Everything on Set Theory

A set represents a collection of items

none - empty set It contains nothing

Int Set of integers

A <u>signature</u> introduces a new set → sig fruit{}

An <u>enumeration</u> introduces a new set containing specific items → enum Vegetable{Tomato}

Multiplicities

When we add a set, we can specify how many members it contains by adding a multiplicity constraint

one Exactly one member

lone Less or one

some At least 1 member

If you want to specify a different size, use a fact

Fact FiveFruit{# fruit = 5}

A set can be introduced as a subset of another set. There are 2 ways:

extends sig Apple, Banana, Pear extends Fruit {}

in sig Fresh, Expensive in Fruit{}

Extends is used for subsets that are mutually disjoint, no members in common

In includes subsets which may overlap

An abstract signature introduces a set that contains nothing apart from the members of sets that extend that signature

abstract sig Fruit{}

Operations

+ union Something is in S+T when it's in S or T or both
 & intersection Something is in S&T when its in both S and T
 - difference Something is in S-T when its in S but not T

number/cardinality # S is the number of members in S

| such that $\{i: lnt \mid i > 5\}$ - lnt that is greater than 5

Logical expressions

 $in \rightarrow subset$ S in T is true if every member of S is also in T

 $in \rightarrow membership$ A in S is true if A is a member of S

 $= \rightarrow$ equality S=T is true if S and T have exactly the same members

 $some \rightarrow non-emptiness$ Some S is true if S has at least one member

 $no \rightarrow emptiness$ No P is true if p has no members

General laws

$$A + A = A$$

 $A & A = A$

A - A = none

Commutative laws:

$$A + B = B + A$$

A & B = B & A

· Associative laws:

$$A + (B + C) = (A + B) + C$$

 $A & (B & C) = (A & B) & C$

Distributive laws:

$$A + (B \& C) = (A + B) \& (A + C)$$

$$A \& (B + C) = (A \& B) + (A \& C)$$

Everything on Using Sigs and Sets

A signature may include fields

A signature may have <u>constraints</u>. These are boolean expressions and may refer directly to the fields. Constraints impose restrictions on the possible models

A <u>predicate</u> is a parameterised boolean expression. It can be run to find the example that makes the predicate true

A <u>fact</u> is a boolean expression that imposes some additional constraint on the specification

An <u>assertion</u> is a boolean expression, expressing some property that we think should follow our specification. Allow can <u>check</u> if there are any counterexamples to this assertion