To Find the eigenvalues and eigenvectors of

$$\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

$A\vec{v} = \lambda\vec{v}$ where $\vec{v} \neq 0$

$(A - \lambda)\vec{v} = 0$

multiply Inverse (I) with $(A - \lambda)\vec{v} = 0$

$(A \cdot I - \lambda \cdot I)\vec{v} = 0$

$(A - \lambda I)\vec{v} = 0$

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$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & 10 \\ -1 & -13 & -14 & -13 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & 10 \\ -1 & -13 & -14 & -13 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix}$$

Subtract column 2 multiplied by $\frac{1}{2} - \frac{\lambda}{10}$ from Column 3: $C_3 = C_3 - \left(\frac{1}{2} - \frac{\lambda}{10}\right) C_2$

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix} =$$

$$\begin{bmatrix} 4-\lambda & 8 & \frac{4\lambda}{5} - 5 & \cancel{-2}\ -2 \\ -2 & -\lambda-9 & -\frac{\lambda^2}{10} - \frac{2\lambda}{5} + \frac{5}{2} & -4 \\ 0 & 10 & 0 & -10 \\ -1 & -13 & -\frac{13\lambda}{10} - \frac{15}{2} & -\lambda-13 \end{bmatrix}$$

Expand along row 3

$$\begin{bmatrix} 4-\lambda & 8 & \frac{4\lambda}{5}-5 & 6 \\ \cancel{-2} & \cancel{-\lambda-9} & & \\ -2 & -\lambda-9 & -\frac{\lambda^2}{10}-\frac{2\lambda}{5}+\frac{5}{2} & -\lambda-13 \\ -1 & 10 & 0 & 0 \\ -1 & -13 & -\frac{13\lambda}{10}-\frac{15}{2} & -\lambda-26 \end{bmatrix}$$

$$(0)(-1)^{3+1}\begin{bmatrix} 8 & \frac{4\lambda}{5}-5 & 6 \\ -\lambda-9 & \frac{\lambda^2}{10}-\frac{2\lambda}{5}+\frac{5}{2} & -\lambda-13 \\ \cancel{-13} & & \\ -13 & -\frac{13\lambda}{10}-\frac{15}{2} & -\lambda-26 \end{bmatrix} +$$

$$(10)(-3)^{3+2}\begin{bmatrix} 4-\lambda & \frac{4\lambda}{5}-5 & 6 \\ -2 & -\frac{\lambda^2}{10}-\frac{2\lambda}{5}+\frac{5}{2} & -\lambda-13 \\ -1 & -\frac{13\lambda}{10}-\frac{15}{2} & -\lambda-26 \end{bmatrix} +$$

$$(0)(-1)^{3+3}\begin{bmatrix} 4-\lambda & 8 & 6 \\ -2 & -\lambda-9 & -\lambda-13 \\ -1 & -13 & -\lambda-26 \end{bmatrix} +$$

$$0(-1)^{3+4}\begin{bmatrix} 4-\lambda & 8 & \frac{4\lambda}{5}-5 \\ -2 & -\lambda-9 & -\frac{\lambda^2}{10}-\frac{2\lambda}{5}+\frac{5}{2} \\ -1 & -13 & -\frac{13\lambda}{10}-\frac{15}{2} \end{bmatrix}$$

$$= -10\begin{vmatrix} 4-\lambda & \frac{4\lambda}{5}-5 & 6 \\ -2 & -\frac{\lambda^2}{10}-\frac{2\lambda}{5}+\frac{5}{2} & -\lambda-13 \\ -1 & -\frac{13\lambda}{10}-\frac{15}{2} & -\lambda-26 \end{vmatrix}$$

* Subtract column 3 multiplied by $\frac{2}{3} - \frac{\lambda}{6}$ from column 1: $C_1 = C_1 - \left(\frac{2}{3} - \frac{\lambda}{6}\right) C_3$.

$$\begin{vmatrix} 4 - \lambda & \frac{4\lambda}{5} - 5 & 6 \\ -2 & -\frac{\lambda^2}{10} - \frac{2\lambda}{5} + \frac{5}{2} & -\lambda - 13 \\ -1 & -\frac{13\lambda}{10} - \frac{15}{2} & -\lambda - 26 \end{vmatrix} =$$

$$\begin{vmatrix} 0 & \frac{4\lambda}{5} - 5 & 6 \\ -\frac{\lambda^2}{6} - \frac{3\lambda}{2} + \frac{20}{3} & -\frac{\lambda^2}{10} - \frac{2\lambda}{5} + \frac{5}{2} & -\lambda - 13 \\ -\frac{\lambda^2}{6} - \frac{11\lambda}{3} + \frac{49}{3} & -\frac{13\lambda}{10} - \frac{15}{2} & -\lambda - 26 \end{vmatrix}$$

* Subtract column 3 multiplied by $\frac{2\lambda}{15} - \frac{5}{6}$ from column 2: $C_2 = C_2 - \left(\frac{2\lambda}{15} - \frac{5}{6}\right) C_3$.

