Benchmarking Solvers, SAT-style

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Abstract

The SAT community, and hence the SMT community, have substantial experience in benchmarking solvers against each other on large sample sets, and publishing summaries, whereas the computer algebra community tends to time solvers on a small set of problems, and publishing individual times.

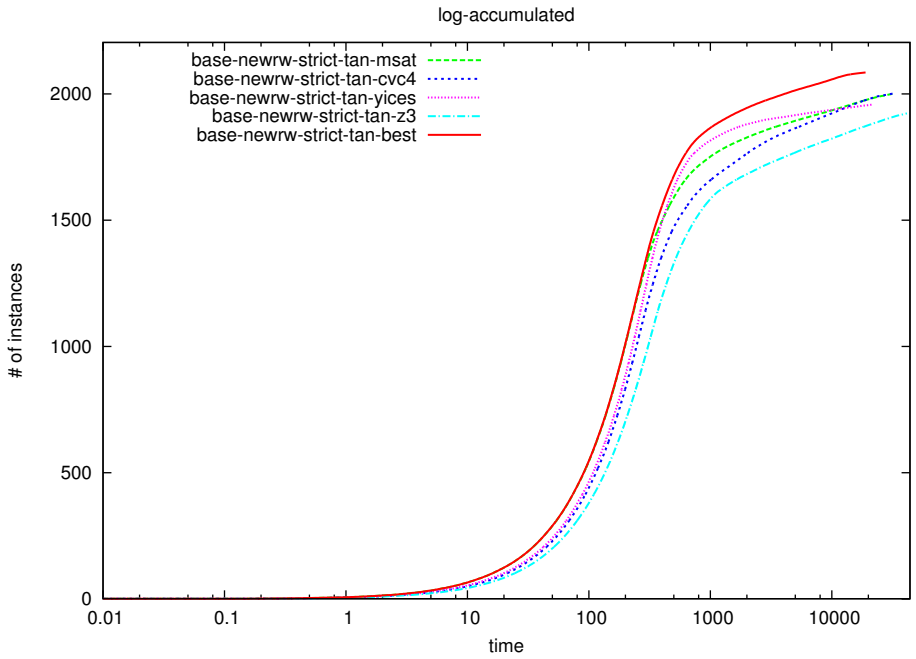
This paper aims to document the SAT community practice for the benefit of the computer algebra community.

Implicitly underlying benchmarking for solvers is the following hypothesis.

Hypothesis 1. *Our solvers will be faced in practice (read "the next competition") with problems whose time-to-solve distribution is the same as that for the benchmark set.*

A necessary condition is that one has a large benchmark set, ideally in the thousands. Sampling appropriately is always a question of judgement.

Figure 1: A Survival Plot



1 "Cactus" or "Survival" plots

Figure 1 is a typical survival[1] plot produced in the SMT community. The methodology for producing these, given a large benchmark set of problems, is as follows.

1. For each method separately
   (a) Solve each problem p_i, noting the time t_i (up to some threshold T).
   (b) Sort the t_i into increasing order (discarding the time-out ones).
   (c) Plot the points $(t_1, 1)$, $(t_1 + t_2, 2)$ etc., and in general $(\sum_{i=1}^{k} t_i, k)$.
2. Place all the plots on the same axes, optionally (as in Figure 1) using a logarithmic scale for time.

N.B. There is therefore no guarantee that the same problems were used to produce time results from different solvers.

From 1 we can deduce that, up to 100 seconds, the solvers are pretty similar, and with a time budget of 100 seconds, can solve *at most* around 500 problems. At a time budget of 1000 seconds, the differences are more marked, and the worst solved around 1600 problems and the best around 2000.

2 CDF plots

An alternative is (still after sorting the t_i) to plot $(t_1, 1)$, $t_2, 2)$ etc., and in general (t_k, k). If we normalise the y axis to $[0, 1]$ (*not* discarding the timeouts) we have an approximation to the cumulative distribution function. See [3] for some examples. Hence from these plots we can ask questions like "what is the probability of solving a random problem in t seconds".

