a real environment.

However, the degree to which communication was synchronous was heavily dependant on the distance between the students in the virtual environment. As students separated, they tended to speak less, even though the voice chat allowed them to communicate from anywhere in the environment. It was noted that even when working on the same problem, if one student moved to another part of that problem, communication would often cease. This behaviour is most likely a result of being unaccustomed with this type of communication: we are not used speaking with someone who appears to be 10 or 15 meters away. It would be interesting to implement a communication system in which the volume drops off as the distance between students in the space is increased. This would add to the immersive experience and may discourage students from separating from each other, but might also discourage collaborative interactions as once the students separate in the VRLE they might not communicate effectively. This would be an interesting experiment to conduct nonetheless.

4.2.3 General Observations

It was also noted that the rate at which students became familiar with the user interface and controls impacted the extent to which the interactions were collaborative. For Pair Three there was a difference in ability to use controls, which resulted in one student exploring the space efficiently while the other student struggled to keep pace. As the students separated, the amount of collaborative interactions decreased rapidly.

One solution mentioned earlier would be to implement an initial lesson on navigating the environment followed by a test which both students would have to pass before either student could continue to explore the environment.

4.2.4 Final Remarks on User Collaboration

Ultimately, the interactions in the environment were not consistently collaborative. The main barriers to collaborative interactions were an asymmetry in knowledge between the participants, the relationship between distance in the learning environment and communication, and a disparity in the ability to navigate the environment.

Going forward, these issues could be addressed with careful selection of student pairs and a revised implementation of voice chat which encourages students to communicate continually. It would be worthwhile to consider how the presence of an instructor in the virtual space would impact the issues raised above. Ultimately this technology as it is currently conceptualised would be implemented in the classroom, where an instructor could scaffold the interactions to ensure an asymmetry in knowledge is not manifest in disunited interactions.

4.3 Future Work

Moving forward, both HCI researchers and educational professors should continue to investigate the benefits that a mixed reality mathematics classroom may have on both learning experiences and learning outcomes. This paper has raised some important questions that need to be addressed. Pilot studies should be conducted, investigating the effects on learning outcomes over the course of an entire curriculum.

It is necessary to develop an understanding of how culture in the virtual learning environment would evolve, similar to our understanding of classroom dynamics and culture. Pilot studies should be conducted, investigating the effects over the course of an entire curriculum. It would be interesting to introduce an educational instructor into the space and see how this affects learning in the virtual environment. The applications of VRLE's in distance learning has massive potential to transform the way we think about education. As the availability of VR technologies continues to increase, it is vital that we understand how this technology may can educate students who are unable to reach the classroom, due to illness or otherwise.

The paths of research outlined above ultimately rely on our ability to effectively develop virtual reality learning environments. Presently, only computer scientists and game developers which possess the skills to build and program interactive virtual environments. Most often, they do not have sufficient understanding of the relevant educational psychology and will struggle to construct a space which is truly optimised for learning and teaching. As such, it is important that research going forward is interdisciplinary, drawing from computer science, HCI, educational psychology, and practicing teachers.

A software which allows educators to construct their own virtual learning environments should be developed, one which does not require a deep understanding of game development or programming to be used effectively. The abstraction of technical details would allow the educator to focus on educating, and would increase the receptivity to this new technology among teachers.