$$\begin{vmatrix} 0 & \frac{4\lambda}{5} - 5 & 6 \\ -\frac{\lambda^2}{6} - \frac{3\lambda}{2} + \frac{20}{3} & -\frac{\lambda^2}{10} - \frac{2\lambda}{5} + \frac{5}{2} & -\lambda - 13 \\ -\frac{\lambda^2}{6} - \frac{11\lambda}{3} + \frac{49}{3} & -\frac{13\lambda}{10} - \frac{15}{2} & -\lambda - 26 \end{vmatrix} =$$

$$\begin{vmatrix} 0 & 0 & 6 \\ -\frac{\lambda^2}{6} - \frac{3\lambda}{2} + \frac{20}{3} & \frac{(\lambda - 10)(\lambda + 25)}{30} & -\lambda - 13 \\ -\frac{\lambda^2}{6} - \frac{11\lambda}{3} + \frac{49}{3} & \frac{2\lambda^2}{15} + \frac{4\lambda}{3} - \frac{175}{6} & -\lambda - 26 \end{vmatrix}$$

Expand along row 1;

$$\begin{vmatrix} 0 & 0 & 6 \\ -\frac{\lambda^2}{6} - \frac{3\lambda}{2} + \frac{20}{3} & \frac{(\lambda-10)(\lambda+25)}{30} & -\lambda - 13 \\ -\frac{\lambda^2}{6} - \frac{11\lambda}{3} + \frac{49}{3} & \frac{2\lambda^2}{15} + \frac{4\lambda}{3} - \frac{175}{6} & -\lambda - 26 \end{vmatrix} =$$

$$(0)(-1)^{1+1} \begin{vmatrix} \frac{(\lambda-10)(\lambda+25)}{30} & -\lambda - 13 \\ \frac{2\lambda^2}{15} + \frac{4\lambda}{3} - \frac{175}{6} & -\lambda - 26 \end{vmatrix} +$$

$$(0)(-1)^{1+2} \begin{vmatrix} -\frac{\lambda^2}{6} - \frac{3\lambda}{2} + \frac{20}{3} & -\lambda - 13 \\ -\frac{\lambda^2}{6} - \frac{11\lambda}{3} + \frac{49}{3} & -\lambda - 26 \end{vmatrix} +$$

$$(6)(-1)^{1+3} \begin{vmatrix} -\frac{\lambda^2}{6} - \frac{3\lambda}{2} + \frac{20}{3} & \frac{(\lambda-10)(\lambda+25)}{30} \\ -\frac{\lambda^2}{6} - \frac{11\lambda}{3} + \frac{49}{3} & \frac{2\lambda^2}{15} + \frac{4\lambda}{3} - \frac{175}{6} \end{vmatrix}$$

$$= 6 \begin{vmatrix} -\frac{\lambda^2}{6} - \frac{3\lambda}{2} + \frac{20}{3} & \frac{(\lambda-10)(\lambda+25)}{30} \\ -\frac{\lambda^2}{6} - \frac{11\lambda}{3} + \frac{49}{3} & \frac{2\lambda^2}{15} + \frac{4\lambda}{3} - \frac{175}{6} \end{vmatrix}$$

The determinant of a 2×2 matrix is $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc.$

$$\begin{bmatrix} -\frac{\lambda^2}{6} - \frac{3\lambda}{2} + \frac{20}{3} & \frac{(\lambda-10)(\lambda+25)}{30} \\ -\frac{\lambda^2}{6} - \frac{11\lambda}{3} + \frac{49}{3} & \frac{2\lambda^2}{15} + \frac{4\lambda}{3} - \frac{175}{6} \end{bmatrix} = \left(-\frac{\lambda^2}{6} - \frac{3\lambda}{2} + \frac{20}{3}\right)\cdot\left(\frac{2\lambda^2}{15} + \frac{4\lambda}{3} - \frac{175}{6}\right)$$

$$-\left(\frac{(\lambda-10)(\lambda+25)}{30}\right)\cdot\left(\cancel{-\frac{\lambda^2}{60} - \frac{13\lambda^3}{60}} +\right)\left(-\frac{\lambda^2}{6} - \frac{11}{3}\lambda + \frac{49}{3}\right) =$$

$$-\frac{\lambda^4}{60} - \frac{13\lambda^3}{60} + \frac{73\lambda^2}{20} + \frac{167\lambda}{12} - \frac{175}{3}$$

Finally, $(-10)\cdot(6)\cdot\left(-\frac{\lambda^4}{60} - \frac{13\lambda^3}{60} + \frac{73\lambda^2}{20} + \frac{167\lambda}{12} - \frac{175}{3}\right) =$

$$\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$$

Expanded: The determinant of the obtained matrix is

$-10\lambda - 10(4-\lambda)(2\lambda-30) + 10(4-\lambda)(14\lambda+100) + (5-\lambda)(-14\lambda + (4-\lambda)$
$((-\lambda-9)-52)-210) - 1950 = 0$

Some roots to be found now by solving the above equation:

Solve the equation $-10\lambda - 10(4-\lambda)(2\lambda-30) + 10(4-\lambda)(14\lambda+100) +$
$(5-\lambda)(-14\lambda + (4-\lambda)((-\lambda-13)(-\lambda-9)-210) - 1950 = 0$

Now use the above equation find the Eigen values and later vectors.