3 Virtual Best Solver

Though not shown in Figure 1, the SAT competition has taken to including a "virtual best solver" (VBS) which is synthesised from the other results by taking the minimum (across all solvers tested) time taken to solve every given benchmark. Thus the VBS is always equal to the time of some solver, but which one will change by the benchmark (measuring how often each solver is the VBS is also an interesting metric). The VBS can be added to the survivor/cactus plot to get a feeling for the variability between solvers.

We have therefore added this to our solvers on the diagrams,and counted how often a solver is the VBS. A variation on counting is provided by [1], who measure how often a solver is within one second of being VBS. Their justification is "The constant of one second was chosen since we consider a smaller difference as insignificant, especially in the context of a one-second time-out". This is open to the argument that it is just as subject to random fluctuation as the original, but in a different place. One could consider scoring "VBS points": 1 if VBS, 0 if more than 5% slower[2] than VBS, and linear interpolation in between.

[1] "Cactus" plots are the same with the axes flipped. Some cactus plots can be seen at http://fmv.jku.at/hwmcc15/Biere-HWMCC15-talk.pdf.

[2] A percentage-based approach is probably more appropriate than a fixed time, as differences in time tend to come from consistent features. But this could also do with more experimentation.

4 Running multiple copies

One of the effects of having a solution process whose running time is widely variable[3] is that one may well not be best served by just running the process to termination. In the case of a single processor, this issue was considered by [2], who suggested (and indeed proved almost-optimality) running the process up to certain time limits and then starting afresh, where the limits were of the form $T, T, 2T, T, T, 2T, 4T, T, T, 2T, T, T, 2T, 4T, 8T, \ldots$, where T is some arbitrary unit.

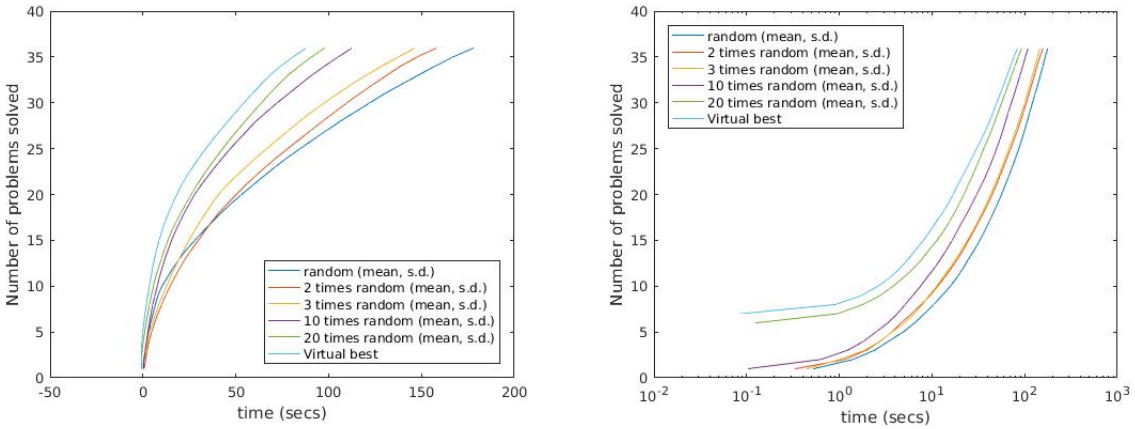
This is in fact the default behaviour in MiniSAT 2.2.0, where it is known as `Luby` (though T is in fact measured in terms of ?? rather than time, and it's not a complete restart that is performed, as certain learned clauses are kept).

These days, with processors getting more numerous rather than faster, we might consider running multiple copies in parallel. To see how this might help, consider the trivial case of a process whose running time is $1, K, K^2$ with equal probability. Then the average time to solution is $\frac{1}{3}(1+K+K^2) = 37$ when $K = 10$. Running two copies and aborting the other when one finds the solution has an average time to solution of $\frac{1}{9}(5+3K+K^2) = 15$ when $K = 10$, so the CPU cost is 30 units, still less than the sequential cost. The break even point for two-fold parallel running is $K = \frac{1}{2}\left(1+\sqrt{37}\right) \approx 4.5$. It is worth noting, though, that a single Luby process with $T = \frac{1}{3}$ (to avoid $T = 1$ getting lucky) achieves an average time to solution of ≈ 9.

