Elliptic PDEs of 2nd Order, Gilbarg and Trudinger Errata Sheet

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I found a few possible misprints on Prof. Gilbarg and Prof. Trudinger's famous textbook on elliptic PDEs. It's not official. If you find any error in this file, please send me the email (d04221001@ntu.edu.tw). It's really helpful for me. Thanks!!

Page 23 last paragraph (i) and Page 103 second line, I think we should assume $u \in C^0(\overline{\Omega})$ and Ω is bounded to ensure the maximum is attained in the interior.

Page 112, line -3, $\beta \in \mathbb{R} \to \beta \leq 2$.

Page 120, (6.59), change one $v_i(x_0) \to v_j(x_0)$.

Page 170 line -12, I think we should take u smooth up to the boundary $\{x_n = 0\}$, that is, $u \in C^\infty(\overline{B_1 \cap \mathbb{R}^n_+}) \cap W^{k,p}(B_1 \cap \mathbb{R}^n_+)$. This is the remark to Theorem 7.9, page 155.

Page 179 and 181, or more geneally, for the theorems whose proof relies on weak maximum principle (Section 8.1) and Rellich compactness theorem (Section 7.10 and 7.12); I think we should assume Ω is bounded and $\partial\Omega$ is C^1 or Lipschitz to make the trace and trace-zero theorem holds (for example, to ensure $v = \max\{u - l, 0\} \in W_0^{1,2}(\Omega)$ in the last line of page 179.) By the way, the boundedness of domain is also necessary to the Rellich compactness theorem for general Sobolev space.

Page 224, (9.14). $\int_{\Gamma^+} \left(1 + \frac{|b|^n}{\mathscr{D}}\right) \to \left(1 + \int_{\Gamma^+} \frac{|b|^n}{\mathscr{D}}\right)$.

Page 255, line 7, Section 9.4 $\to$ 9.5.

Page 255, line -2, [TA 5] $\to$ [TA 4].

Page 263, Theorem 10.1 and hence to every proof which apply this theorem, for example, section 10.2; I think we should assume the domain Ω is bounded in the $Qu \geq Qv$ case since we

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are going to apply the small perturbation technique, that is, by considering $u + \epsilon e^{\alpha x_1}$, in the proof of weak maximum principle. I also think we should also assume (i') Q is locally strictly elliptic w.r.t. u or v instead of uniform ellipticity since we need to check $\frac{|b^i|}{\lambda}$ is locally bounded for some $i = 1, \cdots n$.

Page 264, line 15, Chapter 13 $\to$ Chapter 14.

Page 267, last line of Section 10.2, $C^0(\overline{\Omega}) \cap W^{2,n}_{\text{loc}}(\Omega) \to C^0(\overline{\Omega}) \cap C^1(\Omega) \cap W^{2,n}_{\text{loc}}(\Omega)$.

Page 277, line 5 $C^2(\Omega) \cap C^0(\overline{\Omega}) \to C^1(\Omega) \cap C^0(\overline{\Omega})$

Page 283, (11.9), I think we can use Theorem 10.1 instead of 10.3.

Page 283, (11.10), I think it should be $\sup_{\partial\Omega} |D_\nu u| \le \sigma K$, where ν is the outer normal, not $|Du|$. This is because of the difference quotient at $x_0 \in \partial\Omega$ is well-defined in $\nu(x_0)$ direction only. But for Step III, we did need the a-priori bound for $\sup_{\partial\Omega} |Du|$. Note that $u \in C^{2,\alpha}(\overline{\Omega})$ and hence Du can be defined at the boundary. My way is to analyze what ν is:

Since $C^{2,\alpha}(\overline{\Omega}) \ni u - \varphi = 0$ on $\partial\Omega$, $\nu = D(u-\varphi)/|D(u-\varphi)|$. Hence for each $x_0 \in \partial\Omega$,

$$
\begin{aligned}
|Du(x_0)| &\le |D(u-\varphi)(x_0)| + |D\varphi(x_0)| \le D(u-\varphi)(x_0) \cdot \nu(x_0) + \|D\varphi\|_\infty \\
&\le Du \cdot \nu(x_0) + 2\|D\varphi\|_\infty \le \sigma K + 2\|D\varphi\|_\infty.
\end{aligned}
$$

Page 287, the condition (iii) on Q_σ; Since there does not exist "norm" on the $C^\alpha(\overline{\Omega} \times \mathbb{R} \times \mathbb{R}^n)$ (see Rudin's Functional analysis, Section 1.44 and 1.46), it should be careful to interpret the continuity, that is, what is the topology imposed on the space ? One way is to interpret C^α as BC^α, the set of bounded Hölder function on $\Omega \times \mathbb{R} \times \mathbb{R}^n$. If one interpret in such way, some meaningful function such as $a^{ii}(x, z, p) = z$ (resp, the operator is div $(u\nabla u)$) can not be included.

My way to get rid of such situation is to rephrase (iii) as follows:

For each $M > 0, a^{ij}, b \in C^0(C^\alpha(\overline{\Omega \times (-M, M) \times (-M, M)^n}); [0, 1])$

I think one should check the proof for Theorem 11.8 still work.

Page 289, line 6, $C^{0,1}(\Omega) \to C^{0,1}(\overline{\Omega})$.

Page 289, 2 lines before Theorem 11.9, Theorem 10.1 $\to$ Theorem 10.7. We also note this theorem has stronger assumption on the regularity of F and u, but it's can be relaxed to the case $F \in C^2(\Omega \times \mathbb{R} \times \mathbb{R}^n)$ and $u \in C^{0,1}(\overline{\Omega})$. We present a proof mentioned in the second paragraph of Page 271:

Page 514, quasilinear elliptic equation 257ff $\to$ 259ff.