MSCI

Barra Optimizer 9.2 User Guide

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1 About This Guide

Barra Optimizer is a software package that handles a wide variety of portfolio construction and optimization problems. Barra Open Optimizer is the stand-alone version of the same solver (Barra Optimizer) available in Barra PortfolioManager, BarraOne, and Barra Aegis. Barra Open Optimizer provides C++, Java, C#, R, and Python APIs to help you to develop your applications using C++, Java, C#, MATLAB, Python, and R.

The organization of this guide is as follows:

- Chapters 1-4 explain the various types of portfolio optimizations that Barra Optimizer can handle and includes the following information about Barra Optimizer:
  - Brief problem definitions and high-level descriptions of the features and functionalities
  - Insightful mathematical formulations and illustrative examples
- Chapter 5 provides general API information for programmers who would like to develop advanced portfolio optimization applications with ease. It also describes workflow of setting up optimization, as well as input/output information to help users take advantage of the powerful features offered by Barra Optimizer
- Chapters 6-8 give additional specific information about Barra Optimizer's MATLAB, R, and Python interfaces
- Chapter 9 describes how to use the Barra Optimizer command line executable
- Chapter 10 contains many of the frequently asked questions about Barra Optimizer from users
- Appendix B summarizes the availability of various feature combinations
- Other appendixes contain versatile information that the users may find useful

The problem definitions and mathematical formulations in this guide can be used as a reference when either using the Barra Open Optimizer library directly or using Barra Optimizer via Barra's portfolio and risk management products. However, the updated API information here only applies to the standalone Barra Open Optimizer 9.1.

Note: The actual implementation in a given product may also be more or less restrictive than what is described here. If you are interested in the theoretical background of optimization, you may find references listed at the end of this guide.

Typographic Conventions

Throughout this guide, vectors and matrices are in bold fonts, whereas scalar variables are in italicized fonts.

2 Portfolio Optimization with Barra Optimizer

Optimization is the process of locating the best possible answer given a model of your goals (objectives or utility) and the rules as well as restrictions you need to follow (constraints).

Creating an optimization involves the following three important steps:

- Step 1: Assess the goals and restrictions for your portfolio
- Step 2: Quantify and model the goals and restrictions
- Step 3: Formulate Step 2 into objectives and constraints for use in an optimizer

Step 1 is specific to the portfolio and investor. Step 2 is a key component of Barra's modeling offerings, and is discussed extensively on the MSCI Client Support Site. This guide discusses the third step in creating an optimization problem.

Once the optimization problem has been created, it needs to be run and then, analyzed.

2.1 Introducing Barra Optimizer

Barra Optimizer is a software library designed to solve a wide variety of portfolio optimization problems. The goal of optimization is to construct an optimal portfolio, balancing various competing objectives (such as maximizing return, minimizing risk, etc.) while taking into account specified constraints. It is specially designed to create portfolios and trades that maximize an objective function subject to a set of practical trading or investment constraints. The objective function consists of optional return, risk, transaction cost, and other terms. Utility function is an alternative name for the objective function. Barra Optimizer is an integral part of many MSCI products and services.

Barra Optimizer incorporates proprietary solvers developed in-house, resulting from over three decades of dedicated research at MSCI in the area of financial optimization. It provides fast and reliable results for well-defined, convex portfolio construction and rebalancing cases. It also applies innovative, high-quality heuristic techniques to certain ill-behaved, complex portfolio optimization problems. It takes advantage of the special structure of multi-factor risk models employed by many portfolio managers.

Studies show that Barra Optimizer can handle 30,000 assets and more than 4,000 constraints. Its only capacity limitation is the size of available computing memory. Barra Optimizer currently supports the following types of problems:

- Standard Mean-Variance Optimization
- Maximizing the Sharpe Ratio or Information Ratio
- Threshold and Cardinality Optimization
- Risk Parity Portfolio Construction
- Risk-Target or Return-Target Optimization
- Risk-Constrained Optimization
- Portfolio Construction with Diversification Control

- Long/Short (Hedge) Optimization
- Tax-Aware Optimization
- Expected Shortfall Optimization
- Optimal Roundlotting
- 5/10/40 Rules
- Parametric Optimization / Efficient Frontiers
- Multi-Period Optimization
- (General) Multi-Account Optimization
- Multi-Account Tax-Aware Optimization

However, a particular release of Barra Optimizer (either stand-alone or in a particular Barra product, such as BarraOne™) may offer only a subset of the full capabilities.

The easiest way to use Barra Optimizer is through Barra's portfolio and risk management products, such as Barra Aegis, BarraOne, and Barra PortfolioManager. These applications automatically package user inputs into the formats required by Barra Optimizer.

The standalone Barra Open Optimizer, on the other hand, accommodates alternative risk models and offers a much wider range of features and functionalities. It also affords sophisticated users more control over the selection of internal parameters, which makes the product more flexible and powerful.

2.2 Key Features

The following list highlights some of the most prominent features of Barra Optimizer:

- Consists of a fast, efficient, and reliable convex solver for mean-variance optimization
- Employs innovative heuristics to tackle non-convex, non-linear, or discrete problems
- Offers maximizing Sharpe ratio or information ratio as alternative objective functions
- Supports various risk models
- Facilitates multi-criteria optimization by allowing different multipliers for different terms in the objective function
- Supplies two separate risk aversion parameters for the common factor risk and specific risk
- Allows up to two risk models, multiple risk terms, and multiple benchmarks in a given problem
- Allows the benchmarks or covariance matrices in the objective function to be different from those in the constraints
- Accepts any separable, convex, piecewise-linear transaction cost function, both in the objective function and in the constraints
- Accommodates nonlinear and fixed transaction costs in the objective function

- Incorporates residual alpha as an optional term in the objective function
- Assists in tilting portfolios towards specific targets through the use of various penalty functions, including:
  - Symmetric penalties
  - Asymmetric penalties
  - Quadratic penalties
  - Linear-only penalties
- Supports a variety of constraint types and combinations of constraint types:
  - Bounds on assets, factors, or constraint slacks
  - User-defined general linear constraints
  - Beta constraint
  - 5/10/40 rules
  - Turnover upper limit
  - Transaction cost upper limit
  - Maximum or minimum number of assets
  - Maximum or minimum number of trades
  - Minimum holding or transaction size level
  - Risk constraints
  - Leverage (Hedge) constraints
  - Round lot constraints
  - Portfolio concentration limit
  - Total active weights constraint
  - General piecewise-linear constraints
  -
- Affords more flexible user control by offering soft constraints, constraint hierarchy, and an optimality tolerance multiplier
- Supports roundlotting either during the optimization process or post optimization
- Provides comprehensive and flexible modeling choices in By-Side Optimization:
  - Various types of leverage constraints
  - Leverage-by-side constraints
  - Turnover-by-side constraints
  - Cardinality-by-side
  - Modeling of short rebates or costs
  - Threshold-by-side
  - Various risk constraints
  - Risk-target optimization
- Provides sophisticated tax-aware optimization consideration:
  - Supports different tax-rule and capital-gain groups
  - Grants a choice of either one or two tax rates within each tax-rule group
  - Adapts to HIFO, LIFO, or FIFO trading rules
  - Allows targeting on or netting of different types of gains and losses
  - Affords control over the bounds of net as well as gross gains or losses
  - Offers three wash sales handling options

  - Supports the combination with cardinality, threshold, leverage and portfolio-level risk constraints
- Generates efficient frontiers for a range of return, risk, turnover limit, tax, or other constraint bounds at the user's request
- Controls portfolio risk or active risk (i.e., tracking error) via a variety of flexible mechanisms:
  - Upper and lower bounds on portfolio-level total risk, specific risk or factor risk
  - Upper and lower bounds on portfolio-level risk contribution from specific risk or factor risk
  - Upper and lower bounds on risk or risk contribution from a subgroup of assets or factors
  - Risk-by-asset or risk-contribution-by-asset constraints
  - Risk parity constraint
- Supports minimizing or constraining expected shortfall
- Showcases a novel mean-variance approach to Multi-Period Optimization for look-ahead portfolio construction and trade scheduling
- Offers two approaches to Multi-Account Optimization—Total Welfare Maximization and Nash Equilibrium—for dealing with joint market-impact transaction costs and cross-account constraints. Multi-Account Tax-Aware Optimization is also supported.

3 Basic Optimization Concepts

This section first presents the traditional mean-variance portfolio optimization problem and discusses its classifications. Then, it describes the individual terms in the objective function (also known as the utility function) as well as the constraint categories. In addition, it describes the basic settings and controls that you can view or edit in the Barra Optimizer workspace.

3.1 Portfolio Optimization Definition

3.1.1 Problem Formulation

From the user's perspective, a portfolio optimization problem in Barra Optimizer can be represented as:

Maximize:

$$\text{Objective Function} = \text{Return Terms} - \text{Risk Terms} - \text{Transaction Cost} - \text{Other Terms}$$

Subject to:

$$\text{Standard Constraints} \begin{cases} \text{Portfolio Balance Constraint} \\ \text{General Linear Constraints} \\ \text{Factor Constraints} \\ \text{Beta Constraint} \\ \text{(Convex) Turnover Limit Constraints} \\ \text{(Convex) Pieewise-Linear Transaction Cost Limit Constraints} \\ \text{General Convex Piecewise Linear Constraints} \\ \text{Bounds on Asset Weights and Factor Exposures} \end{cases}$$

$$\text{Special Constraints} \begin{cases} \text{Paring Constraints} \\ \text{Round Lot Constraints} \\ \text{5/10/40 Constraints} \\ \text{Leverage (Hedge) Constraints} \\ \text{Turnover-by-Side Constraints} \\ \text{Non-Convex Pieewise-Linear Transaction Cost Limit Constraints} \\ \text{General Non-Convex Piecewise Linear Constraints} \\ \text{Tax-Related Constraints} \\ \text{Risk Constraints} \\ \ldots \end{cases}$$

Among all terms in the objective function, the return ones are linear, the risk ones are quadratic, and the transaction cost ones can be pure linear, piecewise-linear, quadratic, fixed per trade, or even special convex nonlinear. The additional terms include the capital-gain tax cost, residual alpha, and so on, which may be complicated and sometimes highly non-linear depending on the additional constraints that define them. Most terms allow a multiplier in the front to adjust its impact in the objective function. All terms are optional.

Similarly, all constraints are optional except the Portfolio Balance Constraint (see Section 3.3). The standard constraints consist of only linear or convex piecewise-linear constraints. The special constraints are more complicated, usually discrete, or nonlinear in nature.

Mathematically, a typical mean-variance portfolio optimization can be formulated as follows:

$$\underset{\mathbf{h}}{\text{Maximize}}: \quad f(\mathbf{h}) = \lambda_r \mathbf{r}^{\mathrm{T}}\mathbf{h} - 100(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}(\lambda_D \mathbf{D} + \lambda_F \mathbf{X}\mathbf{F}\mathbf{X}^{\mathrm{T}})(\mathbf{h}-\mathbf{h}_B) - \lambda_{Tc} Tc(\mathbf{h},\mathbf{h}_0) - \lambda_{Pen} Pen(\mathbf{s},\mathbf{w}) + \mathbf{a}^{\mathrm{T}}\mathbf{h} - g(\mathbf{h}) \qquad (1)$$

Subject to:

$\sum_{i \neq futures} h_i = \sum_{i \neq futures} h_{0i} + CF/BV$	Portfolio Balance Constraint	(2)
$\mathbf{l}_h \leq \mathbf{h} \leq \mathbf{u}_h$	Asset ranges	(3)
$\mathbf{s} = \mathbf{A}\mathbf{h}$	Definitional Constraints for General Slack Variables	(4)
$\mathbf{l}_s \leq \mathbf{s} \leq \mathbf{u}_s$	Ranges for Slack Variables	(5)
$\mathbf{w} = \mathbf{X}^{\mathrm{T}}\mathbf{h}$	Definitional Constraints for Factor Slack Variables	(6)
$\mathbf{l}_X \leq \mathbf{w} \leq \mathbf{u}_X$	Factor Exposure Ranges	(7)
$l_\beta \leq \beta(\mathbf{h}) \leq u_\beta$	Beta Constraint	(8)
$To(\mathbf{h},\mathbf{h}_0) \leq u_{To}$	Turnover Constraint	(9)
$Tc(\mathbf{h},\mathbf{h}_0) \leq u_{Tc}$	Transaction Cost Constraint	(10)
$L_p^i \leq P^i(\mathbf{h},\mathbf{h}_0) \leq U_p^i, \quad i = 1, 2, \cdots, n_p$	General Piecewise-Linear Constraints	(11)
$\mathbf{l}_{Con} \leq Con(\mathbf{h}) \leq \mathbf{u}_{Con}$	Additional Constraints	(12)

where,

λ_F	Common factor risk aversion	
λ_D	Specific risk aversion	
λ_r	Return multiplier	
λ_{TC}	Transaction cost multiplier	
λ_{pen}	Penalty multiplier	
$\mathbf{h}$	Portfolio holding weights	(nx1)
$\mathbf{h}_0$	Initial portfolio weights	(nx1)
$\mathbf{h}_B$	Benchmark holding weights	(nx1)

$\mathbf{r}$	Alpha or expected asset return	(nx1)
$\mathbf{w}$	Factor exposure slacks	(kx1)
$\mathbf{A}$	Coefficient matrix for general constraints	(mxn)
$\mathbf{D}$	Specific covariance matrix	(nxn)
$\mathbf{X}$	Factor exposures	(nxk)
$\mathbf{F}$	Factor covariance matrix	(kxk)
$\mathbf{l}$	Lower bounds, per constraint	
$\mathbf{u}$	Upper bounds, per constraint	
$Pen(\mathbf{s},\mathbf{w})$	Penalty functions on constraint and factor slacks	
$Tc(\mathbf{h},\mathbf{h}_0)$	Transaction cost function	
$To(\mathbf{h},\mathbf{h}_0)$	Turnover function	
$\mathbf{a}$	coefficient vector of the extra linear term	(nx1)
$g(\mathbf{h})$	Additional non-standard objective terms	
$Con(\mathbf{h})$	Additional non-standard constraints	
$P^i(\mathbf{h},\mathbf{h}_0)$	The i^{th} general piecewise-linear function	
n	Number of assets in investment universe	
m	Number of general constraints	
k	Number of factors	
n_p	Number of general piecewise-linear constraints	

Notes:

- All the above notations are used throughout this guide, unless otherwise noted.
- All the constraints are optional, except for the portfolio balance constraint, which is required and automatically created.
- Unless otherwise specified, all definitions will be applied to the assets in the investment universe only.
- Currently, $g(\mathbf{h})$ is only supported in the form of a power function, although any differentiable function may be considered in the future.

Some of the other constraints (for example, threshold, cardinality, and long/short leverage constraints, etc.) that are referenced as non-standard constraints are described later in this document.

3.1.2 Problem Classification

We often distinguish a long-short optimization from a standard one. Typically, Standard (Portfolio) Optimization consists of only the standard constraints. For information about standard constraints, see the Portfolio Optimization Definition section.

The presence of any special constraints would render the optimization non-standard, or a special type. On many occasions, we also classify optimization problems into convex and non-convex categories. It should be noted that standard optimization might not be convex. Similarly, convex optimization may not be standard. We also want to emphasize that Barra Optimizer cannot guarantee a global optimal solution in the case of non-convex problems.

A convex portfolio optimization problem enjoys a special property—any convex combination of two feasible portfolios will remain a feasible portfolio. Convexity is desirable because it makes the search for an optimal solution easy and predictable. Convex problems are well-behaved, can be solved with less difficulty, and are guaranteed to produce a global optimal portfolio, if one exists. Non-convex problems are usually very complicated mathematically and more challenging computationally.

All the standard constraints are convex, and many special constraints are non-convex. Convex functions, convex constraints, and convex problems are separate yet related concepts. To qualify an optimization problem as convex, not only must all constraints be convex, the objective function must also be convex if it is a minimization problem or concave if it is a maximization problem.

Because of their complexity and difficulty, most non-convex portfolio optimization problems are tackled by heuristic procedures. Even though Barra Optimizer strives to employ innovative, intuitive, efficient, and effective heuristics, and the solution quality is usually good in many cases, there is no guarantee that the heuristic solution it generates for a particular non-convex case is indeed global optimal.

3.2 Units

Barra Optimizer uses *fractions* rather than *percentages* for units of input. This means that all input, including the alphas, variances, covariances, as well as holdings, transaction costs, and tax costs are entered as decimals relative to the base value used to calculate portfolio weights. For example, an alpha of 2% is input as 0.02. Similarly, if the specific variance of an asset is $(30\%)^2$, then it is entered as 0.09. Raw alpha scores need to be scaled or normalized before inputting into the Barra Optimizer.

Everything associated with the holdings (for example, upper and lower bounds on assets, coefficients of constraints, and so on) should be scaled accordingly.

Transaction costs and tax costs should be expressed as the fraction of their dollar amount to the base value. An exception to this is the breakpoints for piecewise-linear transaction costs, which are entered in high-level APIs as dollar amounts.

3.3 Portfolio Balance Constraint

The portfolio balance constraint is an important constraint that is automatically enforced by Barra Optimizer. Portfolio optimization will not change the value of the initial portfolio plus the cash flow value:

$$PV = PV_0 + CF \tag{13}$$

where,

PV	the optimal portfolio value
PV_0	the initial portfolio value
CF	the cash flow value

Since $PV = BV \cdot \sum_{i \neq futures} h_i$, where, BV is the base value used for computing asset weights, the optimal holding $\mathbf{h}$ must satisfy the following portfolio balance constraint:

$$\sum_{i \neq futures} h_i = \frac{PV_0 + CF}{BV} \tag{14}$$

Alternatively, the same constraint can be expressed as a function of the initial weights:

$$\sum_{i \neq futures} h_i = \sum_{i \neq futures} h_{0i} + CF / BV \tag{15}$$

Notes:

- Typically, you can select the initial portfolio value or the initial portfolio value plus the cash flow value as the base value.
- You can also select an assigned value as the base value. To avoid numerical issues during optimization, the base value should not be far away from $PV_0 + CF$.
- You do not set the portfolio balance constraint explicitly as **Barra Optimizer will add it automatically**. This constraint can be disabled and replaced with a user-defined linear constraint (although this is not recommended).
- The portfolio balance constraint enforces the total net trade to be equal to the cash flow if no cash asset is used. If no cash flow is set, the total net trade will be 0. A cash asset is used to control the net total trade value. For more information, see the Turnover Functions and Constraints section.

3.4 Asset Types

3.4.1 Regular Assets

An asset that is not classified as any other asset type is referred to as a regular asset in Barra Optimizer. It can represent an equity asset, a non-equity instrument, or even an asset class. The only requirement is that the asset must be consistent with the risk model used.

3.4.2 Cash Assets

To leave some of the portfolio as uninvested, a cash asset should be added to the workspace. Bounds on the cash asset can be set in the same way as bounds on any other asset, and can be used to specify a maximum or minimum level of cash. The cash asset can have a negative weight. The optimal results from the optimization will obey the specified cash bounds. However, post-optimal roundlotting may lead to a breach of the bounds.

By default, cash flow and change in cash asset position both affect the amount of turnover. An option is available to exclude cash in turnover definitions. For more information, see the Turnover Functions and Constraints section.

3.4.3 Currencies

Currencies are treated as a regular asset for all purposes outside non-linear transaction costs. Normally a currency asset has exposure to just its currency factor. Multiple currencies can occur within a single portfolio, for example, when using a global or regional model.

3.4.4 Futures-Type Assets

Futures-type assets are used to reflect the special nature of futures and other contracts that are entered into with zero or minimal investment. They are treated differently than other assets. Major differences include the following:

- Futures-type assets are NOT counted in the portfolio balance constraint. The portfolio market value will not change even though the position of futures may change before and after optimization.
- By default, futures-type assets are NOT counted in:
  - Turnover computation
  - Threshold or cardinality constraints
  - Leverage constraints in hedge (long/short) optimization
  - Roundlotting optimization

  We have added separate options to allow futures in turnover, threshold or cardinality, hedge and roundlotting constraints. See Note 22 in Section 5.5.3 for more details.

3.4.5 Composites

Composites are used to reflect the look-through exposure gained when using an instrument that has exposure to underlying components of the risk model. For example, S&P 500 ETF or Future would be a composite when used with a US-stock model. By using the composite functionality, the optimizer correctly accounts for the exposure to the underlying stocks within the risk control functions. Effectively, the weight in the composite is passed through to the weight of the underlying assets when using the risk aversion or risk constraints. For other constraints and objective terms, the composite is treated as a normal asset.

An asset can be both a composite and futures-type asset.

3.5 Return

The (portfolio) return ($\mathbf{r}^{\mathrm{T}}\mathbf{h}$) in Barra Optimizer is generally defined as the inner product of the asset return vector ($\mathbf{r}$) and the asset holding vector ($\mathbf{h}$). As such, the return term is linear in $\mathbf{h}$. For cases with no benchmark, the individual asset returns are total return. For cases with a benchmark, the individual asset returns may represent either a total or an active return with respect to the corresponding return of a benchmark—the difference between the two is a just a constant, which should not affect the optimization outcome (see Appendix G). In the latter case, the asset return vector $\mathbf{r}$ is also known as the asset alpha, denoted as α.

3.6 Bounds and Linear or Piecewise-Linear Constraints

By definition, standard constraints are linear or convex piecewise-linear constraints. In Barra Optimizer, they are classified into a number of categories according to their roles or applications, as described in the following sections.

3.6.1 Asset and Factor Bounds

Barra Optimizer allows users to set lower and upper bounds on each asset's weight in the portfolio. For example, one can require the portfolio weight of IBM, $\mathbf{h_1}$, to satisfy the inequality $0 \le \mathbf{h}_1 \le 0.05$ or the weight of Intel, $\mathbf{h}_2$ to be in the range $-0.01 \le \mathbf{h_1} \le 0.02$.

With the composite look-through capability, the asset bounds are on the total weight

$$\boldsymbol{h}_i + \sum_{j=1}^{n_c} \boldsymbol{c}_{ij}\boldsymbol{h}_j^c$$

where $\boldsymbol{c}_{ij}$ is the weight of asset i in the composite j and $\boldsymbol{h}_j^c$ is the weight of the composite j in the portfolio.

Users can also control the magnitude of a factor exposure by using an upper or lower bound or both. For instance, one can demand that a portfolio's exposure to the "size" factor be less than one standard deviation of the benchmark's exposure to the same factor.

3.6.2 General Linear Constraints

Many real-world investment rules or restrictions may be modeled or approximated by a general linear constraint. For example, the requirement that the sum of weights of all assets that have exposure to the Oil Industry factor must be less than 3% of the total portfolio value can be represented by a general linear constraint. Barra Optimizer supports all general linear constraints. These constraints may be set directly or by using a slack variable.

Barra Optimizer may accommodate user-defined linear constraints. For example, the return constrained problems can be easily set up with a user-defined constraint of the form: $return \ge \text{Lower Bound}$, where $return$ is defined as $\mathbf{r}^{\mathrm{T}}\mathbf{h}$. However, different interpretations of "return" would require different values of "Lower Bound" (see Appendix G).

3.6.3 Constraint Bounds and Slack Variables

In the context of optimization, a **slack variable** is a variable that is added to an inequality constraint to transform it to equality. Introducing a slack variable replaces an inequality constraint with an equality constraint and a bound constraint on the slack variable. Slack variables are introduced internally in Barra Optimizer to make user-defined inequality constraints conform to a standard equality format.

For example, equations (4)-(5) are internal replacements of the user-specified general linear constraints

$$\mathbf{l}_s \le \mathbf{Ah} \le \mathbf{u}_s \tag{16}$$

Similarly, equations (6)-(7) are replacements of the factor constraints

$$\mathbf{l}_X \le \mathbf{X}^{\mathrm{T}}\mathbf{h} \le \mathbf{u}_X \tag{17}$$

The lower bound, upper bound, both, or neither can be set for any linear constraint, either in inequality or equality format. If a fixed value is desired on the constraint, set both the lower bound and the upper bound to that value.

3.6.4 Beta Constraint

A Beta constraint is, technically, a general linear constraint. However, Barra Optimizer does provide users with an option to set a beta constraint with the betas calculated within the optimization. Beta measures a portfolio's volatility, or systematic risk, with respect to the benchmark. Beta is calculated using the risk model and benchmark:

$$\boldsymbol{\beta} = (\mathbf{XFX}^{\mathrm{T}} + \mathbf{D})\mathbf{h}_B \big/ \mathbf{h}_B{}^{\mathrm{T}}(\mathbf{XFX}^{\mathrm{T}} + \mathbf{D})\mathbf{h}_B \tag{18}$$

The $\beta(\mathbf{h})$ term in the Beta constraint (8) is given by:

$$\beta(\mathbf{h}) = \boldsymbol{\beta}^{\mathrm{T}}\mathbf{h} \tag{19}$$

Note that a benchmark may contain assets outside the investment universe. For more information, see the Benchmarks section.

3.6.5 General Piecewise-Linear Constraints

Many useful constraints such as the turnover limit constraint, transaction cost limit constraints, leverage constraints, and tax-related constraints are special cases of the general, separable piecewise-linear constraints. Some of these constraints are discussed in detail in the Transaction Cost, Holding Cost, and Turnover Control section and the Tax-Aware Optimization section.

Mathematically, these constraints can be represented by:

$$l_{gpw} \le gpw(h_1, h_2, \cdots, h_n) = \sum_{i=1}^{n} gpw(h_i) \le u_{gpw} \tag{20}$$

where for integer k and real numbers b_{ij}'s and c_{ij}'s,

$$gpw(h_i) = \begin{cases} c_{i,k}(h_i - b_{i,k}) + \sum_{j=0}^{k-1} c_{i,j}\left[b_{i,(j+1)} - b_{i,j}\right] & k \ge 0 \;\; and \;\; b_{i,0} \le \cdots \le b_{i,k} \le h_i \le b_{i,k+1} \\ c_{i,k}(h_i - b_{i,(k+1)}) + \sum_{j=k+1}^{-1} c_{i,j}\left[b_{i,j} - b_{i,(j+1)}\right] & k < 0 \;\; and \;\; b_{i,k} \le h_i \le b_{i,k+1} \le \cdots \le b_{i,0} \end{cases} \tag{21}$$

Intuitively, the b_{ij} 's are the break points and the c_{ij} 's are the slopes of the individual piecewise-linear functions (see Figure 1). The special break point $b_{i,0}$ is where the piecewise-linear function for h_i is zero, i.e., $gpw(h_i) = 0$. Note that,

$$\begin{array}{lll} k = 0 & ==> & gpw(h_i) = c_{i,0}(h_i - b_{i,0}) \\ k = 1 & ==> & gpw(h_i) = c_{i,1}(h_i - b_{i,1}) + c_{i,0}\left[b_{i,1} - b_{i,0}\right] \\ k = 2 & ==> & gpw(h_i) = c_{i,2}(h_i - b_{i,2}) + c_{i,0}\left[b_{i,1} - b_{i,0}\right] + c_{i,1}\left[b_{i,2} - b_{i,1}\right] \\ k = -1 & ==> & gpw(h_i) = c_{i,-1}(h_i - b_{i,0}) \\ k = -2 & ==> & gpw(h_i) = c_{i,-2}(h_i - b_{i,-1}) + c_{i,-1}\left[b_{i,-1} - b_{i,0}\right] \\ k = -3 & ==> & gpw(h_i) = c_{i,-3}(h_i - b_{i,-2}) + c_{i,-1}\left[b_{i,-1} - b_{i,0}\right] + c_{i,-2}\left[b_{i,-2} - b_{i,-1}\right] \end{array} \tag{22}$$

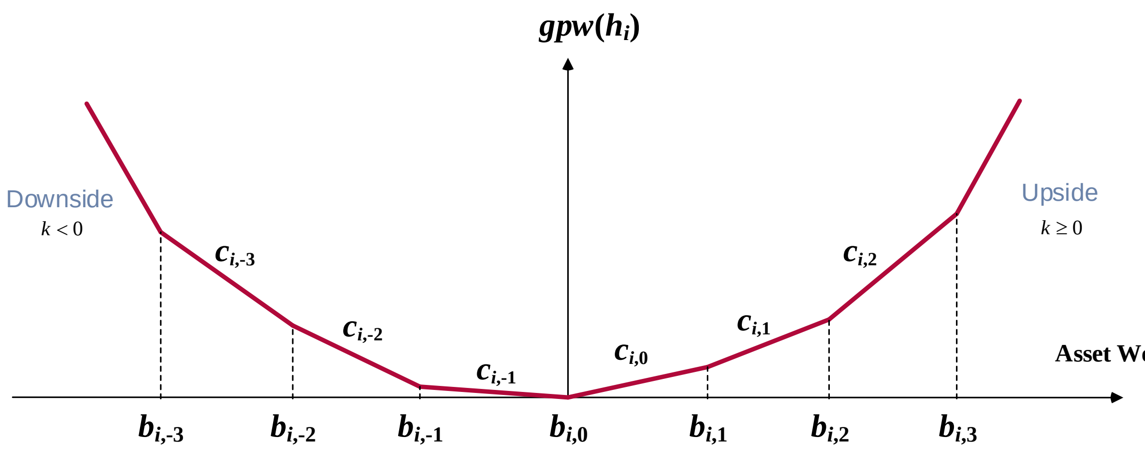


Figure 1. General Piecewise-Linear Function

An upper bound on a piecewise-linear function with monotonically increasing slopes or a lower bound on a piecewise-linear function with monotonically decreasing slopes is a convex piecewise-linear constraint. Conversely, a lower bound on a piecewise-linear function with monotonically increasing slopes or an upper bound on a piecewise-linear function with monotonically decreasing slopes is not a convex constraint.

3.6.6 Total Active Weights Constraint

The total active weights constraint is another special case of the general, separable piecewise-linear constraints. Mathematically, it is defined as:

$$l_{taw} \le \sum_i \left| h_i - h_{Bi} \right| \le u_{taw} \tag{23}$$

where l_{taw} and u_{taw} are the lower and upper bound of the constraint, respectively. Intuitively, total active weights represent the sum of the absolute active weights with respect to the benchmark. A binding lower bound may render this constraint non-convex.

Note that constraint (23) is defined on assets in the investment universe only. To consider benchmark assets outside the investment universe, users need to pre-adjust the lower and upper bounds accordingly as follows:

$$\underbrace{l_{taw} - \sum_i \left|\tilde{h}_{Bi}\right|}_{\text{new lower bound}} \le \sum_i \left|h_i - h_{Bi}\right| \le \underbrace{u_{taw} - \sum_i \left|\tilde{h}_{Bi}\right|}_{\text{new upper bound}} \tag{24}$$

where $\tilde{h}_{Bi}$ is the weight of an outside-of-universe benchmark asset (see Section 3.7.3.2).

3.6.7 Issuer Constraints

Issuer constraints refer to the bounds on net or absolute total holdings of an issuer. Let h_{K_i} be the portfolio weight of the *i*th asset of issuer *K*, and n_K be the number of distinctive assets belonging to issuer *K*. Then, a "net" issuer constraint on issuer *K* is defined as

$$l_K \le \sum_{i=1}^{n_K} h_{K_i} \le u_K \tag{25}$$

while an "absolute" issuer constraint on issuer K is given by:

$$l_K \le \sum_{i=1}^{n_K} \left|h_{K_i}\right| \le u_K \tag{26}$$

where l_K and u_K are the lower and upper bound of the constraint, respectively.

Mathematically, (25) is a linear constraint and (26) is a piecewise-linear constraint. For long-only optimization problems, all issuer constraints are convex. However, if shorting is allowed, the "absolute" issuer constraints may not be convex when the lower bound is positive.

Barra Optimizer's definition of an "issuer" is much broader than the traditional meaning of a security issuer. It extends the concept of an issuer to any group of assets based on certain rules. In other words, issuer constraints can be used to handle certain group constraints.

Users can set both global bounds (that are applied to all issuers) and individual bounds (that are applied to specific issuers) on a particular issuer simultaneously. However, only the more restrictive one prevails.

Issuer constraints can be combined with almost all optimization problems except the following:

- Multi-Period Optimization
- Multi-Account Optimization
- Tax-Aware Optimization
- Maximizing Sharpe Ratio or Information Ratio

Notes:

- For each issuer, users are only allowed to set at most one net total holding constraint AND at most one absolute total holding constraint. If users set multiple issuer constraints of the same type on an issuer, only the last one will prevail.
- Both upper and lower bounds on net total holding are supported. However, only upper bound on absolute total holding is supported.
- Soft bounds or penalties on issuer constraints are not supported.

3.6.8 Ratio Constraints

Many useful constraints can be expressed as:

$$l \le \frac{\sum_{i \in G_n} a_i h_i}{\sum_{i \in G_d} b_i h_i} \le u \tag{27}$$

where G_n and G_d are two groups of assets, and a_i and b_i are arbitrary coefficients.

Assuming the weights h_i in G_n and G_d are the same, and that

$$\sum_{i \in G_d} b_i \cdot h_i > 0 \tag{28}$$

then the ratio constraint (27) is equivalent to the following two linear constraints:

$$\sum_{i \in G_n \cup G_d} \left(a_i - l \cdot b_i\right) \cdot h_i \ge 0 \tag{29}$$

$$\sum_{i \in G_n \cup G_d} \left(u \cdot b_i - a_i\right) \cdot h_i \ge 0 \tag{30}$$

with $a_i := 0$ if $i \notin G_n$ and $b_i := 0$ if $i \notin G_d$.

This is how Barra Optimizer treats ratio constraints internally.

When all b_i is 1, the ratio constraint (27) reduces to an important special case:

$$l \le \frac{\sum_{i \in G} a_i h_i}{\sum_{i \in G} h_i} \le u \tag{31}$$

This represents a weighted average of some asset attribute a_i in a group.

For more details on how to set up the ratio constraints in the Barra Optimizer, please refer to Sections 8.5.20 and 8.5.21 in this document.

3.7 Risk Control in Optimization—Aversions, Constraints, and Grouping

Risk control in Barra Optimizer can be achieved in a number of ways. The simplest and recommended method is to use a risk aversion within the objective. This requires two aversion parameters λ_D and λ_F, which can be set to the same value. Risk control using aversions is a Quadratic Programming Problem and produces a more stable problem than risk control using constraints.

Alternatively, risk control may be achieved using risk constraints. In addition, Barra Optimizer supports dual risk models, multiple benchmarks, as well as factor or asset groupings for fine control of risk in the portfolio.

3.7.1 Risk Aversions

In standard portfolio optimization (1), the importance of risk relative to return may be adjusted implicitly by altering the risk aversion values in the objective function. In contrast to the return term, the risk terms are quadratic in $\mathbf{h}$:

$$\begin{aligned} &100(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}(\lambda_D\mathbf{D}+\lambda_F\mathbf{XFX}^{\mathrm{T}})(\mathbf{h}-\mathbf{h}_B) \\ &=100\left(\underbrace{\mathbf{h}^{\mathrm{T}}(\lambda_D\mathbf{D}+\lambda_F\mathbf{XFX}^{\mathrm{T}})\mathbf{h}}_{quadratic}-\underbrace{2\mathbf{h}_B{}^{\mathrm{T}}(\lambda_D\mathbf{D}+\lambda_F\mathbf{XFX}^{\mathrm{T}})\mathbf{h}}_{linear}+\underbrace{\mathbf{h}_B{}^{\mathrm{T}}(\lambda_D\mathbf{D}+\lambda_F\mathbf{XFX}^{\mathrm{T}})\mathbf{h}_B}_{constant}\right) \end{aligned} \quad (32)$$

where the benchmark vector $\mathbf{h}_B$ may contain assets outside the investment universe. As shown in (32), the impact of risk on the objective is influenced by the two risk aversion parameters λ_D and λ_F, and amplified by the multiplier 100. For more information about the effects of risk aversion on portfolio optimization, see Liu and Xu (2010).

3.7.2 Risk Models

Barra Optimizer accepts risk models in either standard factor model form, or in asset-covariance form.

3.7.2.1 Factor Risk Models

Factor model form can be written as:

$$\Sigma=\mathbf{D}+\mathbf{XFX}^{\mathrm{T}} \quad (33)$$

Barra Optimizer takes advantage of this special structure in its internal algorithms.

The asset-specific covariance matrix can be further represented by:

$$\mathbf{D}=\begin{bmatrix}\mathbf{D}_r & \mathbf{E}^{\mathrm{T}} \\ \mathbf{E} & \mathbf{V}\end{bmatrix} \quad (34)$$

where,

$\mathbf{D}_r$	$(n_r \times n_r)$ (Block) diagonal covariance matrix representing specific risk for non-composite assets
$\mathbf{E}$	$(n_c \times n_r)$ Covariance matrix of composites with non-composite assets
$\mathbf{V}$	$(n_c \times n_c)$ Covariance matrix of composites with composites
n_r	The number of non-composite assets in the investment universe

n_c	The number of composites in the investment universe. Note that $n_r + n_c = n$.

Furthermore, the matrices $\mathbf{E}$ and $\mathbf{V}$ in (34) are usually computed using the following formulae:

$$\mathbf{E} = \mathbf{W}\mathbf{D}_m\mathbf{M}^{\mathrm{T}} \tag{35}$$

$$\mathbf{V} = \mathbf{W}\mathbf{D}_m\mathbf{W}^{\mathrm{T}} \tag{36}$$

where

$\mathbf{D}_m$	$(n_m \times n_m)$ (Block) diagonal specific covariance matrix for all non-composite assets in the market
$\mathbf{W}$	$(n_c \times n_m)$ Composition (or weight) matrix for composites in which $\mathbf{W}_{ij}$ is the fraction of composite i made up by regular asset j
$\mathbf{M}$	$(n_r \times n_m)$ Identification matrix which selects assets from the market to the investment universe $\mathbf{M}_{ij} = \begin{cases} 1, & \text{asset } j \text{ in the market is selected as asset } i \text{ in the investment universe} \\ 0, & \text{otherwise} \end{cases}$
n_m	The number of non-composite assets in the market[1]

As noted above, the specific covariance matrix $\mathbf{D}_r$ is usually diagonal. It becomes block diagonal only when there are "linked" assets, which are groups of non-composite assets with common exposure to a source of specific risk. Typical linked assets include different classes of stocks of the same company (for example, common stocks, or preferred stocks) or different bonds issued by the same company. The common, underlying source of specific risk is the root—in this case, the underlying company.

Without composite and linked assets, the specific covariance matrix $\mathbf{D}$ reduces to a simple diagonal matrix $\mathbf{D}_r$.

[1] "market" refers to the collection of all assets in existence.

3.7.2.2 Asset-Covariance Risk Models

In addition to factor risk models, Barra Optimizer also supports risk models using asset-by-asset covariance matrices. When uploading such risk models into Barra Optimizer, we recommend using no factors, and using the specific covariance functionality to populate $\mathbf{D}$ as an $n \times n$ positive semi-definite general covariance matrix among all assets.

3.7.2.3 Secondary Risk Model

The risk model embedded in (32) is considered as the primary risk model. In addition to it, Barra Optimizer supports an optional secondary risk model. The secondary risk model may appear either in the objective function or in risk constraints.

When used in the objective function, an additional risk term as shown below is added:

$$-100(\mathbf{h}-\mathbf{h}_{B_2})^{\mathrm{T}}(\lambda_{D_2}\mathbf{D}_2+\lambda_{F_2}\mathbf{X}_2\mathbf{F}_2\mathbf{X}_2^{\mathrm{T}})(\mathbf{h}-\mathbf{h}_{B_2}) \tag{37}$$

where,

λ_{D_2}	Specific risk aversion parameter of the secondary risk model
λ_{F_2}	Common factor risk aversion parameter of the secondary risk model
$\mathbf{h}_{B_2}$	Secondary benchmark holding weights (may or may not be the same as the primary one)
$\mathbf{D}_2$	Specific covariance matrix of the secondary risk model
$\mathbf{X}_2$	Factor exposures matrix of the secondary risk model
$\mathbf{F}_2$	Factor covariance matrix of the secondary risk model

Note that the factor constraints as defined by (6)-(7) are for the primary risk model only. To set similar constraints for the secondary risk model, general linear constraints must be used.

3.7.3 Benchmarks

Benchmarks may appear either in the objective function or in risk constraints. The presence of a benchmark alters the definition of risk.

3.7.3.1 Active Risk vs. Portfolio Risk

When the benchmark is non-empty, the associated risk term represents an *active risk*, or the tracking error with respect to this benchmark. When the benchmark vector $\mathbf{h}_B$ is NULL, the risk term in (1) simply reduces to the *portfolio risk*:

$$100\mathbf{h}^{\mathrm{T}}(\lambda_D\mathbf{D}+\lambda_F\mathbf{XFX}^{\mathrm{T}})\mathbf{h} \tag{38}$$

which represents the variance of the portfolio's total return or its return with respect to cash.

3.7.3.2 Benchmark with Non-Investment Universe Assets

Benchmark assets that are not in the investment universe are still included in the risk calculation. Internally, Barra Optimizer is often able to reduce the number of decision variables by efficient treatment of the non-investment universe assets.

More specifically, it is straightforward to show that:

$$(\bar{\mathbf{h}}-\bar{\mathbf{h}}_B)^{\mathrm{T}}(\lambda_D\bar{\mathbf{D}}+\lambda_F\bar{\mathbf{X}}\mathbf{F}\bar{\mathbf{X}}^{\mathrm{T}})(\bar{\mathbf{h}}-\bar{\mathbf{h}}_B) \tag{39}$$

$$=\underbrace{\mathbf{h}^{\mathrm{T}}(\lambda_D\mathbf{D}+\lambda_F\mathbf{X}\mathbf{F}\mathbf{X}^{\mathrm{T}})\mathbf{h}}_{quadratic}-\underbrace{2[\lambda_D(\mathbf{h}_B{}^{\mathrm{T}}\mathbf{D}+\tilde{\mathbf{h}}_B{}^{\mathrm{T}}\mathbf{C})+\lambda_F\bar{\mathbf{h}}_B{}^{\mathrm{T}}\bar{\mathbf{X}}\mathbf{F}^{\mathrm{T}}\mathbf{X}^{\mathrm{T}}]\mathbf{h}}_{linear}+\underbrace{\bar{\mathbf{h}}_B{}^{\mathrm{T}}(\lambda_D\bar{\mathbf{D}}+\lambda_F\bar{\mathbf{X}}\mathbf{F}\bar{\mathbf{X}}^{\mathrm{T}})\bar{\mathbf{h}}_B}_{constant} \tag{40}$$

where

$\tilde{\mathbf{h}}_B$	$(\tilde{n}\times 1)$ Benchmark weights containing only the out-of-universe assets
$\bar{\mathbf{h}}_B=\begin{bmatrix}\mathbf{h}_B\\ \tilde{\mathbf{h}}_B\end{bmatrix}$	$(\bar{n}\times 1)$ Augmented vector of all benchmark weights
$\bar{\mathbf{h}}=\begin{bmatrix}\mathbf{h}\\ \mathbf{0}\end{bmatrix}$	$(\bar{n}\times 1)$ Augmented vector of asset holdings
$\bar{\mathbf{D}}=\begin{bmatrix}\mathbf{D} & \mathbf{C}\\ \mathbf{C}^{\mathrm{T}} & \tilde{\mathbf{D}}\end{bmatrix}$	$(\bar{n}\times\bar{n})$ Augmented specific covariance matrix $(\bar{n}\times\bar{n})$ The off-diagonal term $\mathbf{C}$ is often zero, but need not be so (e.g., when linked assets or composites are present)
$\bar{\mathbf{X}}=\begin{bmatrix}\mathbf{X}\\ \tilde{\mathbf{X}}\end{bmatrix}$	$(\bar{n}\times k)$ Augmented factor exposure matrix
$\tilde{n}$	The number of assets inside the benchmark yet outside the universe
$\bar{n}$	The number of all benchmark assets, $\bar{n}=n+\tilde{n}$

From (40), we see that out-of-universe assets only appear in the linear and constant parts. In other words, they do not affect the quadratic component of the risk terms. Therefore, only the linear component needs to be adjusted for these assets. The adjustment to constant component is optional but not essential since it would not affect either the optimization process or optimal solution.

3.7.3.3 Multiple Risk Terms and Multiple Benchmarks

Additional control can be achieved by deriving new risk terms from the existing primary or secondary risk models and by weighting specific risks, assets, or factors to form new covariance terms. Different benchmarks are allowed in different risk control terms.

For accuracy, the additional risk terms not expressed explicitly in (1) already, are absorbed into the last term $g(\mathbf{h})$. In general, these terms take on one of the following forms:

Weighted Specific Risk: $$100\,\lambda_{D_*}(\mathbf{h}-\mathbf{h}_B^*)^{\mathrm{T}}\mathbf{W}_{D_*}\mathbf{D}_*\mathbf{W}_{D_*}^T(\mathbf{h}-\mathbf{h}_B^*) \tag{41}$$

Factor-Weighted Factor Risk: $100\lambda_{F_*}(\mathbf{h}-\mathbf{h}_B^*)^T \mathbf{X}_* \mathbf{W}_{F_*} \mathbf{F}_* \mathbf{W}_{F_*}^T \mathbf{X}_*^T (\mathbf{h}-\mathbf{h}_B^*)$ (42)

Asset-Weighted Factor Risk: $100\lambda_{X_*}(\mathbf{h}-\mathbf{h}_B^*)^T \mathbf{W}_{X_*} \mathbf{X}_* \mathbf{F}_* \mathbf{X}_*^T \mathbf{W}_{X_*}^T (\mathbf{h}-\mathbf{h}_B^*)$ (43)

where

$\mathbf{D}_*$	Specific covariance matrix or the full asset-by-asset covariance matrix of the primary or secondary risk model
$\mathbf{F}_*$	Factor covariance matrix of the primary or secondary risk model
$\mathbf{X}_*$	Factor exposure matrix of the primary or secondary risk model
$\mathbf{W}_*$	Diagonal weight matrix with appropriate dimensions
λ_*	Risk aversion for the new risk term
$\mathbf{h}_B^*$	Benchmark vector for the new risk term, which could be unique or NULL

Multiple covariance terms of each type can be added, but they must reference either the primary or the secondary risk model.

3.7.4 Risk Constraints—Risk-Constrained Optimization

From a user's perspective, imposing an explicit upper or lower bound on risk is a straightforward way to control risk. From the Optimizer's point of view, however, risk constraints are much more complex to handle than risk aversions in general. While Barra Optimizer supports lower bounds on most risk constraints, *users should avoid risk lower bounds whenever possible* due to the theoretical and computational complexities associated with them.

Risk constraints are considered as special constraints by the classification scheme outlined in Section 3.1, since they are all non-linear, and some of them are non-convex as well. Non-convexity may also lead to suboptimal solutions. Portfolio optimization with risk constraints is also known as Risk Constrained Optimization. Risk constraints may also be combined with threshold, cardinality, and hedge constraints.

To construct a risk-constrained problem, one or more risk constraints of the following format are to be added to the standard portfolio optimization problem:

$$l_{risk} \le risk(\mathbf{h}) \le u_{risk} \tag{44}$$

where

l_{risk}	the given lower bound of the risk constraint
u_{risk}	the given upper bound of the risk constraint
$risk(\mathbf{h})$	one of the risk or risk contribution terms defined in Appendix H.

The risk terms in a risk constraint can be either identical to or different from the ones in the objective. When different, the terms may represent risk from a different risk model or different aspects of risk from the same risk model—for example, with respect to different benchmarks or from different subgroups.

Risk constraints can be classified into three broad categories—portfolio-level, subgroup-level and individual-asset-level risk constraints. In the following sections, we provide further descriptions and examples to illustrate different combinations of these risk constraints supported in the Barra Optimizer.

3.7.4.1 Portfolio-Level Risk Constraints

Barra Optimizer can be used to constrain one risk term while minimizing another, or to constrain multiple sources of risk with varying definitions. In general, upper bounds on portfolio-level risk are convex and can be set on the total risk, factor risk, or specific risk. Up to two risk models can be used in multiple risk constraints. A lower bound on portfolio total risk or active total risk is also supported; however, the lower bound will make the risk constraint non-convex.

3.7.4.1.1 Constraining Total Risk

In the following example, an active risk relative to a benchmark is included in the objective while an active total risk with respect to an in-house model portfolio is set as a constraint:

$$\text{Maximize:}\quad \lambda_r\,\mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda\,\sigma^2(\mathbf{h},\mathbf{h}_{B_1}) - \text{Other Terms} \tag{45}$$

$$\text{Subject to:}\quad l_\sigma \le \sigma_2(\mathbf{h},\mathbf{h}_{B_2}) \le u_\sigma \tag{46}$$

$$\text{Other Constraints}$$

where,

$\mathbf{h}_{B_1}$, $\mathbf{h}_{B_2}$	weight vectors for the index and in-house model portfolios, one or both vectors could be empty
$\sigma^2(\mathbf{h},\mathbf{h}_{B_1})$	active variance with respect to the benchmark portfolio
	$= (\mathbf{h}-\mathbf{h}_{B_1})^{\mathrm{T}}\mathbf{V}(\mathbf{h}-\mathbf{h}_{B_1})$
$\sigma_2(\mathbf{h},\mathbf{h}_{B_2})$	active total risk with respect to the in-house model portfolio
	$= \sqrt{(\mathbf{h}-\mathbf{h}_{B_2})^{\mathrm{T}}\Sigma(\mathbf{h}-\mathbf{h}_{B_2})}$
$\mathbf{V}, \Sigma$	two covariance matrices, which could also be either identical or different, and could be from the same or different risk models
λ	a risk aversion parameter
l_σ	the given lower bound on the active total risk with respect to the in-house model portfolio
u_σ	the given upper bound on the active total risk with respect to the in-house model portfolio

3.7.4.1.2 Constraining Factor Risk

An example of constraining two risks, one of which is the portfolio's factor risk, is given below:

$$\text{Maximize:} \quad \lambda_r \mathbf{r}^{\mathrm{T}}\mathbf{h} - \text{Other Terms} \tag{47}$$

$$\text{Subject to:} \quad l_\sigma \le \sigma(\mathbf{h}, \mathbf{h}_{B_1}) \le u_\sigma \tag{48}$$

$$l_F \le \sigma_F(\mathbf{h}, \mathbf{h}_{B_2}) \le u_F \tag{49}$$

Other Constraints

where,

$\sigma(\mathbf{h}, \mathbf{h}_{B_1})$	total risk
	$= \sqrt{(\mathbf{h} - \mathbf{h}_{B_1})^{\mathrm{T}} \mathbf{V} (\mathbf{h} - \mathbf{h}_{B_1})}$
$\sigma_F(\mathbf{h}, \mathbf{h}_{B_2})$	factor risk
	$= \sqrt{(\mathbf{h} - \mathbf{h}_{B_2})^{\mathrm{T}} \mathbf{X}_2 \mathbf{F}_2 \mathbf{X}_2^T (\mathbf{h} - \mathbf{h}_{B_2})}$
$\mathbf{F}_2$	factor covariance matrix
$\mathbf{X}_2$	exposure matrix
l_F	the given lower bound on the factor risk of the portfolio
u_F	the given upper bound on the factor risk of the portfolio

Again, the benchmarks $\mathbf{h}_{B_1}$ and $\mathbf{h}_{B_2}$ here could be empty or identical, and the $\mathbf{F}_2$ and $\mathbf{X}_2$ matrices could be either related to or independent of the asset-by-asset covariance matrix $\mathbf{V}$.

3.7.4.1.3 Constraining Specific Risk

In the following example, we maximize a factor-risk-adjusted return in the objective while constraining a total risk and a specific risk:

$$\text{Maximize:} \quad \lambda_r \mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda \sigma_F{}^2(\mathbf{h}, \mathbf{h}_{B_1}) - \text{Other Terms} \tag{50}$$

$$\text{Subject to:} \quad l_\sigma \le \sigma(\mathbf{h}, \mathbf{h}_{B_2}) \le u_\sigma \tag{51}$$

$$l_D \le \sigma_S(\mathbf{h}, \mathbf{h}_{B_3}) \le u_D \tag{52}$$

Other Constraints

where,

$\sigma_F{}^2(\mathbf{h}, \mathbf{h}_{B_1})$	factor variance
	$= (\mathbf{h} - \mathbf{h}_{B_1})^{\mathrm{T}} \mathbf{X}_1 \mathbf{F}_1 \mathbf{X}_1^{\mathrm{T}} (\mathbf{h} - \mathbf{h}_{B_1})$
$\sigma(\mathbf{h}, \mathbf{h}_{B_2})$	total risk

	$= \sqrt{(\mathbf{h}-\mathbf{h}_{B_2})^{\mathrm{T}}\mathbf{V}_2(\mathbf{h}-\mathbf{h}_{B_2})}$
$\sigma_S(\mathbf{h},\mathbf{h}_{B_3})$	specific risk
	$= \sqrt{(\mathbf{h}-\mathbf{h}_{B_3})^{\mathrm{T}}\mathbf{D}(\mathbf{h}-\mathbf{h}_{B_3})}$
$\mathbf{D}$	specific covariance matrix, which could be either related to or independent from the matrices $\mathbf{F}_1$ and $\mathbf{V}_2$, as long as they all come from at most two risk models.
l_D	the given lower bound on the specific risk of the portfolio
u_D	the given upper bound on the specific risk of the portfolio

Again, one or more of the three benchmarks here could be empty, and they could all be identical.

3.7.4.1.4 Constraining Factor or Specific Risk Contribution

In addition to constraining the absolute levels of factor or specific risk, users may constrain the ratio of factor or specific risk to the total risk. Mathematically, a factor risk contribution constraint takes on the form:

$$l_{F_c} \le \frac{\sigma_F(\mathbf{h},\mathbf{h}_B)}{\sigma(\mathbf{h},\mathbf{h}_B)} \le u_{F_c} \tag{53}$$

where l_{F_c} and u_{F_c} are the given lower and upper bound, respectively, on the ratio of factor to total risk at the portfolio level. Similarly, a specific risk contribution constraint has the form:

$$l_{S_c} \le \frac{\sigma_S(\mathbf{h},\mathbf{h}_B)}{\sigma(\mathbf{h},\mathbf{h}_B)} \le u_{S_c} \tag{54}$$

where l_{S_c} and u_{S_c} are the given lower and upper bound, respectively, on the ratio of specific to total risk at the portfolio level. As before, the benchmark vector $\mathbf{h}_B$ is allowed to be NULL. The risk models used in both the numerator and denominator of (53) or (54) are required to be the same.

3.7.4.2 Subgroup Risk Constraints

Sometimes, investors are more concerned with the risk from a subgroup than with the overall risk of the entire portfolio. For example, users may wish to limit the risk attributed to the style factor to be below 10 basis points, or alternatively, set an upper bound on the risk attributed to the technology sector at 0.05. Risk here refers to either the portfolio risk or the active risk, depending on whether there is a benchmark present in the constraint.

3.7.4.2.1 Additive vs. Non-Additive Definitions of Subgroup Risk

Barra Optimizer supports two definitions of subgroup risk: an additive definition and a non-additive definition. The subgroup risk computed under the additive definition is the same as computed using MSCI’s x-sigma-rho paradigm.

The non-additive definition, on the other hand, is a simple legacy product as it totally or partially[2] ignores covariances outside of the subgroup, which may or may not be desirable to the user.

Risk from all partitioning subgroups under the additive definition sums to the corresponding portfolio-level risk. This may not be true under the non-additive definition. The advantage of the non-additive definition is convexity. The additive definition, on the other hand, makes the problem non-convex and hence more difficult to solve, among other consequences.

Note that the two definitions differ only when applied to the risk constraints defined on a subgroup of assets or factors. They are equivalent for portfolio-level risk constraints.

Subgroup Risk – Additive Definition

Let $\boldsymbol{I}$ be a subset of asset indexes and $\boldsymbol{J}$ be a subset of factor indexes.

Subscripting a vector by $\boldsymbol{I}$ (or $\boldsymbol{J}$) denotes replacing its elements with zeroes if the position indexes are not in $\boldsymbol{I}$ (or $\boldsymbol{J}$).

For completeness, let

$$\sigma(\mathbf{h},\mathbf{h}_B)=\sqrt{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}(\mathbf{D}+\mathbf{XFX}^{\mathrm{T}})(\mathbf{h}-\mathbf{h}_B)} \qquad (55)$$

$$\sigma_F(\mathbf{h},\mathbf{h}_B)=\sqrt{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}\mathbf{XFX}^{\mathrm{T}}(\mathbf{h}-\mathbf{h}_B)} \qquad (56)$$

$$\sigma_S(\mathbf{h},\mathbf{h}_B)=\sqrt{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}\mathbf{D}(\mathbf{h}-\mathbf{h}_B)} \qquad (57)$$

Then, the total risk from asset group $\boldsymbol{I}$ under the additive definition is:

$$\sigma^A(\mathbf{h},\mathbf{h}_B,\boldsymbol{I})=\frac{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}(\mathbf{D}+\mathbf{XFX}^{\mathrm{T}})(\mathbf{h}-\mathbf{h}_B)_{\boldsymbol{I}}}{\sigma(\mathbf{h},\mathbf{h}_B)} \qquad (58)$$

The factor risk from factor group $\boldsymbol{J}$ under the additive definition is:

$$\sigma_F^A(\mathbf{h},\mathbf{h}_B,\boldsymbol{J})=\frac{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}\mathbf{XF}(\mathbf{X}^{\mathrm{T}}(\mathbf{h}-\mathbf{h}_B))_{\boldsymbol{J}}}{\sigma_F(\mathbf{h},\mathbf{h}_B)} \qquad (59)$$

The specific risk from asset subgroup $\boldsymbol{I}$ under the additive definition is:

$$\sigma_S^A(\mathbf{h},\mathbf{h}_B,\boldsymbol{I})=\frac{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}\mathbf{D}(\mathbf{h}-\mathbf{h}_B)_{\boldsymbol{I}}}{\sigma_s(\mathbf{h},\mathbf{h}_B)} \qquad (60)$$

A subgroup risk constraint under the additive definition is an upper bound on any one of the risk terms defined by (58)-(60).