4.4 Concluding Remarks

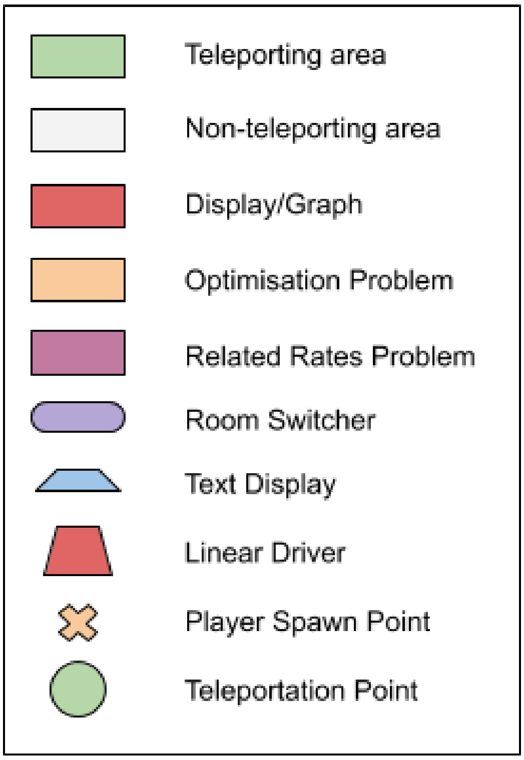
This paper documents the motivation, development and evaluation of a virtual learning environment for calculus. The application presented serves primarily as a proof of concept which can be used as a basis for further developments. The next stage in the process could be to iteratively improve and evaluate the application, expanding on the concepts illustrated to include a more comprehensive view of introductory calculus, ultimately culminating in a carefully designed pilot study to assess the effects of this technology on learning outcomes.

As access to virtual reality technology continues to increase, educators and entrepreneurs alike are responding to the potential that VR has in mathematics education, not only for its ability to teach abstract concepts such as those presented in this paper, but also as a means of increasing student engagement and facilitating synchronous distance learning. The next generation of immersive technologies presents an exciting challenge for HCI research not just in mathematics, but in many educational fields.

