A Breif Description of the Pigeonhole Principle

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1 Pigeonhole Principle

The Pigeonhole Principle is a mathematical principle used in Discrete math that states that if n items are put into m containers, when $n > m$, then at least one container must contain more than one item.

ex. If 3 pigeons (items) are put into 2 holes (containers) then one of the pigeons must go into the same hole as another pigeon.

Seems easy enough, however, why don't we start to apply some math to this concept. In Discrete Math we use things called Sets. Now a set is just a collection of elements, and elements belonging to the set are called members. The important thing to remember is that all the members of a set are distinct or unique. So each of the members are different, and in a set the order of the list of elements doesn't matter.

Now that we understand what sets are we can discuss what a Finite set is. Finite sets are sets that can in principle be listed as full. The thing about Finite sets is they hace cardinality, that is shown by using,$|_|$.

For example Lets day D represents the days of the week so the set D would equal Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday. To put this mathmatically.

$$D = Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday$$

So applying this to a Finite set we can say that:

$$|D| = 7$$

Or the Cardinality of D in this Sequence is 7.

Now that we understand sets a little more we can start talking about Functions. A Function from one set to another is a rule that will associates each member of the first set with exactly one member in the second set. To say that in some more math terms if f is a function from x to y and $x \in X$ then $f(x)$ is the member of Y that function associates with x. We call these functions mapping as we're mapping elements to other elements.

Lets walk through a problem that will tie all of this together:

$$f : X \to Y, |X| > |Y| \Rightarrow \exists x_1, x_2 \in X : x_1 \neq x_2 \wedge f(x_1) = f(x_2)$$