5 Distributions

5.1 Normal Distribution of Times

Figure 2: Data from Section 5.1 — Normal distribution



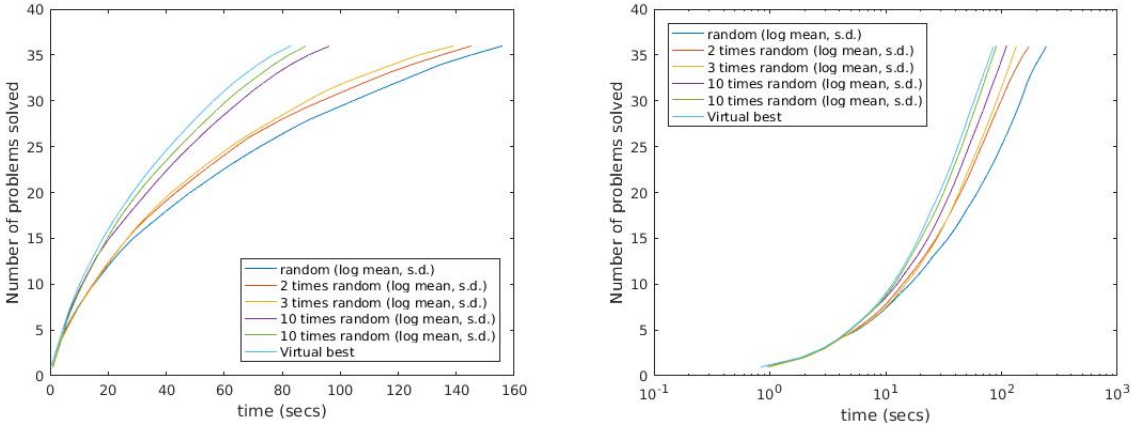
It is far from clear what sort of distribution the running times of SAT, and even less SMT, solvers have, but it would be foolish to ignore the normal distribution. We took 36

[3] These algorithms are widely called *Las Vegas* algorithms. However, the term has different connotations in the different fields. In Symbolic Computation, most Las Vegas algorithms are ones that normally produce the answer in a deterministically-bounded time, but may occcasionally fail and have to try again, and effort goes into bounding the error probability, and proving that the algorithm will terminate. Modular algorithms are a classic example. But here we are considering algorithms whose running time is intrinsically variable.

hypothetical cases, with 9 different running times $t_0 = 1, \ldots, 9$, and standard deviations $t_0/10, 2t_0/10, 3t_0/10, 4t_0/10$ for each running time.

We used five (hypothetical) possible solvers. The base line one just took a time t at random from the relevant normal distribution: $t \in N(t_0, kt_0/10)$. The second one ran two copies in parallel, terminating when the first one did, the third ran three copies, and the next two 10 and 20. In this case, the VBS is in fact the equivalent of running 36 copies. The data are in Figure 2. Note that we are *not* charging twice for the cost of running two copies, i.e. we are loookingat "time to solution" not "cost of solution"

Figure 3: Data from Section 5.1 — log Normal distribution



It is also not clear whether we should assume t or $\log t$ is normally distributed. Hence we re-ran these experiments, but applied the normal distribution in $\log t$-space. The standard deviation was scaled so that it bore the same ratio to the mean as before, i.e. $\sigma_{\log} = \sigma \frac{\mu_{\log}}{\mu}$, where μ and σ represent the mean and standard deviation. Again, the VBS is in fact the equivalent of running 36 copies. The data are in Figure 3.

5.2 Uniform Distribution of Times

We then considered uniform distributions, with the lower bounds being $t_0 = 1, \ldots, 10$, and the upper bounds being 10 times that. We used the same hypothetical solvers as before. The data are in Figure 4.

