Lot Sizes, Welfare and Urban Structure: A View from the United States*

James Macek†

July 14, 2024

Abstract

In this paper, I consider the effect of minimum lot size regulation on welfare and urban structure. I show that minimal lots are the most expensive in the low-density neighborhoods of productive cities relative to others, and this can explain the sorting on income into these cities and neighborhoods. Motivated by this evidence, I construct a general equilibrium model in which households of heterogeneous incomes choose cities and neighborhoods, value affluent neighbors, and are burdened differently by regulation. A counterfactual deregulation exercise shows significant and progressive welfare gains for renters (9% of income) that offset the losses to landowners (17% of land values). The exercise also reveals two surprising results. First, any productivity gains that occur from the expansion of productive cities is largely nullified by the out-migration of affluent households who prefer regulated neighborhoods. Second, the neighborhood choice externality arising from the demand for affluent neighbors matters little for every household. These results suggest that the most important consequence of deregulating housing markets is increasing housing affordability.

*I am indebted to Nathaniel Baum-Snow, William Strange, Kevin Lim and Joseph Steinberg for their patience, guidance and support. I have benefited substantially from discussions with David Albouy, Boaz Abramson, Devin Dziadyk, Ruben Gaetani, Kris Gerardi, Ingrid Gould Ellen, Stephan Heblich, Guangbin Hong, Hans Koster, Gabriel Kreindler, Timothy McQuade, Peter Morrow, David Krisztian Nagy, Albert Saiz, Aradhya Sood and Yue Yu. I also thank all participants from the Toronto Urban, Trade and Macro seminars, as well as the 2024 AREUEA Poster Session, for comments and feedback.

†University of Toronto, Canada.

1 Introduction

In recent decades, the rapid rise of US housing prices has been ascribed to strict housing regulation (Molloy et al., 2022; Gyourko et al., 2013). However, these regulations have implications that extend beyond the issue of high housing prices; they have been found to slow aggregate growth by limiting density in big cities (Hsieh and Moretti, 2019; Duranton and Puga, 2023). A particular type of regulation – the minimum lot size – also causes differences in opportunity and affluence across cities and neighborhoods by excluding those who cannot afford large lots (Song, 2021; Kulka, 2019). In this paper, I ask how these minimum lot sizes shape housing affordability, welfare, inequality, and income segregation within and across cities. Understanding minimum lot size restrictions in a way that accommodates migration both within and across labour markets is important because they are prevalent and vary substantially across the United States (Gyourko et al., 2021). They are also an actionable policy lever.

Previous work in the macroeconomics of housing regulation ignores the importance of the income sorting that these regulations cause. The standard analysis, one emphasized by Hsieh and Moretti (2019) and Duranton and Puga (2023), is that regulations slow aggregate growth by preventing workers from accessing productive cities that are responsible for that growth. However, loosening regulation in expensive cities in the presence of sorting causes high income, productive households to leave, attenuating productivity growth that would have been achieved in the absence of such sorting. Moreover, these migration responses may be reinforced by endogenous changes to residential amenity value, a point emphasized in a recent literature (Diamond, 2016; Amalgro and Dominguez-Iino, 2021). To motivate this view, I show empirically that the prices of minimal lots are higher in more productive cities; and this explains some positive sorting on income into these cities. These demand side effects have also received little attention in computing the aggregate welfare impacts of housing regulations. In this paper, I ask by how much large scale deregulation affects aggregate productivity, and in particular relative to the accompanying increases in housing affordability.

Focusing on income sorting across cities also masks important sorting happening across neighborhoods *within* these cities. I show empirically that there is negative income sorting on residential density within cities, and that this sorting is significantly stronger in expensive cities after controlling for other income correlates. Variation in the prices of minimal lots explain these differences in the income-density gradient across cities. This suggests a mechanism driving income sorting that differs from access to public transportation (Glaeser et al., 2008), topographical and historical amenities (Brueckner et al., 1999) or filtering dynamics (Brueckner and Rosenthal, 2009). Accounting for this income sorting within cities is crucial for gauging the welfare impacts of these regulations because they alter neighborhood quality; conferring external costs or benefits on residents. The typically held view is that housing regulation is a tool to limit the negative externalities associated with too many lower income households free riding off amenities in rich neighborhoods (Calabrese et al., 2007; Hamilton, 1975). In this paper, I ask by how much these externalities contribute to the costs of large scale deregulation, and whether this outweighs the accompanying increases in housing affordability.

To evaluate the welfare consequences of minimum lot size restrictions, I construct a general equilibrium model encompassing the metropolitan United States. In the model, households differ on income and choose cities and neighborhoods subject to a varying intensity of regulation and non-homothetic demand for housing. Minimum lot sizes impose a floor on housing consumption required to live in a neighborhood, as in Kulka (2019) or Calabrese et al. (2007). Tight regulation excludes the poor by constraining their choices over small and affordable housing, thereby increasing neighborhood affluence. The model also incorporates rich heterogeneity across locations along two dimensions. First, cities differ on labour productivity, so that any changes in labour supply across cities affects aggregate productivity. Second, neighborhoods differ both exogenously and endogenously on amenity values by income; increases in average income cause increases in amenity value as in Brueckner et al. (1999) or Guerrieri et al. (2013) with elasticities that vary by income. I show theoretically that this externality can justify the use of minimum lot sizes if income segregation in the absence of regulation is strong. In such a setting, regulating rich neighborhoods can induce the movement of the poorest households in rich neighborhoods to become the richest households in poorer neighborhoods, increasing average incomes and amenity values everywhere. If income segregation in the absence of regulation is instead weak, I show that stringent minimum lot sizes produce segregation patterns that benefit rich neighborhoods necessarily at the expense of poor ones. This same logic has been used to argue that fiscal centralization is typically more efficient than fiscal decentralization (Calabrese et al., 2011). I contribute by taking a model to the data that is rich enough to nest both theoretical predictions at differing parameter values to which the data can speak.

Using this model to study regulation is challenging because of two methodological issues. First, minimum lot sizes are difficult to measure, especially with broad geographic coverage, because they vary by local jurisdiction and are in most cases not publicly available. I use a similar procedure to detect minimum lot sizes to that in Song (2021) and Cui (2023), leveraging CoreLogic's property assessment database. These minimum lot sizes enter directly into the calibration of the model, along with the estimates of housing supply elasticities from Baum-Snow and Han (2021).

Second, inferring the causal effect of neighborhood affluence on amenities is difficult because unobserved amenities likely cause income sorting. Correctly identifying the strength of this relationship matters for the welfare analysis, as it widens the scope for regulation to correct the neighborhood choice externality. I address this endogeneity issue by proposing an instrument based around terrain slopes. It has long been known that neighborhoods with steeper slopes have higher income residents (Saiz, 2010), but these slopes are likely natural amenities and thus cannot be used as instruments alone. Instead, I assume the amenity value of sloped terrain decays rapidly when moving away from a neighborhood. This justifies the use of a neighborhood-level "donut" design; the income of a neighborhood is instrumented with the slopes of other neighborhoods that are within some distance band. Donut identification designs are prevalent in the IO literature (Bayer et al., 2007), and have been used for different applications in the housing regulation context (Anagol et al., 2021). Using the instrument, I find that doubling income increases neighborhood value by approximately 23% for an average household. Moreover, this elasticity is increasing with income. Doubling income increases neighborhood value by 13.5% for the lowest income households and 30% for the highest, suggesting that the endogenous amenity response to neighborhood income causes self-reinforcing sorting, as in Diamond (2016) or Su (2022). The instrument corrects for a large downward bias and the results are robust to a host of different controls, donut definitions, calibration strategies, and a placebo test that exploits time variation in neighborhood income changes.

I use the model to first study the long-run implications of a nationwide elimination of lot size restrictions, paying special attention to the relative importance of its effect on housing affordability, the external costs of neighborhood choice, and aggregate labour productivity. The policy change delivers an average gain of 9.4% for renters[1] and is strongly progressive, while absentee landowners lose because land values fall by 17.6% nationally in comovement with rents[2]. Renters of all incomes are made better off. At odds with evidence in the literature, the counterfactual also reveals very little aggregate productivity gains associated with the expansion of productive cities, at 0.26%[3]. Instead, renters benefit primarily from the opportunity to consume smaller and more affordable homes. On the other hand, I find minor evidence that regulation is correcting the neighborhood choice externality. Renters of all incomes gain less after deregulation when allowing amenities to respond endogenously to the income composition of a neighborhood. Aided by theory, this suggests that neighborhoods that would be rich absent regulation are imposing stringent restrictions and preventing free-riding. However, the welfare gains are attenuated by only 1.2 percentage points for the average household. This means that minimum lot sizes are inefficient at correcting the externality: they do not appear to increase the amenity value of neighborhoods by much relative to accompanying distortions to housing consumption. This is in contrast to quantitative findings in the local public finance literature (Calabrese et al., 2007, 2011). Instead, households of all income levels move in response to deregulation, causing many different neighborhoods to either increase or decrease their amenity value with little change to the average neighborhood. Migration patterns within expensive cities after deregulation look very similar to recent US gentrification; affluent households move away from previously regulated neighborhoods and toward dense ones. This result is motivated by the empirical observations, which suggest that the strong income-density gradient in expensive cities is caused by minimum lot size regulation[4]. Taken as a whole, these results suggest that housing affordability is by far the most important consequence of large-scale deregulation.

Motivated by recent policy changes, I also use the model to study a unilateral halving of minimum lot sizes in San Francisco. An average renter in the country benefits approxi-

[1] This is measured using the population-weighted compensating variation expressed as a percentage of income.

[2] To weigh landower losses against renter welfare, I model the disutility associated with capital losses on land by income level, with more details in Section 6. Aggregate welfare with this approach is smaller but still sizable, at 5.6%.

[3] Income sorting responses to deregulation drive this low value. Assuming cities changed at their predicted levels and holding city income fixed, aggregate productivity growth would instead be 1.41%. These results are also robust to considering agglomeration economies at typical values (Combes and Gobillon, 2015), production complementarities between low and high skill labour (Card, 2009), and skill-augmenting agglomeration economies (Diamond, 2016; Baum-Snow et al., 2018).

[4] In particular, I find that the highest density neighborhoods in expensive cities observe a 20% increase in incomes after deregulation; this represents a complete flattening of the income density gradient in them.

mately 0.1% from this policy change. In contrast, I find evidence that national land values fall by 0.2% and the highest income renters are made worse off by 0.02%[5]. This is driven entirely by income sorting and the subsequent deterioration of amenity values within the city. If amenities were exogenous, welfare of all renters would additionally increase by an average of 0.08 percentage points, along with increasing land values both in the city and nationwide.

This paper builds upon several strands of literature within macroeconomics, housing regulation, urban, and public economics. First, this paper challenges the idea that housing deregulation must lead to the growth of productive cities. This is recognized by virtually all work in the macroeconomics of housing regulation as a large benefit of deregulation (Hsieh and Moretti, 2019; Duranton and Puga, 2023; Parkhomenko, 2023; Herkenhoff et al., 2018; Bunten, 2017). My model yields an opposing conclusion because of household heterogeneity. Stringent regulation in productive cities affect both the intensive and extensive margin of household labour supply in opposing directions.

On the other hand, this paper complements theory and evidence of the effects of housing regulation on housing prices, city structure and income segregation (Molloy, 2020; Gyourko and Molloy, 2015; Turner et al., 2014; Glaeser and Gyourko, 2018; Brueckner and Singh, 2020; Anagol et al., 2021; Bertaud and Brueckner, 2005; Mills, 2005; Hilber and Robert-Nicoud, 2013; Ortalo-Magné and Prat, 2014), and particularly the minimum lot size (Zabel and Dalton, 2011; Song, 2021; Kulka, 2019; Cui, 2023; Molloy et al., 2022; Kulka et al., 2022; Grieson and White, 1981; White, 1975). Using both a discontinuity design and a model, Song (2021) and Kulka (2019) show extreme income and racial sorting on minimum lot sizes, and Kulka et al. (2022) show that height restrictions and unit density restrictions work in tandem to increase housing prices. I differ from this work on two dimensions; these papers do not incorporate endogenous amenities[6] and they do not model heterogeneous city technologies that determine aggregate productivity.

This paper also builds upon relatively recent work studying sorting on income and other demographics (Diamond, 2016; Baum-Snow and Hartley, 2020; Couture and Handbury, 2020; Couture et al., 2019; Gyourko et al., 2013; Su, 2022; Baum-Snow and Pavan, 2011; Brueckner and Rosenthal, 2009; Fogli and Guerrieri, 2019; Glaeser et al., 2008; Brueckner et al., 1999; Lee and Lin, 2017). I add to this literature by providing theory and evidence that regulation causes income sorting on density in cities with high housing prices. In addition, a major lesson from this literature is that exogenous demographic changes can be positively reinforced by the endogenous supply of amenities. I also highlight the role of endogenous amenities in both sorting patterns and welfare gains after deregulation, and think carefully about identifying this relationship.

[5]That is, renting households that make more than $\$200,000$ annually.

[6]Song (2021) directly models preferences over minimum lot sizes in a discrete choice model. This may be thought of as a stand-in for endogenous amenities. However, minimum lot sizes are likely correlated with unobserved demand factors, and no instrument is used in the structural model to address identification of demand parameters. Moreover, directly modeling preferences over minimum lot sizes results in a model with no fiscal externalities, and this misses a key channel that affects welfare (Hamilton, 1976; Calabrese et al., 2007). On the other hand, Kulka et al. (2022) use a discontinuity design to identify neighborhood demand for a low density of housing units. I posit an alternative specification – demand for high income neighbors.

Lastly, this paper builds on the local public finance literature, particularly the idea of housing regulation as an efficient substitute for head taxation (Hamilton, 1976; Calabrese et al., 2007; Fernandez and Rogerson, 1997; Epple and Platt, 1998; Calabrese et al., 2011; Barseghyan and Coate, 2016). In my model, the relationship between amenities and neighborhood income composition can be similarly interpreted in the context of local public goods, and creates a similar neighborhood choice externality. However, I do not endogenize the choice of regulation, and instead consider it as a lever than can be freely changed by a social planner. This paper makes three innovations. First, my model is calibrated using direct measures of minimum lot sizes, rather than inferred indirectly in a political economy equilibrium. I find that regulation has distorted housing consumption considerably more in the aggregate in comparison. Second, my model can match any observed level of income sorting in the data, and I show both theoretically and with the calibrated model that this affects the external costs of neighborhood choice. The logic behind this theoretical result has been used in Calabrese et al. (2011) to argue why fiscal centralization (the pooling of public good provision across space) is frequently better than decentralization in the absence of head taxes or regulation. Third, I allow for imperfect labour mobility, which amplifies the amount of regulation needed to implement an efficient neighborhood income distribution. These reasons can explain why I find smaller costs associated with the neighborhood choice externality when compared to affordability benefits.

This draft is organized as follows. Section 2 introduces the data sources and motivating facts, Section 3 introduces the model, Section 4 calibrates the model to US cross section, 5 estimates the relationship between neighborhood amenity value and income, and 6 performs counterfactuals.

2 Data and Motivating Evidence

In this section, I argue that variation in the stringency of minimum lot size regulation both within and across cities can explain broad patterns of income sorting we see in the data. To establish these facts, I draw on two main sources of data outlined below, with a full description of how they are constructed in Appendix A.

2.1 Data

Geography and Demographics The primary unit of analysis for both the model and the empirical work is the 2020 definition census block group, which I define to be a *neighborhood*. I think of these block groups as representing the smallest geographical unit by which there is meaningful variation in location characteristics that factor into housing demand. Block groups are also often adhere to political boundaries, and are likely to have little variation in both regulation and the choices of residential structures. Moreover, block groups are small enough to reasonably capture demand spillovers that may arise from the presence of affluent neighbors. Each block group is associated with one *city*, and these are defined as 2013-definition Metropolitan Statistical Areas (MSAs). I think of these cities as self-contained labour markets. There are approximately 196,000 block groups allocated to 377 cities in the main sample used to derive the empirical facts. For each block group, I take household income distributions and other demographic information from the pooled 2016-2020 American Community Survey (ACS)[7], housing unit counts from the 2020 Census, and various neighborhood data from both the ACS and the 2016-2017 National Neighborhood Data Archive (NaNDA) for use as controls in estimation. I also use the 2008-2012 ACS and 2007-2010 NaNDA for ancillary robustness checks.

Property Assessments and Transactions Local jurisdictions collect detailed data on residential structures, such as the lot size, construction material, heating, water, and AC systems to calculate property taxes. These data are digitized and harmonized by CoreLogic, which I leverage in this paper. I take the most recent assessment of each residential property as of December 2022, and use CoreLogic's internal assignment of the coordinates of each property to match them to 2020 definition block groups. I combine these assessments with CoreLogic's universe of arms-length housing transactions from 2016-2022. I also use transactions from 2008-2012 for robustness checks. For the forthcoming empirical work, I use both these datasets to construct measures of land value per acre, as well as measure minimum lot sizes.

Minimum lot sizes Studying minimum lot sizes is challenging because of the difficulty of collecting and harmonizing data that vary at small spatial scales and potentially within jurisdictions. In recent work, Song (2021) and Cui (2023) infer regulation using only the observed distribution of lot sizes within some geographical boundary where regulation is assumed uniform. The method predicates itself on the idea that, if building on a lot below the minimum were costly from the perspective of a developer, we'd observe a "bunching" of lots around that minimum. This means that the mode of the observed lot size distribution is close to the level of regulation we observe in the data. Figure 12 in Appendix A.3 provides an example from Hayward, California where this bunching around the mode is both visible and accurately suggests the 5000 square foot minimum lot size. I adapt this method for my empirical setting, and test its performance on new data. I provide a broad description of the algorithm in this section, and provide all other details in Appendix A. In what follows and throughout the paper, I assume that regulation only applies to what I call *regulated structures*, henceforth defined to be single family homes, duplexes, triplexes and fourplexes.

Constructing Zoning Districts The first challenge to observe lot size bunching is to accurately construct a geographic boundary that is as large as possible subject to regulation being uniform within it. I call these geographic boundaries *zoning districts*. For approximately 66,000 block groups (one third of the sample), I use populated zoning codes from the assessments to identify districts[8]. These codes only identify geography and convey no information about regulatory stringency themselves.

[7]This includes all sources of income elicited in the ACS, including reported capital gains and income on rental properties.

[8]I use the modal zoning code in a block group to aggregate from the parcel level. Missing codes are omitted from the calculation of the mode.

To complete coverage, I construct zoning districts from the remaining block groups using a data-driven approach. There is a key trade off. If the choice of geography is too large, we would potentially observe multiple instances of bunching reflecting different levels of regulation and have no way to distinguish between them. If the choice of geographic unit was too small, we would potentially observe spurious discontinuities in the distribution of lot sizes because of the lack of observations. To get the right level of aggregation, I first start by identifying local jurisdictions that are typically responsible for setting regulation – these tend to be incorporated municipalities. CoreLogic reports the municipality associated with an assessment[9]. Within each municipality, I cluster block groups into zoning districts using the algorithm of Chavent et al. (2018). This algorithm allows for the weighing of the importance of geographical proximity when defining clusters. Apart from geographic proximity, I cluster on the mode of the lot size distribution of regulated structures. In Appendix A.2, I detail how my approach differs from Song (2021), how I select the size of an average cluster, as well as test the algorithm on alternative definitions of a jurisdiction. Each of these decisions is associated with a hyperparameter that I validate on new data, and importantly, the following facts are robust to a large range of these parameters.

Detecting Minimum Lot Sizes The zoning districts define a set of lot size distributions that can be used to detect bunching. Recall that I assume four types of residential structures to be *regulated*: single family homes, duplexes, triplexes and quadriplexes. These structures tend to be regulated differently within a jurisdiction; for example, the minimum lot size associated with a duplex is often greater than that of a single family home. I construct lot size distributions for each of these types of structures separately[10]. For each structure type and zoning district, I calculate the minimum lot size as the smallest mode of the associated lot size distribution[11]. Then, I adjust the physical minimum lot size by the implied number of units per lot for each structure type[12]. I call the resulting statistic the *unit density restriction*. Finally, I select the unit density restriction to be the smallest adjusted of these restrictions across all structure types. In a minority of block groups, the calculated mode of the distribution is sometimes large relative to the mean. To ensure these do not drive results, I set unit density restrictions in block groups whose modal lot size is 2 times greater than the mean to zero; the following facts are robust to a host of different thresholds for which to perform this cleaning, including none at all.

Validating Minimum Lot Sizes To select among clustering hyperparameters and to test the performance of the algorithm in general, I use two data sources. The first is the Terner California Land Use Survey, which covers practically all muncipalities in California. At the optimal set of hyperparameters, the algorithm predicts this data with a 4.5% me-

[9]Although rare, block groups cross municipality borders, so I take the modal municipality across parcels to aggregate to the block group. For properties with missing municipality information, I assume the county is responsible for setting regulation. This is typical for many unincorporated locations. Finally, I also restrict zoning districts that are constructed from codes to never cross municipal borders.

[10]I exclude condominiums from this construction if the assessment does not provide information on the associated number of housing units.

[11]This approach differs from Song (2021), who uses a structural break estimator. I find that taking the mode performs as well. Cui (2023) uses what is effectively an excess mass estimator, and measures regulatory changes over time.

[12]For example, a duplex allows for two housing units per lot, so I divide the minimum lot size in half to arrive at a measure of the minimum amount of land per housing unit.

dian absolute error, which is in line with similar exercises in Song (2021) and Cui (2023). However, minimum lot sizes in this data are aggregated at the municipality level, while in most cases they vary within municipalities. Understanding within-city variation in minimum lot sizes is a big purpose of this paper. To test how well the algorithm works at capturing sub-municipality variation, I hand collect and overlay official zoning maps from 13 large US cities. The median absolute error on this data is larger but very accurate, at 16%. In Appendix A.4, I detail how the estimated minimum lot sizes are merged to each of these data sources. Figure 11 in Appendix A.2 shows the optimal set of zoning districts for Hayward, California. The algorithm places high weight on geographic proximity in defining clusters and has an average cluster size of roughly 3 block groups, with a standard deviation of approximately 10. A majority of block groups are their own zoning district.

2.2 Motivating Evidence

I establish two broad facts. The first links residential density (the number of housing units per unit of land) to income within cities, hearkening back to literature that seeks to explain why US downtowns are poor (Brueckner and Rosenthal, 2009; Glaeser et al., 2008; Brueckner et al., 1999). In particular, I find that the negative income-density gradient is larger in expensive ”superstar” cities, such as New York or San Francisco. Since residential density is directly affected by lot sizes, the second fact links both observations to variation in the stringency of lot size regulation, both within and across cities. Taken together, these suggest that regulation has fundamentally altered urban structure, and suggests that deregulation will cause high density neighborhoods in expensive cities to gentrify. This also motivates a model that can fit this urban structure and understand its implications for welfare analysis. I start with Fact 1.

Fact 1. *(The geography of residential income sorting)*

1. *Within cities, there is a negative relationship between household income and residential density.*

2. *Expensive cities exhibit a stronger income-density gradient, especially after controlling for income correlates, such as building age, public transportation use, and various observed amenities.*

To show the first part of Fact 1, I order neighborhoods in each city by the density of housing units (dividing the 2020 census household count with the measured land mass of the neighborhood). Since block groups are drawn to have roughly similar populations, larger cities have more neighborhoods in the sample. I normalize this ranking so that it forms an even partition of the unit interval within each city, ensuring cities of differing sizes are comparable. A linear regression of log average household income against this ranking (with city fixed effects) reveals a striking negative relationship – the least dense neighborhood is associated with half as much income compared to the most dense neighborhood in an average city.

Residential density is an endogenous object: it reflects several underlying factors that must be driving income sorting. High density neighborhoods may contain buildings that are old and dilapidated, thus attracting the poor (Brueckner and Rosenthal, 2009). These neighborhoods might also be easily accessible by public transportation, which the poor are more likely to use (Glaeser et al., 2008). The second part of Fact 1 says that controlling for these underlying factors reveals a leftover income-density gradient that is *stronger* in dense cities. To show this, I split the main estimation sample into a "superstar" sample containing cities that are in the top quartile of unadjusted housing prices and housing unit density, and a sample containing all other cities[13]. I consider the following partial linear model over neighborhoods i and cities c in each sample

$$\log(\text{Income}_{ic}) = \mathbf{S}(\text{RankedHousingDensity}_{ic}) + \sum_{x \in \textbf{Controls}} \beta_x x_{ic} + \epsilon_{ic} \tag{1}$$

where $\mathbf{S}$ is a cubic spline and the β_x are coefficients to be estimated. **Controls** include building age, shares of individuals using cars and public transport, density of public transportation, average commute time, household size, density of bars and coffee shops, among others; I include the full list below. All variables are demeaned by the city average. I estimate two versions of Equation (1) on each sample: one with no controls ($\beta_x = 0$ for every x), and one including the full set. The objective is to document the shape of the function $\mathbf{S}$ across samples, and I do so in Figure 1.