[2] Benchmark assets outside the subgroup may still influence the covariances.

Subgroup Risk – Non-Additive Definition

The non-additive definition of the subgroup risk in Barra Optimizer is simply based on elimination of the entries not belonging to the subgroup from the covariance matrices.

For example, let $\boldsymbol{I} = \{2, 5, 8\}$ be a subset of asset or factor indexes. Then, the covariance matrix for subgroup $\boldsymbol{I}$ would be:

$$\mathbf{V}_{\boldsymbol{I}} = \begin{bmatrix} \sigma^2_{2,2} & \sigma^2_{2,5} & \sigma^2_{2,8} \\ \sigma^2_{5,2} & \sigma^2_{5,5} & \sigma^2_{5,8} \\ \sigma^2_{8,2} & \sigma^2_{8,5} & \sigma^2_{8,8} \end{bmatrix} \tag{61}$$

where, $\sigma^2_{i,j}$ is an element in the i^{th} row and j^{th} column of the original covariance matrix $\mathbf{V}$.

More formally, let $\boldsymbol{I}$ be a subset of asset indexes and $\boldsymbol{J}$ be a subset of factor indexes. Then, the total risk from sub-group {$\boldsymbol{I}$, $\boldsymbol{J}$ } under the non-additive definition is defined as:

$$\sigma(\mathbf{h}, \mathbf{h}_B, \boldsymbol{I}, \boldsymbol{J}) = \sqrt{(\mathbf{h}_{\boldsymbol{I}} - \mathbf{h}_B)^T (\mathbf{D} + \mathbf{X}\,\mathbf{F}_{\boldsymbol{J}}\mathbf{X}^T)(\mathbf{h}_{\boldsymbol{I}} - \mathbf{h}_B)} \tag{62}$$

the factor risk from sub- group {$\boldsymbol{I}$, $\boldsymbol{J}$ } is defined as:

$$\sigma_F(\mathbf{h}, \mathbf{h}_B, \boldsymbol{I}, \boldsymbol{J}) = \sqrt{(\mathbf{h}_{\boldsymbol{I}} - \mathbf{h}_B)^T \mathbf{X}\,\mathbf{F}_{\boldsymbol{J}}\mathbf{X}^T (\mathbf{h}_{\boldsymbol{I}} - \mathbf{h}_B)} \tag{63}$$

and the specific risk from sub-group $\boldsymbol{I}$ is defined as:

$$\sigma_S(\mathbf{h}, \mathbf{h}_B, \boldsymbol{I}) = \sqrt{(\mathbf{h}_{\boldsymbol{I}} - \mathbf{h}_B)^T \mathbf{D} (\mathbf{h}_{\boldsymbol{I}} - \mathbf{h}_B)} \tag{64}$$

where,

$\mathbf{h}_{\boldsymbol{I}}$	an $n \times 1$ holding vector in which all positions are zeroes if they are not in $\boldsymbol{I}$
$\mathbf{F}_{\boldsymbol{J}}$	the modified factor covariance matrix $\mathbf{F}$ in which rows and columns are all zeroes if their indexes are not in $\boldsymbol{J}$
$\mathbf{h}_B$	the original benchmark vector which contains all benchmark assets

A benefit of this simple definition is that an upper bound on subgroup risk defined this way would be a convex constraint. A drawback of this definition is that risk is non-additive—the sum of risks from all subgroups does not equal to the overall risk.

Note: When defining risk from a subgroup under the non-additive definition, factors that do not belong to the subgroup are removed from both the portfolio exposures and the benchmark exposures. This is not true for assets. Assets that do not belong to the subgroup are removed from the portfolio, but not from the benchmark. If the users desire to subgroup the benchmark weights as well, a custom benchmark containing weights for just the subgroup assets should be created.

3.7.4.2.2 Subgroup Risk Contribution Constraints

In addition to constraining the absolute risk level from a subgroup, Barra Optimizer allows users to set an upper bound on the fractional risk contribution from a subgroup, which is the ratio of the subgroup risk to the portfolio-level risk. The presence of these constraints renders the optimization problem both non-convex and nonlinear. In general, Barra Optimizer may fail or return a suboptimal solution when solving a case containing non-convex risk constraints. Caution should be exercised when imposing such a constraint.

Subgroup Risk Contribution – Additive Definition

The following three types of subgroup risk contribution constraints are supported under the additive definitions:

$$l \le \frac{\sigma^A(\mathbf{h},\mathbf{h}_B,\boldsymbol{I})}{\sigma(\mathbf{h},\mathbf{h}_B)} \le u \tag{65}$$

$$l \le \frac{\sigma_F^A(\mathbf{h},\mathbf{h}_B,\boldsymbol{J})}{\sigma_F(\mathbf{h},\mathbf{h}_B)} \le u \tag{66}$$

$$l \le \frac{\sigma_S^A(\mathbf{h},\mathbf{h}_B,\boldsymbol{I})}{\sigma_S(\mathbf{h},\mathbf{h}_B)} \le u \tag{67}$$

where $\sigma^A(\mathbf{h},\mathbf{h}_B,\boldsymbol{I})$, $\sigma_F^A(\mathbf{h},\mathbf{h}_B,\boldsymbol{J})$, $\sigma_S^A(\mathbf{h},\mathbf{h}_B,\boldsymbol{I})$, $\sigma(\mathbf{h},\mathbf{h}_B)$, $\sigma_F(\mathbf{h},\mathbf{h}_B)$ and $\sigma_S(\mathbf{h},\mathbf{h}_B)$ are as defined in (55)-(60).

Subgroup Risk Contribution – Non-Additive Definition

The following three types of subgroup risk contribution constraints are supported under the non-additive definitions:

$$l \le \frac{\sigma(\mathbf{h},\mathbf{h}_B,\boldsymbol{I},\boldsymbol{J})}{\sigma(\mathbf{h},\mathbf{h}_B)} \le u \tag{68}$$

$$l \le \frac{\sigma_F(\mathbf{h},\mathbf{h}_B,\boldsymbol{I},\boldsymbol{J})}{\sigma(\mathbf{h},\mathbf{h}_B)} \le u \tag{69}$$

$$l \le \frac{\sigma_S(\mathbf{h},\mathbf{h}_B,\boldsymbol{I})}{\sigma(\mathbf{h},\mathbf{h}_B)} \le u \tag{70}$$

where $\sigma(\mathbf{h},\mathbf{h}_B)$, $\sigma(\mathbf{h},\mathbf{h}_B,\boldsymbol{I},\boldsymbol{J})$, $\sigma_F(\mathbf{h},\mathbf{h}_B,\boldsymbol{I},\boldsymbol{J})$ and $\sigma_S(\mathbf{h},\mathbf{h}_B,\boldsymbol{I})$ are as defined in (55), and (62)-(64).

3.7.4.3 Individual Asset-Level Risk Constraints

An individual asset-level risk constraint is a special case of the subgroup risk constraints where the subgroup contains only one asset. Barra Optimizer 8.9 and later versions allow users to constrain either the total risk or total risk contribution coming from an individual asset. We call these constraints risk-by-asset or risk-contribution-by-asset constraints. These constraints may be set either globally or locally. When both global and local bounds are applied on the same asset, the tighter bound prevails.

3.7.4.3.1 Risk-by-Asset Constraints

Let $\mathbf{h}_{\{i\}}$ be an $n \times 1$ holding vector in which all elements are zeroes except at the i^{th} position. Then the total risk from asset i under the additive-definition is:

$$\sigma^A(\mathbf{h},\mathbf{h}_B,i) = \frac{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}(\mathbf{D}+\mathbf{XFX}^{\mathrm{T}})(\mathbf{h}-\mathbf{h}_B)_{\{i\}}}{\sigma(\mathbf{h},\mathbf{h}_B)} \tag{71}$$

whereas under the non-additive definition is:

$$\sigma(\mathbf{h},\mathbf{h}_B,i) = \sqrt{(\mathbf{h}_{\{i\}}-\mathbf{h}_B)^T(\mathbf{D}+\mathbf{X}\,\mathbf{F}\,\mathbf{X}^T)(\mathbf{h}_{\{i\}}-\mathbf{h}_B)} \tag{72}$$

As usual, the benchmark vector $\mathbf{h}_B$ could be NULL. A risk-by-asset constraint takes on the form:

$$l \le \sigma^A(\mathbf{h},\mathbf{h}_B,i) \le u \tag{73}$$

or

$$l \le \sigma(\mathbf{h},\mathbf{h}_B,i) \le u \tag{74}$$

In other words, a range can be specified, not just an upper bound. However, a lower bound will render the problem non-convex and make it harder to solve.

3.7.4.3.2 Risk-Contribution-by-Asset Constraints

The total risk contribution from asset i under the additive-definition is:

$$\frac{\sigma^A(\mathbf{h},\mathbf{h}_B,i)}{\sigma(\mathbf{h},\mathbf{h}_B)} \tag{75}$$

whereas under the non-additive definition is:

$$\frac{\sigma(\mathbf{h},\mathbf{h}_B,i)}{\sigma(\mathbf{h},\mathbf{h}_B)} \tag{76}$$

A risk-contribution-by-asset constraint takes on the form:

$$l \le \frac{\sigma^A(\mathbf{h},\mathbf{h}_B,i)}{\sigma(\mathbf{h},\mathbf{h}_B)} \le u \tag{77}$$

or

$$l \le \frac{\sigma(\mathbf{h},\mathbf{h}_B,i)}{\sigma(\mathbf{h},\mathbf{h}_B)} \le u \tag{78}$$

3.7.4.4 Upper Bound on a General Convex Quadratic Function

Some risk measures can take the form of a general convex quadratic function, and users may have the desire to set an upper bound on such a function. Starting with Barra Optimizer 9.2, the following new type of risk constraint is supported:

$$(\mathbf{h}-\mathbf{h}_B)^T\mathbf{Q}(\mathbf{h}-\mathbf{h}_B)+\mathbf{q}^T(\mathbf{h}-\mathbf{h}_B) \le u \tag{79}$$

where,

$\mathbf{Q}$	an $n \times n$ symmetric positive semi-definite matrix
$\mathbf{q}$	an $n \times 1$ vector of linear coefficients

$\mathbf{h}_B$	a benchmark vector (optional)
u	the upper bound on the convex quadratic function.

3.7.5 Risk-Target Optimization

As an alternative to the typical mean-variance portfolio optimization described in Section 3.1.1, Risk Target Optimization allows users to set a specific target on the portfolio risk level. Mathematically, the problem can be represented as:

$$\text{Maximize:} \quad \lambda_r \mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda_{Tc} Tc(\mathbf{h},\mathbf{h}_0) - \lambda_{Pen} Pen(\mathbf{s},\mathbf{w}) - \text{Other Terms} \tag{80}$$

$$\text{Subject to:} \quad \sigma(\mathbf{h}) = \sigma_{Target} \tag{81}$$

$$\text{Other Constraints}$$

where $\sigma(\mathbf{h})$ is the standard deviation of the portfolio return or active return, and has the following flexible form:

$$\sigma(\mathbf{h}) = \sqrt{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}(\theta_D\,\mathbf{D} + \theta_F\,\mathbf{XFX}^{\mathrm{T}})(\mathbf{h}-\mathbf{h}_B)} \tag{82}$$

where θ_D and θ_F are two scalars for controlling the relative impact of specific and factor risk, and the benchmark $\mathbf{h}_B$ could be empty. Risk Target Optimization is a special case of Varying Risk in the Risk-Return Efficient Frontiers section.

On successful exit, Barra Optimizer reports an implied risk aversion, λ_{imp} for each risk target problem. Assuming no non-convex features are present, using this value as the risk aversion in the following standard optimization problem would produce an optimal risk value identical or similar to the original risk target.

$$\text{Maximize:} \quad \lambda_r \mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda_{imp}\cdot\sigma(\mathbf{h})^2 - \lambda_{Tc} Tc(\mathbf{h},\mathbf{h}_0) - \lambda_{Pen} Pen(\mathbf{s},\mathbf{w}) - \text{Other Terms} \tag{83}$$

$$\text{Subject to:} \quad \text{Other Constraints}$$

By default, risk target acts as a soft constraint. Risk Target Optimization will only return a portfolio on the efficient frontier. If the given risk target is higher than the risk of the maximum-return portfolio on the efficient frontier, Barra Optimizer will return the maximum-return portfolio with a message indicating that the risk target is too high.

Starting with Barra Optimizer 8.8, however, users have an option to treat risk target as a hard bound on risk. When the option “CHECK_EFFICIENCY” is set to “false” (default is “true”), Barra Optimizer will return a portfolio with the targeted risk even if it is not on the efficient frontier, as long as such a feasible portfolio exists. When no feasible portfolio with the targeted risk exists, Barra Optimizer will return a portfolio as close to the given target as possible and declare the problem as “infeasible”. This option may be useful for risk target optimization without alpha.

Risk Target Optimization differs from the general Risk-Constrained Optimization in several aspects, including:

- It has no risk term in the objective. By default, if the risk target is on the efficient frontier, then constraint (81) must be binding.
- Users do not supply the risk aversion values. Instead, Barra Optimizer will derive and output an implied risk aversion value. Note that if the risk target optimization is not successful, this implied risk aversion value may not be meaningful and the default value "-1" may be returned.
- When the risk target or the lower bound on risk is not on the efficient frontier (i.e., the required risk is either too high or too low), Risk Target Optimization and Risk-Constrained Optimization may output different messages.

For more information on Risk Target Optimization and its relationship with Mean-Variance Optimization, see Kopman and Liu (2009).

3.8 Transaction Cost, Holding Cost, and Turnover Control

Transaction cost (in decimals) may appear not only in the objective function, but also in a constraint. Barra Optimizer allows an upper limit on the total transaction cost in the form of constraint. (10) For example, users may require that the portfolio's total transaction cost be no more than 3% of the portfolio value. Separate transaction cost limits on buying and selling equities may also be imposed by using a number of general piecewise-linear constraints. However, transaction cost limit constraints may contain only piecewise-linear functions. Constraints on nonlinear or fixed transaction costs are currently not supported.

Barra Optimizer accommodates a wide variety of transaction cost functions in the objective. As explained more in detail in the subsequent subsections, transaction costs can be modeled either by traditional piecewise-linear functions ($Tc_p(\mathbf{h},\mathbf{h}_0)$) or by nonlinear power functions ($Tc_n(\mathbf{h},\mathbf{h}_0)$), or as a fixed cost per trade ($Tc_f(\mathbf{h},\mathbf{h}_0)$). The transaction cost term in (1) can be written as the sum of three optional terms:

$$Tc(\mathbf{h},\mathbf{h}_0) = Tc_p(\mathbf{h},\mathbf{h}_0) + Tc_n(\mathbf{h},\mathbf{h}_0) + Tc_f(\mathbf{h},\mathbf{h}_0) \tag{84}$$

3.8.1 Piecewise-Linear Transaction Costs

A simple piecewise-linear transaction cost function (e.g., the left part of Figure 2) has the following form:

$$Tc_p(\mathbf{h},\mathbf{h}_0) = \sum_{i=1}^{n}\left[c_{i\,buy} \cdot \max(\mathbf{h}_i - \mathbf{h}_{0i}, 0) + c_{i\,sell} \cdot \max(\mathbf{h}_{0i} - \mathbf{h}_i, 0)\right] \tag{85}$$

where, $c_{i\,buy}$ and $c_{i\,sell}$ are unit transaction costs for buying and selling, respectively. To ensure convexity, it must be true that $c_{i\,buy} \geq 0$ and $c_{i\,sell} \geq 0$.

The Barra Market Impact Model provides more sophisticated transaction cost handling, and may generate complicated piecewise-linear functions with multiple break points at both the buy and sell sides (e.g., the right part of Figure 2). More details on such functions are provided in Section 3.6.5.

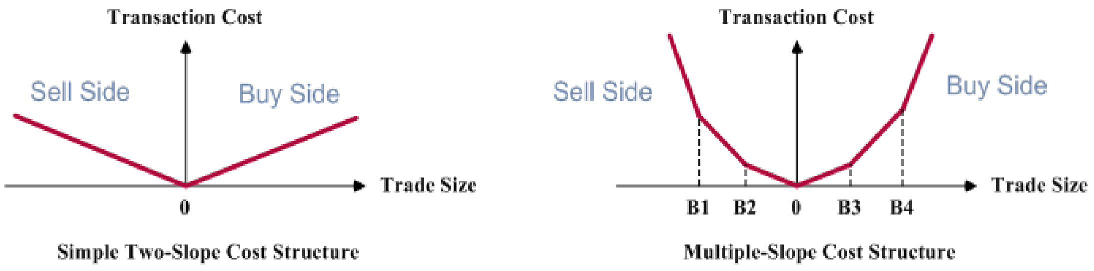


Figure 2: Piecewise-Linear Transaction Cost

3.8.2 Non-Linear Transaction Costs

Barra Optimizer allows non-linear transaction costs in the following form:

$$Tc_n(\mathbf{h}, \mathbf{h}_0) = \sum_{i=1}^{n} r_i \left|\mathbf{h}_i - \mathbf{h}_{0i}\right|^{p_i} \tag{86}$$

where $r_i > 0$ is the non-linear transaction cost multiplier and $p_i > 1$ is the exponent of the asset-level power function. Figure 3 provides an illustration of such a function.

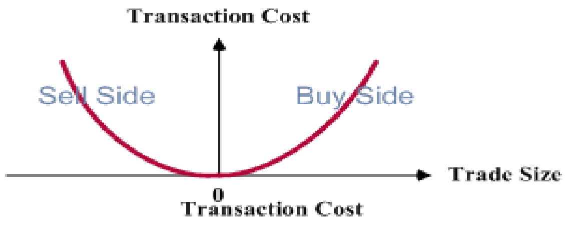


Figure 3: Non-linear Transaction Cost

To specify a proper value for the coefficient r_i, you may use the following formula:

$$r_i = \frac{c_i}{q_i^{p_i}} \tag{87}$$

This means that the transaction cost would be c_i (in decimal), if the trading amount $\left|\mathbf{h}_i - \mathbf{h}_{0i}\right|$ is q_i (in decimal).

Notes:

- Cash and currencies are not included in non-linear transaction costs.
- Barra Optimizer currently does not support upper bounds on non-linear transaction costs.

3.8.3 Fixed Transaction Costs

Barra Optimizer also supports general fixed transaction costs in the following form:

$$Tc_f(\mathbf{h}, \mathbf{h}_0) = \sum_{i=1}^{n} fc_i \tag{88}$$

where, fc_i is the fixed transaction cost for asset i and is defined as:

$$fc_i = \begin{cases} c_{f\,b_i}, & h_i > h_{0i} \\ 0, & h_i = h_{0i} \\ c_{f\,s_i}, & h_i < h_{0i} \end{cases} \tag{89}$$

In (89), $c_{f\,b_i}$ and $c_{f\,s_i}$ are two constants representing the fixed cost for asset i at the buy side and sell side, respectively. Figure 4 shows an example of the impact of such a function.

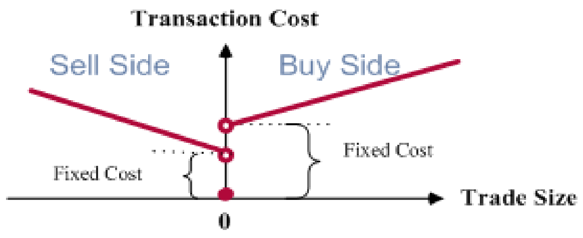


Figure 4: Piecewise-Linear Plus Fixed Transaction Cost

A crossover trade makes an asset change from long position to short position, or vice versa. A crossover trade may be counted as one trade or two trades. For example, if you count a crossover trade as two trades, an asset with an initial position of 2% is sold with a final position of -1%, and the fixed transaction costs would double the amount of the sell cost.

Note: Barra Optimizer currently does not support upper bounds on fixed transaction costs.

3.8.4 Fixed Holding Costs

Similar to the definition of fixed transaction costs, fixed holding costs can be modeled as follows:

$$Hc_f(\mathbf{h},\mathbf{h}_0) = \sum_{i=1}^{n} fh_i \tag{90}$$

where, fh_i is the fixed holding cost for asset i and is defined as:

$$fh_i = \begin{cases} c_{u_i}, & h_i > h_{ri} \\ 0, & h_i = h_{ri} \\ c_{d_i}, & h_i < h_{ri} \end{cases} \tag{91}$$

In (91), c_{u_i} (or c_{d_i}) is a constant representing the fixed cost for holding asset i more (or less) than asset i's weight in a reference portfolio— h_{ri}. The reference portfolio could be the initial or a benchmark portfolio, or even empty. Figure 5 shows an example of the impact of such a function.

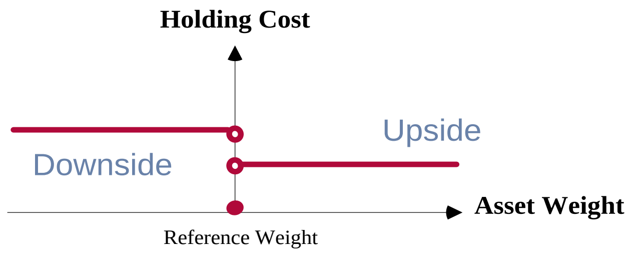


Figure 5: Fixed Holding Cost

The fixed holding costs $Hc_f(\mathbf{h},\mathbf{h}_0)$ enter into the objective (1) as part of the term $g(\mathbf{h})$.

Note: Barra Optimizer currently supports fixed holding costs only in the objective function, not in the constraints. Additionally, combination of fixed transaction costs and fixed holding costs is not allowed.

3.8.5 Turnover Functions and Constraints

Barra Optimizer allows users to set an upper bound on the amount of overall portfolio turnover[3] or turnover by group. When there is no cash flow (either cash infusion or cash withdrawal), the total amount of equity buys must equal the total amount of equity sells. Under such circumstances, the overall turnover in Barra Optimizer is defined as:

$$\frac{Equity\ Buys + Equity\ Sells + |Change\ in\ Cash\ Position|}{2 \cdot TurnoverBaseValue} \tag{92}$$

When cash flow does occur, the overall turnover is defined as:

$$\frac{Equity\ Buys + Equity\ Sells + |Change\ in\ Cash\ Position| - |Cash\ Flow|}{2 \cdot TurnoverBaseValue} \tag{93}$$

In terms of weights, a turnover function can be rewritten as shown below:

$$TO(\mathbf{h},\mathbf{h}_0) = \frac{1}{2}\left(\sum_{j \in J}\left|h_j - h_{0j}\right| - \left|CFW\right|\right) / TBVW \tag{94}$$

where, $TBVW = TurnoverBaseValue / BV$ is the weight of the turnover base value

3.8.5.1 Turnover Base Value

Barra Optimizer offers the following two possible definitions for TurnoverBaseValue:

1. Default definition:

$$\begin{aligned}\text{TurnoverBaseValue} = \text{InitialLongPositions} + \left|\ \text{InitialShortPositions}\right| \\ + \left|\text{InitialCashPositions}\right| + \text{CashFlow}\end{aligned} \tag{95}$$

$TBVW = \sum_{j \in J}\left|h_{0j}\right| + CFW$ and the turnover function can be written as:

$$TO(\mathbf{h},\mathbf{h}_0) = \frac{1}{2}\left(\sum_{j \in J}\left|h_j - h_{0j}\right| - \left|CFW\right|\right) / \left(\sum_{j \in J}\left|h_{0j}\right| + CFW\right) \tag{96}$$

Optionally, futures can be included.

2. Use the same base value BV that is used for asset weight computation:

$$\text{TurnoverBaseValue} = \text{BV} \tag{97}$$

[3] In API, overall turnover is also called net turnover, in contrast to long-side turnover and short-side turnover.

$TBVW = 1$ and the turnover function can be written as shown below:

$$TO(\mathbf{h}, \mathbf{h}_0) = \frac{1}{2}\left(\sum_{j \in J} \left|h_j - h_{0j}\right| - \left|CFW\right|\right) \tag{98}$$

When there is no cash flow, the numerator of the turnover can be written as:

$$\max(Equity\,buys, Equity\,sales) \tag{99}$$

For multi-period optimization, only the second option is supported.

3.8.5.2 Upper Bound on Buy or Sell Turnover

In addition to the overall turnover, Barra Optimizer also supports buy-side and sell-side turnover limit constraints:

Buy-Side Turnover Constraint:

$$\frac{Equity\ Buys}{TurnoverBaseValue} \le MaxBuyTurnover\,; \tag{100}$$

Sell-Side Turnover Constraint:

$$\frac{Equity\ Sells}{TurnoverBaseValue} \le MaxSellTurnover\,. \tag{101}$$

In terms of weights, these constraints are:

$$\begin{aligned} &\sum_{i \in I} Max(h_i - h_{0i}, 0.0) \le MaxBuySideTurnover \cdot TBVW \\ &\sum_{i \in I} Max(h_{0i} - h_i, 0.0) \le MaxSellSideTurnover \cdot TBVW \end{aligned} \tag{102}$$

where I is the index set of assets, excluding cash and futures. Optionally, futures can be included.

Furthermore, for long/short optimization, Barra Optimizer allows you to specify additional turnover limits by side. For details, see the By-Side Optimization section.

3.8.5.3 Upper Bound on Turnover by Group

In addition to an upper bound on turnover at the whole portfolio-level, Barra Open Optimizer also supports an upper bound on turnover at the group-level. Examples of such constraints are, "turnover from asset group *K* is at most 20%", or "total buys in asset group J is at most 10%". The definition for turnover by group is same as the definition for overall turnover as defined above except that the numerator will be defined on the given asset group only.

Note that users can set multiple turnover-by-group constraints simultaneously. Futures are excluded by default and can be included optionally.

3.8.5.4 Option to Exclude Cash

In the default definitions above, cash is included in the numerator.

Barra Open Optimizer now provides an option to exclude cash. With this option, both cash position change and cash flow will be excluded from the numerator of the turnover definition. In other words, turnover will be simply defined as:

$$\frac{Equity\ Buys + Equity\ Sells}{2 \cdot TurnoverBaseValue}. \tag{103}$$

To further exclude cash from the denominator of the turnover definition, you may use a user-defined turnover base value that does not include any cash.

3.9 Constraint Flexibility—Penalties, Soft Bounds, and Hierarchy

3.9.1 Penalty Functions

Penalty functions are used to tilt portfolios toward user-specified targets on constraint or factor slacks, as well as targets on asset weights (either ordinary or active weights). Deviation away from a target causes a negative impact on the objective function. Some of the penalty terms are part of the $g(\mathbf{h})$ in (1) .

One method to specify a penalty function is to use three user-specified parameters—*Target*, *Upper*, and *Lower*. *Target* is the desired value of the constraint slack in question. *Upper* (*Lower*) is a given value above (below) *Target* at which a 1% deduction of the objective value, before any multiplier adjustment, will be imposed. At values other than these three parameters, different penalty functions usually affect the objective value differently.

Barra Optimizer supports both quadratic penalties and pure linear penalties. For pure linear penalties, the target can be either a single point or a free range. Figure 6 illustrates the six types of penalty functions that Barra Optimizer supports. The following sections provide more details on these penalty functions.

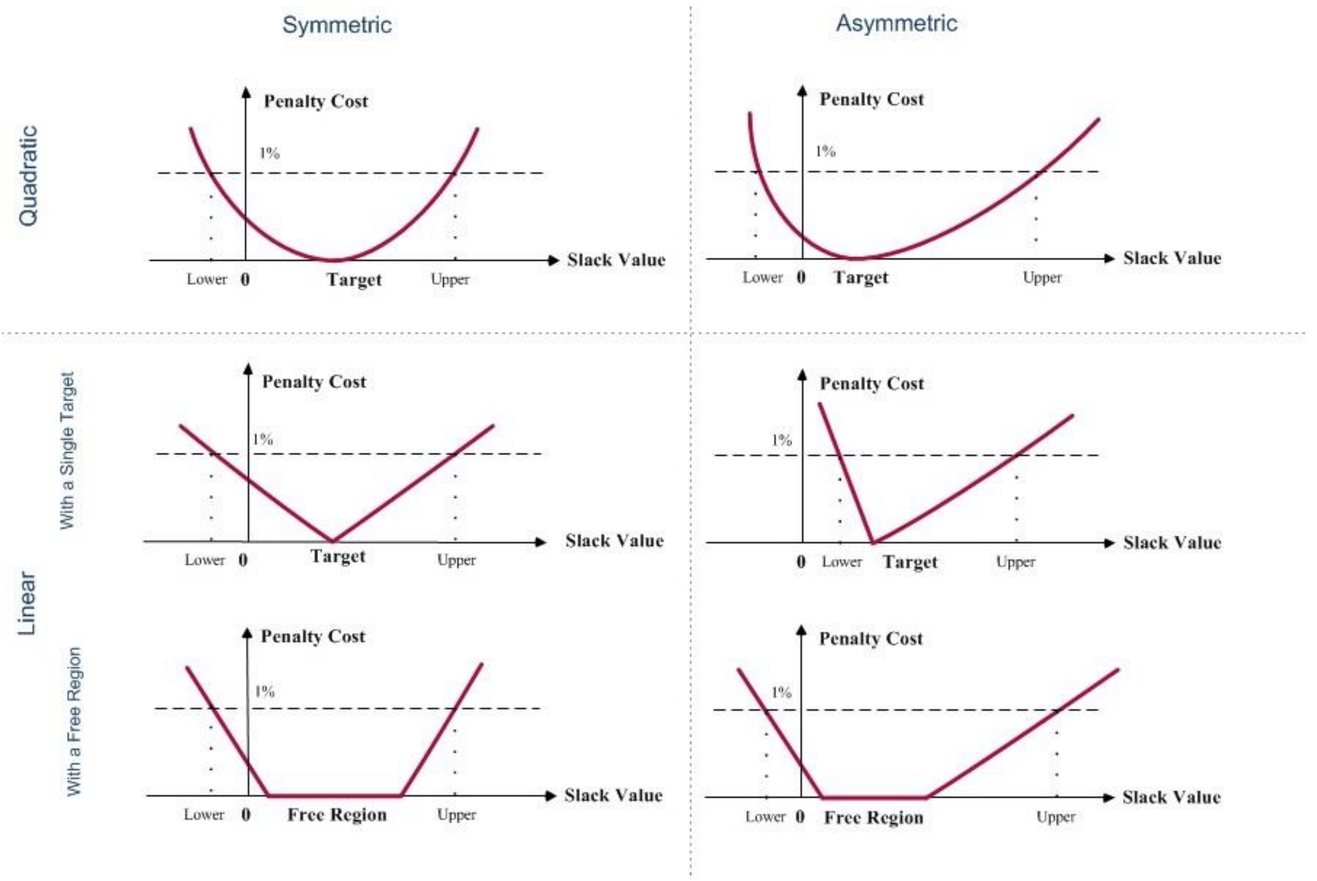


Figure 6: Different Types of Penalty Functions

3.9.1.1 Quadratic vs. Linear Penalties

Let v denote a generic constraint slack variable. Then, pure quadratic penalties are in the form:

$$Pen_s(v) = \rho_q \cdot (v - Target)^2 \,, \tag{104}$$

whereas pure linear penalties with a single target are represented by:

$$Pen_u(v) = \rho_{up} \cdot \max(v - Target,\ 0) \tag{105}$$

$$Pen_d(v) = \rho_{down} \cdot \max(Target - v,\ 0)\,. \tag{106}$$

To ensure the 1% deduction in objective at the two points— $v = Upper$ or $v = Lower$, it must be true that the slopes in the above penalty functions are given by:

$$\rho_q = 0.01/(Upper - Target)^2 \tag{107}$$

$$\rho_{up} = 0.01/(Upper - Target) \tag{108}$$

$$\rho_{down} = 0.01/(Target - Lower)\,. \tag{109}$$

In the case of pure quadratic penalties, "$Upper - Target = Target - Lower$" must also hold.

3.9.1.2 Symmetric vs. Asymmetric Penalties

A symmetric penalty is characterized by the property:

$$Upper - Target = Target - Lower\,. \tag{110}$$

By definition, a pure quadratic penalty is a symmetric penalty. The converse is not true. A symmetric penalty is not necessarily quadratic because it can be pure linear or piecewise-linear.

In contrast, an asymmetric penalty has the property:

$$Upper - Target \neq Target - Lower \tag{111}$$

It is usually synthesized by a pure (symmetric) quadratic penalty plus two pure linear penalties, and can be represented by:

$$Pen_a(v) = Pen_s(v) + Pen_u(v) + Pen_d(v) \tag{112}$$

Let $BigRange = \max(Upper - Target,\ Target - Lower)$. It is not difficult to show that to ensure the 1% deduction in objective at $v = Upper$ and $v = Lower$, the slopes implied in the three penalty terms in (112) must be the following:

$$\rho_q \quad = 0.01/BigRange^2 \tag{113}$$

$$\rho_{up} \quad = \begin{cases} 0, & if\ Upper - Target = BigRange \\ 0.01\left[1/(Upper - Target) - (Upper - Target)/BigRange^2\right], & otherwise \end{cases} \tag{114}$$

$$\rho_{down} = \begin{cases} 0, & \text{if } Target - Lower = BigRange \\ 0.01\left[1/(Target - Lower) - (Target - Lower)/BigRange^2\right], & otherwise \end{cases} \quad (115)$$

3.9.1.3 Pure Linear Penalties with a Single Target vs. with a Free Region

Pure linear penalties with a single target are given by (105) - (106). Similarly, pure linear penalties with a free region are in the form:

$$Pen_u(v) = \rho_{up} \cdot \max(v - TargetHigh,\ 0) \quad (116)$$

$$Pen_d(v) = \rho_{down} \cdot \max(TargetLow - v,\ 0) \quad (117)$$

Note that when $TargetHigh = TargetLow$, the penalty target is reduced from a free range to a single point.

To specify a pure linear penalty function with a free range, users are asked to provide the two end points of the free region—$TargetLow$ and $TargetHigh$, as well as the slopes at each side of the free region—ρ_{up} and ρ_{down} directly. To ensure the convexity of the penalty function, the slopes should satisfy the condition $-\rho_{down} \le 0 \le \rho_{up}$. It is permissible for both slopes to be zero in this case.

3.9.1.4 Summary

When penalty target is a single point, a penalty function $Pen(v)$ has the following characteristics:

- $Lower < Target < Upper$
- $Pen(Target)=0$
- $Pen(Lower)=Pen(Upper)=0.01$
- The smaller the value of $Target - Lower$ (i.e. the closer $Lower$ is to $Target$), the larger the penalty value $Pen(v)$ will be when $v < Target$
- Similarly, the smaller the value of $Upper - Target$ (i.e. the closer $Upper$ is to $Target$), the larger the penalty value $Pen(v)$ will be when $v > Target$
- To avoid numerical difficulties, the values of $Lower$ and $Upper$ should not be too close to the value of $Target$. In the actual implementation, the quadratic penalty function (104) is modified to be,

$$Pen_s(v) = \min\left(1.0e+5,\quad \lambda_{Pen} \cdot 0.01/BigRange^2\right) \cdot (v - Target)^2 \quad (118)$$

The user may apply penalties only for the following constraint categories:

- Asset bounds (i.e., bounds on ordinary or active asset weights)
- General linear constraints (including beta constraint and group linear constraints)
- Factor constraints

- Hedge (leverage) constraints
- Total leverage constraints

3.9.2 Soft Constraints

Soft constraints are useful for reducing the possibility of Barra Optimizer not finding a feasible solution. They are another set of valuable tools for users to manage their constraints. All constraints in the usual sense are hard requirements that an optimal portfolio ought to obey. Hard constraints are usually inflexible and may often lead to infeasibility, leaving the portfolio optimization problem unsolved. On many occasions, infeasibility is due to conflicts among a small group of constraints. Relaxing or "softening" some of the hard constraints may help yield a feasible solution.

Most constraints can be specified as soft constraints in Barra Optimizer. The few exceptions include asset bounds, 5/10/40 rules, and tax constraints. The upper and lower bounds of a soft constraint are treated as a nice-to-have but not a must. For well-behaved convex problems, Barra Optimizer will return an optimal portfolio satisfying the bounds if such a portfolio exists. Otherwise, it will try to find a portfolio with minimal violation of the constraint. For non-convex problems, Barra Optimizer will return the best available feasible solution if it can find one. Otherwise, it will return the portfolio that has the least violation of the constraint among all the portfolios at its disposal. Again, there is no guarantee that the solutions are globally optimal in the non-convex cases.

A soft constraint may be violated due to conflicts with other constraints, particularly in cases where a large number of other constraints and penalties are defined. Barra Optimizer internally applies a penalty to the violation of the soft constraint to discourage the constraint slack moving away from the soft bound. This is not applicable to soft constraints on thresholds, cardinality, roundlotting, and risk constraints.

Barra Optimizer produces a log of the violated soft constraints as part of the optimization output. It supports soft constraints for the following constraint categories:

- General linear constraints (including beta constraint and group constraints)
- Factor constraints
- Turnover constraints
- Transaction cost constraint
- Hedge (leverage) constraints, including total leverage constraint, short/long ratio constraint, and net/total leverage constraint
- Threshold constraints
- Cardinality constraints
- Risk constraints
- Roundlotting constraint

3.9.3 Constraint Hierarchy

A constraint hierarchy is designed to provide users with greater flexibility in arranging and managing their constraints. It allows users to specify constraint priorities in the event of infeasibility.

When no solution can be found that satisfies all constraints simultaneously, it will systematically and sequentially soften the constraints, one, or several at a time, according to their relaxation priorities. In such cases, it will search for a solution with minimal violation with respect to the original constraints. Constraints in the lower levels of the hierarchy are softened first. Constraints not pre-set in the hierarchy will not be relaxed at all. Constraints softened in a lower level of the hierarchy will remain soft for higher levels.

This procedure continues until a feasible solution to the softened problem for a given level of the hierarchy is found, or until all levels of the hierarchy are exhausted.

Barra Optimizer allows you to prioritize the following constraint categories:

- General linear constraints
- Factor constraints
- Turnover constraints
- Transaction cost constraint
- Holding cardinality constraints
- Holding threshold constraints
- Transaction threshold constraints
- Trade cardinality constraints
- Paring constraints, which include all of the above four threshold and cardinality categories
- Hedge (leverage) constraints, including total leverage constraint, short/long ration constraint, and net/total leverage constraint
- Risk constraints
- Roundlotting constraints

Barra Optimizer also allows you to set the priority for individual constraints from categories of general linear constraints and factor constraints. You can categorize your constraints based on priority. They are relaxed in the following order:

Priority	Relax Order
Low	Relax first
Medium	Relax next
High	Relax last
Not in Hierarchy	Never Relax

With constraint hierarchies defined, Barra Optimizer will resolve optimization problems sequentially as follows:

- Tries to solve the original problem; returns successfully if the problem is solved.
- If finding an optimal solution fails, relaxes constraints with low priority and tries to solve the problem; returns successfully if the problem is solved.
- If finding an optimal solution still fails, relaxes constraints with medium priority and tries to solve the problem; returns successfully if the problem is solved.
- If finding an optimal solution still fails, relaxes constraints with high priority and tries to solve the problem; returns successfully if the problem is solved.
- If finding an optimal solution still fails, terminates and reports failure.

Notes:

- More than one constraint category may be in the same hierarchy (with the same priority).
- When relaxing a constraint (except threshold, cardinality, roundlotting, and risk constraints), Barra Optimizer will set soft bounds for the category so that violations of these constraints are discouraged by soft-bound penalties.
- If the problem has no constraints in a category that is specified in the constraint hierarchies, Barra Optimizer will ignore that category.
- Constraint categories not specified in the hierarchy will not be relaxed at all.
- This feature is especially useful if you do not want a lengthy back-testing process interrupted by infeasibilities caused by restrictive constraints

Constraint Hierarchy Example 1:

Constraint Category	Priority	Relax Order
General linear constraints	Low	Relax first
Turnover constraints	High	Relax last
Transaction cost constraint	Low	Relax first
Hedge constraints	Medium	Relax next
Trade cardinality constraints	Medium	Relax next
Paring constraints	High	Relax last

Considering Example 1:

1. Barra Optimizer first attempts to solve the problem without relaxing any constraints.
2. If the first attempt fails, it tries to solve a modified problem with general linear constraints and the transaction cost constraint relaxed.
3. If the second attempt fails, it tries to solve a modified problem with hedge constraints and trade cardinality constraints relaxed (in addition to constraints relaxed in the first attempt).
4. If the third attempt fails, it tries to solve a modified problem with turnover constraints and all threshold and cardinality constraints relaxed (in addition to constraints relaxed in the previous attempts).
5. Other constraints (for example, factor constraints) are not in the hierarchy and are not relaxed at all.

Constraint Hierarchy Example 2:

Constraint Category/Individual Constraints	Priority	Relax Order
Holding cardinality	Medium	Relax next
Holding threshold	Medium	Relax next
Transaction threshold	Low	Relax first
Factor Momentum constraint	High	Relax last

Considering Example 2:

1. Barra Optimizer first attempts to solve the problem without relaxing any constraints.
2. If the first attempt fails, it tries to solve a modified problem with minimum transaction size threshold relaxed.
3. If the second attempt fails, it tries to solve a modified problem with all three types of threshold and cardinality constraints relaxed.
4. If the third attempt fails, it tries to solve a modified problem with the Momentum factor constraint relaxed (in addition to constraints relaxed in the previous attempts).
5. Instead of softening the threshold and cardinality constraints during the optimization, they are simply relaxed.

3.10 Optimization Inspection

3.10.1 Optimality Conditions

An optimal portfolio ought to fulfill the following three sets of conditions, also known as the Karush-Kuhn-Tucker (KKT) conditions:

- Primal Feasibility Conditions—All primal constraints defined by the original portfolio optimization problem should be satisfied;
- Dual Feasibility Conditions—All conditions for the implied dual variables should be satisfied; and
- Complementary Slackness Conditions—All relationships between the slackness in a primal constraint and the sign of the associated dual variable should be satisfied.

On the other hand, a heuristic portfolio may not satisfy some of the conditions in the last two sets.

Barra Optimizer employs a number of solution methods, some of which are primal algorithms. These algorithms start with a portfolio that satisfies the primal and complementary slackness conditions, and then keep searching for better portfolios within the primal feasible region. The algorithms terminate when a portfolio satisfying the dual feasibility conditions is found.

In contrast, dual algorithms start with a portfolio that satisfies the dual feasibility conditions as well as the complementary slackness conditions. These algorithms keep searching for better portfolios within the dual feasible space until they find a portfolio that also satisfies the primal feasibility conditions.

Primal algorithms have an advantage over others in that they always return a portfolio satisfying all original constraints, even if they have to be terminated prematurely. For more information about the algorithms employed in Barra Optimizer, see Liu (2004).

3.10.2 Optimality Tolerances

Optimality tolerances affect the termination criteria of the optimization. Let $\mathbf{h}^*$ be the true theoretical optimal solution to a general Convex Quadratic Programming (CQP) problem, and $u(\mathbf{h}^*)$ be the corresponding true maximum objective. Suppose the optimality tolerance used by an optimizer is ε. Then, the optimizer may terminate its process at any time as long as the solution ($\mathbf{h}^{**}$) it returns satisfies:

$$\left| u(\mathbf{h}^{**}) - u(\mathbf{h}^*) \right| \le \varepsilon \tag{119}$$

In general, the tighter the optimality tolerance, the higher the quality of the optimal solution in terms of the objective function value, but the longer it may take for optimizer to reach its "tolerance-adjusted" optimality.

In practice, however, the true optimal objective is unknown. Different solvers may adopt different surrogate optimality evaluation criteria in lieu of (119). The optimality tolerances used may also be different. The impact of these optimality tolerances on speed will depend on the solver used.

To afford users some control over the tradeoffs between the speed and solution quality, Barra Optimizer offers an optimality tolerance multiplier to scale the default optimality tolerance. The default optimality tolerance multiplier is 1.0. To prevent unreasonable or unpractical settings, there is a permissible range for this multiplier. This range depends on the solver being used. Specifically, the permissible range for our quadratic solver and nonlinear programming solver is [0.01, 10000] and for our second-order cone programming solver is [0.01, 100].

If the user-defined optimality tolerance multiplier is outside the permissible range, the closest permissible value for the selected solver will be used. For example, if users set the multiplier to 1000, and the optimizer decides that the second-order cone programming solver is best suited for the given case, then the optimizer will reset the multiplier to 100 before applying it.

The effectiveness of the optimality tolerance multiplier will depend on the solver used as well as the case characteristics. In some cases, the effect may be counter-intuitive. For example, with cardinality or threshold constraints, larger optimality tolerance multipliers may result in a slower performance because the searching paths might be different.

Under usual circumstances, **leaving the default optimality tolerance multiplier unchanged is strongly recommended.**

3.10.3 Maximum Time Limit

Users can set a maximum time limit on solving a given optimization case. However, this limit is only treated as a guideline by the Barra Optimizer. Sometimes, an internal procedure or process may not be stopped once it starts, causing the time limit to be exceeded.

3.10.4 Indication of the Optimization Problem Type

Barra Optimizer can indicate whether an optimization problem is convex.

An optimization problem is convex if all of the following conditions hold true:

- Objective function is quadratic (not information ratio or Sharpe ratio).
- Fixed transaction costs are absent.
- Roundlotting constraint is not enabled.
- Threshold constraints are absent.
- Cardinality constraints are absent.
- 5/10/40 rule is not enabled.
- Optimization problem is not a tax-aware optimization.
- Turnover-by-side constraint is absent.
- L/S leverage constraint is absent, or all of the following conditions hold:
  - There is no S/L leverage ratio constraint.
  - There is no net/total leverage ratio constraint.
  - There is no positive lower bound on any of the long side leverage constraint(s).
  - There is no negative upper bound on any of the short side leverage constraint(s).

  - There is no positive lower bound on the total leverage constraint.
- There are no positive lower bounds on risk, no risk contribution constraints and no subgroup risk constraints or risk-by-asset constraints using the additive definition.

3.10.5 Indication of the Output Portfolio as Heuristic or Optimal

Barra Optimizer can indicate whether the output portfolio is heuristic or optimal. An output portfolio is optimal if the following conditions hold true:

- The problem is convex and there is no soft bound violation, or
- The objective is information/Sharpe ratio and the objective value is non-negative

3.10.6 Trade List Information

Barra Optimizer outputs the following trade list information for the optimal or roundlotted portfolio for a given asset:

- Trade type (hold, buy, sell, covered buy, short sell, crossover buy, and crossover sell)
- Initial shares
- Traded shares
- Final shares
- Price
- Traded value
- Traded value in percentage
- Fixed transaction cost
- Non-linear transaction cost
- Piecewise-linear transaction cost
- Total transaction cost

3.10.7 Solver Information

Starting with Barra Optimizer 8.0, users can obtain the solver information associated with the final solution. Such information may be used to understand why a case takes a long time and may also help decide which constraints to drop to speed up the optimization process. For example, if the solver associated with the final solution is the “Non-Linear Programming Solver”, risk constraints may be the cause for excessive run time. If they are not essential, consider relax or remove them.

3.10.8 Impact to Utility

Impact-to-utility is a value computed by the optimizer, based on duality theory, estimating the instantaneous rate of utility change if a binding bound is relaxed.

- It is a diagnostic tool helping to identify the most influential binding constraint.

- It is not always available for non-convex cases.

3.10.9 Small Weights or Trades

Due to numerical precisions used by different optimization engines, the optimal portfolio may contain small holdings or small trades that are within the tolerances specified. This phenomenon may be especially prominent with interior-point-method-based solvers. Starting with Barra Optimizer 8.6, users have three options if they want to eliminate these small weights or trades post-optimization:

1. Add the dropped weights to cash position or treat them as a cash flow.
2. Allocate the dropped holdings or trades to the remaining holdings or trades equally.
3. Allocate the dropped holdings or trades to the remaining holdings or trades proportionally to their relative sizes.

Note that the resulting portfolio after these adjustments may slightly violate some constraints or asset bounds. Starting with Barra Optimizer 9.0.3, an extra in-optimization procedure is added to help eliminate small weights or trades. The essence of this new procedure is to use our Quadratic Solver to clean up the solution produced by the Second-Order Cone Solver. More specifically,

- This procedure will only be triggered when the flag “DROP_TINY_TRADES” is turned on.
- When this procedure is on assets with weights below “ToleranceForZero” will be locked to zero; while assets with trades below “TINY_TRADE_THRESHOLD” will be locked to their initial values (so that they will have a zero trade). The tolerance used in locking these tiny weights or trades is “TINY_TRADE_TOLERANCE”, with a default value of 0. For example, suppose an asset’s optimal weight is 5.000005 and its initial weight is 5.0. Therefore, it has a tiny trade that is below the default value of 5e-5 for “TINY_TRADE_THRESHOLD”. We will lock this asset by setting its upper bound and lower bound both at 5.0 during the clean-up re-optimization.
- All other assets will be locked to their optimal solution with a tolerance level of “NON_TINY_TRADE_TOLERANCE”. For example, suppose an asset has an optimal weight of x and it is not a “tiny-trade” asset. Using the default value of 1e-5 for “NON_TINY_TRADE_TOLERANCE”, we will set this asset’s lower bound at x-1.0e-5, and its upper bound at x+1.0e-5 during the clean-up re-optimization.
- This procedure may fail since the clean-up re-optimization may result in infeasibility.

Please refer to Section 5.6 on how to set up these options.

3.10.10 Portfolios and Messages Returned

In general, for well-defined convex portfolio optimization problems, Barra Optimizer will return an optimal portfolio if the problem is feasible. For others, only a heuristic portfolio will be provided.

- Appendix A summarizes the types of portfolios and return messages that will be provided for various portfolio optimization problems.

- Appendix B shows which combinations of features and functions are currently supported in Barra Optimizer.

4 Advanced Optimization Features

4.1 Threshold and Cardinality Constraints—Paring Optimization

Threshold constraints are used to ensure non-zero trades or positions are sufficiently large. Cardinality constraints restrict the number of non-zero trades or positions to be less than or greater than a set bound. Both can be used to ensure the optimizer makes economically meaningful decisions. Barra Optimizer uses similar techniques for solving these two types of constraints, and thus they are often grouped together.

Inclusion of these constraints makes the optimization problem discrete and non-convex. These problems are well known to be difficult to solve to optimality. For a convex optimization problem, Barra Optimizer will be able to either correctly detect the problem as infeasible or return an optimal portfolio. This is not true for non-convex problems. The following analogy compares the nature and complexity of the two problems.

Imagine a glass marble is slipping down the side of an empty pool. If the pool is smoothly curved without any craters in the concrete, the initial position of the marble does not matter. One can always count on the marble to settle into a single spot, which must be the lowest point in the pool. This process resembles solving a well-behaved optimization case without discrete or non-convex constraints.

Introducing discrete constraints is like roughing up the smooth surface of the empty pool. The marble slipping down the side of such a pool might settle into a crater in the shallow end without ever reaching the deepest spot in the entire pool. In other words, it might get stuck at a local minimum without ever reaching the global minimum. Where the marble will finally end depends on its initial position and the height of the pool, as well as the configuration details of the pool bottom. Locating the lowest point in the cratered pool with a marble becomes trickier and less reliable in such cases.

Barra Optimizer often refers to threshold and cardinality constraints as paring constraints and the optimization problems that contain them as pairing problems, because Barra Optimizer uses a paring algorithm to solve them. These terms originated from Barra. Barra Optimizer's paring algorithm is a combination of heuristic procedures aimed at finding an approximate optimal solution within a reasonable amount of time. The traditional pare-down heuristic emphasizes optimality. It starts with a standard optimal solution and iteratively pares down excessive assets. By contrast, the build-up heuristic stresses feasibility. It forms a small feasible portfolio that satisfies the paring constraints first, and then gradually adds more assets to improve the objective, or utility. For more detailed discussions of the pare-down and build-up heuristics, as well as their performance results, see Liu and Xu (2016), Liu and Xu (2011) and Xu and Liu (2010).

The quality of heuristic paring solutions can be evaluated using a Branch-and-Bound algorithm in Barra Optimizer based on Lagrangian relaxation and cost splitting. This algorithm is also capable of providing truly optimal solutions for small-sized paring problems, as well as possibly improved heuristic solutions for large ones. For more information, see Kopman, Liu, and Shaw (2009).

The following sections describe cardinality and threshold constraints in detail.

Notes:

- Cash and futures are not counted in the threshold and cardinality constraints. Futures can optionally be counted in the threshold and cardinality constraints.
- Paring is implemented using heuristic approaches that try to find a good solution within a reasonable amount of time; it is not guaranteed that global optimal solutions will always be achieved.
- All combinations of cardinality and threshold constraints are allowed.
- Soft min/max bounds are allowed for cardinality constraints
- Soft thresholds are allowed for minimum level constraints.
- A crossover trade makes an asset change from a long position to a short position, or vice versa, and a crossover trade may be counted as one trade or two trades.

4.1.1 Minimum Holding Thresholds/Holding Level Paring

Minimum holding threshold constraints specify that the holding level for any asset cannot be non-zero and below a certain level. They are also known as holding level paring constraints in Barra Optimizer. Mathematically, they may be represented as:

$$h_i \geq \xi_i^+ \;\; or \;\; h_i \leq -\xi_i^- \;\; or \;\; h_i = 0, \quad i = 1, 2, \cdots n \quad \text{(Holding Threshold Constraints)} \tag{120}$$

where ξ_i^+ and ξ_i^- are positive real constants. Note that constraints (120) require that the minimum holding level for asset i be ξ_i^+ if it takes on a long position, or ξ_i^- if it takes on a short position.

However, starting with Barra Optimizer 8.4, an “EnableGrandfatherRule” option is provided. If you select this option, $h_i \geq h_{0i}$ is always considered feasible when $h_{0i} > 0$, even if $h_{0i} < \xi_i^+$; and similarly, $h_i \leq h_{0i}$ is always considered feasible when $h_{0i} < 0$, even if $|h_{0i}| < \xi_i^-$.

Mathematically, the holding threshold constraints with the grandfather rule can be represented by:

$$\begin{cases} h_i \geq \text{Min}(\xi_i^+, h_{0i}) \;\; or \;\; h_i \leq -\xi_i^- \;\; or \;\; h_i = 0, \;\; if \;\; h_{0i} > 0, \\ h_i \geq \xi_i^+ \;\; or \;\; h_i \leq \text{Max}(-\xi_i^-, h_{0i}) \;\; or \;\; h_i = 0, \;\; if \;\; h_{0i} < 0, \quad i = 1, 2, \cdots n. \\ h_i \geq \xi_i^+ \;\; or \;\; h_i \leq -\xi_i^- \;\; or \;\; h_i = 0, \;\; if \;\; h_{0i} = 0, \end{cases} \tag{121}$$

4.1.2 Minimum Trade Thresholds/Transaction Level Paring

Minimum trade threshold constraints specify that the transaction size for any non-zero buy or sell trades cannot be below a certain level. These constraints are also known as Transaction Level Paring. Mathematically, they may be represented as:

$$h_i - h_{0i} \geq \zeta_i^+ \;\; or \;\; h_i - h_{0i} \leq -\zeta_i^- \;\; or \;\; h_i - h_{0i} = 0, \quad i = 1, 2, \cdots n \quad \text{(Trade Threshold Constraints)} \tag{122}$$

where ζ_i^+ and ζ_i^- are positive real constants. Note that constraints (122) require that the minimum transaction size for asset i be ζ_i^+ if the transaction is a buy trade, or ζ_i^- if it is a sell trade. Under this definition, $h_i = 0$ may not be feasible if $0 \le h_{0i} < \zeta_i^-$ or $-\zeta_i^+ < h_{0i} \le 0$.

However, starting with Barra Optimizer 8.0, an "Allow Close-Out" option is provided. If you select this option, $h_i = 0$ is always considered feasible even if the transaction size is below the minimum requirement.

Mathematically, the trade threshold constraints with the close-out option can be represented by:

$$h_i - h_{0i} \ge \zeta_i^+ \quad or \quad h_i - h_{0i} \le -\zeta_i^- \quad or \quad h_i - h_{0i} = 0 \quad or \quad h_i = 0, \qquad i = 1, 2, \cdots n \tag{123}$$

4.1.3 Holding Cardinality/Asset Paring

Holding cardinality constraints specify that the number of assets in the whole portfolio or in an asset group cannot be more or less than a certain number. These are also known as asset paring constraints.