Again, one could say that we should be uniform in $\log(t)$, and we did these computations. The data are in Figure 5. It might seen from these that running twice and running thrice were very similar, and in fact that running twice was almost half the time of running once, thus meaning that they were almost equivalent in cost. In fact, this model is susceptible to algebraic treatment, and the formulae (running from 1 to B seconds, with numeric values for $B = 10$) are as follows:

$$\begin{array}{rcll} \text{once} & = & \frac{B-1}{\log B} & \approx 3.9087 \\ \text{twice} & = & \frac{2}{(\log B)^2}(B - (\log B + 1)) & \approx 2.5264 \\ \text{thrice} & = & \frac{6}{(\log B)^3}\left(B - (\frac{1}{2}\log B + \log B + 1)\right) & \approx 1.9887 \end{array}$$

Hence in fact the "running thrice" number is approximately correct, at one-half the elapsed time of running once.

Figure 4: Data from Section 5.2 — Uniform distribution

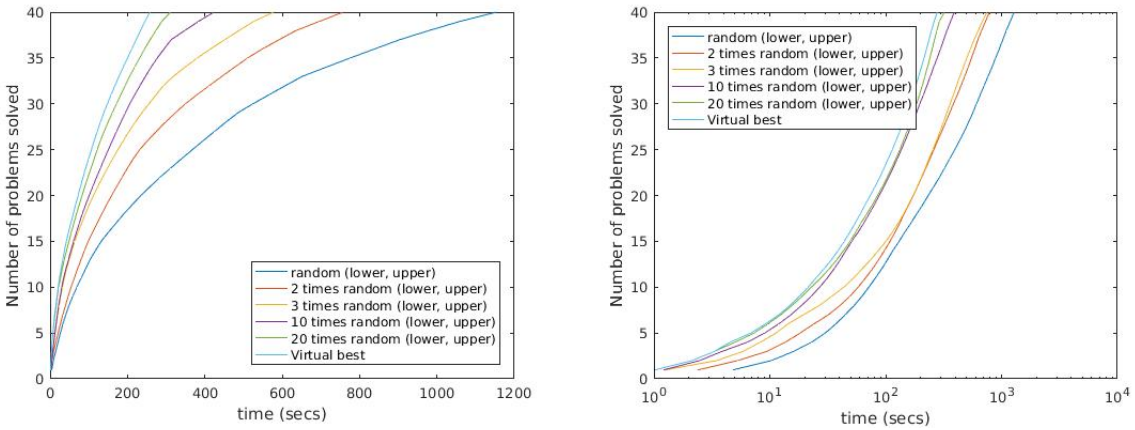


Figure 5: Data from Section 5.2 — log Uniform distribution

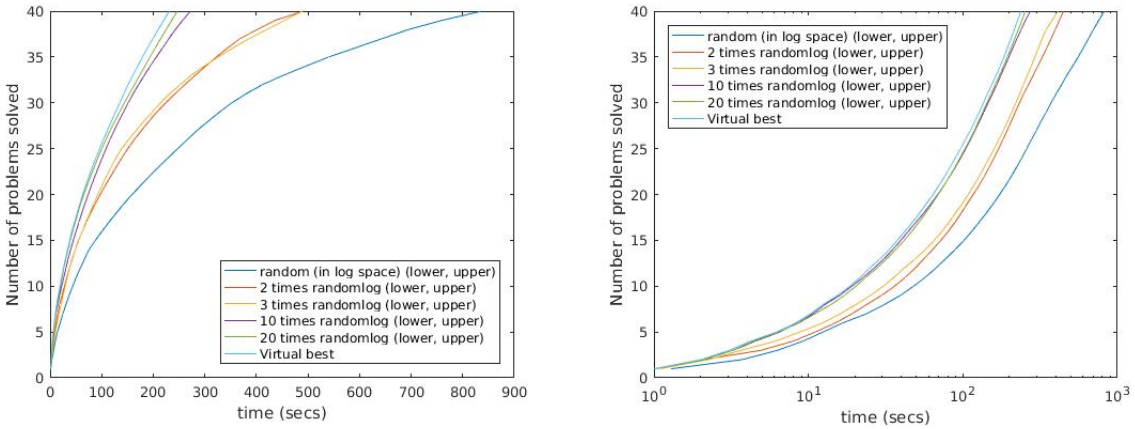
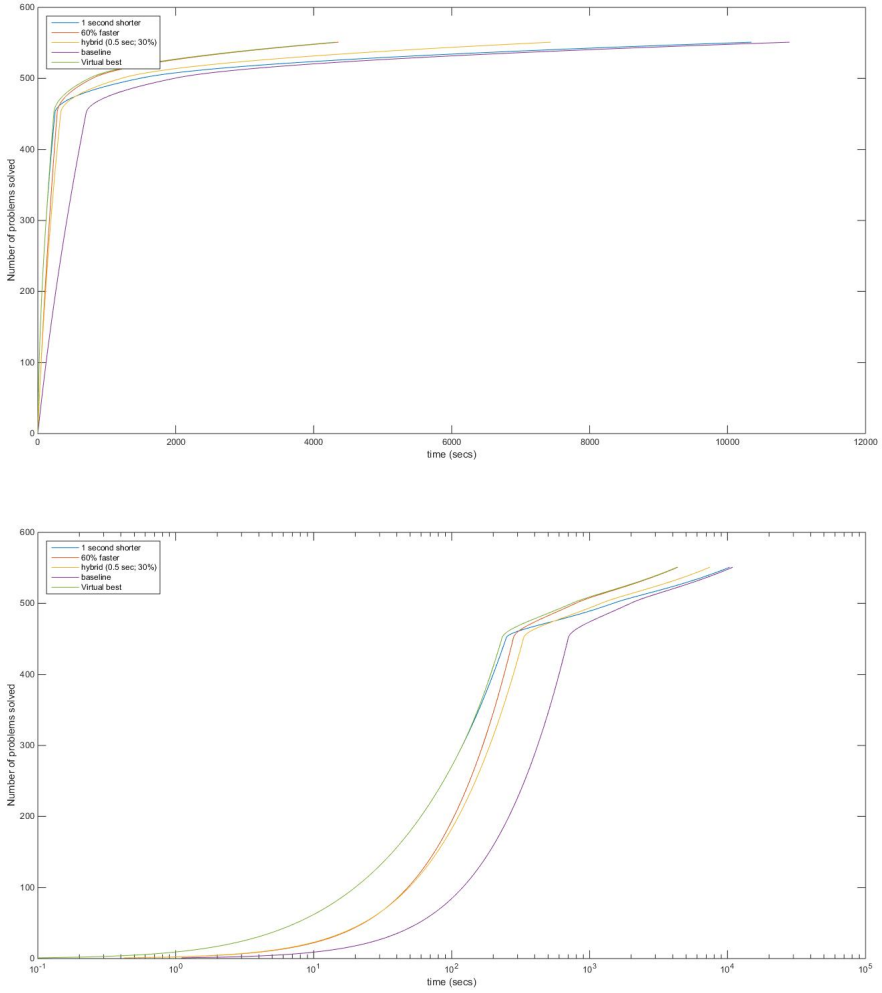


Figure 6: Data from Section 6.1



6 Case Studies

For the first three tests we used a vector of baseline times (notionally in seconds) of `cat(2,[1.1:0.002:2],[2:1:50],[50:5:300]);` in MatLab speak, i.e. 1.1 to 2 in steps of 0.002, 2 to 50 in step of 1, and 50 to 300 in steps of 5.

6.1 Predictable

