Assignment 6

James Lawton
Physics Department, Virginia Tech,
Blacksburg, Virginia 24061, USA

Abstract: Using the Relaxation, Crank-Nicholson, Euler, and Monte Carlo Methods in order to solve different equations.

I. PROBLEM 1

Here the Relaxation method is used to plot the potential of the Poisson equation:

$$\nabla^2\phi(x,y) = -\rho(x,y)/\epsilon_0 \quad (1)$$

for the following two sets of conditions:

(a)

$\rho(x,y) = 0$,

$\phi(0,y) = \phi(L_x,y) = \phi(x,0) = 0$,

$\phi(x,L_y) = 1V, L_x = 1m$,

$L_y = 1.5m$

(b)

$\rho(x,y)/\epsilon_0 = 1V/m^2$,

$\phi(0,y) = \phi(L_x,y) = \phi(x,0) = \phi(x,L_y) = 0$,

$L_x = L_y = 1m$

Then each scenario is plotted in 3D with these two starting conditions:

(a)

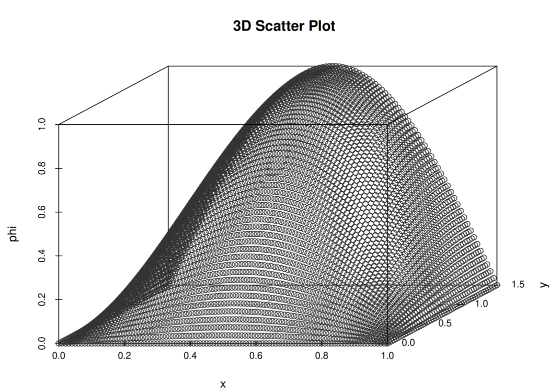

FIG. 1. Condition (a) Initial

(b)

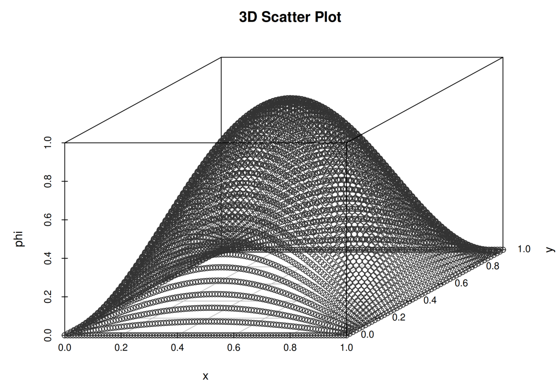

FIG. 2. Condition (b) Initial

After running the algorithm we get the following results

(a)

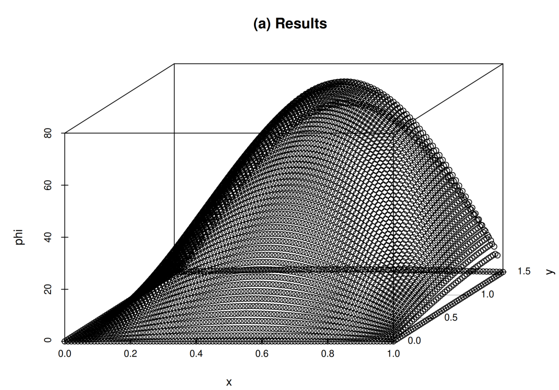

FIG. 3. Condition (a) Results

(b)

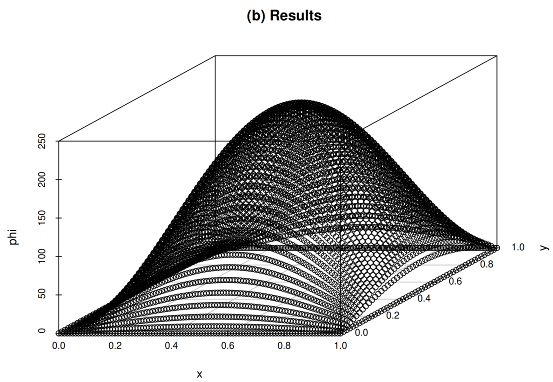

FIG. 4. Condition (b) Results

We can see that in both scenarios the amplitude of the plot grew, with the plot (a) growing to a peak of 80 with a sharp drop back to zero, and plot (b) growing to a peak of 250 with relatively similar proportions.

