from the polynomial $\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3900$,

there are no real roots to the equation. using complex roots.

$$\lambda = -\frac{13}{4} - \frac{\sqrt{\frac{753}{4} + \frac{6807}{\sqrt[3]{\frac{440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}} + 2\sqrt[3]{\frac{440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}}}{2} - \frac{\sqrt{\frac{753}{2} - 2\sqrt[3]{\frac{440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}} + \frac{6905}{4\sqrt{\frac{753}{4} + \frac{6807}{\sqrt[3]{\frac{440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}} + 2\sqrt[3]{\frac{440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}}} - \frac{6807}{\sqrt[3]{\frac{440569 + 5\sqrt{17468281445}\,i}{4}}}}}{2}$$

$= -21.1246202553390136.$

Note: calculating for the complex roots was handled by a scientific calculator.

Finding the eigenvector for $\lambda_1 = -21.1246202553390136$

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix} =$$

$$\begin{bmatrix} 25.1246202553390136 & 8 & -1 & -2 \\ -2 & 12.1246202553390136 & -2 & -4 \\ 0 & 10 & 26.124620553390136 & -10 \\ -1 & -13 & -14 & 8.12462055390136 \end{bmatrix}$$

solving for the null space ($Ax=0$) was also handled by a scientific calculator as performing reduced row reduction manually is extremely error prone.

$\therefore$ The null space (eigenvector) of this matrix is

$$= \begin{bmatrix} -0.027109401495545 \\ 0.365498540062851 \\ 0.24287490257633 \\ 1 \end{bmatrix}$$

Solve the equation

$$-10\lambda + (5-\lambda)(-141\lambda + (4-\lambda)((-\lambda-15)(-\lambda-9)-52)-210) - (40-10\lambda)(2\lambda-30) + (40-10\lambda)(44\lambda+100) - 1950 = 0$$

using complex roots.

$$\lambda = -\frac{13}{4}\,\frac{\sqrt{\frac{753}{4} + \frac{6807}{\sqrt[3]{\frac{4440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}} + 2\sqrt[3]{\frac{4440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}}}{2} + \frac{\sqrt{\frac{753}{2} - 2\sqrt[3]{\frac{4440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}} + \frac{6905}{\sqrt{\frac{753}{4} + \frac{6807}{\sqrt[3]{\frac{4440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}} + 2\sqrt[3]{\frac{4440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}}} - \frac{6807}{\sqrt[3]{\frac{4440569}{4} + 5\sqrt{17468281445}\,i}}}}{2}$$

$$= -5.6040206079 6358$$

$\therefore$ the eigenvalue is $-5.6040\ldots\ldots$

To Find the eigenvector,

$\lambda = -5.60402060796358$

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix}$$

$$\begin{bmatrix} 9.60402060796358 & 8 & -1 & -2 \\ -2 & -3.39597939203642 & -2 & -4 \\ 0 & 10 & 10.60402060796358 & -10 \\ -1 & -13 & -14 & -7.39597939203642 \end{bmatrix}$$

The null space of this matrix is

$$\begin{bmatrix} -16.863828697994474 \\ 18.439325329054799 \\ -16.445955712268166 \end{bmatrix}$$

~~Therefore~~

Therefore:

The eigen ~~values~~

by multiply by 1

$$\begin{bmatrix} -16.863828697994474 \\ 18.439325329054799 \\ -16.445955712268166 \\ 1 \end{bmatrix}$$

Solve the equation:

$$-10\lambda + (5-\lambda)(-14\lambda + (4-\lambda)((-\lambda-13)(-\lambda-9)-52)-210) - (40-10\lambda)(21-30) + (40-10\lambda)(14\lambda+100) - 1950 = 0 \text{ for } \lambda.$$

$$\lambda = -\frac{13}{4} - \frac{\sqrt{\frac{753}{2} - 2\sqrt[3]{\frac{440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}} - \frac{6905}{4\sqrt{\frac{753}{4} + \frac{6807}{\sqrt[3]{\frac{440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}} + 2\sqrt[3]{\frac{440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}}} - \frac{6807}{\sqrt[3]{\frac{440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}}}}{2}$$

$$+ \frac{\sqrt{\frac{753}{4} + \frac{6807}{\sqrt[3]{\frac{440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}} + 2\sqrt[3]{\frac{440569}{4} + \frac{5\sqrt{17468281445}\,i}{4}}}}{2} \approx 2.674595970951094$$

$$\lambda_3 = 2.674595970951094$$

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix} = \begin{bmatrix} 1.325404029048906 & 8 & -1 & -2 \\ -2 & -11.674595977095109 & -2 & -4 \\ 0 & 10 & 2.35404029048906 & -10 \\ -1 & -13 & -14 & -15.67459597095109 \end{bmatrix}$$

The null space of this matrix is $\left\{ \begin{bmatrix} -8.49436150586954 6 \\ 1.4275057522560 19 \\ -1.838414946029271 \\ 1 \end{bmatrix} \right\}$

James Jok

Eigenvalue $\lambda_4 = ?$

$$\lambda = \frac{-\frac{13}{4} + \sqrt{\frac{753}{2} - 2\sqrt[3]{\frac{440569}{4} + \sqrt[5]{\frac{174682814445i}{4}}} - \frac{6905}{4\sqrt{\frac{753}{4} + \frac{6807}{\sqrt[3]{\frac{440569}{4} + 5\sqrt{174682814445i}}} + 2\sqrt[3]{\frac{440569}{4} + 5\sqrt{\frac{174682814445i}{4}}}}} - \frac{6807}{3\sqrt[3]{\frac{440569}{4} + 5\ 174682814445i}}} + \sqrt{\frac{733}{3} + \frac{6807}{3\sqrt{\frac{440569}{4} + 5\sqrt{\frac{174682814445i}{4}}}} + 2\sqrt[3]{\frac{44056}{4} + 5\sqrt{\frac{174682814445i}{4}}}}}{2}$$

$\therefore \lambda_4 \approx 11.054044892402621.$

Eigenvector ?

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix} = \begin{bmatrix} -7.054044892402621 & 8 & -1 & -2 \\ -2 & -20.054044892402621 & -2 & -4 \\ 0 & 10 & -6.054044892402621 & -10 \\ -1 & -13 & -14 & 24.054044892402621 \end{bmatrix}$$

The null space of this matrix is $\left\{ \begin{bmatrix} -0.07074435068439 \\ -0.023758192802241 \\ -1.69103171680636 4 \end{bmatrix} \right\}$ and this is the eigenvector four (4).

Summary of the eigen value and eigen vectors.

$\lambda_1 = -21.124620255390136$, multiplicity : 1, eigen vector:

$$\begin{bmatrix} -0.027109401495545 \\ 0.365498540062851 \\ 0.24287490257633 \\ 1 \end{bmatrix}$$

$\lambda_2 = -5.60402060796358$, multiplicity : 1, eigen vector:

$$\begin{bmatrix} -16.863828697994474 \\ 18.439325329054799 \\ -16.445955712268166 \\ 1 \end{bmatrix}$$

$\lambda_3 = 2.674595970951094$, multiplicity : 1, eigen vector:

$$\begin{bmatrix} -8.494361505869546 \\ 1.427505752256019 \\ -1.838414946029271 \\ 1 \end{bmatrix}$$

$\lambda_4 = 11.054044892402621$, multiplicity : 1, eigen vector:

$$\begin{bmatrix} -0.07074435068439 \\ -0.023758192802241 \\ -1.691031716806364 \\ 1 \end{bmatrix}$$