Panel A of Figure 1 reports $\mathbf{S}$ for superstar cities (in blue) and non-superstar cities (in red) with excluded controls. There is a clear negative relationship across samples for most points along the density distribution. There is also a significant difference in the income-density gradient; we see that the high density neighborhoods in dense cities are relatively less affluent, with the exception of block groups in the top 10 % of the density distribution. Visually, this corresponds to the blue curve being below the red curve for most neighborhoods the top half of the density distribution. The pattern becomes even more stark in Panel B after accounting for other observable characteristics of these neighborhoods that cause income sorting. Panel B also shows that the differences in the gradients are large. For example, a neighborhood that is equivalent on observables in the 75th percentile of the density distribution would be roughly 10% poorer relative to the mean in dense cities, and similarly 7.5% richer relative to the mean for neighborhoods in the 25th percentile.

Fact 1 suggests a mechanism that is correlated with density and drives income sorting. This mechanism must also act differently in expensive cities in order to rationalize differences in the income-density gradients. It has long been argued that productive cities are more likely to impose stringent regulation (Hilber and Robert-Nicoud, 2013; Parkhomenko, 2023; Duranton and Puga, 2023). I echo this message and make an additional argument: within-city variation in regulation across neighborhoods is fundamentally different in dense cities. This leads to Fact 2.

Fact 2. *(The geography of minimum lot sizes)*

1. *Low density neighborhoods in expensive cities exhibit relatively higher regulatory strin-*

[13] There are many ways to define "superstar" cities – using housing prices, wages and density alone – that yield the same conclusions.

gency than cheap cities. Conversely, high density neighborhoods in dense cities are relatively less stringent. This explains the stronger income-density gradient in expensive cities.

2. *The stringency of lot size regulation is significantly higher in expensive cities.*

To show Fact 2, I first propose an intuitive measure of regulatory stringency that has foundations in the forthcoming model. This measure relies on two simplifying assumptions. First, the measured unit density restriction must be uniform within neighborhood i. Second, I assume that housing services are supplied at a rate proportional to the size of a lot within i. I define the novel *stringency of minimum lot size regulation I_{ic}* as

$$I_{ic} = \text{LandValueDensity}_{ic} \times \text{UnitDensityRestriction}_{ic} \times \text{FractionRegulated}_{ic} \tag{2}$$

where $\text{LandValueDensity}_{ic}$ is the housing value per acre in neighborhood i and city c constructed from the CoreLogic data. $\text{FractionRegulated}_{ic}$ is the ACS fraction of households who live in *regulated structures*, which I define as those between 1 and 4 housing units per lot. I measure housing values per acre using a interpolation procedure in Appendix A.3. Under the assumption that all other structure types do not face any regulatory constraints, I_{ic} can be interpreted as the expected value of a minimal lot faced by a randomly selected household in (i, c). In the theoretical model, higher stringency levels imply stronger income sorting – households with low income are forced to spend a fraction of their income to rent the minimal lot beyond what they would if they could choose their housing consumption freely. Empirically, this measure is extremely powerful at predicting neighborhood income both within and across cities. A log-linear regression of income on (2) across block groups yields an R^2 of 0.43[14].

The first part of Fact 2 says that the stronger income-density gradient in expensive cities is met with a similarly stronger stringency-density gradient, suggesting that regulation is an underlying explanation. To show this, I alter the regression in Equation (1) such that the dependent variable is instead I_{ic} demeaned by the average in c. The objective is the same as in the exercise in Panel A of Figure 1: to plot the function S associated with this regression for both city samples and compare them. I do so in Panel A of Figure 2, with controls excluded.

Panel A of Figure 2 reveals a similar pattern to Panel B of Figure 1: high density neighborhoods in expensive cities exhibit relatively less stringency, and low density neighborhoods relatively higher stringency. A qualitatively similar pattern holds when conditioning on controls, so the associated plot is omitted. These differences are large. The highest density neighborhood in a expensive city has the value of a minimal lot that is almost $\$125,000$ lower relative to the city mean. At the 25th percentile, expensive cities have a value that is roughly $\$25,000$ greater. These differences represent a significant proportion of the average value of a home, especially in high density neighborhoods. Conceptually, the differences in relative stringency at different points along the density distribution accompany differences in the *types* of residential structures that appear. Think about the

[14] I provide summary statistics for this measure and its components in Appendix A.5; they are additionally broken down by superstar city status, reported for select cities, and plotted against measures of city productivity.

densest neighborhood in Los Angeles – most of the housing units are in large multifamily structures, where there is no concept of a minimum lot size[15]. Contrast this with a low density city like Abilene in Texas, where the densest neighborhoods consist of marginally smaller single family homes.

The second part of Fact 2 says that expensive cities have higher levels of stringency in an average neighborhood. This observation can explain some income sorting into dense cities to the extent that this reflects sorting on skills or other household attributes (Diamond, 2016; Baum-Snow and Pavan, 2011). In Panel B of Figure 2, I show this by repeating the same regression as in Panel A while allowing the average stringency across neighborhoods to vary across the two samples. The figure shows that, in practically every neighborhood, average stringency levels are higher in dense cities. A typical neighborhood in the middle of the density distribution has a price of a minimal lot approximately $\$300,000$ greater in expensive cities. This gap in stringency disappears only when comparing the highest density neighborhoods[16].

Implications for deregulation Facts 1 and 2 inform how large scale deregulation might affect the welfare of spatially mobile households. The literature emphasizes that housing regulation limits aggregate labour productivity because it limits the size of productive cities. This assertion is muddied by the fact that labour supply to any given city depends both on the number of households and the labour supply per household. If regulation causes productive cities to have a high labour supply per household as suggested by Fact 2, then any productivity gains from deregulation could be offset by the out-migration of affluent households. Indeed, eliminating all minimum lot sizes in the forthcoming model finds gains to aggregate labour productivity of about 0.26%, which is less than one-tenth of outcomes of similar exercises, in particular that of Hsieh and Moretti (2019) and Duranton and Puga (2023).

On the other hand, within-city variation in income and lot size stringency highlighted in Facts 1 and 2 suggest a correlation between neighborhood income and regulatory stringency along the urban density gradient, particularly in expensive cities. This is important if the sorting of high income households are associated with positive externalities. I argue in the following section that if regulation does not cause most of the observed neighborhood sorting on income, then imposing regulation in high income neighborhoods can correct an externality associated with demand spillovers caused by high income neighbors[17]. If such strong income sorting characterizes the world be observe, then these facts suggest that low density neighborhoods in expensive cites are imposing stringent regulation in a way that increases amenity values of both rich and poor neighborhoods. On the other hand, if the income density gradient we observe in Fact 1 would be flatter in

[15]Regulatory authorities could impose minimum floorspace requirements for units in large residential structures. However, these units tend to be consistently smaller than single family homes.

[16]Facts 1 and 2 are robust to alternative weights, clustering schemes, definitions of superstar cities using housing prices, density or productivity (wages) alone, and various combinations of control variables and time periods. However, Fact 1 does not hold when replacing the use of density rankings with distances to the central business district. I discuss these robustness checks with more detail in Appendix A.6.

[17]In other words, the imposition of minimum lot sizes in rich neighborhoods would cause reallocations of the lowest income households in rich neighborhoods to become the highest income households in poor neighborhoods; increasing average income everywhere. I show this with a simple model in Section 3.

the absence of regulation (in other words, if income sorting in the absence of regulation is weak), I show that stringent minimum lot sizes cause largely distributional consequences, benefiting high income neighborhoods necessarily at the expense of lower income households and increasing segregation overall. A necessary consequence of the latter scenario is that deregulation will cause initially poor neighborhoods to gentrify significantly. Facts 1 and 2 suggest that the poorer neighborhoods are precisely the higher density neighborhoods of expensive cities. Counterfactuals in Section 6 predict elements of both these scenarios: the gentrification of the high density neighborhoods of expensive cities and the deterioration of neighborhood amenity values in all types of neighborhoods.

3 Theoretical framework

In this section, I introduce a quantitative framework that can be taken to the data to assess the consequences of deregulation, particularly the relative importance of its effect on housing affordability, aggregate labour productivity, and the external costs associated with neighborhood choice. I analyze a few stylized versions of the model and provide two broad messages. First, the model can qualitatively rationalize Facts 1 and 2 with little structure. Second, and related, I show that minimum lot size regulation can be used to correct the neighborhood choice externality when regulation does not explain a majority of income sorting in the data. I also show that they cause largely distributional consequences when income sorting is instead weak absent regulation. The full quantitative framework is designed to be able to match the structure of regulation and nest these competing hypotheses for which the data can speak.

3.1 The Quantitative Spatial Model

Geography I consider a finite set of cities $\mathbb{C}$ indexed by c, which map to MSAs in the data. These cities are self-contained labour markets; that is, I do not allow households to access productive technologies outside of the city for which they reside. Each city c has an exogenous finite set of neighborhoods $N(c)$. I use the index i to denote a typical neighborhood from any city, $i \in \cup_{c\in\mathbb{C}} N(c)$, and define the map $\mathbb{C}(i)$ to be the city associated with i. Neighborhoods are 2020-definition census block groups. Each of these neighborhoods have an exogenous amount of land $T^R(i)$ zoned for *regulated structures* and land $T^U(i)$ zoned for any type of structure. Regulated structures are those with between 1-4 housing units per lot. I use the notation $\boldsymbol{o} \in \{R, U\}$ to index unregulated and regulated zones, respectively. Land in each zone will be calibrated to target the observed share of households who reside in regulated structures in each neighborhood. This distinction is important, as high density locations (like downtown New York City) appear to have expensive single family homes but a disproportionately small share of units that comprise them.

Landowners Given a parcel in i, absentee landowners choose the total amount of housing services $A(i)$ that occupy it in a standard way. That is, they use a neighborhood-varying Cobb-Douglas technology over land and capital, facing a perfectly elastic supply of that capital at some exogenous rate r. This yields the neighborhood-zone level housing supply function per unit of land

$$A_{\boldsymbol{o}}(i) = \lambda(i) P_{\boldsymbol{o}}(i)^{\epsilon(i)} \tag{3}$$

where $P_{\boldsymbol{o}}(i)$ is the price of a unit of housing services in zone $\boldsymbol{o}$, $\epsilon(i)$ is the supply elasticity and $\lambda(i)$ is an exogenous supply shifter. With Cobb-Douglas production,

In a world without minimum lot sizes, the landowner is indifferent to allocating housing services across housing units; there may be many small houses or few large ones, provided the density of supplied services is given by (3). Instead, if landowners in zone R respect the minimum lot size $l(i)$[18], the minimum amount of housing services per housing unit must be

$$A_R(i)l(i) = \lambda(i) P_R(i)^{\epsilon(i)} l(i) \tag{4}$$

Define the quantity

$$R(i) = P_R(i) A_R(i) l(i) \tag{5}$$

which is the cost to rent housing services on a minimal lot in the regulated zone. This is the model equivalent to the observed stringency of regulation in Equation (2), and suggests an immediate mapping between the model and the data. In the unregulated zone U, I assume there is no minimum lot size. This is equivalent to imposing $l(i) = 0$.

Equation (4) reveals the material difference between lot size regulation and other regulations studied in recent quantitative models. Contrast the equation with the standard Floor Area Ratio restriction studied in Brueckner and Singh (2020), which puts limits on the density of floorspace in each parcel. In this framework, there are only restrictions on the number of housing units (or households) that can occupy a given parcel of land. This distinction is forcefully argued in Grieson and White (1981). Most work studying the aggregate implications of housing regulation assume that regulation affects the floorspace supply elasticity $\epsilon(i)$ or construction productivity $\lambda(i)$ (Hsieh and Moretti, 2019; Parkhomenko, 2023; Herkenhoff et al., 2018). In this paper, minimum lot sizes block the supply of low quality housing units that would otherwise appear on small lots[19]. The interpretation is that minimum lot size regulation affects only the supply elasticity of low quality units.

Household's problem Households have Stone-Geary preferences over a freely traded, homogeneous good g (with a normalized price of 1 dollar) and housing services A. Households differ on *skill level* indexed by $z \in Z$, where Z is finite, and hold no land wealth. Alternatively, z is the labour supply of the household. Deferring location choice for a moment, suppose a household of type z has chosen neighborhood i and zone $\boldsymbol{o}$. Given the city $\mathbb{C}(i)$, households receive a wage $w(\mathbb{C}(i)) := w(i)$ per effective unit, and enjoy a

[18]More generally, $l(i)$ can be interpreted as a restriction on the density of housing units. For example, a neighborhood that allows for unrestricted development of duplexes has an effective density restriction of half the minimum lot size.

[19]By low quality, I mean a small amount of housing services associated with a housing unit.

neighborhood-type specific *amenity value* $b(i,z)$. Given the wage and amenity, the household of type z chooses numeraire good g and housing services A to maximize

$$V_{\boldsymbol{o}}(i,z) := \max_{A,g} \underbrace{\kappa(z)\beta^{-\beta}(1-\beta)^{-(1-\beta)}(A-\bar{A})^{\beta}g^{1-\beta}}_{\text{Consumption value}} + \underbrace{\log b(i,z)}_{\text{Amenity value}} \tag{6}$$

subject to

$$P_{\boldsymbol{o}}(i)A \geq R(i) \text{ if } \boldsymbol{o} = R \text{ and}$$
$$P_{\boldsymbol{o}}(i)A + g \leq w(i)z$$

where $\bar{A}$ is a minimum level of housing services that must be consumed. If a household chooses a minimally sized lot whose price exceeds income at z, I set $V_{\boldsymbol{o}}(i,z) = 0$ and assume that the household spends all their income on housing. Similarly, I set $V_{\boldsymbol{o}}(i,z) = 0$ if a household cannot afford $\bar{A}$ units of housing services in i irrespective of regulation[20]. The function $\kappa(z)$ allows for the marginal utility of consumption to differ arbitrarily across income levels, and will govern how high and low income households trade off higher rents for higher amenity neighborhoods.

To see how regulation in zone R distorts housing consumption, $V_R(i,z)$ can be decomposed into two components when preferences are Cobb-Douglas ($\bar{A} = 0$) and the minimum lot size is binding:

$$V_R(i,z) = \underbrace{\kappa(z)\frac{w(i)z}{P_R(i)^{\beta}}}_{\text{Undistorted utility}} \times \underbrace{\left[\frac{\frac{R(i)}{w(i)z}}{\beta}\right]^{\beta}\left[\frac{1-\frac{R(i)}{w(i)z}}{1-\beta}\right]^{1-\beta}}_{\text{Distortion factor}} + \log b(i,z) \tag{7}$$

whenever $\beta w(i)z < R(i)$, so that the desired spending on housing is smaller than the cost to rent a minimal lot. The distortion factor is 1 when the minimum lot size is nonbinding, and is decreasing in the distance between the desired spending share on housing β and the spending share on housing $\frac{R(i)}{w(i)z}$ induced by regulation. I derive this formula for the general case when $\bar{A} > 0$ in Appendix B.1.

Regulation distorts housing consumption, but less so for affluent households. To see this, consider how much a household of type z needs to be compensated in terms of reduced housing prices $P_R(i)$ to live in a neighborhood with marginally higher regulatory stringency $R(i)$, holding amenity values fixed. This is defined implicitly by the slope of the indifference curve $\frac{\partial V_R(i,z)}{\partial R(i)} / \frac{\partial V_R(i)}{\partial P_R(i)}$. In appendix B.2, I show that

$$\frac{\partial}{\partial z}\left[\frac{\partial V_R(i,z)}{\partial R(i)} / \frac{\partial V_R(i,z)}{\partial P_R(i)}\right] < 0 \tag{8}$$

whenever regulation is binding at z and $w(i)z > R(i)$. This means that a higher income household requires a lower reduction in rents per unit of housing services to be indifferent to any increase in the cost of a minimal lot.

[20] In the data, some households appear in neighborhoods where the model predicts the cost of a minimal lot exceeds their income. This is an artifact of unobserved wealth or permanent income. This assumption on the utility of these households will allow the model to rationalize observations as location choices in the presence of these unobserved factors.

Zone choice I assume households are perfectly mobile across zones. In other words, households choose zones to solve

$$V(i,z) := \max_{\boldsymbol{o}\in\{R,U\}} V_{\boldsymbol{o}}(i,z) \tag{9}$$

Define $C(i,z) := V(i,z) - \log b(i,z)$ to be the *consumption value* of i for type z households. When choosing zones within a neighborhood, there is an implicit trade off that varies across income types. In spatial equilibrium, the disutility of a large lot must be compensated by a relatively lower price per unit of housing services in the regulated zone. This logic will not hold when making comparisons of regulation across neighborhoods. This is because neighborhood amenities will endogenously respond to the level of regulation, and thus serve as another compensating differential. I introduce that portion of the model later in the section.

Neighborhood choice As in Equation (6), neighborhoods offer an amenity value $b(i,z)$ for households of type z. Crucially, these amenities are flexible enough to rationalize any local population and income distributions across neighborhoods that may be observed in the data. The structure of these amenities will imply a rich set of counterfactual predictions of deregulation, as I show at the end of this section. Along with these amenities, households draw idiosyncratic preference shocks over neighborhoods, and these shocks are distributed Gumbel. The mass of z households who choose neighborhood i is

$$L(i,z) = \left[\frac{W(\mathbb{C}(i),z)}{\boldsymbol{W}(z)}\right]^{\theta}\left[\frac{e^{V(i,z)}}{W(\mathbb{C}(i),z)}\right]^{\rho}\bar{L}(z) \tag{10}$$

where

$$W(\mathbb{C}(i),z) = \left[\sum_{i'\in N(\mathbb{C}(i))}\left[e^{V(i',z)}\right]^{\rho}\right]^{\frac{1}{\rho}}$$

and

$$\boldsymbol{W}(z) = \left[\sum_{c\in\mathbb{C}} W(c,z)^{\theta}\right]^{\frac{1}{\theta}} \tag{11}$$

is the expected utility of a type z household before drawing a shock, with $\bar{L}(z)$ being the mass of households of type z nationally. This is the standard measure of renter welfare moving forward. θ governs how responsive migration flows are to changes in value across cities. ρ governs the responsiveness of migration flows across neighborhoods within any given city.

Endogenous Amenities So far, regulation only decreases utility because it constrains the housing consumption possibilities of certain households. This is not true when neighborhood quality responds endogenously to regulation. I assume that amenity values $b(i,z)$ depend on the average income of a neighborhood:

$$\log b(i,z) = \Omega(z)\log \text{Inc}(i) + \log \nu(i,z) \tag{12}$$

where $\text{Inc}(i) := \frac{\sum_{z' \in Z} w(i) z' L(i', z')}{\sum_{z' \in Z} L(i', z')}$ is average income[21]. I refer to the $\nu(i, z)$ as *fundamental amenities*, which contain all other observed or unobserved demand factors that can be reasonably taken as exogenous with respect to this model; including commuting time, and natural amenities. Equation (12) gives rise to an externality because residents of a neighborhood are not compensated for the location decisions of low and high income households. $\Omega(z)$ governs the elasticity of amenity values to income, and will be estimated for each type using a donut strategy in Section 4.

There are at least two main channels that I have emphasized thus far that would proximally give rise to (12). Firstly, local income could increase local amenities through variety effects in a Dixit-Stigliz style model (Amalgro and Dominguez-Iino, 2021; Couture et al., 2019), while local population could decrease the amenity value through congestion effects or direct preferences for density. When these two forces operate at the same elasticity $\Omega(z)$, amenity values depend only on income per capita. Secondly, local governments could provide a congested public good financed through income or property taxes (Calabrese et al., 2007, 2011). In the latter case, income per capita would be replaced with property tax revenue per capita. In a model with Cobb-Douglas preferences over housing, no minimum lot sizes and random heterogeneity in property tax rates, this essentially identical to income per capita. With Stone-Geary preferences and minimum lot sizes, the relationship between property tax revenue and incomes need not be linear, but nevertheless they would still be highly correlated. In Appendix B.3, I provide microfoundations for each of these mechanisms, but I do not limit the interpretation of (12) to them. Instead, I assume $\Omega(z)$ reflects all factors that could be caused by the compositional effects of affluence. Apart from the above, these may include reduced crime, peer effects (Chetty and Hendren, 2018), or a general taste for affluent neighbors (Guerrieri et al., 2013; Brueckner et al., 1999).

How households trade off rents and amenities

Production In each city c, production of the numeraire good g takes place competitively with a constant-returns technology

$$g(c) = \iota(c) \left[\sum_{i \in N(c)} \sum_{z \in Z} z L(i, z) \right] \tag{13}$$

where $\iota(c)$ is the exogenous labor productivity in city c. In equilibrium, it must be that $\iota(c) = w(c)$ and so I refer to both interchangeably. Aggregate labour productivity is thus

$$\frac{\sum_{c \in C} g(c)}{\bar{L}}$$

where $\bar{L}$ is the total mass of households nationally. Differences in labor productivity and populations across cities will be crucial for understanding aggregate labour productivity, as in Hsieh and Moretti (2019). Additionally, I argue that differences in the labour supply

[21] Alternatively, I could have specified that amenities depend only on the average type (and not labour income). In the baseline model, city wages $w(c)$ are exogenous as I assume below. As a result, this alternative model yields numerically identical counterfactual implications after calibration.

per household across cities matters when assessing the impacts of deregulation because this regulation cannot be decoupled from income sorting.

Understanding why marginal impacts matter Explain result on why income sorting matters because of marginal impacts on aggregate productivity *here*.

With housing and labour markets defined, I turn to the definition of an equilibrium with exogenous productivity.

Equilibrium An equilibrium is defined as a set of housing prices $P_{\boldsymbol{o}}(i)$, neighborhood allocations $L_{\boldsymbol{o}}(i,z)$, amenities $b(i,z)$ such that

1. Labour Markets clear: Given indirect utility $V(i,z)$ solving (9), amenities $b(i,z)$ solving (12), labour supply per household type $L(i,z)$ solves (10) at wages $w(c)=\iota(c)$.
2. Housing Markets clear: Given $A^{\star}(i)$ solving (4) and population $L(i,z)$, the neighborhood demand for housing services derived from (6) equals the supply of housing services derived from (3) in every neighborhood i and zone $\boldsymbol{o}$.

In Section 6, I use this definition of equilibrium in a fully-quantified model to predict the effects of deregulation.

3.2 Model Extensions

Endogenous productivity The production technology in Equation (13) is restrictive in three important ways. First, population flows across cities cause changes in city productivity via agglomeration effects (Combes and Gobillon, 2015). I extend the baseline production technology so that wages respond to city size with elasticity α, or

$$\iota(c)=\tilde{\iota}(c)L(c)^{\alpha} \tag{14}$$

where $\tilde{\iota}(c)$ is an exogenous component of city productivity and $L(c)$ is the population of city c. Second, relative flows of high and low income workers alter the relative wage they earn if these workers are not perfect substitutes in production (Card, 2009). I also extend the model to allow for households to differ on both income z and education $s\in S:=\{College, NonCollege\}$ and modify the technology (13):

$$g(c)=\left[\sum_{s\in S}\left[\iota(c,s)\sum_{z\in Z}zL(c,s,z)\right]^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{15}$$

where σ is the elasticity of substitution between education levels, $\iota(c,s)$ is an education-augmenting productivity term, and $L(c,s,z)$ is the population of (s,z) types in c. Third, these agglomeration effects may be biased in their benefit to college workers, as is suggested by evidence in Diamond (2016) and Baum-Snow et al. (2018). I allow education-augmenting productivity $\iota(c,s)$ to respond arbitrarily to a city's educational composition,

$$\iota(c,s)=\tilde{\iota}(c,s)\prod_{s'\in S}L(c,s')^{\alpha(s',s)} \tag{16}$$

where $L(c, s)$ is the population of s types in c. Since deregulation changes both the population and skill composition of cities, each of these extensions may be quantitatively important. However, I show that the main message of this paper is robust to each of these extensions.

Incorporating capital gains and losses So far, I have abstracted from land ownership. Deregulation will have large impacts on equilibrium land values. This affects the lifetime consumption of many households because most of them are heavily invested in housing, especially low and medium wealth homeowners (Greaney, 2023). To this end, I allow households to differ by a designation of *renter* or *homeowner*. Homeowners purchase shares in a national land portfolio that they sell when a regulatory change occurs. Total household income then includes capital gains:

$$w(i)z + \underbrace{s^{\text{Nat}}(z)(\Pi - \Pi^{\text{Initial}})}_{\text{Capital gains on national housing portfolio}} \tag{17}$$

where Π is total value of land in a counterfactual equilibrium, Π^{Initial} is the nationwide value in a baseline equilibrium that matches data, and $s^{\text{Nat}}(z)$ is the fraction of total land wealth owned by a z-homeowner. Renters do not observe capital gains or losses, and are equivalent to the definition of a household in the baseline model.

3.3 Replicating the facts with the model

Facts 1 and 2 say that expensive cities are on average higher income and exhibit stronger negative income sorting on density, and that this can be explained by the spatial variation in the prices of minimal lots. With little structure on the model, I construct an equilibrium in which a chosen set of these values reproduce both of the facts.

To this end, I assume there are N cities indexed by $c \in \{1 \dots N\}$, with two income types z_0 and z_1 such that $z_0 < z_1$. Cities differ on productivity, which is ordered as $w(c) < w(c')$ whenever $c < c'$. Each city has two neighborhoods indexed by $i \in \{0, 1\}$, each with unit land mass, unit amenity value, and identical production technology for housing (net of regulation). Cities also differ on value of a minimal lot in city c and neighborhood i in the following way:

$$I(i, c) = \alpha(c)i$$

where $\alpha(c)$ is a function such that $\frac{\alpha(c)}{w(c)}$ is a strictly increasing function in c. I also assume that $\beta < \max_{i,c,z} \frac{I(i,c)}{w(c)z} < 1$, so all neighborhoods are affordable by at least one skill level and the minimum lot size is binding for at least one. These assumptions take as given the spatial structure of regulation in Fact 2. Within cities, neighborhoods will be ordered by affluence and inversely ordered by density because of this variation in regulation between them. This variation also causes sorting into high-productivity cities.

