INDENG 250 Homework 4

Due on Sunday 11/17/2024 11:59 pm

Submit your typed solution via bCourses - Assignment - HW4.

1 Production Planning: Standard vs Delayed Product Differentiation (40pt)

An e-commerce furniture company called FlexiSpot offers a certain style of standing desk to a custom shape of desktop. The bulk of FlexiSpot sales are direct-to-customer; we will ignore its sales through third-party retailers. A long time ago, the company offered desks in four standard desktop shapes, including $55'' \times 28''$, $55'' \times 28''$ curved, $48'' \times 30''$, and $48'' \times 30''$ curved. Consider a new student targeted desk, which is sold only during the fall. FlexiSpot would have to place a one-time order from its supplier in Asia in April and the order will arrive in early July.

In the old system (i.e., standard differentiation), the company orders desks in the above 4 standard shapes seperately. For the desks manufactured in above 4 standard shapes, the manufacturing cost (you can treat this cost as an overall cost of replenishing one desk in its inventory storage) is \$100 each desk. The retail price of the desk is \$250. If there are leftover desks at the end of the season, the company can resell them in a lower-end market at a net selling price of \$75 (taking into account a much lower selling price and additional transportation costs).

Now consider a new ordering system (i.e., delayed differentiation). The manufacturing cost of one desk with uncut desktop is also \$100, on average, and this uncut desktop can be further cut to any of the above 4 shapes easily. The primary manufacturing steps were/are performed in Asia. For custom-shaped desktop, it is done in Wisconsin at a cost of \$5 per desk. Cutting can be done quickly and has a negligible effect on the delivery lead time to the final customer. Thus, we cut the desk only after we receive an order of a specific shape.

Management is preparing to place this year's order. Management would assume demand for each shape follows a discrete distribution, and the demand would be 400 with a 0.5 probability and 200 with a 0.5 probability. Management would also assume the demands for the four different shapes are statistically independent because they do not have a good way to estimate the correlation.

For below four questions, state clearly the notation and formulation of your optimization model.

1.1 Standard System Solution (10 pt)

Determine the optimal order quantities under the old system (ordering desktops in 4 standard shapes separately). Calculate the expected revenue under your decision.

1.2 Standard System SP Formulation (10 pt)

Formulate the two-stage stochastic programming model for (1.1). You don't have to solve this SP. If by coincidence you used two-stage SP formulation to solve for the optimal solution in (1.1) already, you can ignore this question. If you used a different approach for solving (1.1), please present your two-stage SP formulation here.

Hint: you can start with identifying the stochastic model parameters, constructing the set of scenarios S, and defining first-stage decisions and second-stage decisions.

Hint: you can think about whether your formulation is decomposable.

1.3 New System Solution (10 pt)

Determine the optimal order quantities under the new system (desks ordered uncut from Asia and cut "just-in-time" in Wisconsin). Calculate the expected revenue under your decision. Estimate how much money can be saved by using the new system.

1.4 New System SP Formulation (10 pt)

Formulate the two-stage stochastic programming model for (1.3). You don't have to solve this SP. If by coincidence you used two-stage SP formulation to solve for the optimal solution in (1.3) already, you can ignore this question. If you used a different approach for solving (1.3), please present your two-stage SP formulation here.

Hint: you can start with identifying the stochastic model parameters, constructing the set of scenarios S, and defining first-stage decisions and second-stage decisions.

2 Integrated Routing and Inventory Management (60pt)

The area of the continental US is approximately 3.1 million square miles, and there are approximately 18,000 automobile dealers who sell a total of about 14.4 million cars per year.

Consider you are operating one manufacturing site and several dealerships. Let's first develop a simplified version of vehicle delivery plan. You are transporting new vehicles from this manufacturing site (denoted as O) to your dealer locations (denoted as set I). Without loss of generality, we denote the distance between any two locations $i \in I \cup \{O\}$ and $j \in I \cup \{O\}$ as d_{ij}.

Each dealer works for 6 days per week and on average, each auto dealer sells approximately D number of cars per day. Let's consider a deterministic demand case and thus we need to satisfy all demand. The dealers have to pay holding cost h per night for each unsold car in the dealer location. (It is traditional for US automobile dealers to "own" the new vehicles for all or most of the time until they are sold.)

An auto carrier (open truck that carries automobiles on two levels) can hold at most q vehicles and costs about c per mile to operate, including labor while the truck is traveling. Consider each auto carrier only departs from the manufacturing site once every day. Each auto carrier can visit at most N dealers in one single trip as multiple cars may need to be unloaded to access the car(s) that are delivered at a location and any cars bound for later stops need to be reloaded. We assume that in each day, any single trip of one auto carrier can be done before the demand arrives in that day.

We decide to simply take one week (6 working days) as a operational cycle, and determine the best vehicle delivery plan to minimize the total transportation costs and inventory holding cost. Basically we assume there is no car in each dealership in the beginning of Day 1, and we expect to have no car by end of Day 6. The delivery plan means, in each day (Day 1, ..., Day 6) in a week, how many auto carriers are driving out from the manufacturing site, how many vehicles are loaded to them, what are the routes of them, and how many vehicles to unload in each stop of the route.

For the below two questions, you can ignore the other cost or restrictions if they are not mentioned above. Or you can also make your own assumptions over other cost and constraints, and present both your assumptions and your formulation accordingly.

2.1 Model Formulation (30 pt)

Formulate a Mixed Integer Linear Program (MILP) for solving this problem with general parameter and decision notations.

Hints:

- Without loss of generality, you can assume a fleet of auto carriers K for you to use and they can satisfy all possible delivery plans you can come up with.
- You can consider these decisions in your formulation (you can also define decision variables in another way):
  - $x_{ijkt} \in \{0, 1\}$, where 1 indicates that carrier k is moving from location i to location j in a direct route in day t. (You need to think about and specify that i and j are in which sets.)
  - $y_{ijkt} \geq 0$, for carriers k, if it is is moving from location i to location j in a direct route in day t, it unloads y_{ijkt} cars in site j. (You need to think about and specify that i and j are in which sets.)
  - $s_{it} \geq 0$, the inventory of dealer i by end of day t. That's the number of cars you need to pay for inventory holding cost. (You need to think about and specify that i is from which set.)
- You may consider these constraints in your formulation:
  - In each day, one auto carrier $k \in K$ starts from the manufacturing site and returns to the manufacturing site.
  - In each day, the demand in each dealership site must be satisfied by inventory in the beginning of the day and newly arrived cars during this day.
  - Each auto carrier can visit at most two dealers.
  - Each auto carrier can carry at most q cars each day.

2.2 Numerical Example (30 pt)

Consider below numerical example and solve it with Gurobi. Report the optimal solution, translate the optimal solution to the optimal delivery plan, and report the optimal objective value. Attach your code and results as screenshot in your solution.

- Consider two dealers with distance 100 miles from the manufacturing site and 20 miles from each other.
- Transportation cost of an auto carrier is \$10 per mile.
- Each auto carrier can move at most 8 cars each time.
- The holding cost is \$100 per car per day.

- The daily demand of each dealer is 6 cars.