Mathematically, they may be represented as:

$$K_{\min}^J \le \sum_{i \in G_j} \delta_i \le K_{\max}^J, \ \forall J, \ \delta_i = \begin{cases} 0, & if \ h_i = 0 \\ 1, & otherwise \end{cases} \qquad \text{(Holding Cardinality Constraints)} \tag{124}$$

where, $K_{\max}^J$ and $K_{\min}^J$ are positive integers indicating the maximum or minimum number, respectively, for group G_J. An asset group can be comprised of the whole portfolio, or its predefined subset. Barra Optimizer also supports counting long versus short assets separately, through a more complex optimization procedure known as hedge optimization. For more information, see the By-Side Optimization—Leverage, Turnover by Side, Cardinality by Side section.

Note that long or short assets are NOT pre-defined. Rather, they are the output of optimization.

4.1.4 Trade Cardinality/Paring

Trade cardinality constraints specify that the number of trades involved in transacting an initial portfolio into an optimal one cannot be greater or less than a certain number. These are also known as trade paring constraints. Mathematically, they can be represented as:

$$T_{\min}^J \le \sum_{i \in G_J} \sigma_i \le T_{\max}^J, \quad \forall J, \quad \sigma_i = \begin{cases} 0, & if \ h_i - h_{0i} = 0 \\ 1, & otherwise \end{cases} \qquad \text{(Trade Cardinality Constraints)} \tag{125}$$

where, $T_{\max}^J$ and $T_{\min}^J$ are positive integers representing the maximum and minimum, respectively, for group G_J.

A trade can either be a buy or sell trade. In addition to constraining the total number of trades, Barra Optimizer allows setting a maximum or minimum on the number of buy or sell trades separately. Mathematically, these constraints are variations of (125), namely,

$$T_{\min}^J \le \sum_{i \in G_J} \sigma_i \le T_{\max}^J, \quad \forall J, \quad \sigma_i = \begin{cases} 1, & if \ h_i > h_{0i} \\ 0, & otherwise \end{cases} \text{(Trade Cardinality Constraints for Buy Trades)} \tag{126}$$

or

$$T_{\min}^J \le \sum_{i \in G_J} \sigma_i \le T_{\max}^J, \quad \forall J, \quad \sigma_i = \begin{cases} 1, & \text{if } h_i < h_{0i} \\ 0, & \text{otherwise} \end{cases} \text{ (Trade Cardinality Constraints for Sell Trades)} \tag{127}$$

For more information, see Xu and Liu (2011).

4.1.5 Paring Tolerances

Numerical tolerances are used by the Barra Optimizer to determine whether a paring constraint is satisfied or violated. Users have the flexibility to use three different tolerances for different types of paring constraints:

- torA—Tolerance for Asset Paring
- torL—Tolerance for Level Paring
- torT—Tolerance for Trade Paring

The default value for all three tolerances is 5.0e-6. With these tolerances, the paring constraints can be re-written as follows:

- Holding Threshold Constraints:

$$h_i \ge \xi_i^+ - torL \quad or \quad h_i \le torL - \xi_i^- \quad or \quad -torL \le h_i \le torL, \qquad i = 1, 2, \cdots n \tag{128}$$

- Trade Threshold Constraints:

$$h_i - h_{0i} \ge \zeta_i^+ - torL \quad or \quad h_i - h_{0i} \le torL - \zeta_i^- \quad or \quad -torL \le h_i - h_{0i} \le torL, \qquad i = 1, 2, \cdots n \tag{129}$$

- Holding Cardinality Constraints:

$$K_{\min}^J \le \sum_{i \in G_j} \delta_i \le K_{\max}^J, \ \forall J, \ \delta_i = \begin{cases} 1, & if \ h_i > torA \ or \ h_i < -torA \\ 0, & otherwise \end{cases} \tag{130}$$

- Trade Cardinality Constraints:

$$T_{\min}^J \le \sum_{i \in G_J} \sigma_i \le T_{\max}^J, \quad \forall J, \quad \sigma_i = \begin{cases} 1, & if \ h_i - h_{0i} > torT \quad or \quad h_i - h_{0i} < -torT \\ 0, & otherwise \end{cases} \tag{131}$$

- Trade Cardinality Constraints for Buy Trades:

$$T_{\min}^J \le \sum_{i \in G_J} \sigma_i \le T_{\max}^J, \quad \forall J, \quad \sigma_i = \begin{cases} 1, & if \ h_i > h_{0i} + torT \\ 0, & otherwise \end{cases} \tag{132}$$

- Trade Cardinality Constraints for Sell Trades:

$$T_{\min}^J \le \sum_{i \in G_J} \sigma_i \le T_{\max}^J, \quad \forall J, \quad \sigma_i = \begin{cases} 1, & if \ h_i < h_{0i} - torT \\ 0, & otherwise \end{cases} \tag{133}$$

4.1.6 Linear Penalties on Paring Constraints

In the earlier Barra Optimizer versions, cardinality and threshold constraints could be specified either as hard or soft constraints, but no penalty functions were allowed on these constraints. When a cardinality or threshold constraint was specified as hard, then any solution violating the constraint would be deemed as infeasible. On the other hand, when a constraint was specified as soft, all else being equal, a solution satisfying the constraint would be deemed as infinitely better than a solution violating the constraint, regardless of the utility values of the two solutions.

Starting with Barra Optimizer 8.4, users can set linear penalties on cardinality and threshold constraints. Violations on these constraints are translated through penalty rates into reductions on the utility level. More specifically, the optimization problem with penalties on paring constraints becomes:

$$\begin{aligned} \underset{\mathbf{h}}{\text{Maximize:}} \quad & U(\mathbf{h}) - Pen_{paring}(\mathbf{h}) \\ \text{Subject to:} \quad & \text{Paring constraints} \\ & \text{Other Constraints} \end{aligned} \tag{134}$$

where,

$U(\mathbf{h})$	the original utility function when paring penalties were not allowed
$Pen_{paring}(\mathbf{h})$	the sum of paring penalties penalizing the violations of all cardinality or threshold constraints

Different penalty rates can be applied to different types of cardinality or threshold constraints. Let us assume the following:

P_A = penalty per extra asset or penalty due to lack of one asset

P_T = penalty per extra trade or penalty due to lack of one trade

P_{HL} = penalty per unit below the minimum holding threshold

P_{TL} = penalty per unit below the minimum trade threshold

Then, the total paring penalties can be represented by:

$$\begin{aligned} Pen_{paring}(\mathbf{h}) = P_A \cdot \{ & Max(\sum_{i=1}^{n} \delta_i - K_{\max}^J, 0.0) + Max(K_{\min}^J - \sum_{i=1}^{n} \delta_i, 0.0)\} \\ & + P_T \cdot \{\max(\sum_{i=1}^{n} \sigma_i - T_{\max}^J, 0.0) + \max(T_{\min}^J - \sum_{i=1}^{n} \sigma_i, 0.0)\} \\ & + P_{HL} \cdot \sum_{i=1}^{n} \left(\theta_i^+ \cdot (\xi_i^+ - h_i) + \theta_i^- \cdot (h_i - \xi_i^-) \right) \\ & + P_{TL} \cdot \sum_{i=1}^{n} \left(\varphi_i^+ \cdot (h_{0i} + \zeta_i^+ - h_i) + \varphi_i^- \cdot (h_i - h_{0i} + \zeta_i^-) \right) \end{aligned} \tag{135}$$

where,

$K_{\max}^J$ and $K_{\min}^J$ = the upper and lower bounds of the holding cardinality constraint, respectively

$T_{\max}^J$ and $T_{\min}^J$ = the upper and lower bounds of the trade cardinality constraint, respectively

ξ_i^+ and ξ_i^- = the long- and short-side minimum holding thresholds, respectively

ζ_i^+ and ζ_i^- = the buy- and sell-side minimum trade thresholds, respectively

$$\delta_i = \begin{cases} 1, & if\ \ h_i > torA\ \ or\ h_i < -torA \\ 0, & otherwise \end{cases}$$

$$\sigma_i = \begin{cases} 1, & if\ \ h_i > h_{0i} + torT\ \ or\ h_i < h_{0i} - torT \\ 0, & otherwise \end{cases}$$

$$\theta_i^+ = \begin{cases} 1, & if\ \ torL < h_i < \xi_i^+ - torL \\ 0, & otherwise \end{cases}$$

$$\theta_i^- = \begin{cases} 1, & if\ \ -\xi_i^- + torL < h_i < -torL \\ 0, & otherwise \end{cases}$$

$$\varphi_i^+ == \begin{cases} 1, & if\ \ h_{0i} + torL < h_i < h_{0i} + \zeta_i^+ - torL \\ 0, & otherwise \end{cases}$$

$$\varphi_i^- == \begin{cases} 1, & if\ \ h_{0i} - \zeta_i^- + torL < h_i < h_{0i} - torL \\ 0, & otherwise \end{cases}$$

Notes:

- For group-level paring constraints, the summations in (135) should be over all assets in the particular group rather than over all assets in the entire portfolio.
- When the penalty rate for a specific paring type is positive, all paring constraints of that type will be relaxed, no matter whether they were input as hard or soft constraints. In other words, violations on the paring constraints with a positive penalty rate will not trigger infeasibility. Instead, they will be penalized according to the penalty rates provided by the user. When comparing two paring solutions involving positive penalty rates, all else equal, the one with higher utility will always be preferred.
- When the penalty rate for a specific paring type is zero, then paring penalties for that type will be ignored, and all paring constraints of that type will behave the same as before. That is, when a paring constraint is specified as hard, then any violation of it will trigger infeasibility. When it is specified as soft, then a solution violating the constraint will be deemed as infinitely worse than a solution satisfying the constraint, all else equal, regardless of the utility values of the two solutions.

4.1.7 Upper Bound on Utility and Branch-and-Bound Algorithm for Paring Cases

Starting with Barra Optimizer 8.4, users have the option to obtain an upper bound on utility for some paring cases. This upper bound is estimated by Barra Optimizer using the perspective reformulation approach. It can be used to estimate the utility gap between the unknown optimal and known heuristic solutions:

$$Gap_{uti} = UB_{uti} - Uti(\mathbf{h}^*) \tag{136}$$

where $\mathbf{h}^*$ is the heuristic portfolio returned by Barra Optimizer, $Uti(.)$ is the utility function of the paring case in question, and UB_{uti} is the upper bound on utility.

Note: Currently, the scope for option is limited.

This option only applies to cases that are simply standard portfolio optimization cases and limited cardinality or threshold constraints (see the Problem Classification section for the definition of a standard case). In other words, no other special constraints are allowed besides paring. For example, the following features CANNOT be present in the cases:

- Non-convex leverage constraints
- Non-linear transaction costs
- Risk constraints, with one exception described below
- Tax-aware optimization
- Roundlotting constraints
- Parametric optimization

In addition, note the following:

- The specific covariance matrix must be diagonal or block-diagonal. In particular, composite assets are not allowed in the specific covariance matrix of these cases.
- Specific risk aversion must be positive, and/or there must be exactly one hard constraint on risk or tracking error.
- Only hard maximum cardinality constraints and hard threshold constraints are currently supported. Minimum cardinality constraints, penalties on cardinality and threshold constraints, soft cardinality, and threshold constraints are not supported.

Starting with Barra Optimizer 8.6, users have an option to solve a paring problem by calling a newly developed branch-and-bound (BB) algorithm. The BB algorithm is based on the bounding routine mentioned above and currently has the same scope limitations. The BB algorithm may take longer than the heuristics described in the beginning of this section, but may potentially produce better results, as was demonstrated in internal testing.

The BB algorithm is tuned, however, to produce a good solution as fast as possible, so it is not guaranteed to produce the optimal solution. Additionally, failure of the BB algorithm does not mean that the feasible solution does not exist. BB algorithm might exit earlier than the user-defined timeout if it deems it is not likely to find a feasible solution. The users will need to explicitly specify that they want BB algorithm to run, instead of the standard heuristics. This option is mutually exclusive with the option of getting an upper bound on utility. If both are specified, the one specified last will take effect. Both options will be ignored when the problem features are out of scope.

4.2 Roundlotting—Optimal and Post-Optimization

In general, solutions to portfolio optimization problems are real numbers. The resulting trade list may contain many small or odd trades—for example, to buy 11.51 shares of IBM, or to sell 3603 shares of AAPL. In practice, however, stocks are typically traded in multiples of round lots rather than in sporadic odd ones. The size of a round lot may vary. For most stocks on the New York Stock Exchange, one round lot is 100 shares.

4.2.1 Optimal Roundlotting or Constraint-Aware Roundlotting

Round lot constraints impose restrictions on the size of the trades leading to an optimal portfolio. These constraints may be represented by:

$$h_i = h_{0i} + \omega_i r_i, \qquad i = 1, 2, \ldots, n \tag{137}$$

where,

r_i	round lot trade size (in terms of portfolio weight) for assets i
ω_i	an integer number to be determined

Enforcing the above n constraints ensures that the trade list contains only round lot trades. Note that the final optimal holdings may not be round lot shares.

Unless h_{0i} is a round lot position itself, $h_i = 0$ may not be a feasible solution to an optimal roundlotting case because $h_i = h_{0i}$ may not be a round lot trade. However, if the option "AllowOddCloseOut" is set to be true, then the optimizer will consider $h_i = 0$ as a feasible solution. Note that this is just an option, not a requirement for the optimizer to close out positions. The optimizer will only "close out" a position if doing so improves utility.

Similar to the threshold and cardinality constraints discussed in the Threshold and Cardinality Constraints—Paring Optimization section, the presence of the round lot constraints makes the feasible region discontinuous and complicated. Optimal Roundlotting is a hard-to-solve mixed-integer programming problem. A solution may not exist. Even if an optimal solution does exist, it is usually difficult to find.

Barra Optimizer employs heuristic algorithms to solve the Optimal Roundlotting problem. These algorithms seek a portfolio that is near optimal within a reasonable amount of time. The solution portfolios, in general, are feasible yet not globally optimal. For more information about constraint-aware roundlotting, see Xu and Liu (2013).

4.2.2 Post-Optimization

Instead of imposing the round lot constraints, Barra Optimizer now provides users with a utility function to round the optimal trades to their nearest round lots after the optimization process. This is a simple, one-step mechanical procedure. The resultant trade list will have no odd lots, yet one or more of the original constraints in the problem may be violated.

Notes:

- One or more of the original constraints in the problem may be violated.
- The in-optimization roundlotting constraint need not be enabled to run post-optimization roundlotting.
- Post-optimization roundlotting is not applicable to frontier optimization.
- If crossover trade is not allowed and the nearest roundlot trade results in a crossover trade:
  - If closeout with odd lot trade is allowed, the final position is set to zero.
  - If closeout is not allowed, make a roundlot trade in reverse direction to avoid crossover.
- Cash position may be adjusted to offset those post optimization roundlotting adjustments.

4.3 Risk Parity

4.3.1 Definitions

Let $\mathbf{V}$ be a covariance matrix of asset returns. Further, let $\mathbf{X}$ and $\mathbf{F}$ be a matching pair of a factor exposure matrix and a factor covariance matrix, respectively. Given a benchmark $\mathbf{h}_B$, let

$$Ta_i = \frac{\mathbf{h}^{\mathrm{T}}\mathbf{V}\mathbf{h}_{\{i\}}}{\mathbf{h}^{\mathrm{T}}\mathbf{V}\mathbf{h}} \tag{138}$$

$$Aa_i = \frac{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}\mathbf{V}(\mathbf{h}-\mathbf{h}_B)_{\{i\}}}{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}\mathbf{V}(\mathbf{h}-\mathbf{h}_B)} \tag{139}$$

$$Tf_j = \frac{\mathbf{h}^{\mathrm{T}}\mathbf{X}\mathbf{F}(\mathbf{X}^{\mathrm{T}}\mathbf{h})_{\{j\}}}{\mathbf{h}^{\mathrm{T}}\mathbf{X}\mathbf{F}\mathbf{X}^{\mathrm{T}}\mathbf{h}} \tag{140}$$

$$Af_j = \frac{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}\mathbf{X}\mathbf{F}(\mathbf{X}^{\mathrm{T}}(\mathbf{h}-\mathbf{h}_B))_{\{j\}}}{(\mathbf{h}-\mathbf{h}_B)^{\mathrm{T}}\mathbf{X}\mathbf{F}\mathbf{X}^{\mathrm{T}}(\mathbf{h}-\mathbf{h}_B)} \tag{141}$$

Note that Ta_i and Aa_i are the (additive) total risk contribution and active risk contribution of asset *i*, respectively. Similarly, Tf_j and Af_i are the total risk contribution and active risk contribution of factor *j*, using the additive definition. (See Section 3.7.4 and Appendix H for more information on these risk terms.)

Using the above notations, we can define the following risk parity constraints:

$$Ta_{i1} = Ta_{i2} \quad \text{for all } i1,\ i2 \in S_A \tag{142}$$

$$Aa_{i1} = Aa_{i2} \quad \text{for all } i1,\ i2 \in S_A \tag{143}$$

$$Tf_{j1} = Tf_{j2} \quad \text{for all } j1,\ j2 \in S_F \tag{144}$$

$$Af_{j1} = Af_{j2} \quad \text{for all } j1,\ j2 \in S_F \tag{145}$$

where S_A is a risk parity subset of assets and S_F is a risk parity subset of factors, called the risk parity set of the constraint.

A portfolio that satisfies one of the above risk parity constraints (142)-(145) is called a risk parity portfolio. In risk parity optimization, the goal is to find a risk parity portfolio for a given constraint type.

4.3.2 Excluded Assets or Factors

By default, any assets/factors excluded from the risk parity subsets S_A or S_F are also allowed to contribute to risk. In other words, this means excluded assets can have a non-zero weight in the optimal portfolio (for asset risk parity) or the exposure of excluded factors can be non-zero (for factor risk parity). Note that the optimizer may report a risk parity portfolio where the risk contributions of included assets/factors are all 0. This can be addressed with a second run with an additional constraint or penalty included.

However, users have an option to disable risk contribution from excluded assets/factors. In case of asset risk parity, the exclusion does not apply to risk-free assets – this means that a risk parity portfolio will consist of elements of subset S_A and, potentially, cash. Similarly, for factor risk parity, the exposure of factors outside of S_F will be 0.

4.3.3 Scope and Limitations

To set up a risk parity optimization problem in Barra Optimizer, users must specify a subset S_A or S_F of assets or factors, select the risk model to be used (i.e. primary or secondary) and possibly select a benchmark (for constraining active risk contribution).

Prior to Barra Optimizer version 9.1, risk parity optimization was more restrictive, with only the risk parity constraint type (142) supported, along with the additional requirements that the optimization must be long only. It can be shown that if $\mathbf{V}$ is non-singular, this special case has a unique solution. This has the following implications:

- Imposing additional constraints on a optimization case besides the risk parity constraint has a definite potential of making the portfolio construction problem infeasible.
- Trying to influence portfolio weights in any way by introducing additional terms in the objective function will have no effect if the solution is unique.

Additionally, this special type of risk parity optimization can be formulated as a convex problem (more specifically, as a second-order cone problem) and can be solved efficiently.

While, multiple risk parity portfolios may exist for a general case, it is still true that adding extra constraints may make a problem infeasible and that modification of the objective function may have no effect on the portfolio reported.

The general risk parity problem is also nonconvex, which means that optimization is more resource-intense and the Optimizer may not find the best risk parity portfolio.

Risk parity portfolio construction is now supported in Barra Optimizer with the following limitations:

- Only single period and single account optimization
- No tax-aware features
- No hedge optimization
- There are no discrete elements in the problem (i.e. no cardinality, threshold, roundlotting, 5/10/40 constraints, or fixed transaction costs, no fixed transaction cost or fixed holding cost).
- The objective function is linear or convex quadratic.

Because of the highly restrictive nature of the risk parity constraints, the number of additional constraints and objective terms in the risk parity portfolio construction problem should be kept to the minimum.

Note that in Barra Optimizer 9.1, there are two methods to set up a risk parity problem.

- One is a legacy method from earlier versions, to set up the special type of asset risk parity discussed above. The use of this method is not recommended anymore,
- The new and recommended method is using a new, alternative API function suitable for any kind of risk parity constraint.

4.3.4 Applications

Risk parity optimization may be helpful when a portfolio manager needs a weighting schema for a group of assets or factors and information about the expected return of the group is not available.
Unlike the minimum volatility portfolio which places most of the weight on low-risk assets or factors, or the equal weighting scheme which is indifferent to the differences in risk between elements, the risk parity approach considers each asset's or factor's risk contribution when assigning weights.

Risk parity portfolio construction is often utilized as a first step in a multi-stage process. One common application is to use the resulting optimal portfolio as a benchmark in subsequent portfolio construction. This can be helpful if additional portfolio rules or constraints need to be satisfied in addition to risk parity. The tradeoff between risk parity and other constraints can be controlled by the objective function for subsequent optimization problems such as:

- To minimize risk relative to risk parity → Use a variance aversion with the risk model
- To minimize the weight deviation → Use a quadratic or linear penalty
- To maximize alpha, minimize risk, minimize transaction cost → Use a standard quantitative process

The solution to a risk parity optimization problem can also be used to scale weights in an underlying benchmark. For example, an aggregate sector weighting can be determined through risk parity. In addition, an industry rotation strategy can leverage risk parity portfolio construction to select the weighting within a sector, as an alternative to market cap weighting the sector components.

4.4 Portfolio Concentration Limit Constraint

Several measures of the diversification level of a portfolio have been proposed in literature. Portfolio Concentration is a particular important measure useful in quantitative portfolio construction. Barra Optimizer supports an upper bound on portfolio concentration.

For long-only portfolios, portfolio concentration is defined as the sum of weights of the top k_c largest holdings. A lower concentration implies a higher diversification. To prevent a portfolio from being too concentrated, one may want to set an upper bound on portfolio concentration.

Mathematically, a portfolio concentration limit constraint can be written as:

$$\sum_{j=1}^{k_c} h_{cj} \leq U_{ConP} \tag{146}$$

where, $h_{c1} \geq h_{c2} \geq \cdots \geq h_{cn}$ are the asset weights ranked from the largest to smallest and $1 \leq k_c \leq n$.

For example, a client may want to cap the top 5 holdings of his portfolio at 45% of the total portfolio value. His asset universe is 500. In this case, $k_c = 5$, $n = 500$ and $U_{ConP} = 0.45$. Thus, his portfolio concentration limit constraint is essentially $h_{c1} + h_{c2} + h_{c3} + h_{c4} + h_{c5} \leq 0.45$.

The portfolio concentration limit constraint can be used to meet any regulatory guidelines that limit the concentration of the top holdings.

Note that this constraint is not a simple linear constraint in $\mathbf{h}$, because assets that have the largest holdings are not known in advance.

Notes:

- This constraint is applied to (total) asset weights directly. An exclusion list can be used to specify the assets not to be considered when applying this constraint.
- Shorting is allowed. Negative weights, instead of absolute weights, are used for short positions in this constraint.
- A composite is counted as an asset. In other words, there is no “look through ability”.
- In general, this constraint can be combined with the analytic features that can be used in a standard portfolio optimization.

4.5 Residual Alpha

An asset’s active return, or alpha, may be decomposed into a part that is spanned by the risk exposures— $\boldsymbol{\alpha}_R$ and a part that is residual (orthogonal) to them— $\boldsymbol{\alpha}_{R_\perp}$

$$\boldsymbol{\alpha} = \underbrace{\mathbf{X}\left(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{W}\boldsymbol{\alpha}}_{\boldsymbol{\alpha}_R} + \underbrace{\left(\mathbf{I} - \mathbf{X}\left(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{W}\right)\boldsymbol{\alpha}}_{\boldsymbol{\alpha}_{R_\perp}} \tag{147}$$

To penalize the residual alpha in the portfolio optimization, a quadratic penalty term is added to Barra Optimizer’s objective function $f(\mathbf{h})$:

$$Maximize \quad f(\mathbf{h}) - \lambda_R \cdot \left(\tilde{\alpha}_{R_\perp}^T (\mathbf{h} - \mathbf{h}_B)\right)^2 \tag{148}$$

where,

- $\tilde{\alpha}_{R_1} = \dfrac{\alpha_{R_\perp}}{std\left(\alpha_{R_\perp}\right)}$ is scaled residual alpha
- $std\left(\alpha_{R_\perp}\right) = \sqrt{\dfrac{\sum_{i=1}^{n}\left(\alpha_{R_\perp,i} - \bar{\alpha}_{R_\perp}\right)^2}{n-1}}$, where $\bar{\alpha}_{R_\perp}$ is the average of $\alpha_{R_\perp}$ elements
- $\alpha_{R_1} = \left(I - \mathbf{X}\left(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{W}\right)\alpha$ is a non-scaled residual alpha
- $\mathbf{W}$ is a diagonal matrix that represents a weighting scheme selected by the user
- λ_R is a user-specified multiplier for the residual alpha penalty term. Its default value can be computed using the following formula:

$$\lambda_R = 100 \cdot \lambda_F \cdot \sigma_{\alpha_{R1}}^{\ 2}, \quad \text{where } \sigma_{\alpha_{R1}} = 0.02 \tag{149}$$

For background information and application issues regarding residual alpha, see Bender et al. (March 2009) and Bender et al. (June 2009)

4.6 By-Side Optimization—Leverage, Turnover by Side, Cardinality by Side

Long-Short (Hedge) Optimization is not characterized by just allowing shorting. Instead, it is characterized by at least one of the leverage constraints defined in the Leverage (Hedge) Constraints section. From Barra Optimizer's perspective, an optimization problem with short positions yet without leverage or other special constraints is simply a Standard Optimization problem. For a summary of Long-Short Optimization in the Barra Optimizer, see Liu and Xu (2014).

Long-Short Optimization and Standard Optimization have many features and functions in common. In particular, variance of either the portfolio's total return or active return with respect to a benchmark may be specified in the objective function. Long—Short Optimization can also be combined with Risk Constrained Optimization and Round Lot Optimization. Certain features—for example, turnover-by-side constraints, threshold-by-side constraints, cardinality-by-side constraints, and short rebates/costs—are currently unique to Long-Short Optimization and not available in Standard Optimization, though.

Long-Short Optimization is generally more difficult to solve than Standard Optimization due to the presence of the leverage constraint(s). The algorithms involved are more computationally expensive. Moreover, a lower bound on a leverage constraint may make the problem non-convex. For these reasons, it is best to use Standard Optimization instead of Long-Short Optimization whenever possible.

Notes:

- Tax-aware optimization is currently not supported with hedge optimization.
- You may set penalties on slacks s_{long}, s_{short} and s_{Hi}.

- Barra Optimizer will take into account short rebate costs if you specify them. For information about short rebate costs, see the Short Rebates/Short Costs section.
- By default, futures are excluded from leverage constraints and by side constraints. However, optionally, futures can be included in such constraints.

4.6.1 Leverage (Hedge) Constraints

Leverage constraints limit the total amount of long positions (L) or short positions (S) or both in a portfolio. By definition,

$$L = \sum_{i=1, i\neq cash, i\neq futures}^{n} \max(h_i, 0) \tag{150}$$

and

$$S = \sum_{i=1, i\neq cash, i\neq futures}^{n} \max(-h_i, 0) \tag{151}$$

Note that the Portfolio Balance Constraint can now be represented by:

$$C + L - S = 1 \tag{152}$$

where, C is the total amount of cash in the portfolio. Fixing any two of the three values L, S, and C implicitly determines the third if the holding constraint is enforced. The cash asset C is optional.

From time to time, you may not want to have a cash asset in the investable universe, nor have the typical portfolio balance constraint. Instead, you may want to constrain on the net leverage, which is the difference between total long and total short. In this case, you can drop the portfolio balance constraint and add a general linear constraint as a replacement:

Lower Bound <= Long – Short <= Upper Bound

For example, for dollar-neutral portfolios, you can add a linear constraint:

Long –Short = 0.

The following sections present several types of leverage constraints currently supported in Barra Optimizer. We use l and u to denote the lower and upper bounds throughout the sections.

4.6.1.1 Hedge Scaling Constraint

The Hedge Scaling Constraint has the following form:

$$l_L \leq L \leq u_L \tag{153}$$

In essence, it is a leverage constraint on long positions only.

4.6.1.2 Weighted Long or Short Leverage Constraints

The Weighted Long Leverage Constraints have the following form:

$$l_{L_i} \leq \sum_{j=1, j\neq cash, j\neq futures}^{n} a_{ij} \max(h_j, 0) \leq u_{L_i}, \qquad i = 1, 2, \dots, n \tag{154}$$

where a_{ij} is the weight for h_j in the i^{th} weighted long hedge constraint. Similarly, the Weighted Short Leverage Constraints have the following form:

$$l_{s_i} \le \sum_{j=1,\, j\neq cash,\, j\neq futures}^{n} b_{ij} \min(h_j, 0) \le u_{S_i}, \qquad i = 1, 2, \ldots, n \tag{155}$$

where b_{ij} is the weight for h_j in the i^{th} weighted short hedge constraint.

Note that the scaling constraint (153) can be considered as a special case of the constraints in (154). For example, by setting $a_{ij} = 1$ for $j = 1, 2, \ldots, n$ the i^{th} constraint of (154) reduces to (153). Another example of (154) is a constraint that requires the total long exposure to the banking industry be at most 10% of the given base value. Similarly, constraining the total short exposure to the size factor to be at least 0.02 is an example of (155).

4.6.1.3 Total Leverage Constraint

The Total Leverage Constraint has the following form:

$$l_T \le L + S \le u_T \tag{156}$$

In essence, it is a leverage constraint on the sum of long and short positions.

Note: You can set soft bound and/or penalty on total leverage.

4.6.1.4 Short-Long Leverage Ratio Constraint

The Short-Long Leverage Ratio Constraint has the following form:

$$l_r \le \frac{S}{L} \le u_r \tag{157}$$

Notes:

- This constraint is not defined if $L = 0$.
- You can set a soft bound on the short/long ratio.
- You cannot set a penalty on the short/long ratio.

4.6.1.5 Net-Total Leverage Ratio Constraint

The Net-Total Leverage Ratio Constraint has the following form:

$$l_r \le \frac{L - S}{L + S} \le u_r \tag{158}$$

Note that the numerator of the ratio is the net leverage, whereas the denominator is the total leverage.

From (152), we see that:

$$L - S = 1 - C \tag{159}$$

Thus, any limit on the net leverage $L-S$ can be translated into a bound on the cash asset when holding constraint is present.

Notes:

- You can set a soft bound on the net/total leverage ratio.
- You cannot set a penalty on the net/total leverage ratio.

4.6.1.6 Weighted Total Leverage Constraints

The Weighted Total Leverage Constraints have the following form:

$$l_{T_i} \le \sum_{j\neq cash,\, j\neq futures}^{n} c_{ij}\max(h_j,0) - \sum_{j\neq cash,\, j\neq futures}^{n} d_{ij}\min(h_j,0) \le u_{T_i}, \qquad i=1,2,\ldots,n \tag{160}$$

where the c_{ij} and d_{ij} are the long and short weights, respectively, for h_j in the i^{th} weighted total hedge constraint. The total leverage constraint (156) is a special case of the constraints in (160).

4.6.2 Penalty on Leverage Constraints

Barra Optimizer supports penalties on the hedge constraints (153)-(156) and (160). These penalties can be either quadratic or pure linear and either symmetric or asymmetric, as described in the Constraint Flexibility—Penalties, Soft Bounds, and Hierarchy section. This feature helps users to tilt their portfolios towards specific leverage targets, meeting their compliance requirements set either by regulation or investment policy.

Penalty on either of the leverage ratio constraints (157) and (158) is not currently supported.

4.6.3 Turnover-by-Side Constraints

In Long-Short Optimization, in addition to limiting the overall turnover, users may constrain turnover at either the long or the short side. The long side turnover constraint is defined as:

$$\frac{Long\ buys + Long\ sales}{2\cdot Initial\ long\ positions} \le TurnoverLimit_{long} \tag{161}$$

$$\frac{Short\ covers + Short\ sales}{2\cdot |\,Initial\ short\ positions\,|} \le TurnoverLimit_{short} \tag{162}$$

Let $x^{+} = max(x,0)$ and $x^{-} = \min(x,0)$. Then, in more rigorous mathematical terms:

- $Long\ buys = \sum_i max(h_i - h_{0i}^{+}, 0)$, $\quad Long\ sales = \sum_i max(h_{0i} - h_i^{+}, 0)$
- $Short\ sales = \sum_i max(h_{0i}^{-} - h_i, 0)$, $\quad Short\ covers = \sum_i max(h_i^{-} - h_{0i}, 0)$
- $Initial\ long\ positions = \sum_i h_{0i}^{+}$, $\quad |Initial\ short\ positions| = -\sum_i h_{0i}^{-}$

4.6.4 Threshold- and Cardinality-by-Side Constraints

Barra Optimizer allows multiple threshold or cardinality constraints to appear in the Long-Short Optimization. In addition to the usual constraints with respect to the overall holdings—maximum or minimum number of assets, minimum holding levels or transaction sizes, and maximum or minimum number of trades—Barra Optimizer supports these constraints by the long or short side separately.

4.6.4.1 Maximum or Minimum Number of Assets by Side

For holding cardinality, Barra Optimizer allows users to constrain the number of long names and short names separately. For instance, one may limit the maximum number of long assets to 100, and the minimum number of short assets to 10. As in Standard Optimization, one may also set a limit on the total number of assets in the portfolio.

4.6.4.2 Minimum Holding Level by Side

For holding thresholds, users may set different thresholds on the long and short side holdings simultaneously. For example, users may require that any long position be at least 1%, but any short position be at most -0.5%.

4.6.4.3 Minimum Transaction Level by Side

For trade thresholds, users may simultaneously set different thresholds on the long side transactions (i.e., for long buys and long sales) and short side transactions (i.e., for short covers and short sales). For example, one may specify that any trade size on the long side should be at least 0.5%, and any trade size on the short side should be at least 1%.

By definition, if an asset has an initial long position, the minimum long side transaction level is applied to its buy and sell transactions. Similarly, if any asset has an initial short position, then the minimum short side transaction level is applied to all its transactions. However, if an asset's initial position is zero, then the minimum long side transaction level will be applied to its buy transactions, while the minimum short side transaction level will be applied to its sell transactions.

4.6.4.4 Maximum or Minimum Number of Trades by Side

For trade cardinality, users have the flexibility to limit the number of long trades or short trades separately. A long trade occurs when an asset has a positive initial position and its final position differs from this initial, or when the initial position is zero and the final position is positive. Conversely, a short trade arises when an asset has a negative initial position with a different final position, or a zero initial position but a negative final position. In the case where an asset has a negative initial position and a positive final position, the transaction actually involves two segments—a short trade (covering a short position) plus a crossover long trade (buying a long position). Similarly, the transaction going from a positive initial position to a negative final position consists of a long trade (selling a long position) plus a crossover short trade (initiating a short position). Barra Optimizer gives the user a choice to treat a transaction with a crossover segment either as one trade or as two trades.

4.6.5 Short Rebates/Short Costs

In a short sale of a security, a short-seller first borrows and then sells the stock. Typically, the proceeds from the short sale earn interest for the seller, but the seller also pays various fees to the lender and prime broker for providing the stock. The amount of the interest earned on proceeds, minus the sum of all costs associated with short-selling, is the short rebate the seller actually gets back. A negative short rebate implies a net short cost to the short-seller.

Short rebates can be modeled directly in Barra Optimizer's Long-Short Optimization. When short rebates are present, the objective function becomes:

$$\text{Objective Function} = \text{Return} - \text{Risk} - \text{Transaction Cost} + \text{Short Rebates} - \text{Others} \tag{163}$$

For individual assets, the short rebate is defined as:

$$ShortRebate_i = SRate_i.\text{MAX}(-h_i, 0) \tag{164}$$

where $SRate_i$ is the user-provided short-rebate rate for asset i. Either $SRate_i$ can be passed directly to the optimizer, or the user can pass its components:

$$SRate_i = r_c - c - p_i \tag{165}$$

where,

r_c	the interest rate earned on cash proceeds from short-selling this stock.
c	the cost of leverage, referring to the standard fees applying uniformly to all short stocks.
p_i	the hard-to-borrow penalty specific to this stock, referring to the additional borrowing cost incurred by small cap or illiquid stocks.

Explicitly modeling short rebates/short costs in Long-Short Optimization allows users to see how optimal portfolios vary with different short-rebate rates, or different components of the rate. This feature helps users fine tune their long-short investment strategies.

4.6.6 Parametric Optimization with Leverage Constraints

Parametric optimization (discussed in the Optimal Frontiers—Parametric Optimization section) is now supported in the presence of piecewise-linear leverage constraints. In other words, risk-return efficient frontiers and risk target or return target problems can be combined with the constraints (153)-(156) and (160) in long-short optimization. For Leverage-Utility frontiers, only varying the upper bound of a (weighted) long-leverage, the lower bound of a (weighted) short-leverage, or the upper bound of a (weighted) total-leverage constraint is allowed. See the Varying Leverage section for more details.

4.7 Solving a General Linear or Convex Quadratic Program

A general Convex Quadratic Programming (CQP) problem has the following mathematical form:

$$\underset{\mathbf{x}}{\text{Maximize}}: \quad \mathbf{c}^{\mathrm{T}}\mathbf{x} - \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{Q}\,\mathbf{x} \tag{166}$$

$$\text{Subject to}: \quad \mathbf{A}\,\mathbf{x} \le \mathbf{b} \tag{167}$$

$$\mathbf{l_x} \le \mathbf{x} \le \mathbf{u_x} \tag{168}$$

where, $\mathbf{c}$, $\mathbf{x}$, $\mathbf{l_x}$, $\mathbf{u_x}$ are $n \times 1$ vectors, $\mathbf{Q}$ is an $n \times n$ positive semi-definite matrix, $\mathbf{A}$ is an $m \times n$ coefficient matrix, and $\mathbf{b}$ is an $m \times 1$ vector. If $\mathbf{Q} = \mathbf{0}$ the problem reduces to a Linear Programming (LP) problem.

Barra Optimizer can be used to solve a generic LP or CQP problem by transforming it into the format of (1)-(12) with $\mathbf{h} = \mathbf{x}$, $\mathbf{r} = \mathbf{c}$, $\Sigma = \mathbf{Q}$, and so on.

4.8 Tax-Aware Optimization

Barra Optimizer's Tax-Aware Optimization extends the features and functions of Standard Optimization by taking the capital gain taxes into consideration. More specifically, the Tax-Aware Optimization allows users to:

- Penalize the realization of any capital gain tax in the objective
- Set explicit bounds on the total capital gain tax incurred
- Specify different tax rates or tax-related rules for different groups of assets
- Vary trading rules at the tax-lot level (e.g., sell none) or at the asset level (e.g., FIFO)
- Place bounds on the gross gains or losses as well as on net gains or losses at the portfolio level or by groups of assets
- Target net gains or losses and reap benefits from tax harvesting
- Account for, disallow, or ignore wash sales
- Examine the summary gains/losses/tax information and detailed trading information for each tax lot in the output
- Take short positions on assets
- Combine with leverage or cardinality/threshold constraints

In particular, threshold and cardinality constraints are supported (more details are in the Cardinality and Threshold Constraints in Tax-Aware Optimization section). Tax-utility frontiers (defined in the Constraint-Utility Frontiers section) are also offered.

Barra Optimizer employs a couple of heuristic procedures to handle tax-aware optimization problems.

4.8.1 Mathematical Formulation

In general, a Tax-Aware Optimization problem has the following form:

$$\begin{aligned} \underset{\mathbf{h}}{\text{Maximize:}} \quad & f_{tax}(\mathbf{h}) = f(\mathbf{h}) - \lambda_{Tax}\, Tax(\mathbf{h}) + \lambda_{Benefit}\, Benefit(\mathbf{h}) - TaxPen(\mathbf{h}) \\ \text{Subject to:} \quad & Tax - Related\ \ Constraints \\ & \text{Other Constraints} \end{aligned} \tag{169}$$

where,

$f(\mathbf{h})$	the typical mean-variance objective function (1) which ignores taxes
$Tax(\mathbf{h})$	total tax cost (as a fraction of the portfolio base value) as defined by (198) in Section 4.8.1.
λ_{Tax}	multiplier specifying the importance of the tax cost relative to other terms in the objective. It is a scaling factor under the user's control and may be construed as a linear penalty on the capital gains tax. The default value is 1.
$Benefit(\mathbf{h})$	total loss benefit as defined by (199) in Section 4.8.1.
$\lambda_{Benefit}$	multiplier specifying the importance of the loss benefit term relative to other terms in the objective. The default value is 0.
$TaxPen(\mathbf{h})$	tax-related penalties, such as the tax loss harvesting penalties defined in Section 4.8.5.

The tax-related constraints include

- For Each Capital-Gain Group q:

$$lgg_q = \sum_{i\in C_q}\left(\sum_{l\in\{l:(G_{il}\geq 0)\cap(A_{il}>D_i)\cap(f_{il}=0)\}} G_{il}h_{il}^-\right)$$ Long-Term Gross Gain Definition (170)

$$lgl_q = \sum_{i\in C_q}\left(\sum_{l\in\{l:(G_{il}<0)\cap(A_{il}>D_i)\cap(f_{il}=0)\}} G_{il}h_{il}^-\right)$$ Long-Term Gross Loss Definition (171)

$$sgg_q = \sum_{i\in C_q}\left(\sum_{l\in\{l:(G_{il}\geq 0)\cap(A_{il}\leq D_i)\cap(f_{il}=0)\}} G_{il}h_{il}^-\right)$$ Short-Term Gross Gain Definition (172)

$$sgl_q = \sum_{i\in C_q}\left(\sum_{l\in\{l:(G_{il}<0)\cap(A_{il}\leq D_i)\cap(f_{il}=0)\}} G_{il}h_{il}^-\right)$$ Short-Term Gross Loss Definition (173)

$$fgg_q = \sum_{i\in C_q}\left(\sum_{l\in\{l:(G_{il}\geq 0)\cap(f_{il}=1)\}} G_{il}h_{il}^-\right)$$ Tax-Free Gross Gain Definition (174)

$$fgl_q = \sum_{i\in C_q}\left(\sum_{l\in\{l:(G_{il}<0)\cap(f_{il}=1)\}} G_{il}h_{il}^-\right)$$ Tax-Free Gross Loss Definition (175)

$$lng_q = lgg_q + lgl_q - LLC_q$$ Long-Term Net Gain Definition (176)

$$sng_q = sgg_q + sgl_q - SLC_q$$ Short-Term Net Gain Definition (177)

$fng_q = fgg_q + fgl_q - FLC_q$ Tax-Free Net Gain Definition (178)

$l_{lgg_q} \le lgg_q \le u_{lgg_q}$ Tax Arbitrage Constraint on Long-Term Gross Gain (179)

$l_{lgl_q} \le lgl_q \le u_{lgl_q}$ Tax Arbitrage Constraint on Long-Term Gross Loss (180)

$l_{lng_q} \le lng_q \le u_{lng_q}$ Tax Arbitrage Constraint on Long-Term Net Gain (181)

$l_{sgg_q} \le sgg_q \le u_{sgg_q}$ Tax Arbitrage Constraint on Short-Term Gross Gain (182)

$l_{sgl_q} \le sgl_q \le u_{sgl_q}$ Tax Arbitrage Constraint on Short-Term Gross Loss (183)

$l_{sng_q} \le sng_q \le u_{sng_q}$ Tax Arbitrage Constraint on Short-Term Net Gain (184)

$l_{fgg_q} \le fgg_q \le u_{fgg_q}$ Tax Arbitrage Constraint on Tax-Free Gross Gain (185)

$l_{fgl_q} \le fgl_q \le u_{fgl_q}$ Tax Arbitrage Constraint on Tax-Free Gross Loss (186)

$l_{fng_q} \le fng_q \le u_{fng_q}$ Tax Arbitrage Constraint on Tax-Free Net Gain (187)

- For Each Tax-Rule Group r :

$lgg_r = \sum_{i \in T_r} \left(\sum_{l \in \{l:(G_{il} \ge 0) \cap (A_{il} > D_i) \cap (f_{il}=0)\}} G_{il} h_{il}^- \right)$ Long-Term Gross Gain Definition (188)

$lgl_r = \sum_{i \in T_r} \left(\sum_{l \in \{l:(G_{il} < 0) \cap (A_{il} > D_i) \cap (f_{il}=0)\}} G_{il} h_{il}^- \right)$ Long-Term Gross Loss Definition (189)

$sgg_r = \sum_{i \in T_r} \left(\sum_{l \in \{l:(G_{il} \ge 0) \cap (A_{il} \le D_i) \cap (f_{il}=0)\}} G_{il} h_{il}^- \right)$ Short-Term Gross Gain Definition (190)

$sgl_r = \sum_{i \in T_r} \left(\sum_{l \in \{l:(G_{il} < 0) \cap (A_{il} \le D_i) \cap (f_{il}=0)\}} G_{il} h_{il}^- \right)$ Short-Term Gross Loss Definition (191)

$lng_r = lgg_r + lgl_r - LLC_r$ Long-Term Net Gain Definition (192)

$sng_r = sgg_r + sgl_r - SLC_r$ Short-Term Net Gain Definition (193)

$lcng_r = \max(lng_r, 0) + \delta_r \min(sng_r, 0)$ Long-Term Cross Net Gain Definition (194)

$scng_r = \max(sng_r, 0) + \delta_r \min(lng_r, 0)$ Short-Term Cross Net Gain Definition (195)

$lcnl_r = -\min(lng_r, 0) - \delta_r \max(sng_r, 0)$ Long-Term Cross Net Loss Definition (196)

$scnl_r = -\min(sng_r, 0) - \delta_r \max(lng_r, 0)$ Short-Term Cross Net Loss Definition (197)

- For the Whole Portfolio:

$Tax = \sum_{r=1}^{n_r} \left[ltr_r \cdot \max(lcng_r, 0) + str_r \cdot \max(scng_r, 0) \right]$	Total Tax Cost Definition	(198)
$Benefit = \sum_{r=1}^{n_r} \left[ltr_r \cdot \max(lcnl_r, 0) + str_r \cdot \max(scnl_r, 0) \right]$	Total Loss Benefit Definition	(199)
$Tax \le u_{tax}$	Total Tax Cost Constraint	(200)

where,

h_{il}^{-}	sales amount of tax lot l of asset i
G_{il}	capital gain or loss per dollar sold of tax lot l of asset i $(Price_i - CostBasis_{il}) / Price_i$[4]
A_{il}	age (in terms of days away from the analysis date) of tax lot l of asset i
D_i	holding period (in terms of days) used to classify long- and short-term gain/losses for asset i
f_{il}	$= \begin{cases} 1, & \text{tax lot } l \text{ of asset } i \text{ is a tax-free lot} \\ 0, & \text{otherwise} \end{cases}$
C_q	the collection of assets in capital-gain group q
LLC_q	long-term loss carryforward for capital-gain group q
SLC_q	short-term loss carryforward for capital-gain group q
FLC_q	tax-free loss carryforward for capital-gain group q
T_r	the collection of assets in tax-rule group r
LLC_r	long-term loss carryforward for tax-rule group r (negative means gain carryover)
SLC_r	short-term loss carryforward for tax-rule group r (negative means gain carryover)
δ_r	$= \begin{cases} 1, & \text{netting between long-term and short-term capital gains and losses inside tax-rule group } r \text{ is allowed} \\ 0, & \text{otherwise} \end{cases}$
ltr_r	long-term capital-gain tax rate for tax-rule group r
str_r	short-term capital-gain tax rate for tax-rule group r
l_{*q}, u_{*q}	lower and upper bound, respectively, on various types of capital gains or losses for capital-gain group q
u_{tax}	upper bound on total tax cost

[4] Note that asset prices are required inputs for tax-aware optimization.

Some of the terms mentioned here (e.g., tax-rule groups and capital-gain groups) are explained more in detail in the following sections. To set up tax features and constraints, please refer to Sections 5.5.6, 8.2.5, and 8.5.35 in this document.

4.8.2 Tax-Rule and Capital-Gain Groups

We divide the investment universe into one or more tax-rule groups. A tax-rule group is a group of assets with the same tax rates:

- In the default setting, a single long-term tax rate applies to all taxable tax lots of all assets.
- When "two-rate" is enabled, taxable tax lots are classified into either long-term or short-term, and are taxed accordingly at the same two rates.
- Some tax lots may be designated as tax-free.

Every asset (excluding cash) in the investment universe belongs to one and only one tax-rule group. For example, we may divide the investment universe into US and non-US two tax-rule groups.

Two extreme cases with regard to the tax-rule group are:

1. All assets are in one tax-rule group—All assets in the portfolio share the same long-term and short-term tax rates. No tax-free lots and no assets are excluded from tax consideration. Tax cases supported before Barra Optimizer 8.8 belong to this category.
2. Each asset is a tax-rule group itself—Each asset has its own long-term and short-term tax rates, as well as other tax features.

A capital-gain group is an arbitrary group of assets on which constraints related to various capital gains can be set. Unlike the case with a tax-rule group, an asset in the investment universe may belong to multiple capital-gain groups. For example, if we are interested in controlling the amount of capital gains from the banking industry and from Europe, we may create two separate capital-gain groups, and a European banking asset belongs to both groups at the same time.

While both upper and lower bounds on various types of gains and losses for capital-gain groups are supported, only the upper bound on the portfolio-level total tax is allowed.

4.8.3 Netting Different Types of Gains and Losses

Within each tax-rule group, Barra Optimizer lets users decide whether to net long-term gains (losses) with short-term losses (gains) or not, each choice tailored to a particular investment situation.

The first is characteristic of a limited partnership or a mutual fund. Here, it is not appropriate to focus on the total net tax of a portfolio, either because the portfolio will be combined with others before computing the tax, or because the gains or losses will be passed through to the investors.

The second is more characteristic of an individual investor managing all his taxable (and possibly non-taxable) capital assets as a single portfolio. Here, the focus is on the total net tax of the portfolio. There may also be institutional portfolios for which this is the appropriate model.

The difference in how Barra Optimizer models these situations is straightforward. In the first case, no attempt is made to net short-term losses against long-term gains or vice-versa. In the second case, netting between different types of gains and losses takes place. In terms of constraints (194) and (195) the first situation corresponds to $\delta_r = 0$, whereas the second corresponds to $\delta_r = 1$. Barra Optimizer sets the first case as the default, and makes the second case an option.

Netting different types of gains and losses across different tax-rule groups is not allowed. Also, this type of netting does not apply to those capital-gain-group-level constraints (176)-(178).

4.8.4 Tax Arbitrage

Tax arbitrage refers to the trading activities that take advantage of a difference in tax rates or tax systems as the basis for profit. Barra Optimizer's Tax-Aware Optimization offers users numerous ways to explore their tax arbitrage strategies. In particular, users can set both upper and lower bounds on any of the gross gains, gross losses, and net gains amount, either long-term, short-term, or tax-free, within each capital-gain group. Note that the loss carryforward term in either of the net-gain definitional constraints (176)-(178) is a constant.

Tax arbitrage functionality in Barra Optimizer refers to the upper and lower bounds on the following types of capital gains/losses for a given capital-gain group:

- Long-Term Gross Gain
- Long-Term Gross Loss
- Long-Term Net Gain
- Short-Term Gross Gain
- Short-Term Gross Loss
- Short-Term Net Gain
- Tax-Free Gross Gain
- Tax-Free Gross Loss
- Tax-Free Net Gain
- Total Gross Gain
- Total Gross Loss
- Total Net Gain

where "Total" in the last three types listed above refers to the sum of long-term, short-term, and tax-free capital gains/losses.

4.8.5 Tax-Loss Harvesting

Tax loss harvesting usually refers to the practice of selling stocks that have lost money to offset a capital-gains tax liability. To facilitate tax loss -harvesting strategies, Barra Optimizer allows users to set a specific target on the amount of net capital gains, either short-term or long-term. A quadratic symmetric penalty function of the form (104) is then used to penalize any deviations away from the target.

For example, users may target the net short-term capital gains at $1 million with the penalty value ρ_q set at 100. In contrast to the usual practice of letting users specify the Upper and Lower parameters of the penalty function, here the users are required to supply the penalty values—ρ_q, ρ_{up} or ρ_{down}—explicitly. However, for a quadratic symmetric penalty function, the values for ρ_{up} or ρ_{down} are both zero, so users only need to supply the value ρ_q. For more information about the penalty functions, see the Constraint Flexibility—Penalties, Soft Bounds, and Hierarchy section.

Many investors desire to take full advantage of their annual capital loss deduction allowance and harvest the resulting tax loss benefits. In such cases, they could explicitly set the net capital loss target at the allowance level. In the US, the annual allowance is typically $3000. Thus, the tax loss harvesting problem is a special case of the net capital gains target problem.

Tax-loss harvesting functionality in Barra Optimizer refers to the target-penalty pairs placed on the following types of capital gains/losses for a given capital-gain group:

- Long-Term Net Gain
- Short-Term Net Gain
- Tax-Free Net Gain

4.8.6 Tax Lot Trading Rules

Under the default AUTO setting, Barra Optimizer automatically determines the order in which tax lots for each asset are traded according to the marginal contribution of each tax lot to the objective. The resulting trades usually obey the HIFO rule, but may not be strictly so.

Users may override the AUTO setting with the following selling-order rules:

- FIFO—Tax lots bought earlier are traded before those bought later
- LIFO—Tax lots bought later are traded before those bought earlier
- HIFO—Tax lots with higher cost basis are traded before those with lower cost basis

Barra Optimizer also allows users to specify a minimum holding period for each asset.

In addition, at the tax-lot level, users may specify the following trading rules with the last one being the default:

- SELL NONE—No share in the tax lot may be sold
- SELL ALL —All shares in the tax lot must be sold
- ALLOW ALL—No restriction is placed on selling any shares in the tax lot

Notes:

- The lot-level trading rules “SELL NONE/ SELL ALL” have a higher priority than the “minimum holding period” rule.
- The “minimum holding period” has a higher priority than the selling-order rules FIFO, LIFO and HIFO.

- Rules with a lower priority may be violated in order to fulfill those with a higher priority.

4.8.7 Short Tax Lots

Barra Optimizer also allows shorting in tax-aware optimization. Capital gains or losses from taxable short lots are always treated as short-term. An asset may have either long positions or short positions, but not both. In other words, Barra Optimizer does not allow the simultaneous occurrence of long and short tax lots of the same asset.

4.8.8 Wash Sales

According to the Internal Revenue Service's Publication 550, "A wash sale occurs when you sell or trade stock or securities at a loss and, within 30 days before or after the sale, you buy substantially identical stock or securities…"

From the Optimizer's point of view, a wash sale occurs when you:

- Sell a tax lot at loss while holding another unsold tax lot of the same asset bought within the wash-sales period (i.e., sell a tax lot at loss while a replacement lot exists)
- Buy an asset while a tax lot of this asset was sold at loss within the wash-sales period.

A wash sale would NOT occur when you:

- Sell a tax lot at gain.
- Sell a tax lot at loss AFTER selling all tax lots of the same asset bought within the wash-sales period (i.e., sell a tax lot at loss after no replacement lots remain),
- Buy an asset when no tax lots of this asset were sold at loss within the wash-sales period.

In the event of a wash sale,

1) You cannot deduct the loss from the sale in a wash sale.
2) You can add the disallowed loss to the cost basis of the new stock or securities purchased.
3) Your holding period for the new stock or securities extends to include the period you held the old stock or securities.

The details of the wash sale rules are rather complicated. Barra Optimizer offers three options in handling wash sales, as described in the following sections.

4.8.8.1 Ignore Wash Sales

With this option, Barra Optimizer would do nothing even if a wash sale did occur. Rules pertaining to wash sales are ignored.

4.8.8.2 Disallow Wash Sales

With this option, wash sales are prevented from happening. In particular, this rule implies the following (assuming the wash-sales period is 30 days):

- If no sales or purchases of an asset occurred within the last 30 days, then there is no restriction on the purchase or sale of this asset.
- If an asset was sold for a loss at least once within the last 30 days, and if that loss had not been disallowed previously, then the user would be prohibited from purchasing any shares of this asset, or closing a short sale on the same asset.
- If a short sale was closed for a loss within the last 30 days, and if that loss had not been disallowed previously, then users would be prohibited from any selling (including entering into another short sale on) the same asset.
- If one or more lots of an asset have been purchased within the last 30 days, unless such lots are sold out completely first, users would be prohibited from selling any tax lots of the same asset with unrealized losses.

In the last scenario above, Barra Optimizer may sometimes select those tax lots with an age less than 30 days and sell them first. Doing so enables the Optimizer to sell other tax lots of the same asset freely without triggering any wash sales, and may improve the objective value ultimately.

For example, suppose you have the following four tax lots of IBM:

Lot ID	Age	Shares	Loss or Gain
1	25	10	Loss
2	69	20	Loss
3	6	40	Gain
4	86	60	Gain

Under the Disallow-Wash-Sales option,

- You can sell Lot 3 and Lot 4 freely without violating the wash sale rule
- You cannot sell Lot 2 unless you sell out both Lot 1 and Lot 3 first.
- You cannot sell Lot 1 unless you sell out Lot 3 first.