We first measure four solvers: baseline, baseline less 1 second, 40% of baseline and a hybrid of 70% of (baseline less 0.5 seconds). The results are shown in Figure 6. "1 second faster" was quickest 284 times, and "60% faster" 267 times. However, "60% faster" took 48.3 seconds longer than the Virtual Best (which took 4311 seconds),"1 second faster" 6036 seconds longer, hybrid 3125 seconds longer and the baseline 6572 seconds longer.

6.2 Predictable plus Fuzz

What happens if we multiply each time by a random variable, uniform in [0.8,1.2]? 40 runs of this experiment give a mean VBS time of 4299 seconds, with a standard deviation of 53.455. In the counts of how often each solver was VBS, hybrid appeared, showing up between 5 and 17 times, with corresponding adjustments to the others. "1 second faster" was always the most common, with the ratio of it over "60% faster" ranging from 1.09 to 1.34. The plots (linear and semilogx) are in Figure 7.

6.3 Predictable plus Random

To the previous solvers, we add a "joker", that, on one problem in 10, takes 10% as long as the baseline. The results are shown in Figure 8. The joker was quickest 55 times, "1 second faster" was quickest 256 times, and "60% faster" 240 times. The time differences are that "60% faster" took 402 seconds longer than the Virtual Best, "1 second faster" 6390 seconds, hybrid 3479 seconds, the joker 5865 seconds and the baseline 6941 seconds.

