INDENG 250 Homework 2

Due on Thursday 9/26/2024 11:59 pm

Submit your typed solution via bCourses - Assignment - HW2.

1 Problem 1 (20pt)

EOQ with Zero-Demand Sub-Cycles.

Consider below variant of the EOQ problem. The system has a nominal demand rate λ. WLOG, denote a cycle is the periods between two orders. Each time an order is placed, the demand rate is initially 0 for a fraction β of the cycle, and then the demand occurs at a rate of $\frac{\lambda}{1-\beta}$ for the rest duration of the cycle. One can show (you need not) that the total cycle length is still $\frac{Q}{\lambda}$, just like in the original EOQ model. The fixed cost of each order is denoted as K. The inventory level curve is presented as shown in Figure 1.

Calculate the optimal order quantity Q^*.

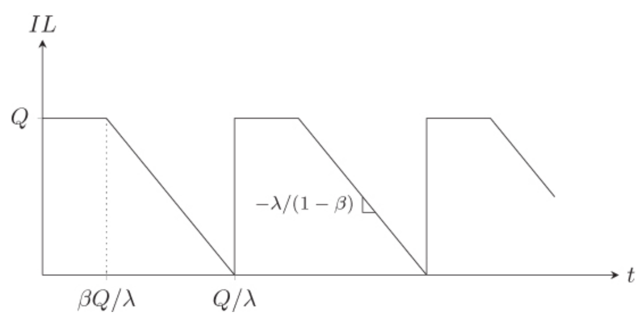

Figure 1: Inventory level curve.

2 Problem 2 (80pt)

Wagner-Whitin Model.

Consider below Wagner–Whitin problem where all notations are the same as lecture notes.

Given

- holding cost $h = 1$ per item per unit time,
- fixed setup cost $K = 20$ per order,
- in total $T = 4$ periods, and deterministic demand $(d_1, ..., d_4) = (25, 15, 15, 30)$.

The objective is to find the optimal ordering strategy with minimum operating cost.

2.1 MILP Approach

Define the variables and write out the MILP formulation.

2.2 Gurobi Implementation for MILP Formulation.

Install *gurobipy* on your local python environment. Solving your above MILP model with *python + gurobi*. You can take the sample codes provided in Wagner_Whitin_MILP.ipynb or Wagner_Whitin_MILP.py as reference.

Attach a screenshot of your output console.

Present the optimal solution, including which periods to place an order, how much to order in each of those periods, and the corresponding total optimal cost.

2.3 DP Approach

Apply the dynamic programming approach to solve this problem.

Basically, following the definition of

- θ_t: optimal cost if running from period t with 0 initial inventory,
- $s(t)$: the period that minimizes the cost in the induction equation,

and algorithmic steps in the lecture notes.

Write out values of θ_t and $s(t)$, $\forall t = 1, \ldots, 5$. Note that $t = 5$ is the boundary condition.

Present the optimal solution, including which periods to place an order, how much to order in each of those periods, and the corresponding total optimal cost.

2.4 Shortest Path Approach

Construct a Wagner-Whitin DP network for this problem (you can find an example in lecture note 3b_), and indicate the corresponding edge length (cost) in the graph. Attach the graph in your answer.

Solve the shortest path from node 1 to nodes 2, 3, 4, and 5, with Dijkstra's algorithm. Write out the values of shortest distance $A(\cdot)$ and the shortest path $B(\cdot)$ of a new node in each iteration (the same notation as used in the lecture note 4a_).

Convert your shortest path solution to the optimal ordering solution, including which periods to place an order, how much to order in each of those periods, and the corresponding total optimal cost.

Note that you are supposed to obtain same optimal solution by utilizing these three approaches and you can double check by yourself before submitting.