The optimizer may pick Lot 3 to sell all of its 40 shares first, so that Lot 1 would be free to sell. If Lot 1 is also sold out, then Lot 4 will also be free to sell. Such a strategy is called "cherry-picking."

4.8.8.3 Trade-Off Wash Sales

With this option, wash sales can take place as long as it helps maximize the objective function. Barra Optimizer would internally do the adjustments (1)-(3) as listed in Section 4.8.8 when a wash sale occurs. In other words, the effects of any wash sales are being taken into consideration when Barra Optimizer weighs the trade-offs of return, risk, and tax.

More specifically, as explained in the beginning of Section 4.8.8, wash sales may be triggered by

- A new sale (i.e., sell-side), due to a previous purchase within the wash-sales period

OR

- A new purchase (i.e., buy-side), due to a realized loss on a previous sale within the last 30 days

For sell-side wash sales, the disallowed losses from new sales are deducted from the corresponding net gains. Take the following three tax lots of an asset as an example (assuming again the wash-sales period is 30 days, and the price of the asset is $40):

Lot ID	Age	Cost Basis	Initial Shares	Optimal Shares	Sold Shares	Net Gain		Adjusted		
						Ignore Wash Sales	Consider Wash Sales	Shares	Age	Cost Basis
1	20	$41	30	20	10	-$10	$0	10	20	$40
2	60	$42	50	0	50	-$100	$0	50	60	$40
3	5	$18	270	270	0	$0	$0	10	25	$19
								50	65	$20
								210	5	$18

Notice that both Lot 1 and Lot 2 are sold at a loss, but Lot 1 is sold partially while Lot 2 is sold out. Since Lot 3 was bought within 30 days and it has plenty of "replacement shares" remaining for both Lot 1 and Lot 2, the sales of Lot 1 and Lot 2 are considered as wash sales. Therefore, the losses from Lot 1 and Lot 2 will be disallowed (effectively by setting the cost basis to the price). Lot 3 will be split into three parts—10 shares will be adjusted for Lot 1, 50 shares will be adjusted for Lot 2, and the remaining 210 shares will be unchanged.

Similarly, for buy-side wash sales, the losses from previous sales will be added back to the corresponding net gains.

These changes in net gains would have ripple effects on the amount of capital gain taxes and on the objective value, which in turn affect the optimal solutions.

In general, for a plain vanilla case (i.e., without special features such as the tax penalty terms), when all else equal, the "Ignore Wash Sales" option is the least restrictive and would result in the best objective value, while the "Disallow Wash Sales" option is the most restrictive and would result in the worst objective value. The "Trade-Off Wash Sales" option is in between, both in terms of the flexibility and the objective value. Computationally, the "Trade-Off Wash Sales" option is most expensive among the three, resulting in the longest CPU time in most cases.

4.8.9 Overlap Constraint—Control of Total Active Benchmark Holdings

For various reasons, a portfolio manager may want to control the sum of the benchmark weights corresponding to his non-zero portfolio weights. We call this an “overlap” constraint. Mathematically, it can be represented as:

$$\sum_{i \in J} h_{B_i} \le U_{overlap}, \qquad J = \{ j \mid h_j \ne 0.0 \} \tag{201}$$

or

$$\sum_{i} \delta_i \cdot h_{B_i} \le U_{overlap}, \qquad \delta_i = \begin{cases} 1, & \text{if } h_i \ne 0.0 \\ 0, & \text{otherwise} \end{cases} \tag{202}$$

where h_i and h_{B_i} are the optimal portfolio weight and benchmark weight, respectively, for asset i. $U_{overlap}$ is an upper bound for the overlap.

The overlap constraint is a hard-to-solve, non-convex, discrete constraint similar to the cardinality constraints. The Barra Optimizer employs heuristics to handle it. In Barra Optimizer 8.8, this feature is only supported in the “compatible mode”.

4.8.10 Non-Convexity

In general, tax-aware optimization problems are non-convex and solutions are heuristic as well as local in nature. Global optimality is not guaranteed.

4.8.11 Cardinality and Threshold Constraints in Tax-Aware Optimization

Barra Optimizer supports the combination of tax-aware optimization with cardinality and threshold constraints. More specifically, the following constraints that are available in standard optimization or long-short optimization are also allowed in tax-aware optimization:

- Maximum/minimum number of names
- Maximum/minimum number of trades
- Maximum/minimum number of buy- or sell-side trades
- Minimum long- or short-side holding threshold
- Minimum long- or short-side trade threshold
- Minimum buy- or sell-side trade threshold

4.8.12 Leverage Constraints in Tax-Aware Optimization

Barra Optimizer also supports most types of leverage constraints in tax-aware optimization. The exceptions are the ratio-type leverage constraints described in Sections 4.6.1.4 and 4.6.1.5.

4.8.13 Risk Constraints in Tax-Aware Optimization

Currently, only a portfolio-level risk constraint is supported. It can be an upper bound or lower bound or both. Please refer to Appendix B for the supported combinations of features.

4.8.14 Tax-Aware Risk- or Return-Target Optimization

Tax-Aware Risk-Target Optimization allows users to set a specific target on the portfolio risk level while taking into account the impact of capital gain taxes. Mathematically, the problem can be represented as:

$$\begin{aligned}
&\text{Maximize:} && \lambda_r \mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda_{Tc} Tc(\mathbf{h},\mathbf{h}_0) - \lambda_{Pen} Pen(\mathbf{s},\mathbf{w}) - Tax(\mathbf{h}) - \text{Other Terms} \\
&\text{Subject to:} && \sigma(\mathbf{h}) = \sigma_{target} \\
& && \text{Other Constraints}
\end{aligned} \tag{203}$$

where, $\sigma(\mathbf{h})$ is the standard deviation of the portfolio return or active return, and "Other Constraints" includes standard and tax-related constraints.

Tax-Aware Return-Target Optimization has the form

$$\begin{aligned}
&\text{Minimize:} && \sigma(\mathbf{h})^2 + \lambda_{Pen} Pen(\mathbf{s},\mathbf{w}) + Tax(\mathbf{h}) + \text{Other Terms} \\
&\text{Subject to:} && \lambda_r \mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda_{Tc} Tc(\mathbf{h},\mathbf{h}_0) = r, \quad r \in [r_{min}, r_{max}] \\
& && \text{Other Constraints}
\end{aligned} \tag{204}$$

Note that the "trade-off wash sales" option is currently not supported with this feature.

4.9 5/10/40 Rule

The 5/10/40 Rule is a shorthand term for the European Community Directive on Undertakings for Collective Investment in Transferable Securities (UCITS). It stems from a German law, but similar regulations exist in France, Switzerland, and Austria. It requires that a balanced portfolio satisfy the following conditions:

- The maximum weight of securities of a single issuer cannot exceed 10% of the portfolio value; and
- The sum of the weights of all issuers representing more than 5% of the portfolio value cannot exceed 40%.

A more flexible definition of the rule is implemented in Barra Optimizer as follows:

- The maximum weight of securities of a single issuer cannot exceed Q% of the portfolio value; and
- The sum of the weights of all issuers representing more than P% of the portfolio value cannot exceed R%.

This general rule is referred to as the "P/Q/R" rule, and the default values are P = 5, Q = 10, and R = 40. It is important to note that Barra Optimizer treats composites separately from other types of assets. A composite is considered its own issuer. The component weights of a composite are not combined with the non-composite asset weights of the same issuer. For example, assume that a portfolio consists of only three equally weighted assets—X, Y, and Z. Assets X and Y are regular assets, both issued by company A. Asset Z is a composite comprised of 30% X and 70% Y. Then, from the perspective of Barra Optimizer, this portfolio has two issuers—A and Z. Issuer A accounts for two-thirds of the portfolio weight, and Z one-third.

Portfolio optimization problems with 5/10/40 rules are discrete and not convex in general. Barra Optimizer employs branch and bound, or heuristic algorithms to tackle the 5/10/40 rules. The solution portfolios, particularly for large-sized problems, are not globally optimal in general.

4.10 Maximizing the Sharpe or Information Ratio

4.10.1 Maximizing the Sharpe Ratio

The Sharpe Ratio (SR) is defined as the excess return of a portfolio divided by its total risk. This can be expressed mathematically as:

$$\underset{\mathbf{h}}{\text{Maximize}}: \quad \frac{\mathbf{r}^{\mathrm{T}}\mathbf{h} - r_f - \lambda_{Tc}\, Tc(\mathbf{h})}{\sqrt{\mathbf{h}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{h}}} \tag{205}$$

where r_f is the risk-free interest rate, $\boldsymbol{\Sigma}$ is defined in (33), and the other variables are defined in (1).

4.10.2 Maximizing the Information Ratio

The Information Ratio (IR) is defined as the active return of a portfolio with respect to its benchmark divided by its active risk. It can be expressed mathematically as:

$$\underset{\mathbf{h}}{\text{Maximize}}: \quad \frac{\mathbf{r}^{\mathrm{T}}(\mathbf{h} - \mathbf{h}_B) - \lambda_{Tc}\, Tc(\mathbf{h})}{\sqrt{(\mathbf{h} - \mathbf{h}_B)^{\mathrm{T}}\boldsymbol{\Sigma}(\mathbf{h} - \mathbf{h}_B)}} \tag{206}$$

For more information, see Kopman & Liu (2009).

4.11 Optimal Frontiers—Parametric Optimization

Standard Optimization, Paring Problems, Risk Constrained Optimization, Long-Short Optimization, and Tax-Aware Optimization can all be regarded as single portfolio optimization because the optimal solution for any one of them is a single portfolio. In contrast, Parametric Optimization consists of sequences of single portfolio optimization. Its output usually contains multiple portfolios.

Generally, Parametric Optimization treats a certain component in a single portfolio optimization problem as a parameter. The optimal solution is a function of the parameter chosen. As the parameter takes on different values, so does the optimal solution. The trajectory of the optimal solution forms a frontier, each point on which indicates the best possible portfolio given a value of the parameter.

Barra Optimizer offers two broad categories of Parametric Optimization—the Risk-Return Efficient Frontiers and the Constraint-Objective Frontiers.

4.11.1 Risk-Return Efficient Frontiers

A Risk-Return Efficient Portfolio has the smallest level of risk among all portfolios with the same level of expected return. It is also the portfolio having the highest expected return among all portfolios with the same level of risk. Return here refers to alpha minus any transaction costs present in the objective function. The (risk-return) efficient frontier is a graph of the risk-return pairs for all efficient portfolios.

Either return or risk may be varied. Assume:

- r is the portfolio net return
- σ is the standard deviation of the portfolio return or active return

Mathematically, the two basic types of risk-return efficient frontier problems are:

4.11.1.1 Varying Return

$$\begin{aligned} \text{Minimize:} \quad & \sigma(\mathbf{h})^2 + \lambda_{Pen} Pen(\mathbf{s},\mathbf{w}) \\ \text{Subject to:} \quad & \lambda_r \mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda_{Tc} Tc(\mathbf{h},\mathbf{h}_0) = r, \quad r \in [r_{min}, r_{max}] \\ & \text{Other Constraints} \end{aligned} \tag{207}$$

4.11.1.2 Varying Risk

$$\begin{aligned} \text{Maximize:} \quad & \lambda_r \mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda_{Tc} Tc(\mathbf{h},\mathbf{h}_0) - \lambda_{Pen} Pen(\mathbf{s},\mathbf{w}) - \text{Other Terms} \\ \text{Subject to:} \quad & \sigma(\mathbf{h}) \in [\sigma_{min}, \sigma_{max}] \\ & \text{Other Constraints} \end{aligned} \tag{208}$$

The Risk-Target Problem in the Risk Target Optimization section is a special case of the Varying Risk frontier problem where the risk range is reduced to a fixed point. Similar, a Return-Target Problem is a special case of the Varying Return frontier problem where the return range is reduced to a fixed point.

Thus, the variance term could be for either total risk or active risk. Net return may incorporate penalties when needed. Users may specify a section of the frontier to examine by entering either the return range

$[r_{min}, r_{max}]$ or the risk range $[\sigma_{min}, \sigma_{max}]$. If no user input is given, a broad section of the frontier will be provided in the output by default.

Barra Optimizer returns the following information about selected efficient portfolios along the frontier:

- Objective function value
- Net return (%), i.e., $\lambda_r \mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda_{Tc} Tc(\mathbf{h}, \mathbf{h}_0)$
- Predicted portfolio risk (%)
- Portfolio turnover (%)
- Implied risk aversion

The implied risk aversion is the risk aversion value that, if applied to a single portfolio optimization with an equivalent problem setting, would result in an optimal portfolio with the specified predicted risk[5]. The reported objective is computed by using the implied risk aversion.

4.11.2 Constraint-Utility Frontiers

The Constraint-Utility Frontier, (or in other words, the Constraint-Objective Frontier), is a graph of the objective levels of an optimal portfolio against different values of a constraint bound. It illustrates the effect of varying the bound of a constraint on the optimal objective value.

For example, how would the objective value change when the turnover limit is increased from 5% to 30%? What would happen to the objective value when the portfolio's exposure to the "growth" factor varies from -1 to 1 relative to the benchmark? The Turnover-Utility Frontier, Tax-Utility Frontier, and Leverage-Utility Frontier are special cases of the Constraint-Utility Frontier.

To use the Constraint-Utility Frontier feature, users need to indicate which constraint is to vary (i.e., whether it is the turnover constraint limit constraint) and over what range. The constraint bound could be an upper bound, a lower bound, or equality bound. For piecewise-linear constraints, however, only the upper bound is permitted to vary.

For a given number of points on the efficient frontier, $\mathrm{N}_{\text{points}}$ Barra Optimizer picks $\mathrm{N}_{\text{points}}$ values of variable t in the user-specified range [lower, upper] and solves a series of optimization problems as shown below:

4.11.2.1 Varying Turnover

$$\begin{aligned} \text{Maximize:} \quad & f(\mathbf{h}) \\ \text{Subject to:} \quad & TO(\mathbf{h}) \le t \\ & \text{Other Constraints} \end{aligned} \tag{209}$$

[5] If the problem is not convex, or it has a penalty or short rebate cost term in the objective function, or soft bound is involved, then this may not be true.

4.11.2.2 Varying Tax Cost

$$\begin{aligned} &\text{Maximize:} && f(\mathbf{h}) \\ &\text{Subject to:} && Tax(\mathbf{h}) \le t \\ &&& \text{Other Constraints} \end{aligned} \tag{210}$$

4.11.2.3 Varying Leverage

$$\begin{aligned} &\text{Maximize:} && f(\mathbf{h}) \\ &\text{Subject to:} && \begin{cases} L_{LLev} \le LongLeverage(\mathbf{h}) \le t & or \\ t \le ShortLeverage(\mathbf{h}) \le U_{SLev} & or \\ L_{TLev} \le TotalLeverage(\mathbf{h}) \le t \end{cases} \\ &&& \text{Other Constraints} \end{aligned} \tag{211}$$

Note that the leverage constraint in (211) is only limited to the types described in Sections 4.6.1.1-4.6.1.3 and section 4.6.1.6. In other words, only varying the upper bound of a (weighted) long-leverage, the lower bound of a (weighted) short-leverage, or the upper bound of a (weighted) total-leverage constraint is allowed. No ratio-type leverage constraints are supported in leverage-utility frontiers.

Similar to the output for Risk-Return Frontiers, Barra Optimizer returns the following information about selected optimal portfolios along a Constraint-Utility Frontier:

- Objective function value
- Net return (%), i.e., $\lambda_r \mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda_{Tc} Tc(\mathbf{h}, \mathbf{h}_0)$
- Predicted portfolio risk (%)
- Portfolio's exposure to the constraint

4.12 Multi-Period Optimization

As its name suggests, Multi-Period Optimization takes into consideration the input information (i.e., return, risk, transaction cost, etc.) over a number of successive periods and produces a set of optimal portfolios for each of these periods. This is in contrast to the traditional single portfolio optimization, which only considers the input information for a single period and produces one optimal portfolio for that period. It also differs from Parametric Optimization, which solves a sequence of single portfolio optimization problems, all for a single period. Multi-Period Optimization solves one single portfolio optimization problem combining multiple periods.

Mathematically, a mean-variance multi-period optimization can be formulated as follows:

$$\underset{\mathbf{h}^1,\mathbf{h}^2,\cdots,\mathbf{h}^{N_t}}{\text{Maximize}}: \quad f(\mathbf{h}^1,\mathbf{h}^2,\cdots,\mathbf{h}^{N_t}) = \sum_{t=1}^{N_t} \lambda_t \, f(\mathbf{h}^t) \tag{212}$$

Subject to:

$\mathbf{e}^T\mathbf{h}^t = 1$	Portfolio Balance Constraint for Period t	(213)
$\mathbf{l}_h^t \le \mathbf{h}^t \le \mathbf{u}_h^t$	Asset Ranges for Period t	(214)
$\mathbf{s}^t = \mathbf{A}^t\mathbf{h}^t$	Definitional Constraints for Period t's General Slack Variables	(215)
$\mathbf{l}_s^t \le \mathbf{s}^t \le \mathbf{u}_s^t$	Ranges for Period t's Constraint Slacks	(216)
$\mathbf{w}^t = (\mathbf{X}^t)^{\mathrm{T}}\mathbf{h}^t$	Definitional Constraints for Period t's Factor Slack Variables	(217)
$\mathbf{l}_X^t \le \mathbf{w}^t \le \mathbf{u}_X^t$	Factor Exposure Ranges for Period t	(218)
$To\left(\mathbf{h}^t,\mathbf{h}_0^t\right) \le u_{To}^t$	Turnover Constraint for Period t	(219)
$\sum_{t=1}^{N_t} To\left(\mathbf{h}^t,\mathbf{h}_0^t\right) \le u_{To}$	Cross-Period Total Turnover Constraint	(220)
$Tc\left(\mathbf{h}^t,\mathbf{h}_0^t\right) \le u_{Tc}^t$	Transaction Cost Constraint for Period t	(221)
$\sum_{t=1}^{Nt} Tc\left(\mathbf{h}^t,\mathbf{h}_0^t\right) \le u_{Tc}$	Cross-Period Total Transaction Cost Constraint	(222)

for all periods $t = 1,\ 2, \cdots,\ N_t$, where

N_t	total number of periods	
λ_t	scalar for each period's utility	
$\mathbf{h}^t$	vector of portfolio weights for period t	(nx1)
$f(\mathbf{h}^t)$	standard mean-variance objective function (1) for period t	

All variables, terms, and constraints in (212)-(222) are essentially the same as defined in the Problem Formulation section except that they are now period-specific.

Notes:

- The total number of periods is at most 5. i.e. $N_t \le 5$.
- The only cross-period constraints allowed are the turnover and transaction cost limit constraints.
- The objective terms and constraints (except the cross-period ones) for each period are separable. The objective for each period is a convex quadratic function. The standard constraints for each period are either linear or convex piecewise-linear.
- No discrete constraints or variables are allowed except cardinality and threshold constraints with limited scope. More specifically, holding cardinality and minimum holding threshold constraints are supported for all periods, but trade cardinality and minimum trade threshold constraints are only supported for the first period.
- No nonlinear constraints are allowed.
- Users specify portfolio base value for the first period and cash flow weights for each period. This base value should equal to the initial portfolio value plus cash flow value for the first period. Barra Optimizer adjusts the portfolio base values and asset initial weights for subsequent periods internally according to the cash flow weights provided. Asset initial weights for each period sum to 1.
- Users are responsible for providing alpha and transaction cost information for all periods. Starting with Barra Optimizer 9.2, non-linear transaction costs (as described in Section 3.8.2) are supported in the objective of each period. However, nonlinear transaction costs are still NOT supported in any of the transaction cost limit constraints.
- Only one risk model is allowed in each period. The same risk model is used for all periods.
- At most one benchmark is allowed in each period.
- Overall as well as buy- and sell-side turnover are supported in per-period turnover constraints. However, buy- and sell-side cross-period turnover constraints are not supported.

To setup multi-period features and constraints in the Barra Optimizer, please refer to Sections 5.7, 8.5.30, and 8.8. For more information about multi-period optimization, see Liu and Xu (2017).

4.13 Multi-Account Optimization

Multi-Account Optimization (or Multi-Portfolio Optimization) simultaneously optimizes a number of accounts based on their individual account information as well as their joint market-impact transaction costs or any cross-account constraints. Barra Optimizer supports two variants of Multi-Account Optimization as described in the following sections.

4.13.1 General Multi-Account Optimization

Mathematically, a general mean-variance Multi-Account Optimization that maximizes the total-welfare of all non-tax accounts can be formulated as follows:

$$\underset{\mathbf{h}^1,\mathbf{h}^2,\cdots,\mathbf{h}^{N_p}}{\text{Maximize}}: \quad f(\mathbf{h}^1,\mathbf{h}^2,\cdots,\mathbf{h}^{N_p}) = \sum_{p=1}^{N_p}\left(\frac{BV_p}{TBV}\ \lambda_p\ f(\mathbf{h}^p)\right) - \lambda_{JTC} JTC(\mathbf{h}^1-\mathbf{h}_0^1,\mathbf{h}^2-\mathbf{h}_0^2,\cdots,\mathbf{h}^{N_p}-\mathbf{h}_0^{N_p}) \tag{223}$$

Subject to:

$\mathbf{e}^T\mathbf{h}^p = 1$	Portfolio Balance Constraint for Account p	(224)
$\mathbf{l}_h^p \le \mathbf{h}^p \le \mathbf{u}_h^p$	Asset Ranges for Account p	(225)
$\mathbf{s}^p = \mathbf{A}^p\mathbf{h}^p$	Definitional Constraints for Account p's General Slack Variables	(226)
$\mathbf{l}_s^p \le \mathbf{s}^p \le \mathbf{u}_s^p$	Ranges for Account p's Constraint Slacks	(227)
$\mathbf{w}^p = (\mathbf{X}^p)^{\mathrm{T}}\mathbf{h}^p$	Definitional Constraints for Account p's Factor Slack Variables	(228)
$\mathbf{l}_X^p \le \mathbf{w}^p \le \mathbf{u}_X^p$	Factor Exposure Ranges for Account p	(229)
$To\left(\mathbf{h}^p,\mathbf{h}_0^p\right) \le u_{To}^p$	Turnover Constraint for Account p	(230)
$Tc\left(\mathbf{h}^p,\mathbf{h}_0^p\right) \le u_{Tc}^p$	Transaction Cost Constraint for Account p	(231)
$\mathbf{l}_g \le \boldsymbol{g}(\mathbf{h}^1,\mathbf{h}^2,\cdots,\mathbf{h}^{N_p}) \le \mathbf{u}_g$	Cross-Account Constraints	(232)

for all accounts $p = 1, 2, \cdots, N_p$, where

N_p	total number of accounts or portfolios
λ_p	scalar for Account p's utility
BV_p	base value for Account p
TBV	total base value for all accounts
λ_{JTC}	multiplier specifying the importance of the joint market-impact term relative to other terms in the objective.

$\mathbf{h}^p$	vector of portfolio weights for Account p	(nx1)
$f(\mathbf{h}^p)$	standard mean-variance objective function (1) for Account p	
$JTC(.)$	joint market-impact transaction-cost term	

What sets Multi-Account Optimization apart from single-portfolio optimization is the extra joint market-impact term at the end of the objective function (223) and the cross-account constraints (232). We will discuss these two items in more detail in the following sections. All other variables, terms, and constraints in (223)-(232) are essentially the same as defined in the Problem Formulation section except that they are now account-specific.

4.13.1.1 Joint Market-Impact Transaction Costs

Different accounts may buy or sell the same asset at the same time. Depending on whether cross-account trading is allowed, the joint market-impact transaction cost term could be different. In most cases, especially with ERISA compliance though, cross-account trading is not allowed. All buy and sell trades may need to be pooled together and trade in the open market.

The joint market-impact transaction cost function for aggregate buys and sells is currently limited to piecewise-linear function. In other words, nonlinear joint transaction cost (power function) is not supported.

Let $x_i^{p+} \geq 0$ and $x_i^{p-} \geq 0$ be the buy and sell dollar amounts for asset i in account p, and $\mathbf{x}^p = \left(x_1^{p+} + x_1^{p-} \quad x_2^{p+} + x_2^{p-} \quad \cdots \quad x_n^{p+} + x_n^{p-} \right)$ be the trade amount vector for account p.

Let $t_i^+ \geq 0$ and $t_i^- \geq 0$ be the total buy and sell dollar amounts for asset i for all accounts. Then, we have the following relationship:

$$\begin{cases} x_i^{p+} = P^p \max(h_i^p - h_{0i}^p, 0) \\ x_i^{p-} = P^p \max(h_{0i}^p - h_i^p, 0) \end{cases}, \qquad i = 1, 2, \cdots, n, \qquad p = 1, 2, \cdots, N_p \tag{233}$$

where P^p is the portfolio value for account p, and

$$\begin{cases} t_i^+ = \sum_{p=1}^{N_p} x_i^{p+}, \\ t_i^- = \sum_{p=1}^{N_p} x_i^{p-}, \end{cases} \qquad i = 1, 2, \cdots, n \tag{234}$$

Transaction costs are separable among assets. Hence, the joint market-impact transaction cost term in (223) can be further written as:

$$\begin{aligned} &JTC(\mathbf{h}^1 - \mathbf{h}_0^1, \mathbf{h}^2 - \mathbf{h}_0^2, \cdots, \mathbf{h}^{N_p} - \mathbf{h}_0^{N_p}) \\ &= JTC(\mathbf{x}^1, \mathbf{x}^2, \cdots, \mathbf{x}^{N_p}) = \sum_{i=1}^{n} JTC_i(t_i^+, t_i^-) = \sum_{i=1}^{n} \left[JTC_i(t_i^+) + JTC_i(t_i^-) \right] \end{aligned} \tag{235}$$

4.13.1.2 Cross-Account Constraints

For cross-account constraints, users will provide input in terms of dollars. The following three types of cross-account constraints are the only ones currently supported:

- Bounds on global position for an asset. For example, total positions for MSCI should be no more than $1B
- Maximum total buy, sell, or trade size per asset over all accounts. For example, aggregate total buys for MSCI should be less than or equal to $500M, or the total trades for IBM should not be more than $280M
- Cross-Account Turnover constraint
  Note especially that cross-account transaction cost constraints are NOT currently supported.

4.13.1.3 Two Basic Approaches

We offer two approaches to Multi-Account Optimization; both approaches are well established in financial research literature. For details, see Yang et al (2013) and O'Cinneide et al (2006).

4.13.1.3.1 Total-Welfare Maximization Approach

This approach considers the optimization problem (223) -(232) as one single large problem. The objective is to maximize the total-welfare (i.e., the joint utility function) of all accounts. The solution of this approach is called a Pareto optimal solution. Multiple optimal solutions may exist.

Although the solution of this approach maximizes the total welfare for all accounts, it may do so at the expense of some individual accounts. In other words, some accounts may be sacrificed more than others for the benefit of maximizing total welfare. For this reason, a Pareto solution may not be a fair solution to all accounts.

4.13.1.3.2 Nash Equilibrium (Game Theory) Approach

This approach decomposes the optimization problem (223) -(232) into successive account-level sub-problems, and solves them iteratively, one at a time, until the trade amounts do not change for each individual account. When solving the sub-problem for account p, we assume that the trades for other accounts are known already and treat them as merely constants. In other words, the objective function of the sub-problem for account p can be written as:

$$\underset{\mathbf{h}^p}{\text{Maximize}}: \ \frac{BV_p}{TBV} \lambda_p \ f(\mathbf{h}^p) - \lambda_{JTC} JTC_p(\mathbf{x}^p, \gamma^{p+}, \gamma^{p-}) \tag{236}$$

where $\mathbf{x}^p$ is the trade amount for account p, and both $\gamma^{p+} = \sum_{i=1}^{n} (t_i^+ - x_i^{p+})$ and $\gamma^{p-} = \sum_{i=1}^{n} (t_i^- - x_i^{p-})$ are constants representing the sum of buy or sell amounts for all other accounts.

When there are no cross-account constraints, Yang et al (2013) have shown that a unique Nash Equilibrium solution exists and can be obtained by simple iterative procedures. If cross-account constraints are present, more sophisticated methods, such as the Lagrange Multiplier and Sub-Gradient Algorithms, will be used.

4.13.1.4 Post-Optimization Transaction Cost Allocation

After solving the Multi-Account Optimization problem, for various reasons, the joint market-impact transaction costs, if any, may need to be allocated back to each account. The following pro-rata allocation formula is used as the current default approach:

$$TC_p = \sum_{i=1}^{n} Buy_p(i)\,\frac{TC_{buy}(\mathrm{i})}{Total_{buy}(\mathrm{i})} + \sum_{i=1}^{n} Sell_p(i)\,\frac{TC_{sell}(\mathrm{i})}{Total_{sell}(\mathrm{i})} \tag{237}$$

where

- $TC_{buy}(\mathrm{i})$ and $TC_{sell}(\mathrm{i})$ are the total buy cost and total sell cost for asset i.
- $Total_{buy}(\mathrm{i})$ and $Total_{sell}(\mathrm{i})$ are the total buy and total sell amount for asset i for all accounts.
- $Buy_p(i)$ and $Sell_p(i)$ are the buy and sell amounts for asset i in account p.

This allocation method does not guarantee fairness. Users are free to apply any other allocation method of their choice because this procedure is post-optimization.

4.13.1.5 Scopes and Limitations

Currently, we limit the scope of Multi-Account Optimization as follows:

- The total number of accounts will be at most 20, i.e., $N_p \le 20$.
- The objective terms and constraints (except the joint and cross-account ones) for each account are separable. The objective for each account is a convex quadratic function. The standard constraints for each account are either linear or convex piecewise-linear. Most features for a standard single-portfolio optimization are supported at the individual account level. An exception to this is that constraint hierarchy will not be supported for Multi-Account Optimization. Some other exceptions are given below.
- Users will be able to set transaction costs EITHER for each individual account OR for the joint market-impact term, but not BOTH. In other words, when the joint market-impact term is present, users will NOT be able to set any transaction cost limit constraint for each individual account.
- When transaction cost terms are set for individual accounts, we assume that the same asset will have the same transaction cost function for all accounts. There is no limit on number of break points for an asset or for aggregate buys or sells.
- We assume cross-account trading is not allowed.
- Besides linear and piecewise-linear constraints, no other type constraints are allowed for individual accounts with the exception of cardinality and threshold constraints. Penalty on cardinality and threshold constraints are not supported. The “EnableGrandfatherRule” and ““Allow Close-Out” options for paring apply to all accounts uniformly.
- Only one risk model is allowed in each account. The same risk model is used for all accounts. However, different accounts may have different benchmarks and risk aversions.

- Each account may have a different portfolio value. However, the portfolio base value for each account must be the same as its portfolio value.
- All constraints and objective function terms (except the break-points for transaction costs) that are for individual accounts only will be given in terms of weights. Joint transaction cost terms and cross-account constraints will be given in terms of dollar amounts.
- It is currently assumed that asset alphas are the same for all accounts.
- Overall turnover is supported as per-account as well as cross-account constraints.
- Universal maximum trade size per asset for all accounts (e.g., no accounts can buy more than $10M MSCI) should be enforced by asset bounds rather than cross-account constraints for efficiency reasons.

Please refer to Sections 5.8, 8.5.34 and 8.9 for the specifics of setting up multi-account features and constraints in the Barra Optimizer.

4.13.2 Multi-Account Tax-Aware Optimization

Multi-account tax-aware optimization is the combination of two existing Barra Open Optimizer features: multi-account optimization and tax-aware optimization. The scope and limitations of multi-account tax-aware optimization are the same as those for general multi-account optimization and general tax-aware optimization combined. Additionally, the “Trade-Off Wash Sales” option is not yet supported either at the individual-account or the cross-account level.

The two approaches to the general multi-account optimization also apply to this optimization.

4.13.2.1 Tax-Aware Total-Welfare Maximization

Mathematically, a mean-variance Multi-Account Tax-Aware Optimization that maximizes the total-welfare of all accounts can be formulated as follows:

$$\underset{\mathbf{h}^1,\mathbf{h}^2,\cdots,\mathbf{h}^{N_p}}{\text{Maximize}}: \quad f_{tax}(\mathbf{h}^1,\mathbf{h}^2,\cdots,\mathbf{h}^{N_p}) = f(\mathbf{h}^1,\mathbf{h}^2,\cdots,\mathbf{h}^{N_p}) - \sum_{p=1}^{N_p}\left(\frac{BV_p}{TBV}\ \lambda_{Tax}^{p}\, Tax(\mathbf{h}^p)\right) \tag{238}$$

$$\text{Subject to:} \quad \text{Standard Constraints for Each Individual Account} \tag{239}$$

$$\text{Cardinality and Threshold Constraints for Each Individual Account} \tag{240}$$

$$\text{Cross-Account Linear or Piecewise-Linear Constraints} \tag{241}$$

$$\text{Tax-Related Constraints for Each Individual Account} \tag{242}$$

$$\text{Cross-Account Tax-Related Constraints} \tag{243}$$

where

$f(\mathbf{h}^1,\mathbf{h}^2,\cdots,\mathbf{h}^{N_p})$	the general multi-account optimization objective as defined in (223)
$Tax(\mathbf{h}^p)$	total tax cost for Account p
λ_{Tax}^{p}	tax multiplier for Account p

All constraints (239)-(243) are optional.

Definitions for the individual-account standard constraints (239) are given by (224)-(231). The scope for the cardinality and threshold constraints (240) are given in Section 4.13.1.5 and for the cross-account linear and piecewise-linear constraints (241) are given in Section 4.13.1.2. Individual-account tax-related constraints (242) are defined by (170)-(200).

The only constraints in the category (243) that are supported currently are:

- Upper bound on cross-account total tax:

$$CTax = \sum_{p=1}^{N_p} Tax(\mathbf{h}^p) \qquad \text{Cross-Account Total Tax Definition} \qquad (244)$$

$$CTax \le u_{ctax} \qquad \text{Cross-Account Total Tax Constraint} \qquad (245)$$

  where u_{ctax} is the given upper bound on the cross-account total tax

- Upper bound on the total tax from each tax-account group, as defined in Section 4.13.2.3 below.
- Internal cross-account constraints associated with the "Disallow Wash-Sales" option. See Section 4.13.2.4 for more details.

4.13.2.2 Tax-Aware Nash Equilibrium

As in the case of General Multi-Account Optimization, the Nash Equilibrium approach decomposes the optimization problem (238)– (243) into successive account-level sub-problems, and solves them iteratively, one account at a time, until the trade amounts do not change for each individual account. The objective function of the sub-problem for account p in a Multi-Account Tax-Aware Nash Equilibrium (game theory) problem can be written as:

$$\underset{\mathbf{h}^p}{\text{Maximize}}: \frac{BV_p}{TBV} \lambda_p \left(f(\mathbf{h}^p) - \lambda_{Tax}^p Tax(\mathbf{h}^p) \right) - \lambda_{JTC} JTC_p(\mathbf{x}^p, \gamma^{p+}, \gamma^{p-}) \qquad (246)$$

or equivalently,

$$\underset{\mathbf{h}^p}{\text{Maximize}}: \frac{BV_p}{TBV} \lambda_p f_{tax}(\mathbf{h}^p) - \lambda_{JTC} JTC_p(\mathbf{x}^p, \gamma^{p+}, \gamma^{p-}) \qquad (247)$$

4.13.2.3 Tax-Account Groups

To give users more flexibility in netting capital gains and control capital-gain taxes, Barra Optimizer now provides an option for users to divide tax accounts into one or more groups. Upper bound on the total tax from each tax-account group is supported.

For example, consider the following family portfolio with three accounts (assuming the capital-gain tax rate is 20%):

Account	Unrealized Gain ($)	Unrealized Loss ($)	Unrealized Net Gain ($)	Potential Tax ($)
A (Father's)	1000	0	1000	200
B (Mother's)	0	500	0	0
C (Child's)	0	1000	0	0
Portfolio Total				200

In the above case, each account is independent when we calculate capital-gain taxes. However, sometimes, we may desire to net the gains and losses between accounts A and B first, before capital-gain taxes are calculated. In this second case, we can add a two-account group (named AB) to achieve this goal, as displayed in the following table:

Account Group	Account	Unrealized Gain ($)	Unrealized Loss ($)	Unrealized Net Gain ($)	Potential Tax ($)
AB		1000	500	500	100
	C (Child's)	0	1000	0	0
Portfolio Total					100

Similarly, if we desire to net the gains and losses of all three accounts, we may create a three-account group to achieve the goal. In this third case, the potential capital-gain tax reduces to zero:

Account Group	Unrealized Gain ($)	Unrealized Loss ($)	Unrealized Net Gain ($)	Potential Tax ($)
ABC	1000	1000	0	0
Portfolio Total				0

Note that tax-account groups are different from the tax-rule and capital-grain groups defined in Section 4.8.2.

- A tax-rule or capital-gain group is a group of assets within one account.
- A tax-account group is a group of accounts.
- A portfolio may consist of multiple tax-account groups and multiple independent accounts.
- However, an account can only appear once in the portfolio, either independently, or in one and only one tax-account group, but not both.

- We require that the accounts in the same tax-account group must share the same tax-rule group definitions.

Mathematically, for a given tax-account group S, its tax-related constraints are identical to those individual-account level constraints (188)-(200), except that the "i" index in (188)-(191) now represents the assets in all accounts inside the tax-account group S. In other words, the gross gains (or losses) for tax-rule group r inside tax-account group S are the sum of the gross gains (or losses) for tax-rule group r from all constituent accounts of S.

When tax-account groups are present, the cross-account total tax definition (244) is modified as follows:

$$CTax = \sum_{s=1}^{N_s} Tax_s + \sum_{p \notin T_s} Tax(\mathbf{h}^p) \tag{248}$$

where

N_s	total number of tax-account groups
T_s	the collection of accounts in all tax-account groups
Tax_s	total tax from tax-account group S

The right-hand side of (248) is the sum of the taxes from all tax-account groups and the taxes from each independent account.

Notes:

- The same asset in different accounts of a tax-account group belongs to the same tax-rule group.
- A tax-free account cannot be included in any tax-account group
- If an account belongs to a tax-account group, its independent total tax will not be reported. Instead, only the total tax from the tax-account group it belongs to will be reported.

4.13.2.4 Disallow Cross-Account Wash Sales

In the context of Multi-Account Tax-Aware Optimization, the wash sales rule applies to individual accounts as well as a group of "wash-sale-related" accounts designated by the user in advance. Under the "Disallow Wash Sales" option, wash sales are not allowed to occur within those related accounts. In particular, it implies the following:

- If no sales or purchases of an asset occurred within the last 30 days in any one of the wash-sale-related accounts, then there is no restriction on the purchase or sale of this asset.
- If an asset was sold for a loss at least once within the last 30 days in one of the wash-sale-related accounts, and if that loss had not been disallowed previously, then we cannot purchase any shares of this asset, or close a short sale on the same asset in any of the wash-sale-related accounts.

- If a short sale was closed for a loss within the last 30 days in one of the wash-sale-related accounts, and if that loss had not been disallowed previously, then we cannot sell (or enter into another short sale on) the same asset in any of the wash-sale-related accounts.
- If one or more lots of an asset have been purchased within the last 30 days in one of the wash-sale-related accounts, unless such lots are sold out completely first, we cannot sell any tax lots of the same asset with unrealized losses in any of the wash-sale-related accounts.

Note: By default, the number of "wash-sale-related" accounts is zero. Users need to specify these accounts if desired.

4.14 Expected Shortfall Optimization

Critics of the mean-variance portfolio optimization often point out that using variance as the risk measure equally penalizes the desirable upside potential and the undesirable downside risk. Investors and managers alike naturally would prefer to construct their portfolios with some downside protection while preserving all upside potentials.

Expected shortfall provides an alternative risk measure which is only concerned with the downside risk. It is particularly useful when the underlying return distributions have fat tails and predictable asymmetries, i.e., when the downside differs remarkably from the upside.

4.14.1 Definitions

Expected shortfall is a downside risk measure, defined as the expected value of a portfolio's loss (L_h) conditioned on the loss being larger than the Value-at-Risk ($VaR_\rho(\mathbf{h})$) of the portfolio for a confidence level ρ:

$$s_\rho(\mathbf{h}) = E\left[L_h \mid L_h = -\mathbf{r}^{\mathrm{T}}\mathbf{h},\ L_h \ge VaR_\rho(\mathbf{h})\right] \tag{249}$$

Expected shortfall may also be defined relative to a target (or benchmark) return ($\boldsymbol{\mu}^T\mathbf{h}$), as the expected value of the difference between the target and portfolio returns, conditioned on the difference being larger than the Value-at-Risk, and the target return is constrained to be equal to a given scalar value r_p:

$$s_\rho(\mathbf{h}) = E\left[\boldsymbol{\mu}^T\mathbf{h} - \mathbf{r}^{\mathrm{T}}\mathbf{h} \mid (\boldsymbol{\mu}-\mathbf{r})^T\mathbf{h} \ge VaR_\rho(\mathbf{h}),\ \boldsymbol{\mu}^T\mathbf{h} = r_p\right] \tag{250}$$

Expected shortfall sometimes is also known as the Conditional Value-at-Risk (CVaR), Average Value-at-Risk (AVaR), Tail Value-at-Risk (TVaR), and Expected Tail Loss (ETL). Even though technical differences exist between these terms, they are conceptually equivalent. See Rachev et al. (2008) for more details.

Given a sample of P returns $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_P$ and the portfolio weight vector $\mathbf{h}$, let $r_{(p)}(\mathbf{h}) = \mathbf{r}_p{}^T\mathbf{h}$ be the portfolio return in period p, We sort $r_{(p)}$ to form the following increasing order:

$$r_{(1)}(\mathbf{h}) \le r_{(2)}(\mathbf{h}) \le \cdots \le r_{(P)}(\mathbf{h})$$

Let $\bar{\mathbf{r}}$ denote the sample mean of $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_P$ and $K = \lfloor (1-\rho)P \rfloor$ which means K is the integer floor value of $(1-\rho)P$. Then, following Bertsimas et al (2004), a non-parametric estimator of $s_\rho(\mathbf{h})$ in Eq. (250) is given by:

$$\hat{s}_\rho(\mathbf{h}) = \bar{\mathbf{r}}^T\mathbf{h} - \frac{1}{K}\sum_{j=1}^{K} r_{(j)}(\mathbf{h}) \tag{251}$$

which does not rely on any distributional assumptions.

4.14.2 Problem Formulations

This section discusses the practical, scenario-based expected shortfall formulations adopted by the Barra Optimizer.

According to Bertsimas et al (2004), the sample mean-shortfall optimization can be stated as follows:

$$\begin{aligned} &\text{Minimize:} \quad \hat{s}_\rho(\mathbf{h}) \\ &\text{Subject to:} \quad \bar{\mathbf{r}}^T\mathbf{h} = r_p, \ \ \mathbf{e}^T\mathbf{h} = 1. \end{aligned} \tag{252}$$

Further, it can be reformatted as the following equivalent linear program:

$$\begin{aligned} &\underset{\mathbf{h},t,\mathbf{z}}{\text{Minimize:}} \quad \bar{\mathbf{r}}^T\mathbf{h} - t + \frac{1}{K}\sum_{i=1}^{P} z_i \\ &\text{Subject to:} \quad \bar{\mathbf{r}}^T\mathbf{h} = r_p, \\ &\qquad\qquad\quad \mathbf{e}^T\mathbf{h} = 1, \\ &\qquad\qquad\quad z_i \ge t - \mathbf{r}_i^T\mathbf{h}, \quad z_i \ge 0, \quad i = 1, 2, \cdots, P. \end{aligned} \tag{253}$$

where

t and z_i	definitional decision variables
ρ	confidence level, $\rho \in (0,1)$
K	an integer constant for any given ρ and P $= \lfloor (1-\rho)P \rfloor$ (i.e., the integer floor of $(1-\rho)P$)
r_p	a target for portfolio return

In Barra Optimizer, we incorporate expected shortfall optimization (253) into mean-variance optimization (1) .The resulting general framework is given as follows:

$$\underset{\mathbf{h},t,\mathbf{z}}{\text{Maximize:}} \quad f(\mathbf{h}) - \theta \cdot \hat{s}_\rho(\mathbf{h}) \tag{254}$$

$$\text{Subject to:} \quad \hat{s}_\rho(\mathbf{h}) = \eta \bar{\mathbf{r}}^T \mathbf{h} - t + \frac{1}{K}\sum_{i=1}^{P} z_i \quad \text{(Definitional Constraint for Expected Shortfall)} \tag{255}$$

$$\mathbf{r}_i^T \mathbf{h} - t + z_i \ge 0, \quad z_i \ge 0, \quad i = 1, 2, \cdots, P \quad \text{(Reformulation Constraint for Shortfall)} \tag{256}$$

$$\xi_L \le \bar{\mathbf{r}}^T \mathbf{h} \le \xi_U \quad \text{(Bounds on the Target Return)} \tag{257}$$

$$\mathrm{L}_{\hat{s}_\rho} \le \hat{s}_\rho(\mathbf{h}) \le U_{\hat{s}_\rho} \quad \text{(Bounds on Expected Shortfall)} \tag{258}$$

Other Constraints

where

$f(\mathbf{h})$	the typical mean-variance objective function (1). This term is optional.
θ	a scalar balancing the importance of the shortfall terms with other terms in the utility. When $\theta = 0$, the shortfall term is ignored in the objective, but may still appear in the constraints.
η	a 0-1 indicator signaling whether target return is included in the expected shortfall definition. When $\eta = 0$, the expected shortfall can be represented as:

$$\hat{s}_\rho(\mathbf{h}) = -t + \frac{1}{K}\sum_{i=1}^{P} z_i \tag{259}$$

$\bar{\mathbf{r}}$	an optional vector of targeted (or benchmark) asset returns
$\mathbf{r}_i$	a required vector of historical or simulated asset or factor returns for scenario i
ρ	the desired confidence level for the value at risk given the return scenarios
P	number of return scenarios
K	the maximum integer number that is less than or equal to $(1-\rho)P$. It needs to be positive, which means $(1-\rho)P \ge 1$ is required.

When return scenarios are provided at the factor level instead of the asset level (assuming asset specific returns are negligible), constraint (256) is replaced with:

$$\mathbf{r}_i^T \mathbf{w} - t + z_i \ge 0, \quad z_i \ge 0, \quad i = 1, 2, \cdots, P \tag{260}$$

For example, suppose we have 150 simulated returns, $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_{150}$, and a target return $\bar{\mathrm{r}}$.
We are interested in minimizing the expected shortfall at the 95% confidence level, $\hat{s}_{0.95}(\mathbf{h})$, with a target return of 8%. In this case, we may set $\theta = 1$, $\eta = 1$, $\rho = 0.95$, $P = 150$, $K = \lfloor (1-\rho)P \rfloor == \lfloor 0.05 \cdot 150 \rfloor = \lfloor 7.5 \rfloor = 7$, and $\xi_L = \xi_U = 0.08$.

Notes:

- Expected shortfall in combination with non-convex terms in the mean-variance objective $f(\mathbf{h})$ is currently a Beta feature.
- Users may provide return scenarios for either assets or factors.
- θ, η, ρ , P , $\mathbf{r}_i$, $\bar{\mathrm{r}}$, ξ_L , ξ_U , $\mathrm{L}_{\hat{s}_\rho}$ and $U_{\hat{s}_\rho}$ are all user inputs
- K is an internally calculated constant.
- t **and** z_i **are the decision variables controlled directly by the optimizer. Eqns. (255) and (256) are set internally by the optimizer.**
- Multi-period or multi-account expected shortfall optimization is NOT supported.

For detailed instructions on how to set up input and parameters for expected shortfall optimization, please refer to Section 5.5.5.

4.14.3 Applications

In this section, we provide several examples to illustrate how expected shortfall optimization may be used in portfolio optimization.

4.14.3.1 Minimizing Expected Shortfall

In this case, users are only interested in minimizing expected shortfall. The optimization problem becomes:

$$\begin{aligned}
&\text{Minimize:} && \hat{s}_\rho(\mathbf{h}) && (261)\\
&\text{Subject to:} && \text{Portfolio Balance Constraint}\\
& && \text{(Internal) Definitional Shortfall Constraints (255), and (256) or (260)}\\
& && \text{User-defined (Optional) Target-Return Constraints (257)}\\
& && \text{Other Constraints}
\end{aligned}$$

4.14.3.2 Maximizing Mean-Expected Shortfall

As in the case of the traditional mean-variance optimization, users are interested in maximizing the risk-adjusted return. In this case, it is to maximize the shortfall-adjusted return:

$$\begin{aligned}
&\text{Maximize:} && \mathbf{r}^{\mathrm{T}}\mathbf{h} - \theta \cdot \hat{s}_\rho(\mathbf{h}) && (262)\\
&\text{Subject to:} && \text{Portfolio Balance Constraint}\\
& && \text{(Internal) Definitional Shortfall Constraints (255), and (256) or (260)}\\
& && \text{User-defined (Optional) Target-Return Constraints (257)}\\
& && \text{Other Constraints}
\end{aligned}$$

where θ is a scalar balancing the importance of portfolio return and Expected Shortfall in objective function. Setting θ as a very large number will essentially minimize the expected shortfall.

4.14.3.3 Maximizing Mean-Variance-Expected Shortfall

In this case, users are still maximizing risk-adjusted return, but risk now includes both variance and expected shortfall:

$$\text{Maximize:} \quad \mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda \mathbf{h}^{T} \sum \mathbf{h} - \theta \cdot \hat{s}_{\rho}(\mathbf{h}) \tag{263}$$

Subject to: Portfolio Balance Constraint
(Internal) Definitional Shortfall Constraints (255), and (256) or (260)
User-defined (Optional) Target-Return Constraints (257)
Other Constraints

where Σ is a covariance matrix and λ is a risk aversion.

4.14.3.4 Constraining Expected Shortfall

In this case, users are interested in maximizing the traditional variance-adjusted return and constraining expected shortfall at the same time:

$$\text{Maximize:} \quad \mathbf{r}^{\mathrm{T}}\mathbf{h} - \lambda \mathbf{h}^{T} \sum \mathbf{h} \tag{264}$$

Subject to: Portfolio Balance Constraint
(Internal) Definitional Shortfall Constraints (255), and (256) or (260)
User-defined (Optional) Target-Return Constraints (257)
User-defined Shortfall Constraints (258)
Other Constraints

When setting the risk aversion $\lambda = 0$, the problem reduces to maximizing return, subject to the expected shortfall being less than or equal to a given bound.

5 Using the Barra Optimizer APIs

Barra Open Optimizer provides C++, Java, and C# APIs to enable you to develop your applications using C++, Java, C#, MATLAB, Python, and R. To use the Barra Optimizer API for running a portfolio optimization, your application typically needs to perform the following steps:

- Create a workspace
- Add assets into the workspace and set per-asset data
- Construct initial portfolio, benchmark(s), and trade universe
- Define risk model(s)
- Prepare an optimization case, including setting up the objective, constraints, etc.
- Run optimization
- Inspect outputs
- Release resources

The following sections describe each step in detail for the C++, Java, and C# APIs. We use <Class Name>::<Function Name> to reference a member function <Function Name> of a class <Class Name>. For example, `CWorkSpace::CreateInstance()` stands for the `CreateInstance()` function of the `CWorkSpace` class.

For applications using Java API, you need to load the native Barra Optimizer C++ library by calling `COptJavaInterface.Init()`.

5.1 Create a Workspace

The first step to run a portfolio optimization is to create an instance of the `CWorkSpace` object by calling `CWorkSpace::CreateInstance()`.

5.2 Add Assets into Workspace

To add assets into the `CWorkSpace` object, call `CWorkSpace::CreateAsset()`. Assets added into the workspace should at least cover those in the initial portfolio, the benchmark(s), and the trade universe.

For each of the added assets, call methods of `CAsset` to set per-asset data, such as alpha, price (optional for non-roundlotting cases), hard-to-borrow penalty, roundlot size, buy/sell costs, tax lots, transaction costs, residual alpha weights, fixed holding cost, etc.

Alternatively, users may load per-asset data using a CSV-formatted file as described below:

1. Create a CSV file with the first column as assetID, the second column as attribute type, and the third column as the value for the attribute type. The following attribute types are available:

Attribute Type	Description
FBTC	Fixed Buy Transaction Cost in decimal format
FSTC	Fixed Sell Transaction Cost in decimal format
P	Price
A	Expected return in decimal format
LS	Round-lot size
AW	Residual alpha weight
ISS	Issuer ID; used in 5/10/40 rule
NSC	Total net cost on short position
TYPE	Asset type 0 = Cash; 1 = Futures; 2 = Currency; 3 = Regular; 4 = Composite; 5 = Composite futures
NLTC_C	Multiplier of the nonlinear transaction cost function
NLTC_P	The exponent of the nonlinear transaction cost function
NLTC_Q	The traded amount in the nonlinear transaction cost function
USFHC	Upside fixed holding cost
DSFHC	Downside fixed holding cost
RWFHC	Reference weight for fixed holding cost

In addition to the above attribute types, the second column may take a format of `GroupName.GroupAttribute` (with the ".GroupAttribute" part being optional) to specify the asset's group association. In this case, the optional third column is the group coefficient. If the third column is absent, the group coefficient defaults to 1.0.

2. Call `CWorkSpace::LoadAssetData()` to load the data from the CSV file into `CWorkSpace`. The assets will be automatically added to `CWorkSpace` eliminating the need to call `CWorkSpace::CreateAsset()` for these assets.

Sample CSV file contents:

```
!AssetID,AttributeType,Value
USA11I1,A,0.00231
USA11I1,P,35.65
USA11I1,LS,1000
USA11I1,NSC,0.003193
USA11I1,FBTC,0.0005
USA13Y1,GICS_SECTOR.Information Technology,
USA1LI1,GICS_SECTOR.Information Technology,
USA3351,GICS_SECTOR.Utilities,0.5
```

5.3 Construct Initial Portfolio, Benchmark, and Trade Universe

To construct an initial portfolio, do the following:

1. Create a `CPortfolio` object by calling `CWorkSpace::CreatePortfolio()`.
2. For optimization problems that are not tax-aware, populate the portfolio object with asset IDs and weights by calling `CPortfolio::AddAsset()`.
3. For tax-aware optimization, populate the portfolio object with tax lot purchase information by calling `CPortfolio::AddTaxLot()`. The method has the following parameters:
    - `assetID`: identifies the asset of the lot.
    - `age`: the number of days the asset is held.
    - `costBasis`: cost basis per share, cannot be negative.
    - `shares`: number of shares in the lot. A negative number indicates a short sell.
    - `disallowed`: true if the lot has already been used to disallow loss on a previous sale.
    - `taxFree`: if true then no tax from selling the lot.

    Note: The optimizer will calculate asset weights from tax lot information, so do not call `CPortfolio::AddAsset()` to add asset weights.
4. For tax-aware optimization, add wash sale records by calling `CPortfolio::AddWashSaleRec()`. Such wash sale records have recent sales information and will be used by the optimizer to count wash sales against new purchases. The method accepts these parameters:
    - `assetID`: identifies the asset.
    - `age`: the number of days the asset is held.
    - `lossPerShare`: loss per share, must be a positive number.
    - `shares`: number of shares in the lot. A negative number for a short position.
    - `disqualified`: true if the loss has already been disqualified due to a previous purchase.

Benchmark portfolios and the trade universe can be constructed by calling `CPortfolio::AddAsset()`. Your application does not need to add weights for the trade universe.

5.4 Define Risk Model

To define a risk model, do the following:

1. Create a `CRiskModel` object by calling `CRiskModel::CreateRiskModel()`.
2. Call member functions of the `CRiskModel` object to populate the object with model data such as D, F, X, and so forth.

Notes:

- To construct another risk model, repeat the above steps.
- To specify a dense risk model without factors, set only the specific covariance.
- To save time for setting up risk model, you may skip setting 0 value covariance and exposure data in D, F, and X.
- For all composite-type assets, do the following:
  - Ensure that covariances and exposures among non-composite assets have been set.
  - Construct a `CPortfolio` object and populate the constituents and weights representing the composite.
  - Call `CAsset::SetCompositePort()` to compute factor exposures of the composite and covariances of the composite with other assets in the universe.
- Optionally, you may provide factor-blocking structure by calling `CRiskModel::AddFactorBlock()`. Barra Optimizer may improve performance by taking advantage of different scarcity of each factor block in the factor exposure matrix X.

To load Barra Models Direct data into `CRiskModel`, do the following:

1. Unzip the Models Direct zip files into a root directory containing all the Models Direct flat files, or a subdirectory with same name as the model name.
   A BIMe model name shall always have a format of “BIMeXY” with “X” being series number and “Y” being the long/short specifier. For example, the model name of a long–term, 3 series BIMe model will be “BIMe3L”).
   MAC equity models are named as “MAC_eq_L” (long-term) or “MAC_eq_S” (short-term).

a. For non-integrated models, the following files should be extracted:

   - <Model>_<Version>_Asset_Data.YYYYMMDD
   - <Model>_<Version>_Asset_Exposure.YYYYMMDD
   - <Model>_<Version>_Asset_LSR.YYYYMMDD*
   - <Model>_<Version>_Covariance.YYYYMMDD
   - <Model>_Factors.dat*

   **Optional files*

b. For integrated models, such as BIMe and MAC, the following files should be extracted:

- <Model>_Asset_Identity.YYYYMMDD*
- <Model>_<Version><L/S>_Currency_Factors.dat*
- <Model>_<Version><L/S>_Factors.dat
- <Model>_<Version><L/S>_FullCovariance.YYYYMMDD* or <Model>_<Version><L/S>_Covariance.YYYYMMDD
- <Model>_<Version><L/S>_Models.dat

**Optional files*

<Version> is empty for MAC model files

Sub-model zip files should be extracted the same way as the non-integrated model files.