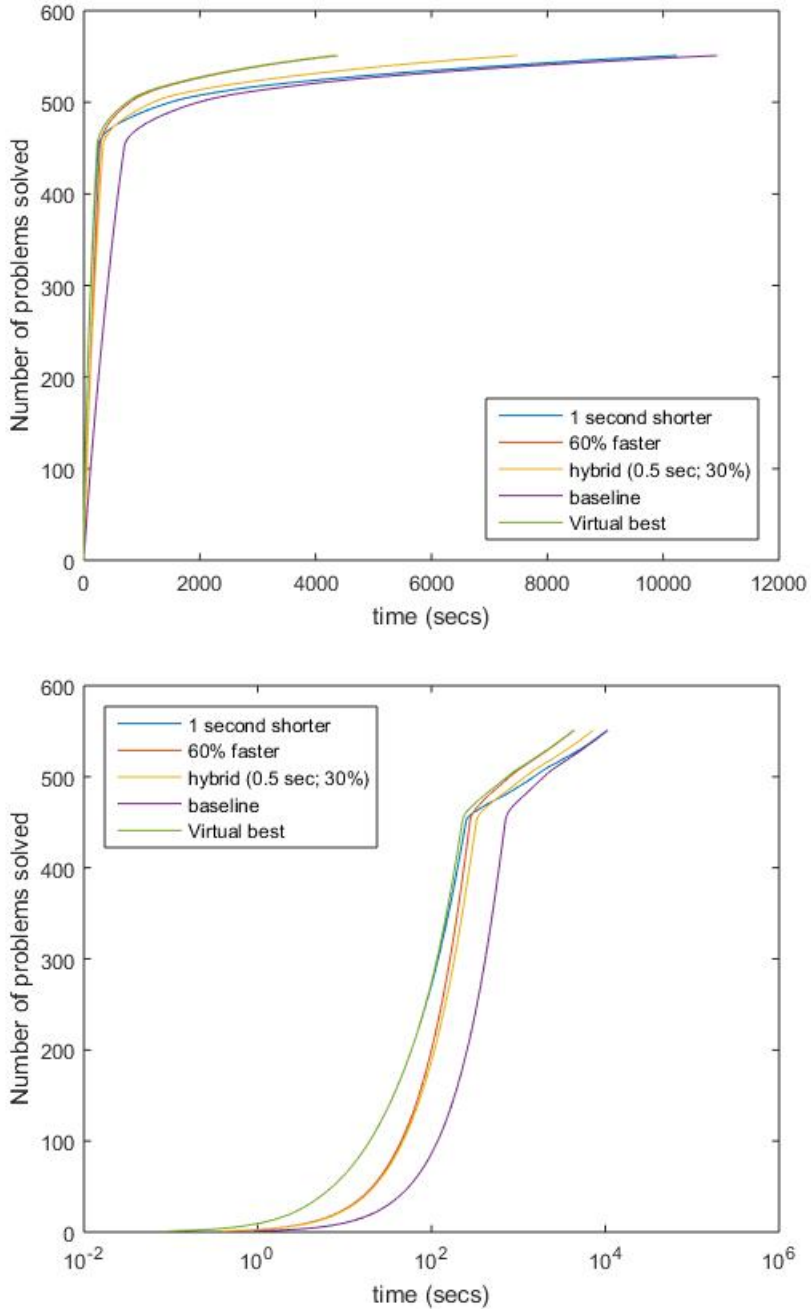
6.4 Judgement

The data used so far had 500 "fast" problem ($<$ 2 seconds), 50 "medium" (between 2 and 50) and 5 hard (over 50). What happens if, instead, we have equal numbers in each bracket. The results from this, otherwise using the same methodology as section 6.3, are in Figure 9: the reader can see the difference from Figure 8: the current figure is dominated by the slow problems. The joker was quickest 593 times, "1 second faster" was quickest 256 times, and "60% faster" 5084 times.