Appendices

Appendix A

A.1 Floor Plans



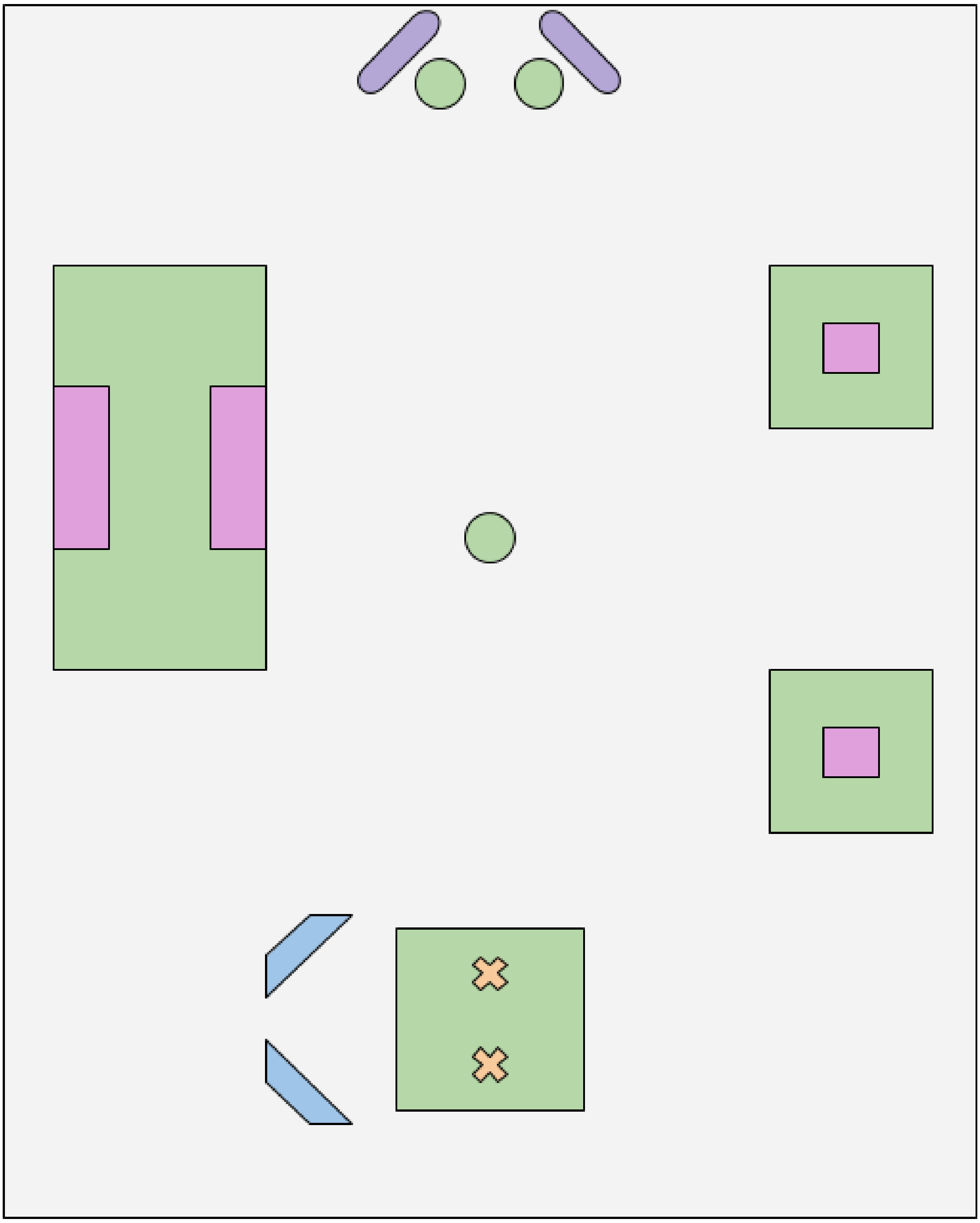


Figure A.1: Floor plan for the Introduction Room.