For integrated models, the Models.dat file specifies the model names of sub-models that the Models Direct loader uses to retrieve specific risk and exposure data.

The FullCovariance file includes Global Equity Model (GEM) factors. If this file is present, it is loaded instead of the Covariance file.

The loader uses the Asset_Identity and Currency_Factor files to find currency exposures of assets. If these files are present, then currency exposures will be added to the risk model.

2. Call `CreateRiskModel()` with same model name as the Models Direct model name.
3. Call `LoadModelsDirectData()` with the following parameters:
    - Root model directory: Path where the flat files are located. The Models Direct loader looks for files under the root model directory, or in a subfolder with the same name as the model name. For example, if the root model directory for USE3L data is \ModelsDirectRepo, then the flat files can be extracted to \ModelsDirectRepo or \ModelsDirectRepo\USE3L.
    - Analysis date: Date starting from which, flat files will be loaded. If a flat file with the exact date is not found, the loader goes back at most 45 calendar days before the analysis date.
    - AssetIDs: List of the asset's BARRIDs for which the asset factor exposures and specific risks are to be loaded. The AssetIDs should contain all of the assets in the investible universe
4. For risk models having a currency block, call `SetNumeraire()` to change the numeraire of the factor covariance matrix. <Model>_Factors.dat is required as it contains factor block information.

To load asset exposures into `CRiskModel` using CSV file:

1. Create a CSV file with first column as asset ID, second column as factor ID, and third column as exposure value. For example:

```
! AssetID,FactorID,Exposure
USA11I1,Factor_1A,0.144
```

```
USA11I1,Factor_1B,-0.666
USA11I1,Factor_1C,1.646
USA11I1,Factor_1D,0.21
USA11I1,Factor_1E,-0.361
```

2. Call `LoadAssetExposures()` with the full path of the CSV file.

To compute the marginal contribution to tracking error or to active risk, do the following:

1. Call `ComputeAssetMCTE()` to compute the MCTE for a single asset.
2. Call `ComputePortAssetMCTE()` to compute the asset MCTE for all assets in the specified portfolio.
3. Call `ComputeFactorMCAR()` to compute the MCAR for a single factor.
4. Call `ComputePortFactorMCAR()` to compute the factor MCAR for all factors in the specified portfolio.

Asset-level MCTE computation is defined as following:

$$Asset\ MCTE(i) = \frac{\left[\left(\mathbf{XFX}^{\mathrm{T}} + \mathbf{D}\right)(\mathbf{h} - \mathbf{h_B})\right]_{\mathrm{i}} - \mathbf{h_B}\left(\mathbf{XFX}^{\mathrm{T}} + \mathbf{D}\right)(\mathbf{h} - \mathbf{h}_{\mathrm{B}})}{\sqrt{(\mathbf{h} - \mathbf{h}_{\mathrm{B}})^{\mathrm{T}}\left(\mathbf{XFX}^{\mathrm{T}} + \mathbf{D}\right)(\mathbf{h} - \mathbf{h}_{\mathrm{B}})}} \tag{265}$$

where,

$\mathbf{X}$ = Asset factor exposures matrix

$\mathbf{F}$ = Factor covariance matrix

$\mathbf{D}$= Specific covariance matrix

$[\ldots]_i$ = the i component of the vector

Factor-level MCAR computation is defined as following:

$$Factor\ MCAR(j) = \frac{\left[\mathbf{FX}^{\mathbf{T}}(\mathbf{h} - \mathbf{h_B})\right]_j}{\sqrt{(\mathbf{h} - \mathbf{h_B})^{\mathrm{T}}\left(\mathbf{XFX}^{T} + \mathbf{D}\right)(\mathbf{h} - \mathbf{h_B})}} \tag{266}$$

where,

$\mathbf{X}$ = Asset factor exposures matrix

$\mathbf{F}$ = Factor covariance matrix

$[\ldots]_j$ = the j component of the vector

1. Call `ComputeAssetPCTE()` to compute the percent contribution to tracking error for a single asset.

2. Call `ComputePortAssetPCTE()` to compute the percent contributions to tracking error for all assets in the specified portfolio.
3. Call `ComputeFactorPCAR()` to compute the percent contribution to active risk for a single factor.
4. Call `ComputePortFactorPCAR()` to compute the percent contributions to active risk for all factors in the specified portfolio.
5. Call `ComputeAssetBeta()` to compute the beta for a single asset.
6. Call `ComputePortAssetBeta()` to compute the betas for all assets in the specified portfolio.
7. Call `ComputePortCorrelation()` to compute the correlation between two portfolios.
8. Call `ComputePortBeta()`to compute the beta for a portfolio.

Asset-level PCTE computation is defined as following:

$$Asset\ PCTE(i) = \left(\mathbf{h}_i - \mathbf{h}_{\mathrm{B}_i}\right) \frac{\left[\left(\mathbf{XFX}^{\mathrm{T}} + \mathbf{D}\right)(\mathbf{h} - \mathbf{h}_{\mathrm{B}})\right]_i - \mathbf{h}_{\mathrm{B}}\left(\mathbf{XFX}^{\mathrm{T}} + \mathbf{D}\right)(\mathbf{h} - \mathbf{h}_{\mathrm{B}})}{(\mathbf{h} - \mathbf{h}_{\mathrm{B}})^{\mathrm{T}}\left(\mathbf{XFX}^{\mathrm{T}} + \mathbf{D}\right)(\mathbf{h} - \mathbf{h}_{\mathrm{B}})} \cdot 100 \tag{267}$$

where,

$\mathbf{X}$ = Asset factor exposures matrix

$\mathbf{F}$ = Factor covariance matrix

$\mathbf{D}$= Specific covariance matrix

$[\ldots]_i$ = the i component of the vector

Factor-level PCAR computation is defined as following:

$$Factor\ PCAR(j) = \left(\mathbf{X}_j - \mathbf{X}_{\mathrm{B}_j}\right) \frac{\left[\mathbf{FX}^{\mathrm{T}}(\mathbf{h} - \mathbf{h}_{\mathrm{B}})\right]_j}{(\mathbf{h} - \mathbf{h}_{\mathrm{B}})^{T}\left(\mathbf{XFX}^{\mathrm{T}} + \mathbf{D}\right)(\mathbf{h} - \mathbf{h}_{\mathrm{B}})} \cdot 100 \tag{268}$$

where

$\mathbf{X}$ = Asset factor exposures matrix

$\mathbf{F}$ = Factor covariance matrix

$[\ldots]_j$ = the j component of the vector

- Asset beta computation is defined as following:

$$\beta = \frac{\left(\mathbf{XFX}^{\mathrm{T}} + \mathbf{D}\right)\mathbf{h}_{\mathrm{B}}}{\mathbf{h}_{\mathbf{B}}\left(\mathbf{XFX}^{\mathrm{T}} + \mathbf{D}\right)\mathbf{h}_{\mathrm{B}}}$$

where

$\mathbf{X}$ = Asset factor exposures matrix

$\mathbf{F}$ = Factor covariance matrix

$\mathbf{D}$= Specific covariance matrix

Portfolio correlation computation is defined as following:

$$\rho(P,Q) = \frac{\mathbf{X}_p\mathbf{F}\mathbf{X}_q + \mathbf{h}_p\mathbf{D}\mathbf{h}_q}{\left(\sqrt{\mathbf{X}_p\mathbf{F}\mathbf{X}_p + \mathbf{h}_p\mathbf{D}\mathbf{h}_p}\right)\left(\sqrt{\mathbf{X}_q\mathbf{F}\mathbf{X}_q + \mathbf{h}_q\mathbf{D}\mathbf{h}_q}\right)} \tag{269}$$

where,

$\mathbf{X}_p$= P (or Q) portfolio factor exposures

$\mathbf{F}$ = Factor covariance matrix

$\mathbf{D}$= Specific covariance matrix

Portfolio beta computation is defined as following:

$$b_p = \frac{\mathbf{X}_p\mathbf{F}\mathbf{X}_b + \mathbf{h}_p\mathbf{D}\mathbf{h}_b}{\mathbf{X}_b\mathbf{F}\mathbf{X}_b + \mathbf{h}_b\mathbf{D}\mathbf{h}_b} \tag{270}$$

where,

$\mathbf{X}_p$= P (or B) portfolio factor exposures

$\mathbf{F}$ = Factor covariance matrix

$\mathbf{D}$= Specific covariance matrix

5.5 Prepare Optimization Case

To prepare an optimization case, call `CWorkSpace::CreateCase()` to create a `CCase` object.

When calling the `CreateCase()` function, your application needs to specify initial portfolio, trade universe, base value (used to compute asset weights) and, optionally, cash flow weight. For long-short portfolios that result in net value of zero, set the base value to long side value of the portfolio for a 100% long portfolio.

Then, call methods of the `CCase` objects to do the following:

- Link risk models
- Set up the objective
- Set up constraints
- Set up efficient frontier
- Set up expected shortfall parameter
- Set up tax-aware optimization parameters
- Set a risk target or a return target.

- Define a non-linear transaction cost function at portfolio level.
- Test whether the optimization problem is convex by calling `CCase::IsConvex()`. For information about the convex portfolio optimization problem, see the Problem Classification section.
- Set the risk scalar for the return-risk frontier or risk-target optimization.

Note the following when preparing an optimization case:

- Unless specified otherwise, use decimals rather than percentages as units of input parameters.
- Use the `OPT_INF` constant when specifying an infinity value.

5.5.1 Link Risk Models

Barra Optimizer supports up to two risk models per optimization case.

- To link a primary risk model to the case, call `CCase::SetPrimaryRiskModel()`.
- Optionally, to link a secondary risk model to the case, call `CCase::SetSecondaryRiskModel()`.

5.5.2 Set Up the Objective

To set up the objective function, also known as the utility function for the optimization case, call `CCase::InitUtility()` to get a `CUtility` object.

When calling this function, your application can pass an appropriate parameter to specify the type of the objective function: standard quadratic, Information Ratio, or Sharpe Ratio. Then, call methods of the `CUtility` object to populate the object with the following parameters used in the objective function:

- Link a benchmark to the primary risk term and/or the optional secondary risk term
- Risk aversions λ_F and λ_D
- Risk aversions λ_{D_2} and λ_{F_2} if the objective has the variance term of the secondary risk model
- Multiplier for alpha λ_α
- Multiplier for tax costs λ_{TAX}
- Multiplier for transaction cost λ_{TC}
- Multiplier for short costs λ_{SC}
- Multiplier for penalties λ_{pen}
- Whether the penalty term is in the numerator or the denominator depends on whether the objective function is Information Ratio or Sharpe Ratio
- Multiplier for the residual alpha penalty term λ_R

- Multiplier for the expected shortfall term θ
- Multiplier for the loss benefit term $\lambda_{Benefit}$
- Risk free rate (a parameter used in Sharpe Ratio)
- Cost of leverage
- Additional covariance terms
- Additional linear terms

5.5.3 Set Up Constraints

To set up constraints, call `CCase::InitConstraint()` to get a `CConstraints` object. Then, call Init methods (for example, `InitLinearConstraints()`) of the `CConstraints` object to get an object of a specific type of constraint (for example, `CLinearConstraints`). Methods of such objects usually return a `CConstraintInfo` object, which in turn can be used for setting up the following:

- Values of lower bound and upper bound
- Soft bounds
- Penalties
- Free range linear penalties
- Whether the bounds are relative to a reference portfolio, for example, benchmark or initial portfolio

Two types of penalty functions are supported. To set up a penalty function specified in the Penalty Functions section, call `CConstraintInfo::SetPenalty()` to specify the parameters Target, Upper, and Lower. To set up a free-range linear penalty, call `CConstraintInfo::SetFreeRangeLinearPenalty()` to specify the lower and upper bound of the free range and slopes for each side outside the range.

Note that not all constraints can have lower bounds, soft bounds, penalties or relative bounds, as listed in the following table:

CConstraintInfo Object Returned By	Lower Bound	Upper Bound	Relativity	Softness	Penalty	Free Range Linear Penalty
`CConstraints::SetTransactionCostConstraint`		Y		Y		
`CConstraints::SetCrossPeriodTransactionCostConstraint`		Y		Y		
`CHedgeConstraints::AddLongSideConstraint`	Y	Y	Y	Y	Y	Y
`CHedgeConstraints::AddLongSideFactorConstraint`	Y	Y	Y	Y	Y	Y
`CHedgeConstraints::AddLongSideGroupConstraint`	Y	Y	Y	Y	Y	Y

CConstraintInfo Object Returned By	Lower Bound	Upper Bound	Relativity	Softness	Penalty	Free Range Linear Penalty
`CHedgeConstraints::AddShortSideConstraint`	Y	Y	Y	Y	Y	Y
`CHedgeConstraints::AddShortSideFactorConstraint`	Y	Y	Y	Y	Y	Y
`CHedgeConstraints::AddShortSideGroupConstraint`	Y	Y	Y	Y	Y	Y
`CHedgeConstraints::SetLongSideLeverageRange`	Y	Y		Y	Y	Y
`CHedgeConstraints::SetShortSideLeverageRange`	Y	Y		Y	Y	Y
`CHedgeConstraints::SetTotalLeverageRange`	Y	Y		Y	Y	Y
`CHedgeConstraints::AddWeightedTotalLeverageConstraint`	Y	Y	Y	Y	Y	Y
`CHedgeConstraints::AddTotalLeverageGroupConstraint`	Y	Y	Y	Y	Y	Y
`CHedgeConstraints::AddTotalLeverageFactorConstraint`	Y	Y	Y	Y	Y	Y
`CHedgeConstraints::SetShortLongLeverageRatioRange`	Y	Y		Y		
`CHedgeConstraints::SetNetTotalLeverageRatioRange`	Y	Y		Y		
`CLinearConstraints::SetBetaConstraint`	Y	Y		Y	Y	Y
`CLinearConstraints::AddGeneralConstraint`	Y	Y	Y	Y	Y	Y
`CLinearConstraints::AddGroupConstraint`	Y	Y	Y	Y	Y	Y
`CLinearConstraints::SetAssetRange`	Y	Y	Y		Y	Y
`CLinearConstraints::SetFactorRange`	Y	Y	Y	Y	Y	Y
`CLinearConstraints::SetAssetTradeSize`		Y				
`CLinearConstraints::SetExpectedShortfallConstraint`	Y	Y				
`CLinearConstraints::SetTargetMeanReturnConstraint`	Y	Y	Y	Y	Y	Y
`CRiskConstraints::AddPLFactorConstraint`		Y				
`CRiskConstraints::AddPLSpecificConstraint`		Y				
`CRiskConstraints::AddPLTotalConstraint`	Y	Y				
`CRiskConstraints::AddFactorConstraint`		Y				
`CRiskConstraints::AddSpecificConstraint`		Y				

CConstraintInfo Object Returned By	Lower Bound	Upper Bound	Relativity	Softness	Penalty	Free Range Linear Penalty
`CRiskConstraints::AddTotalConstraint`	Y	Y				
`CRiskConstraints::AddFactorConstraintByGroup`		Y				
`CRiskConstraints::AddSpecificConstraintByGroup`		Y				
`CRiskConstraints::AddTotalConstraintByGroup`		Y				
`CNewTaxConstraint::SetTaxArbitrageRange`	Y	Y				
`CNewTaxConstraint::SetTotalTaxArbitrageRange`	Y	Y				
`CNewTaxConstraint::SetTaxLimit`		Y				
`CTaxConstraints::SetLongGainArbitrageRange`	Y	Y				
`CTaxConstraints::SetLongLossArbitrageRange`	Y	Y				
`CTaxConstraints::SetLongNetArbitrageRange`	Y	Y				
`CTaxConstraints::SetShortGainArbitrageRange`	Y	Y				
`CTaxConstraints::SetShortLossArbitrageRange`	Y	Y				
`CTaxConstraints::SetShortNetArbitrageRange`	Y	Y				
`CTaxConstraints::SetTaxLimit`		Y				
`CTurnoverConstraints::SetLongSideConstraint`		Y		Y		
`CTurnoverConstraints::SetNetConstraint`		Y		Y		
`CTurnoverConstraints::SetShortSideConstraint`		Y		Y		
`CTurnoverConstraints::SetBuySideConstraint`		Y		Y		
`CTurnoverConstraints::SetSellSideConstraint`		Y		Y		
`CTurnoverConstraints::SetCrossPeriodNetConstraint`		Y		Y		
`CTurnoverConstraints::AddNetConstraintByGroup`		Y		Y	Y	Y
`CTurnoverConstraints::AddLongSideConstraintByGroup`		Y		Y	Y	Y
`CTurnoverConstraints::AddShortSideConstraintByGroup`		Y		Y	Y	Y
`CTurnoverConstraints::AddBuySideConstraintByGroup`		Y		Y	Y	Y
`CTurnoverConstraints::AddSellSideConstraintByGroup`		Y		Y	Y	Y

CConstraintInfo Object Returned By	Lower Bound	Upper Bound	Relativity	Softness	Penalty	Free Range Linear Penalty
`CGeneralPWLinearConstraint::SetConstraint`	Y	Y		Y	Y	Y
`CConstraints::SetOverlapConstraint`		Y				
`CConstraints::SetPortConcentrationConstraint`		Y				
`CConstraints::SetTotalActiveWeightConstraint`	Y	Y		Y	Y	Y
`CConstraints::AddTotalActiveWeightConstraintByGroup`	Y	Y		Y	Y	Y
`CCrossAccountConstraints::SetAssetBoundOnTotalPosition`	Y	Y		Y	Y	
`CCrossAccountConstraints::SetMaxAssetTotalTrades`		Y		Y	Y	
`CCrossAccountConstraints::SetMaxAssetTotalBuys`		Y		Y	Y	
`CCrossAccountConstraints::SetMaxAssetTotalSells`		Y		Y	Y	
`CCrossAccountConstraints::SetNetTurnoverConstraint`		Y		Y		
`CCrossAccountConstraints::SetTaxLimit`		Y				
`CIssuerConstraints::AddHoldingConstraint`	Y[1]	Y	Y	N	N	N
`CRatioConstraints::AddGeneralConstraint`	Y	Y	Y			
`CRatioConstraints::AddGroupConstraint`	Y	Y	Y			
`CQuadraticConstraints::AddConstraint`		Y	Y	Y		

Note: [1] Both upper and lower bounds on net total holding are supported. However, only upper bound on absolute total holding is supported.

1. To set up linear constraints, call `CConstraints::InitLinearConstraints()` to get a `CLinearConstraints` object. Then, call methods of the `CLinearConstraints` object to set up the following:
    - Asset ranges: $\boldsymbol{l}_h \leq \mathbf{h} \leq \boldsymbol{u}_h$
    - Factor exposure constraints: $\boldsymbol{l}_w \leq \boldsymbol{w} \leq \boldsymbol{u}_w$
    - General linear constraints: $\boldsymbol{l}_s \leq \boldsymbol{s} \leq \boldsymbol{u}_s$
    - Beta constraint: $\boldsymbol{l}_\beta \leq \boldsymbol{\beta}^T \boldsymbol{h} \leq \boldsymbol{u}_\beta$, where $\boldsymbol{\beta} = \frac{(\mathbf{XFX}^T + \mathbf{D})\mathbf{h}_B}{\mathbf{h}_B(\mathbf{XFX}^T + \mathbf{D})\mathbf{h}_B}$

- Transaction limit: Buy_None, Sell_None, etc.
- Group constraints
- Expected shortfall constraint
- Target mean return constraint
- Option to enable/disable crossovers

2. To set up risk constraints, call `CConstraints::InitRiskConstraints()` to get a `CRiskConstraints` object.
3. To set up constraints on risk or risk contribution at portfolio-level (see H.1 and H.2 in Appendix H), call the following functions. Then, use methods of the returned `CConstraintInfo` object to set up upper bounds, etc.
   - `CRiskConstraints::AddPLFactorConstraint()`
   - `CRiskConstraints::AddPLSpecificConstraint()`
   - `CRiskConstraints::AddPLTotalConstraint()`
4. To set up constraints on risk or risk contribution from sub-groups (see H.3 through H.6 in Appendix H), call the following functions. Then, use methods of the returned CConstraintInfo object to set up upper bounds, etc.
   - `CRiskConstraints::AddFactorConstraint()`
   - `CRiskConstraints::AddSpecificConstraint()`
   - `CRiskConstraints::AddTotalConstraint()`
   - `CRiskConstraints::AddFactorConstraintByGroup()`
   - `CRiskConstraints::AddSpecificConstraintByGroup()`
   - `CRiskConstraints::AddTotalConstraintByGroup()`
5. To set up constraints on risk or risk contribution by asset (see H.7 and H.8 in Appendix H), call the following function. Then, use methods of the returned `CConstraintInfo` object to set up upper bounds, etc.
   - `CRiskConstraints::AddRiskConstraintByAsset()`
6. To set up risk parity constraints, call `CConstraints::InitRiskConstraints()` to get a `CRiskConstraints` object and call `CRiskConstraints::SetRiskParity(ERiskParityType,…)` with the appropriate risk parity type, the set of included assets/factors and optionally a benchmark.
7. To set up hedge constraints (leverage constraints), call `CConstraints::InitHedgeConstraints()` to get a `CHedgeConstraints` object. Then call methods of the `CHedgeConstraints` object to set up various leverage constraints. For example, call `CHedgeConstraints::AddWeightedTotalLeverageConstraint()` to get a `CConstraintInfo` object and use its methods to set up bounds, penalty, etc. for a weighted total leverage constraint

8. Call `CConstraints::InitTurnoverConstraint()` to get a `CTurnoverConstraints` object. Then, call methods of this object to get `CConstraintInfo` objects for various turnover constraints and use their methods to set up upper bounds, softness, etc.
9. To add ratio constraints, call `CConstraints::InitRatioConstraints()` and then call either `AddGeneralConstraint()` or `AddGroupConstraint()` on the returned object.
   For general ratio constraints, the coefficients of the numerator and denominator are given as `CAttributeSet` objects; for group ratio constraints, they are given by asset attributes.
10. If the new tax-aware API is used (i.e. `CCase::InitNewTax()` is called), call `CConstraints::InitNewTaxConstraint()` to get a `CNewTaxConstraints` object.
    - Call `CNewTaxConstraints::SetTaxArbitrageRange()` to get `CConstraintInfo` objects and use their methods to set up lower/upper bounds, etc.
    - Call `CNewTaxConstraints::SetTotalTaxArbitrageRange()` to get `CConstraintInfo` objects which allow setting bounds on the total gain, loss, or net gain. To exclude tax-free gains and losses from the total, call `CNewTax::SetExcludeTaxFreeFromTotalGain()`.
    - Call `CNewTaxConstraints::SetTaxLimit()` to get a `CConstraintInfo` object and use its method to set up upper bound on total tax.
    - Call `CNewTaxConstraints::SetTaxLotTradingRule()` to set taxlot-level trading rule
    - Call `CNewTaxConstraints::SetMinHoldingPeriod()` to set group-level minimum holding period.
11. If the legacy pre-v8.8 tax-aware API is used (i.e. `CCase::InitTax()` is called), call `CConstraints::InitTaxConstraint()` to get a `CTaxConstraints` object. Then, call methods of this object to get `CConstraintInfo` objects and use their methods to set up lower/upper bounds, etc.
12. To set up transaction costs constraint, call `CConstraints::SetTransactionCostConstraint()` to get a `CConstraintInfo` object and then call methods of the `CConstraintInfo` object to set up a transaction costs upper bound, and optionally its softness.
13. To enable roundlotting optimization, call `CConstraints::EnableRoundLotting()`.
14. To set up threshold or cardinality constraints, also known as paring constraints, call `CConstraints::InitParingConstrinats()` to get a `CParingConstraints` object. Then, call methods of the `CParingConstraints` object to populate the object with parameters such as min/max number of assets/trades, min/max number of buys/sells, minimum holding threshold, minimum transaction size threshold, etc. Limits on the number of assets/trades can be specified on the group-level by calling `AddAssetTradeParingByGroup()` or `AddLevelParingByGroup()`. Minimum holding threshold and minimum transaction size threshold can be specified on the asset-level by calling `AddLevelParingByAsset()`.
    - To allow close out positions if the transaction size is below the minimum threshold, call `CParingConstraints::SetAllowCloseOut()`.
    - To set penalty on cardinality constraints, call `SetPenaltyPerExtraAsset()` or `SetPenaltyPerExtraTrade()`.

- To set penalty on threshold constraints, call `SetPenaltyPerUnitBelowHoldingThreshold()` or `SetPenaltyPerUnitBelowTradeThreshold()`.

15. To enable grandfather rule for minimum holding level threshold, call `EnableGrandfatherRule()`. To set up the 5/10/40 rule, call `CConstraints::Init5_10_40Rule()` to get a `C5_10_40Rule` object. Then, call methods of the `C5_10_40Rule` object to populate the object with 5/10/40 rule-related parameters and issuer/branches data.
16. To set up constraint hierarchy, call `CConstraints::InitConstraintHierarchy()` to get a `CConstraintHierarchy` object. Then, call `CConstraintHierarchy::AddConstraintPriority()` to set priority for constraint categories or call `CConstraintHierarchy::AddConstraintPriority4IndivCon()` to set priority for individual linear or factor constraints.
17. To add a general piecewise-linear constraint, call `CConstraints::AddGeneralPWLinearConstraint()` to get a `CGeneralPWLinearConstraint` object. Then, call methods of `CGeneralPWLinearConstraint` to set the starting point, add break points, and slopes. Call `CGeneralPWLinearConstraint::SetConstraint()` to get a `CConstraintInfo` object and then call methods of the `CConstraintInfo` object to set up bounds, softness, etc.
18. To set total active weight constraint, call `CConstraints::SetActiveWeightConstraint()` to get `a CConstraintInfo` object and then call methods of the `CConstraintInfo` object to set up bounds, softness, etc.
19. To set an active weight constraint by group, call `CConstraints::SetActiveWeightConstraintByGroup()` to get `a CConstraintInfo` object and then call methods of the `CConstraintInfo` object to set up bounds, softness, etc.
20. To set portfolio concentration constraint, call `CConstraints::SetPortConcentrationConstraint()` to get a `CPortConcentrationConstraint` object and then call methods of the `CPortConcentrationConstraint` object to set up upper bound, number of top holdings, etc.
21. To set the overlap constraint, call `CConstraints::SetOverlap()` to get a `CConstraintInfo` object and then call methods of the `CConstraintInfo` object to set up the upper bound on overlap.
22. To allow including futures in turnover, paring, roundlot and hedge constraints, call `CConstraints::SetIncludeFutures()`.
23. To set issuer constraints, call `CConstraints::InitIssuerConstraints()` to get a `CIssuerConstraints` object. Then call `CIssuerConstraints::AddHoldingConstraint()` to get a `CConstraintInfo` object, then call methods of the `CConstraintInfo` object to set up bounds.
24. To set ratio constraints, call `CConstraints::InitRatioConstraints()` to get a `CRatioConstraints` object. Then call `CRatioConstraints::AddGeneralGeneralConstraint()` or `CRatioConstraints::AddGroupConstraint()` to get a `CConstraintInfo` object. Call methods of `CConstraintInfo` to set up bounds.

25. To set quadratic constraints, call `CConstraints::InitQuadraticConstraints()` to get a `CQuadraticConstraints` object. The call `CQuadraticConstraints::AddConstraint()` to get a `CConstraintInfo` object and then call the methods of the `CConstraintInfo` object to set upper bound, softness, etc.

5.5.4 Set Up Efficient Frontier

To set up an efficient frontier, call `CCase::InitFrontier()` to get a `CFrontier` object.

Your application can pass appropriate parameters to specify one of the following types of the frontier:

- Risk-reward frontier varying return
- Risk-reward frontier varying risk
- Utility-turnover frontier varying turnover
- Utility-tax frontier varying tax costs
- Utility-constraint frontier varying the following constraints:
  - Factor constraint
  - General linear constraint
  - Transaction cost constraint
  - Leverage (hedge) constraint

Then, call methods of the `CFrontier` object to populate the object with data, such as the number of frontier data points and frontier range. For the utility-constraint frontier, call `CFrontier::SetFrontierConstraintID()` to set the varying constraint. To account for transaction costs in the return for risk-reward frontier, call `CFrontier::SetIncludeTransactionCost()`. To set the type of bound (lower or upper) to vary for utility-constraint frontier, call `CFrontier::SetFrontierBoundType()`.

5.5.5 Set Up Expected Shortfall Parameters

To set up parameters for expected shortfall optimizations, call `CCase::InitExpectedShortfall()` to get a `CExpectedShortfall` object. Then, call methods of the `CExpectedShortfall` object to set parameters and populate the case with return scenarios.

- Call the `CExpectedShortfall::SetConfidenceLevel()` of the expected shortfall. It must be a positive value less than 1.
- Call `CExpectedShortfall::SetScenarioReturnType()` with parameter `eFACTOR_RETURN`, if the scenario and target returns are specified for factors. If this method is not called, then asset returns are assumed.
- To set target returns, call `CExpectedShortfall::SetTargetMeanReturns()` passing a pointer to a `CAttributeSet` object. The `CAttributeSet` object should map asset/factor IDs to their returns, depending on the previous setting of the return type. If `CExpectedShortfall::SetTargetMeanReturns()` is called with `NULL`, then the target returns are

calculated by averaging the scenario returns. If this method is not called, then target return will not be included in the expected shortfall definition ($\eta = 0$ case in Section 4.14.2.)

- Call `CExpectedShortfall::AddScenarioReturns()` multiple times to add scenario return data. The mandatory `CAttributeSet` parameter maps asset/factor IDs (depending on the return type setting) to their returns. For confidence level ρ, at least $1/(1-\rho)$ scenarios are required. (See Section 4.14.2.)

5.5.6 Set Up Tax-Aware Optimization Parameters

Starting from version 8.8, a new set of API functions is introduced to support new tax-aware features.

To set up tax-aware optimization parameters with the new API functions, call `CCase::InitNewTax()` to get a `CNewTax` object. Then, call methods of the `CNewTax` object to populate the object with tax related data.

- Call `CNewTax::AddTaxRule()` to add one or more `CTaxRule` object(s). Then call methods of the `CTaxRule` object(s) to populate the object with the following data:
  - Tax rate
  - One-rate or two-rate
  - Cross netting gain/loss option
  - Loss carryforward for the tax rule
  - Wash sale rule
- Call `CNewTax::SetTaxHarvesting()` to set group-level tax harvesting parameters
- Call `CNewTax::SetSellingOrderRule()` to set group-level selling order rule
- Call `CNewTax::SetLossCarryForward()` to set group-level loss carryforward
- Call `CNewTax::SetTaxUnit()` to set Tax unit (Default to dollar amount)

Notes:

- For backward compatibility, the API functions compatible with older versions of Optimizer (prior to Optimizer 8.8) are kept intact. See next section for details.
- Do not mix use of the API functions for new tax-aware features with the old ones.

5.5.7 Set Up Tax-Aware Optimization Parameters with legacy API (prior to Optimizer 8.8)

To set up tax-aware optimization parameters with legacy API (prior to Optimizer 8.8), call `CCase::InitTax()` to get a `CTax` object. Then, call methods of the `CTax` object to populate the object with tax related data such as:

- Tax rate
- One-rate or two-rate
- Cross netting gain/loss option

- Loss carryforward
- Wash sale rule
- Tax harvesting targets/penalties
- Selling order rule
- Tax unit (Default to dollar amount)

5.6 Run Optimization

To run an optimization, do the following:

- Call `CWorkSpace::CreateSolver()` with the populated `CCase` object as its parameter to get a `CSolver` object.
- If the interactive approach is used, call `CSolver::SetCallBack()` with a parameter of the pointer to an instance of a client application class that derives from the `CCallback` class. For information about the interactive approach, see the Interactive Approach section.
- Optionally, call `CSolver::SetOptimalityToleranceMultiplier()` to make trade-off of performance and optimality.
- Optionally, call `CSolver::SetCountCrossTradeAsOne()` to count a crossover trade as one or two trades.
- Optionally, call `CSolver::SetOption()` with the string "COMPATIBLE_MODE" to set the solver approach that is backward compatible.
- Optionally, call `CSolver::SetOption()` with the string "MAX_OPTIMAL_TIME" to set the maximum time in seconds for an optimization to run.
- Optionally, call `CSolver::SetOption()`with the string " OPTIMAL_WEIGHT_TOLERANCE" or " OPTIMAL_TRADE_TOLERANCE" to set the tolerance for asset weights or trade sizes in the optimal portfolio. If the absolute value of weights/trades is below the tolerance level, the weights/trades will be dropped from the optimal portfolio and the dropped weights/trades are accumulated and allocated based on next options. When setting this option, make sure to check the constraint diagnostics as constraint violations might occur.
- Optionally, call `CSolver::SetOption()` with the string "OPTIMAL_WEIGHT_ALLOCATION_MODE" OR "OPTIMAL_TRADE_ALLOCATION_MODE" to set the allocation modes to redistribute the accumulated small weights/trades. A value of "0" (default) specifies to convert the accumulated weights/trades to cash or cash flow; A value of "1" specifies to equally allocate the accumulated weights/trades to non-zero holdings or trades; A value of "2" specifies to allocate the accumulated weights/trades proportional to the asset's holding/trade size.
- Optionally, call `CSolver::SetOption()` with the string "ALLOCATE_WEIGHTS_WITHIN_ASSET_BOUNDS" and value "1" to specify that allocation of accumulated weights/trades to non-zero holding or trades should not violate asset range constraints. If the weight calculated according to these strings – "OPTIMAL_WEIGHT_ALLOCATION_MODE" or "OPTIMAL_TRADE_ALLOCATION_MODE" – cannot be added fully to the asset weight, then the excess weight is converted to cash or cash flow.
- Optionally, call `CWorkSpace::Serialize()` to save the input data to a file with extension name .wsp. The wsp file is useful for troubleshooting and is required when sending support requests to Barra Optimizer developers. You may utilize openopt_exe (included in the installation package) to review the input data and then run an optimization based on the wsp file. We recommend that users call `CWorkSpace::Serialize()` after calling `CWorkSpace::CreateSolver()`.

- Optionally, call `CSolver::SetOption("REPORT_UPPERBOUND_ON_UTILITY", 1 )` to direct the optimizer to run heuristics to obtain solution for the problem with cardinality and threshold constraints and to report the estimated upper bound on the utility.
- Optionally, call `CSolver::SetOption("RUN_BRANCH_AND_BOUND", 1 )` to direct the optimizer to run branch-and-bound procedure to obtain the solution for the problem with cardinality and threshold constraints; and report the estimated upper bound on the utility.
- Optionally, call `CSolver::SetOption("CHECK_EFFICIENCY", 0 )` to return a portfolio with the given risk target or risk lower bound.
- Optionally, call `CSolver::SetOption("DROP_TINY_TRADES", 1)` to enable the elimination of small trades during optimization (see Section 3.10.9). A trade is considered to be small if it is below the value of "TINY_TRADE_THRESHOLD" option. To set tolerance levels of the algorithm, call `SetOption()` with "TINY_TRADE_TOLERANCE" or "NON_TINY_TRADE_TOLERANCE" and the tolerance level.
- Call `CSolver::Optimize()` to run the optimization.

5.7 Run Multi-Period Optimization

To run multi-period optimization, you need to first set up asset data and constraints for each period. Barra Optimizer allows up to 5 periods to be added to a case. To specify a specific period, call `CWorkSpace::SwitchPeriod()`.The subsequent (applicable) data being set in this case will be for the particular period only. Call `CWorkSpace::GetPeriod()` to retrieve the current period ID.

If certain asset data or constraints apply to all of the periods, simply call `CWorkSpace::SwitchPeriod()` and pass in the constant ALL_PERIOD. Then the subsequent data being set will be applicable for all of the periods. Note that setting data for ALL_PERIOD will invalidate the per-asset or constraint that was previously set for a particular period. To set a cross-period turnover constraint, call `CTurnoverConstraints::SetCrossPeriodNetConstraint()`, and to set a cross-period transaction cost constraint, call `CConstraints::SetCrossPeriodTransactionCostConstraint()`.

The typical steps to set up a multi-period optimization case are as follows:

1. Create the workspace and add assets to the workspace.
2. Define primary risk model and construct portfolios.
3. To set asset alphas for a period, for example:
    a. Call `CWorkSpace::SwitchPeriod()` to specify a given period as the current period

APIs to Set Up Multi Period Data	Multi Period	Notes
`CAsset::SetAlpha`	Y	
`CAsset::AddPWLinearBuyCost`	Y	
`CAsset::AddPWLinearSellCost`	Y	
`CUtility::SetAlphaTerm`		

APIs to Set Up Multi Period Data	Multi Period	Notes
`CUtility::SetPrimaryRiskTerm`	Y	
`CUtility::SetPenaltyTerm`	Y	
`CUtility::SetTranxCostTerm`	Y	
`CCase::SetCashFlowWeight`	Y	
`CLinearConstraint::SetBetaConstraint`	Y	
`CLinearConstraint::AddGeneralConstraint`	Y	
`CLinearConstraint::AddGroupConstraint`	Y	
`CLinearConstraint::SetAssetRange`	Y	
`CLinearConstraint::SetFactorRange`	Y	
`CTurnoverConstraints::SetNetConstraint`	Y	
`CTurnoverConstraints::SetBuySideConstraint`	Y	
`CTurnoverConstraints::SetSellSideConstraint`	Y	
`CTurnoverConstraints::SetCrossPeriodNetConstraint`	Y	
`CConstraints::SetTransactionCostConstraint`	Y	
`CConstraints::SetCrossPeriodTransactionCostConstraint`	Y	
`CParingConstraints::AddAssetTradeParing`	Y	
`CParingConstraints::AddAssetTradeParingByGroup`	Y	
`CParingConstraints::AddLevelParing`	Y*	Minimum holding threshold supported for multiple periods. Minimum transaction size threshold supported only for first period.
`CParingConstraints::AddLevelParingByAsset`	Y*	See AddLevelParing
`CParingConstraints::AddLevelParingByGroup`	Y*	See AddLevelParing
`CParingConstraints::SetAllowCloseOut`	N	A constraint applies to all periods once set; If set more than once, the latter setting overwrites the previous setting

APIs to Set Up Multi Period Data	Multi Period	Notes
`CParingConstraints::EnableGrandfatherRule`	N	A constraint applies to all periods once set; If set more than once, the latter setting overwrites the previous setting

   b. Call `CWorkSpace::GetAsset()` to obtain a pointer to the `CAsset` object
   c. Call `CAsset::SetAlpha()` to set alphas for the current period

4. Create the case and set up constraints for each period. For example,
   a. Call `CCase::InitConstraints()` or `CCase::GetConstraints()` to obtain a pointer to the `CConstraints` object
   b. Call `CConstraints::InitLinearConstraints()` or `CConstraints::GetLinearConstraints()` to obtain a pointer to the `CLinearConstraints` object
   c. Call `CWorkSpace::SwitchPeriod()`
   d. Call `CLinearConstraints::SetAssetRange()` to set asset ranges for the current period
5. Set up the objective and set risk aversions or multipliers for each period. For example:
   a. Call `CCase::InitUtility()or CCase::GetUtility()` to obtain a pointer to the `CUtility` object
   b. Call `CWorkSpace::SwitchPeriod()`
   c. Call `CUtility::SetAlphaTerm()` to set alpha term for the current period
   d. Call `CUtility::SetPrimaryRiskTerm()` to set benchmark and risk aversions for the current period
6. Create the solver and call `CSolver::AddPeriod()` for each period to be included in multi-period optimization.

5.8 Run Multi-Account Optimization

To run multi-account optimization, you need to first set up constraints and case settings for each account. Before setting data for a specific account, call `CWorkSpace::SwitchAccount()`.The subsequent (applicable) data being set in this case will be for the particular account only. Call `CWorkSpace::GetAccountID()` to retrieve the current account ID.

If some constraint or utility term applies to all of the accounts, call `CWorkSpace::SwitchAccount()` and pass in the constant ALL_ACCOUNT. Then the subsequent data being set will be applicable for all of the accounts.

Note that setting data for ALL_ACCOUNT will invalidate the constraint that was previously set for a particular account.

Accounts can be organized into account groups. Each account can be a member of at most one group. The group of the account can be given when the account is created by the `CSolver::AddAccount()` method. For specifying settings for a given account group, use `CWorkSpace::SwitchAccountGroup()`. Call `CWorkSpace::GetAccountGroup()` to retrieve the ID of the current account group.

The typical steps to set up a multi-account optimization case are as follows:

1. Create the workspace and add assets to the workspace.
2. Define primary risk model and construct portfolios.
3. Set asset alphas or transaction cost which will be applicable for all accounts.
4. Create the case and set up initial portfolio and base value for each account. For example,
    a. Call `CWorkSpace::CreateCase()` to obtain a pointer to a CCase object. Initial portfolio and portfolio base value is not required at this point.
    b. Call `CWorkSpace::SwitchAccount()` to switch to an account
    c. Call `CCase::SetTradeUniverse()` to set the trade universe for the current account, if different from the trade universe for ALL_ACCOUNT
    d. Call `CCase::SetIntialPort()` to set the initial portfolio for the current account
    e. Call `CCase::SetPortBaseValue()` to set the portfolio base value for the current account
5. Call `CCase::InitConstraints()` or `CCase::GetConstraints()` to obtain a pointer to the `CConstraints` object
    a. Call `CConstraints::InitLinearConstraints()` or `CConstraints::GetLinearConstraints()` to obtain a pointer to the `CLinearConstraints` object
    b. Call `CWorkSpace::SwitchAccount()`
    c. Call `CLinearConstraints::SetAssetRange()` to set asset ranges for the current account
6. To set cross-account constraints, first call `CConstraints::InitCrossAccountConstraints` to obtain a pointer to the `CCrossAccountConstraints` object. Then call the following APIs to set the corresponding cross-account constraints:
    a. Call `CCrossAccountConstraints::SetNetTurnoverConstraint()` (and the member functions of the `CConstraintInfo` class) to set cross-account turnover constraint
    b. Call `CCrossAccountConstraints::SetAssetBoundOnTotalPostion()` (and the member functions of the `CConstraintInfo` class) to set bound on total position across all accounts for an asset
    c. Call `CCrossAccountConstraints::SetMaxAssetTotalTrades()` (and the member functions of the `CConstraintInfo` class) to set the maximum total trade size for an asset over all accounts

   d. Call `CCrossAccountConstraints::SetMaxAssetTotalBuys`()(and the member functions of the `CConstraintInfo` class) to set the maximum total buys for an asset over all accounts

   e. Call `CCrossAccountConstraints::SetMaxAssetTotalSells()`(and the member functions of the `CConstraintInfo` class) to set the maximum total sells for an asset over all accounts

   f. Call `CCrossAccountConstraints::SetTaxLimit()` (and the member functions of the `CConstraintInfo` class) to set the maximum total tax over all accounts.

   g. Call `CWorkSpace::SwitchAccountGroup()` and `CCrossAccountConstraints::SetTaxLimit()` (and member functions of the `CConstraintInfo` class) to set a limit for total tax of accounts in a given account group.

7. Set up the objective and set risk aversions or multipliers for each account. For example:

   a. Call `CCase::InitUtility()or CCase::GetUtility()`to obtain a pointer to the `CUtility` object

   b. Call `CWorkSpace::SwitchAccountGroup()` and `CUtility::SetTaxTerm()` to set tax multiplier for an account group in tax-aware cases.

   c. Call `CWorkSpace::SwitchAccount()`

   d. Call `CUtility::SetAlphaTerm()`to set alpha term for the current account

   e. Call `CUtility::SetPrimaryRiskTerm()` to set benchmark and risk aversions for the current account

   f. Call `CUtility::SetUtilityScalar` to set the scalar for utility function for the current account

   g. Call `CUtility::SetJointMarketImpactTerm()` to set the cross-account joint market impact term

   h. Call `CUtility::SetTaxTerm()` to set tax multipliers for an account not belonging to a group.

8. Create the solver and call `CSolver::AddAccount()`for each account to be included in multi-account optimization. To assign the account to an account group, set the `accountGroupID` parameter. If the case is tax-aware, call methods of the returned `CAccount` object to change the default values of account specific settings as follows:

   a. Call `CAccount::SetTaxable(false)` to indicate that all tax-lots of this account are tax-free.

   b. Call `CAccount::SetWashSaleRelated(true)` to indicate that the account is "wash-sale-related".

9. To set up tax-aware optimization parameters, call `CCase::InitMAOTax()` to get a `CMAOTax` object. Then, call methods of the `CMAOTax` object to populate the object with tax related data.

   a. Call `CMAOTax::AddTaxRule()` to add one or more `CTaxRule` object(s). Then, call methods of the `CTaxRule` object(s) to populate the object with the following data:

- Tax rate
- One-rate or two-rate
- Cross netting gain/loss option
- Loss carryforward for the tax rule
- Wash sale rule

b. Call `CMAOTax::SetTaxRule()` to associate a tax rule object returned by `CMAOTax::AddTaxRule()` for a given asset group in the current account or account group.

c. Call `CMAOTax::SetTaxHarvesting()` to set group-level tax harvesting parameters

d. Call `CMAOTax::SetSellingOrderRule()` to set group-level selling order rule

e. Call `CMAOTax::SetLossCarryForward()` to set group-level loss carryforward

f. Call `CMAOTax::SetTaxUnit()` to set Tax unit (Default to dollar amount)

10. Optionally, call `CSolver::SetOption`("MAO_APPROACH", 1) to choose Nash Equilibrium approach or call `CSolver::SetOption`("MAO_APPROACH", 0) to choose Total Welfare approach.

APIs to Set Up Multi-Account Data	Individual Account	Notes
`CUtility::SetAlphaTerm`	Y	
`CUtility::SetPrimaryRiskTerm`	Y	
`CUtility::SetPenaltyTerm`	Y	
`CUtility::SetTranxCostTerm`	Y	
`CUtility::SetTaxTerm`	Y	
`CCase::SetTradeUniverse`	Y	
`CCase::SetInitialPort`	Y	
`CCase::SetPortBaseValue`	Y	
`CCase::SetCashFlowWeight`	Y	
`CUtility::SetJointMarketImpactTerm`	N	Cross-account term
`CCase::SetUtilityScalar`	Y	
`CMAOTax::SetTaxUnit`	N	
`CMAOTax::AddTaxRule`	N	

APIs to Set Up Multi-Account Data	Individual Account	Notes
`CMAOTax::SetTaxRule`	Y	
`CMAOTax::SetTaxHarvesting`	Y	
`CMAOTax::SetSellingOrderRule`	Y	
`CMAOTax::SetLossCarryForward`	Y	
`CLinearConstraint::SetBetaConstraint`	Y	
`CLinearConstraint::AddGeneralConstraint`	Y	
`CLinearConstraint::AddGroupConstraint`	Y	
`CLinearConstraint::SetAssetRange`	Y	
`CLinearConstraint::SetAssetTradeSize`	Y	
`CLinearConstraint::SetFactorRange`	Y	
`CTurnoverConstraints::SetNetConstraint`	Y	
`CTurnoverConstraints::SetBuySideConstraint`	Y	
`CTurnoverConstraints::SetSellSideConstraint`	Y	
`CConstraints::SetTransactionCostConstraint`	Y	
`CParingConstraints::AddAssetTradeParing`	Y	
`CParingConstraints::AddAssetTradeParingByGroup`	Y	
`CParingConstraints::SetAllowCloseOut`	N	A constraint applies to all accounts once set; If set more than once, the latter setting overwrites the previous setting
`CParingConstraints::EnableGrandfatherRule`	N	A constraint applies to all accounts once set; If set more than once, the latter setting overwrites the previous setting
`CNewTaxConstraints::SetTaxLimit`	Y	
`CNewTaxConstraints::SetTaxArbitrageRange`	Y	

APIs to Set Up Multi-Account Data	Individual Account	Notes
`CNewTaxConstraints::SetTaxLotTradingRule`	Y	ALL_ACCOUNT setting is not allowed
`CNewTaxConstraints::SetMinHoldingPeriod`	Y	
`CCrossAccountConstraints::SetAssetBoundOnTotalPosition`	N	Cross-account constraint
`CCrossAccountConstraints::SetMaxAssetTotalTrades`	N	Cross-account constraint
`CCrossAccountConstraints::SetMaxAssetTotalBuys`	N	Cross-account constraint
`CCrossAccountConstraints::SetMaxAssetTotalSells`	N	Cross-account constraint
`CCrossAccountConstraints::SetNetTurnoverConstraint`	N	Cross-account constraint
`CCrossAccountConstraints::SetTaxLimit`	N	Cross-account constraint

5.9 Inspect Output

5.9.1 Non-Interactive Approach

- When using the non-interactive approach, your application obtains control after the `CSolver::Optimize()` method returns. `CSolver::Optimize()` returns a `CStatus` object that can be used for error handling.
- For a portfolio optimization problem, call `CSolver::GetPortfolioOutput()` to get the `CPortfolioOutput` object if the optimization is successful. Then, call methods of the respective objects to get output information such as the optimal portfolio, objective, risk, turnover, and so forth.
- For an efficient frontier problem, call `CSolver::GetFrontierOutput()` to get the `CFrontierOutput` object. Then, call methods of the respective objects to get `CDataPoint` objects, and call `CDataPoint` methods to get portfolios and risk/return values of the frontier data points.
- For multi-period optimization, call `CSolver::GetMultiPeriodOutput()` to get the `CMultiPeriodOutput` object.
  Call `CMultiPeriodOutput::GetPeriodOutput()` to get a pointer to the `CPortfolioOutput` object which can be used to retrieve per period results.
  Call `CMultiPeriodOutput::GetCrossPeriodOutput()` to get a pointer to the `CPortfolioOutput` object which can be used to retrieve cross period results.
- For multi-account optimization, call `CSolver::GetMultiAccountOutput()` to get a pointer to the `CMultiAccountOutput` object.
  Call `CMultiAccountOutput::GetAccountOutput()` to get a pointer to the `CPortfolioOutput` object that can be used to retrieve per account results.
  Call `CMultiAccountOutput::GetCrossAccountOutput()` to get a pointer to the `CPortfolioOutput`

object which can be used to retrieve cross-account results.
Call `CMultiAccountOutput::GetCrossAccountTradeInfo()` to retrieve total buys or sells across all accounts per asset.
Call `CMultiAccountOutput::GetJointMarketImpactBuyCost()`,`GetJointMarketImpactSellCost()` to retrieve joint market impact costs of the optimal portfolio.
Call `CMultiAccountOutput::GetCrossAccountTaxOutput()` to retrieve cross-account tax results.
Call `CMultiAccount::GetAccountGroupTaxOutput()` to retrieve results for each account group.

5.9.2 Interactive Approach

The interactive approach allows your application to process frontier data points/messages generated by Barra Optimizer during the optimization process, before the `CSolver::Optimize()` method returns.

To use the interactive approach, your application needs to derive a class from the `CCallBack` class and override its virtual functions `CCallback::OnDataPoint()` and `CCallback::OnMessage()`. Your application uses the derived class to handle frontier data points and messages sent by Barra Optimizer. Compared to the non-interactive approach, the interactive approach offers more flexibility, but is more difficult to implement.

5.9.3 Basic Output Information

On successful optimization, Barra Optimizer produces the following basic output information:

- Optimal portfolio weights h*
- Objective value
- Return (in %): $100\mathbf{r}^{\mathrm{T}}\mathbf{h}^*$
- Active risk (in %): $100\sqrt{(\mathbf{h}^*-\mathbf{h}_B)^{\mathrm{T}}(\mathbf{D}+\mathbf{XFX}^{\mathrm{T}})(\mathbf{h}^*-\mathbf{h}_B)}$ if a benchmark is added for the primary risk model in the objective
- Total risk (in %): $100\sqrt{\mathbf{h}^{*\mathrm{T}}(\mathbf{D}+\mathbf{XFX}^{\mathrm{T}})\mathbf{h}^*}$, if no benchmark is added for the primary risk model in the objective
- Total penalty costs
- Beta of the portfolio with respect to the benchmark for the primary risk model used in the objective function
- Portfolio turnover (in %)
- Transaction costs (in %)
- Total short rebate costs
- Implied risk aversion (only available for efficient frontier and return/risk target cases)
- Constraint (turnover or tax cost) slack value in the constraint efficient frontier

For portfolio optimization (non-frontier optimization), Barra Optimizer provides the following additional output information via methods of the `CPortfolioOutput`:

- Constraint slack information
- For holding/trade cardinality information such as values of discrete variables (number of assets/trades, etc.), call `CPortfolioOutput::GetAssetTradeParingInfo()` to get a `CAssetTradeParingInfo` object. For holding/trade cardinality information about a group of assets, call `CPortfolioOutput::GetAssetTradeParingGroupInfo()` to get a `CAssetTradeParingInfo` object.
- For threshold violation information (also known as level paring), call `CPortfolioOutput::GetLevelParingViolationInfo()` to get a `CLevelParingInfo` object.
- For post-optimization roundlotted portfolio, use `CPortfolioOutput::GetRoundlottedPortfolio()`.
- For a tax-aware optimization case, if the new tax-aware API is used (i.e. `CCase::InitNewTax()` or `CCase::InitMAOTax()` is called), call `CPortfolioOutput::GetNewTaxOutput()` to get a `CNewTaxOutput` object. Then, call methods of the `CNewTaxOutput` object to get more detailed information on tax related outputs.
- For a tax-aware optimization case, if the legacy tax-aware API is used (i.e. `CCase::InitTax()` is called), call `CPortfolioOutput::GetTaxOutput()` to get a `CTaxOutput` object. Then, call methods of the `CTaxOutput` object to get more detailed information on tax-related outputs.
- For determining if the output portfolio is heuristic or optimal, use `CPortfolioOutput::IsHeuristic()`. For more information, see the Indication of the Output Portfolio as Heuristic or Optimal section.
- For trade list information, call `CPortfolioOutput::GetAssetTradeListInfo()` to get a `CAssetTradeListInfo` object. Alternatively, call `CSolver::GetTradeList()` to get a `CTradeListInfo` object that contains trade list information between any two portfolios.
- For the estimated upper bound on the utility for paring cases, call `CPortfolioOutput::GetUpperBoundOnUtility()`. To compute the heuristic gap, call `CPortfolioOutput::GetUpperBoundOnUtility() - CDataPoint::GetUtility()`.
- KKT table items

Note: You can log output information to a file by calling `CPortfolioOutput::WriteToFile()`.

5.9.4 Additional Statistics for a Given Portfolio

Barra Optimizer allows you to add the following additional statistics for a given portfolio, including the optimal/initial portfolio using the `CSolver::Evaluate()`:

- Portfolio return
- Factor risk
- Specific risk
- Information ratio/Sharpe ratio

For primary risk model, you may obtain the contribution to risk (or tracking error) from a set of assets or factors for a given portfolio, including the optimal/initial portfolio using `CSolver::EvaluateRiskContribution()`.

5.9.5 Constraint Slack Information

With the output, Barra Optimizer also provides information about constraint slacks such as a list of binding constraints, impact to utility, soft bound violations, etc.

- For a given constraint slack, one may call `CPortfolioOutput::GetSlackInfo()` to get `CSlackInfo` and then call methods of the `CSlackInfo` to get more details of the slack.
- Call `CPortfolioOutput::GetBindingSlackIDs()` to get a list of binding constraints. A constraint $l \le g(\mathbf{h}) \le u$ is called binding at the optimal portfolio h* if $l < g(\mathbf{h}^*) < u$ and is called non-binding if $l < g(\mathbf{h}^*) < u$.
- Impact to utility:
  - Call `CSlackInfo::GetDownImpactToUtility()` to get the value of down-side impact to utility.
  - Call `CSlackInfo::GetUpImpactToUtility()` to get the value of up-side impact to utility.
  - Up-side impact to utility usually has the same value as down-side impact to utility. However, they may be different at the break points of the piecewise-linear functions (for example, transaction cost function, penalty functions).
  - Information on impact to utility is not available for some non-standard optimization cases, for example, paring cases, efficient frontier cases, risk/return target cases, and roundlotting cases.
  - Value of impact to utility is usually zero for non-binding constraints.
- Call `CPortfolioOutput::GetSoftBoundSlackIDs()` to get a list of constraints that have soft bound violations. Then, call methods of the `CSlackInfo` objects to get detailed information on how much a soft bound is violated.

The value returned by `CSlackInfo::GetSlackValue()` is always in decimal.

5.9.6 Error Handling

Call `CStatus::GetStatusCode()` to get the status code of EStatusCode type (see the following table for the various error messages). Call `CStatus::GetMessage()` to get a default English message string for the current status code.