Under this parameterization of the model, there is purposefully no reason for income sorting absent regulation. This is because each location provides identical fundamental amenity value for households of all incomes. The restrictions on preferences $\Omega(z) = 0$ and

$\bar{A} = 0$ also mute additional income sorting on rents (and income itself). Consequently, all variation in affluence across neighborhoods will be caused by regulation. This leads to the income sorting patterns observed in the data, as summarized in Proposition 1.

Proposition 1. *Suppose $\Omega(z) = 0$ for every z, preferences are Cobb-Douglas, $\bar{A} = 0$ and households are perfectly mobile ($\Omega, \rho \to \infty$). Also assume $I(i, c)$ takes values as above. Then, the following hold in equilibrium:*

1. *City affluence is increasing in c*
2. *In each city c, the density of housing units is decreasing in i*
3. *Every city has a negative income density gradient. The slope of this gradient is increasing in c.*

Proof. See Appendix B.4. □

3.4 When does regulation correct the neighborhood choice externality?

Under the model structure assumed in Proposition 1, all income segregation across neighborhoods and cities is induced only by regulation. The regulated, high income neighborhoods in productive cities offers cheaper prices per unit of housing services to compensate for the fact that too much housing services are needed to be purchased to live there[22]. Additional inequality between the two income types is induced by the (endogenous) creation of high value neighborhoods. In other words, regulation causes mostly distributional consequences, so that the value of the policy depends solely on how much weight high income households are given in the social welfare function. In this section, I contrast this regressive outcome with another parameterization of a simplified model: one where a specific structure of minimum lot sizes can be used to increase incomes and amenity values in all locations. This is achieved by reallocating the poorest households in rich neighborhoods to become the richest households of poor neighborhoods. This result requires strong income sorting in the absence of regulation, or at the very least that regulation explains a small portion of neighborhood sorting on income. This is in part governed by the fundamental amenities $\nu(i, z)$[23].

To this end, suppose there is instead three income types $z_l < z_m < z_h$ (low, medium and high), one closed city, and neighborhoods $i \in \{0, 1\}$ with an identical production technology for housing. These neighborhoods will be ordered in equilibrium by affluence. I consider a set of *economies* that are parameterized by $t \in [0, 1]$. These economies will vary by the structure of fundamental amenities. The $t = 0$ economy will be characterized by no variation in income across neighborhoods, which can be achieved by setting $\nu_{t=0}(i, z) = 1$ across all neighborhoods and types[24]. Conversely, the $t = 1$ economy is

[22] Or, in the full model with endogenous amenities, these neighborhoods could offer higher amenities as another compensating differential.

[23] The idea that income sorting creates an externality by which too many lower income households crowd rich neighborhoods has previously been used to argue that fiscal centralization across neighborhoods is typically more efficient than decentralization (Calabrese et al., 2011). The reasoning behind this argument is the same reasoning I use here.

[24] Since the scale of amenities do not matter for equilibrium outcomes or welfare measurement, the choice of $\nu(i, z) = 1$ is arbitrary. Any positive number works under the same argument.

associated with the largest variance in neighborhood income absent regulation. This is achieved by restricting $\nu_{t=1}(1, z_h) = 1$, $\nu_{t=1}(0, z_h) = 0$, $\nu_{t=1}(0, z_l) = 1$ and $\nu_{t=1}(1, z_l) = 0$; along with $\nu_{t=1}(i, z_m) = 1$ for every i. Under this parameterization, income sorting is at its strongest because there will be complete segregation of z_l and z_h types in either neighborhood. Finally, for any economy $t \in (0, 1)$, define the fundamental amenity as $\nu_t(i, z) = \kappa_z(1-t)\nu_{t=0}(i, z) + \kappa_z(t)\nu_{t=1}(i, z)$ for some strictly increasing function κ_z satisfying $\kappa_z(0) = 0$ and $\kappa_z(1) = 1$ for every z[25].

I study how the effects of regulation vary with t. In the $t = 1$ economy, introducing regulation in $i = 1$ must reallocate *only* z_m types away from high amenity, low density, rich neighborhoods and toward low amenity, high density poor neighborhoods; increasing income in both locations. Conversely, imposing regulation in the $t = 0$ economy must create low density rich neighborhoods and high density poor neighborhoods; there is no scope for regulation to increase amenity values in all neighborhoods simultaneously. In Proposition 2, I argue that this same logic holds for economies sufficiently close to $t = 1$ and $t = 0$.

Proposition 2. *Suppose preferences are Cobb-Douglas ($\bar{A} = 0$), and assume $\Omega(z)$ is strictly increasing in z and sufficiently small for all z.*
Finally, assume fundamental amenities take the described form for a set of economies parameterized by $t \in [0, 1]$.
Then, the following is true about an equilibrium where regulation does not bind:

1. *The variation in income across neighborhoods is strictly increasing in t.*
2. ***Inclusionary Zoning**: There exists a $t^{\uparrow} \in [0, 1)$ such that, for every $t \in (t^{\uparrow}, 1]$, a small increase in regulation in $i = 1$ increases average income in all locations.*
3. ***Exclusionary Zoning**: There exists a $t_{\downarrow} \in (0, 1]$ such that, for every $t \in [0, t^{\downarrow})$, increasing regulation in any neighborhood(s) does not increase average income in all locations. Instead, average income across neighborhoods weighted by the population of z_h types increases.*

Proof. See Appendix B.5 □

Proposition 2 suggests that the efficacy of regulation as a second best policy varies under different parameterizations of the model[26]. It also hints at different distributional consequences. An important question is which, if any, of these examples better characterizes the actual world we live in; the model can distinguish between these hypotheses precisely because it can fit variation in fundamental amenities. In the main deregulation exercise of Section 6, I find that the contribution of changing amenities to overall welfare is negative for all households, but there are large changes in the relative income of different neighborhoods and desegregation overall. This suggests that the data is best described by some combination of both extremes. This is commensurate with the idea that regulation

[25] I use a specific functional form for $\kappa_z(t)$ that is analytically convenient. I provide details in Appendix B.5.

[26] I want to stress that strong income sorting absent regulation *is not* required for minimum lot size regulation to be desirable. Net welfare gains when fundamental amenities do not differ across space are achievable through classical Tiebout sorting – a better allocation of congested public goods to income levels – as demonstrated in Calabrese et al. (2007).

drives a large portion of income sorting on density within and across cities (Facts 1 and 2). Moreover, if the economy is characterized by little income sorting absent regulation, then mostly all of the correlation between neighborhood income and regulatory stringency should disappear after deregulation. Counterfactuals suggests that, while significantly attenuated, this correlation persists. The model also predicts that fundamental amenities matter for the effects of deregulation: muting all variation across incomes implies that the lowest income households benefit considerably more from increased exposure to higher amenity neighborhoods, as is predicted by Proposition 2.

4 Calibration and Estimation

In this section, I outline how I calibrate and estimate parameters to rationalize observed data on wages, housing prices and populations of varying incomes as an equilibrium of the model after accounting for minimum lot size regulation. All additional details are in Appendix C.

4.1 Productivity and local income distributions

City productivity Central to the model is the sorting of high income households into productive cities. To measure city wages in terms of efficiency units of labour, I follow Baum-Snow et al. (2018) and regress log hourly wages in a set of occupation, sex, race, ancestry, year, quadratic in age and years of education, including MSA fixed effects using the 2015-2019 ACS individual sample[27]. The MSA fixed effects in this regressions define $w(c)$. I normalize $w(c)$ so that it is on average one across cities for each education level, noting that the relevant interpretation of household efficiency units z is then the total labour income that can be earned in an average city.

Local household type distributions Calibrating amenity values $b(i, z)$ requires constructing measures of the local mass of households by type $L(i, z)$. I construct this in two steps. The first step is to construct neighborhood income distributions. The 2016-2020 ACS reports yearly household income distributions at the block group level aggregated to 17 income bins. I aggregate these bins further into 7 to partially address the presence of empty bins in the data[28] and scale the distribution by 2020 census household counts. Second, I construct a reasonable support of the type distribution Z. In the 2015-2019 ACS household sample of employed individuals, I deflate total household income by the corresponding city wage $w(c)$ to arrive at a household measure of efficiency units. I choose the support of efficiency units z so that they roughly correspond to a measure of center for each income bin. There are two issues with the resulting income distributions. First, they are not deflated by the city wage, and thus do not correspond to the distribution of efficiency units. In practice, the distributions exhibit a high degree of aggregation such that any reasonable adjustments do not matter, so I ignore them. Second, a recent litera-

[27]Households in approximately 80 2013-definition MSAs cannot be directly identified in the IPUMS ACS sample. For these MSAs, I use the official IPUMS PUMA-MSA crosswalk, matching PUMAs to MSAs based on the largest land coverage.

[28]These income bins are (in yearly household income) $\$0 - 25,000$, $\$25,000 - 50,000$, $\$50,000 - 75,000$, $\$75,000 - 100,000$, $\$100,000 - 150,000$, $\$150,000 - 200,000$ and $\$200,000+$.

ture has shown that idiosyncratic preference shocks induce spurious variation in location fundamentals, inducing potentially large uncertainty in counterfactual outcomes (Dingel and Tintelnot, 2021). I address the presence of zero counts via a smoothing procedure[29]. Results are insensitive to smoothing.

4.2 Consumption values $C(i, z)$

Constructing consumption values and amenities additionally requires a choice of three sets of parameters. The first is the price per unit of housing services, which is a general measure of housing affordability. Second, the measure of regulatory stringency– the value of a minimal lot– to understand by how much regulation has distorted housing consumption. Thirdly, preference parameters $\bar{A}$ and β, which inform how important housing is in the consumption basket for all income types. I directly infer prices per unit of housing services in regulated zones and the value of a minimal lot from the data. I then *choose* prices in unregulated zones and housing preference parameters to target 1) the observed share of households who choose regulated structures and 2) the observed aggregate spending on housing services by income. I describe each procedure below.

Housing prices in regulated zones The value of a house in the data is not adjusted for how many units of housing services it provides. Adjusting housing prices for quality is particularly important for this paper because it responds to the degree of income sorting and regulation. The assessment data are detailed enough to parse a measure of quality. Following Baum-Snow and Han (2021), I construct them using the following hedonic regression

$$\log[Value_{iht}] = \log[P_R(i)] + Controls_{iht} + \sum_{t \in Year} FE_t + \sum_{t \in Month} FE_t \tag{18}$$

where $Value_{iht}$ is the observed arms-length transaction value of the house h at time t in block group i, $Controls_{iht}$ are a set of observed characteristics, FE_t are year and month fixed effects in the 2016-2022 housing transactions data linked to the 2022 assessments. Identification of housing prices are from block group fixed effects $P_R(i)$. I limit the estimation sample to regulated structures (with 1-4 housing units per lot). Controls include the type of structure (single family, duplex, triplex), floorspace, lot size, number of rooms, bathrooms, types of AC and heating systems, roof and foundation types, sewage systems, and more characteristics recorded to assess the value of the house. I impute missing categorical variables to use as much of the transactions data as possible. Prices are censored at the top and bottom 2.5% of their distribution within each MSA. Using this procedure yields prices $P_R(i)$ in roughly 95% of block groups. To complete coverage, I construct lower-quality indices using residuals of a hedonic regression using block group aggregates in the 2016-2020 ACS. To ensure that these prices are of similar scale, I adjust the ACS prices so that the log mean and variance are identical to the prices derived from the

[29] If I observe a zero population count in the data for a specific type, I impute this count with half the average count in a set of aggregated population bins with incomes $0-50k$, $50-100k$ and $100k+$. This has only minor effects on the overall neighborhood income distribution, and counterfactuals are largely insensitive to this procedure.

transactions and assessments[30].

Converting the value of a minimal lot to a user cost of housing To the construct stringency measure $R(i)$, I use observed home values. These do not correspond to yearly expenses paid by households for maintenance, interest, or property taxes. Attanasio and Pistaferri (2016) and Straub (2019) impute the yearly user cost of housing to 6% of home values. Couture et al. (2019) use price-to-rent ratios of $4.3 - 5\%$ to estimate implicit rents. I use a value of 5% in all analyses and all results are robust to a range of values.

Preference parameters and prices in unregulated zones The calibrate unregulated prices $P_U(i)$ in a way that maintains spatial equilibrium across zones under perfect mobility (see Equation 9). If a neighborhood has an expensive minimal lot and most housing units in unregulated structures, that must mean that prices $P_U(i)$ are prohibitively high; households are willing to bear the cost of regulation. Conversely, if minimal lots are expensive but there are many housing units in unregulated sturctures, this must mean that prices $P_U(i)$ are low; enticing households to substitute away from regulated structures. $P_U(i)$ is chosen such that the number of households who choose regulated structures matches ACS data.

To limit computational burden, I approximate $P_U(i)$ in a model of zone choice with an elasticity κ that is large enough to mimic the spatial equilibrium under perfect mobility. Let the fraction of type z agents who choose zone R in neighborhood i be

$$L_R(i,z) = \frac{e^{\kappa V_R(i,z)}}{\sum_{\boldsymbol{o} \in \{R,U\}} e^{\kappa V_{\boldsymbol{o}}(i,z)}} \tag{19}$$

where $V_{\boldsymbol{o}}(i,z)$ is the solution to Equation (6)[31]. $V_R(i,z)$ is directly calibrated to match data on regulated prices $P_R(i)$, the value of a minimal lot $I(i)$ (as a yearly flow cost) and wages $w(i)$. Moreover, $L_R(i,z)$ is strictly decreasing in unregulated prices $P_U(i)$. I calibrate to the unique $P_U(i)$ such that

$$\sum_{z \in Z} L_R(i,z) = L^R(i)$$

where $L^R(i)$ is the number of households in regulated structures from the ACS sample[32]. In Table 6 of Appendix C.3, I provide summary statistics for prices in both regulated and unregulated zones. I also break these statistics down by "superstar" city status using the same definition from Facts 1 and 2. Prices in regulated zones are normalized to mean 1, and have a standard deviation of approximately 1.1. Prices in unregulated zones have a mean of 1.4 and a standard deviation of 1.6. The higher mean in unregulated zones serves as a compensating differential for the disutility of regulation.

[30]The ACS block group tabulations do not report housing characteristics specifically for regulated structures. Hedonic indices derived from ACS data pool across observations of regulated and unregulated structures.

[31]Since amenity values do not vary across zones, Equation 19 is independent of amenities $b(i,z)$.

[32]In block groups where all housing units are in unregulated structures, I re-estimate the hedonic regression (18) over the entire sample of transactions in all structures. This hedonic index is the calibrated price for these block groups.

Constructing the measure $V_{\boldsymbol{o}}(i,z)$ to solve Equation (19) requires preference parameters β and $\bar{A}$. These are chosen to target spending shares on housing by type[33]. However, they need to be jointly calibrated with each $P_U(i)$ because they directly determine how many households have their housing consumption distorted by regulation. This is computationally intensive, so I calibrate them using 5000 randomly selected block groups. I find plausible values of $\beta = 0.075$ and $\bar{A} = 4500$, meaning that a very high income household spends 7.5% of their expenditure on housing and every household spends at least $\$4500$ a year on housing in a block group with average housing prices.

4.3 Amenities $b(i,z)$ and migration elasticities θ, ρ

The migration elasticities determine how responsive migration flows are to changes in neighborhood value. Hornbeck and Moretti (2018) considers the employment impact of an exogenous increase in wages across US cities, and causally estimate $\theta = 4.16$. Baum-Snow and Han (2021) estimate $\rho = 8.5$ using within-city variation in rents, wages, commuting costs and a Bartik instrument. I leverage these estimates. However, my measure of consumption value $C(i,z)$ differs in scale from those used in these papers because they do not model regulation. To ensure comparability, I rescale my consumption measures $C(i,z)$ so that they have an identical standard deviation to the log of a Cobb-Douglas index with spending share parameter $\beta = 0.2$[34]. This means that an exogenous one standard deviation increase in the consumption index $C(i,z)$ has a similar impact on population growth when compared to these papers. Amenities are chosen to uniquely rationalize data after accounting for observed populations $L(i,z)$, neighborhood values $V(i,z)$, ρ and θ (up to a normalization). This amounts to choosing amenities to satisfy a Rosen-Roback spatial equilibrium condition. That is, for every neighborhood i and type z,

$$e^{V(i,z)} L(i,z)^{-\rho} L(\mathbb{C}(i),z)^{-\frac{1}{\rho}+\frac{1}{\theta}} \bar{L}(z)^{\frac{1}{\theta}} = \boldsymbol{W}(z) \tag{20}$$

where $L(\mathbb{C}(i),z)$ is the city population of type z agents that contains i. This is a simple algebraic manipulation of the neighborhood choice equation (10). Table 6 in Appendix C.3 details summary statistics for both amenity and consumption values by income type, additionally broken down by superstar city status. The standard deviations of the amenities are comparable to Hsieh and Moretti (2019). The table shows that both consumption values and amenities are higher for higher income households in superstar cities relative to other cities. This reflects the income sorting that regulation causes.

[33]To construct spending shares on housing by income type, I take the sample of all renters from the 2015-2019 ACS microdata. Following Finlay and Williams (2022), I construct net income after taxes and transfers using the tax progressivity parameter $\tau = 0.171$ (similar values are estimated in Heathcote et al. (2017)). I then divide a measure of housing value by an estimated national price to rent ratio to arrive at a measure of imputed rents for owner occupied homes. Pooling renters and homeowners, I divide the measure of after tax and transfer income by the imputed (or actual) rent and take the median by income type nationally. This yields following spending shares by household income: 0.389 for 0-25k, 0.232 for 25-50k, 0.177 for 50-75k, 0.153 for 75k-100k, 0.138 for 100-150k, 0.122 for 150k-200k, and 0.089 for 200k+.

[34]This index is evaluated using a hedonic index from a sample pooled across both regulated and unregulated structures.

4.4 Housing supply parameters

There are four sets of parameters in each neighborhood that govern housing supply in the model: zone-invariant elasticities $\epsilon(i)$, productivity shifters $\lambda(i)$, and land used in production in each zone, $T_{\boldsymbol{o}}(i)$ for $\boldsymbol{o} \in \{R, U\}$. Using direct estimates of the housing supply elasticity from Baum-Snow and Han (2021), I show that the remaining parameters can be chosen to rationalize prices $P_{\boldsymbol{o}}(i)$ as an equilibrium in each zone and neighborhood. Given data on the value of a minimal lot $I(i)$ and regulated prices $P_R(i)$, I choose $\lambda(i)$ to uniquely solve the identity $I(i) = \lambda(i)P_R(i)^{1+\epsilon(i)}l(i)$ (Equation 5). In other words, $\lambda(i)$ is identified off of the density of housing services on a minimal lot beyond what would be predicted by prices or the physical minimum lot size. Then, I choose $T_{\boldsymbol{o}}(i)$ to directly equate the supply of housing services with demand identified from $I(i)$, $P_{\boldsymbol{o}}$ in each zone, and preference parameters[35].

5 Estimating $\Omega(z)$

Equation (12) specifies a relationship between neighborhood affluence and amenities, and the strength of this relationship is governed by the set of elasticities $\Omega(z)$. These are crucial to gauge the welfare effects of increasing incomes in the average neighborhood, as in Proposition 2. Identification of these parameters is challenging because of at least two endogeneity issues. The first is a reverse-causality bias. Large unobserved amenities imply high housing prices, and thus a high price for the minimal lot; putting a disproportionate penalty on local consumption for low income households and driving sorting patterns. This is a very similar mechanism in models that consider how housing markets and non-homothetic preferences cause residential sorting by income (Lee and Lin, 2017; Couture et al., 2019). The second arises because high amenity locations may be disproportionately valued by the rich irrespective of outcomes in the local housing market. Alternatively, the opposite might be true, as is suggested by the negative relationship between income and density within cities (Fact 1).

To address endogeneity concerns, I propose a donut strategy by which characteristics of other neighborhoods are used as instruments; this follows a large literature (Bayer et al., 2007; Anagol et al., 2021; Almagro et al., 2023). I use local terrain slopes as this characteristic, as it is a strong predictor of neighborhood income (Lee and Lin, 2017; Saiz, 2010). Why this is true is not well understood, but is likely driven by a combination of supply and demand side factors. Slopes make residential development costly, but also may be directly demanded by households because they may be associated with nice views[36]. The donut design is meant to address the possibility that localized slopes are, or are correlated with, local demand factors.

Let $S(d)$ be the average slopes over a set d of block groups. The estimating equation is

[35] For neighborhoods with no housing units in regulated structures, I set $T_U(i)$ equal to the observe land mass of the block group. This identifies $\lambda(i)$ in a housing market equilibrium.

[36] The idea that housing supply constraints are not orthogonal to neighborhood amenities has been used to argue that they are invalid instruments to estimate neighborhood demand parameters (Davidoff, 2016).

$$\log\left[b(i,z)\right] = \Omega(z)\log\text{Inc}(i) + \beta_1(z)S[d_1(i)] + \log\tilde{\nu}(i,z) \tag{21}$$

where $\text{Inc}(i)$ is average income and $d_1(i)$ is the set of block groups whose centroids are within distance d_1 of the boundary of i[37]. I use average slopes within a second buffer of length d_2 with $d_1 < d_2$ as an instrument for average income in (21). There are two identification assumptions. First, households do not demand sloped terrain outside of the buffer of length d_1. Second, slopes may be correlated with excluded demand factors in ν (such as lakefront views) insofar as slopes in $d_2(i)$ are uncorrelated with these demand factors *conditonal on slopes in* $d_1(i)$. In other words,

$$S[d_2(i)] \perp \log\tilde{\nu}(i,z) \mid S[d_1(i)] \quad \text{for all } z \in Z. \tag{22}$$

In Appendix D.1, I present an econometric model which incorporates latent and spatially correlated demand factors that may themselves be correlated with slopes. I use this to explain the meaning of the identification assumption in more detail.

Table 1 presents the baseline IV results of Equation (21) across three aggregated income groups: low, medium and high with $d_1 = 0.75$ kilometres and $d_2 = 1.25$ kilometres. These income groups correspond to households making between $\$0-\$50,000$, $\$50,000-100,000$, and $\$100,000+$, respectively[38]. All specifications control for slopes in the own block group, as well as slopes in an outer buffer reaching from d_2 to 10 kilometers. In addition, all specifications partial out MSA fixed effects, as well as unreported additional controls for block group land mass, topographic features, commuting time, share of public transport in commuting and CBD distance rankings, among others. Standard errors are clustered using a Bartlett kernel reaching 35 kilometres, with large Kleibergen-Paap F-statistics. Block groups with no neighboring block group centroids between distances d_1 and d_2 are dropped from the sample; these correspond to block groups with large land mass and little population density.

The results in Table 1 provide two messages. First, the amenity value of income is large for the average household. Estimates suggest that doubling income increases neighborhood value by approximately 28% for the medium income type. Alternatively, the estimates can be interpreted in terms of relative populations. Compare two identical neighborhoods with the same consumption value in the same city but one has twice as much income. Estimates for the medium type household suggest that the high income neighborhood would have approximately $28 \times \rho = 28 \times 8.5 = 238\%$ more medium income type households, where $\rho = 8.5$ is the within-city migration elasticity estimated in Baum-Snow and Han (2021). Second, households with high income have an elasticity $\Omega(z)$ that is two times greater than low income households, suggesting that the types of amenities that are created by neighborhood affluence are disproportionately valued by the rich.

[37]Here, I define $\log\tilde{\nu}(i,z) = \log\nu(i,z) - \beta_1(z)S\left[d_1(i)\right]$; these are the fundamental amenities of a neighborhood orthogonal to slopes within the buffer region d_1.

[38]I aggregate the 7 income bins used in the model into these three *only for estimation*. This is meant to address the presence of zero populations by income in the data (especially for high income types), which is only commensurate with a fundamental amenity value of $\nu(i,z) = 0$. Neighborhood demand (net of amenities) for these aggregated groups is a simple population-weighted average of all types in the group.

First Stage Table 7 in Appendix D.2 estimates the first stage regression pooled over each income type across various controls[39]. Column (2) corresponds to the set of controls used in the baseline estimates of Table 1. In all specifications, both slopes within the block group and slopes within the donut are associated with higher income block groups.

Comparison with OLS Table 8 in Appendix D.3 contain the OLS counterparts to the baseline IV estimates in Table 1. Point estimates of $\Omega(z)$ are all considerably smaller for each income type, suggesting that the IV corrects for downward bias induced by the negative correlation between income and unobserved demand factors. This downward bias makes sense. Recall that each $\Omega(z)$ is estimated using only variation within MSAs. Unobserved demand factors are positively correlated with residential density within cities, and residential density is negatively correlated with income (Fact 1). This means that unobserved demand factors are negatively correlated with income. This negative bias is strong enough to make the OLS coefficient for low income households negative and significant from zero. This negative value is likely driven by the mechanical negative correlation between income and the relative amenity value for low income households. Since the OLS estimate is negative, the downward bias that persists for low income households cannot be driven my measurement error alone.