7 Pairwise comparisons

Scatter plots are used to compare pairs of solvers. For each benchmark you plot (sometimes using different colours or marks for SAT and UNSAT) a point with x location the time taken by solver 1 and y the time taken by solver 2. To make things easier to follow, people commonly add the diagonal (sometimes annotated with "solver 1 is faster" and "solver 2 is faster" on the relevant sides / corners) and the time-out lines. An example is shown in Figure 10.

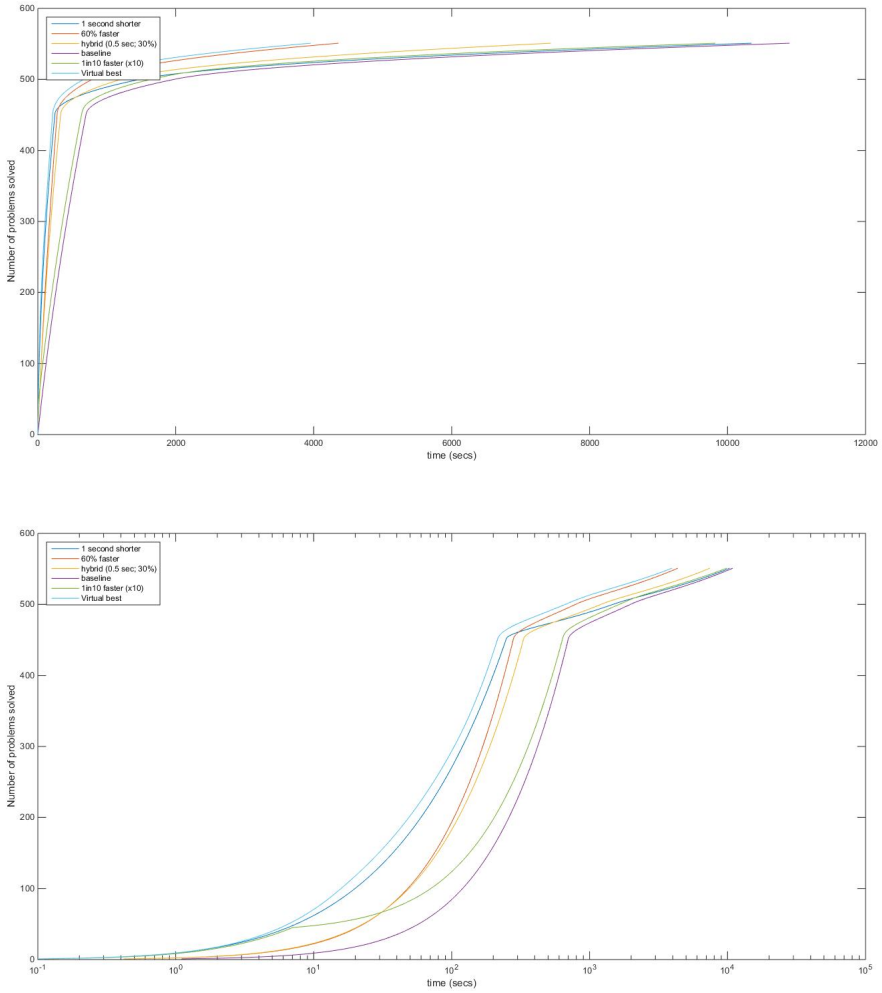
Figure 7: Data from Section 6.2



References

[1] M. Janota, I. Lynce, and J. Marques-Silva. Algorithms for computing backbones of propositional formulae. *AI Communications*, 28:161–177, 2016.