EStatusCode	Message
eOK	Optimization terminated normally
eDATA_ERROR	Data Error

EStatusCode	Message
eNOT_ENOUGH_MEMORY	Memory error
eUSER_CANCELED	Optimization process canceled by the user
eLICENSE_ERROR	License check failed
eOTHER_ERROR	Other Error (%s)
eTIME_LIMIT_EXCEEDED	Optimization exceeded time limit set by the user. The default time limit is 1 hour.
eINTERNAL_ERROR	Internal Error
eINFEASIBLE	Infeasible Case
eINFEASIBLE_DUETO_51040	Infeasible due to 5/10/40
eINFEASIBLE_DUETO_ROUNDLOTTING	Infeasible due to roundlotting
eINFEASIBLE_DUETO_RISK_CONSTRAINTS	At least one of the risk constraint bounds is too tight
eINFEASIBLE_DUETO_PARING	A paring constraint such as threshold or cardinality is too restrictive
eINFEASIBLE_DUETO_PARING_AND_ROUNDLOTTING	The problem is infeasible due to paring and roundlotting constraints
eINFEASIBLE DUETO_OVERLAP	The problem is infeasible due to overlap constraint
eINFEASIBLE_DUETO_ISSUER_CONS	The problem is infeasible due to issuer constraint(s)
eMAYBE_INFEASIBLE_DUETO_OVERLAP	The problem may be infeasible due to overlap constraint
eMAYBE_INFEASIBLE_DUETO_RISK_CONSTRAINTS	At least one of the risk constraint bounds may be too tight

EStatusCode	Message
eMAYBE_INFEASIBLE_DUETO_HEDGE_AND_PARING	The problem may be infeasible due to hedge and paring constraints
eMAYBE_INFEASIBLE_DUETO_TAX_AND_PARING	The problem may be infeasible due to tax and paring constraints
eMAYBE_INFEASIBLE_DUETO_TURNOVERBYSIDE	The problem may be infeasible due to turnover-by-side constraints
eMAYBE_INFEASIBLE_DUETO_TAX_CONS	Tax constraints may be too tight
eMAYBE_INFEASIBLE_DUETO_51040	The problem may be infeasible due to 5/10/40 rule.
eMAYBE_INFEASIBLE_DUETO_RISKPARITY	The problem may be infeasible due to risk parity constraint.
eMAYBE_INFEASIBLE_DUETO_PARING_AND_ROUNDLOTTING	The problem may be infeasible due to paring and roundlotting constraints.
eMAYBE_INFEASIBLE_DUETO_GENERAL_PWLI	The problem may be infeasible due to general piecewise-linear constraints
eMAYBE_INFEASIBLE_DUETO_HEDGE_OR_GENERAL_PWLI	The problem may be infeasible due to hedge constraints or general piecewise-linear constraints
eMAYBE_INFEASIBLE_DUETO_ISSUER_CONS	The problem may be infeasible due to issuer constraint(s)
eRISK_TARGET_OR_RANGE_UNATTAINABLE	Risk target/range is unattainable. The closest risk we can get is x%

EStatusCode	Message
eRETURN_TARGET_OR_RANGE_UNATTAINABLE	Return target/range is unattainable. The closest return we can get is x%
eCONSTRAINT_RANGE_UNATTAINABLE	Constraint range is unattainable. The closest one we can get is x%
eTARGET_OR_RANGE_UNATTAINABLE_DUETO_PARING	Target/range is unattainable because of paring constraints. The best guess is x%
eTARGET_OR_RANGE_UNATTAINABLE_DUETO_NONCONVEXITY	Failed to reach the target/range because of non-convex features other than paring constraints. The best guess is x%
eRISK_TARGET_OR_RANGE_NOT_ON_FRONTIER	Risk target/range is outside efficient frontier range. Better utility with a smaller risk (x%) achieved.
eRETURN_TARGET_OR_RANGE_NOT_ON_FRONTIER	Return target/range is outside efficient frontier range. Better utility with a higher return (x%) achieved.
eCONSTRAINT_RANGE_NOT_ON_FRONTIER	Constraint range is outside efficient frontier range. Better utility with a tighter constraint (x) achieved.
eRISK_TARGET_FAILED	Risk target optimization failed.
ePARING_HEURISTIC_FAILED	Paring failed. The threshold or cardinality constraint may be too restrictive.
eHEDGE_HEURISTIC_FAILED	Hedge heuristic approach failed
eROUNDLOT_HEURISTIC_FAILED	Roundlotting heuristic approach failed

EStatusCode	Message
eMAO_HEURISTIC_FAILED	Multi-Account Optimization heuristic approach failed

With the status code, your application knows whether the optimization is successful or unsuccessful and can act accordingly. Your application may use the status code along with the additional information (obtained by calling `CStatus::GetAdditionalInfo()`) to display a customized message string that may contain more details about the case including timing and solver information.

5.9.7 Information for Infeasible Cases

If an optimization case proves infeasible, Barra Optimizer usually provides enough information to determine which constraints have been violated, and by how much. This information allows you to loosen constraints and make the problem feasible.

Some Restrictions:

- If the infeasibility was caused by overly restrictive asset bounds, Barra Optimizer may not be able to indicate which asset bounds were violated, and may only indicate that the portfolio balance constraint was violated.
- Barra Optimizer may not be able to provide information about infeasibility caused by some special constraints, such as threshold or holding constraints.

When an optimization case is determined to be infeasible, you may call: `CPortfolioOutput::GetInfeasibleSlackIDs()` to review slack IDs and determine which constraints may cause the infeasibility. After obtaining the slack IDs, call `CPortfolioOutput::GetSlackInfo()` to produce more detailed information on how much to relax bounds to make the case feasible.

When the balance constraint is violated, its ID is not added to the set returned by `CPortfolioOutput::GetInfeasibleSlackIDs()`.

In this case, call `CPortfolioOutput::GetSlackInfo4BalanceCon()` to retrieve the `CSlackInfo` object.

For infeasible cases with threshold or holding (also known as paring) constraints, it may be useful to call the following for hints on relaxing the constraints:

- `CPortfolioOutput::GetAssetTradeParingInfo()`
- `CPortfolioOutput::GetLevelParingViolationInfo()`

5.9.8 KKT Table Items

Call the following functions to retrieve KKT table items for the objective terms:

- `CPortfolioOutput::GetPrimaryRiskModelKKTTerm()`
- `CPortfolioOutput::GetSecondaryRiskModelKKTTerm()`
- `CPortfolioOutput::GetResidualAlphaKKTTerm()`

- `CPortfolioOutput::GetTransactioncostKKTTerm()`

Call the following functions to retrieve KKT table items for the constraint slacks

- `CSlackInfo::GetKKTTerm()`
- `CSlackInfo::GetPenaltyKKTTerm()`

5.9.9 Asset-level Transaction Costs

Call the following functions to retrieve asset-level transaction costs from `CAssetTradeListInfo` object:

- `CAssetTradeListInfo::GetFixedTransactionCost()` - For more information, see the Fixed Transaction Costs section.
- `CAssetTradeListInfo::GetNonLinearTransactionCost()` - For more information, see the Non-Linear Transaction Costs section.
- `CAssetTradeListInfo::GetPiecewiseLinearTransactionCost()` - For more information, see the Piecewise-Linear Transaction Costs section
- `CAssetTradeListInfo::GetTotalTransactionCost()`

5.10 Data Serialization

Barra Open Optimizer supports serialization of the workspace, which includes any data loaded into the workspace. Prior to version 2.0, data of a `CWorkSpace` object could be serialized into the binary format with the wsp file extension. Starting with version 2.0, additional binary (protobuf) and XML formats are available via the OpenOpt APIs. The OpenOpt APIs take advantage of Protocol Buffers technology developed by Google, which encodes the workspace data in an efficient yet extensible format. The Protocol Buffer (abbreviated as protobuf) is platform-neutral and is portable across Windows and Linux platforms. The Protobuf format is fully backward compatible.

To run optimization with the new formats, we recommend users to load data into the workspace using the OpenOpt APIs for a more streamlined process and user-friendly experience. Alternatively, the existing `CworkSpace` data can be converted into the following formats by calling `CWorkSpace::Serialize()`.

EFileFormat	Description
PB_XML, eXML	XML format created by OpenOpt APIs
eJSON	JSON format created by OpenOpt APIs
ePB	Binary format created by OpenOpt APIs
eWSP	Wsp file containing binary data of a `CWorkSpace` object

For more information, see the Using the Barra Optimizer XML and Using the Command Line Executable chapters.

5.11 Release Resources

Finally, your C++ or Java application must call the `CWorkSpace::Release()` method to release memory resources when the workspace is no longer needed. For C# API, call the `CWorkSpace::Dispose()` method to release the memory resources.

Once `CWorkSpace::Release()` or `CWorkSpace::Dispose()` is called, memory resources associated with the `CWorkSpace` object and all other objects (for example, `CAsset`, `CPortfolio`, `CSolver`, `CCase` objects) will be automatically released.

5.12 Thread Safety

Barra Open Optimizer APIs are thread-safe in the context of a multi-threaded environment in which each thread creates a separate instance of the `CWorkSpace` object without the need for external synchronization. Each thread should run to completion and produces consistent result as if each optimization is invoked by a separate process.

5.13 Reference and Tutorial Sample Codes

API reference guides:

- C++ API Reference Guide
- Java API Reference Guide
- C# API Reference Guide
- Python API Reference Guide

Tutorial sample codes:

- C++ API tutorials
- Java API tutorials
- C# API tutorials
- MATLAB tutorials

After installing Barra Optimizer, you can see the complete documentation set by navigating to:

<installation directory>/doc/index.html

You can also find the latest versions of some of the documents on the MSCI Client Support site.

6 Using MATLAB and R Interface

Barra Open Optimizer provides an Application Programming Interface (API) to MATLAB and R Language, in addition to the standard C++, Java, and C# APIs.

MATLAB and R Language are powerful tools for data visualization, data analysis, and numeric computation. Through the APIs described here, these tools can fully utilize the portfolio optimization features of Barra Optimizer.

This section explains the setup steps for MATLAB and R Language. Their interface to Barra Optimizer is derived from the existing Java and C++ APIs. Developers can use this information with the C++ or Java API information to derive the interface format. Some tutorial samples are provided to demonstrate the interface framework. Developers should be able to build their application by applying similar changes to all the other interface classes.

6.1 Working with MATLAB Interface

The Barra Optimizer functions can be accessed from MATLAB through their Java interface mechanism. MATLAB includes a Java Virtual Machine (JVM) enabling developers to use the Java interpreter via MATLAB commands, create, and run programs that create and access Java objects. You can invoke all the functions available in the Barra Optimizer Java API.

One small difference (coding with MATLAB vs. coding with Java) is in the method of passing a null object to the API functions. MATLAB does not have a null object. In these cases, you may pass in '' to denote the null object. For example:

```
% do not want to set benchmark portfolio
% benchmark portfolio object is null, use ''
ut.SetPrimaryRiskTerm('');
```

For detailed information, see the Barra Optimizer Java API Reference Guide.

6.1.1 Setting Up MATLAB

To use the Barra Optimizer functions, you need to make the Barra Optimizer Java classes available in the system. For MATLAB to correctly find and load the Barra Optimizer Java jar file OptJAVAAPI.jar, perform the following setup steps.

6.1.1.1 Verify Your Java Version

Barra Optimizer supports Java 1.8 or higher. To verify the JVM version installed in your MATLAB system, type *version –java* in the command window.

6.1.1.2 Verify PATH (Windows) or LD_LIBRARY_PATH (Linux)

For Barra Optimizer to find the 32-bit (or 64-bit) run time library on the Windows platform, the environment variable PATH should include the path to the *bin\ia32* or *bin\intel64* subfolder.
For example:

```
       C:\Program Files\MSCI Barra\Barra Optimizer\bin\ia32
```

For Barra Optimizer to find the run time library on a Linux platform, the environment variable LD_LIBRARY_PATH should include the path to the *lib/intel64* (64-bit) subfolder.
For example:

```
       /BarraOptimizer/lib/intel64
```

If using MATLAB R2013 **or later on Linux,** MATLAB has its own version of Intel MKL BLAS library that conflicts Barra Open Optimizer. You will need to set the environment variable as follows:

export BLAS_VERSION=libblas.so

For earlier versions of MATLAB on Windows, if certain operations such as matrix multiplication are not working, add the environment variable:

BLAS_VERSION=refblas.dll

6.1.1.3 Quick Start to Set Up the Java Class Path and Library Path

On Windows, Barra Optimizer provides a batch file, in the tutorials\matlab directory, to set up the necessary paths to import Java API classes into MATLAB.

To run the batch script, do the following:

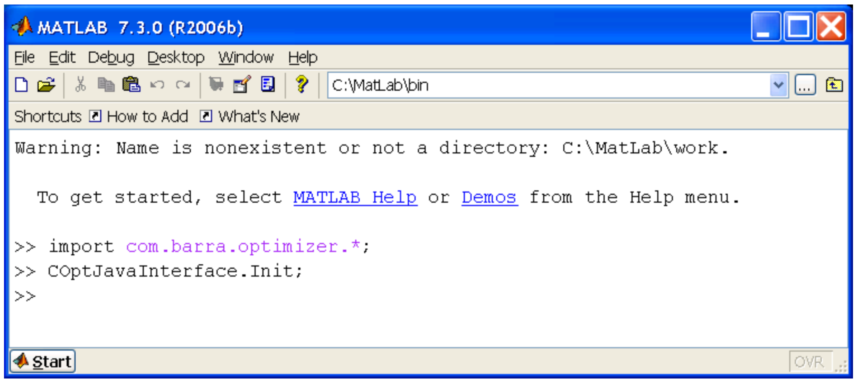
1. Open a command prompt window and navigate to tutorials\matlab.
2. Type *setup.bat 32* or *setup.bat 64* and then press Enter.

where, provide parameter 32 or 64 based on your MATLAB installation architecture.

Note: To know about the MATLAB architecture (32-bit or 64-bit), see Help > About MATLAB.

The script sets up the static class and library paths, which are loaded by MATLAB at startup. Specifically, it appends the Barra Optimizer JAR file path to javaclasspath.txt and the Barra Optimizer executable path to javalibrarypath.txt. Both the text files are under the user folder on MATLAB's search path. (In MATLAB, you can type `userpath` to locate the mentioned user folder.)

Then, to verify that the Barra Optimizer interface is working properly, type the following instructions in the command window:


MATLAB 7.3.0 (R2006b)
File Edit Debug Desktop Window Help
C:\MatLab\bin
Shortcuts How to Add What's New
Start
OVR

```
Warning: Name is nonexistent or not a directory: C:\MatLab\work.

  To get started, select MATLAB Help or Demos from the Help menu.

>> import com.barra.optimizer.*;
>> COptJavaInterface.Init;
>>
```

These instructions initialize the Barra Optimizer Java interface classes. If the system returns no errors, then the interface is ready.

6.1.2 MATLAB Tutorials

The set of MATLAB tutorials are located in the *tutorials/matlab* directory. They consist of three class definitions—TutorialData, TutorialBase, and TutorialApp—and the TutorialDriver script to drive through each individual tutorial.

Tutorial Files	Description
TutorialData.m	Defines a TutorialData class to handle simulation data used for all tutorials, including reading models from files
TutorialBase.m	Defines a TutorialBase class that contains shared routines for all tutorials
TutorialApp.m	Defines a TutorialApp class that implements individual tutorials
TutorialDriver.m	User access point/interface to set up simulation data and run through each individual tutorial

As the user interface for the tutorials, you may want to run the TutorialDrive function at the MATLAB prompt to run all the tutorials. The runtime results will appear on your MATLAB terminal as the TutorialDriver function completes individual tutorials. The example tutorial 1a's result is shown below:

```
>> TutorialDriver
======== Running Tutorial 1a  ========
Minimize Total Risk
Optimization terminated normally.
Initializing Barra Optimizer 8.2.0
CPU by COptCore::Optimize() = 0.003 seconds
Quadratic Solver

Optimized Portfolio:
Risk(%)     = 19.3642
Return(%)   = 0.0000
Utility     = -0.0281
Turnover(%) = 7.7663
Penalty     = 0.0000
TranxCost(%)= 0.0000
Beta        = 0.0000

Asset Holdings:
USA11I1: 0.4829
```

```
USA13Y1: 0.5171


Balance constraint KKT term
USA11I1: -0.0562
USA13Y1: -0.0562
```

Alternatively, a runtutorial.bat (for Windows) or a runtutorial.sh (for Linux) is provided to run the tutorials at your operating system's command prompt.

6.1.3 Java APIs to for setting up workspace data

To improve performance, Barra Optimizer provides a number of Java interfaces to accept arrays of data in order to minimize the number of API calls in MATLAB. The Java APIs allow you to set factor and specific covariance matrices, asset exposures, portfolio weights, and asset attributes. For example, instead of calling SetFactorExposure() in nested loops to set the asset exposure per asset and per factor:

```
for i = 1 : size(data.id,1)
    for j = 1 : size(data.expData,2)
        rm.SetFactorExposure(data.id(i),data.factor(j),data.expData(i,j));
    end
end
```

You can set the asset exposures with one API call by passing arrays of data:

```
rm.SetFactorExposures(assetIDs, factorIDs, exposures);
```

The following Java APIs set the workspace data with the specified arrays of values:

Java API	Description
`CRiskModel::SetFactorCovariances()`	Sets the covariance matrix.
`CRiskModel::SetFactorExposures()`	Sets the exposures for a list of assets. The ith asset is exposed to the ith factor with ith exposure.
`CRiskModel::SetFactorExposuresForAsset()`	Sets the exposures of an asset.
`CRiskModel::SetSpecificCovariances()`	Sets the specific covariances for list of assets. The ith covariance is the covariance between ith assetIDs1 and ith assetIDs2.
`CRiskModel::SetSpecificVariances()`	Sets the specific variances for list of assets.

Java API	Description
`CPortfolio::AddAssets()`	Adds a list of assets and their weights to the portfolio.
`CWorkSpace::CreateAttributeSet()`	Creates a `CAttributeSet` object with the arrays of keys and values.

6.1.4 MATLAB Toolbox

For the MATLAB users, Barra Open Optimizer has an additional support toolbox, which is provided with the installation package in the \docs\matlabref directory. With the MATLAB toolbox, all the documentation and tutorials are easily available while using Barra Open Optimizer.

To use the toolbox, you need to set \docs\matlabref as the current folder in MATLAB. Once the current folder is set, navigate to the home page in the MATLAB Help browser, and then click Supplemental Software at the bottom of the page.

6.2 Working with R Language Interface

R Language is a free software environment for statistical computing and graphics. It provides a wide variety of graphical and statistical techniques (for example, linear and nonlinear modeling, classical statistical tests, time-series analysis, classification, clustering, and so on). R Language is a GNU project available from http://www.r-project.org.

The Barra Optimizer API can be accessed from R via the external C/C++ DLL interface mechanism. We provide the API functions in a set of C wrapper classes packaged in barraopt_wrap shareable library. This library can be dynamically loaded in R together with an R macro package barraopt_wrap.R. You can invoke all the functions available in the Barra Optimizer C++ API.

For detailed information, see the Barra Optimizer C++ API Reference Guide.

The main differences between the R wrapper interface and the C++ API are:

- All the methods in the Barra Optimizer C++ API are available as function calls: `<class name>_<method name> ( object variable, method arguments )`
- For instance, in C++:

```
workspace->CreatePortfolio( "Portfolio Name" );
```

- In R:

```
CWorkSpace_CreatePortfolio( workspace, "PortfolioName" );
```

- All enumeration values are passed in as a String:
- For instance, in C++:

```
CRiskModel *riskmodel = workspace->CreateRiskModel( "BIM", eEQUITY );
```

- In R:

```
riskmodel = CWorkSpace_CreateRiskModel( workspace, "BIM", "eEQUITY" );
```

6.2.1 Setting Up the R Language Environment

To set up the R Language environment, perform the following steps.

6.2.1.1 Verify PATH (Windows) or LD_LIBRARY_PATH (Linux)

For Barra Optimizer to find the run time library on the Windows platform, the environment variable PATH should include the path to the *bin\ia32* (32-bit) or *bin\intel64* (64-bit) subfolder. For example:

```
C:\Program Files\MSCI Barra\Barra Optimizer\bin\ia32
```

For Barra Optimizer to find the run time libraries on a Linux platform, the environment variable LD_LIBRARY_PATH should include the path to the *lib/intel64* (64-bit) subfolder. For example:

```
/BarraOptimizer/lib/32
```

6.2.1.2 Initialize Barra Optimizer Access

Before calling your R program, do the following:

- Set the environment variable BARRAOPT_WRAP to point to the path of the library file barraopt_wrap.dll (Windows) or barraopt_wrap.so (Linux).
- Set the environment variable BARRAOPT_WRAP_R with the value being the path of the barraopt_wrap.R file.

At the beginning of your R program, load the barraopt_wrap library and the barraopt_wrap.R program:

```
dyn.load(Sys.getenv("BARRAOPT_WRAP"))
source(Sys.getenv(“BARRAOPT_WRAP_R”))
```

6.2.2 R Tutorials and Sample Code

The set of R tutorials is located in the *tutorials/R* directory. They consist of three class definitions—TutorialData, TutorialBase, and TutorialApp—and the TutorialDriver script to drive through each individual tutorial. Classes with Reference Semantics are used in the R tutorials and R6 package is therefore required to run tutorials[6].

Tutorial Files	Description
TutorialData.r	Defines a TutorialData class to handle simulation data used for all tutorials, including reading models from files
TutorialBase.r	Defines a TutorialBase class that contains shared routines for all tutorials

[6] As of R version 3.1.2 and R6 version 2.2.1, the class reference semantics may not work with some of the Barraopt R APIs. However, the semantics described in the Working with R Language Interface section always work. Some of these APIs are also illustrated in the R tutorials.

Tutorial Files	Description
TutorialApp.r	Defines a TutorialApp class that implements individual tutorials
TutorialDriver.r	User access point/interface to set up simulation data and run through each individual tutorial

A runtutorial.bat (for Windows) or a runtutorial.sh (for Linux) is provided to run the tutorials at your operating system's command prompt. The results will appear on the terminal and will be saved in TutorialDriver.r.Rout file.

In the same directory, the R Language sample code sample.r is also provided to illustrate the access of R interface without Classes with Reference Semantics. The provision of the additional command line parameter "sample" to the batch/shell command will run the sample script instead of the tutorials:

On Windows,

```
> runtutorial.bat sample
```

Or on Linux,

```
> runtutorial.sh sample
```

7 Using Python Interface

Barra Optimizer provides pre-compiled extension module that can be loaded into the Python interpreter version 3.4, 3.5, 3.6, 3.7 and 3.8. On Windows, the DLL is called _barraopt.pyd available under the bin\ia32 (32-bit) and bin\intel64 (64-bit) folders.

To use the Barra Optimizer API in Python, perform the following steps:

1. Set the environment variable PYTHONPATH to where the extension dll (_barraopt.pyd) and python interface (barraopt.py) are located. For example:
   - If you are running 32-bit Python, then set `PYTHONPATH= C:\Program Files (x86)\MSCI Barra\Barra Optimizer\bin; C:\Program Files (x86)\MSCI Barra\Barra Optimizer\bin\ia32`
   - If you are running 64-bit Python, then set `PYTHONPATH= C:\Program Files (x86)\MSCI Barra\Barra Optimizer\bin; C:\Program Files (x86)\MSCI Barra\Barra Optimizer\bin\intel64`
2. Launch Python.
3. Import the Barra Optimizer module into Python: `>>> import barraopt`
4. If no errors are displayed, then the module has been imported successfully.
   - If you get the “ImportError: No module named barraopt” error, then check if the paths set in PYTHONPATH environment variable contain the _barraopt.pyd and barraopt.py files.

The interface is ready for use:

```
>>> ws = barraopt.CWorkSpace.CreateInstance()
>>> asset1 = ws.CreateAsset("A1")
>>> asset1.SetPrice(100)
>>> asset1.GetPrice()
100.0
```

For detailed information about the Python APIs, refer to Python API Reference Guide.

8 Using the Barra Optimizer XML

Barra Optimizer provides the flexibility of serializing optimization inputs into a readable XML format, which can be edited and parsed by OpenOpt executable, and invoke optimization. The XML schema is designed with the goal of reusability, such that the user can update a particular input and rerun optimization without having to rebuild the entire optimization profile. The XML format also allows for the possibility of building individual repositories for risk model data, asset data, and portfolios. The XML interface is easy to set up, provides additional features such as attribute and grouping schemes, and can be converted via OpenOpt executable into a binary format for improved performance and smaller file size.

The major schema elements are listed in the following table. For more information about each element, click on the element name.

WorkSpace	Data	Attribute
		Grouping_Scheme
		Transaction_Costs
		Attribute_Assignment
		Tax Lots
		Wash Sales
		Expected_Shortfall_Scenarios
		Symmetric_Matrix
		Data_File
	Portfolios	
	Risk_Models	
	Rebalance_Profiles	Utility
		Linear_Constraint
		Cash_Asset_Bound
		NonCash_Asset_Bound
		Asset_Bounds
		Asset_Bounds_By_Group

		Asset_Bound_With_Look_Through
		Factor_Constraint
		Factor_Constraint_By_Group
		Custom_Constraint
		Constraint_By_Group
		Beta_Constraint
		NonCash_Asset_Penalty
		Asset_Penalty_By_Group
		Asset_Penalty
		Asset_Free_Range_Penalty
		Asset_Trade_Size
		Cardinality_Constraint
		Cardinality Constraint By Group
		Threshold_Constraint
		Threshold Constraint By Group
		Threshold Constraint By Asset
		Transaction_Cost_Constraint
		Roundlot_Constraint
		Turnover_Constraint
		Turnover_Constraint_By_Group
		Turnover_Definition
		Leverage_Constraint
		Weighted_Total_Leverage_Constraint
		Risk_Constraint

		Risk_Constraint_By_Group
		Risk_Parity_Constraint
		Five_Ten_Forty_Rule
		General_Ratio_Constraint
		Group_Ratio_Constraint
		Quadratic_Constraint
		Constraint_Priority
		Penalty
		Free_Range_Penalty
		Portfolio_Concentration_Constraint
		Total_Active_Weight_Constraint
		Total_Active_Weight_Constraint_By_Group
		Multi-Period Optimization: Cross-Period Constraints
		General_PWLI_Constraint
		Overlap_Constraint
		Issuer Constraint
		Include_Futures
		Multi-Account Optimization: Cross-Account Constraints
		Tax Constraints
	Rebalance_Job	Rebalance_Profile_ID
		Initial_Portfolio_ID
		Universe_Portfolio_ID
		Reference_Portfolios

		Portfolio_Base_Value
		Cashflow_Weight
		Primary_Risk_Model_ID
		Secondary_Risk_Model_ID
		Attribute Assignment
		Optimization_Setting
	Rebalance_Result	
	Multi-Period Optimization	
	Multi-Account Optimization	

8.1 <WorkSpace>

WorkSpace is the root node of the XML schema and contains the <Data>, <Portfolios>, <Risk_Models>, <Rebalance_Profiles>, and <Rebalance_Job> elements.

- The <Data> element contains attribute and grouping scheme definitions, transaction costs, and additional asset-level data such as prices, alphas, roundlots, issuer IDs, and asset type assignments.
- The <Portfolios> element contains set of portfolios that represent the initial portfolio, multiple benchmark portfolios, and investment universe.
- The <Risk_Models> element contains a set of risk models. There is no limit on the number of risk model elements, but at most two risk models can be specified for an optimization job. The OpenOpt executable can be used to load Barra ModelsDirect flat files and convert them into XML format that represent the risk model element.
- The <Rebalance_Profiles> element contains a set of rebalance profiles. Each rebalance_profiles element consists of parameters in the objective function and user constraints. The attributes and grouping schemes can be used in conjunction with certain constraints for flexibility in specifying a group of assets or factors which the constraint applies to.
- The <Rebalance_Job> element ties the optimization inputs together and allows the user to pick specific pieces of data to run an optimization. The rebalance_job element also controls the type of optimization to run, such as utility maximization, risk or return target, or frontier optimization, and contains rebalance_result element, which organizes the optimization outputs into profile diagnostics, portfolio summary, asset details, and trade list information.

8.2 <Data>

8.2.1 <Attribute>

The <attribute> element contains the attribute_id and attribute_value_type attributes. The attribute_id is the unique identifier of the attribute element and attribute_value_type sets the data type of the attribute values, which can be text, integer, or double. The attribute element contains a list of elements, and each element is identified by the element_id and one of the text_value, double_value, or integer_value attribute. An attribute for asset prices is defined as shown:

```
<attribute attribute_id="##default##price" attribute_value_type="double">
      <element double_value="18.879" element_id="#BRAA221|BARRAID"/>
      <element double_value="15.971" element_id="#BRAA222|BARRAID"/>
</attribute>
```

The following table lists each attribute and its value type:

Attribute	Attribute Value Type	Element attribute tag	Element value description
Price	double	double_value	Asset price
Alpha	double	double_value	Asset alpha
Residual Alpha	double	double_value	Asset residual alpha
Round lot	integer	integer_value	Asset round lot size
Issuer ID	text	text_value	Asset issuer ID, used when 5/10/40 rule is set
Asset Type	integer	integer_value	Asset type: 1=Futures, 2=Currency, 3=Regular, 4=Composite, 5=Composite Futures
Fixed Buy/Sell Cost	double	double_value	Asset fixed transaction cost
Piecewise-Linear Buy/Sell Cost	double	double_value	Asset piecewise-linear transaction cost
Net Short Cost	double	double_value	Asset net cost on short position
Absolute Asset Bounds or Relative Asset Bounds to one benchmark	double	double_value	Asset upper/lower bounds

Attribute	Attribute Value Type	Element attribute tag	Element value description
Asset Relative Bounds to multiple benchmarks	text	text_value	Formatted expression for asset relative bounds
Covariance Term Weight	double	double_value	Diagonal weight matrix in the covariance term
Custom Constraint Attribute	double	double_value	Asset attribute values to control portfolio-level exposure
ConstraintByGroup Attribute	double	double_value	Asset coefficient values for group constraint
Weighted Total Leverage Constraint Long/Short Side Attribute	double	double_value	Asset coefficient values for long/short side constraint
Fitted Cost	double	double_value	Fitted cost for the asset nonlinear transaction cost function
Exponent	double	double_value	Exponent for the asset nonlinear transaction cost function
Traded amount	double	double_value	Traded amount for the asset nonlinear transaction cost function
Upside/Downside Fixed Holding Cost	double	double_value	Upside/Downside fixed holding cost
Reference Weight for Fixed Holding Cost	double	double_value	Reference weight for fixed holding cost

8.2.2 <Grouping_Scheme>

The <grouping_scheme> element groups assets based on their alphanumeric attributes. In the grouping_scheme element, the grouping_scheme_id uniquely identifies the grouping scheme, and contains a list of group elements. Each group element is identified by the group_id attribute and contains a list of values that corresponds to the attribute values that have to be grouped.

For example, you can define an attribute that contains a list of assets and their local markets, and a grouping scheme to group multiple local markets into Europe, North America, and Asia. You can then control the weight of a specific group in the portfolio by setting a group constraint. The following attribute and grouping scheme definitions illustrate this.

Define an attribute for local markets:

```
   <attribute attribute_id="LocalMarket" attribute_value_type="text">
      <element text_value="US" element_id="MSCI"/>
      <element text_value="UK" element_id="HSBA.L"/>
   </attribute>
```

Define a grouping scheme for local markets:

```
<grouping_scheme grouping_scheme_id="LocalMarket_GroupingScheme">
      <group group_id="North America">
        <value>US</value>
        <value>Canada</value>
      </group>
      <group group_id="Europe">
        <value>UK</value>
        <value>Germany </value>
      </group>
   </grouping_scheme>
```

For information about applying the grouping scheme to group constraint, see the <Constraint_By_Group> section.

Grouping scheme can also be used to group a list of factors, and applied to factor and risk constraints. For example, to group the USE3L oil services and oil refining factors into the same group:

```
   <grouping_scheme grouping_scheme_id="Factor_GroupingScheme">
      <group group_id="Oil Industry Factors">
        <value>USE3L_OILREF</value>
        <value>USE3L_OILSVCS</value>
      </group>
   </grouping_scheme>
```

For information about applying the factor grouping scheme to the factor constraint, see the <Factor_Constraint_By_Group> section.

8.2.3 <Transaction_Costs>

To apply the asset fixed transaction cost to an optimization, first define an attribute for buy and sell costs, and set the attribute ID in fixed_buy_cost_attribute_id and fixed_sell_cost_attribute_id:

```
<attribute attribute_id="FixedBuyCost" attribute_value_type="double">
      <element double_value="0.01" element_id="MSCI"/>
      <element double_value=”0.01" element_id="HSBA.L"/>
```

```
   </attribute>
   <transaction_costs>
      <fixed_buy_cost_attribute_id>FixedBuyCost</fixed_buy_cost_attribute_id>
   </transaction_costs>
```

The piecewise-linear transaction cost is set in the piecewise_linear_buy_cost and piecewise_linear_sell_cost elements. Each element contains an asset_id attribute and a list of cost elements that have the slope and breakpoint attributes:

```
    <transaction_costs>
      <piecewise_linear_buy_cost asset_id="USA11I1">
        <cost break_point="10000" slope="0.00283368"/>
        <cost break_point="1e+010" slope="0.00383368"/>
      </piecewise_linear_buy_cost>
</transaction_costs>
```

The profile-level nonlinear transaction cost is set in nonlinear_transaction_cost element, which has fitted_cost, exponent, and traded_amount attributes:

```
    <transaction_costs>
      <nonlinear_transaction_cost exponent="1.1" fitted_cost="1e-005"
traded_amount="0.01"/>
    </transaction_costs>
```

To set the asset nonlinear transaction cost, first define the attribute for fitted cost, and optionally define the attributes for exponent and traded amount. The attribute IDs are set in asset_nonlinear_transaction_cost element, which has fitted_cost_attribute_id, exponent_attribute_id, and traded_amount_attribute_id attributes:

```
    <attribute attribute_id="FittedCost" attribute_value_type="double">
      <element double_value="5e-005" element_id="USA11I1"/>
      <element double_value="2.5e-005" element_id="USA13Y1"/>
    </attribute>
    <attribute attribute_id="Exponent" attribute_value_type="double">
      <element double_value="1.1" element_id="USA11I1"/>
      <element double_value="0.55" element_id="USA13Y1"/>
    </attribute>
    <attribute attribute_id="TradedAmount" attribute_value_type="double">
      <element double_value="0.01" element_id="USA11I1"/>
      <element double_value="0.005" element_id="USA13Y1"/>
    </attribute>
    <transaction_costs>
      <asset_nonlinear_transaction_cost exponent_attribute_id="Exponent"
fitted_cost_attribute_id=" FittedCost" traded_amount_attribute_id="TradedAmount"/>
    </transaction_costs>
```

8.2.4 <Attribute_Assignment>

The <attribute_assignment> element defines which attribute to use for the optimization job, and can be set under <Data> or <Rebalance_Job>. The attribute assignment set under <Rebalance_Job> is given higher priority. It has the following elements:

Attribute	Attribute_Assignment Element
Price	price_attribute_id
Alpha	alpha_attribute_id
Residual Alpha	residual_alpha_weight_attribute_id
Round lot	round_lot_size_attribute_id
Issuer ID	issuer_attribute_id
Asset Type	asset_type_attribute_id
Upside Fixed Holding Cost	upside_fixedHC_attribute_id
Downside Fixed Holding Cost	downside_fixedHC_attribute_id
Reference Weight for Fixed Holding Cost	reference_weight_fixedHC_attribute_id

To identify a particular asset as cash, set the cash_asset_id attribute to the cash asset ID:

```
<attribute_assignment cash_asset_id="CASH"/>
```

8.2.5 <Tax_Lots>

For tax-aware optimization, tax lots can be assigned in the <tax_lots> element. Each child <tax_lot> element represents a tax lot and has the following attributes:

Attribute	Attribute type	Description
asset_id	string	Asset ID
age	integer	The holding period in number of days
cost_basis	double	Cost basis per share
shares	double	Number of shares in the lot
trading_rule	enum	Trading rule of the lot. Optional, default is "default".

Attribute	Attribute type	Description
used_to_disqualify	boolean	True if already used to disallow loss on a previous sale. Optional, default is “false”.
tax_free	boolean	True if the lot is tax free. Optional, default is “false”.

Possible values of the trading_rule attribute:

Trading_rule Attribute	Trading Rule
default	Allow all transactions
keepLot	Keep the lot from selling/covering
sellLot	Must sell/cover the whole tax lot

The following example illustrates the usage of this element:

```
    <tax_lots>
      <tax_lot asset_id="USA11I1" age="937" cost_basis="28.22" shares="50"/>
      <tax_lot asset_id="USA11I1" age="832" cost_basis="25.37" shares="50"/>
      <tax_lot asset_id="USA13Y1" age="1941" cost_basis="15.19" shares="20"
tax_free="true"/>
      <tax_lot asset_id="USA13Y1" age="295" cost_basis="18.9" shares="35"
trading_rule="keepLot"/>
      <tax_lot asset_id="USA1LI1" age="301" cost_basis="54.12" shares="-13"/>
      <tax_lot asset_id="USA1TY1" age="20" cost_basis="43.12" shares="25"
used_to_disqualify="true"/>
    </tax_lots>
```

Here, the first two lots are purchases of the same shares at different times, the third lot can be sold without paying taxes, the fourth one cannot be sold, the fifth lot is a short sell, and the last one has already been sold out in a wash sale.

If tax lots are assigned, then the asset weights and the portfolio value are calculated using these tax lots and used in the optimization.

For more information about tax lots, see the Tax Lot Trading Rules section.

8.2.6 <Wash_Sales>

Tax-aware optimization needs the records of the recent purchases for applying the Wash Sale rules. Such data can be added to the <data> section in a <wash_sales> element. Each child element describes a purchase within the wash sale period by the following attributes:

Attribute	Attribute type	Description
asset_id	string	Asset ID
age	integer	The holding period in number of days
loss_per_share	double	Loss per share
shares	double	Number of shares in the lot
used_to_disqualify	boolean	True if the loss has already been disqualified due to a previous purchase. Optional, default is “false”.

The following example illustrates the usage of the element:

```
    <wash_sales>
      <wash_sale asset_id="USA11I1" age="9" loss_per_share="14.71" shares="77"/>
      <wash_sale asset_id="USA11I1" age="3" loss_per_share="15.23" shares="12"/>
      <wash_sale asset_id="USA1TY1" age="10" loss_per_share="1.51" shares="25"
used_to_disqualify="true"/>
      <wash_sale asset_id="USA1LI1" age="2" loss_per_share="179.05" shares="-13"/>
    </wash_sales>
```

8.2.7 <Expected_Shortfall_Scenarios>

Expected Shortfall optimizations need historical records of returns. The <expected_shortfall_scenarios> element collects the IDs of <attribute> elements which hold such returns. The attributes either map asset IDs or factor IDs to their returns.

The following example illustrates the usage of the element:

```
<data>
  <attribute attribute_id="scen-1" attribute_value_type="double">
    <element element_id="USA11I1" double_value="-0.159" />
    <element element_id="USA13Y1" double_value="-0.296" />
    …
  </attribute>
  <attribute attribute_id="scen-2" attribute_value_type="double">
    <element element_id="USA11I1" double_value="0.066" />
    <element element_id="USA13Y1" double_value="-0.083" />
    …
  <attribute>
  …
  <expected_shortfall_scenarios scenarios_id="Scenarios">
```

```
    <scenario_return_attribute_id>scen-1</scenario_return_attribute_id>
    <scenario_return_attribute_id>scen-2</scenario_return_attribute_id>
    …
  </expected_shortfall_scenarios>
</data>
```

8.2.8 <Symmetric_Matrix>

The <symmetric_matrix> element represents a mapping of ID pairs to double values, where the pair (id1, id2) is mapped to the same value as (id2, id1).

The XML element enumerates the IDs of the rows/columns of the matrix, and then contains the elements of the upper part of the matrix in row-major order (i.e., $a_{11}, a_{12}, \dots a_{1n}, a_{22}, a_{23}, .. a_{2n}, \dots a_{nn}$).

For example, the following element

```
<symmetric_matrix id="Q">
  <row_id>USA11I1</row_id>
  <row_id>USA13Y1</row_id>
  <row_id>USA1LI1</row_id>
  <v>1</v>
  <v>2</v>
  <v>3</v>
  <v>4</v>
  <v>5</v>
  <v>6</v>
</symmetric_matrix>
```

encodes the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}.$$

In quadratic constraints, such matrices are used for specifying the quadratic coefficients of the constraint form.

8.2.9 <Data_File> Attribute

The <data_file> attribute can be used to reference external XML file containing the Data section. For example, if the XML file for the data section is stored in a separate directory from the WorkSpace XML:

```
<data data_file="C:\Optimizer\20140201\data.xml"/>
```

Alternatively, the data_file attribute can be used to reference a CSV-formatted file containing asset attribute data:

```
<data data_file="C:\Optimizer\20140201\asset_data.csv"/>
```

For information about the CSV-formatted file, see the Add Assets into Workspace section.

8.3 <Portfolios>

The <portfolios> element contains a list of portfolio elements, and each portfolio element has a portfolio_id attribute, which is a unique identifier for the portfolio, and a list of holding elements. Each holding element has asset_id and amount attribute, which defines the weight of each asset in the portfolio. For the investable universe portfolio, set the amount for all assets to zero.

```
<portfolios>
    <portfolio portfolio_id="Initial Portfolio">
      <holding amount="0.3" asset_id="USA11I1"/>
      <holding amount="0.7" asset_id="USA13Y1"/>
</portfolio>
</portfolios>
```

8.4 <Risk_Models>

The <risk_models> element contains the definition of each risk model element. A risk model element has a risk_model_id attribute, which specifies the name of the risk model and an optional risk_model_type attribute, which may improve the efficiency of optimization.

The factor covariance matrix is defined by factor_covariances element, each factor_covariance element has factor_id1, factor_id2, and value attributes to set the covariance between two factors. For improved performance and reduced file size, there is no need to set the covariance if it is zero.

The factor_exposures element has the factor exposures for a list of assets. Inside the factor_exposure element, the exposure elements specify the set of factor ID and exposure value for a given asset. For improved performance, if an asset has zero exposure to a factor, the exposure element does not need to be set.

The specific risk matrix is set in specific_variances element that has the diagonal values of the matrix, and specific_covariances element that contains the specific covariances between two assets.

To define the factor block structure of the factor covariance matrix, add the factor_block elements with unique factor_block_id and the group of factor_id elements associated with the factor block. The factor_blocks element is optional and speeds up optimization by taking advantage of block sparsity of the matrix.

```
<risk_models>
    <riskmodel risk_model_id="GEM" risk_model_type="equity">
       <factor_covariances>
          <covariances factor_id1="Factor_1A" factor_id2="Factor_1A" value="0.00294054"/>
          <covariances factor_id1="Factor_1A" factor_id2="Factor_1B" value="-
0.000211062"/>
       </factor_covariances>
       <factor_exposures>
          <asset_factor_exposure asset_id="USA11I1">
            <exposure factor_id="Factor_1A" value="0.144"/>
            <exposure factor_id="Factor_1B" value="-0.666"/>
          </asset_factor_exposure>
       </factor_exposures>
```

```
        <specific_variances>
           <specific_variance asset_id="USA11I1" value="0.032472"/>
           <specific_variance asset_id="USA13Y1" value="0.0347077"/>
        </specific_variances>
        <factor_blocks>
          <factor_block factor_block_id="A">
            <factor_id>Factor_1A</factor_id>
            <factor_id>Factor_2A</factor_id>
          </factor_block>
        </factor_blocks>
     </riskmodel>
</risk_models>
```

8.5 <Rebalance_Profiles>

Multiple rebalance profiles can be set up and the <rebalance_job> attribute dictates that rebalance profile to use for an optimization. A rebalance profile contains the parameters for the objective function and any constraints on the optimization.

A constraint has attributes such as constraint ID that uniquely identifies a constraint, lower and upper bound, relative bound mode, a benchmark portfolio ID, whether the constraint is a soft constraint, and a penalty element. Most constraint bounds can be set as relative to a benchmark portfolio.

8.5.1 <Utility>

The <utility> element has utility_type attribute that sets the type of objective function, and allows you to set the following parameters through its child elements:

Element	Description
alpha_term	Multiplier of the linear alpha term
residual_alpha_term	Multiplier of the quadratic residual alpha penalty term
penalty_term	Constraint penalty term multiplier
risk_free_rate	Risk-free rate; applicable only for the Sharpe ratio optimization
transaction_cost_term	Transaction cost term multiplier
short_rebate_term	Short rebate term
risk_term	Quadratic risk term associated with a risk model
covariance_term	Additional covariance term
fixed_holding_cost_term	Fixed holding cost term
utility_scalar	Scalar for utility function for the current account in multi-account optimization

Element	Description
joint_market_impact_term	Joint market impact term for multi-account optimization
tax_term	Multiplier of the tax term
linear_term	Additional linear terms
loss_benefit_term	Multiplier of the loss benefit term
expected_shortfall_term	Multiplier of the expected shortfall term

A risk term has parameters for common factor and specific risk aversions, a benchmark portfolio ID, and whether the term is associated with the primary or secondary risk model.

At most two risk terms can be specified as two risk models are supported for an optimization job. The following code defines multiple risk terms for primary and secondary risk models without a benchmark:

```
     <utility utility_type="quadratic">
        <risk_term common_factor_risk_aversion="0.0075" specific_risk_aversion="0.0075"/>
        <risk_term common_factor_risk_aversion="0.0075" is_primary_risk_model="false"
specific_risk_aversion="0.0075"/>
     </utility>
```

Additional covariance terms can be added to the objective function. If the term has diagonal weight matrix, an attribute for the weights can be created using the <Attribute> element. For more information, see the <Attribute> section. The term's weight_matrix_attribute_id can be set to the attribute ID. The following defines the additional covariance term for the primary risk model with a benchmark:

```
  <data>
    <attribute attribute_id="CovTerm_Weight_Attribute_1" attribute_value_type="double">
      <element double_value="1" element_id="USA11I1"/>
      <element double_value="1" element_id="USA13Y1"/>
   </attribute>
  </data>
  <rebalance_profiles>
    <rebalance_profile rebalance_profile_id="Case 16a">
      <utility utility_type="quadratic">
        <covariance_term benchmark_portfolio_id="Benchmark" covariance_term="0.0075"
covariance_term_type="wxfxw" weight_matrix_attribute_id="CovTerm_Weight_Attribute_1"/>
      </utility>
    </rebalance_profile>
  </rebalance_profiles>
```

For multi-account optimization, the joint_market_impact_term element has joint_market_impact_multiplier:

```
  <joint_market_impact_term joint_market_impact_multiplier="1" />
```

8.5.2 <Linear_Constraint>

Linear constraints allow setting asset bounds, factor constraint, custom constraint, group constraint, and beta constraint. It has the transaction_type attribute that sets the type of transaction strategy to be used in the optimization, and the enable_crossover attribute that allows asset position to change from short to long side and from long to short side.

8.5.2.1 <Cash_Asset_Bound>

The cash asset bound sets a holding constraint on the cash asset in the portfolio. To define the cash asset weight to be in between 5% and 10% of portfolio:

```
        <cash_asset_bound lower_bound="0.05" lower_bound_mode="absolute" upper_bound="0.1"
upper_bound_mode="absolute"/>
```

8.5.2.2 <NonCash_Asset_Bound>

The noncash bound sets a general holding constraint on all noncash assets in the investable universe. For example, you might be tracking a model portfolio and want to be sure you hold positions in all its assets. You can specify upper and lower bounds for all non-cash assets in the optimal portfolio. If you set a lower bound, the optimizer must include these assets and weights, or it will not produce an optimal portfolio. The following limits the weight of any asset (excluding cash) in the optimal portfolio to 30%:

```
        <noncash_asset_bound upper_bound="0.3" upper_bound_mode="absolute"/>
```

The bounds can be set in addition to individual asset bounds or asset bounds by group. If conflicting bounds are defined for the same asset, a more restrictive bound is determined based on the asset_bound_priority attribute. The lower priority bound will be relaxed to where it is no longer conflicting with the higher priority bound.

For example, resolving the following bounds on the same asset results in the upper bound on the lower priority noncash assets being relaxed to the lower bound on the higher priority asset:

Constraint	asset_bound_priority	lower_bound	upper_bound
asset_bound	highest	0.1	0.2
assest_bound_by_group	medium	0.05	0.15
noncash_asset_bound	lowest	0	0.05
Final Asset Bound		0.1	0.1

8.5.2.3 <Asset_Bounds>

Asset bounds allow you to set asset-level bound for specific assets in the optimal portfolio. There are two ways to set the bounds depending on the number of benchmarks used if you have relative bounds defined.

If all bounds are absolute, or if all bounds are relative and only one benchmark is used, first create a double type attribute with the bound values:

```
<attribute attribute_id="asset_upper_bound_attribute" attribute_value_type="double">
  <element element_id="USA11I1" double_value="0.03"/>
</attribute>
```

To limit the weight of USA11I1 to no more than its weight in the benchmark plus 3%, set the bound mode, attribute ID and reference portfolio ID in asset_bound element:

```
    <asset_bounds upper_bound_attribute_id="asset_upper_bound_attribute"
upper_bound_mode="plus" reference_portfolio_id="Benchmark"/>
```

To limit the weight of USA11I1 to no more than 3% in the optimal portfolio, set the attribute ID:

```
    <asset_bounds upper_bound_attribute_id="asset_upper_bound_attribute"/>
```

If bounds consist of both absolute and relative bounds, or if bounds are relative to multiple benchmarks, first create a text type attribute with the bound expression in the following format

[prefix] [operator] [constant]

where,

[prefix] is p, b, b2, b3, b4, or b5

where,

- p = initial portfolio
- b = primary benchmark
- b2 = secondary benchmark
- [operator] is a standard math operator, such as +, -, *
- [constant] is a numeric value (enter percentages as decimal values)

You can also enter absolute numbers. For example, to limit the weight of USA11I1 to no more than its weight in the benchmark plus 3%:

```
<attribute attribute_id="asset_upper_bound_attribute" attribute_value_type="text">
  <element element_id="USA11I1" text_value="b + 0.03"/>
</attribute>
```

The reference_portfolios element under <RebalanceJob> allows you to set the reference portfolio ID based on the following mapping:

Bound Expression Prefix	Reference_Portfolios Attributes
b	primary_benchmark
b2	secondary_benchmark
b3	b3

Bound Expression Prefix	Reference_Portfolios Attributes
b4	b4
b5	b5

To set the benchmark portfolio ID to “Benchmark” in the above example:

```
  <rebalance_job>
<reference_portfolios primary_benchmark="Benchmark"/>
  </rebalance_job>
```

Set the asset bound as:

```
  <asset_bounds upper_bound_attribute_id="asset_upper_bound_attribute"/>
```

8.5.2.4 <Asset_Bounds_By_Group>

Asset bounds by group constraint allows you to specify upper and lower bounds for groups of assets in the investable universe. If you set a lower bound, the optimal portfolio must include all assets within a group at the given weight. This can be a way to ensure that you do not sell particular assets. You can also force a sell on an asset during rebalance by setting an upper bound of zero. You can use this constraint more than once in order to specify multiple grouping schemes. To use this constraint, an attribute and grouping scheme must be created first. For more information, see the <Grouping_Scheme> section.

The following sets the upper bound for all the assets in the North America group to be 30% from the example in the <Grouping_Scheme> section:

```
    <asset_bounds_by_group grouping_attribute_id="LocalMarket"
grouping_scheme_id="LocalMarket_GroupingScheme" group_id="North America"
upper_bound="0.3"/>
```

8.5.2.5 <Asset_Bound_With_Look_Through>

Asset bound with look through constraint allows you to define a lower and upper bound of an asset weight with composite look-through capability. With this constraint you can limit the total weight of the asset including contributions from composite assets.

For example, to limit the weight of the USA11I1 equity to 1%:

```
    <asset_bound_with_look_through assetID="USA11I1" lower_bound="0" upper_bound="0.01”/>
```

8.5.2.6 <Factor_Constraint>

Factor constraint allows you to set bounds for portfolio exposure to specific factors in the risk model. To set the factor constraint for the secondary risk model, set the attribute is_primary_risk_model to false. The by_side attribute allows you to apply the constraint to long, short, or all positions. For example, to limit the exposure to primary risk model’s factor FACTOR_1A in the optimal portfolio to 1%:

```
    <factor_constraint factor_id="Factor_1A" lower_bound="0" lower_bound_mode="absolute"
upper_bound="0.01" upper_bound_mode="absolute"/>
```

8.5.2.7 <Factor_Constraint_By_Group>

Factor constraint by group allows you to define bounds for group-level factor exposure from the primary risk model. You must define a factor grouping scheme first. For more information, see the <Grouping_Scheme> section.

The following limits the group exposure of “Oil Industry Factors” to no more than 10%:

```
    <factor_constraint_by_group grouping_scheme_id="Factor_GroupingScheme" group_id="Oil
Industry Factors" upper_bound="0.1"/>
```

8.5.2.8 <Custom_Constraint>

Custom constraints allow you to define an optimal portfolio by setting minimum and maximum values for portfolio-level exposure to asset attributes. The by_side attribute sets if the bound should apply to long, short, or all positions in the optimal portfolio. For custom constraints, you must first import your asset attributes. For more information, see the <Attribute> section.

8.5.2.9 <Constraint_By_Group>

Group constraints enable you to set bounds for group-level exposures. The by_side attribute sets the bound for long, short, or all positions in the optimal portfolio.

You can define groups for an alphanumeric grouping schemes such as GICS or user-defined industry groups. A grouping scheme is required to be created first. For more information, see the <Grouping_Scheme> section.

An attribute for asset coefficients is also required. For more information, see the <Attribute> section.

8.5.2.10 <Beta_Constraint>

Beta constraint specifies the desired market exposure of the optimal portfolio with respect to the benchmark set in the primary risk term. For example, to set the beta to a range from 0.9 to 1.0:

```
    <beta_constraint lower_bound="0.9" upper_bound="1"/>
```

8.5.2.11 <Noncash_Asset_Penalty>

Noncash asset penalty sets the asset-level penalty for all assets in the universe except cash. The penalty targets, lower and upper, are absolute weights. For example:

```
    <noncash_asset_penalty target="0.1" lower="0.05" upper="0.15"/>
```

8.5.2.12 <Expected_Shortfall_Constraint>

Expected shortfall constraint specifies lower and upper bounds for the expected shortfall of the optimal portfolio. For example:

```
    <expected_shortfall_constraint constraint_info_id="ES" lower_bound="0.1"
upper_bound="0.3"/>
```

8.5.2.13 <Target_Mean_Return_Constraint>

The target mean return constraint specifies lower and upper bounds for the target return of the optimal portfolio. The target return of the portfolio is calculated from the target mean returns of assets or factors specified for expected shortfall. For example:

```
    <target_mean_return_constraint_constraint_info_id="TMR" lower_bound="0.03"/>
```

8.5.3 <Asset_Penalty_By_Group>

Asset penalty by group sets the asset-level penalty for a group of assets in the universe. If noncash asset penalty is also set, asset penalty by group has higher priority for the assets in the group.

An attribute and grouping scheme must be created first. For more information, see the <Grouping_Scheme> section.

For example, to set asset penalty for all assets in the North America group:

```
    <asset_penalty_by_group grouping_attribute_id="LocalMarket"
grouping_scheme_id="LocalMarket_GroupingScheme" group_id="North America" target="0.2"
upper="0.25" lower="0.15"/>
```

8.5.3.1 <Asset_Penalty>

Asset penalty sets asset-level penalty for specific assets. It has higher priority for the same assets contained in asset penalty by group and noncash asset penalty. The penalty targets, lower and upper, are absolute weights. For benchmark-relative asset penalty, the slack values need to be converted to absolute weights by shifting the benchmark weights by the relative penalty.

For example, to set asset penalty as absolute weights, with target weight of 10% and 1% disutility at weight of 5% or 15%:

```
    <asset_penalty asset_id="USA11I1" target="0.1" lower="0.05" upper="0.15"/>
```

To set a penalty of ±5% relative to benchmark, first find out the weight of the asset in the benchmark (for example, 7%), then shift the benchmark weight by the relative value (lower=7% minus 5%, upper=7% plus 5%):

```
    <asset_penalty asset_id="USA11I1" target="0.07" lower="0.02" upper="0.12"/>
```

8.5.3.2 <Asset_Free_Range_Penalty>

Asset free range linear penalty penalizes assets whose bounds are outside of the specified free range. The free range penalty bound is in units of absolute weights. For benchmark-relative asset free range penalty, the slack values need to be converted to absolute weights by shifting the benchmark weights by the relative penalty.

For example, to set asset penalty as absolute weights with free range of 0.05 to 0.1 and penalty rate of 0.01 on both sides:

```
    <asset_free_range_penalty asset_id="USA11I1" target_low="0.05" target_high="0.1"
downside_slope="-0.01" upside_slope="0.01"/>
```

8.5.3.3 <Asset_Trade_Size>

Asset trade size constraints specify a maximum trade size in dollar amount on the asset-level. This constraint can be set for single portfolio as well as multi-account and multi-period optimizations.