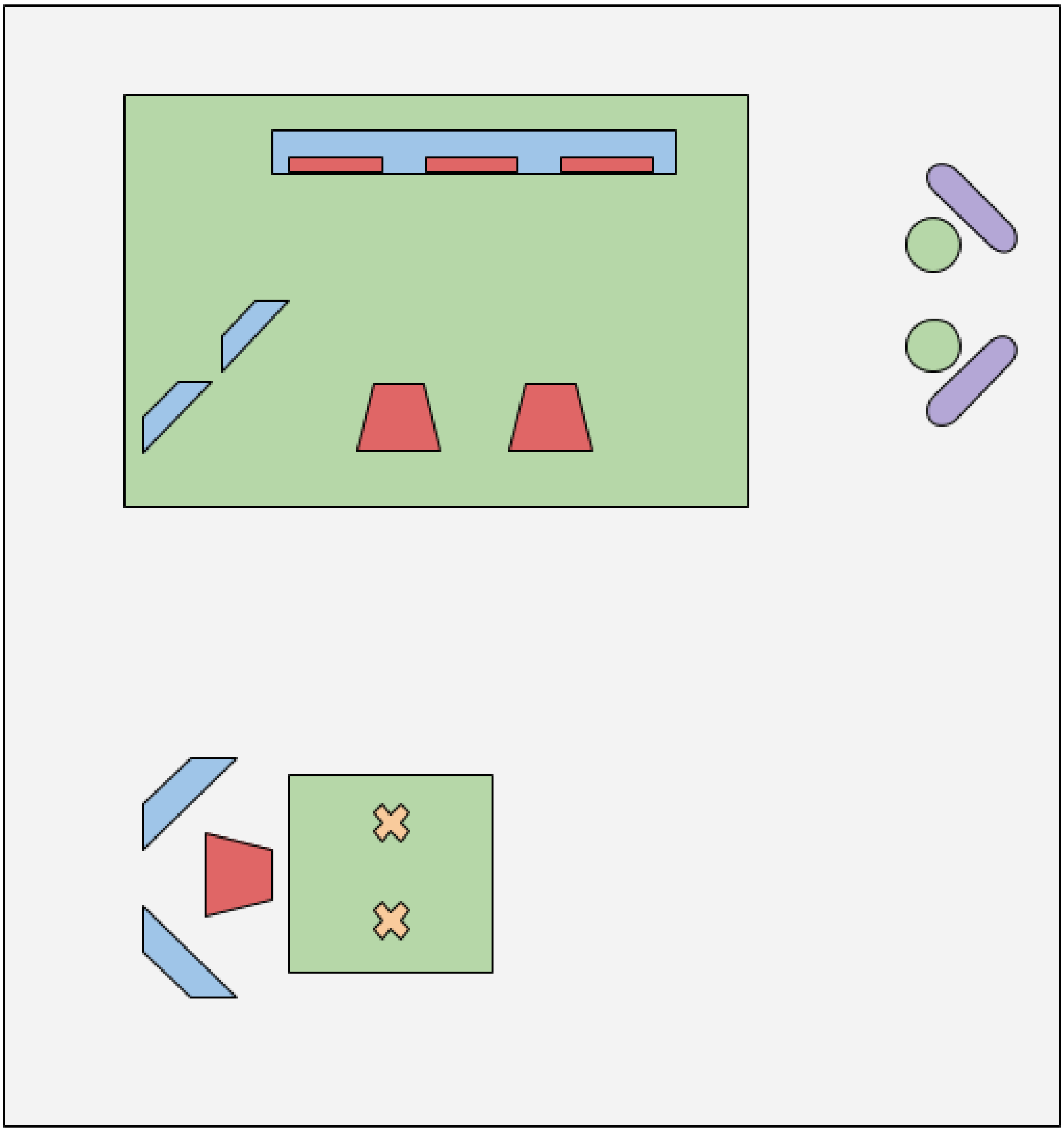


Figure A.2: Floor plan for the Theory Room.