II. PROBLEM 2

Here we look to model a diffusion equation for a rod of nuclear waste as depicted in the diagram below:

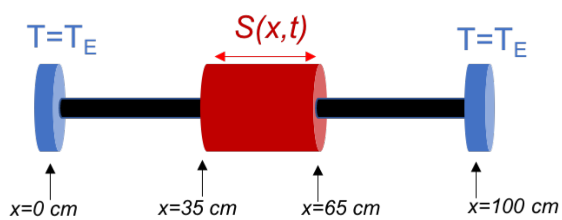

FIG. 5. Nuclear waste diagram

The following equation models the temperature diffusion across the horizontal axis:

$$\frac{1}{\kappa}\frac{\partial T(x,t)}{\partial t} - \frac{\partial^2 T(x,t)}{\partial^x x} = S(x,t) \tag{2}$$

The following boundary conditions will be used reflecting the diagram:
$S(x,t) = \frac{T_0}{a^2} e^{-t/\tau_0}$, for $x > 35$cm, and $x < 65$cm, but $S(x,t) = 0$ otherwise

A. The Crank-Nicholson Method

Here is the solution having used the Crank-Nicholson method using the distance across rather than the distance from the center:

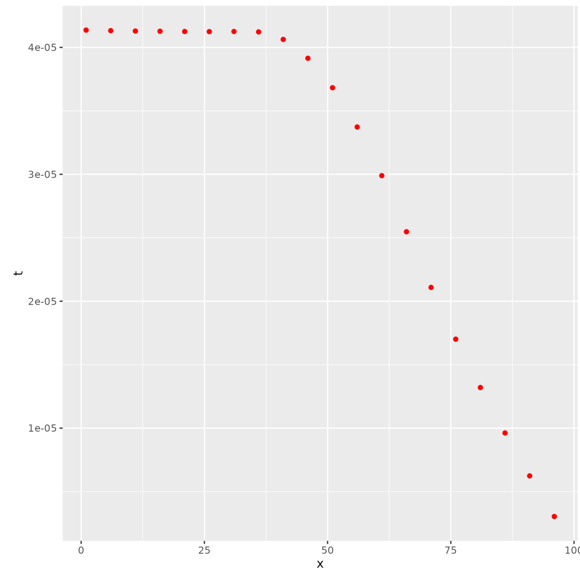

FIG. 6. Temperature of nuclear rod across x

We can see that this is not the expected output as it should be symmetrical across $x = 50$ with both ends tapering off towards zero. This graph still looks like it's measuring the temperature relative to the radius similar to a bell curve. The modifications made to the java file were not effective enough in changing it to plot the x rather than the radius.

Another unexpected result is the very small values obtained with all the values being within the magnitude of $4.5 * 10^{-5}$

III. PROBLEM 3

We will calculate the following integral using the Monte-Carlo Method using the Metropolis Algorithm:

$$S = \int_{-\infty}^{\infty} e^{-r^2/2}(xyz)^2 dr \tag{3}$$

Here are the solutions to the integral plotted over the number of Monte-Carlo steps:

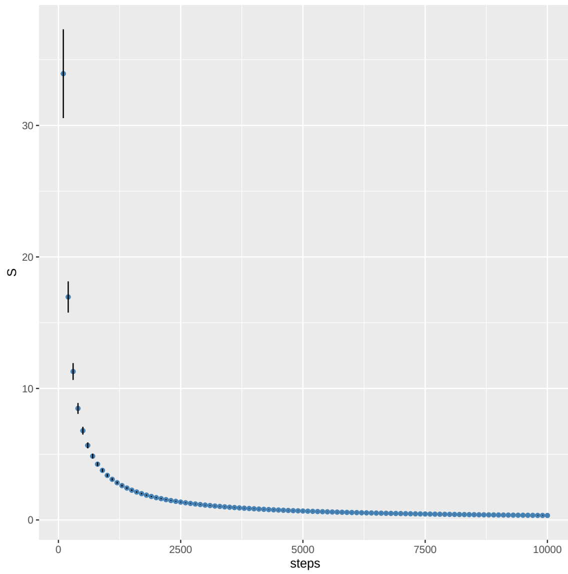

FIG. 7. Integral solved by Monte-Carlo plotted against number of steps

We can observe that as the number of steps increases, the integral approaches a single value, presumably the correct solution. The error bars also get closer and closer to zero.

[1] T. Pang, *Introduction to Computational Physics*, Cambridge University Press (2006).