In Appendix D.4, I show that these estimates are reasonably robust to a host of alternative controls, alternative calibrations of other parameters used to construct amenity values, and larger donut radii up to 1.5 kilometres. Radii beyond this limit yield weak instruments conditional on controls.

5.1 A simple placebo test

For further validation, I construct a placebo test that exploits the timing of neighborhood income changes. It relies on the assumption that future incomes cannot cause past amenity values. This is a plausible assumption if either 1) moving costs are low[40], so location decisions do not respond to expectations over future amenity values or 2) households cannot form expectations over future income growth[41].

Let the t subscript denote time. The placebo test centers around the following specification:

$$\log \nu_{t=0}(i,z) = \tilde{\beta}(z) \log \text{Inc}_{t=1}(i) + \beta_1(z) S[d_1(i)] + \log \tilde{\nu}_{t=0}(i,z) \tag{23}$$

where $\nu_{t=2010}(i,z)$ are the fundamental amenities at time zero and $\text{Inc}_{t=1}(i)$ are incomes at $t=1$. $t=0$ corresponds to a cross-section of the 2008-2012 ACS, 2008-2012 CoreLogic transactions, and the 2010 census; details are in Appendix A.1. Time $t=1$ is the period of the main 2016-2020 sample used in baseline estimates of $\Omega(z)$. If the identification

[39] Pooled estimates regress the average log amenity $b(i,z)$ across all aggregated income bins (low, medium and high). Observations with zero population by group are omitted when calculating this average.

[40] That is, one time moving costs that appear in dynamic models of location choice (Caliendo et al., 2019; Bayer et al., 2016).

[41] This assumption can be relaxed by instead requiring that households can informatively predict future incomes only with past incomes. Another implicit assumption is that the effect of income on amenity values is not dynamic in the sense that it depends on lagged incomes. This would be invalidated if, for example, local public goods that were financed in rich communities of the past depreciate slowly over time.

assumption holds in addition to the assumptions above, then $\tilde{\beta}(z) = 0$. If $\tilde{\beta}(z) \neq 0$ and the placebo test assumptions hold, then the instrument is correlated with unobserved $t = 0$ demand factors – and likely also with $t = 1$ demand factors. To estimate (23), I first fit fundamental amenities under the assumption that $\Omega(z)$ is identified: $\log \hat{\nu}_{t=0}(i, z) = \log b_{t=0}(i, z) - \hat{\Omega}(z) \log \text{Inc}_{t=0}(i)$, where hats denote estimated values. Then, I estimate $\tilde{\beta}(z)$ treating fitted fundamentals as data with the same identification strategy.

Table 11 of Appendix D.4 reports various estimates of $\tilde{\beta}(z)$. Column (1) corresponds to an IV specification that is pooled over income types. The estimated coefficient on income is positive, but small and insignificant. Column (2) corresponds to the OLS counterpart of (1), with coefficients that are negative and significant. This is evidence that the placebo test has statistical power. The remaining columns report IV estimates of (23) broken down by income type, leading to the same conclusions. Reported standard errors represent a lower bound because they assume $\Omega(z)$ was not estimated in a previous stage.

6 Counterfactuals

How would deregulation affect the aggregate economy, and how would those gains be realized by households of varying affluence? In this section, I study the impacts of deregulation on both welfare outcomes and income sorting across neighborhoods in cities. I pay special attention to three different channels:

1. Gains from the expansion of productive cities (Hsieh and Moretti, 2019; Duranton and Puga, 2023)
2. Losses from the neighborhood choice externality (Hamilton, 1976; Calabrese et al., 2007)
3. Increased housing affordability through smaller lots (Song, 2021; Kulka, 2019)

The main message of this paper is that the gains from the expansion of productive cities and losses from the neighborhood choice externality are negligible relative to the increases in housing affordability. I perform two counterfactual exercises. The first is a nationwide deregulation exercise. The second halves the stringency of regulation in San Francisco, mirroring the recent elimination of single family zoning in the State of California.

6.1 Complete deregulation

I start by studying the equilibrium impacts of complete deregulation; amounting to finding an equilibrium with $l(i) = 0$ in all neighborhoods. To define renter welfare, I take the ordinal measure of utility $\boldsymbol{W}(z)$ from Equation (11). From this, I measure welfare gains using the (inverse of) the compensating variation as a percentage of income; that is, how much income would have to grow in all neighborhoods for a household to be as well off as they would be before the reform, holding all other equilibrium values fixed[42]. Figure 3 reports each measurement by household type for models where amenities are endogenous and exogenous, respectively. Social welfare is calculated by taking the population weighted average of each compensating variation.

Figure 3 shows that the social welfare of renters increases by a sizable 9.4%, and that this is primarily driven by the high share of households who make less than $\$50,000$ annually in a city with average productivity. Households who make less than $\$25,000$ gain the most, at 28% of income, while those who make more than $\$200,000$ gain less than 1%. Welfare effects are similar by type when adopting an equivalent variation measure, albeit slightly attenuated to 8% (unreported). The results by income type are comparable to less ambitious reforms where there are no neighborhood choice externalities, e.g. Song (2021) and Kulka (2019). Moreover, the fact that all renters benefit in a presence of endogenous amenities challenges fiscal zoning exercises in the local public finance, e.g. Calabrese et al. (2007) and Calabrese et al. (2011). These typically find aggregate welfare losses even when focusing only on households with no initial housing wealth. Finally, I find that the national land portfolio loses 17.7% in value after deregulation. This makes sense. In the theory, regulation makes housing consumption artificially large, and housing prices need to adjust to induce a supply response. An equivalent view is that deregulation increases land availability, which is accompanied by a decrease in the density of housing services (and rents) nationally.

Weighing renters against landowners Housing is one of the most widely held assets. This means that capital losses from deregulation affect the average household. So far, I have provided no way to weigh landowner losses against renter gains. Doing this in a model that abstracts from both tenure choice and wealth accumulation over the life cycle is difficult. Notwithstanding, I make two assumptions to this end: 1) renters observed in the data do not own any housing wealth and 2) homeowners own a portion of a national land portfolio that is proportional to their expenditure on rents at the initial equilibrium[43]. Welfare for homeowners of type z is then calculated as

$$\big[1 - s(z)\big]\text{CompVar}(z) + s(z)\text{LandValueChange} \tag{24}$$

where LandValueChange is the capital loss on the national portfolio (-17.7%) and CompVar(z) is the renter compensating variation measure from Figure 3. The $s(z)$ are designed to measure the fraction of total household income associated with imputed rents and income from rental properties. I provide more details on the construction of $s(z)$ in Appendix E.2. Renters retain the same welfare measure as in Figure 3.

Figure 4 reports the welfare measure (24) by income type and pooled over renters and homeowners[44]. The increase in social welfare after deregulation is cut in half but still

[42] Note that exogenously increasing income of all households would induce changes in rents and location decisions, altering the equilibrium. The compensating variation measure ignores this general equilibrium effect.

[43] These assumptions do not appreciate heterogeneity in land value appreciation across neighborhoods after deregulation. As a result, they will mask heterogeneity in local welfare outcomes.

[44] A renter and homeowner receive equal weight in this procedure, so this involves weighing renter and homeowner welfare measures by their share of the population by income type. Owner-occupied shares are calculated from the data and are: 0.375 for 0-25k, 0.511 for 25-50k, 0.618 for 50-75k, 0.708 for 75-100k, 0.793

sizable, at 5.2%. This is driven by large gains to lower income households, who are mostly renters. The losses to high income households are small because the share of income from rents $s(z)$ is small. This reflects the fact that the share of wealth in housing is decreasing in wealth for a large portion of the wealth distribution (Greaney, 2023; Goldsmith-Pinkham and Shue, 2023).

Neighborhood choice externalities How has the neighborhood choice externality affected the gains to deregulation we see in Figure 3? I propose two different methods to parse the welfare consequences of changing amenities $b(i, z)$ and neighborhood consumption values $C(i, z)$. The first method asks what the benefits of deregulation would otherwise be if amenities were exogenous; this amounts to repeating the counterfactual with $\Omega(z) = 0$ for every z. Another way of interpreting this is to say that all variation in amenity value by income type inferred from the data come from factors that correlated with income, but not caused by it. Figure 3 reports the consequences of this counterfactual by income type in turquoise. The figure highlights two important things. First, the social welfare of renters is lower when amenities are endogenous compared to when they are not, but only by 1.2% for an average household; this comprises a relatively small portion of welfare gains. Note that all welfare gains of regulation for renting households must come from endogenous amenities, as otherwise regulation can only distort housing consumption relative to the unregulated competitive equilibrium. The small differences in social welfare are in stark contrast to other papers studying regulatory reforms quantitatively, namely Calabrese et al. (2007). Second, social welfare is lower for all income types when amenity values are endogenous.

The second method relies on decomposing welfare changes in the baseline model. Consider the welfare measure $\boldsymbol{W}(z)$, defined in Equation (11). I contribute changes in $\boldsymbol{W}(z)$ to changes in consumption and amenities separately using a Shapely value decomposition, with details of its construction in Appendix E.3. I report the results of this decomposition for each type in Figure 5. The sum of each component is equivalent to the compensating variation measure reported in Figure 3. The results are remarkably similar to those obtained when comparing outcomes under exogenous and endgenous amenity responses. The contribution of amenities to overall welfare changes is negative for all households, with some distributional consequences. This should be interpreted in the context of the theory in Section 3.4, where income sorting absent regulation increases the scope of regulation to increase incomes in all neighborhoods simultaneously. The model suggests that this is happening, but in an inefficient way: distortions to housing consumption strongly outweigh any benefit. Both methods point toward the idea that the neighborhood choice externality matters relatively little when compared to housing affordability for the average household.

The endogenous amenity response also exacerbates losses to the average landowner, but not by much. If amenities were exogenous, the aggregate land values would fall by 16.1% instead of 17.7%. This reflects the fact that highly regulated neighborhoods, which distort the housing consumption of their residents, endogenously offer higher amenities

for 100-150k, 0.857 for 150-200k and 0.89 for 200k+. More details are in Appendix E.2.

and tend to offer higher wages. This makes these neighborhoods more populous, increasing aggregate spending on housing services and thus land values overall.

The role of fundamentals and income sorting The evidence in Figure 3 and Figure 5 point to deregulation decreasing neighborhood amenity value for all households. The theory of Section 3.4 justifies the role of fundamental amenities $\nu(i, z)$ in rationalizing this outcome. Ideally, regulation should be stronger in neighborhoods with relatively higher fundamental amenities for the rich. There is direct evidence that this selection occurs in the data. Recall that the empirical measure of stringency from Equation (2) can explain 43% of the variation in incomes across neighborhoods. After complete deregulation, this same measure can explain an attenuated but sizable 14%[45].

To further probe the importance of fundamentals, I consider the same counterfactual in an alternative world where fundamental amenity values are equal across all income types, preserving variation in average amenity values across neighborhoods and cities. This means that, absent regulation, there is little motive for income sorting. However, locations retain similar populations to the data. Theory suggests that the contribution of amenities to overall welfare will be larger under this counterfactual because increases in the amenity value of initially poor neighborhoods will be offset by the deterioration of rich neighborhoods. That is, changing amenities contribute to the distributional consequences of the reform, and this may be efficient if low income households are given adequate weight in the social welfare calculation. I repeat the Shapely decomposition under this restriction on fundamental amenities, with results reported in Figure 6. The amenity contribution to overall welfare changes is now *positive*, in contrast to the baseline model. This is driven entirely by large gains in the amenity values of neighborhoods populated by households who make less than $\$50,000$ annually.

Gentrification While deregulation does reduce aggregate amenity values, there may be large distributional consequences across neighborhoods. Facts 1 and 2 suggest that high density neighborhoods benefit relative to low density neighborhoods, especially within superstar cities. The model also makes this prediction. In Figure 7, I compare the income density gradient for both the observed and counterfactual equilibrium separately for the superstar and non-superstar samples[46]. Each panel corresponds to a sample of cities. Within each panel, I plot flexible regressions of income (demeaned by MSA) on the baseline density ranking. Purple and yellow lines correspond to counterfactual and baseline income data, respectively. In Figure 16 of Appendix E.4, I additionally reconstruct Figure 1 treating counterfactual incomes as data.

Figure 7 shows the stark prediction made by the model. The highest density neighborhoods in superstar cities experience a 20% increase in their income relative to the city mean. Low density neighborhoods experience relatively less income declines spread out over more neighborhoods. On the other hand, cities in the non-superstar sample see little change to their income-density gradient, suggesting that minimum lot sizes tend to not

[45]These statistics are similar when considering how regulation predicts within-city variation in income.

[46]Note that the model closely matches the average income of all neighborhoods in the ACS, so these differences under the counterfactual are directly comparable to observed neighborhood income distributions.

be very binding in them or there is very little variation across space. To examine how much the income-density gradient has increased in superstar cities, I perform a linear regression of demeaned income on density in the superstar sample for both baseline and counterfactual data. I find that the differences in the income density gradient between superstars and non-superstars (Fact 1) disappears entirely. This suggests that variation in regulation across space can account for *all* differences in income sorting on density in expensive cities. In absolute terms, the average income density gradient in superstar cities increases from approximately -0.65 to -0.2 log points, a drop in magnitude of over half. This is commensurate with the idea that income sorting in the absence of regulation is not strong in these cities, which puts a limit on the negative externalities associated with too many low income households crowding rich neighborhoods (Section 3.4).

Aggregate labour productivity I find that complete deregulation increases aggregate labour productivity by only 0.26%– equivalent to roughly an eighth of a typical year of growth in the US. In contrast to the literature, Duranton and Puga (2023) find that eliminating housing supply restrictions in superstar cities would increase output per person by around 8.2%. Hsieh and Moretti (2019) estimate the number to be 3.2% in an exercise where they increase the housing supply elasticity in San Jose, San Francisco and New York.

I show that income sorting responses drive this low value in two ways. First, I plot the growth rate in the number of households against the growth rate in the average household income type z across all cities in Figure 8, with each city representing a circle that is coloured by productivity and has a radius proportional to the city population. In general, we see a net inflow of households in cities with higher wages and housing prices[47]. Second, I calculate what aggregate productivity would be if 1) cities had household growth as predicted by deregulation and 2) average income in each city remained at observed levels. This exercise yields an aggregate productivity of 1.41%, which is four times higher than what is predicted when income sorting occurs. The endogenous amenity response is also contributes to low productivity growth, driven by deteriorating amenity values offered in productive cities. Counterfactual productivity growth when amenities are assumed exogenous would be almost twice as high, at 0.44%.

These results are robust to a multitude of assumptions about externalities an production technologies. First, I allow city productivity $\iota(c)$ to respond endogenously to the city population as in Equation (14) with an elasticity $\alpha = 0.05$ (Combes and Gobillon, 2015). Aggregate productivity growth under the counterfactual increases from 0.26% to 0.5%[48]. Second, I introduce education as an additional dimension by which households differ as in Equation (15); households with varying education levels are substitutes with elasticity $\sigma = 1.3$ (Card, 2009). Under the same counterfactual, aggregate productivity growth increases by 0.25%. Finally, I allow households of differing education levels to arbitrar-

[47] Correlations between population flows and changes in average household type with housing prices and city productivity are 69% and 45%, respectively.

[48] If we took the observed population flows under this new counterfactual and assumed no changes in average income across cities, aggregate productivity growth would have been approximately 1.82%.

ily affect labour productivity by education, as in Equation (16) (Diamond, 2016)[49]. This counterfactual yields an aggregate productivity growth of 0.46%. In Table 2, I organize counterfactual productivity changes by each of these model extensions.

Landlords in regulated neighborhoods Given that all renters are better off, how can the model rationalize why minimum lot sizes are frequently imposed? I find that land values on average decrease by a larger amount in initially stringent neighborhoods. This is commensurate with the idea that homeowners impose land use restrictions to maximize their land values (Parkhomenko, 2023; Hilber and Robert-Nicoud, 2013; Fischel, 2001). Theoretically, there are two competing effects that determine the relationship between land value growth and initial stringency. On one hand, stringent regulation distorts housing consumption choices and thus decreases neighborhood demand. On the other hand, neighborhood demand increases when stringent regulation increases neighborhood affluence. I find that, in an equilibrium where amenities are endogenous, land values are negatively related to initial levels of neighborhood stringency, but *positively* related when amenities are assumed exogenous[50]. In Figure 9, I plot log differences in land values from baseline to counterfactual against the empirical measure of stringency (2). On the same plot, I include a linear fit using data generated from the endogenous amenities counterfactual (red) and exogenous counterfactual (blue). The red curve slopes downward and the blue curve upward. When amenities are endogenous, this downward sloping relationship is driven by large income responses in the upper tail of the stringency distribution.

6.2 Unilateral deregulation

Finally, I simulate the effects of halving the value of a minimal lot in the San Francisco metropolitan area. This policy change is designed to mirror the recent elimination of single family zoning in the State of California. I study welfare effects, as well as how incomes, land values, and productivity change both in San Francisco and nationally.

Deregulation causes San Francisco to densify by an influx of primarily lower income households. The number of households in the city grows by 12%, but the income of an average household falls by 8%. The city grows in terms of total output, which causes aggregate labour productivity to increase by 0.02%, and assuming away all income-composition effects of city migration implies that aggregate labour productivity would increase by 0.19%. Overall, San Francisco becomes a better place to live for a randomly selected renter. To show this, note that Equation (11) defines the utility value of living in San Francisco: a CES aggregate of the utility offered by its neighborhoods under substitution elasticity ρ. Deregulation increases this utility value for the average renter by about 5% in equivalent variation terms. However, a Shapely decomposition suggests that deteriorating amenities

[49] For this counterfactual, I use the calibrated estimates of $\alpha_{s's}$ from Fajgelbaum and Gaubert (2020). Let C index college households and N index non-college households. Using estimates from Diamond (2016), they calibrate $\alpha(N, N) = 0.003$, $\alpha(N, C) = 0.02$, $\alpha(C, C) = 0.053$ and $\alpha(C, N) = 0.044$. These estimates suggest that productivity spillovers mostly come from the presence of college educated workers.

[50] Parkhomenko (2023) provides analytical conditions on parameters to predict when land use regulation will increase or decrease land values (and rents). An important message is that larger congestion externalities in part drive the positive relationship between land use regulation and rents. In this paper, the amenity value of income has an identical effect: it mediates this relationship.

attenuated these welfare gains by about 3 percentage points for all renters, representing a large fraction of overall gains. This explains why welfare decreases by 1% for renters who make more than $\$200,000$ annually. Increases in renter welfare are also offset by a fall in city land value of 2%. If amenity values were exogenous, this type of renter would gain slightly and city land values would have instead increased by 13%[51]. Overall, considerably more households are made worse off relative to nationwide deregulation, underscoring the challenge when regulation is decided by local governments. Moreover, the general equilibrium effects of this policy change on renters and landowners nationally are similar in sign, but unsurprisingly attenuated[52].

The discussion of city-wide outcomes mask significant heterogeneity across neighborhoods. I map this heterogeneity in Figure 10. Panel A plots the initial income distribution before deregulation. Low density neighborhoods opposite the northeastern central city are on average higher income. Panel C plots the observed measure of regulatory stringency from (2) before deregulation. This follows a similar pattern to the income distribution, and is particularly high in both the low-density Presidio and the neighborhoods of Forest Hill and Edgehill Heights. Panels B and D show counterfactual changes in income and land values after deregulation, respectively. Both incomes and land values decrease substantially in these aforementioned neighborhoods, and increase near the less-regulated, high-density central city. That is, these neighborhoods gentrify, as is suggested by Facts 1 and 2.

6.3 Socially optimal policy

7 Conclusion

Recent work studying housing regulation asserts that these regulations have implications that extend beyond the issue of housing affordability. First, they are thought to decrease aggregate labour productivity by restricting the size of America's most productive cities (Glaeser and Gyourko, 2018; Hsieh and Moretti, 2019; Duranton and Puga, 2023; Parkhomenko, 2023). Second, they are argued to limit the externalities associated with lower income households free riding off amenities in rich neighborhoods (Hamilton, 1976; Calabrese et al., 2011; Brueckner, 2021), or general congestion externalities (Parkhomenko, 2023; Glaeser and Gyourko, 2018).

In this paper, I ask how a particular type of regulation – the minimum lot size – has impacted both welfare, inequality and income segregation. To do so, I build a model of minimum lot sizes that accompanies rich heterogeneity across cities, households and neighborhoods. This model informs a novel measure of regulatory stringency, and I use that measure to show that the low density neighborhoods of expensive cities are most stringent. This measure of stringency explains positive income sorting into these neighborhoods. Theory shows that patterns of income sorting matter crucially for the welfare

[51] This is in line with conclusions drawn from Figure 9. Endogenous amenities are required to generate a positive relationship between regulation and land values.

[52] These welfare statistics are reported in the introduction.

impacts of deregulation, which motivates the use of a model that can fit variation in stringency both within and across cities that I show exists in the data.

I use this model assess the extent to which the additional implications of deregulation – both aggregate labour productivity and neighborhood choice externalities – matter relative to housing affordability. I find little aggregate productivity gains to deregulation because of the ensuing outflow of affluent households. I also find evidence that regulation does not increase the amenity values of neighborhoods much on average. Instead, households of all income levels move in response to deregulation, causing many different neighborhoods to either increase or decrease their amenity value, and these changes in urban structure have distributional consequences. The neighborhoods that observe the largest increases in affluence and amenity values are the high density neighborhoods of expensive cities. Taken together, I argue that housing affordability is the most important consequence of large-scale deregulation.

In future iterations of this paper, I want to systematically explore which neighborhoods and cities *should* be regulated. Theory suggests that deregulation is beneficial if it causes the poor households of rich neighborhoods to become the rich households of poor neighborhoods, increasing amenity value overall, and that these reallocations can be achieved if regulating neighborhoods that would be rich absent regulation. Facts 1, 2 and the model's predictions about gentrification of high density neighborhoods point toward optimal policy with two distinct features. First, since there is some income sorting into productive "superstar" cities on fundamentals, these cities should have at least some uniform level of regulation. Second, since regulation explains mostly all of the income-density gradient in these cities, within-city variation in regulation appears exclusionary and should be made more uniform across space.

References

Almagro, M., E. Chyn, and B. A. Stuart (2023, January). Urban renewal and inequality: Evidence from chicago's public housing demolitions. Working Paper 30838, National Bureau of Economic Research.

Amalgro, M. and T. Dominguez-Iino (2021, 12). Location sorting and endogenous amenities: Evidence from amsterdam. *Working paper*.

Anagol, S., F. V. Ferreira, and J. M. Rexer (2021, October). Estimating the economic value of zoning reform. Working Paper 29440, National Bureau of Economic Research.

Attanasio, O. P. and L. Pistaferri (2016, May). Consumption inequality. *Journal of Economic Perspectives 30*(2), 3–28.

Barseghyan, L. and S. Coate (2016, August). Property taxation, zoning, and efficiency in a dynamic tiebout model. *American Economic Journal: Economic Policy 8*(3), 1–38.

Baum-Snow, N., M. Freedman, and R. Pavan (2018, October). Why has urban inequality increased? *American Economic Journal: Applied Economics 10*(4), 1–42.

Baum-Snow, N. and L. Han (2021, 05). The microgeography of housing supply. *Working paper*.

Baum-Snow, N. and D. Hartley (2020). Accounting for central neighborhood change, 1980–2010. *Journal of Urban Economics 117*, 103228.

Baum-Snow, N. and R. Pavan (2011, 08). Understanding the City Size Wage Gap. *The Review of Economic Studies 79*(1), 88–127.

Bayer, P., F. Ferreira, and R. McMillan (2007). A unified framework for measuring preferences for schools and neighborhoods. *Journal of Political Economy 115*(4), 588–638.

Bayer, P., R. McMillan, A. Murphy, and C. Timmins (2016). A dynamic model of demand for houses and neighborhoods. *Econometrica 84*(3), 893–942.

Bertaud, A. and J. K. Brueckner (2005). Analyzing building-height restrictions: predicted impacts and welfare costs. *Regional Science and Urban Economics 35*(2), 109–125.

Brueckner, J. K. (2021). Jue insight: Zoning and property taxation revisited—was hamilton right? *Journal of Urban Economics*, 103393.

Brueckner, J. K. and S. S. Rosenthal (2009, 11). Gentrification and Neighborhood Housing Cycles: Will America's Future Downtowns Be Rich? *The Review of Economics and Statistics 91*(4), 725–743.

Brueckner, J. K. and R. Singh (2020). Stringency of land-use regulation: Building heights in us cities. *Journal of Urban Economics 116*, 103239.

Brueckner, J. K., J.-F. Thisse, and Y. Zenou (1999). Why is central paris rich and downtown detroit poor?: An amenity-based theory. *European Economic Review 43*(1), 91–107.

Bunten, D. M. (2017, 6). Is the rent too high? aggregate implications of local land-use regulation. Working Paper 2017-064, Board of Governors of the Federal Reserve System.

Calabrese, S., D. Epple, and R. Romano (2007). On the political economy of zoning. *Journal of Public Economics 91*(1-2), 25–49.

Calabrese, S. M., D. N. Epple, and R. E. Romano (2011, 12). Inefficiencies from Metropolitan Political and Fiscal Decentralization: Failures of Tiebout Competition. *The Review of Economic Studies 79*(3), 1081–1111.