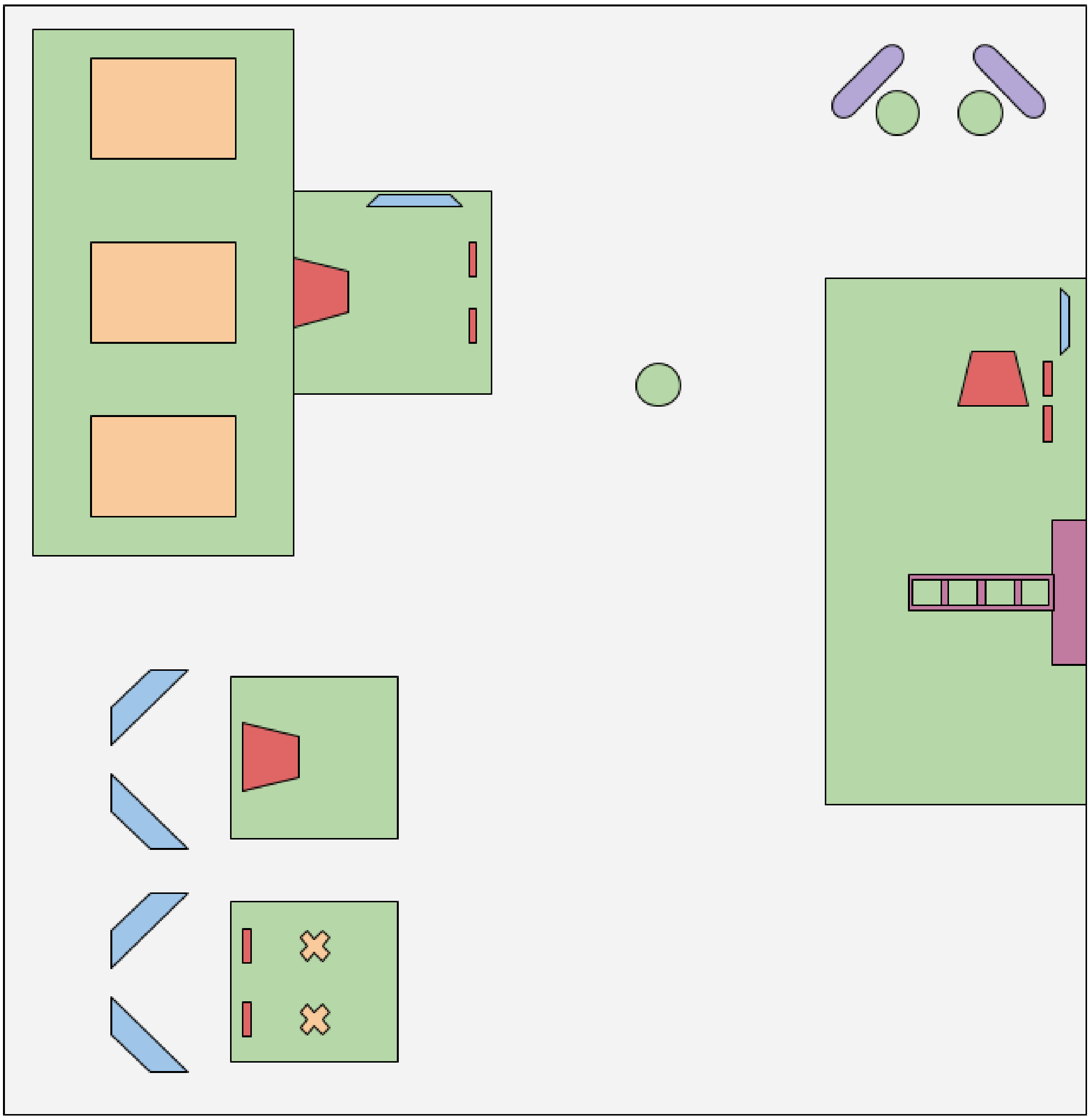


Figure A.3: Floor plan for the Application Room.

A.2 Problem Sketches

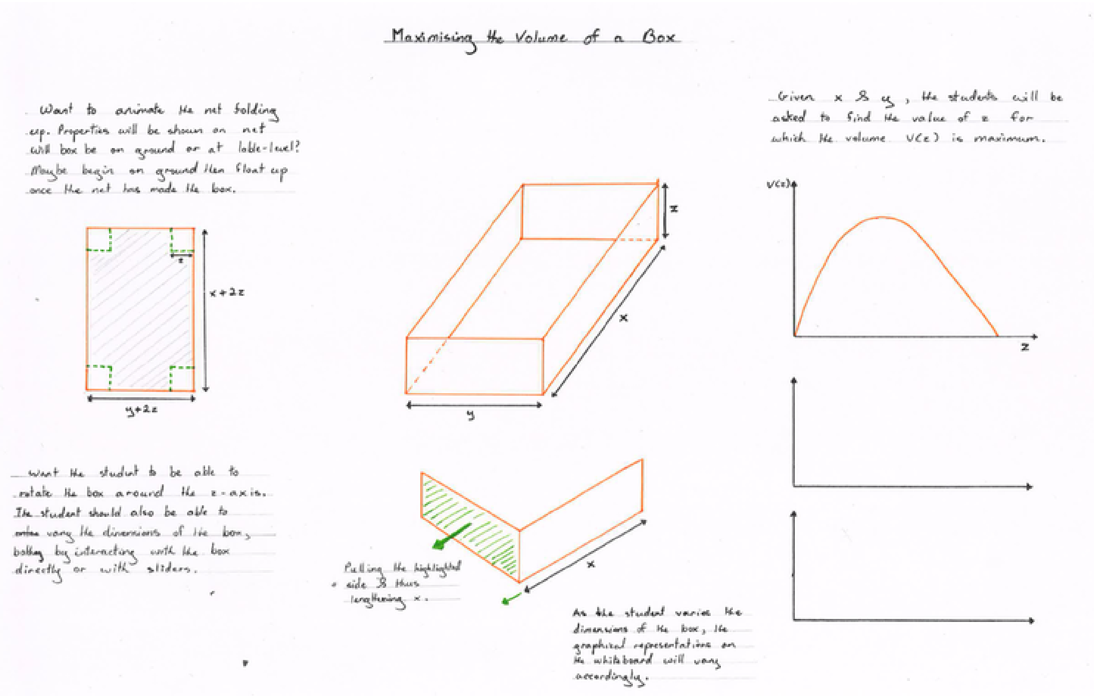


Figure A.4: Original sketches for the box problem.

