Everything on Model-Checking

First Order Logic

Alloy is based on first order logic. Every specification contains some undefined elements these form the signature of the specification

A structure of a signature is an assignment of specific values to each of the variables in the signature

```
sig Student { }

one sig JaneDoe extends Student { }

sig Module {
    class : set Student
}

one sig CSC9P6 extends Module { }

fact { JaneDoe in CSC9P6.class }

There are students.

There is a student called Jane Doe.

There is a relation, class, between Module and Student.

There is a module called CSC9P6.

Jane Doe is in the CSC9P6 class.
```

```
Student = {JaneDoe, Student0, Student1}

JaneDoe = {JaneDoe}

Module = {CSC9P6, Module0}

Class = {CSC9P6 -> Student0, CSC9P6 -> JaneDoe}

CSC9P6 = {CSC9P6}
```

```
Student = {JaneDoe, Student0, Student1}
JaneDoe = {JaneDoe}
Module = {CSC9P6, Module0}
Class = {CSC9P6 -> Student0, CSC9P6 -> Student1}
CSC9P6 = {CSC9P6}
```

A model of a specification is a structure for the specification which makes all the statements in the specification true.

A logical statement is consistent if there exists at least one model of that statement. There's at least one possible world in which this statement is true

A logical statement is valid if every structure is a model of that statement. The statement is true in all possible worlds.

```
Run a predicate → check if it is consistent
Check an assertion → check if assertion is valid
```

Run a predicate to find out if something can possible be true. Check an assertion to find out if something must always be true

Alloy only checks models up to a certain size \rightarrow scope. The default scope is 3. This can be modified

Limitations of model checking

Because of the use of a finite scope, model-checking in alloy has some limitations

- 1) When a predicate is run, if a model instance is found we can be certain that the predicate is consistent, but if no instance is found, we cannot conclude that the predicate is inconsistent
 - It may be inconsistent or a model instance could be found if a larger scope was used
- 2) When an assertion is checked, if a counter-example is found, we can be certain the assertion is invalid, but if no counter-example is found, we cannot be sure that the assertion is valid.

It may be valid or maybe a counter-example can be found by using a larger scope

Small Scope Hypothesis

Despite the limitation, it can still be argued that alloy is a useful tool

The basic statement to support that is the small scope hypothesis:

Most bugs have small counter-examples. If an assertion is invalid, it possible has a small counter-example. If you examine all small cases, you're likely to find a counter-example