- To set the constraint for a specific account or period include the constraint under that account's or period's rebalance profile
- To set the constraint for all accounts or periods include the constraint under the rebalance profile with account_id="-1" or period_id=-1

To specify the maximum trade size values, first create a double type attribute under the <data> section. For example, the following attribute sets trade size of 10 million for asset USA11I1:

```
    <attribute attribute_id="asset_max_trade_size_attribute"
attribute_value_type="double">
      <element element_id="USA11I1" double_value="10000000"/>
    </attribute>
```

Then set the trade size constraint as:

```
    <asset_trade_size upper_bound_attribute_id="asset_max_trade_size_attribute"/>
```

8.5.4 <Cardinality_Constraint>

Cardinality constraints set the minimum and maximum number of long, short, or all asset or trades in the optimal portfolio. The constraint can be either mandatory or soft. Soft constraints can be relaxed if the problem is otherwise infeasible. The attributes penalty_per_extra_asset and penalty_per_extra_trade are used to set penalty on violations of paring constraints. The following limits the number of assets the optimal portfolio can hold to 6:

```
    <cardinality_constraint>
      <num_assets is_max_soft="false" maximum="6"/>
    </cardinality_constraint>
```

8.5.5 <Cardinality_Constraint_By_Group>

Cardinality constraints by group set the minimum and maximum number of long, short, or all asset or trades for a specific group of assets.

For this, first create a grouping scheme. For more information, see the <Grouping_Scheme> section.

For example, the following limits the number of assets in the "Information Technology" group to 20 assets and sets the constraint as soft:

```
    <cardinality_constraint_by_group>
        <num_assets_by_group group_id="Information Technology"
grouping_attribute_id="GICS_SECTOR" grouping_scheme_id="GICS_SECTOR_GroupingScheme"
is_max_soft="true" maximum="20"/>
    </cardinality_constraint_by_group>
```

8.5.6 <Threshold_Constraint>

Threshold constraints specify a minimum level for long and short side holding and transaction size, or a minimum level for buy and sell trades. By setting the "allow_closeout" attribute to true, optimizer may close out a position even if the transaction size is below the minimum threshold. The constraint can be either mandatory or soft. Soft constraints can be relaxed if the problem is otherwise infeasible. The attributes penalty_per_unit_below_holding_threshold and penalty_per_unit_below_trade_threshold are used to set penalty on threshold violations. If the enable_grandfather_rule attribute is set to true, then the grandfather rule is enabled for minimum holding level threshold constraint. The following sets a minimum holding threshold of 4% for both long and short sides, and a minimum trade size of 2% of initial portfolio for both long and short sides:

```
<threshold_constraint>
  <long_side_holding_level is_min_soft="false" minimum="0.04"/>
  <short_side_holding_level is_min_soft="false" minimum="0.04"/>
  <long_side_tranx_level is_min_soft="false" minimum="0.02"/>
  <short_side_tranx_level is_min_soft="false" minimum="0.02"/>
</threshold_constraint>
```

8.5.7 <Threshold_Constraint_By_Group>

Threshold constraint by group specifies a minimum level for long and short side holding and transaction size, or a minimum level for buy and sell trades, for a group of assets. The following sets a minimum long side holding threshold of 1% for all assets in the “Information Technology” group:

```
      <threshold_constraint_by_group>
        <long_side_holding_level_by_group group_id="Information Technology"
grouping_attribute_id="GICS_SECTOR" grouping_scheme_id="GICS_SECTOR_GroupingScheme"
is_min_soft="false" minimum="0.01"/>
      </threshold_constraint_by_group>
```

8.5.8 <Threshold_Constraint_By_Asset>

Threshold constraint by asset specifies a minimum level for long and short side holding and transaction size for a particular asset. The following sets a minimum long side holding threshold of 10% for the asset “USA13Y1”:

```
      <threshold_constraint_by_asset>
        <long_side_holding_level_by_asset asset_id="USA13Y1" is_min_soft="false"
minimum="0.1"/>
      </threshold_constraint_by_asset>
```

8.5.9 <Transaction_Cost_Constraint>

Transaction cost constraint sets a limit on the total transaction cost as a percentage of portfolio value. The constraint can be either mandatory or soft. For information about how to set asset transaction costs, see the <Transaction_Cost> section.

The following limits the transaction cost to be no more than 0.05% of the portfolio:

```
<transaction_cost_constraint is_soft="false" upper_bound="0.0005"/>
```

8.5.10 <Roundlot_Constraint>

Roundlot constraint ensures that the optimal portfolio consists of only roundlotted trades. You are required to set asset prices and roundlot sizes by creating an attribute. For more information, see the <Attribute> section.

The portfolio base value is also required. For more information, see the <Rebalance_Job> section.

The allow_odd_lot_closeout attribute allows reducing an odd lot position to zero, and the constraint may be mandatory or soft. The following enables the roundlot constraint disallowing oddlot close out:

```
<roundlot_constraint allow_odd_lot_closeout="false" is_soft="false"/>
```

8.5.11 <Turnover_Constraint>

Turnover constraint limits the maximum turnover levels for short, long, buys, sells, or all positions of the optimal portfolio. This considers all transactions, including buys, sells, and short sells. The following sets a long side turnover soft constraint with maximum turnover of 20%:

```
<turnover_constraint by_side="long" is_soft="true" upper_bound="0.2" />
```

8.5.12 <Turnover_Constraint_By_Group>

Turnover constraint by group limits the maximum turnover levels for a group of assets and supports net, long, short positions, as well as buy and sell transactions. You will first need to create a grouping scheme. For more information, see the <Grouping_Scheme> section. The following sets maximum turnover of 3% for group of assets in the “Information Technology” group:

```
<turnover_constraint_by_group by_side="net" group_id="Information Technology"
grouping_attribute_id="GICS_SECTOR" grouping_scheme_id="GICS_SECTOR_GroupingScheme"
upper_bound="0.03"/>
```

8.5.13 <Turnover_Definition>

Turnover definition contains the attributes use_portfolio_base_value and exclude_cash that controls how turnover is calculated for the turnover constraint. If the use_portfolio_base_value attribute is set to true, turnover is measured as a percentage of the portfolio base value. If use_portfolio_base_value attribute is set to false, the default turnover base value is used, calculated as Long Equity + |Short Equity| + |Change in Cash Position| + Cash Flow. If exclude_cash is set to true, then change in cash position and cashflow are excluded from the numerator in the definition of overall turnover when turnover constraint is set. The following uses the default turnover base value and excludes cash from the turnover calculation:

```
<turnover_definition exclude_cash="true" use_portfolio_base_value="false"/>
```

8.5.14 <Leverage_Constraint>

Leverage constraints are for long-short rebalances. They allow you to limit the amount of debt used to acquire additional assets. With the by_side attribute, you can constrain the use of leverage on the long side, the short side, the total portfolio, and two ratios between short and long sides. The leverage ratios do not include cash. The constraint can be mandatory or soft, and if the no_change attribute is set to true, the lower and upper bound will be set to the leverage value of the reference portfolio during optimization.

The following table lists the by_side attribute and the respective type of leverage constraint:

By_side Attribute	Type of Leverage Constraint
long	Leverage constraint for the long side of the optimal portfolio
short	Leverage constraint for the short side of the optimal portfolio
total	Leverage constraint for the optimal portfolio as a whole
short_to_long	A ratio of leverage in short and long positions in the optimal portfolio
net_to_total	A ratio of net leverage to total leverage in the optimal portfolio

To set a leverage constraint for 130/30 strategy with long position up to 130% and short positions up to 30% of the portfolio base value:

```
   <leverage_constraint by_side="long" is_soft="false" lower_bound="1"
lower_bound_mode="absolute" no_change="false" upper_bound="1.3"
upper_bound_mode="absolute"/>
   <leverage_constraint by_side="short" is_soft="false" lower_bound="-0.3"
lower_bound_mode="absolute" no_change="false" upper_bound="0"
upper_bound_mode="absolute"/>
```

8.5.15 <Weighted_Total_Leverage_Constraint>

Weighted total leverage constraint provides more flexibility than the total leverage constraint and allows you to specify asset-level coefficients for the long and short sides.

Attributes for the asset long and short side coefficients are required. For more information, see the <Attribute> section.

The following sets the asset coefficient attributes for long and short sides:

```
   <attribute attribute_id="Hedge_weighted_total_leverage0_Long"
attribute_value_type="double">
      <element double_value="1" element_id="USA11I1"/>
      <element double_value="1" element_id="USA13Y1"/>
   </attribute>
   <attribute attribute_id="Hedge_weighted_total_leverage0_Short"
attribute_value_type="double">
      <element double_value="1" element_id="USA11I1"/>
      <element double_value="1" element_id="USA13Y1"/>
   </attribute>
```

The following sets a weighted total leverage range between 100% and 130% of portfolio base value:

```
   <weighted_total_leverage_constraint constraint_info_id="Hedge_weighted_total_leverage0"
long_side_attribute_id="Hedge_weighted_total_leverage0_Long" lower_bound="1"
lower_bound_mode="plus" short_side_attribute_id="Hedge_weighted_total_leverage0_Short"
upper_bound="1.3">
```

```
</weighted_total_leverage_constraint>
```

8.5.16 <Risk_Constraint>

Risk constraint sets an upper bound on the level of total or active risk for the optimal portfolio. You can limit total risk, factor risk, or specific risk with the risk_source_type attribute, and the is_primary_risk_model attribute toggles which risk model to use. If the relative_risk attribute is set to true, the upper bound limits the contribution of a particular risk source to the portfolio's total risk. If the reference_portfolio_id is set, the risk constraint limits active risk of the optimal portfolio. The constraint can be either soft or mandatory.

If the `is_by_asset` attribute is set to true, then the total risk from each asset is constrained separately. In this case users can specify both lower and upper bounds.

The following limits the total active risk in the secondary risk model to 10% in the optimal portfolio:

```
      <risk_constraint is_primary_risk_model="false" is_relative_risk="false"
is_soft="false" reference_portfolio_id="Benchmark2" risk_source_type="totalRisk"
upper_bound="0.1"/>
```

The following specifies a risk constraint using additive definition:

```
       <risk_constraint is_primary_risk_model="true" is_additive_definition="true"
is_relative_risk="false" is_soft="false" reference_portfolio_id="Benchmark2"
risk_source_type="totalRisk" upper_bound="0.1"/>
```

8.5.17 <Risk_Constraint_By_Group>

Risk constraint by group sets an upper bound on the level of total or active risk for a group of assets or factors.

An attribute and grouping scheme is required. For more information, see the <Grouping_Scheme> section.

Using the example from the <Grouping_Scheme> section, the following limits the total risk to 10% in the primary risk model for assets in the "North America" group:

```
   <risk_constraint_by_group asset_grouping_attribute_id="LocalMarket"
asset_grouping_scheme_id="LocalMarket_GroupingScheme" asset_group_id="North America"
is_primary_risk_model="true" risk_source_type="totalRisk" upper_bound="0.1"/>
```

Using the example from the <Grouping_Scheme> section, the following limits the total risk to 10% in the primary risk model for factors in the Oil Industry Factors group:

```
   <risk_constraint_by_group grouping_scheme_id="Factor_GroupingScheme" group_id="Oil
Industry Factors" is_primary_risk_model="true" risk_source_type="totalRisk"
upper_bound="0.1"/>
```

The following example specifies the same constraint on total risk for the group using additive definition:

```
   <risk_constraint_by_group grouping_scheme_id="Factor_GroupingScheme" group_id="Oil
Industry Factors" is_additive_definition="true" is_primary_risk_model="true"
risk_source_type="totalRisk" upper_bound="0.1"/>
```

8.5.18 <Risk_Parity_Constraint>

Risk parity constraint enables the construction of a risk parity portfolio in which included assets or factors have equal marginal contribution to portfolio total risk in the primary or secondary risk model.

To enable the risk parity constraint on all assets using the primary risk model:

```
<risk_parity_constraint type="assetRiskParity"/>
```

To use the secondary risk model, set the is_primary_risk_model attribute to false:

```
<risk_parity_constraint type="assetRiskParity" is_primary_risk_model="false"/>
```

If a benchmark portfolio is given, then the risk parity constraint makes contributions to the total active risks equal:

```
<risk_parity_constraint type="assetRiskParity” benchmark_portfolio_id="benchmark">
```

You can specify the assets which are included in risk parity by creating an attribute.For more information, see the <Attribute> section.

The following attribute defines a group containing assets USA11I1, USA13Y1 and USA1LI1. Specifying the name of the attribute in the `<risk_parity_constraint>` element equates only the risk of the assets in the group:

```
<attribute attribute_id="RiskParityGroup1" attribute_value_type="integer">
   <element element_id="USA11I1" integer_value="1"/>
   <element element_id="USA13Y1" integer_value="1"/>
   <element element_id="USA1LI1" integer_value="1"/>
</attribute>
<risk_parity_constraint type="assetRiskParity"
grouping_attribute_id="RiskParityGroup1"/>
```

To enable the risk parity constraint on factors:

```
<risk_parity_constraint type="factorRiskParity"/>
```

Secondary risk model, benchmark portfolio, or the group of included factors can be specified in the same way as for the assets.

8.5.19 <Five_Ten_Forty_Rule>

The 5/10/40 rule sets a holding constraint that limits the total weight of all issuers that represent more than 5% of the optimal portfolio to 40%, and limit any single issuer weight to 10% of the optimal portfolio.

The issuer ID attribute is required. For more information, see the <Attribute> section.

The following attribute sets the issuer IDs:

```
<attribute attribute_id="##default##issuer_id" attribute_value_type="text">
   <element element_id="USA11I1" text_value="EBAY"/>
   <element element_id="USA13Y1" text_value="NYSE"/>
</attribute>
```

To enable the 5/10/40 rule:

```
<five_ten_forty_rule five="5" forty="40" ten="10"/>
```

8.5.20 <General_Ratio_Constraint>

General ratio constraints allow you to set bounds on the ratio of portfolio-level exposures to asset attributes. For these constraints, you must first import your asset attributes. You can specify the attributes whose values are used in the numerator and denominator. If the denominator attribute is missing, then all coeffients will be 1 in the denominator. For more information, see the <Attribute> section.

This example sets lower and upper bounds on the ratio defined by the “NumeratorAttr” and “DenominatorAttr” asset attributes:

```
<general_ratio_constraint numerator_attribute_id="NumeratorAttr"
   denominator_attribute_id="DenominatorAttr" lower_bound="0.9" upper_bound="1.1"/>
```

In the following example, the `denomitator_attribute_id` is not set, thus, all coefficients will be 1 in the denominator.

```
<general_ratio_constraint numerator_attribute_id="Duration" lower_bound="-0.1"
   lower_bound_mode="plus" upper_bound="0.1" upper_bound_mode="plus"
   reference_portfolio_id="Benchmark" />
```

8.5.21 <Group_Ratio_Constraint>

Group ratio constraints enable you to set bounds for ratios of group-level exposures.

You can define groups for an alphanumeric grouping schemes such as GICS or user-defined industry groups. You must first create a grouping scheme. For more information, see the <Grouping_Scheme> section.

You can specify different asset groups for the numerator and the denominator of the ratio. The denominator group is optional; if missing, then it defaults to the asset group of the numerator.

Attributes for asset coefficients in the numerator and denominator are also required. The attribute for denominator coefficients is optional; if missing, then all denominator coefficients will be 1. For more information, see the <Attribute> section.

```
<group_ratio_constraint numerator_attribute_id="Coeffs1"
   numerator_grouping_attribute_id="GrpAttr1"
   numerator_grouping_scheme_id="GroupingScheme1" numerator_group_id="Grp1"
   denominator_attribute_id="Coeffs2" denominator_grouping_attribute_id="GrpAttr2"
   denominator_grouping_scheme_id="GroupingScheme2" denominator_group_id="Grp2"
   lower_bound="0.1" upper_bound="0.2"/>
```

This example given above sets lower and upper bounds on the ratio of weights of assets in “Grp1” with coefficients “Coeffs1” and the weights of assets in “Grp2” with coeffiecients “Coeffs2”. The asset groups “Grp1” and “Grp2” are defined by the “GroupingScheme1”/”GrpAttr1” and “GroupingScheme2”/”GrpAttr2” attributes respectively.

If the same group of assets are used in the denominator as in the numerator, then the grouping attributes for the denominator can be omitted, as shown below:

```
<group_ratio_constraint numerator_attribute_id="Duration"
   numerator_grouping_attribute_id="GICS_SECTOR"
   numerator_grouping_scheme_id="GICS_SECTOR_GroupingScheme"
   numerator_group_id="Information Technology" lower_bound="-0.2"
   lower_bound_mode="plus" upper_bound="0.2" upper_bound_mode="plus"
   reference_portfolio_id="Benchmark" />
```

In this example, both numerator and denominator assets are from the "Information Technology" sector, and the denominator coefficients are 1.

8.5.22 <Quadratic_Constraint>

Quadratic constraint allows you to set upper bounds on a quadratic form of asset weights or on their active part.

In the <Data> section, you need to define the symmetric matrices and asset attributes which are used as coefficients in quadratic constraints. Then you can add <quadratic_constraint> elements to <rebalance_profile>:

```
<quadratic_constraint quadratic_term_matrix_id="Q" linear_term_attribute_id="q"
    upper_bound="2.0" benchmark_portfolio_id="Benchmark"/>
```

This example sets 2.0 as an upper limit to $(h - h_B)^T Q(h - h_B) + q^T(h - h_B)$, where "Q" is the id of the matrix for quadratic coefficients, and "q" is the id of the asset attribute for linear coefficients.

The following elements sets the upper bound of a quadratic form relative to a reference portfolio:

```
<quadratic_constraint quadratic_term_matrix_id="Q1" linear_term_attribute_id="q1"
    upper_bound="0.2" upper_bound_mode="plus" reference_portfolio_id="Reference"/>
```

8.5.23 <Constraint_Priority>

Constraint priority allows you to build a constraint hierarchy. If the problem becomes infeasible, the optimization algorithm will relax the constraints in the specified order until a solution can be found or infeasibility will be reported. The constraint priority can also apply to certain linear constraints by setting the relax_order attribute within the constraint elements.

8.5.24 <Penalty>

Penalty inside the applicable constraint elements sets the penalty term in the objective function. The target attribute sets the desired value of the slack, lower/upper attribute sets the slack value below/above target, is_pure_linear attribute sets if the penalty function is pure linear, and the multiplier attribute adjusts the impact of the individual slack.

The following sets a penalty that helps restrict market exposure:

```
<beta_constraint is_soft="false">
        <penalty is_pure_linear="true" lower="0.8" multiplier="1" target="0.95"
upper="1.2"/>
</beta_constraint>
```

8.5.25 <Free_Range_Penalty>

Free-range penalty inside the applicable constraint elements sets the free range linear penalty term in the objective function. The target_low and target_high attributes set the lower and upper bounds of the free range where penalty is zero, and the downside_slope and upside_slope attributes set the penalty rate of each side when the slack variable is outside the free range.
The following sets a free-range penalty that helps restrict market exposure when beta is outside the range of 0.9 to 1.1:

```
   <beta_constraint is_soft="false">
         <free_range_penalty target_low="0.9" target_high="1.1" downside_slope="-0.01"
upside_slope="0.01"/>
   </beta_constraint>
```

8.5.26 <Portfolio_Concentration_Constraint>

Portfolio concentration constraint limits the sum of the weights of top X names in the optimal portfolio. The upper_bound attribute sets the limit on the sum of weights and the num_top_holdings attribute sets the number of top holdings whose total weight cannot exceed the upper bound. To exclude certain assets from the constraint, first create an attribute for the list of excluded assets, and then set the attribute excluded_assets_attribute_id. For more information about how to create an attribute, see the <Attribute> section.

The following sets the constraint limiting total weight of top 5 holdings to no more than 70% of optimal portfolio:

```
<portfolio_concentration_constraint num_top_holdings="5" upper_bound="0.7"/>
```

8.5.27 <Total_Active_Weight_Constraint>

Total active weight constraint sets the bound on the sum of the absolute active weights with respect to a benchmark. For example, the following constrains the sum of the absolute active weights in the optimal portfolio to less than 1%:

```
<total_active_weight_constraint reference_portfolio_id="Benchmark" upper_bound="0.01"/>
```

8.5.28 <Total_Active_Weight_Constraint_By_Group>

Total active weight constraint by group sets the total active weight constraint for a group of assets. An attribute and grouping scheme must be created first. For more information, see the <Grouping_Scheme> section.

8.5.29 <Issuer_Constraint>

Issuer constraints set lower and/or upper bounds on net or absolute total holdings of a specific issuer. Users can also set global bounds (that are applied to all issuers) using the wildcard "*" for the issuer ID.

The following limits the absolute holding of any issuer (global) not to exceed 95%, and net holding of issuer "1" between the range of 26.75% and 50.65% in the optimal portfolio:

```
    <issuer_constraint>
        <holding_constraint constraint_info_id="Issuer_Abs_1" constraint_type="abs"
issuer_attribute_id="*" upper_bound="0.95"/>
        <holding_constraint constraint_info_id="Issuer_Net_2" constraint_type="net"
issuer_attribute_id="1" lower_bound="0.2675" upper_bound="0.5065"/>
    </issuer_constraint>
```

8.5.30 Multi-Period Optimization: Cross-Period Constraints

Two types of cross-period constraints are supported for multi-period optimization:

- The cross_period_transaction_cost_constraint element limits the total transaction cost for all of the periods
- The cross_period_turnover_constraint limits the total turnover for all of the periods.

The following example sets the maximum cross-period turnover to 50%:

```
<cross_period_turnover_constraint by_side="net"
constraint_info_id="Turnover_CrossPeriod_net" is_soft="false" upper_bound="0.5"/>
```

8.5.31 <General_PWLI_Constraint>

General piecewise-linear constraint allows you to define upside and downside segments of a piecewise-linear function for each asset. For each upside or downside element, you can set the slope and break point for a particular asset. The optional starting_point element sets the weight of the asset at which point the piecewise-linear function evaluates to zero.

8.5.32 <Overlap_Constraint>

Overlap constraint limits the sum of the benchmark weights corresponding to the non-zero portfolio weights. The benchmark is the primary_benchmark attribute defined in <reference_portfolios> element under <rebalance_job> section. The following limits the overlap to 1%:

```
<overlap_constraint upper_bound="0.01"/>
```

8.5.33 <Include_Futures>

By default, cardinality/threshold constraint, leverage constraint, turnover constraint, and roundlot constraint excludes futures-type asset. To include futures asset for a particular type of constraint, set the corresponding attribute to true. The following includes futures asset in cardinality/threshold constraint:

```
<include_futures cardinality_threshold_constraint="true"/>
```

8.5.34 Multi-Account Optimization: Cross-Account Constraints

Multi-Account Optimization supports the following cross-account constraints:

- net turnover
- asset bound on total positions
- maximum total trades/buys/sells per asset

All cross-account constraint bounds are in units of dollar amount.

Some examples of setting these constraints are:

```
      <cross_account_net_turnover_constraint
constraint_info_id="Turnover_CrossAccount_net" is_soft="false" upper_bound="1000000000"/>
      <cross_account_asset_bound_on_total_position asset_id="USA11I1" is_soft="false"
lower_bound="10000000" upper_bound="20000000"/>
      <cross_account_asset_max_total_trades asset_id="USA11I1" is_soft="false"
upper_bound="10000000"/>
      <cross_account_asset_max_total_buys asset_id="USA11I1" is_soft="false"
upper_bound="10000000"/>
```

```
      <cross_account_asset_max_total_sells asset_id="USA11I1" is_soft="false"
upper_bound="10000000"/>
```

8.5.35 <Tax_Constraints>

The <tax_constraints> element supports the following constraints:

- upper limit on total tax of the portfolio
- tax arbitrage bounds on groups of assets
- minimum holding periods

The <tax_limit> element allows you to specify an upper bound on the total tax of the portfolio. The value is given in the unit that was specified in the <tax_setting> element.

```
      <tax_constraints>
        <tax_limit upper_bound="3000"/>
      </tax_constraints>
```

To place bounds on the gross/net gains or losses at the portfolio level or by group of assets, add <tax_arbitrage_range> elements to the tax constraints. The lower and upper bounds are set in the <tax_constraints> elements.

```
      <tax_constraints>
        <tax_arbitrage_range gain_type="capitalGain" tax_category="longTerm">
          <tax_constraint upper_bound="1000"/>
        </tax_arbitrage_range>
        <tax_arbitrage_range grouping_attribute_id="LocalMarket"
grouping_scheme_id="LocalMarket_GroupingScheme" group_id="North America"
gain_type="capitalLoss" tax_category="shortTerm">
          <tax_constraint lower_bound="100" upper_bound="300"/>
        </tax_arbitrage_range>
      </tax_constraints>
```

In this example, the grouping_attribute_id, grouping_scheme_id, and group_id attributes define the assets for which the tax bounds are applied. If they are omitted, then the constraint is applied to the whole portfolio. See the Grouping Scheme section for the definition of these attributes.

For total gain/loss constraints, use the same type of element, but without a tax_category attribute:

```
      <tax_constraints>
        <tax_arbitrage_range gain_type="capitalGain">
          <tax_constraint upper_bound="1000"/>
        </tax_arbitrage_range>
        <tax_arbitrage_range grouping_attribute_id="LocalMarket"
grouping_scheme_id="LocalMarket_GroupingScheme" group_id="North America"
gain_type="capitalLoss">
          <tax_constraint lower_bound="100" upper_bound="300"/>
        </tax_arbitrage_range>
      </tax_constraints>
```

To exclude tax-free gains/losses from totals, set the `exclude_tax_free_from_total attribute` of the <tax_setting> element.

For setting minimum holding period constraints, add `<min_holding_period>` elements to <tax_constraints>. You can use the grouping attributes to select the assets which this setting is applied to. If the asset groups overlap, then the last setting is applied to an asset.

```
      <tax_constraints>
        <min_holding_period min_holding_period="365"/>
        <min_holding_period grouping_attribute_id="GICS_SECTOR"
grouping_scheme_id="GICS_SECTOR_GroupingScheme" group_id="Information Technology"
min_holding_period="30">
      </tax_constraints>
```

8.6 <Rebalance_Job>

The <rebalance_job> element contains information on the optimization job. You can set the rebalance profile, risk model, the initial portfolio and investable universe, and the type of optimization to run.

8.6.1 <Rebalance_Profile_ID>

The rebalance profile ID for the optimization job, and it should match the rebalance_profile_id attribute defined in the <Rebalance_Profile> element. For more information, see the <Rebalance_Profile> section.

8.6.2 <Initial_Portfolio_ID>

The initial portfolio ID for the optimization job, and it should match the portfolio_id attribute defined in the <Portfolio> element. For more information, see the <Portfolios> section.

8.6.3 <Universe_Portfolio_ID>

The investable universe for the optimization job, and it should match the portfolio_id attribute defined in the <Portfolio> element. For more information, see the <Portfolios> section.

8.6.4 <Reference_Portfolios>

The reference portfolios from the asset bound constraints. For more information, see the <AssetBounds> section.

8.6.5 <Portfolio_Base_Value>

The initial portfolio base value used to compute weights.

8.6.6 <Cashflow_Weight>

Cash flow weight is the percentage of cash contribution to the initial portfolio, entered as decimals.

8.6.7 <Primary_Risk_Model_ID>

The primary risk model ID for the optimization job, and it should match the risk_model_id attribute defined in the <RiskModel> element. For more information, see the <Risk_Model> section.

8.6.8 <Secondary_Risk_Model_ID>

The secondary risk model ID for the optimization job, and should match the risk_model_id attribute defined in the <RiskModel> element. For more information, see the <Risk_Model> section.

8.6.9 <Attribute_Assignment>

Attribute_assignment element defines which attribute to use for the optimization job. For more information, see the <Attribute_Assignment> section under the <Data> section.

8.6.10 <Optimization_Setting>

The optimization setting allows you to set the type of optimization to run, which can be maximum utility (default), risk or return target, or frontier optimization. The tolerance_mutiplier attribute is used for adjusting the optimality tolerance, and count_cross_over_trade_as_one determines whether to count crossover trades as one or two transactions, considering fixed transaction costs and constraining number of trades. For risk target or risk-return frontier, the factor_risk_scalar and specific_risk_scalar elements set the weights for factor risk and specific risk respectively.

The following sets a risk target of 14%:

```
   <optimization_setting optimization_type="riskTarget">
      <risk_target>0.14</risk_target>
    </optimization_setting>
```

The following runs a risk-return frontier optimization with 10 points and upper bound of 10%:

```
    <optimization_setting optimization_type="efficientFrontier">
      <frontier frontier_type="riskReturn" lower_bound="0" max_data_points="10"
upper_bound="0.1"/>
    </optimization_setting>
```

To run a utility-constraint frontier optimization, additionally set the attribute <is_utility_constraint_frontier> to “true” for the supported constraints, such as <Factor Constraint>, <Factor Constraint By Group>, <Custom Constraint>, or <Beta Constraint>. To set the type of bound to vary for utility-factor or utility-general linear constraint, set the attribute bound_type to upper_bound or lower_bound.

```
The option element allows you to specify a number of settings passed to the solver, such
as compatibility mode and maximum time for optimization. For example, the following sets
the solver approach to prior version 8.0 and maximum time allowed for optimization to 5
minutes:    <optimization_setting optimization>
      <option option_id="COMPATIBLE_MODE" value="1"/>
      <option option_id="MAX_OPTIMAL_TIME" value="300"/>
    </optimization_setting>
```

To run multi-period optimization, set for each period the multi_period element that has attributes cashflow_weight and period:

```
<optimization_setting optimization>
  <multi_period cashflow_weight="0" period="0"/>
  <multi_period cashflow_weight="0" period="1"/>
</optimization_setting>
```

8.6.10.1 <Tax_Setting>

The <tax_setting> element allows you to specify tax rules, selling order rules, tax loss harvesting parameters, and loss carry forwards. See the Tax-Aware Optimization section for how these parameters are used. The <tax_setting> element has a tax_unit attribute which defines the unit of tax-related parameters. Its possible values are "dollar" (for absolute amounts) and "decimal" (for amounts relative to the base value). If not specified, then "dollar" is assumed as the default value.

The following example shows a simple tax setup in which there are different tax rates for short-term and long-term gains, wash sales are disallowed, and the selling order of the lots is FIFO:

```
<optimization_setting>
  <tax_setting tax_unit="decimal">
    <tax_rule enable_two_rate="true" long_term_rate="0.243" short_term_rate="0.423"
short_term_period="365" wash_sale_period="30" wash_sale_rule="disallowed"
enable_cross_netting_gain_loss="true"/>
    <selling_order_rule rule="fifo"/>
  </tax_setting>
</optimization_setting>
```

It is possible to add multiple <tax_rule> elements. In this case the grouping attributes must be given, and each asset must belong to exactly one group. See <Grouping_Scheme> section for further information how to group assets.

```
<tax_setting>
  <tax_rule grouping_attribute_id="LocalMarket”
grouping_scheme_id="LocalMarket_GroupingScheme” group_id="US" enable_two_rate="true"
long_term_rate="0.243" short_term_rate="0.423" short_term_period="365"/>
  <tax_rule grouping_attribute_id="LocalMarket”
grouping_scheme_id="LocalMarket_GroupingScheme” group_id="non-US" long_term_rate="0.4"/>
</tax_setting>
```

It is also possible to specify a different selling order for a group of assets by adding additional <selling_order_rule> elements with the appropriate grouping attributes.

Tax harvesting and loss carry forward data can be given after the selling order rules:

```
<tax_setting>
        …
  <tax_harvesting_by_group grouping_attribute_id="LocalMarket”
grouping_scheme_id="LocalMarket_GroupingScheme” group_id="US">
    <tax_harvesting tax_category="shortTerm" target="1000" penalty="500">
    <tax_harvesting tax_category="longTerm" target="2000" penalty="1000">
  </tax_harvesting_by_group>
```

```
      <loss_carry_fowards grouping_attribute_id="LocalMarket”
grouping_schme_id="LocalMarket_Groupingchme” group_id="US" long_term_lcf="412"
short_term_lcf="123" tax_free_lcf="0"/>
    </tax_setting>
```

8.6.10.2 <Expected_Shortfall_Setting>

The <expected_shortfall_setting> element allows you to specify parameters and data for expected shortfall optimizations. See the Expected Shortfall Optimization section how these parameters are used.

The confidence_level attribute specifies the confidence level of the shortfall, allowed values are within the (0,1) interval.

Target mean returns and scenario returns can be specified for assets or factors as <attribute> elements in the <data> section. If a target mean return term is included in the definition of the shortfall, then set the `has_target_mean_return` attribute to “true”. In addition, set `target_mean_return_attribute_id` to the id of the <attribute> element.

Similarly, enumerate the attributes containing scenario returns as <scenario_return_attribute_id> elements:

```
    <optimization_setting>
      <expected_shortfall_setting confidence_level="0.95" return_type="assetReturn"
has_target_mean_return="true" target_mean_attribute_id="tmr-1" scenarios_id="scenarios-
1"/>
    <optimization_setting>
```

If the target_mean_return_attribute_id attribute is omitted, the optimizer uses the average scenario returns as the target:

```
    <optimization_setting>
      <expected_shortfall_setting confidence_level="0.95" return_type="assetReturn"
has_target_mean_return="true" scenarios_id="scenarios-2">
    </expected_shortfall_setting>
    <optimization_setting>
```

8.7 <Rebalance_Result>

The rebalance results contain the outputs from an optimization job, which include profile diagnostics, portfolio summary, asset details, and trade list. The status of a job can be found in the optimization_status element, which contains a detailed message of the status, any additional information on the status, and logging from the optimization solver. The validation and data errors in the contents of the XML file are logged in the input_data_error element.

The profile_diagnostics element contains slack information on the constraints, threshold violation information on the assets, and characteristics of the discrete variables in the asset cardinality information.

The portfolio_summary element contains information on the optimal portfolio, such as its risk, return, objective value, turnover, holdings, and tax output. For frontier optimization, the portfolio_summary contains a set of optimal portfolios corresponding to each frontier point.

The tax_output element contains information on the calculated tax of the portfolio, such as the total tax, total disallowed loss due to wash sale rule, number of shares in the new tax lots, and new wash sale records. It also holds the long and short-term gains, losses, and taxes for each group defined by the tax rules.

The asset_details element contains asset-level information such as weights in the initial and optimal portfolio, its residual alpha, and roundlotted weight if roundlotting is enabled.

The trade_list element contains a set of transaction information, such as trade type, traded number of shares, market values, and transaction costs, for each asset in the optimal portfolio. In addition to the total transaction cost, the trade list reports fixed transaction cost, nonlinear transaction cost, and piecewise-linear transaction cost for each asset. The trade list is based on post-optimization roundlotted portfolio, which rounds the trades to their nearest roundlots after the optimization process. For more information, see Roundlotting—Optimal and Post-Optimization.

8.8 Multi-Period Optimization Using XML

For multi-period optimization, asset-level and profile-level data can be per period, which is controlled by the “period” attribute under <data> and <rebalance_profile> elements. The default value for the period attribute is -1, which denotes that the data is applicable for all of the periods. The following example illustrates setting alphas and asset-level bounds for different periods:

```
<data period="1">
    <attribute attribute_id="##default##alpha" attribute_value_type="double">
      <element double_value="0.01576034" element_id="USA11I1"/>
      <element double_value="0.002919658" element_id="USA13Y1"/>
    </attribute>
    <attribute attribute_id="asset_lower_bound_attribute" attribute_value_type="text">
      <element element_id="USA11I1" text_value="0.1"/>
    </attribute>
    <attribute_assignment alpha_attribute_id="##default##alpha"/>
</data>
<data period="2">
    <attribute attribute_id="##default##alpha" attribute_value_type="double">
      <element double_value="0.001459658" element_id="USA11I1"/>
      <element double_value="0.01849966" element_id="USA13Y1"/>
    </attribute>
    <attribute attribute_id="asset_lower_bound_attribute" attribute_value_type="text">
      <element element_id="USA13Y1" text_value="0.2"/>
    </attribute>
    <attribute_assignment alpha_attribute_id="##default##alpha"/>
</data>

  <rebalance_profiles>
    <rebalance_profile period="-1" rebalance_profile_id="Case 22">
      <utility utility_type="quadratic">
        <risk_term benchmark_portfolio_id="Benchmark"/>
      </utility>
      <linear_constraint enable_crossover="true" transaction_type="allowAll"/>
    </rebalance_profile>
    <rebalance_profile period="1" rebalance_profile_id="Case 22">
      <utility>
        <alpha_term>1</alpha_term>
        <risk_term common_factor_risk_aversion="0.0075" specific_risk_aversion="0.0075"/>
      </utility>
      <linear_constraint>
        <asset_bounds lower_bound_attribute_id="asset_lower_bound_attribute"/>
      </linear_constraint>
    </rebalance_profile>
    <rebalance_profile period="2" rebalance_profile_id="Case 22">
      <utility>
        <alpha_term>1.5</alpha_term>
        <risk_term common_factor_risk_aversion="0.0075" specific_risk_aversion="0.0075"/>
      </utility>
      <linear_constraint>
        <asset_bounds lower_bound_attribute_id="asset_lower_bound_attribute"/>
      </linear_constraint>
    </rebalance_profile>
  </rebalance_profiles>
```

To include the desired periods to run the multi-period optimization, add the element <multi_period> under <optimization_setting> section. The <multi_period> element contains cashflow_weight attribute that allows you to set different cashflow weight for each period, and the period attribute. For example, to add period 0 and period 1:

```
  <optimization_setting>
    <multi_period cashflow_weight="0" period="1"/>
    <multi_period cashflow_weight="0" period="2"/>
  </optimization_setting>
```

The output will be saved under <rebalance_result>, which will contain a <optimal_portfolio> section for each period and will have information on optimal portfolio's statistics and holdings. The cross-period result will have period equal to -1. The <profile_diagnostics> element will also contain constraint slack information for each period. Multi-period optimization does not report post-optimization roundlotted portfolio and asset-level transaction costs, therefore <asset_details> element will not be available.

8.9 Multi-Account Optimization Using XML

For multi-account optimization, profile-level data can be per account, which is controlled by the "account_id" attribute under <data> and <rebalance_profile> elements. The default value for the account_id attribute is -1, which denotes that the data is applicable for all of the accounts. The following example illustrates setting asset-level bounds and case inputs for different accounts:

```
<data account_id="1">
    <attribute attribute_id="asset_lower_bound_attribute" attribute_value_type="text">
```

```
      <element element_id="USA11I1" text_value="0.1"/>
    </attribute>
</data>
<data account_id="2">
    <attribute attribute_id="asset_lower_bound_attribute" attribute_value_type="text">
      <element element_id="USA13Y1" text_value="0.2"/>
    </attribute>
</data>

<rebalance_profiles>
    <rebalance_profile account_id="-1" rebalance_profile_id="Case 25">
      <utility utility_type="quadratic">
        <alpha_term>1.5</alpha_term>
        <risk_term/>
        <joint_market_impact_term joint_market_impact_multiplier="0.1"/>
      </utility>
      <linear_constraint enable_crossover="true"
enable_portfolio_balance_constraint="true" transaction_type="allowAll"/>
      <cross_account_net_turnover_constraint
constraint_info_id="Turnover_CrossAccount_net" is_soft="false" upper_bound="200000"/>
    </rebalance_profile>
    <rebalance_profile account_id="1" rebalance_profile_id="Case 25">
      <utility>
        <risk_term benchmark_portfolio_id="Benchmark" common_factor_risk_aversion="0.0075"
specific_risk_aversion="0.0075"/>
        <utility_scalar>0.25</utility_scalar>
      </utility>
      <linear_constraint>
        <asset_bounds lower_bound_attribute_id="asset_lower_bound_attribute"/>
      </linear_constraint>
    </rebalance_profile>
    <rebalance_profile account_id="2" rebalance_profile_id="Case 25">
      <utility>
        <risk_term benchmark_portfolio_id="Benchmark2"
common_factor_risk_aversion="0.0075" specific_risk_aversion="0.0075"/>
        <utility_scalar>0.75</utility_scalar>
      </utility>
      <linear_constraint>
        <asset_bounds lower_bound_attribute_id="asset_lower_bound_attribute"/>
      </linear_constraint>
    </rebalance_profile>
  </rebalance_profiles>
```

To include the desired accounts to run the multi-account optimization, add the element <multi_account> under <optimization_setting> section. Each <multi_account> element contains universe_portfolio_id, account_id, initial_portfolio_id, portfolio_base_value, and cashflow_weight attributes that allow you to set them for each account. For example, to add account_id 1 and account_id 2:

```
  <optimization_setting>
      <multi_account account_id="1" cashflow_weight="0" initial_portfolio_id="Initial
Portfolio" portfolio_base_value="100000" universe_portfolio_id="Trade Universe"/>
      <multi_account account_id="2" cashflow_weight="0" initial_portfolio_id="Initial
Portfolio2" portfolio_base_value="300000" universe_portfolio_id="Trade Universe2”/>
  </optimization_setting>
```

The universe_portfolio_id attribute may be omitted if the account's trade universe is the same as that of ALL_ACCOUNT.

In Multi-Account Tax-Aware Optimizations,

- The tax rules, selling order rules, tax harvesting, and loss carry-forwards can be specified at the account-level by adding multiple `<tax_setting>` elements to `<optimization_setting>`.
- The `<tax_setting>` element without `account_id` contains settings which refers to ALL_ACCOUNT.
- All accounts must use unit for tax-related parameters, so the `tax_unit` attribute is only accepted on the ALL_ACCOUNT setting.
- Additional `<tax_setting>` elements contain parameters specific to the account identified by the `account_id` attribute:

```
<optimization_setting>
  …
  <tax_setting tax_unit="decimal"/>
  <tax_setting account_id="1">
    <tax_rule enable_two_rate="true" short_term_period="365" long_term_rate="0.243"
short_term_rate="0.423"/>
  </tax_setting>
  <tax_setting account_id="2">
    <tax_rule enable_two_rate="true" short_term_period="365" long_term_rate="0.1"
short_term_rate="0.2" wash_sale_rule="ignored" wash_sale_period="30" />
  </tax_setting>
</optimization_setting>
```

Account groups can be defined by adding an account_group_id attribute to <multi_account> element:

```
  <optimization_setting>
      <multi_account account_id="1" … account_group_id="1"/>
      <multi_account account_id="2" … account_group_id="1"/>
      <multi_account account_id="3"/>
  </optimization_setting>
```

Account which belongs to a group must use the same tax rules. The tax rule for an account group can be given by adding a <tax_setting> element and setting its account_group_id attribute:

```
<optimization_setting>
  …
  <tax_setting account_group_id="1">
    <tax_rule enable_two_rate="true" short_term_period="365" long_term_rate="0.243"
short_term_rate="0.423"/>
  </tax_setting>
</optimization_setting>
```

Constraints related to the account group should be put into their own <rebalance_profile> elements. Currently, only tax terms and tax constraints can be specified for an account group:

```
<rebalance_profiles>
```

```
    <rebalance_profile account_group_id="1" rebalance_profile_id="Case 25">
      <utility>
        <tax_term>1.5</tax_term>
      </utility>
      <tax_constraints>
        <tax_limit constraint_info_id="Tax_Limit" upper_bound="25" />
      </tax_constraints>
    </rebalance_profile>
</rebalance_profiles>
```

The output will be saved under <rebalance_result>, which will contain a <optimal_portfolio> section for each account and will have information on optimal portfolio's statistics and holdings. The cross-account result will have account_id equal to -1. The <profile_diagnostics> element will also contain constraint slack information for each account. Multi-Account Optimization does not report post-optimization roundlotted portfolio and asset-level transaction costs, therefore <asset_details> element will not be available.

8.10 Using XML APIs

All the optimizer XML data can be created via Java and C# APIs. The Java library can be imported into MATLAB. Each XML element is represented by a message class and a corresponding builder class for creating message class instances. The message object is immutable, and you should use the builder for any modification to the message object. After the optimizer message objects are constructed, they can be serialized and passed to optimizer, which creates a RebalanceResult message that stores the results from an optimization.

To create a Message object in Java, using RebalanceProfile as an example, do the following:

1. Create a Builder object for RebalanceProfile:

```
RebalanceProfile.Builder profileBuilder = RebalanceProfile.newBuilder();
```

2. Set the rebalance profile data through the Builder. The set methods return back a reference to the modified Builder object, allowing multiple set method calls in one line:

```
profileBuilder .setRebalanceProfileId("Case 1a");
profileBuilder.setUtility(Utility.newBuilder()
            .setUtilityType(EUtilityType.QUADRATIC)
            .addRiskTerm(RiskTerm.newBuilder()
                            .setIsPrimaryRiskModel(true)
                            .setCommonFactorRiskAversion(0.0075)
                            .setSpecificRiskAversion(0.0075)));
```

3. After all the necessary data has been set, call the Build method to create RebalanceProfile object and add it to the WorkSpace message:

```
wsBuilder.setRebalanceProfiles(RebalanceProfiles.newBuilder()
                    .addRebalanceProfile(profileBuilder.build()));
```

4. Serialize the WorkSpace message to byte array and run optimization:

```
byte[] out = OpenOptimizer.Run(wsBuilder.build().toByteArray());
```

5. Convert the byte array to RebalanceResult message:

```
RebalanceResult result = RebalanceResult.parseFrom(out);
```

Below are some of the commonly used methods for Message and Builder classes:

Description	Java	C#
Create a builder object	<Message>.newBuilder()	<Message>.CreateBuilder()
Build a message object	<Builder>.build()	<Builder>.Build()
Create a builder from message	<Message>.toBuilder()	<Message>.ToBuilder()
Write to output stream	<Message>.writeTo()	<Message>.WriteTo()
Read from input stream	<Message>.parseFrom()	<Message>.ParseFrom()
Package to import	com.barra.openopt.protobuf.*	optimizer.proto, Google.ProtocolBuffers

For more information about the Message interfaces, refer to:

- Java - https://developers.google.com/protocol-buffers/docs/reference/java/index
- C# - http://code.google.com/p/protobuf-csharp-port/downloads/list

Additional reference:

- XML Schema Reference Guide
- XML/JAVA API Reference Guide
- Protocol Buffers Interface File (<Installation Folder>doc/optimizer.proto)
- Google Protocol Buffers API Reference

The OpenOptimizer class enables you to pass data to the optimizer and creates the output. The following methods are provided within the class:

Parameters:

data_file	Serialized binary file from Data message
portfolio_file	Serialized binary file from Portfolios message
risk_model_file	Serialized binary file from RiskModels message
rebalance_profile_file	Serialized binary file from RebalanceProfiles message
rebalance_job_file	Serialized binary file from RebalanceJob message
rebalance_result_file	Result returned from Optimizer, in serialized binary file from RebalanceResult message

Returns:

Status code of optimization

```
static EStatusCode com.barra.openopt.OpenOptimizer.Run  ( String  data_file,
  String  portfolio_file,
  String  risk_model_file,
  String  rebalance_profile_file,
  String  rebalance_job_file,
  String  rebalance_result_file
 )
```

Parameters:

workspace_file	Serialized binary file from WorkSpace message
rebalance_result_file	Result returned from Optimizer, in serialized binary file from RebalanceResult message

Returns:

Status code of optimization

```
static EStatusCode com.barra.openopt.OpenOptimizer.Run  ( String  workspace_file,
  String  rebalance_result_file
 )
```

Parameters:

workspaceByteArray	Serialized byte array from WorkSpace message

Returns:

Serialized byte array from RebalanceResult message

```
static byte [] com.barra.openopt.OpenOptimizer.Run  ( byte[]  workspaceByteArray   )
```

8.10.1 Setting Up in C#

To use the APIs in C#, you will need to add the following references to your C# project:

- opsproto_cs.dll

- Google.ProtocolBuffers.dll
- Google.ProtocolBuffers.Serialization.dll

The interfaces to import for Optimizer APIs are:

```
using optimizer.proto;
using Google.ProtocolBuffers;
```

8.10.2 Setting Up in MATLAB

Follow the steps in the Working with MATLAB Interface section using Java jar file OpenOptJAVAAPI.jar instead of OptJAVAAPI.jar. Then, import and initialize the interfaces by calling:

```
import com.barra.openopt.*;
COptJavaInterface.Init;
```

The message and builder methods are identical to the Java interface. Because schema objects are structured in Java as inner classes, the MATLAB function javaMethod should be called first to obtain the inner class object. For example, to create a RebalanceProfile builder:

```
profileBuilder = javaMethod('newBuilder', 'com.barra.openopt.protobuf$RebalanceProfile');
```

Alternatively, you can import a helper function file newBuilder.m (under tutorials\matlab_proto), to shorten the method call by passing just the class name:

```
profileBuilder = newBuilder('RiskModel');
```

Then, you can directly call the setter and getter methods and set data through the builder object:

```
 profileBuilder.setRebalanceProfileId ('Case 1');
```

To obtain the value of an enumeration type, for AttributeValueType.DOUBLE:

```
doubleAttrType =
javaMethod('valueOf','com.barra.openopt.protobuf$AttributeValueType','DOUBLE');
```

For faster performance by reducing number of method calls to Java from MATLAB, COptJavaInterface class provides several convenience methods that can take arrays of native MATLAB data type and populate the Builder object.

Data Type	Method	Description
`static boolean`	`SetDoubleAttributeElements(javaref_proto/com/barra/openopt/protobuf.Attribute.Builder.html attribute, java.lang.String attributeId, java.lang.String[] assetIds, double[] values)`	Sets the double attribute elements given an Attribute builder.
`static boolean`	`SetFactorCovariances(javaref_proto/com/barra/openopt/protobuf.FactorCovariances.Builder.html factorCovariances, java.lang.String[] factorIds1,`	Sets the factor covariances given a FactorCovariances builder.

Data Type	Method	Description
	java.lang.String[] factorIds2, double[] covariances)	
static boolean	SetFactorExposureForAsset(javaref_proto/com/barra/openopt/protobuf.FactorExposure.Builder.html factorExposure, java.lang.String assetId, java.lang.String[] factorIds, double[] exposures)	Sets the factor exposure array for an asset given a FactorExposure builder.
static boolean	SetIntegerAttributeElements(javaref_proto/com/barra/openopt/protobuf.Attribute.Builder.html attribute, java.lang.String attributeId, java.lang.String[] assetIds, int[] values)	Sets the integer attribute elements given an Attribute builder.
static boolean	SetPortfolioHoldings(javaref_proto/com/barra/openopt/protobuf.Portfolio.Builder.html portfolio, java.lang.String portfolioId, java.lang.String[] assetIds, double[] amounts)	Sets the portfolio holdings given a Portfolio builder.
static boolean	SetSpecificCovariances(javaref_proto/com/barra/openopt/protobuf.SpecificCovariances.Builder.html specificCovariances, java.lang.String[] assetIds1, java.lang.String[] assetIds2, double[] covariances)	Sets the specific covariances given a SpecificCovariances builder.
static boolean	SetSpecificVariances(javaref_proto/com/barra/openopt/protobuf.SpecificVariances.Builder.html specificVariances, java.lang.String[] assetIds, double[] variances)	Sets the specific variances given a SpecificVariances builder.
static boolean	javaref_proto/com/barra/openopt/COptJavaInterface.html#SetTextAttributeElements(com.barra.openopt.protobuf.Attribute.Builder, java.lang.String, java.lang.String[], java.lang.String[])(javaref_proto/com/barra/openopt/protobuf.Attribute.Builder.html attribute, java.lang.String attributeId, java.lang.String[] assetIds, java.lang.String[] values)	Sets the text attribute elements given an Attribute builder.

9 Using the Command Line Executable

You can use the OpenOpt.exe program to perform the following operations:

1. Load Barra ModelsDirect data and convert it into binary/XML format.
2. Convert serialized workspace data from one format to another.

3. Run optimization and output result in the binary/XML format.

The executable determines the file type by its extension. If the file name ends with .xml, then it is determined as XML format, otherwise binary format. To load ModelsDirect data, an assetID universe file should always be provided to avoid loading asset risk data that is not part of the optimization universe.

9.1 Loading Barra Models Direct Data

Option	Parameter	Description
-md	See below	Load ModelsDirect files and convert to protobuf/xml format
-path	<ModelsDirect path>	Location of ModelsDirect files
-m	<model name>	Name of the risk model
-d	<analysis date>	Start date from which, ModelsDirect files will be loaded
-id	<assetID file>	Text file containing list of asset IDs to load exposures and specific risk for, with new line starting with each BARRAID
-o	<output file>	Output risk model filename

9.2 Running Optimization

You can run an optimization either with each element of the workspace or with a single workspace. The following sections describe both the methods of running optimization.

9.2.1 Optimize with Each Element of the Workspace

Option	Parameter	Description
-run	See below	Run optimization given input protobuf/xml files
-data	<data file>	Filename containing the data section
-pf	<portfolios file>	Filename containing the portfolios section
-rm	<risk models file>	Filename containing the risk models section
-rp	< rebalance profile file>	Filename containing the rebalance_profile section
-rj	< rebalance job file>	Filename containing the rebalance_job section
-csv	<asset attribute file>	CSV file containing asset attribute data
-o	<output file>	Filename containing the rebalance_result section

9.2.2 Optimize with a Single Workspace

Option	Parameter	Description
-run	See below	Run optimization given input protobuf/xml files
-ws	<workspace file>	Filename containing the workspace section
-csv	<asset attribute file>	CSV file containing asset attribute data
-o	<output file>	Filename containing the rebalance_result section

Alternatively, optimization can be run in conjunction with ModelsDirect loader. The ModelsDirect risk model will be the primary risk model and overrides any existing risk model specified in the workspace XML. To run optimization with ModelsDirect loader, specify the following additional switches:

Option	Parameter	Description
-path	<ModelsDirect path>	Location of ModelsDirect files
-m	<model name>	Name of the risk model
-d	<analysis date>	Date starting from which, the ModelsDirect files will be loaded
-id	<assetID file>	Text file containing list of asset IDs to load exposures and specific risk for, with new line starting with each BARRAID

Examples:

- Run optimization with each element of workspace and USE4L ModelsDirect data:

  openopt.exe –run –data data.xml –pf portfolios.xml –rp profile.xml –rj job.xml –path \ModelsDirect –m USE4L –d 20140201 –id Universe.txt –o result.xml

- Run optimization with a single WorkSpace and USE4L ModelsDirect data:

  openopt.exe –run –ws workspace.xml –path \ModelsDirect –m USE4L –d 20140201 –id Universe.txt –o

  result.xml

9.3 Converting Protobuf ↔ XML

Option	Parameter	Description
-convert	See below	Convert a protobuf binary file to XML file and vice versa
-msg	<message name>	Full name of protobuf message including namespace, for ex. optimizer.proto.RiskModel
-i	<input file to convert>	Input protobuf/xml filename
-o	<converted output file>	Output protobuf/xml filename

9.4 Converting WSP → Protobuf/XML

Option	Parameter	Description
-convert	See below	Convert a WSP file to protobuf/XML file
-i	<input file to convert>	Input WSP filename
-o	<converted output file>	Output protobuf/xml filename

9.5 Converting WorkSpace Protobuf/XML → WSP

Option	Parameter	Description
-convert	See below	Convert a WorkSpace Protobuf/XML file to WSP file
-i	<input file to convert>	Input WorkSpace Protobuf/XML filename
-o	<converted WSP file>	Output WSP filename

9.6 Converting WSP → Legacy XML format

Option	Parameter	Description
-convert	See below	Convert a WSP file to legacy XML file
-i	<input file to convert>	Input WSP filename
-oxml	<converted output file>	Output xml filename

9.7 Checking license status

Option	Parameter	Description
-license		Displays license validity and expiry date.

9.8 Additional Options

Option	Parameter	Description
-v	<verbose level for logging>	0=Error, 1=Warning, 2=Info, 3=Debug
-l	<log file>	Full path and filename of log file
-debug	<debug file>	WSP file for troubleshooting in optimization mode

10 Frequently Asked Questions

10.1 Why do I get different portfolios from different Optimizer versions?

This is because Barra Optimizer's goal is to maximize the objective function. It pays no special attention to the optimal weights in the resulting portfolio unless there is a constraint, penalty, or transaction cost to limit the turnover or any deviation. Two feasible portfolios are considered equivalent if their objective value difference is within tolerance, even if they differ greatly in weight.

For convex problems, multiple optimal solutions may exist due to numerical precision and the nature of the problem. Barra Optimizer does not guarantee to return a particular solution because all these optimal solutions are considered equivalent and are globally optimal. Differences in computing architecture, risk models, or optimality tolerances between versions may lead to a different optimal solution being returned. The objective values should be the same or slightly different.

For non-convex, nonlinear, or discrete problems, path-dependent heuristics are employed. Barra Optimizer may get stuck with a different local optimal portfolio when it tunes or enhances the heuristics from one version to another. These heuristic portfolios may be drastically different from one version to another. Even though the overall heuristic quality should improve in a newer version, it is possible to see degenerating quality in a particular heuristic solution.

10.2 Why do I get a suboptimal or counterintuitive solution?

For non-convex, nonlinear, or discrete problems, it is impossible or impractical to obtain a global optimal solution. This is due to the complex nature of the problem and the limits of computing power. Barra Optimizer employs heuristics to tackle these problems.