A.3 Multiplayer Virtual Reality Development Resources

- Basics of multiplayer virtual reality: Youtube Tutorial
- Photon Networking: Unity Asset Store Photon Networking
- Voice chat for multiplayer VR: Photon voice
- Photon documentation and tutorial: Photon Documentation

A.4 Texts From the Application

In each room, the students are presented with texts which explain different elements of the virtual environment. Each of these texts are included below.

Introductory Text

"In this room there are some objects for you to interact with. If you look down at your hands you should see a controller. Both your middle and index fingers should be resting on the triggers. Pulling on either of the triggers will let you grab objects."

"If you press the thumbstick in you should hear a click. When you hear the click a targeter should appear. You can use this targeter to teleport around the space. The targeter will turn green whin you can teleprt to an area. To the right ther are some floating cubes. Teleport to them and interact with them with your partner."

"Once you have familiarised yourself with manipulating objects and teleporting in the virtual classroom, you can proceed to the theory room by hitting the button at the end of the room." *"Welcome! If this is your first time in VR, take a moment to look around. VR might seem a little strange at first, but it quickly becomes familiar."*

"In this room there are some objects for you to interact with. If you look down at your hands you should see a controller. Both your middle and index fingers should be resting on the triggers. Pulling on either of the triggers will let you grab objects."

"If you press the thumbstick in you should hear a click. When you hear the click a targeter should appear. You can use this targeter to teleport around the space. The targeter will turn green whin you can teleprt to an area. To the right ther are some floating cubes. Teleport to them and interact with them with your partner."

"Once you have familiarised yourself with manipulating objects and teleporting in the virtual classroom, you can proceed to the theory room by hitting the button at the end of the room."

Linear Driver Interaction Tutorial

"To grab the handle, put your hand near the handle and squeeze and hold the trigger with your middle finger."

"Once you have familiarised yourself with manipulating objects in the virtual classroom, press down on the thumbstick and teleport to the problem on your right.""In this virtual classroom you will interact with objects via sliding handles. Each handle is connected to one or more objects. As you move the handle, the properties of these objects will change"

"To grab the handle, put your hand near the handle and squeeze and hold the trigger with your middle finger."

"Once you have familiarised yourself with manipulating objects in the virtual classroom, press down on the thumbstick and teleport to the problem on your right."

Theory Room Problem

"together with your partner, use the handles to interact with these graphs. Try to relate the behaviour of these graphs to what you have learned in differential calculus. What does the derivative represent? How does the original graph relate to its first and second derivative? Where is the point of inflection and how does it relate to the other graphs? How do tangents behave at maxima and minima?"

"This is all about learning maths in a collaborative way, working through and discusing ideas with your partner.""In front of you are a set of graphs. The graphs show a cubic function, along with its first and second derivatives. The handles allow you to interact with each of the graphs. The left handle controls the value of x, while the right handle controls the tangent line. The display to your right shows the observed values. Play around to get an understanding of how they work"

"together with your partner, use the handles to interact with these graphs. Try to relate the behaviour of these graphs to what you have learned in differential calculus. What does the derivative represent? How does the original graph relate to its first and second derivative? Where is the point of inflection and how does it relate to the other graphs? How do tangents behave at maxima and minima?"

"This is all about learning maths in a collaborative way, working through and discusing ideas with your partner."

Graphic Display Interaction Tutorial

"These displays can be interacted with. Try to pick up a display and move it around. Displays will stay wherever you decide to leave them and will float if you let go. You can also pass graphs to each other and teleport while holding a graph. When working with these problems, move the graphs to find the most comfortable setup." "Welcome to the Application Room! Here you can explore and interact with some real world calculus problems."

"These displays can be interacted with. Try to pick up a display and move it around. Displays will stay wherever you decide to leave them and will float if you let go. You can also pass graphs to each other and teleport while holding a graph. When working with these problems, move the graphs to find the most comfortable setup."

Optimisation Problem Text

"Keeping this in mind, use the handle to vary x and try to relate the graphs behind you to the objects in front of you. Remember you can pick up the graphs and move them to where you feel works best."

