Elliptic PDEs, 2nd Edition, Qing Han and FangHua Lin

Errata Sheet

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I found some possible misprints on Prof. Han and Prof. Lin's famous textbook on elliptic PDEs, it's not official. If you find any error in this file, please send me the email.

For the reader of first edition: The positions of these errata in the second edition are a little different from yours, but I think you can easily found them in ± 5 lines.

Page 17, line 6; add "$\eta \equiv 1$ on $B_{\frac{1}{2}}$" and change "$(1.1) \to (1.6)$".

Page 17, third expression; it's not a big deal, but I think we should change $\frac{1}{n} \to \frac{1}{2n}$, or any positive number less $\frac{1}{n}$.

Page 17, line -10; delete $\leq w(x_0)$.

Page 17, line -5; I don't think we need to assume a_{ij} is continuous !

Page 19, line -12; $3.1 \to 1.29$.

Page 22, line 10 and 12; the constant of unit n-dim sphere ω_n is missed in the sup-norm term.

Page 25, line 9; $2.4 \to 2.5$.

Page 26, line 16, some subscript is misprinted.

Page 29, line -3, In theorem 2.11, I think we should assume strongly that w is $C^2(\overline{\Omega})$. since we are going to apply the Hopf lemma, which relies on the fact that the zero-order coefficient is bounded, which is guaranteed if we know it's in $C(\overline{\Omega})$, that is, $Lw \in C(\overline{\Omega})$.

Page 32, line 9, by Theorem $2.5 \to 2.4$.

Page 40, line 4, change $\Omega \to \Gamma^+$ to make sure that $-a_{ij}D_{ij}u \geq 0$.

Page 40, line 9, $-a_{ij}D_{ij} \to -a_{ij}D_{ij}u$.

Page 40, line 18, $|b^n| \to |b|^n$.

Page 40, line -5, the second $f \not\equiv 0$ should be $f \equiv 0$.

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Page 41, line -8, $\int_{\mathbb{R}^n} \frac{d}{p}(1+|p|^2)^{\frac{n+2}{2}} = \omega_n \to \int_{\mathbb{R}^n}(1+|p|^2)^{-\frac{n+2}{2}}\,dp = \frac{\omega_n}{n}$.

Page 41, line -4, $\omega_n \to \frac{\omega_n}{n}$.

Page 43, line 2, $\int_\Omega |f|^n \to \int_\Omega |f|$.

Page 43, line 7, $\omega_n \to \frac{\omega_n}{n}$.

Page 56, line -2; multiply a constant c in front of the biggest brace.

Page 62, case 1 and so on, I think there is a gap on showing the function

$$\phi(r) := \int_{B_r(x_0)} |Du(y) - (Du)_{x_0,r}|^2\,dy,$$

is non-decreasing in r, I provide a proof in the second method and an alternative way to show the desired inequality holds in the first method :

The First Method:

Fix the reasonable radius R_0 and $\tau < 1$ as found in case 2-3, we find that, for any $R \in (\tau R_0, R_0]$ and for each $k = 0, 1, \cdots$

$$\begin{aligned}\phi(\tau^k R) &\le (\tau^k)^{n+2\alpha+\epsilon}\phi(R) + C(\tau^{k-1}R)^{n+2\alpha}\|f\|_q^2 \frac{1}{(1-\tau^\epsilon)}) \\ &\le (\tau^k R)^{n+2\alpha}\big(\phi(R)\frac{1}{R^{n+2\alpha}} + C\|f\|_q^2 \frac{1}{\tau^{n+2\alpha}(1-\tau^\epsilon)}\big).\end{aligned}$$

Since for each $\rho < R_0, \rho = \tau^k R$ for some $k = 0, 1, \cdots$ and $R \in (\tau R_0, R_0]$,

$$\phi(\rho) = \phi(\tau^k R) \le \rho^{n+2\alpha}\big(\phi(R)\frac{1}{(\tau R_0)^{n+2\alpha}} + C\|f\|_q^2 \frac{1}{\tau^{n+2\alpha}(1-\tau^\epsilon)}\big)$$

Note that $\phi(R) \le 4\|Du\|^2_{L^2(B_1(0))}$.

The Second Method:

I learn this from a lecture by Spencer Frei that the nondecreasing property can be proved by the following theorem:

Theorem 1. *If $f \in L^2$, then*

$$\int_{B(x_0,R)} |f - f_{x_0,R}|^2 = \inf_{a\in\mathbb{R}} \int_{B(x_0,R)} |f-a|^2$$

Proof. Let $F(a) := \int_{B(x_0,R)} |f-a|^2$. Then $F'(a) = 2\int_{B(x_0,R)} a - f$ with only one critical point $A = f_{x_0,R}$. By definition, F is convex and hence the critical point is a global minimizer by the following theorem. $\square$

Theorem 2. *Let $C \subset \mathbb{R}^n$ convex. Let $F : C \to \mathbb{R}$ has continuous first partial derivative. Then F is convex if and only if $f(x) + \nabla f(x)\cdot(y-x) \le f(y)$ for all $x, y \in C$.*

Proof. ($\Rightarrow$) By convexity,

$$\frac{f(x+\lambda(y-x))-f(x)}{\lambda} \leq f(y)-f(x).$$

Letting $\lambda \to 0$, we have the desired inequality.

($\Leftarrow$) This is proved by the following two inequalities:

$$f(x+\lambda(y-x))+\nabla f(x+\lambda(y-x))\cdot\lambda(y-x) \leq f(x)$$

$$f(x+\lambda(y-x))+\nabla f(x+\lambda(y-x))\cdot(1-\lambda)(x-y) \leq f(y)$$

□

Page 70, second line; I think it's

$$|A(k,R)| \leq \frac{1}{k}\int_{A(k,R)} u^+ \leq \frac{1}{k}\|u^+\|_{L^2}|A(k,R)|^{1/2}.$$

Page 72, line 14-15, (1) I don't know how to apply the dominate convergence theorem if we do NOT assume Φ' is bounded. (Note that the original assumption only promise it's locally bounded.)

(2) Instead assuming Φ' is bounded, we can assume u is bounded in Lemma 4.6 without any hurt in our applications (Theorem 4.11 and 4.15) since we have the local boundedness theorem in previous section and ϕ has compact support.

(3) I remark that the a.e. convergence is up to the subsequence (since $\Phi'_\epsilon \to \Phi'$ in $L^1([-k,k])$ for each $k \in \mathbb{N}$.)