OPLSS PL Background Day 2

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1 Products and Sums

Products and sums, and logical relations, will both be very important tomorrow, so if you haven't finished the Day 1 assignment, continuing with

1. progress and preservation for products and sums
2. termination for products and sums by logical relations

are a good use of time (the implementation task is less important).

2 Type Isomorphisms

In addition, the following will be very helpful for tomorrow's discussion of recursive types. Work in a total (terminating) language with products and sums for this problem.

"Prove" that $\mathsf{nat} \cong 1 + \mathsf{nat}$: write functions

- $\mathsf{show} : \mathsf{nat} \to (1 + \mathsf{nat})$
- $\mathsf{hide} : (1 + \mathsf{nat}) \to \mathsf{nat}$

and informally convince yourself that they compose to the identity in both directions:

- for all values $n : \mathsf{nat}$, $\mathsf{hide}(\mathsf{show}(n)) = n$
- for all values $s : 1 + \mathsf{nat}$, $\mathsf{show}(\mathsf{hide}(s)) = s$

Bonus exercise: Give rules for a type list representing lists of natural numbers and "prove" that $\mathsf{list} \cong 1 + (\mathsf{nat} \times \mathsf{list})$ in the same style.