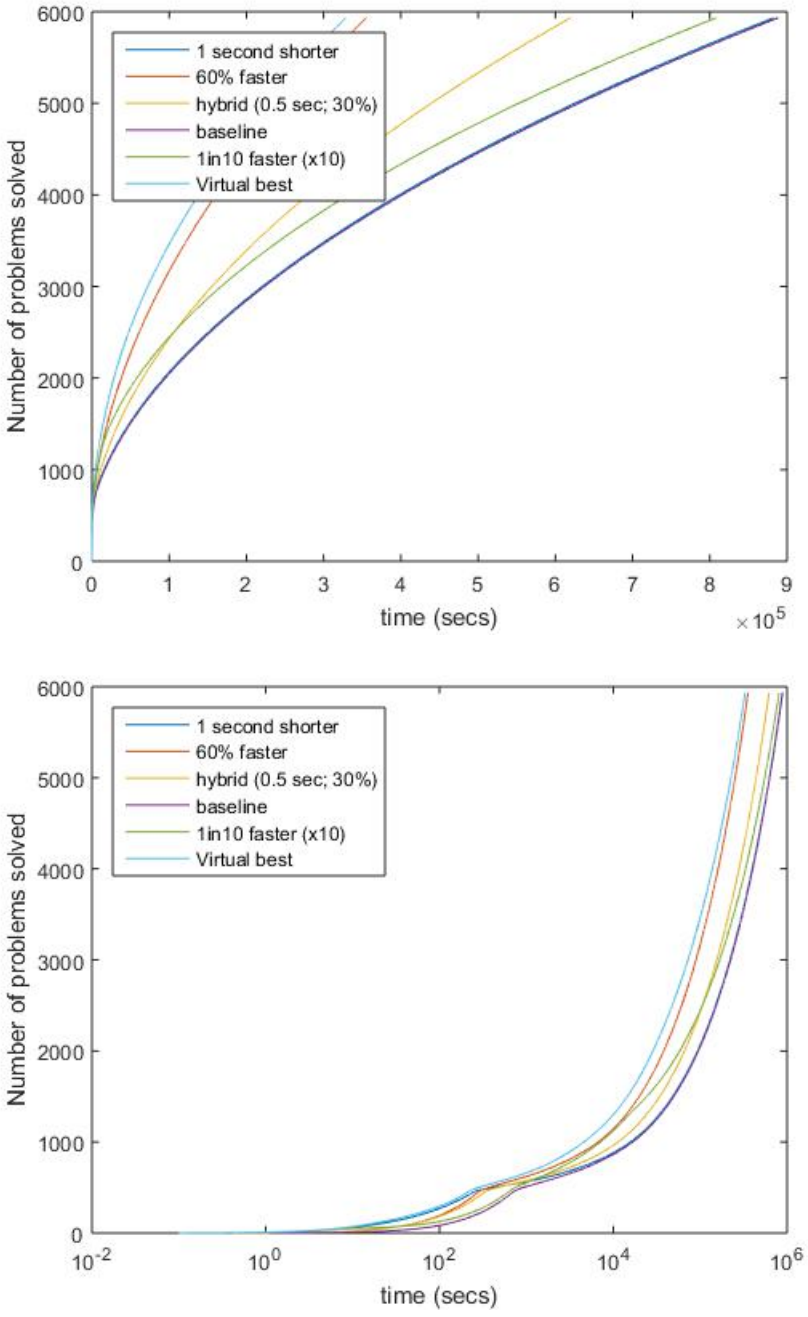
Figure 8: Data from Section 6.3



[2] M. Luby, A. Sinclair, and D. Zuckerman. Optimal Speedup of Las Vegas algorithms. *Information Processing Letters*, 47:173–180, 1993.

[3] L. Xu, F. Hutter, H.H. Hoos, and K. Leyton-Brown. SATzilla: Portfolio-based Algorithm Selection for SAT. *Journal of Artificial Intelligence Research*, 32:565–606, 2008.

Figure 9: Data from Section 6.4



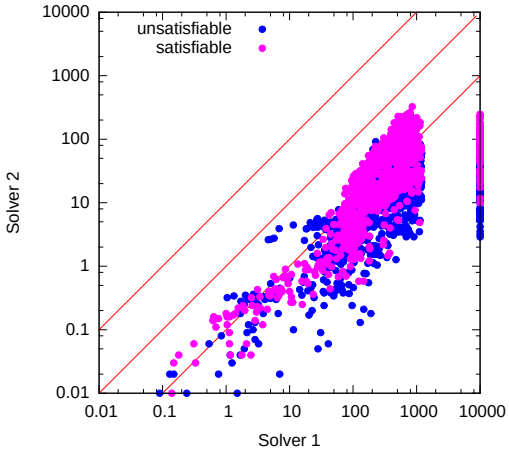


Figure 10: Scatter Plot Example.