"Some questions to consider are:

1. *What are the constraints for x? And why are these constraints appropriate?*
2. *When is the volume at its maximum and how does this relate to the differential graph?*
3. *There are two times when the volume is zero, looking at the objects in front of you, try to make sense of this."*

"This problem asks you to maximise the volume of a box. You might remember from the class that to maximise this volume, you must first differentiate the equation and then set x to zero."

"Keeping this in mind, use the handle to vary x and try to relate the graphs behind you to the objects in front of you. Remember you can pick up the graphs and move them to where you feel works best."

"Some questions to consider are:

1. *What are the constraints for x? And why are these constraints appropriate?*
2. *When is the volume at its maximum and how does this relate to the differential graph?*
3. *There are two times when the volume is zero, looking at the objects in front of you, try to make sense of this."*

Related Rates of Change Problem Text

"This is an applied calculus problem about a ladder sliding down a wall. If you look at the graphs you'll note the variables 'a' and 'b' are used. Here 'a' is the distance of the top of the ladder from the floor and 'b' is the distance of the bottom of the ladder to the wall. In this question you are asked to find the speed of the bottom of the ladder away from the wall at a given time. With your partner, discuss these graphs and how they relate to the problem."

"As the bottom of the ladder moves from the wall, does it speed up or slow down? How do you know this from the graph?"

Appendix B

B.1 User Engagement Results

Table B.1: User Engagement Questionnaire Results.

Item	User 1	User 2	User 3	User 4	User 5	User 6
FA1	3	4	5	3	4	3
FA2	3	4	4	4	5	3
FA3	2	3	3	5	4	4
FA4	2	5	4	4	4	4
FA5	3	4	4	3	4	4
FA6	5	4	4	4	4	4
FA7	3	4	4	4	3	3
PU1	5	5	5	3	5	3
PU2	5	4	5	2	5	3
PU3	5	5	5	4	5	4
PU4	5	5	5	4	5	5
PU5	5	5	5	5	5	3
PU6	3	5	5	4	4	5
PU7	5	4	4	3	4	2
PU8	5	4	3	3	4	3
AE1	5	4	5	3	3	5
AE2	5	4	3	4	5	4
AE3	5	4	4	4	5	5
AE4	5	4	4	5	4	5
AE5	5	4	4	4	3	4
RW1	5	4	4	4	5	4
RW2	5	4	5	4	5	4
RW3	5	5	5	4	5	3
RW4	5	4	3	4	4	3
RW5	5	4	5	4	5	4
RW6	5	5	3	3	4	3
RW7	5	5	5	4	5	4
RW8	5	5	5	4	5	4
RW9	5	5	5	4	4	5
RW10	5	5	5	4	5	5

Age:

1. I am.
○ Male ○ Female

2. I lost myself in this experience.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

3. I was so involved in this experience that I lost track of time.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

4. I blocked out things around me when I was using this application.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

5. When I was using this application, I lost track of the worlds around me.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

6. The time I spent using this application just slipped away.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

7. I was absorbed in this experience.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

8. During this experience I let myself go.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

9. I felt frustrated while using this application.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

10. I found this application confusing to use.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

11. I felt annoyed while using this application.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

12. I felt discouraged while using this application.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

13. Using this application was taxing.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

14. This experience was demanding.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

15. I felt in control while using this application.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

16. I could not do some of the things that I wanted to do while using this application.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

17. This application was attractive.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

18. The application was aesthetically appealing.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

19. I liked the graphics and images in this application.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

20. This application appealed to my visual senses.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

21. The screen layout of this application was visually appealing.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

22. Using this application was worthwhile.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

23. I consider my experience a success.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

24. This experience did not work out the way I had planned.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

25. My experience was rewarding.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

26. I would recommend this application to my family and friends.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

27. I continued to use this application out of curiosity.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

28. The content of this application incited my curiosity.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

29. I was really drawn into this experience.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

30. I felt involved in this experience.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

31. This experience was fun.
○ Strongly disagree ○ Disagree ○ Neither agree nor disagree ○ Agree ○ Strongly agree

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