Caliendo, L., M. Dvorkin, and F. Parro (2019). Trade and labor market dynamics: General equilibrium analysis of the china trade shock. *Econometrica 87*(3), 741–835.

Card, D. (2009, May). Immigration and inequality. *American Economic Review 99*(2), 1–21.

Chavent, M., V. Kuentz-Simonet, A. Labenne, and J. Saracco (2018, Dec). Clustgeo: an r package for hierarchical clustering with spatial constraints. *Computational Statistics 33*(4), 1799–1822.

Chetty, R. and N. Hendren (2018, 02). The Impacts of Neighborhoods on Intergenerational Mobility I: Childhood Exposure Effects*. *The Quarterly Journal of Economics 133*(3), 1107–1162.

Combes, P.-P. and L. Gobillon (2015). Chapter 5 - the empirics of agglomeration economies. In G. Duranton, J. V. Henderson, and W. C. Strange (Eds.), *Handbook of Regional and Urban Economics*, Volume 5 of *Handbook of Regional and Urban Economics*, pp. 247–348. Elsevier.

Couture, V., C. Gaubert, J. Handbury, and E. Hurst (2019, 08). Income growth and the distributional effects of urban spatial sorting. Working Paper 26142, National Bureau of Economic Research.

Couture, V. and J. Handbury (2020). Urban revival in america. *Journal of Urban Economics 119*, 103267.

Cui, T. (2023, 11). The emergence of exclusionary zoning across american cities. *Working paper*.

Davidoff, T. (2016). Supply constraints are not valid instrumental variables for home prices because they are correlated with many demand factors. *Critical Finance Review 5*(2), 177–206.

Davis, M. A., J. Gregory, and D. A. Hartley (2023, 06). Preferences over the racial composition of neighborhoods: Estimates and implications. *Working paper*.

Diamond, R. (2016, March). The determinants and welfare implications of us workers' diverging location choices by skill: 1980-2000. *American Economic Review 106*(3), 479–524.

Dingel, J. I. and F. Tintelnot (2021, January). Spatial economics for granular settings.

Duranton, G. and D. Puga (2023). Urban growth and its aggregate implications. *Econometrica 91*(6), 2219–2259.

Epple, D. and G. J. Platt (1998). Equilibrium and local redistribution in an urban economy when households differ in both preferences and incomes. *Journal of Urban Economics 43*(1), 23–51.

Fajgelbaum, P. D. and C. Gaubert (2020, 01). Optimal Spatial Policies, Geography, and Sorting*. *The Quarterly Journal of Economics 135*(2), 959–1036.

Fernandez, R. and R. Rogerson (1997). Keeping people out: Income distribution, zoning, and the quality of public education. *International Economic Review 38*(1), 23–42.

Finlay, J. and T. Williams (2022, 09). Housing demand, inequality, and spatial sorting. *Working paper*.

Fischel, W. A. (2001). *The Homevoter Hypothesis: How Home Values Influence Local Government Taxation, School Finance, and Land-Use Policies*. Harvard University Press.

Fogli, A. and V. Guerrieri (2019, 08). The end of the american dream? inequality and segregation in us cities. Working Paper 26143, National Bureau of Economic Research.

Glaeser, E. and J. Gyourko (2018, February). The economic implications of housing supply. *Journal of Economic Perspectives 32*(1), 3–30.

Glaeser, E. L., M. E. Kahn, and J. Rappaport (2008). Why do the poor live in cities? the role of public transportation. *Journal of Urban Economics 63*(1), 1–24.

Goldsmith-Pinkham, P. and K. Shue (2023). The gender gap in housing returns. *The Journal of Finance 78*(2), 1097–1145.

Greaney, B. (2023, 10). Homeownership and the distributional effects of uneven regional growth*. *Working paper*.

Grieson, R. E. and J. R. White (1981). The effects of zoning on structure and land markets. *Journal of Urban Economics 10*(3), 271–285.

Guerrieri, V., D. Hartley, and E. Hurst (2013). Endogenous gentrification and housing price dynamics. *Journal of Public Economics 100*, 45–60.

Gyourko, J., J. S. Hartley, and J. Krimmel (2021). The local residential land use regulatory environment across u.s. housing markets: Evidence from a new wharton index. *Journal of Urban Economics 124*, 103337.

Gyourko, J., C. Mayer, and T. Sinai (2013, 11). Superstar cities. *American Economic Journal: Economic Policy 5*(4), 167–99.

Gyourko, J. and R. Molloy (2015). Chapter 19 - regulation and housing supply. In G. Duranton, J. V. Henderson, and W. C. Strange (Eds.), *Handbook of Regional and Urban Economics*, Volume 5 of *Handbook of Regional and Urban Economics*, pp. 1289–1337. Elsevier.

Hamilton, B. W. (1975). Zoning and property taxation in a system of local governments. *Urban Studies 12*(2), 205–211.

Hamilton, B. W. (1976). Capitalization of intrajurisdictional differences in local tax prices. *The American Economic Review 66*(5), 743–753.

Heathcote, J., K. Storesletten, and G. L. Violante (2017, 06). Optimal Tax Progressivity: An Analytical Framework*. *The Quarterly Journal of Economics 132*(4), 1693–1754.

Herkenhoff, K. F., L. E. Ohanian, and E. C. Prescott (2018). Tarnishing the golden and empire states: Land-use restrictions and the u.s. economic slowdown. *Journal of Monetary Economics 93*, 89–109. Carnegie-Rochester-NYU Conference on Public Policy held at the Stern School of Business at New York University.

Hilber, C. A. and F. Robert-Nicoud (2013). On the origins of land use regulations: Theory and evidence from us metro areas. *Journal of Urban Economics 75*, 29–43.

Hornbeck, R. and E. Moretti (2018, May). Who benefits from productivity growth? direct and indirect effects of local tfp growth on wages, rents, and inequality. Working Paper 24661, National Bureau of Economic Research.

Hsieh, C.-T. and E. Moretti (2019, April). Housing constraints and spatial misallocation. *American Economic Journal: Macroeconomics 11*(2), 1–39.

Kulka, A. (2019, 12). Sorting into neighborhoods: The role of minimum lot sizes. *Working paper*.

Kulka, A., A. Sood, and N. Chiumenti (2022, 1). How to reduce housing costs? understanding local deterrents to building multifamily housing. *Working paper*.

Lee, S. and J. Lin (2017, 03). Natural Amenities, Neighbourhood Dynamics, and Persistence in the Spatial Distribution of Income. *The Review of Economic Studies 85*(1), 663–694.

Mills, E. S. (2005). Why do we have urban density controls? *Real Estate Economics 33*(3), 571–585.

Molloy, R. (2020). The effect of housing supply regulation on housing affordability: A review. *Regional Science and Urban Economics 80*(C).

Molloy, R., C. G. Nathanson, and A. Paciorek (2022). Housing supply and affordability: Evidence from rents, housing consumption and household location. *Journal of Urban Economics 129*, 103427.

Ortalo-Magné, F. and A. Prat (2014, February). On the political economy of urban growth: Homeownership versus affordability. *American Economic Journal: Microeconomics 6*(1), 154–81.

Parkhomenko, A. (2023). Local causes and aggregate implications of land use regulation. *Journal of Urban Economics 138*, 103605.

Saiz, A. (2010, 08). The Geographic Determinants of Housing Supply*. *The Quarterly Journal of Economics 125*(3), 1253–1296.

Song, J. (2021, 11). The effects of residential zoning in u.s. housing markets. *Working paper*.

Straub, L. (2019). Consumption, savings, and the distribution of permanent income. Download paperRevise and resubmit at Econometrica.

Su, Y. (2022, February). The rising value of time and the origin of urban gentrification. *American Economic Journal: Economic Policy 14*(1), 402–39.

Turner, M. A., A. Haughwout, and W. van der Klaauw (2014). Land use regulation and welfare. *Econometrica 82*(4), 1341–1403.

White, M. J. (1975). The effect of zoning on the size of metropolitan areas. *Journal of Urban Economics 2*(4), 279–290.

Wood, S. (2012, October). mgcv: Mixed gam computation vehicle with gcv/aic/reml smoothness estimation.

Zabel, J. and M. Dalton (2011). The impact of minimum lot size regulations on house prices in eastern massachusetts. *Regional Science and Urban Economics 41*(6), 571–583.

Figures

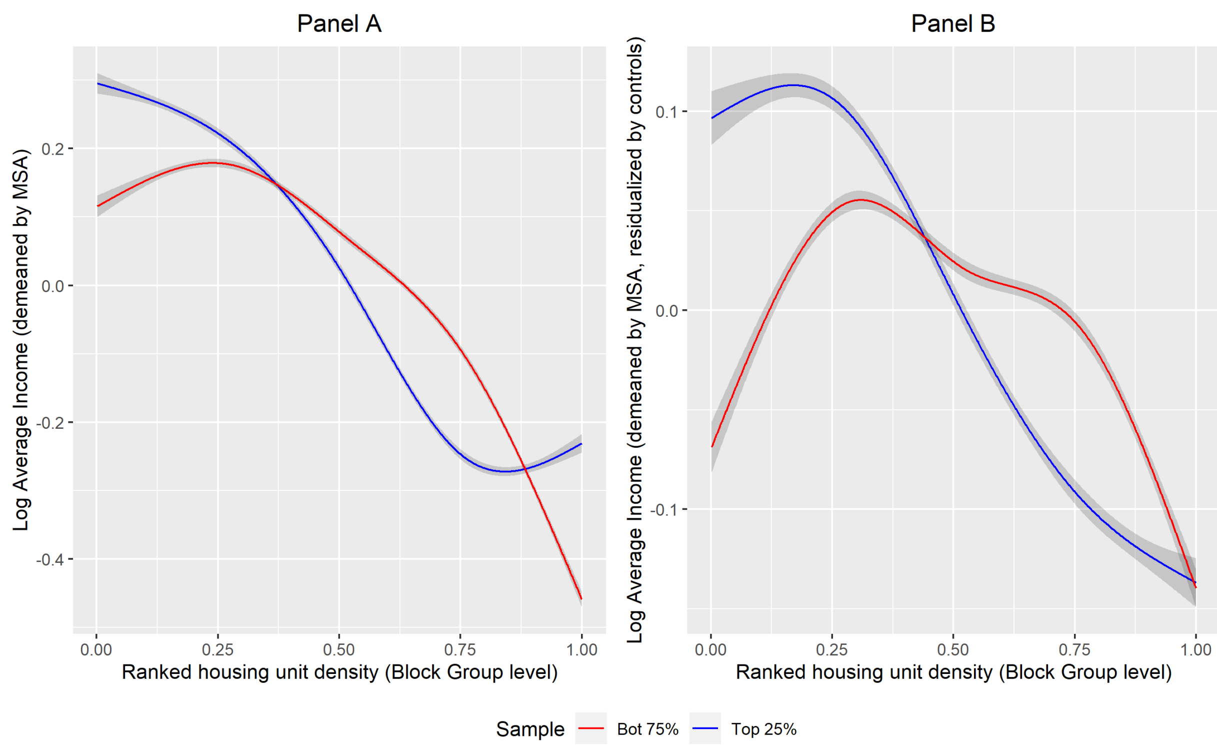


Figure 1:
Plot of cublic spline **S** from Equation (1) with 5 knots, estimated with the *mgcv* package in R (Wood, 2012). 95% confidence sets are reported. Panel A reports the model with controls, while Panel B includes the model with the full set. The full set of controls are CBD distance, share of white households, building age, household size, share of households using public transport to commute, share of households using cars to commute, and average commuting time time from the ACS; density of performing arts, spectator sports, casinos, recreational activities, restaurants, fast food, bars and coffee shop establishments from NaNDA; and the density of EPA toxic releases and the density of public transport stops from NaNDA. All variables from the ACS vary at the block group level, and at the tract level for NaNDA.

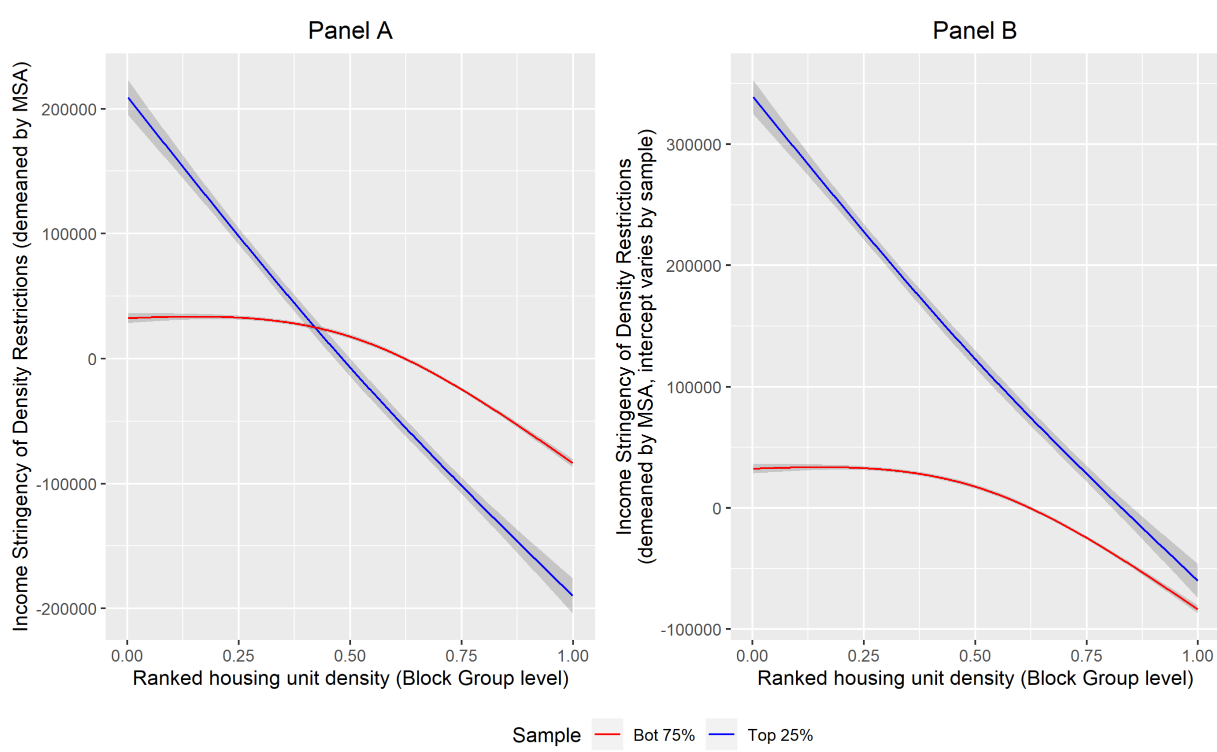
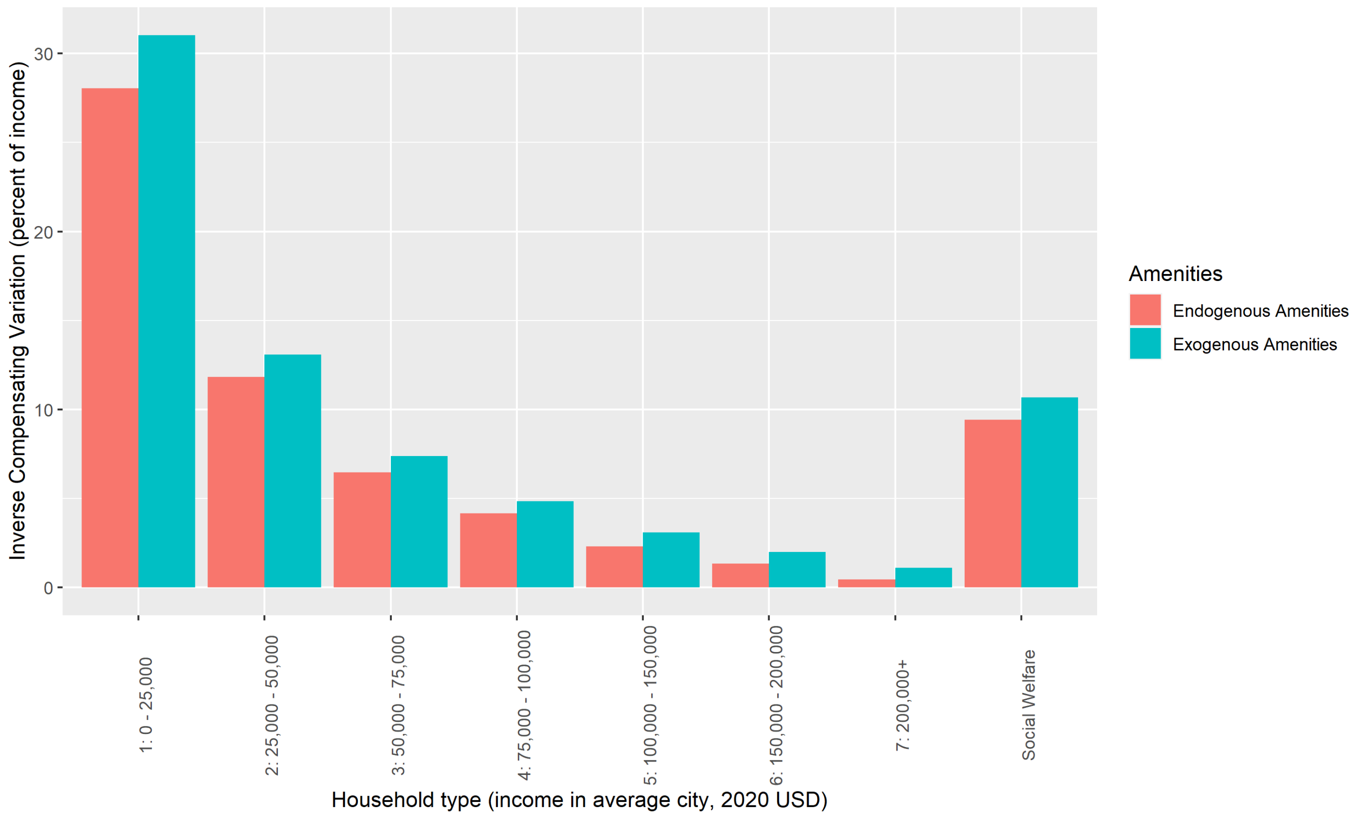


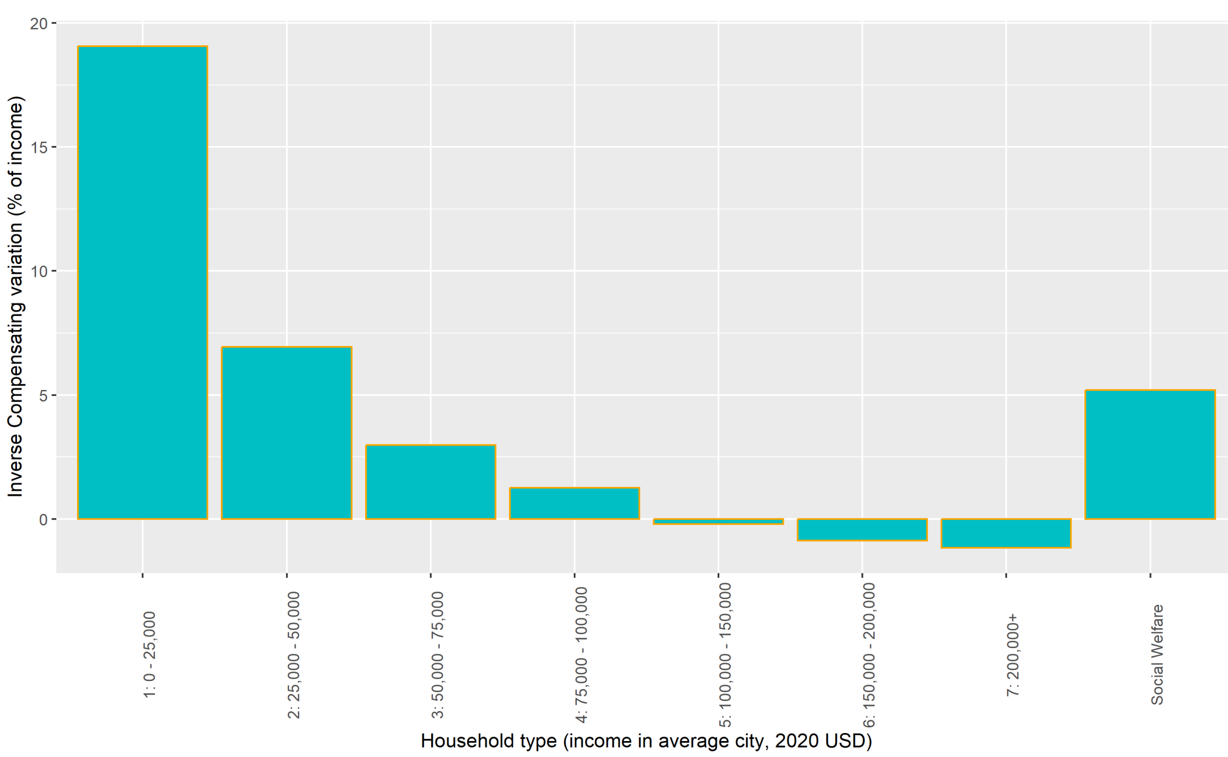
Figure 2:
Plot of cublic spline **S** with 5 knots, estimated with the *mgcv* package in R (Wood, 2012). The specification in Panel A replaces log income in Equation (1) with regulatory stringency in Equation (2) and *excluded* controls, demeaned by city and measured in 2020 US dollars. The residualized version of this regression (mirroring Panel B of Figure 1) is omitted as they look qualitatively identical under the same set of controls used in Figure 1. 95 % confidence sets are reported. Panel B repeats the same regression while allowing the average stringency to vary by sample. In other words, the blue curve is shifted upward by a constant.

Figure 3:
Compensating variation for deregulation by household type.



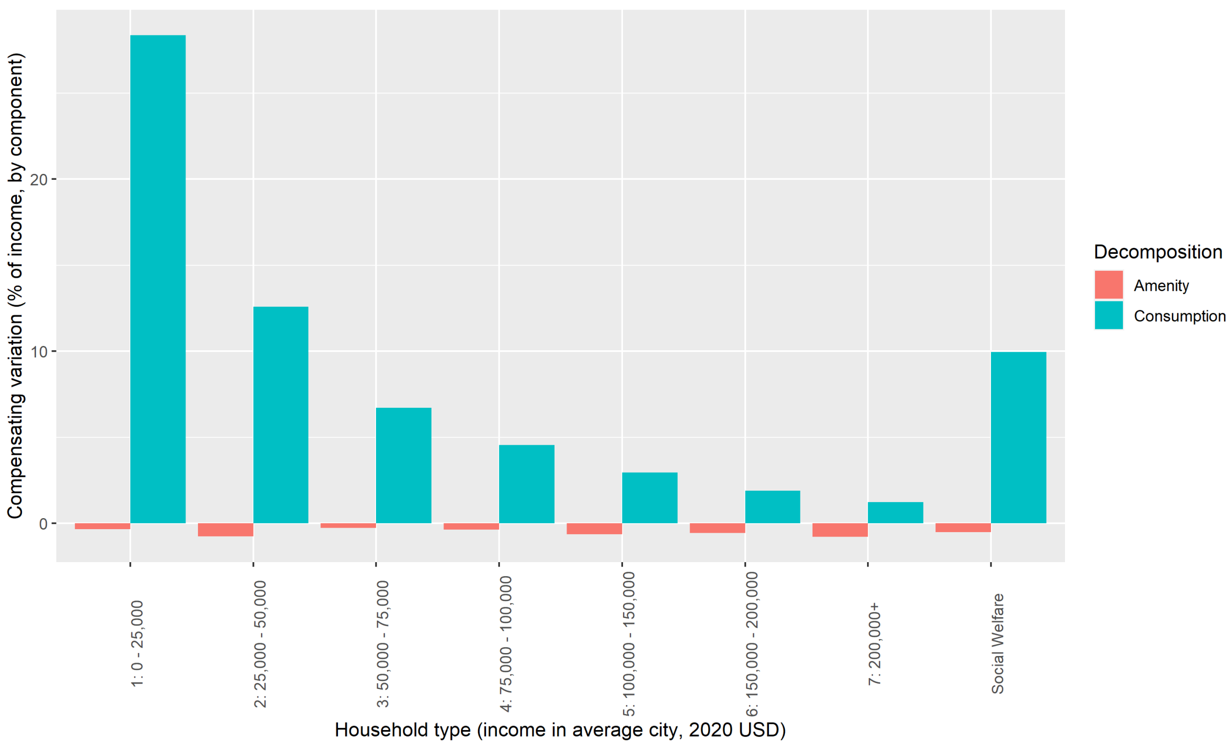
Welfare is measured as the inverse compensating variation as a percentage of initial income. Higher values mean higher welfare gains. Social welfare is the population weighted average of welfare by type. "Endogenous Amenities" refers to counterfactual outcomes under the baseline model. "Exogenous" refers to counterfactuals under the parameter restriction $\Omega(z) = 0$ for all z.

Figure 4:
Compensating variation for deregulation by household type,
pooled over renters and homeowners and including capital losses.