10.3 Why setting a hard constraint (e.g., turnover) to soft constraint may result in a worse solution for paring cases?

This falls under the suboptimal/counterintuitive solution category as explained above. Setting a continuous constraint (e.g., turnover) as soft constraint widens the feasible region at each iteration of the paring heuristic. As a result, it may lead to different search paths and eventually end up with a worse final solution. Unless and until we have an efficient global optimization algorithm, we are not able to guarantee a globally optimal solution for paring cases.

10.4 Which features may lead to suboptimal or counterintuitive solutions?

Features that return a heuristic portfolio may lead to suboptimal or counterintuitive solutions from time to time. For general guidelines, refer to Appendix A.

10.5 Which feature combinations are supported in Barra Open Optimizer?

For information about the supported features, refer to Appendix B.

10.6 Is there any limit on the number of risk terms in the objective function?

No, as long as all risk terms come from not more than two risk models.

10.7 Is there any limit on the number of risk constraints?

No. However, the number of risk constraints may negatively impact the speed.

10.8 Is there any limit on the number of risk models?

Yes. The number of risk models underlying a given problem, either in the objective or in the constraints, is limited to two. Currently, we are investigating applications with more than two risk models.

10.9 Is there any limit on the number of benchmarks?

No. Multiple benchmarks are allowed in either the objective or constraints.

10.10 Can we expect the optimizer to always produce the same results for the same optimization problem on two machines with different architectures?

No. The optimizer calls Intel MKL routines and therefore is subject to the known numerical reproducibility issue with Intel MKL. According to this Intel article,

"Intel MKL does run-time processor dispatching in order to identify the appropriate internal code paths to traverse for the Intel MKL functions called by the application. The code paths chosen may differ across a wide range of Intel processors and Intel architecture compatible processors and may provide differing levels of performance. For example, an Intel MKL function running on an Intel® Pentium® 4 processor may run one code path, while on the latest Intel® Xeon® processor it will run another code path. This happens because each unique code path has been optimized to match the features available on the underlying processor. One key way that the new features of a processor are exposed to the programmer is through the instruction set architecture (ISA). Because of this, code branches in Intel MKL are designated by the latest ISA they use for optimizations: from the Intel® Streaming SIMD Extensions 2 (Intel® SSE2) to the Intel® Advanced Vector Extensions 2 (Intel® AVX2). The feature-based approach introduces a challenge: if any of the internal floating-point operations are done in a different order or are re-associated, the computed results may differ".

10.11 Why do we sometimes see large discrepancies in optimization results for the optimization problem on two different machines?

For non-convex cases (e.g. case with paring constraints), there may be multiple local optimal solutions. The optimizer uses heuristic algorithms and may stop at any of such local optimal solutions. Small differences in intermediate results (due to Intel MKL) may cause different heuristic search paths and therefore produce significantly different final results.

10.12 Is it possible to achieve run-to-run reproducibility on the same machine?

Yes. Follow the instructions in this Intel article to set environment variable of MKL_CBWR.

Appendix A: Explanation of Outputs

Problem Type	Output Portfolio(s)		Notes
	Optimal	Heuristic	
Standard Portfolio Optimization (Single- or Multi-Period)	√		If a problem is infeasible, Barra Optimizer will return the best portfolio it is able to find. It will also provide information on which constraints to relax in order for the problem to be feasible.
General Linear or Convex Quadratic Programming	√		
Optimization with Soft Constraints			Barra Optimizer will provide information on which, if any, soft constraints are violated.
• Soft Standard Constraints	√		
• Soft Special Constraints		√	
Paring Optimization		√	1. If no solution satisfying all *standard* constraints can be found, the problem will be declared as infeasible. 2. If Barra Optimizer can detect that these special constraints are too tight, it will report so and return the best available portfolio. 3. If it cannot be sure whether these special constraints are too tight, yet still cannot find a solution satisfying these constraints as well as the other standard constraints, it will return the best available portfolio and report the optimization as failed.
Round Lot Optimization		√	
5/10/40 Optimization		√	
Risk Constrained Optimization			Barra Optimizer will report a risk upper bound as being too tight, or a desired lower bound as being unreasonable if it can detect so for sure.
• Convex Risk Constraints	√		
• Non-Convex Risk Constraints		√	
• Risk Parity Constraint	√		
Concentration Limit Constraint	√		Barra Optimizer will provide a solution satisfying the concentration constraint if the problem is feasible.
Long/Short (Hedge) Optimization			Barra Optimizer may report hedge optimization as failed if it cannot find a feasible solution satisfying the leverage constraints.
• Convex Long/Short Constraints	√		
• Non-Convex Long/short Constraints		√	
Tax-Aware Optimization		√	Barra Optimizer may report tax constraints as "maybe too tight", if numerical problems occur.

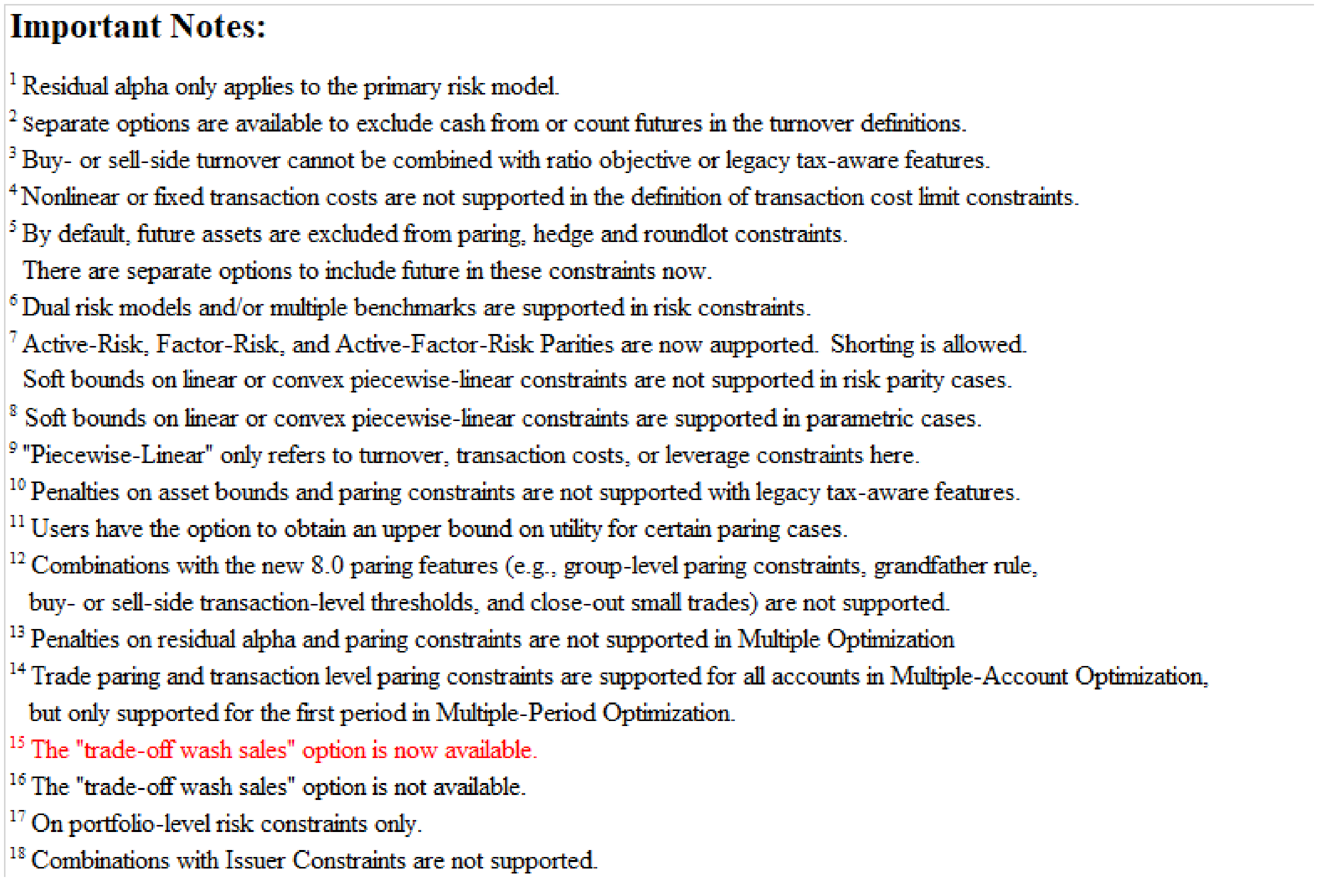
Problem Type	Output Portfolio(s)		Notes
	Optimal	Heuristic	
Parametric Optimization • With Standard Constraints • With Special Constraints	√	√	1. If the range is too high for the return or too low for the risk, Barra Optimizer will return the portfolio closest to the range. It will also output a message indicating the range is unattainable, and suggest the best possible range. 2. If a feasible portfolio with lower risk (or higher return) and a better objective value exists outside the given range, Barra Optimizer will return the better portfolio and indicate that the given range is not on the efficient frontier.
Optimization with Ratio Objectives • Non-Negative Numerator • Negative Numerator	√	√	If the numerator is non-negative and a solution exists, the problem is convex and Barra Optimizer will be able to return an optimal solution. Otherwise, a heuristic solution will be returned.

Appendix B. Availability of Features and Functions

Type	Constraints / Features		ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
Objective Items	Mean (Linear) - Variance (Quadratic)		1	Y																																											
	Dual Risk Models and/or Dual Benchmarks and/or Multiple Risk Terms		2	Y	Y																																										
	Information or Sharpe Ratio		3	▨	▨	Y																																									
	Transaction Costs	Piecewise-Linear	4	Y	Y	Y	Y																																								
		Nonlinear	5	Y	Y		Y	Y																																							
		Fixed	6	Y	Y		Y	Y	Y																																						
	Fixed Holding Costs		7	Y	Y		Y	Y		Y																																			
	Penalties on Asset Bounds, Linear or Piecewise-Linear Constraints, Factor Constraints, Residual Alpha, or Paring Constraints		8	Y	Y[1]		Y	Y	Y	Y	Y																																		
Linear Constraints	General Linear Constraints (Upper and/or Lower Bounds), e.g., "Net" Issuer Constraints		9	Y	Y	Y	Y	Y	Y	Y	Y	Y																																	
	Factor or Beta Constraints (Upper and/or Lower Bounds)		10	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y																																
	Asset Bounds (Upper and/or Lower Bounds)		11	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y																															
Piecewise Linear Constraints	Upper Bound on the Total Turnover, Buy- or Sell-Side Turnover, as well as Turnover by Group[2]		12	Y	Y	Y[3]	Y	Y	Y	Y	Y	Y	Y	Y	Y																														
	Upper Bound on the Total Piecewise-Linear Transaction Cost[4]		13	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y																													
	Convex General Piecewise-Linear Constraints, Upper Bound on the Total Active Weights or "Absolute" Issuer Constraints		14	Y	Y		Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y																												
	Non-Convex General Piecewise-Linear Constraints, Lower Bound on the Total Active Weights		15	Y	Y		Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y																													
Paring Constraints[5,11]	Asset Paring (i.e., Max # or Min# of Assets)		16	Y	Y		Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y																												
	Level Paring (i.e., Min Holding or Transaction Thresholds, including grandfather rule and close-out options)		17	Y	Y		Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y																											
	Trade Paring (i.e., Max # or Min# of Trades, Buys, or Sells)		18	Y	Y		Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y																										
Risk Constraints[6]	Portfolio-Level Risk Constraints (Upper Bounds, and/or Lower Bounds)		19	Y	Y		Y	Y			Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y																									
	Subgroup Risk Constraints (Two Definitions, Upper Bounds, and/or Lower Bounds)		20	Y	Y		Y				Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y																								
	Risk Contribution by Asset (Both Lower and Upper Bounds)		21	Y	Y		Y				Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y																							
	General quadratic constraints (Upper Bound Only)		22	Y	Y		Y	Y			Y	Y	Y	Y	Y	Y	Y								Y																						
	Risk Parity Constraints[7]		23	Y	Y		Y				Y	Y	Y	Y	Y	Y	Y									Y																					
Long/Short Features[5]	(Weighted) Long, Short, or Total Leverage Constraints and Short Rebates		24	Y	Y		Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				Y																				
	Turnover-by-Side and Paring-by-Side Constraints		25	Y	Y		Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				Y	Y																			
	Constraint on the ratio of "Short to Long" or "Net to Total" Leverage (ub and/or lb)		26	Y	Y		Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				Y	Y	Y																		
Risk Target / Return Target[8]	Risk Target		27	Y		▨	Y				Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y						Y	Y	Y	Y																	
	Return Target		28	Y		▨	Y				Y	Y	Y	Y	Y	Y	Y		Y	Y	Y							Y			Y																
Parametric[8] Optimization	Varying Return (Risk-Return Efficient Frontiers)		29	Y		▨	Y				Y	Y	Y	Y	Y	Y	Y		Y	Y	Y						Y	Y		▨	▨	Y															
	Varying Risk (Risk-Return Efficient Frontiers)		30	Y		▨	Y				Y	Y	Y	Y	Y	Y	Y		Y	Y	Y						Y	Y		▨	▨	▨	Y														
	Varying a Linear or Piecewise-Linear[9] Constraint (Constraint-Utility Frontiers, e.g., Leverage-Utility Frontiers)		31	Y		▨	Y				Y	Y	Y	Y	Y	Y	Y		Y	Y	Y						Y	Y		▨	▨	▨	▨	Y													
	Varying Tax (Utility-Tax Frontiers)		32	Y		▨	Y				Y[10]	Y	Y	Y	Y	Y	Y		Y	Y	Y									▨	▨	▨	▨	▨	Y												
Round Lots[5]	Optimal Roundlotting		33	Y	Y		Y				Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y						Y	Y	Y	Y						Y											
	Post-Optimization Roundlotting		34	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				Y	Y	Y	Y	Y	▨	▨	▨	▨	Y	Y										
Tax-Aware Optimization	New Tax-Aware Opt (Multiple Tax-Rule and Capital-Gain Groups, Fractional Shares, and Maximizing Loss-Benefit etc.)[15]		35	Y			Y				Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y[17]					Y	Y		Y	Y				Y		Y	Y									
	Legacy Tax-Aware Optimization (Dual Rates, Trading Rules, Tax Arbitrage, Tax Harvesting, Wash Sales, Overlap, etc.)		36	Y			Y				Y[10]	Y[18]	Y	Y	Y[3]	Y			Y[12]	Y[12]	Y[12]									Y[16]					Y		Y	▨	Y								
Multiple Optimization	Multiple-Period Optimization		37	Y			Y	Y			Y[13]	Y[18]	Y	Y	Y	Y			Y	Y[14]	Y[14]																			Y							
	Multiple-Account Optimization		38	Y			Y				Y[13]	Y[18]	Y	Y	Y	Y			Y	Y	Y																	Y[16]			Y						
Special Constraints / Features	5/10/40 (or P/Q/R) Constraints		39	Y	Y		Y	Y			Y	Y	Y	Y	Y	Y	Y		Y	Y	Y									Y							Y					Y					
	Expected Shortfall Optimization		40	Y	Y		Y				Y	Y	Y	Y	Y	Y	Y																										Y				
	Upper Bound on Portfolio Concentration		41	Y	Y		Y				Y	Y	Y	Y	Y	Y	Y		Y	Y	Y																Y							Y			
	Penalty Related to The Particular Constraint/Feature		42	▨	▨	▨	▨	▨	▨	▨	▨	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y					▨	Y			▨	▨	▨	▨	▨	▨					▨	▨		▨		▨		
	Soft Bound on The Particular Constraint/Feature		43	▨	▨	▨	▨	▨	▨	▨	▨	Y[18]	Y		Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		Y	Y	Y	▨	▨	▨	▨	▨	▨	Y				▨	▨		▨		▨	▨	
	Constraint Hierarchy on The Particular Constraint		44	▨	▨	▨	▨	▨	▨	▨	▨	Y	Y		Y	Y			Y	Y	Y	Y	Y	Y		▨	Y	Y	Y	▨	▨	▨	▨	▨	▨	Y				▨	▨		▨		▨	▨	▨
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44

Please See Important Footnotes on Next Page

Note: Features highlighted in red indicate new or enhanced in Barra Optimizer 9.2. Not applicable feature combinations are greyed out.



Important Notes:

[1] Residual alpha only applies to the primary risk model.
[2] Separate options are available to exclude cash from or count futures in the turnover definitions.
[3] Buy- or sell-side turnover cannot be combined with ratio objective or legacy tax-aware features.
[4] Nonlinear or fixed transaction costs are not supported in the definition of transaction cost limit constraints.
[5] By default, future assets are excluded from paring, hedge and roundlot constraints.
There are separate options to include future in these constraints now.
[6] Dual risk models and/or multiple benchmarks are supported in risk constraints.
[7] Active-Risk, Factor-Risk, and Active-Factor-Risk Parities are now aupported. Shorting is allowed.
Soft bounds on linear or convex piecewise-linear constraints are not supported in risk parity cases.
[8] Soft bounds on linear or convex piecewise-linear constraints are supported in parametric cases.
[9] "Piecewise-Linear" only refers to turnover, transaction costs, or leverage constraints here.
[10] Penalties on asset bounds and paring constraints are not supported with legacy tax-aware features.
[11] Users have the option to obtain an upper bound on utility for certain paring cases.
[12] Combinations with the new 8.0 paring features (e.g., group-level paring constraints, grandfather rule,
buy- or sell-side transaction-level thresholds, and close-out small trades) are not supported.
[13] Penalties on residual alpha and paring constraints are not supported in Multiple Optimization
[14] Trade paring and transaction level paring constraints are supported for all accounts in Multiple-Account Optimization,
but only supported for the first period in Multiple-Period Optimization.
[15] The "trade-off wash sales" option is now available.
[16] The "trade-off wash sales" option is not available.
[17] On portfolio-level risk constraints only.
[18] Combinations with Issuer Constraints are not supported.

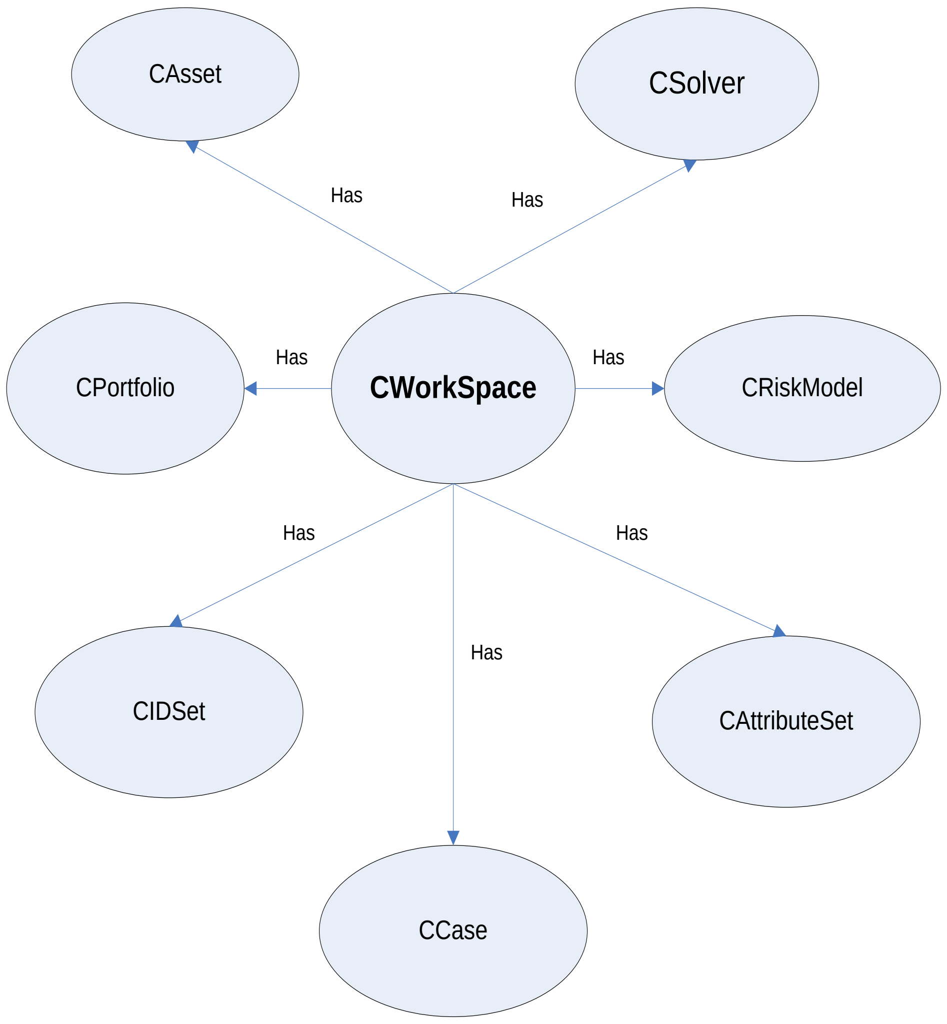
Appendix C: Glossary

Term	Definition
Trade universe	A list of assets that can be considered for inclusion in the optimal portfolio.
Initial portfolio	The portfolio to be optimized. Assets in the initial portfolio that are not in the trade universe can be considered for inclusion in the optimal portfolio. When there is no trade universe, only assets in the initial portfolio can be considered for inclusion in the optimal portfolio.
Investment universe	The union of the initial portfolio and the trade universe; a list of all assets eligible for inclusion in the optimal portfolio.
Benchmark	A portfolio of assets that the optimal portfolio will be compared against. Note: The assets in a benchmark may fall out of the investment universe.
Constraint slack	A constraint $l \le g(h) \le u$ can be rewritten as $g(h) - s = 0, \quad l \le s \le u$ where, s is called a constraint slack variable.
Asset weight	The position of an asset value compared with the base value: the asset value is divided by the base value and represented as a decimal number.
Base value	A reference number used in computing asset weights. Typically, you can choose the initial portfolio value, or the initial portfolio value plus the cash flow value, as the base value. You can also choose an arbitrary “assigned value” as the base value.
Active risk	Tracking error; a measurement of portfolio risk with respect to a benchmark.
Total risk	The standard deviation of the portfolio returns.

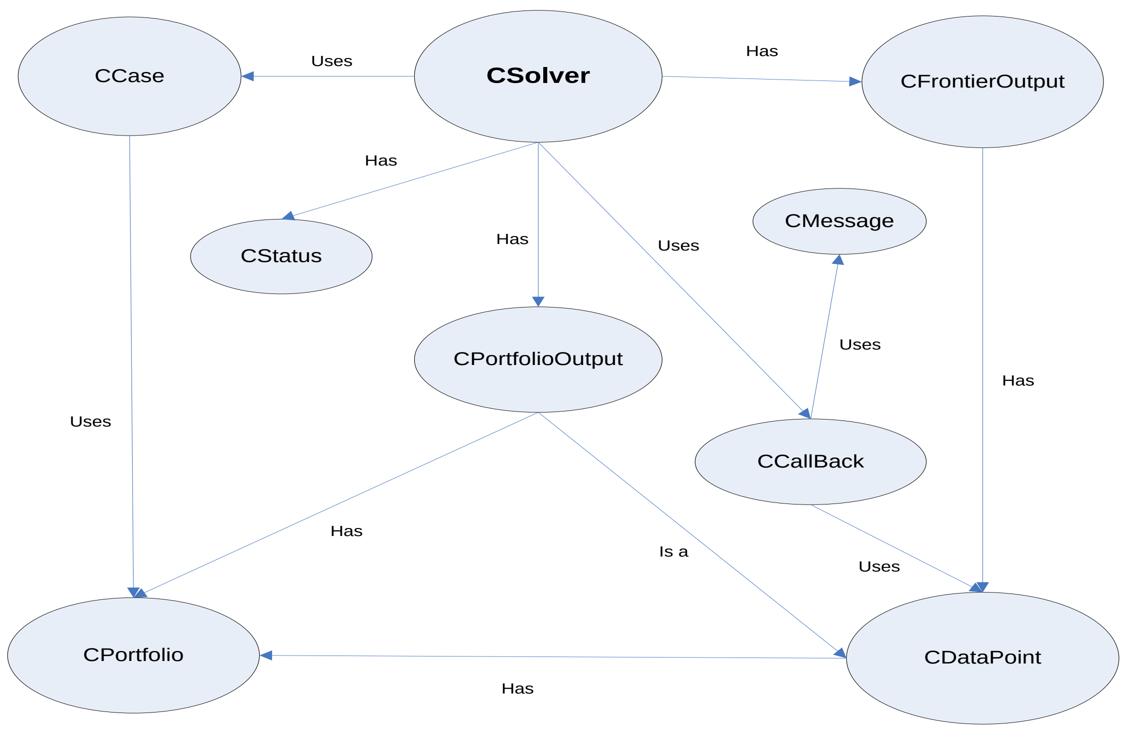
Appendix D: Object Model

The Barra Optimizer C++ API provides a set of classes representing entities used in portfolio optimization. By instantiating these classes and calling their methods, your application can define and solve a portfolio optimization case.

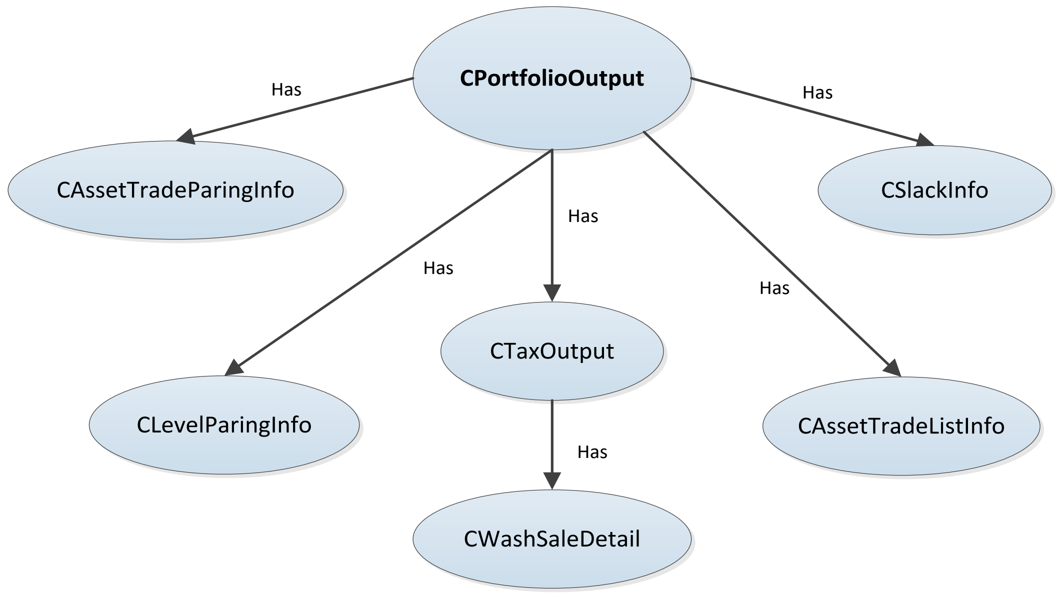
The following figure gives an overview of the major classes in the API:



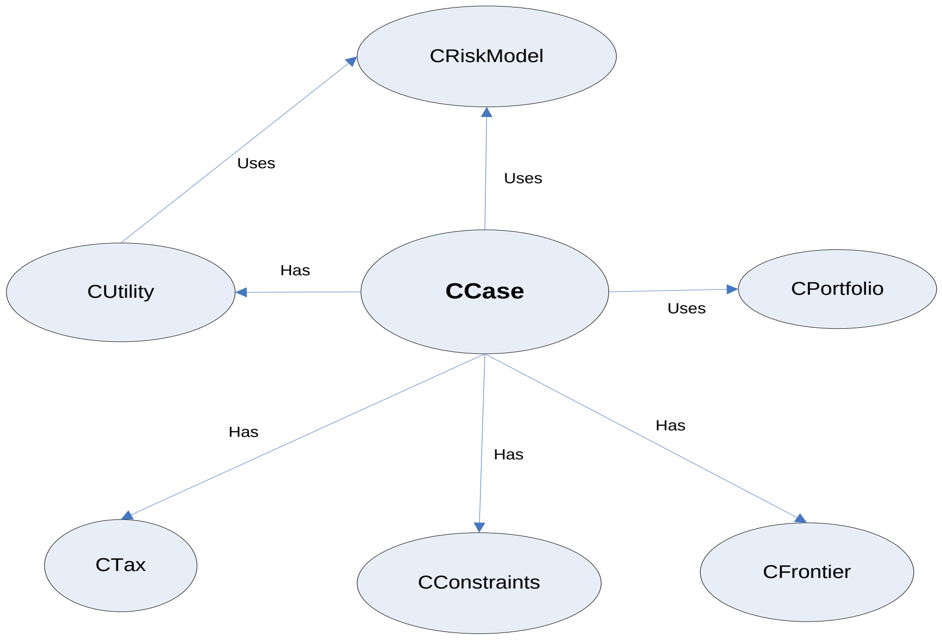
The following figure shows the relationships among the solver-related classes:



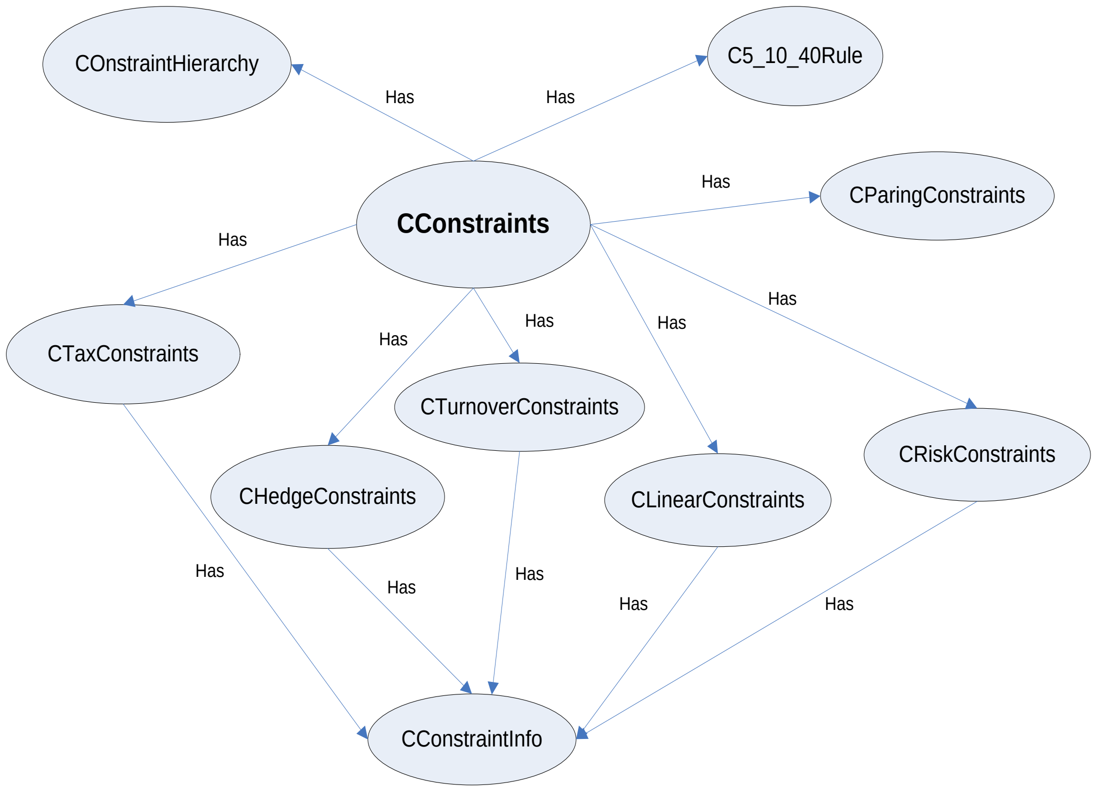
The following figure shows the relationships among the portfolio output-related classes:



The following figure shows the relationships among the case-related classes:



The following figure shows the relationships among the constraints-related classes:



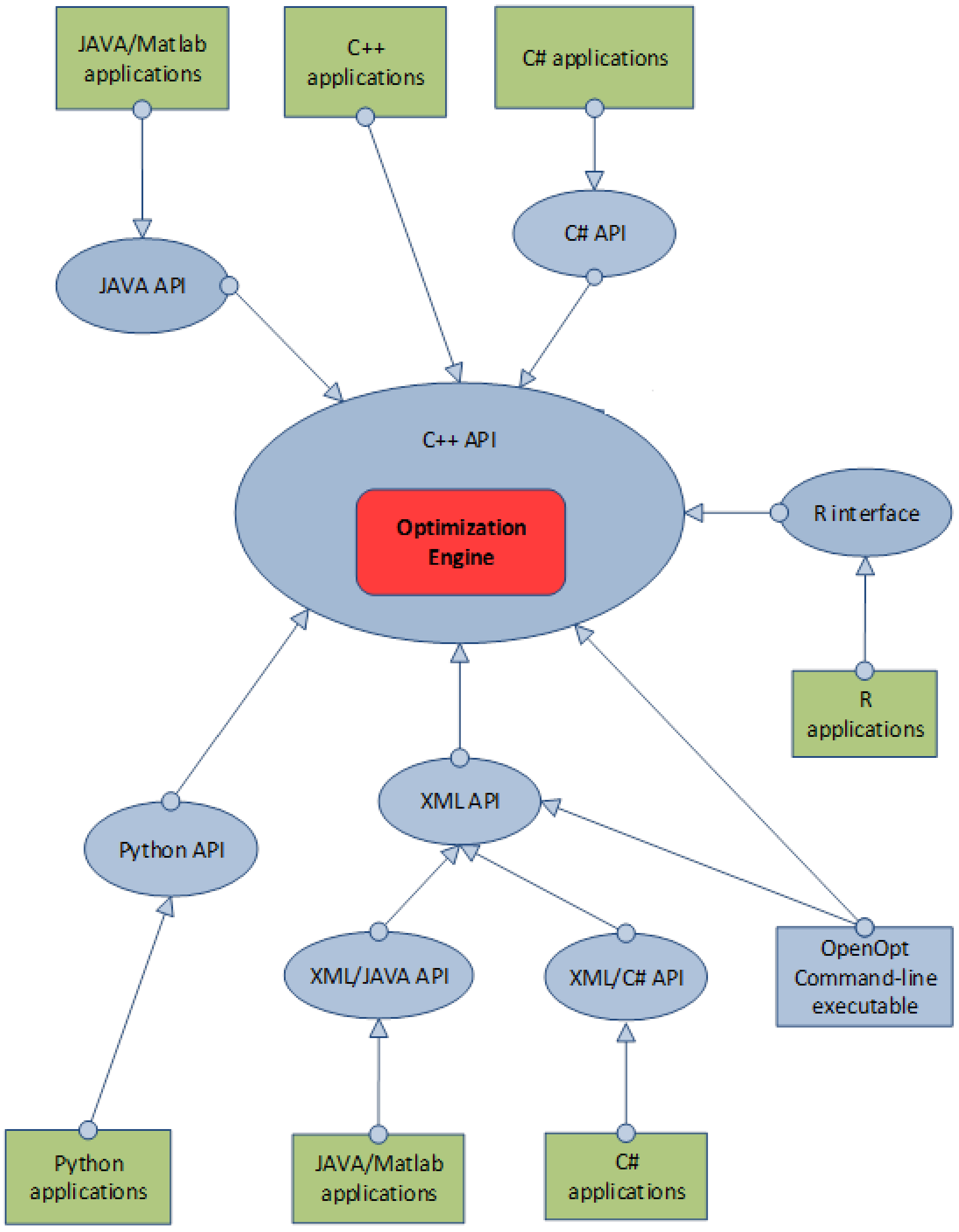
Appendix E: API Class References

Class	Description	Class References		
C5_10_40Rule	Represents 5/10/40 rule	C++	Java	C#
CAccount	Represents an account in multi-account optimization	C++	Java	C#
CAccountGroupTaxOutput	Contains tax related results for an account-group	C++	Java	C#
CAsset	Contains details of an asset	C++	Java	C#
CAssetTradeListInfo	Contains trade list information	C++	Java	C#
CAssetTradeParingInfo	Describes asset/trade paring information of the optimal portfolio	C++	Java	C#
CAttributeSet	Represents a set of key-value pairs	C++	Java	C#
CCallBack	Used for setting callback functions	C++	Java	C#
CCase	Represents an optimization case	C++	Java	C#
CConstraintHierarchy	Represents constraint hierarchy	C++	Java	C#
CConstraintInfo	Contains constraint settings	C++	Java	C#
CConstraints	Represents a collection of constraints	C++	Java	C#
CCrossAccountConstraints	Represents cross-account constraints for Multi-Account Optimization	C++	Java	C#
CDataPoint	Represents an optimal point on the efficient frontier	C++	Java	C#
CExpectedShortfall	Represents parameters of Expected Shortfall optimizations	C++	Java	C#
CFrontier	Represents an efficient frontier	C++	Java	C#
CFrontierOutput	Contains output data for an efficient frontier	C++	Java	C#
CHedgeConstraints	Represents long/short (hedge) constraints	C++	Java	C#
CIDSet	Represents a set of IDs	C++	Java	C#

Class	Description	Class References		
CLevelParingInfo	Describes threshold-violation information	C++	Java	C#
CLinearConstraints	Represents linear constraints	C++	Java	C#
CIssuerConstraints	Represents issuer constraints	C++	Java	C#
CMAOTax	Represents settings for multi-account tax-aware optimization	C++	Java	C#
CMessage	Represents intermediate messages	C++	Java	C#
CMultiPeriodOutput	Represents output of multiple period optimization	C++	Java	C#
CNewTax	Represents settings for tax-aware optimization	C++	Java	C#
CNewTaxConstraints	Represents tax constraints	C++	Java	C#
CNewTaxOutput	Contains tax-related results	C++	Java	C#
CParingConstraints	Represents paring (threshold and cardinality) constraints	C++	Java	C#
CParingRange	Represents constraint settings for asset/trade paring constraint	C++	Java	C#
CPortConcentrationConstraint	Represents portfolio concentration constraint	C++	Java	C#
CPortfolio	Represents a portfolio	C++	Java	C#
CPortfolioOutput	Represents output of the optimal portfolio	C++	Java	C#
CPWLinearFunction	Represents a piecewise linear function	C++	Java	C#
CQuadraticConstraints	Represents quadratic constraints	C++	Java	C#
CRatioConstraints	Represents ratio constraints	C++	Java	C#
CRiskConstraints	Represents risk constraints	C++	Java	C#
CRiskModel	Describes a factor risk model	C++	Java	C#
CRiskParityInfo	Describes risk parity information	C++	Java	C#

Class	Description	Class References		
CRiskTerm	Contains parameters of risk terms in the utility	C++	Java	C#
CSlackInfo	Represents slack information for a constraint	C++	Java	C#
CSolver	Executes the optimization and retrieves the results	C++	Java	C#
CStatus	Describes the status of an optimization	C++	Java	C#
CSymmetricMatrix	Represents a symmetric matrix	C++	Java	C#
CTax	Represents settings for tax-aware optimization	C++	Java	C#
CTaxConstraints	Represents tax constraints	C++	Java	C#
CTaxLot	Represents a tax lot for an asset	C++	Java	C#
CTaxOutput	Contains tax-related results	C++	Java	C#
CTaxRule	Represents tax rules	C++	Java	C#
CTradeListInfo	Contains trade list information between two portfolios	C++	Java	C#
CTurnoverConstraints	Represents turnover constraints	C++	Java	C#
CUtility	Represents the utility function	C++	Java	C#
CWashSaleDetail	Contains wash sale details	C++	Java	C#
CWorkSpace	Represents a workspace	C++	Java	C#

Appendix F: API Diagram



Appendix G: Different Types of Returns

Term	Definition
Total Return	$\mathbf{r}^{\mathrm{T}}\mathbf{h}$, where $\mathbf{r} = (r_1, r_2, \ldots, r_n)^T$ is the vector of asset returns, $\mathbf{h}$ is the vector of asset weights, and n is the number of assets in the investment universe.
Excess Return	$(\mathbf{r} - r_f\mathbf{e})^{\mathrm{T}}\mathbf{h}$, where $\mathbf{e}$ is the vector of 1's and r_f is the rate of return of a hypothetical investment with zero risk (e.g., the interest rate on a three-month US Treasury Bill). It is the leftover after subtracting risk-free return from the portfolio total return.
Active Return	$\mathbf{r}^{\mathrm{T}}(\mathbf{h} - \mathbf{h}_B)$ where $\mathbf{h}_B$ is the vector of benchmark weights. It is the difference between the total returns of the portfolio and benchmark.
Active Excess Return	$(\mathbf{r} - r_f\mathbf{e})^{\mathrm{T}}(\mathbf{h} - \mathbf{h}_B)$. It is the difference between the excess returns of the portfolio and benchmark.

G.1 Which return should be used for the standard mean-variance optimization?

From the optimization point of view, which type of return used does not matter, as long as the portfolio balance constraint is enforced. Any one of the four returns: Total Return, Excess Return, Active Return, or Active Excess Return can be used in the objective, since the difference between them is a constant and would not affect the optimal solution:

Excess Return = $(\mathbf{r} - r_f\mathbf{e})^{\mathrm{T}}\mathbf{h} = \mathbf{r}^{\mathrm{T}}\mathbf{h} - r_f\mathbf{e}^{\mathrm{T}}\mathbf{h} = \mathbf{r}^{\mathrm{T}}\mathbf{h} - r_f \cdot c$[7] = Total Return - $\boxed{r_f \cdot c}$.

Active Return = $\mathbf{r}^{\mathrm{T}}(\mathbf{h} - \mathbf{h}_B) = \mathbf{r}^{\mathrm{T}}\mathbf{h} - \mathbf{r}^{\mathrm{T}}\mathbf{h}_B$ = Total Return - $\boxed{\mathbf{r}^{\mathrm{T}}\mathbf{h}_B}$.

Active Excess Return = $(\mathbf{r} - r_f\mathbf{e})^{\mathrm{T}}(\mathbf{h} - \mathbf{h}_B) = \mathbf{r}^{\mathrm{T}}(\mathbf{h} - \mathbf{h}_B) - r_f\mathbf{e}^{\mathrm{T}}(\mathbf{h} - \mathbf{h}_B) = \mathbf{r}^{\mathrm{T}}\mathbf{h} - \mathbf{r}^{\mathrm{T}}\mathbf{h}_B - r_f(\mathbf{e}^{\mathrm{T}}\mathbf{h} - \mathbf{e}^{\mathrm{T}}\mathbf{h}_B)$

= Total Return - $\boxed{\mathbf{r}^{\mathrm{T}}\mathbf{h}_B - r_f(c - \mathbf{e}^{\mathrm{T}}\mathbf{h}_B)}$.

[7] The constant c is the right-hand side of the portfolio balance constraint (14).

However, the bounds on return-related constraints should be carefully set, and may need to be adjusted to be consistent with the return term in the objective. For example, if you are using Total Return in the objective, but you want to constrain on the active return, then your bounds on the return constraint needs to be adjusted upward by the constant $\boxed{\mathbf{r}^{\mathrm{T}}\mathbf{h}_B}$.

Note that when you choose to remove the portfolio balance constraint (not recommended), using different returns in the objective may lead to different optimal solutions. You can decide upon the appropriate return to be used based on your requirements.

G.2 Which return should be used for maximizing the Sharpe or Information Ratio?

According to their respective definitions, the excess return should be used to maximize the Sharpe ratio and active return should be used for the information ratio.

G.3 Which return should be used for obtaining the efficient frontiers?

All four types of the returns may be used, as long as the return range specified for the efficient frontier is consistent with the return provided for the objective. For example, if the excess return is provided the input file, but the user desires an efficient frontier produced by varying the total return from $[r_{min}, r_{max}]$, then the range specified for the efficient frontier should be adjusted by the constant $\boxed{r_f \cdot c}$ to be $[r_{min} + r_f \cdot c,\ r_{max} + r_f \cdot c]$.

Appendix H: Definitions of Various Risk Terms

This document summarizes the definitions of various risk terms that can be used when constructing risk constraints with the Barra Open Optimizer API.

H.1 Portfolio-Level Risk Terms

Risk	Portfolio	Active
Factor	(Portfolio) Factor Risk: $\sigma_F(\boldsymbol{h}) = \sqrt{\boldsymbol{h}^T\boldsymbol{XFX}^T\boldsymbol{h}}$	Active Factor Risk: $\sigma_F(\boldsymbol{h},\boldsymbol{h}_B) = \sqrt{(\boldsymbol{h}-\boldsymbol{h}_B)^T\boldsymbol{XFX}^T(\boldsymbol{h}-\boldsymbol{h}_B)}$
Specific	(Portfolio) Specific Risk: $\sigma_s(\boldsymbol{h}) = \sqrt{\boldsymbol{h}^T\boldsymbol{Dh}}$	Active Specific Risk: $\sigma_s(\boldsymbol{h},\boldsymbol{h}_B) = \sqrt{(\boldsymbol{h}-\boldsymbol{h}_B)^T\boldsymbol{D}(\boldsymbol{h}-\boldsymbol{h}_B)}$
Total (Factor & Specific)	(Portfolio) Total Risk: $\sigma(\boldsymbol{h}) = \sqrt{\boldsymbol{h}^T(\boldsymbol{D}+\boldsymbol{XFX}^T)\boldsymbol{h}}$	Active Total Risk: $\sigma(\boldsymbol{h},\boldsymbol{h}_B) = \sqrt{(\boldsymbol{h}-\boldsymbol{h}_B)^T(\boldsymbol{D}+\boldsymbol{XFX}^T)(\boldsymbol{h}-\boldsymbol{h}_B)}$

H.2 Portfolio-Level Risk Contribution Terms

Risk Contribution	Portfolio	Active
Factor	(Portfolio) Factor Risk Contribution: $\dfrac{\sigma_F(\boldsymbol{h})}{\sigma(\boldsymbol{h})}$	Active Factor Risk Contribution: $\dfrac{\sigma_F(\boldsymbol{h},\boldsymbol{h}_B)}{\sigma(\boldsymbol{h},\boldsymbol{h}_B)}$
Specific	(Portfolio) Specific Risk Contribution: $\dfrac{\sigma_s(\boldsymbol{h})}{\sigma(\boldsymbol{h})}$	Active Specific Risk Contribution: $\dfrac{\sigma_s(\boldsymbol{h},\boldsymbol{h}_B)}{\sigma(\boldsymbol{h},\boldsymbol{h}_B)}$

H.3 Subgroup Risk (Additive Definition) Terms

Subgroup Risk (from Asset Group *I* or Factor Group *J*)	Portfolio	Active
Factor	Factor Risk from Factor Group *J*: $\sigma_F^A(\boldsymbol{h}, J) = \frac{\boldsymbol{h}^T \boldsymbol{XF}(\boldsymbol{X}^T\boldsymbol{h})_J}{\sigma_F(\boldsymbol{h})}$	Active Factor Risk from Factor Group *J*: $\sigma_F^A(\boldsymbol{h}, \boldsymbol{h}_B, J) = \frac{(\boldsymbol{h}-\boldsymbol{h}_B)^T \boldsymbol{XF}(\boldsymbol{X}^T(\boldsymbol{h}-\boldsymbol{h}_B))_J}{\sigma_F(\boldsymbol{h}, \boldsymbol{h}_B)}$
Specific	Specific Risk from Asset Group *I*: $\sigma_S^A(\boldsymbol{h}, I) = \frac{\boldsymbol{h}^T \boldsymbol{D}\boldsymbol{h}_I}{\sigma_S(\boldsymbol{h})}$	Active Specific Risk from Asset Group *I*: $\sigma_S^A(\boldsymbol{h}, \boldsymbol{h}_B, I) = \frac{(\boldsymbol{h}-\boldsymbol{h}_B)^T \boldsymbol{D}(\boldsymbol{h}-\boldsymbol{h}_B)_I}{\sigma_S(\boldsymbol{h}, \boldsymbol{h}_B)}$
Total (Factor & Specific)	Total Risk from Asset Group *I*: $\sigma^A(\boldsymbol{h}, I) = \frac{\boldsymbol{h}^T(\boldsymbol{D}+\boldsymbol{XFX}^T)\boldsymbol{h}_I}{\sigma(\boldsymbol{h})}$	Active Total Risk from Asset Group *I*: $\sigma^A(\boldsymbol{h}, \boldsymbol{h}_B, I) = \frac{(\boldsymbol{h}-\boldsymbol{h}_B)^T(\boldsymbol{D}+\boldsymbol{XFX}^T)(\boldsymbol{h}-\boldsymbol{h}_B)_I}{\sigma(\boldsymbol{h}, \boldsymbol{h}_B)}$

- I is a subset of asset indexes, and J is a subset of factor indexes
- $(\boldsymbol{X}^T\boldsymbol{h})_J$ is the modified vector $\boldsymbol{X}^T\boldsymbol{h}$ in which rows are all zeroes if their indexes are not in J
- When I includes all asset indexes and J includes all factor indexes (if applicable), risk terms in Table H.3 are identical to those in Table H.1.

H.4 Subgroup Risk Contribution (Additive Definition) Terms

Subgroup Risk Contribution (from Asset Group *I* or Factor Group *J*)	Portfolio	Active
Factor	Factor Risk Contribution from Factor Group *J*: $\frac{\sigma_F^A(\boldsymbol{h}, J)}{\sigma_F(\boldsymbol{h})}$	Active Factor Risk Contribution from Factor Group *J*: $\frac{\sigma_F^A(\boldsymbol{h}, \boldsymbol{h}_B, J)}{\sigma_F(\boldsymbol{h}, \boldsymbol{h}_B)}$
Specific	Specific Risk Contribution from Asset Group *I*: $\frac{\sigma_S^A(\boldsymbol{h}, I)}{\sigma_S(\boldsymbol{h})}$	Active Specific Risk Contribution from Asset Group *I*: $\frac{\sigma_S^A(\boldsymbol{h}, \boldsymbol{h}_B, I)}{\sigma_S(\boldsymbol{h}, \boldsymbol{h}_B)}$
Total (Factor & Specific)	Total Risk Contribution from Asset Group *I*: $\frac{\sigma^A(\boldsymbol{h}, I)}{\sigma(\boldsymbol{h})}$	Active Total Risk Contribution from Asset Group *I*: $\frac{\sigma^A(\boldsymbol{h}, \boldsymbol{h}_B, I)}{\sigma(\boldsymbol{h}, \boldsymbol{h}_B)}$

H.5 Subgroup Risk (Non-Additive Definition) Terms

Subgroup Risk (from Asset Group *I* and Factor Group *J*)	Portfolio	Active
Factor	Factor Risk from Asset Group *I* and Factor Group *J*: $\sigma_F(\boldsymbol{h}, I, J) = \sqrt{\boldsymbol{h}_I{}^T \boldsymbol{X}\boldsymbol{F}_J\boldsymbol{X}^T\boldsymbol{h}_I}$	Active Factor Risk from Asset Group *I* and Factor Group *J*: $\sigma_F(\boldsymbol{h}, \boldsymbol{h}_B, I, J) = \sqrt{(\boldsymbol{h}_I - \boldsymbol{h}_B)^T \boldsymbol{X}\boldsymbol{F}_J\boldsymbol{X}^T(\boldsymbol{h}_I - \boldsymbol{h}_B)}$
Specific	Specific Risk from Asset Group *I*: $\sigma_S(\boldsymbol{h}, I) = \sqrt{\boldsymbol{h}_I{}^T \boldsymbol{D}\boldsymbol{h}_I}$	Active Specific Risk from Asset Group *I*: $\sigma_S(\boldsymbol{h}, \boldsymbol{h}_B, I) = \sqrt{(\boldsymbol{h}_I - \boldsymbol{h}_B)^T \boldsymbol{D}(\boldsymbol{h}_I - \boldsymbol{h}_B)}$
Total (Factor & Specific)	Total Risk from Asset Group *I* and Factor Group *J*: $\sigma(\boldsymbol{h}, I, J) = \sqrt{\boldsymbol{h}_I{}^T (\boldsymbol{D} + \boldsymbol{X}\boldsymbol{F}_J\boldsymbol{X}^T)\boldsymbol{h}_I}$	Active Total Risk from Asset Group *I* and Factor Group *J*: $\sigma(\boldsymbol{h}, \boldsymbol{h}_B, I, J) = \sqrt{(\boldsymbol{h}_I - \boldsymbol{h}_B)^T (\boldsymbol{D} + \boldsymbol{X}\boldsymbol{F}_J\boldsymbol{X}^T)(\boldsymbol{h}_I - \boldsymbol{h}_B)}$

- Subscripting a vector by I (or J) denote replacing its components with zeros if their positions are not in I (or J)
- $\boldsymbol{h}_I$ is the modified holding vector in which all elements are zeros if their positions are not in I
- $\boldsymbol{F}_J$ is the modified factor covariance matrix $\boldsymbol{F}$ in which rows are all zeroes if their indexes are not in J
- When I includes all asset indexes and J includes all factor indexes (if applicable), risk terms in Table H.5 are identical to those in Table H.1.

H.6 Subgroup Risk Contribution (Non-Additive Definition) Terms

Subgroup Risk Contribution (from Asset Group *I* or Factor Group *J*)	Portfolio	Active
Factor	Factor Risk Contribution from Asset Group *I* and Factor Group *J*: $\frac{\sigma_F(\boldsymbol{h}, I, J)}{\sigma(\boldsymbol{h})}$	Active Factor Risk Contribution from Asset Group *I* and Factor Group *J*: $\frac{\sigma_F(\boldsymbol{h}, \boldsymbol{h}_B, I, J)}{\sigma(\boldsymbol{h}, \boldsymbol{h}_B)}$
Specific	Specific Risk Contribution from Asset Group *I*: $\frac{\sigma_S(\boldsymbol{h}, I)}{\sigma(\boldsymbol{h})}$	Active Specific Risk Contribution from Asset Group *I*: $\frac{\sigma_S(\boldsymbol{h}, \boldsymbol{h}_B, I)}{\sigma(\boldsymbol{h}, \boldsymbol{h}_B)}$
Total (Factor & Specific)	Total Risk Contribution from Asset Group *I* and Factor Group *J*: $\frac{\sigma(\boldsymbol{h}, I, J)}{\sigma(\boldsymbol{h})}$	Active Total Risk Contribution from Asset Group *I* and Factor Group *J*: $\frac{\sigma(\boldsymbol{h}, \boldsymbol{h}_B, I, J)}{\sigma(\boldsymbol{h}, \boldsymbol{h}_B)}$

H.7 Risk-by-Asset Terms

Risk from Asset *i*		Portfolio	Active
Total (Factor & Specific)	Additive Definition	(Additive) Total Risk from Asset *i:* $\sigma^A(\boldsymbol{h}, i) = \frac{\boldsymbol{h}^T(\boldsymbol{D} + \boldsymbol{XFX}^T)\boldsymbol{h}_{\{i\}}}{\sigma(\boldsymbol{h})}$	(Additive) Active Total Risk from Asset *i*: $\sigma^A(\boldsymbol{h}, \boldsymbol{h}_B, i) = \frac{(\boldsymbol{h} - \boldsymbol{h}_B)^T(\boldsymbol{D} + \boldsymbol{XFX}^T)(\boldsymbol{h} - \boldsymbol{h}_B)_{\{i\}}}{\sigma(\boldsymbol{h}, \boldsymbol{h}_B)}$
	Non-Additive Definition	(Non-Additive) Total Risk from Asset *i:* $\sigma(\boldsymbol{h}, i) = \sqrt{\boldsymbol{h}_{\{i\}}{}^T(\boldsymbol{D} + \boldsymbol{XFX}^T)\boldsymbol{h}_{\{i\}}}$	(Non-Additive) Active Total Risk from Asset *i*: $\sigma(\boldsymbol{h}, \boldsymbol{h}_B, i) = \sqrt{(\boldsymbol{h}_{\{i\}} - \boldsymbol{h}_B)^T(\boldsymbol{D} + \boldsymbol{XFX}^T)(\boldsymbol{h}_{\{i\}} - \boldsymbol{h}_B)}$

$\boldsymbol{h}_{\{i\}}$ is the modified holding vector in which all elements are zeros except the *i*-th position

H.8 Risk-Contribution-by-Asset Terms

Risk Contribution from Asset *i*		Portfolio	Active
Total (Factor & Specific)	Additive Definition	(Additive) Total Risk Contribution from Asset *i:* $\frac{\sigma^A(\boldsymbol{h}, i)}{\sigma(\boldsymbol{h})}$	(Additive) Active Total Risk Contribution from Asset *i*: $\frac{\sigma^A(\boldsymbol{h}, \boldsymbol{h}_B, i)}{\sigma(\boldsymbol{h}, \boldsymbol{h}_B)}$
	Non-Additive Definition	(Non-Additive) Total Risk Contribution from Asset *i:* $\frac{\sigma(\boldsymbol{h}, i)}{\sigma(\boldsymbol{h})}$	(Non-Additive) Active Total Risk Contribution from Asset *i*: $\frac{\sigma(\boldsymbol{h}, \boldsymbol{h}_B, i)}{\sigma(\boldsymbol{h}, \boldsymbol{h}_B)}$

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