INDENG 250   Homework 3

Due on Sunday 10/6/2024 11:59 pm

Submit your typed solution via bCourses - Assignment - HW3.

1 Problem 1 (20pt)

A clothing company sells ski jackets every winter but must decide in the summer how many jackets to produce. Each jacket costs \$65 to produce and ship, and sells for \$129 at retail stores. (For the sake of simplicity, assume the jacket is sold in a single store.) Customers who wish to buy this jacket but find it out of stock will buy a competitor's jacket; in addition to the lost revenue, the company also incurs a loss-of-goodwill cost of \$15 for each lost sale.

At the end of the winter, unsold jackets are sold to a discount clothing store for \$22 each.

(1.1 10 pt) Suppose that the demand for the ski jackets this winter will be distributed as a normal random variable with mean 900 and standard deviation 60. What is the optimal number of jackets to produce? (Need an integer solution.)

(1.2 10 pt) Now suppose that the demand is distributed as a Poisson random variable with mean 900. What is the optimal number of jackets to produce? (Need an integer solution.)

2 Problem 2 (20pt)

On day t, an electricity utility company must decide how much generation capacity to prepare for the electricity it will generate on day $t+1$. Each megawatt-hour (MWh) of capacity prepared costs the utility r. Let S_{t+1} be the generation capacity chosen on day t for generation on day $t+1$. The demand for day $t+1$, denoted D_{t+1}, is stochastic, with pdf $f(\cdot)$ and cdf $F(\cdot)$. D_{t+1} is not observed until day $t+1$, although for simplicity we will assume that the entire day's demand is revealed at the beginning of the day.

Once D_{t+1} is observed, the utility generates $\min\{D_{t+1}, S_{t+1}\}$ MWh of electricity. Each MWh of electricity actually generated incurs a cost of c per MWh (in addition to the cost r already incurred to prepare the capacity). If $D_{t+1} > S_{t+1}$, the utility must purchase electricity on the spot market to make up the difference. (The spot market is a marketplace in which the utility can purchase an unlimited quantity of electricity with no advance notice required.) The price per MWh of electricity purchased on the spot market is m, with $m > c + r$.

(2.1 10 pt) Derive the expression for S^*_{t+1}, the optimal number of MWh of capacity to prepare.

(2.2 10 pt) Suppose r = \$5/MWh, c = \$2/MWh, m = \$20/MWh, and $D_{t+1} \sim N(150, 20^2)$ MWh. What is S^*_{t+1}?

3 Problem 3 40 pt

A candy bar is deciding the inventory management plan, and they plan to implement a continuous review (s, Q) policy. The customers are fiercely loyal, so if the store is out of stock, they are willing to wait for their candy. (That is, demands are backordered, not lost.) However, each stockout costs

\$0.50 in lost profit and \$7 in loss of goodwill. It costs \$8 to place an order to the candy bar supplier. Each candy bar costs the store \$0.75. Holding costs are estimated to be 30% per year. The lead time is 1/12 year. Based on historical observations, the bar owners report that the demand during the lead time is normally distributed with a mean of 108.3 and a standard deviation of 43.3.

(3.1 10 pt) What are the optimal s and Q? (Using the approximation of holding cost as discussed in the lecture. You can stop once $s_{i+1} - s_i < 0.5$, and report s_{i+1} and Q_{i+1}).

(3.2 10 pt) What are the service levels the owner can provide under the policy determined in (3.1)?

(3.3 10 pt) Regardless of the backorder cost, the owners wish to ensure a type-1 service level of 98%. What are the optimal s and Q?

(3.4 10 pt) Regardless of the backorder cost, the owners wish to ensure a type-2 service level of 98%. What are the optimal s and Q?

4 Problem 4 (20pt)

EOQ with Fixed Batch Sizes - EOQ Sensitivity Analysis.

Suppose that in the EOQ model we can only order batches that are an integer multiple of some number Q_B; that is, we can order a batch of size $Q_B, 2Q_B, 3Q_B$, etc. Let $g(Q)$ be the EOQ cost function.

(4.1 10 pt) Prove that, for the optimal order quantity $\hat{Q} = mQ_B$,

$$\sqrt{\frac{m-1}{m}} \leq \frac{Q^*}{\hat{Q}} \leq \sqrt{\frac{m+1}{m}},$$

where $Q^* = \sqrt{\frac{2K\lambda}{h}}$ is the optimal (non-integer-multiple) EOQ quantity.

(4.2 10 pt) Suppose that $m \geq 2$ for $\hat{Q}$. Prove that $g(\hat{Q}) \leq 1.32 g(Q^*)$.

(4.3, Bonus, 10 pt) Prove that $g(\hat{Q}) \leq 1.06 g(Q^*)$. (still assuming $m \geq 2$).