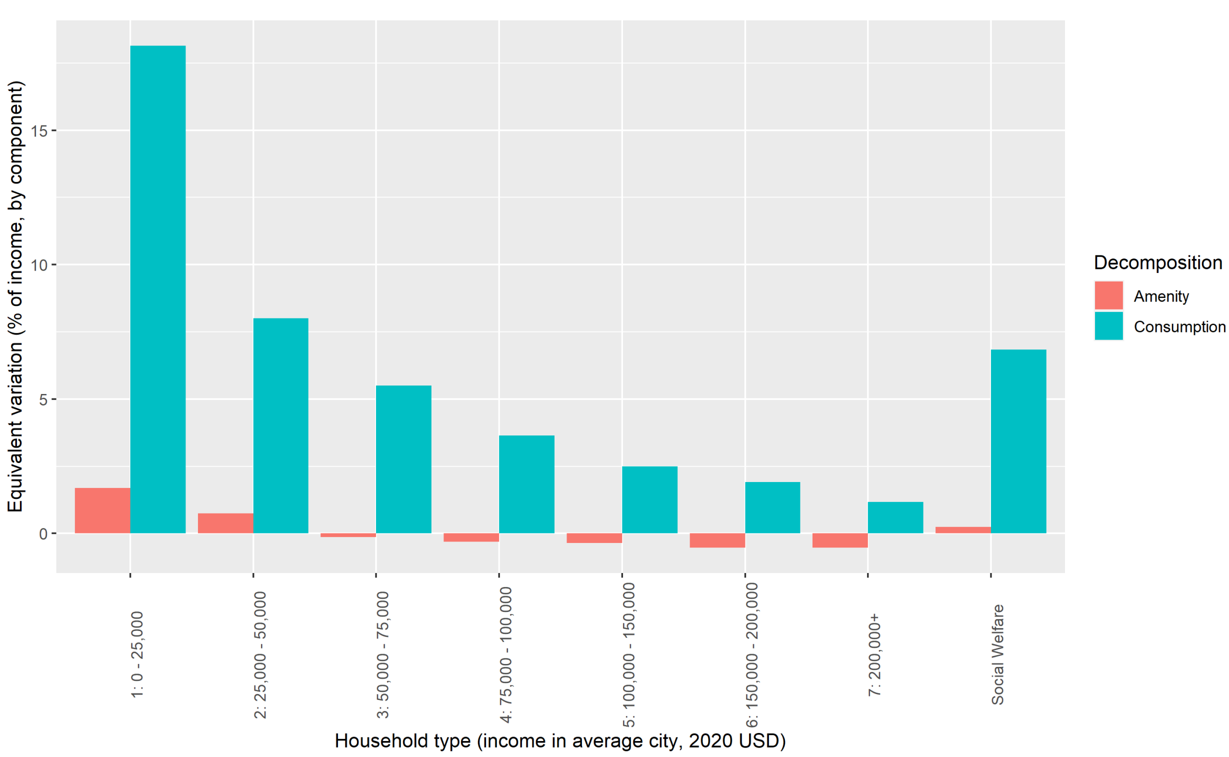
Welfare is measured as the inverse compensating variation as a percentage of initial income. Higher values mean higher welfare gains. Social welfare is the population weighted average of welfare by type. Results incorporate capital losses for homeowners by income type, using a procedure outlined in Appendix E.2. Results include equilibrium adjustments to neighborhood amenity value as in the baseline counterfactual. Renters and homeowners are pooled by type using a population-weighted average of welfare changes.

Figure 5:
Shapely decomposition of welfare into consumption and amenities



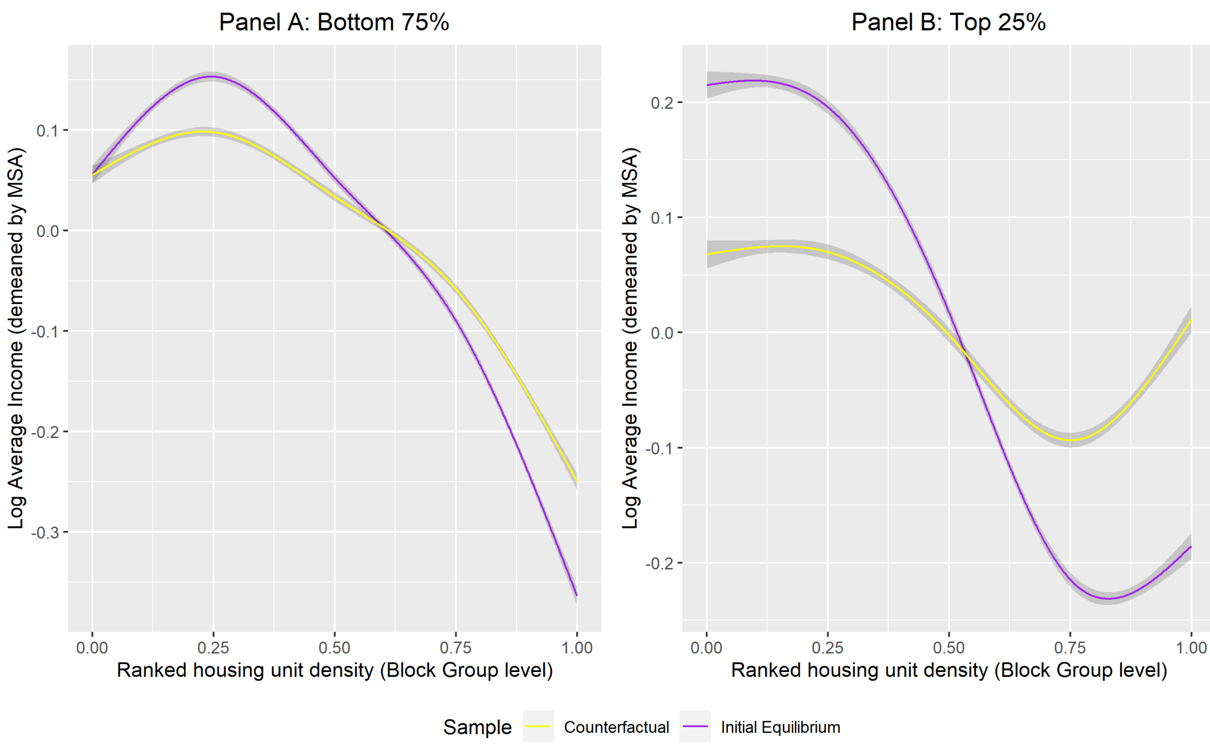
Welfare is measured as the inverse compensating variation as a percentage of initial income. Higher values mean higher welfare gains. Social welfare is the population weighted average of welfare by type. "Amenity" and "Consumption" components are constructed using Shapely values, with a procedure outlined in Appendix E.3. The sum of components add to the compensating variation reported in Figure 3.

Figure 6:
Shapely decomposition of welfare into consumption and amenities,
fundamental amenities are equal across income types



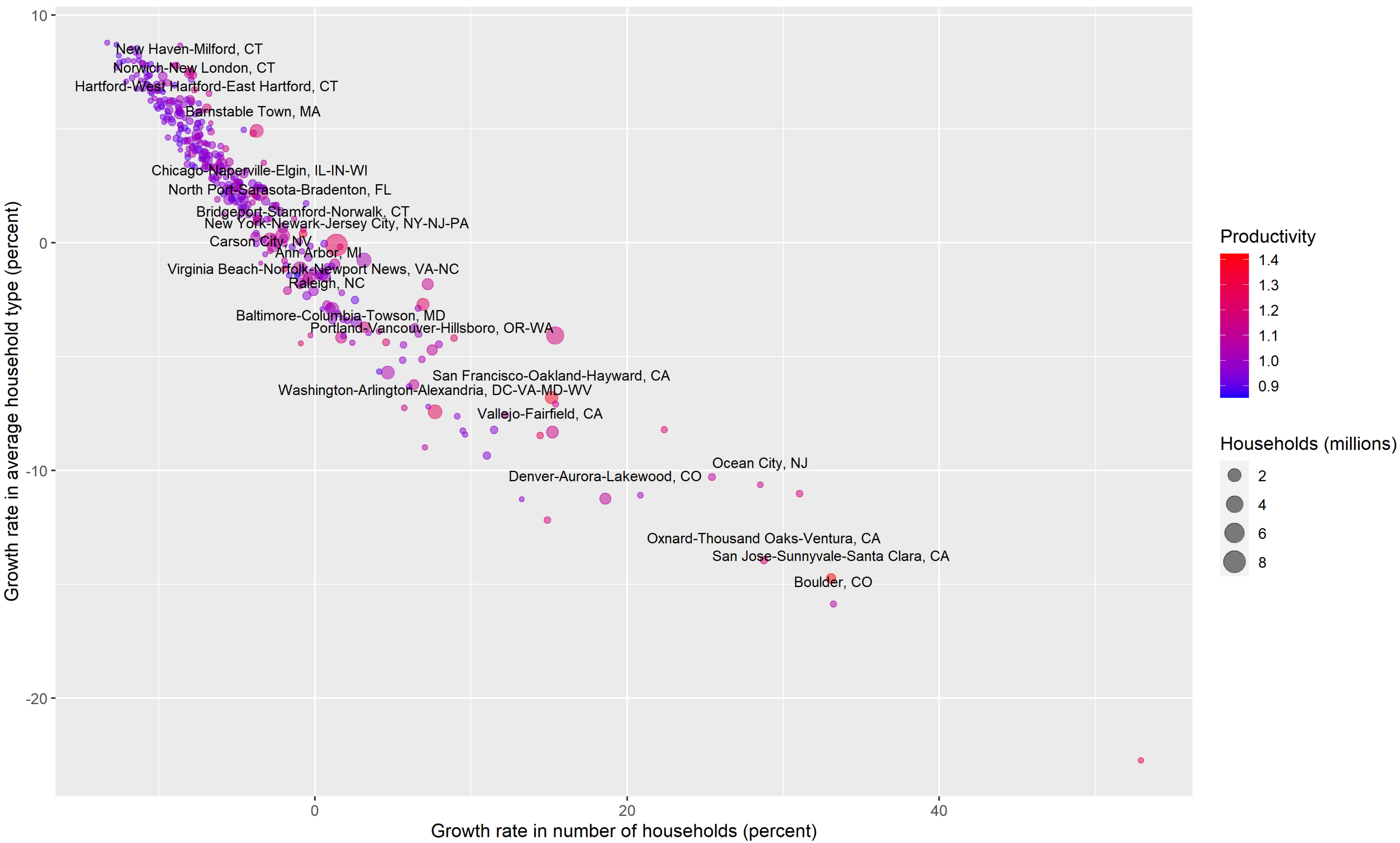
Welfare is measured as the inverse compensating variation as a percentage of initial income. Higher values mean higher welfare gains. Social welfare is the population weighted average of welfare by type. "Amenity" and "Consumption" components are constructed using Shapely values, with a procedure outlined in Appendix E.3. The sum of components add to the equivalent variation associated with full deregulation with no fundamental amenity differences across different income types.

Figure 7:
Gentrification in superstar cities post deregulation.



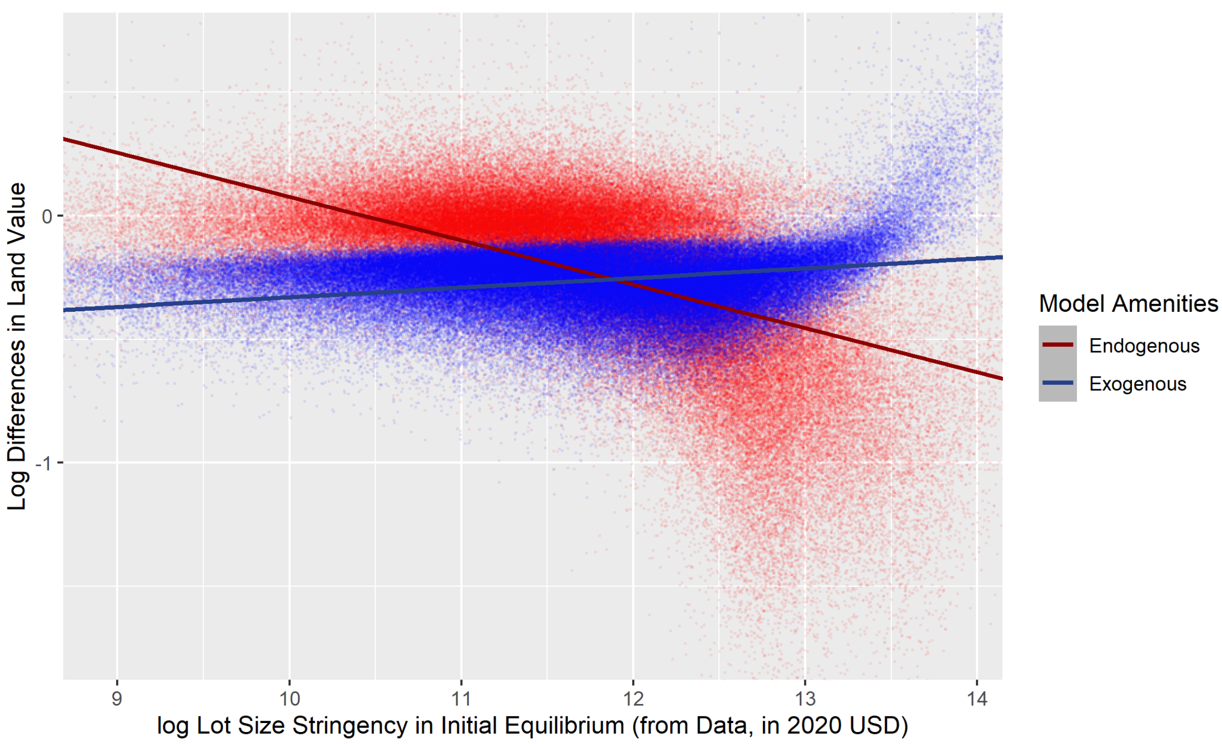
For each panel, demeaned log average income at the neighborhood level is regressed against the observed density ranking of neighborhoods at baseline. The purple regression uses data from the baseline equilibrium that matches data, as in Figure 1. The yellow regression uses data generated from the counterfactual with no minimum lot sizes. Each panel corresponds to ”superstar” sample cities (Top 25%) and ”non-superstar” sample cities (Bottom 75%), as in Figure 1.

Figure 8: City Income Sorting



The y axis is defined as the change in the average income that a household could earn in an average city from baseline to counterfactual. Correlation between the growth rate in the number of households productivity is approximately 50%, and this correlation is 70% for unadjusted housing prices. Cities on the top left (high income, negative population growth) tend to have low productivity, and cities with higher productivity tend to be in the bottom left.

Figure 9: Changes in land values by regulatory stringency in the baseline equilibrium.



Changes in land values by regulatory stringency in the baseline equilibrium (from Equation 2). Data generated from the endogenous amenities counterfactual is scattered in black. Linear fits on data generated by endogenous and exogenous amenity counterfactuals are in red and blue, respectively.

Figure 10:
San Francisco, initial data and counterfactual outcomes.

San Francisco's initial data on income and regulatory stringency, along with changes in incomes and land values after halving the minimum lot size. Regulatory stringency is the empirical measure introduced in Equation (2).

Tables

Table 1: Baseline IV Estimates by income group.

VARIABLES	(1) ln Amenity (Low)	(2) ln Amenity (Med)	(3) ln Amenity (High)
ln Income	0.1341***	0.2859***	0.3004***
	(0.0326)	(0.0445)	(0.0330)
Slope Control	-0.0020**	-0.0029***	-0.0015*
	(0.0008)	(0.0011)	(0.0008)
Local Slope Control	-0.0002	-0.0009	-0.0017***
	(0.0007)	(0.0008)	(0.0006)
Outer Slope Control	0.0063*	0.0052	0.0038
	(0.0036)	(0.0042)	(0.0026)
Observations	170,951	171,045	165,317
R^2	-0.3828	-0.5837	0.2756
Specification	IV	IV	IV
Donut	0.75-1.25km	0.75-1.25km	0.75-1.25km
Base Controls	Yes	Yes	Yes
Amen/Topo Controls	No	No	No
Density Control	No	No	No
FStat Bart c 35 km	63.4	63.4	58.5

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Columns are ordered by income group. All specifications include MSA fixed effects and standard errors are clustered using a 35km Bartlett kernel. "Donut Slope Control" is the average slope within the block group plus a buffer with length equal to d_1. "Local Slope Control" is the average slope within the block group. "Outer slope control" is the average slope from d_2 to 10km. ln Income is instrumented with the average slopes of block groups that have centroids within buffer d_1 and d_2. "Base Controls" include travel time, building age, public transport and bus shares in commuting and CBD distance. "Amen/Topo" controls include various amenities (density of coffee shops, parks, restaurants) and various topographic features (cover of different types of forest such as deciduous or evergreen, wetlands, perennial snow cover). "Density Control" is the within-MSA density ranking of the block group.

Table 2: Productivity changes from counterfactual by model assumptions.

End. Amenities	End. Productivity	Education	Prod. Growth	Prod. Growth, no income sorting
Yes	No	No	0.26%	1.41%
No	No	No	0.44%	2.14%
Yes	Yes	No	0.5%	1.82%
Yes	No	Yes	0.25%	1.3%
Yes	Yes	Yes	0.46%	1.61%

Productivity changes from counterfactual by model assumptions. "Prod. Growth" refers to productivity changes from counterfactual. "No income sorting" refers to calculating what productivity growth would be if cities grew at levels determined by counterfactual, but with no compositional effects by income or education. For the full model (last row), I ignore how city wages change when calculating the no-income-sorting counterfactual. This is because uniform city growth when productivity is endogenous by education causes income sorting. This is because agglomeration forces are education-augmenting, as in Baum-Snow et al. (2018).

A Appendix: Data and Facts Continued

A.1 Data Construction

In this section, I focus on data used in both the empirical work and estimation of the structural model. This paper uses two broad sources of data:

1. CoreLogic:
   (a) Universe of property assessments (most current assessments as of December 2022)
   (b) 2008-2012 and 2015-2022 universe of transactions
2. Other data:
   (a) 2008-2012 and 2016-2020 American Community Survey (ACS) tabulations, harmonized to 2020 block group definitions
   (b) 2010 and 2020 census housing counts harmonized to 2020 block groups.
   (c) 2007-2010 and 2016-2017 National Neighborhood Data Archive (NaNDA) data at the tract level.
   (d) The US Geological survey's EDNA database (2003 version)

where the separated data ranges are used to construct two panels over block groups. These two sources are used to construct a block-group level panel:

1. **Minimum lot sizes and land value density**. I defer a discussion of how minimum lot sizes and land value densities are constructed to Appendices A.2 and A.3, as this requires detail.
2. **The share of housing units in regulated structures**. These enter into the stringency measure in (2). The ACS reports housing counts for structures between 1, 2, and 3-4 housing units, labeled as **Units in Structure**. These are aggregated to arrive at household counts in regulated structures. I do not use CoreLogic data to calculate these shares as many assessments in large multifamily structures do not have information on the number of housing units.
3. **Block group level densities of housing units and incomes**. Average incomes and shares of housing units in regulated structures are directly calculated from the ACS tabulations for each panel. Average incomes in each block group are calculated by dividing tabulated aggregate neighborhood income by the number of sampled households[53]. Housing counts used to construct density measures are from the 2020 Census.
4. **Various controls used in estimation**. These include (from the ACS) median building age, household size; share of cars in commuting, buses and other public transport; shares of households that are families, average travel time, white and college

[53] These tabulations are labeled **Aggregate Household Income in the Past 12 Months** and **Housing Units**

shares; (from NaNDA) density of performing arts facilities, sports, casinos, recreational, transit stops, fast food, restaurants, coffee shops and bars; fraction of land area in parks, covered by perrenial snow, deciduous forest, evergreen, mixed forest, shrubs, herbaceous, woody and herbaceous wetlands.

5. **Slopes**. I use the USGS EDNA database at 30×30m resolution to create average slope measures at the block group level. This data is used solely in the estimation of the effect of income on neighborhood amenity value in Section 5.

There are roughly $196,000$ block groups in 2013-definition Metropolitan Statistical Areas (MSAs). To estimate the regressions in 1 and 2, I remove block groups with less than 25 housing units per square mile, representing about 8000 block groups (4% of the sample). The facts are robust to censoring at a wide range of densities, including not censoring at all. Summary statistics for reported variables above given in Table 4.

A.2 Constructing Zoning Districts

Block groups generally do not correspond to areas by which local governments assign uniform land use regulation. To measure ”bunching” of minimum lot sizes around regulatory levels, I need to take a stand on the geographic unit by which to construct lot size distributions. I call these geographic units ”Zoning Districts”. More often than not, zoning districts can be inferred from zoning codes reported in the CoreLogic data[54]. I assign a zoning code to a block group by taking the modal code across parcels in the block group. Block groups with no populated zoning code data are assigned missing.

For roughly $66,000$ block groups (one third of the sample), there is no data on zoning codes to apply regulation. To extend coverage of zoning districts, I cluster the remaining block groups. The fundamental challenge in doing so is a trade-off when choosing the size of these zoning districts: on one hand, large zoning districts may pool together multiple levels of regulation, and there is no way to distinguish between them. On the other hand, zoning districts that are too small may result in spurious measurements due to the lack of observations. I perform the clustering algorithm within each US municipality, as these are typically responsible for setting regulation. I assign at most one municipality to a block group by matching municipalities to geocoded parcels, and taking the modal municipality across parcels. I test the algorithm on two different definitions of a municipality: the municipality reported by CoreLogic for assessment purposes, and the municipality from the US Place Shapefiles.

To aggregate block groups into zoning districts, I employ a highly flexible clustering algorithm, which I describe here. This algorithm nests the clustering algorithm of Chavent et al. (2018) and is implemented via their R package. This is useful because it allows for the weighing of two types of variables to assign clusters: geographic proximity and other non-geographic variables. I use block-group modal lot size and first decile of the lot size distribution as these variables. The algorithm allows me to consider how important geographic proximity needs to be relative to other location based characteristics that might signal similar minimum lot sizes.

The number of clusters to assign is a parameter in Chavent et al. (2018)'s algorithm. This governs how large zoning districts are. I provide a maximum level of a *targeted average number of block groups per cluster* to as a hyperparameter in the algorithm. This parameter defines the minimum number of clusters that must appear in a municipality. For example, in a municipality with 100 block groups without zoning codes, and if I impose that the maximum cluster size is roughly 25 block groups, then the minimum number of clusters is roughly 4. Subject to this minimum number of clusters, I select the actual number of clusters to minimize the *silhouette score* on non-geographic variables, which is a metric that assesses both within-cluster similarity and across-cluster dissimilarity and is typically used to assess clustering performance. I consider a range of targeted maximum cluster sizes of 5, 15, 25, 100 and 250 block groups, respectively.

[54] However, no information on specific regulation can be extracted from these codes.

Summarizing, my algorithm incorporates three important hyperparameters:

1. Two definitions of a municipality. The first is the municipality reported in the assessments. The second is the incoporated city taken from the US Place shapefiles. If block groups are not in a reported municipality, I treat the county as a municipality instead[55].

2. The weight of geographic proximity relative to non-geographic variables when assigning clusters (Chavent et al., 2018).

3. Targeted maximum sizes (in number of block groups) for the average cluster.

The algorithm can be roughly described as follows:

Algorithm 1 Clustering

For all hyperparameters do:

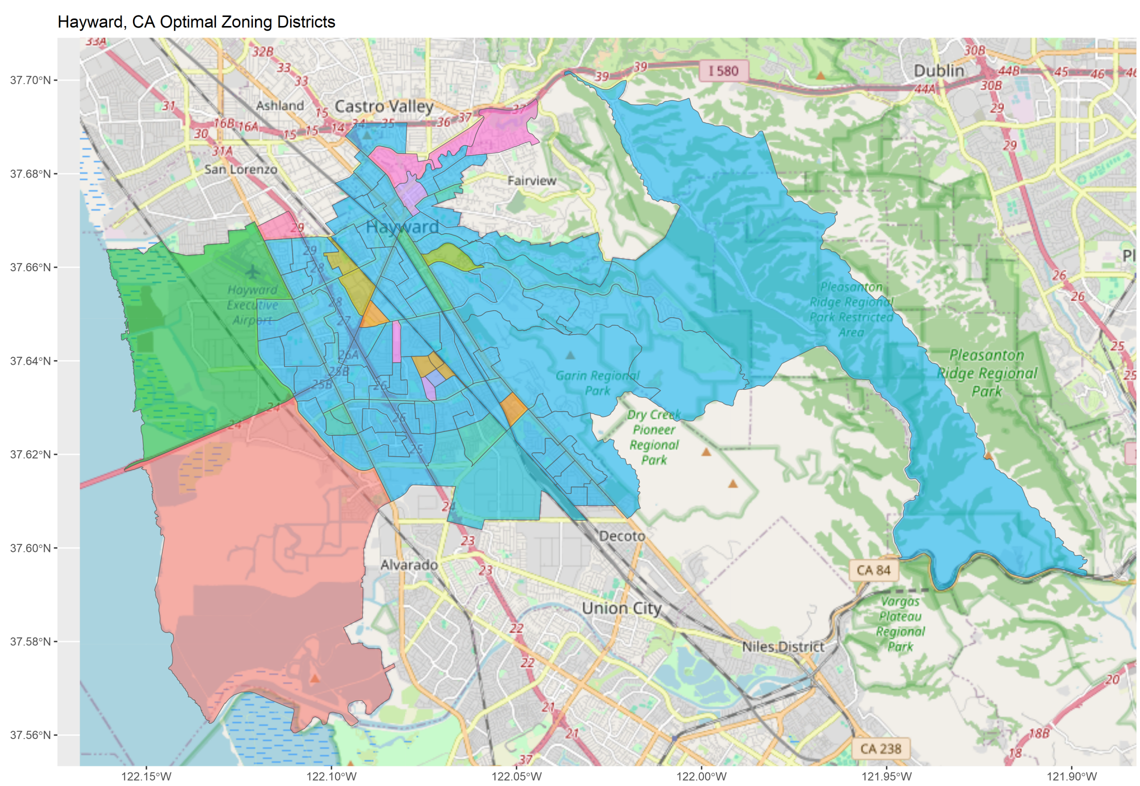
For all municipalities do:

1. Assign block groups with same non-missing zoning code to district
2. Cluster remaining block groups choosing number of clusters between 2 and $N - 1$, where $N - 1$ is the number of block groups to be clustered
3. Select desired number of clusters to minimize sillhouette score subject to the minimum number of clusters constraint. All block groups in a cluster are assigned a different zoning code from other clusters.

