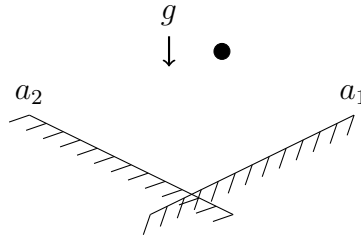


Homework 12: Time-stepping

24-760 Robot Dynamics & Analysis
Fall 2022

Name: _____

Problem 1) Falling Ball



Consider a point particle that can make plastic frictionless impact with several constraints. Assume the particle is mass 1 and gravity is 9.8. Let the constraints be $a_1(x, y) = 2y - x$ and $a_2(x, y) = 2y + x$.

1.1) Simulate the system with an event-based simulation in Matlab using `ode45` and an event function. You are encouraged to reuse any code that you would like from past homeworks.

1.2) Run a simulation from initial condition $(0.2, 1)$ for 3 seconds. What contact mode transitions occur and at what times? Discuss any problems that arise.

1.3) Now simulate the same system with a time-stepping simulation in Matlab using `solve`. At each timestep, pass the full system of equations (including the difference equations for \dot{q} and q as well as all inequality and complementarity constraints) to `solve` in order to find the next q , \dot{q} , and λ . The following Matlab resources may help:

<https://www.mathworks.com/help/symbolic/solve.html>

<https://www.mathworks.com/help/symbolic/solve-a-system-of-algebraic-equations.html>

1.4) Run a simulation from initial condition $(0.2, 1)$ for 3 seconds with time-step sizes h of 40, 20, and 10 ms. What contact mode transitions occur and at what times? In time-stepping the “contact mode” can be interpreted as those constraints who provided a positive force or impulse over the time step (as we cannot differentiate between these), though the system may not meet all of the requirements for the domain D_I in the hybrid system sense. Discuss any differences between these and the event-based simulation.

1.2) Run a simulation from initial condition (0.2,1) for 3 seconds. What contact mode transitions occur and at what times? Discuss any problems that arise.

Initial condition: [0.2 1]'.
Transition to contact mode {1} at time t = 0.42857 s.
Transition to contact mode {2} at time t = 0.53452 s.
Transition to contact mode {1} at time t = 1.1759 s.
Transition to contact mode {2} at time t = 1.5608 s.
Transition to contact mode {1} at time t = 1.7917 s.
Transition to contact mode {2} at time t = 1.9303 s.
Transition to contact mode {1} at time t = 2.0134 s.
Transition to contact mode {2} at time t = 2.0633 s.
Transition to contact mode {1} at time t = 2.0932 s.
Transition to contact mode {2} at time t = 2.1112 s.
Transition to contact mode {1} at time t = 2.1219 s.
Transition to contact mode {2} at time t = 2.1284 s.
Transition to contact mode {1} at time t = 2.1323 s.
Transition to contact mode {2} at time t = 2.1346 s.
Transition to contact mode {1} at time t = 2.136 s.
Transition to contact mode {2} at time t = 2.1368 s.
Transition to contact mode {1} at time t = 2.1373 s.
Transition to contact mode {2} at time t = 2.1376 s.
Transition to contact mode {1} at time t = 2.1378 s.
Transition to contact mode {2} at time t = 2.1379 s.
Transition to contact mode {1} at time t = 2.138 s.
Final time reached

- There was a problem @ the end of the sim where the ball stopped transitioning to b/tw contacts and phased through {2} to fall off screen.

1.4) Run a simulation from initial condition $(0.2, 1)$ for 3 seconds with time-step sizes h of 40, 20, and 10 ms. What contact mode transitions occur and at what times? In time-stepping the "contact mode" can be interpreted as those constraints who provided a positive force or impulse over the time step (as we cannot differentiate between these), though the system may not meet all of the requirements for the domain D_I in the hybrid system sense. Discuss any differences between these and the event-based simulation.

$h = 40 \text{ ms}$:

Initialize in mode $\{0\}$.
Transition from mode $\{0\}$ to mode $\{2\}$ at $t = 0.4$.
Transition from mode $\{2\}$ to mode $\{1 \ 2\}$ at $t = 0.48$.
Transition from mode $\{1 \ 2\}$ to mode $\{1\}$ at $t = 0.52$.
Transition from mode $\{1\}$ to mode $\{2\}$ at $t = 1$.
Transition from mode $\{2\}$ to mode $\{1 \ 2\}$ at $t = 1.16$.
Terminate in mode $\{1 \ 2\}$ at $t = 3$.

$h = 20 \text{ ms}$:

Initialize in mode $\{0\}$.
Transition from mode $\{0\}$ to mode $\{2\}$ at $t = 0.4$.
Transition from mode $\{2\}$ to mode $\{1\}$ at $t = 0.52$.
Transition from mode $\{1\}$ to mode $\{1 \ 2\}$ at $t = 1.1$.
Transition from mode $\{1 \ 2\}$ to mode $\{2\}$ at $t = 1.12$.
Transition from mode $\{2\}$ to mode $\{1 \ 2\}$ at $t = 1.42$.
Transition from mode $\{1 \ 2\}$ to mode $\{1\}$ at $t = 1.44$.
Transition from mode $\{1\}$ to mode $\{1 \ 2\}$ at $t = 1.54$.
Terminate in mode $\{1 \ 2\}$ at $t = 3$.

$h = 10 \text{ ms}$:

Initialize in mode $\{0\}$.
Transition from mode $\{0\}$ to mode $\{2\}$ at $t = 0.42$.
Transition from mode $\{2\}$ to mode $\{1\}$ at $t = 0.52$.
Transition from mode $\{1\}$ to mode $\{2\}$ at $t = 1.12$.
Transition from mode $\{2\}$ to mode $\{1\}$ at $t = 1.45$.
Transition from mode $\{1\}$ to mode $\{1 \ 2\}$ at $t = 1.62$.
Transition from mode $\{1 \ 2\}$ to mode $\{2\}$ at $t = 1.63$.
Transition from mode $\{2\}$ to mode $\{1 \ 2\}$ at $t = 1.69$.
Transition from mode $\{1 \ 2\}$ to mode $\{1\}$ at $t = 1.7$.
Transition from mode $\{1\}$ to mode $\{1 \ 2\}$ at $t = 1.71$.
Terminate in mode $\{1 \ 2\}$ at $t = 3$.

- The event-based simulation has a lot more contact transitions & never transitions to $\{1, 2\}$ as opposed to the time stepping sim. The event based sim also fails to end w/ the ball @ both constraints which violates the system constraints. The time stepping sim does not violate any of the constraints @ any point.