Math 480 Homework SVD updates

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May 22, 2013

1 Introduction

Consider the real matrix $X \epsilon R^{pxq}$. Then the singular value decomposition (SVD) diagnolizes X with orthonormal matrices $U \epsilon R^{pxp}$ and $V \epsilon R^{qxq}$ Note that $S = U^T XV$.

1.1 Formula's

For Additive Modification, see Section 2. For Rank-1 Modification, see Section 3.

2 Additive Modification

Let $A \epsilon R^{pxc}$ and $B \epsilon R^{pxc}$. Then

$$X + AB^T = \begin{bmatrix} U & A \end{bmatrix} \begin{bmatrix} S & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} V & B \end{bmatrix}^T$$

Furthermore, let P be the orthogonal basis of the column space of $(I - UU^T)A$ and $R_A = P^T(I - UU^T)A$

$$\begin{bmatrix} U & A \end{bmatrix} = \begin{bmatrix} U & P \end{bmatrix} \begin{bmatrix} I & U^T A \\ 0 & R_A \end{bmatrix}$$

.

Let $QR_B = (I - VV^T)B$. Then

$$X + AB^T = \begin{bmatrix} U & P \end{bmatrix} K \begin{bmatrix} V & Q \end{bmatrix}^T$$

and

$$K = \begin{bmatrix} I & U^T A \\ 0 & R_A \end{bmatrix} \begin{bmatrix} S & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} I & V^T B \\ 0 & R_B \end{bmatrix}^T = \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} U^T A \\ R_A \end{bmatrix} \begin{bmatrix} V^T B \\ R_B \end{bmatrix}^T$$

.

Diagonalizing K as $U'^T K V' = S'$ give the rotations U' and V' of the extended subspaces $\begin{bmatrix} U & P \end{bmatrix}$ and $\begin{bmatrix} V & Q \end{bmatrix}$ such that

$$X + AB^T = \left(\begin{bmatrix} U & P \end{bmatrix} U'\right) S' \left(\begin{bmatrix} V & Q \end{bmatrix} V'\right)^T$$

is the desired SVD.

3 Rank-1 Modification

3.1 Update

3.2 Downdate

3.3 Revise

3.4 Recenter

For each operate in subsection 3.1 - 3.4

1. Known for 3.1:
$$US \begin{bmatrix} V^T & 0 \end{bmatrix} = \begin{bmatrix} X & 0 \end{bmatrix}$$

2. Known for 3.2:
$$USV^T = \begin{bmatrix} X & c \end{bmatrix}$$

3. Known for 3.3:
$$USV^T = \begin{bmatrix} X & c \end{bmatrix}$$

4. Known for 3.4:
$$USV^T = X$$

4 Reference

All work within this paper is taken from Matthew Brand's article on "Fast low-rank modifications of the thing singular value decomposition."