Figure 11 shows the optimal set of zoning districts for Hayward in the San Francisco metropolitan area (optimal will be defined shortly). The optimal algorithm places largest weight on geographic proximity when assigning clusters. The algorithm also uses a maximum cluster size of 5 block groups, as well as CoreLogic's municipality definition. The average zoning district (including block groups that have missing and non-missing zoning codes) has 3.3 block groups, with a standard deviation of 11. The median size of a zoning district is one block group. Clusters are not sensitive to choices of other hyperparameters; I provide more discussion when assessing the performance of the algorithm in Appendix A.4.

[55]Unincorporated locations typically have land use regulation set by the county they are in.

Figure 11: Zoning Districts in Hayward, California

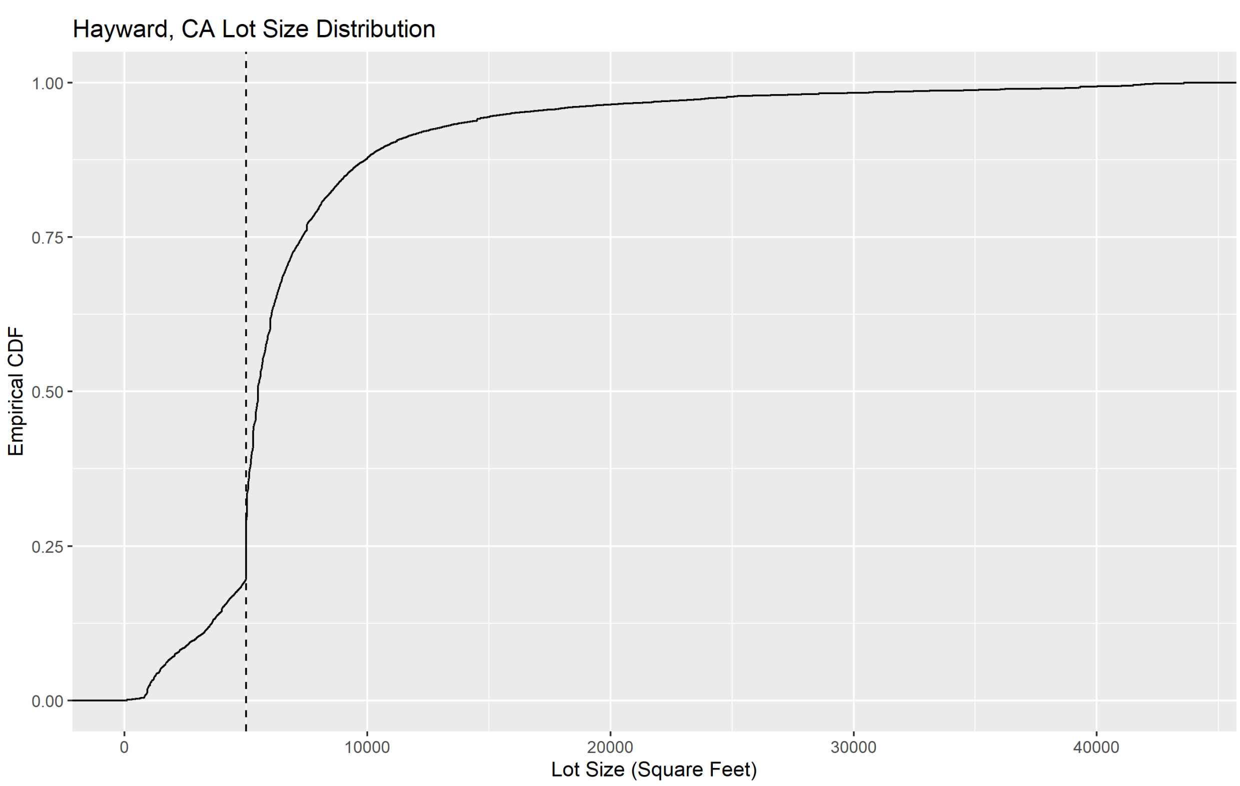


A.3 Measuring Minimum Lot Sizes and Stringency

The zoning districts give a set of geographic boundaries for which to measure ”bunching” in the lot size distribution. I use the mode of the distribution as a measure of this bunching. Within each zoning district, I take the modal lot size for single family homes, duplexes, triplexes and fourplexes, and adjust the lot size by the implied number of housing units per lot (for example, dividing the minimum lot size by 2 for duplexes). This is useful because municipalities often assign different lot size restrictions to these four different types of structures. I define the *baseline unit density restriction* as the minimum of these to be as conservative as possible, as detecting bunching alone cannot measure regulations pertaining to which of these four structures are allowed to be built. This minimum is the unit density restriction that is used both in the facts and in the calibration of the model.

To build intuition behind why the mode is an accurate measure of bunching, consider the distribution of lot sizes in Hayward, California. The official minimum lot size for most of Hayward's zoning districts is 5000 square feet. The mode of the observed distribution of lots in Hayward is also 5000 square feet, as shown below. Mostly all block groups in Hayward are assigned the 5000 square foot minimum lot size with this measurement procedure.

Figure 12: Lot size distribution in Hayward, California



Land value density Next, I outline how I measure the density of land values, which enters into the measure of regulatory stringency in Equation (2). I limit the sample to *regulated structures* (single family, duplex, triplex and fourplexes) as the only use for land value density pertains to the stringency of regulation (in both the model and the empirical work). I deviate from directly calculating the density of land values (for example, by taking the median sales value across regulated structures and dividing by the median lot size for each block group). This is because land value density varies significantly within block groups between houses with small lots and those with large lots; this leads to large measurement error in regulatory stringency when the measured minimum lot size deviates from the median lot size within a neighborhood.

Instead, I predict land value densities *at the minimal lot* using a linear model. Within each MSA, I estimate the following regression over assessments indexed by a in neighborhood i and city c

$$\text{LandValue}_{aic} = \beta_{0ic} + \beta_{1c}\text{LotSize}_{aic} + \epsilon_{aic} \tag{25}$$

The slope parameter of this model informs about the relative price of houses on small lots relative to large lots within the same MSA. I then use the β_{1c} parameters from these regressions to predict the land value of the minimal lot in every neighborhood i using the formula

$$\text{PredLandValue}_{ic} = \text{MedianLandValue}_{ic} - \beta_{1c}\text{MedianLotSize}_{ic} + \beta_{1c}\text{MinLotSize}_{ic} \tag{26}$$

where MinLotSize_{ic} is the measured minimum lot size (ignoring implicit unit density restrictions, such as dividing by 2 if duplexes are allowed). This formula ensures that if the minimum lot size is also at the median in the lot size distribution, then its land value is also predicted to be the median land value of the neighborhood[56]. Finally, I measure land value densities as

$$\text{LandValueDensity}_{ic} = \text{PredLandValue}_{ic}/\text{MinLotSize}_{ic} \tag{27}$$

which enter directly into the stringency measure, Equation (2).

Additional cleaning I perform additional cleaning to ensure my measure of regulatory stringency is not wrought with measurement error and other pathologies. I list each procedure below:

1. Transactions data are particularly sparse in small and inexpensive cities. This means that there are select block groups in small cities that report massive home values relative to nearby block groups, which makes the measure of regulatory stringency large. This is driven by measurement error when using observed transactions to estimate the transaction value of a median home in these block groups. In cities that report a median transaction below 300k USD (below the US average), I impute block groups median transaction value to the MSA median if it is above 1 million USD. This imputation has the effect of limiting measurement error that compounds in model counterfactuals, making some cities look artificially more stringent then they plausibly are; driven by only a handful of block groups that appear extremely stringent for spurious reasons. Conclusions of counterfactual exercises remain the same whether or not I do this cleaning.

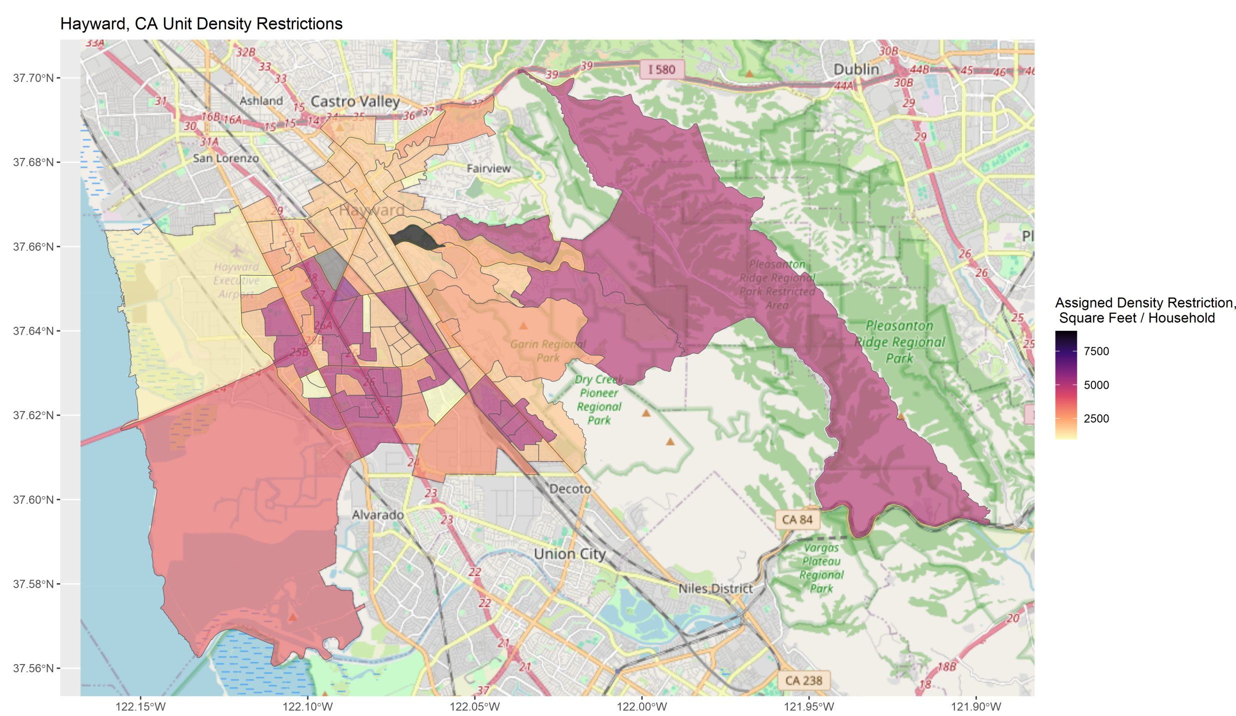
2. Sometimes, the algorithm fails in the sense that the size of the measured minimum lot size is well above the average lot size in some block groups. This usually happens

[56]This often occurs in neighborhoods with extreme "bunching" – where most lots are built at the minimum.

in remote locations where parcels are sparse and their size idiosyncratic. For model counterfactuals and to estimate the facts, I set the observed minimum lot size to zero if the assigned minimum lot size is twice the average. The facts and model counterfactuals are robust to different thresholds for which to perform this cleaning; including no cleaning at all.

In Table 4, I report summary statistics of the measured unit density restrictions, land value densities, and the accompanying measure of regulatory stringency from Fact 2. The average unit density restriction across block groups is 0.272 acres per housing unit with a standard deviation of 3.74. Table 3 additionally reports boxplots of stringency for select cities. Figure 13 shows the measured unit density restrictions for Hayward, California by zoning district.

Figure 13: Hayward Unit density restrictions



A.4 Validating Minimum Lot Sizes

To validate the algorithm and to select clustering hyperparameters, I introduce two sources of official data on minimum lot sizes (or more generally, unit density restrictions). The first source, which mirrors validation exercises in Song (2021) and Cui (2023), is the Terner California Land Use Survey (TCLUS). This source of data is aggregated to the municipality level and cannot be used to test how well the algorithm performs on micro-geographic variation in regulation. The second data source – official zoning maps for 13 large US cities – addresses this limitation. I address how the validation procedure works for each source below.

TCLUS The TCLUS reports minimum lot sizes for both single family homes and "multifamily homes" at the municipality level for over 200 Californian municipalities. As recognized by Song (2021), performing validation when the constructed measure of regulation varies within municipalities is difficult. Notwithstanding, I assume the TCLUS data reports the median minimum lot size (weighted by population) for each of these structure types (single family and multifamily).

I take, at the municipality level, the median modal lot across block groups separately for single family homes, duplexes, triplexes and fourplexes (recall that the measured unit density restriction in a neighborhood is the minimum of each of these modes after adjusting for the implied number of units per lot). In each municipality, I match the TCLUS-reported minimum lot size for single family homes with the modal single family lot. I also match the TCLUS-reported minimum lot size for multifamily structures with the closest modal lot amongst duplexes, triplexes, and fourplexes. I then calculate the *error rate* for each municipality level as the smallest error between reported and observed amongst the errors calcuated for both single family and multifamily structures. On this sample of municipalities, the median error on the best set of clustering hyperparameters is 4.5% of the official minimum lot size. For single family homes alone, this error rate is approximately 10%. This is very accurate.

Note that the unit density restriction actually used in the paper is the minimum across single family homes, duplexes, triplexes and fourplexes, as described in Appendix A.3. By definition, this is conservative relative to the minimum lot sizes used to compare to the TCLUS.

Official Zoning Maps I hand collect official zoning maps in 13 large US cities[57] and manually match official minimum lot sizes (from their respective online ordinances) to as many zoning codes in these maps as possible. I take into account that some codes allow multifamily structures by calculating the implied unit density restriction in the usual way.

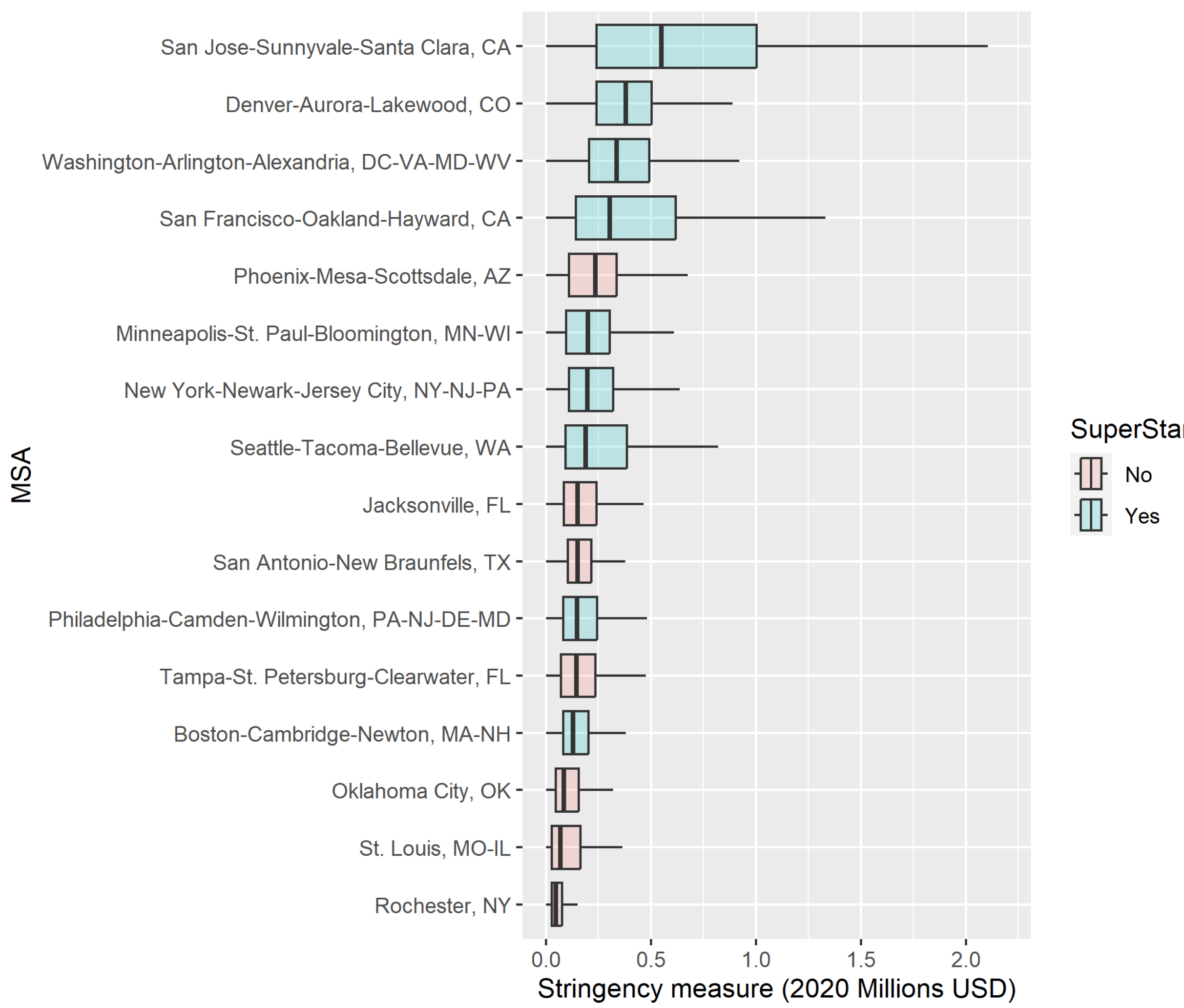
The difficulty with comparing the measured minimum lot sizes with those reported in the official maps is that my data varies at the zoning district level (a union of block

[57] These cities are Albany, GA, Atlanta, GA, Berkeley, CA, Cleveland, OH, Columbus, OH, Hayward, CA, Mesa, AZ, Miami, FL, Minneapolis, MN, New Orleans, LA, Oakland, CA and Scottsdale, AZ

groups), while the official zoning maps vary by official zoning code boundaries. I overlay my zoning districts on the official zoning maps. I then match the closest modal lot amongst single family homes, duplexes, triplexes and fourplexes to data, and calculate an error rate using this closest modal lot. To calcuate the error rate across the entire data, I take the median error across all intersected geography derived from the overlay (the median is weighted such that each block group receives equal weight). This error rate is 16% of the official unit density restriction at the optimal set of clustering hyperparameters, which is less accurate compared to the TCLUS exercise but still very accurate. For Hayward, California, this error rate is less than 1%. This suggests that the efficacy of the algorithm will vary across space, favouring municipalities where the minimum lot size is likely to bind. Reassuringly, performance of the algorithm doesn't vary much across hyperparameters, which implies we are not over-fitting the data used to test.

A.5 Summary Statistics

Table 3: Regulatory stringency boxplot for select cities.



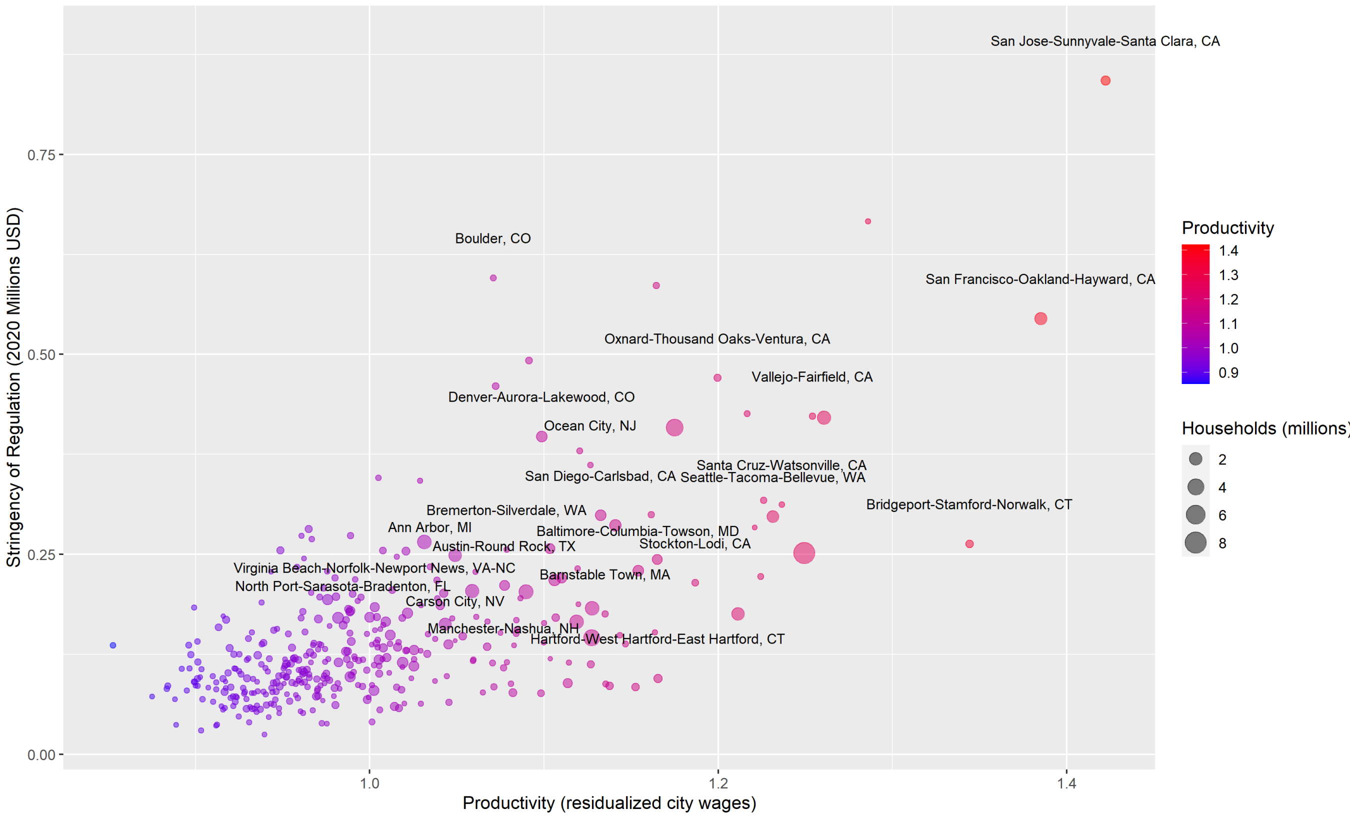
Regulatory stringency boxplot for select cities. Reports stringency measure introduced in Equation (2) for select cities. Cities are colored based on whether they are included in the superstar sample used to construct Facts 1 and 2.

Table 4: Summary Statistics for Key Variables,
disaggregated by superstar city status

SuperStar city	Aggregate				No				Yes			
Variable	N	Mean	Sd	Median	N	Mean	Sd	Median	N	Mean	Sd	Median
ln Average Income	194533	11.2	0.567	11.2	83861	11	0.524	11	110672	11.4	0.556	11.4
Unit Density Restriction (acres)	182564	0.272	3.74	0.114	78743	0.338	5.18	0.138	103821	0.222	2.08	0.0861
Stringency measure (2020 millions USD)	179879	0.198	0.458	0.129	77316	0.124	0.111	0.0911	102563	0.254	0.592	0.172
Land Value Density (2020 millions USD/acre)	187654	4.02	52.7	1.5	81657	1.53	4.82	0.859	105997	5.94	70	2.46
Regulated housing units (share)	190553	0.793	0.254	0.899	82988	0.811	0.222	0.894	107565	0.778	0.275	0.904

Summary statistics. "Unit Density Restriction" refers to the measured physical unit density restriction that enters into Equation (2), and is measured in Appendix A.3. "Regulated Housing Units" refers to the share of housing units in single family homes, duplexes, triplexes and fourplexes. Land Value Density is measured in Appendix A.3 and enters into (2). "Stringency measure" is the empirical regulatory stringency measure introduced in Equation (2). Variation in nonmissing data are a result of either 1) incomplete transactions coverage or 2) additional cleaning procedures described in Appendix A.3.

Table 5: Relationship between regulatory stringency and productivity.



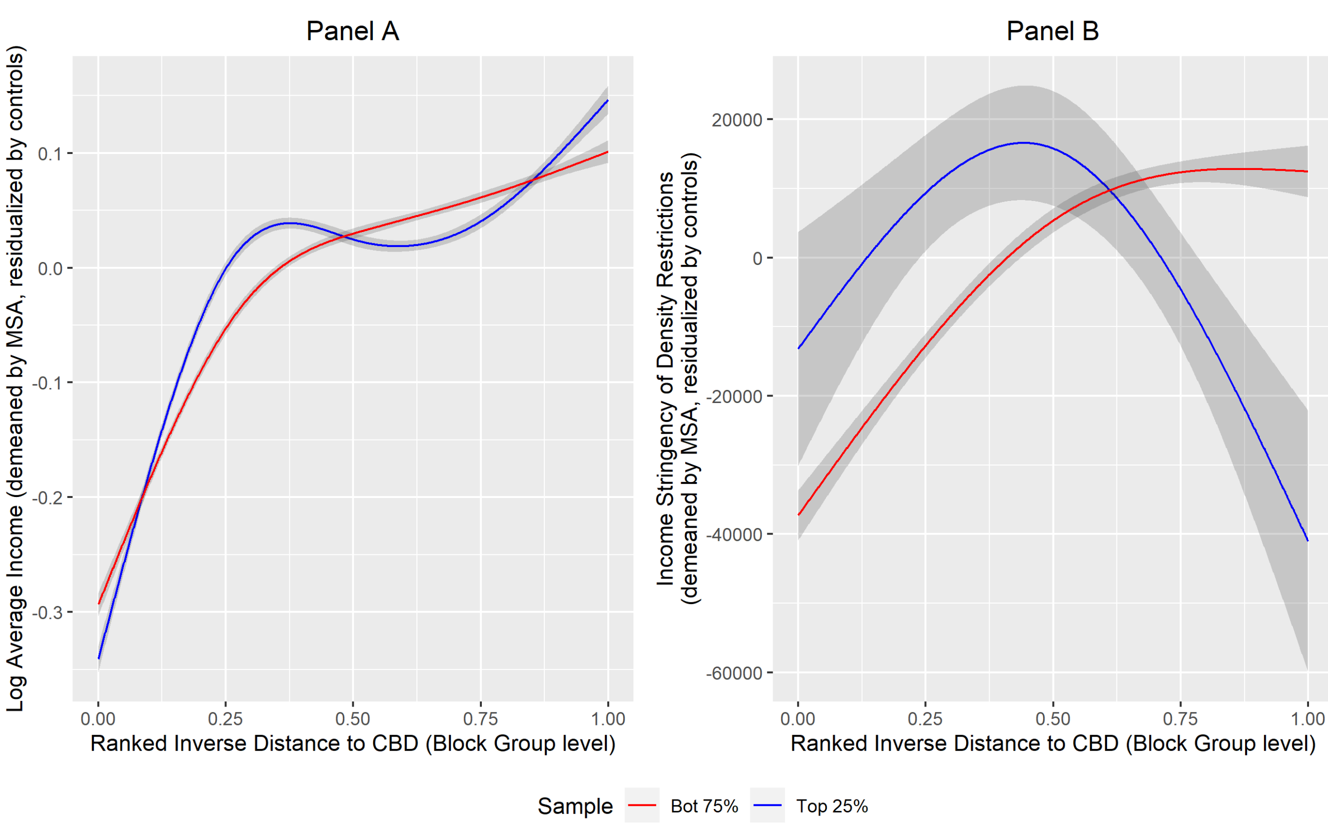
Reports stringency measure introduced in Equation (2) plotted against residualized city wages with some additional city characteristics. For methodology behind construction of wages, see Section 4. Productivity is normalized to be on average one across cities.

A.6 Discussion of Alternative Specifications

In this section, I provide additional robustness checks for the facts. I also discuss instances where Facts 1 and 2 do not hold.

Distance to CBD Fact 1 is not robust when considering distance to the CBD instead of the density ranking. In Figure 14, I reproduce regressions of demeaned income (Panel A) and regulatory stringency (Panel B) on the distance to CBD. We observe a generally increasing relationship between income and CBD distance. Panel B shows that there is evidence that high density neighborhoods close to the CBD are less stringent in superstar cities. However, a key difference is that the relationship is not monotone for each city sample.

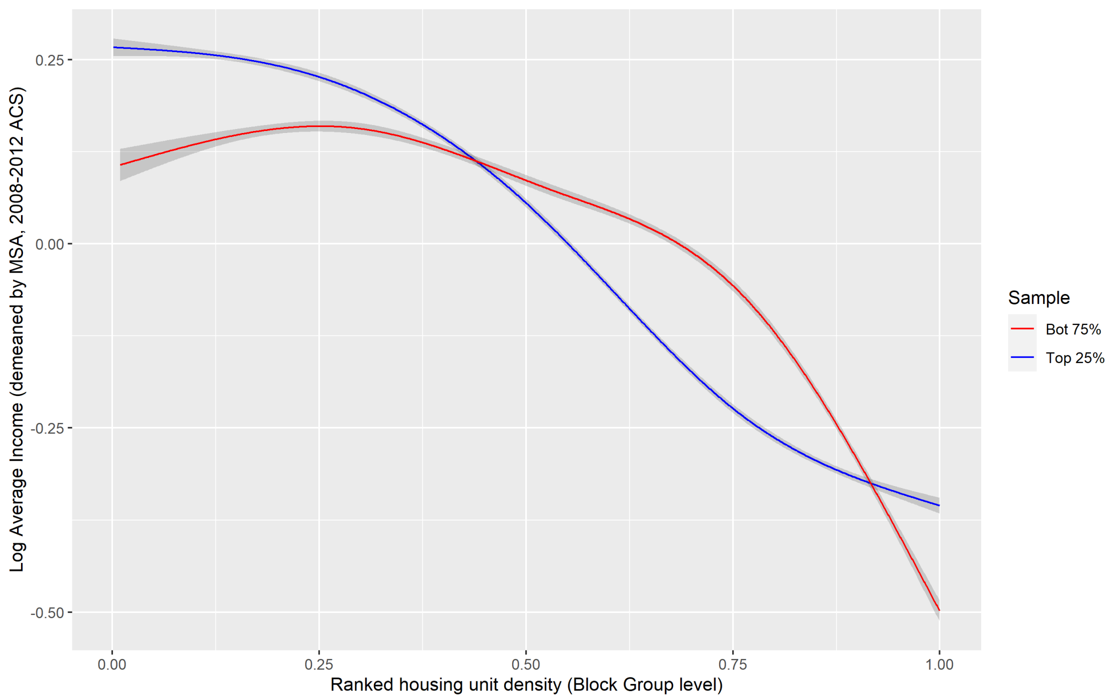
Figure 14: Facts 1 and 2 when considering distance to CBD



Relationship over time Fact 1 is robust to various time periods. I repeat the same exercise using 2008-2012 ACS data, with results reported in Figure 15. The difference in the negative income-density gradient across samples is somewhat stronger in this sample relative to the 2016-2020 ACS. This likely reflects the recent gentrification of high density neighborhoods nationwide. Results also hold when residualizing regressions by the standard set of controls. Unfortunately, my measure of regulation does not vary over time, and so Fact 2 cannot be checked historically. While there appears to be some recent adoption of minimum lot sizes between 2007-2020 (Gyourko et al., 2021), there is probably little variation in regulatory stringency over time that could otherwise invalidate the relationship.

Alternative measures of superstars Facts 1 and 2 look quantitatively identical when adopting three different measures of "superstar" cities: the top 25% and bottom 75% of

Figure 15: Fact 1 in 2008-2012 ACS sample



density, housing prices, and a measure of city productivity alone (see Section 4 for details on productivity measurement). Moreover, these facts are robust to alternative measures of superstars that use 10% and 50% thresholds in their definition (rather than the baseline 25%).

Weights Facts 1 and 2 use regressions that weigh block groups evenly, roughly corresponding to a population-weighted regression. Both 1 and 2 are not driven by larger weights ascribed to bigger cities; they hold when each city receives equal weight. Each figure also looks identical when weighting the regression by the number of households in each block group.

Various hyperparameters There are a large amount of hyperparameters that characterize zoning districts, and thus the measure of regulation used to show Fact 2. Reassuringly, these facts look quantitatively identical for the entire hyperparameter space considered. This is likely driven by the fact that there are only minimal differences in clusters across hyperparameters (after all, two-thirds of all block groups are assigned regulation by populated zoning codes).

B Appendix: Theory

B.1 Derivation of the distortion factor

I start with solving the households maximization problem, restated here

$$\underbrace{k(z)wz\left[\frac{1-\frac{P(i)\bar{A}}{wz}}{P(i)^{\beta}}\right]}_{\text{Undistorted utility}}\underbrace{\left[\frac{\left(1-\frac{R(i)}{wz}\right)\left(1-\frac{P(i)\bar{A}}{wz}\right)^{-1}}{1-\beta}\right]^{1-\beta}\left[\frac{\left(R(i)-P(i)\bar{A}\right)\left(wz-P(i)\bar{A}\right)^{-1}}{\beta}\right]^{\beta}}_{\text{Distortion Factor}} \tag{28}$$

Assuming $\bar{A}=0$, Equation (28) reduces down to what is reported in the text:

$$\underbrace{\kappa(z)\frac{w(i)z}{P_R(i)^{\beta}}}_{\text{Undistorted utility}} \times \underbrace{\left[\frac{\frac{R(i)}{w(i)z}}{\beta}\right]^{\beta}\left[\frac{1-\frac{R(i)}{w(i)z}}{1-\beta}\right]^{1-\beta}}_{\text{Distortion factor}}$$

B.2 Derivation of Equation (8)

Recall the household's problem (6). The objective is to prove this expression is supermodular when $k(z) = 1$, or

$$\frac{\partial V_R(i,z)}{\partial R(i)\partial z} > 0$$

as claimed in Equation (8). We can write the solution to the households maximization problem generally as

$$V_R(i,z) = \tilde{V}\left[\frac{R(i)}{P_R(i)}, z - R(i)\right] \tag{29}$$

for the function $\tilde{V}(A,g) := \beta^{-\beta}(1-\beta)^{-(1-\beta)}(A-\bar{A})^{\beta}g^{1-\beta}$, and when the minimum lot size binds. $\frac{R(i)}{P(i)}$ is housing consumption and $z - R(i)$ is the numeraire consumption.

First, consider a marginal change in $R(i)$ affects utility (29), holding all other variables fixed. This can be expressed as

$$\frac{\partial V_R(i,z)}{\partial R(i)} = V_R(i,z)[\tilde{V}_A P_R(i)^{-1} - \tilde{V}_g]$$

where $\tilde{V}_A = \frac{\partial \tilde{V}}{\partial A}\tilde{V}^{-1}$ is the elasticity of utility with respect to housing consumption and $\tilde{V}_g = \frac{\partial \tilde{V}}{\partial g}\tilde{V}^{-1}$ is the elasticity of utility with respect to numeraire consumption. This implies that

$$\frac{\partial V_R(i,z)}{\partial R(i)\partial z} = \frac{V_R(i,z)}{\partial z}[\tilde{V}_A P_R(i)^{-1} - \tilde{V}_g] + V_R(i,z)[\tilde{V}_{Az} P_R(i)^{-1} - \tilde{V}_{gz}]$$

This quantity is positive if and only if

$$\frac{V_R(i,z)}{\partial z}$$

Recall Equation (7), which describes how to decompose the utility function when preferences are Cobb-Douglas into an undistorted utility factor and a distortion factor:

$$\frac{wz}{P_R(i)^\beta}\left[\frac{\frac{I(i)}{w(i)z}}{\beta}\right]^\beta\left[\frac{1-\frac{I(i)}{w(i)z}}{1-\beta}\right]^{1-\beta}$$

whenever the minimum lot size is binding for a type z household. The objective is to prove this expression is supermodular, or

$$\frac{\partial V_R(i,z)}{\partial I(i)\partial z}>0$$

as claimed in Equation (8).

I start by deriving $\frac{\partial V_R(i,z)}{\partial I(i)}$, which is equivalent to $V_R(i,z)\frac{\partial \log V_R(i,z)}{\partial I(i)}$. Log-differentiating $V_R(i,z)$ yields the expression

$$\frac{\partial \log V_R(i,z)}{\partial I(i)}=\beta\frac{1}{I(i)}-(1-\beta)\frac{\frac{1}{wz}}{1-\frac{I(i)}{wz}}$$

which can be shown to be strictly negative whenever $\beta<\frac{I(i)}{wz}<1$ (equivalently, when the minimum lot size is binding). This implies the obvious result that minimum lot sizes decrease utility by constraining consumption decisions.

Taken together, we know that $\frac{\partial V_R(i,z)}{\partial I(i)}=V_R(i,z)[\beta\frac{1}{I(i)}-(1-\beta)\frac{\frac{1}{wz}}{1-\frac{I(i)}{wz}}]$. Differentiating with respect to z yields

$$\frac{\partial V_R(i,z)}{\partial I(i)\partial z}=V_R(i,z)\left[(1-\beta)\frac{w}{(wz-I(i))^2}\right]+\frac{\partial V_R(i,z)}{\partial z}[\beta\frac{1}{I(i)}-(1-\beta)\frac{\frac{1}{wz}}{1-\frac{I(i)}{wz}}]$$

I ask what conditions must be true for this expression to be strictly positive. It requires that (after some manipulation and using the fact that $V_R(i,z)>0$)

$$\left[(1-\beta)\frac{\frac{1}{wz}}{1-\frac{I(i)}{wz}}-\beta\frac{1}{I(i)}\right]\frac{\partial \log V_R(i,z)}{\partial z}<(1-\beta)\frac{w}{(wz-I(i))^2}$$

The expression for $\frac{\partial \log V_R(i,z)}{\partial z}$ can be derived as follows

$$\frac{\partial \log V_R(i,z)}{\partial z}=-\beta\frac{1}{z}+(1-\beta)\frac{\frac{I(i)}{wz^2}}{1-\frac{I(i)}{wz}}$$

so that supermodularity is satisfied if and only if

$$\left[(1-\beta)\frac{\frac{1}{wz}}{1-\frac{I(i)}{wz}}-\beta\frac{1}{I(i)}\right]\left[-\beta\frac{1}{z}+(1-\beta)\frac{\frac{I(i)}{wz^2}}{1-\frac{I(i)}{wz}}\right]<(1-\beta)\frac{w}{(wz-I(i))^2}$$

This is an ugly expression, which can be reduced down for further analysis. The left hand side of the inequality can be expressed as

$$\frac{1}{I(i)z}\left[\beta-(1-\beta)\frac{\frac{I(i)}{wz}}{1-\frac{I(i)}{wz}}\right]^2$$

so that

$$\frac{1}{I(i)z}\left[\beta-(1-\beta)\frac{\frac{I(i)}{wz}}{1-\frac{I(i)}{wz}}\right]^2<(1-\beta)\frac{w}{(wz-I(i))^2}$$

The right hand side of the inequality can equivalently be expressed as

$$(1-\beta)\frac{1}{wz^2}\frac{1}{(1-\frac{I(i)}{wz})^2}$$

multiplying both sides of the inequality by $z(1-\frac{I(i)}{wz})^2$

$$\frac{1}{I(i)}\left[\beta(1-\frac{I(i)}{wz})-(1-\beta)\frac{I(i)}{wz}\right]^2<(1-\beta)\frac{1}{wz}$$

or

$$\left[\beta-\frac{I(i)}{wz}\right]^2<(1-\beta)\frac{I(i)}{wz}$$

So utility is supermodular if and only if this inequality holds. Let $s=\frac{I(i)}{wz}$. I claim that

$$[\beta-s]^2<(1-\beta)s$$

for all $s\in(\beta,1)$. One can verify that there are no zeros in the equation $(1-\beta)s-\left[\beta-s\right]^2$ over $s\in(\beta,1)$, and this inequality is satisfied when $s=\beta$. This completes the proof.

B.3 Microfoundations for endogenous amenities channel

In this section, I provide an approximate microfoundations for the relationship in Equation (12). I abstract away from different zones within a neighborhood i.

Local public goods financed through income taxes Suppose each neighborhood implements a property tax rate $t(i)$ on the value of a home (in terms of numeraire consumption). The revenue from this tax is rebated equally to all residents. Let $S(i)$ be total spending on housing in neighborhood i by all residents. The average value of a home (which is equal to average spending on a home) from the housing market clearing condition is

$$\frac{S(i)}{L(i)}$$

where $L(i)$ is the neighborhood population and so the consumption rebate is $t_i \frac{S(i)}{L(i)}$ for each household irrespective of income. Suppose households amenity value $b(i, z)$ is a composite of the public good K and fundamential amenities $\nu(i, z)$, or $b(i, z) = K^{\Omega(z)}\nu(i, z)$. Then provision of the public good K is $t_i \frac{S(i)}{L(i)}$. Then, amenity values are $b(i, z) = t_i^{\Omega(z)} \left[\frac{S(i)}{L(i)}\right]^{\Omega(z)} \nu(i, z)$. When housing consumption is Cobb-Douglas and no minimum lot sizes are binding, this means that $S(i) = \beta Y(i)$, where $Y(i)$ is total income in neighborhood i. Putting this all together,

$$b(i, z) = \beta^{\Omega(z)} t_i^{\Omega(z)} \left[\frac{Y(i)}{L(i)}\right]^{\Omega(z)} \nu(i, z)$$

where $\frac{Y(i)}{L(i)}$ property tax rate terms $t(i)^{\Omega(z)}$ and spending shares $\beta^{\Omega(z)}$ are absorbed into the fundamental amenities term $\nu(i, z)$ (if they can reasonably be taken as exogenous). Obviously, property tax rates are endogenous in the local public finance literature, and respond to the changing income composition of the neighborhood to facilitate Tiebout sorting (Calabrese et al., 2007, 2011). Since I take a broad view of this neighborhood choice externality and include many reasons for why it occurs, I abstract away from these specific concerns.

Local love of variety and a disutility of density Suppose preferences over housing is Cobb-Douglas ($\bar{A} = 0$) and suppose the amenity component can be written as $b(i, z) = c^{\Omega'(z)} L(i)^{-\Omega(z)} \nu(i, z)$ where $L(i)$ is the population of neighborhood i and c is a composite consumption good (separate from the numeraire) that is produced in a Dixit-Stiglitz style market with some elasticity of substitution over varieties σ_{Variety}. This Dixit-Stiglitz style market only operates within the neighborhood i (these goods are untraded across neighborhoods). I assume each variety that comprises the consumption good has an equilibrium price of 1 (the numeraire).

Let $S(i)$ be the total spending in neighborhood i on the consumption good c. Then, the number of varieties is proportional to total spending. The utility value of an additional variety is then $S(i)^{\frac{\sigma_{\text{Variety}}}{\sigma_{\text{Variety}}}-1}$. Hence, we can write

$$b(i, z) = S(i)^{\Omega'(z) \frac{\sigma_{\text{Variety}}}{\sigma_{\text{Variety}}-1}} L(i)^{-\Omega(z)} \nu(i, z)$$

Also note that preferences over the numeraire, housing and amenities can be described as (taking a log transformation)

$$A^{\beta}g^{1-\beta} + \log b(i,z) = A^{\beta}g^{1-\beta} + \Omega'(z)\log c - \Omega(z)\log L(i) + \log \nu(i,z)$$

so that preferences are quasilinear in amenities $b(i,z)$. Consider a consumers problem to choose spending to maximize utility over A, g and c, assuming consumption c has a price of 1. Spending on c is then pinned down via the first order condition (assuming no corner solution)

$$\Omega'(z)\frac{1}{c} = 1 \implies c = \Omega'(z)$$

Suppose $\Omega'(z) \propto z$. This means that $S(i)$ is proportional to total neighborhood income (and thus average income) in each neighborhood i[58]. Let $Y(i)$ be this total neighborhood income, with $Y(i) = \alpha S(i)$ for some constant α. Finally, $b(i,z)$ can be written as $b(i,z) = (\alpha Y(i))^{\Omega(z)} L(i)^{-\Omega(z)}\nu(i,z)$. If we assume $\Omega'(z)\frac{\sigma_{\text{Variety}}}{\sigma_{\text{Variety}}-1} = \Omega(z)$, we can lastly write

$$b(i,z) = [\frac{Y(i)}{L(i)}]^{\Omega(z)}\tilde{\nu}(i,z)$$

where α is absorbed in the unobserved amenities component. $\frac{Y(i)}{L(i)}$ is average income in i.

[58] I empirically estimate a positive relationship between $\Omega(z)$ and z in the data, though it is less-than-proportional to income. See Figure 1.

B.4 Proof of Proposition 1

Proposition 1 has three parts. First, it says that cities with higher productivity are relatively more affluent because of variation in regulatory stringency. Second, it says that, within cities, neighborhoods that have higher regulatory stringency in equilibrium also have lower density. Thirdly, regulation generates a negative income-density gradient that is stronger in more productive cities. For this proof only, I adopt a separate index for cities and neighborhoods – c and i, respectively. In what follows, note that we assume a perfect mobility equilibrium for this proof and thus a spatial equilibrium is a set of allocations across neighborhoods $L(i,c,z)$ such that $V(i,c,z) \leq V(z)$ for some $V(z)$ and $V(i,c,z) < V(z) \implies L(i,c,z) = 0$ for all (i,c) pairs. Also note that preferences are Cobb-Douglas, $\bar{A} = 0$ and $\Omega(z) = 0$ for every z, each neighborhood provides unit amenity value and has an identical production technology for housing. Lastly, recall that I make no distinction between regulated and unregulated zones to make the exposition as simple as necessary.

For simplicity, I assume i and c and z are continuous which will directly imply all results on discrete space. For a location with regulatory stringency $I(i,c)$ that offers wage $w(c)$, let $s(i,c,z) := \frac{I(i,c)}{w(c)z}$. I start with two lemmas. The first lemma characterizes the bid rent curve – the maximal rent willing to be paid to live in an arbitrary neighborhood offering wages w and regulation levels I.

Lemma 1. *The bid rent curve θ satisfies $\theta(i,c,z) = V(z)^{-\frac{1}{\beta}} w(c)^{\frac{1}{\beta}} s(i,c,z)\big(1 - s(i,c,z)\big)^{\frac{1-\beta}{\beta}}$ if $s(i,c,z) \in [\beta, 1)$, $\theta(i,c,z) = V(z)^{-\frac{1}{\beta}} w(c)^{\frac{1}{\beta}}$ if $s(i,c,z) < \beta$. If $s(i,c,z) \geq 1$ then $\theta(i,c,z) = 0$*

Proof. This follows directly from rearranging Equation (7) to determine what the price of housing services must be to rationalize utility $V(z)$ at wages w and regulation I. In addition, recall that we set utility to zero if the price of a minimal lot exceeds income. □

The second lemma says that the slope of the bid rent curve across cities is strictly increasing income, all else equal, which will eventually imply income sorting on regulation. This result is essentially implied by the supermodularity property of (8) combined with regulatory stringency increasing faster than wages in c.

Lemma 2. *If $V(i,c,z) = V(z)$ then $\frac{\partial \theta}{\partial c}$ is strictly increasing in z when $s(i,c,z) \in (\beta, 1)$ or is constant when $s(i,c,z) < \beta$ or $s(i,c,z) \geq 1$.*

Proof. The slope of the bid rent curve when $\beta < s(I,w,z) < 1$ can be expressed as

$$\frac{\partial \theta}{\partial c} = \theta(i,c,z)\left[\frac{1}{\beta}\frac{w'(c)}{w(c)} + \frac{\partial s}{\partial c} s(i,c,z)^{-1} - \frac{1-\beta}{\beta}\frac{\partial s}{\partial c}(1 - s(i,c,z))^{-1}\right]$$

Recall that $s(i,c,z) = \frac{I(i,c)}{w(c)z} = \frac{\alpha(c)i}{w(c)z}$ which is strictly increasing in c by one of the Proposition's assumptions. This can be used to show $\frac{\partial \theta}{\partial c}$ is increasing in z using similar arguments from Appendix B.2.

Lastly, if $s(i,c,z) < \beta$ then $\theta(i,c,z) = V(z)^{-\frac{1}{\beta}} w(c)^{\frac{1}{\beta}}$ which is constant in z because of the assumption $V(z) = \max_{i,c} V(i,c,z) = \frac{w(c)}{P(i)^{\beta}}$. The case where $s(i,c,z) \geq 1$ is trivial.

□

The third lemma deals with across-neighborhood comparisons of the bid rent function.

Lemma 3. *If $V(i, c, z) = V(z)$ then $\frac{\partial \theta}{\partial i}$ is strictly increasing in z when $s(i, c, z) \in (\beta, 1)$ or is constant when $s(i, c, z) < \beta$ or $s(i, c, z) \geq 1$.*

Proof. Follows from roughly the same algebraic arguments as in Lemma 2. □

Using these three lemmas will help with proving the three statements in Proposition 1.

1. High productivity cities are (weakly) more affluent:

   Proof. Recall that there are two income types $z_1 > z_0$. Fix any two cities c_0 and c_1 with $c_0 < c_1$. This result can be shown if one can prove that, for all neighborhoods i where the minimum lot size in either city is binding for at least one agent

   $$\text{If } L(i, c_0, z_1) > 0 \text{ then } L(i, c_1, z_1) > 0 \text{ and } L(i, c_1, z_0) = 0$$

   I will prove this. $L(i, c_0, z_1) > 0$ implies that type z_1 can successfully bid in the neighborhood, so that $\theta(i, c_0, z_1) = \max_z \theta(i, c_0, z)$. Suppose for contradiction that $L(i, c_1, z_1) = 0$ or $L(i, c_1, z_0) > 0$. Then, in either case, $\theta(i, c_1, z_0) = \max_z \theta(i, c_1, z)$. But this is impossible, since Lemma 2[59] and $\theta(i, c_0, z_1) = \max_z \theta(i, c_0, z)$ implies that $\theta(i, c_1, z_1) > \theta(i, c_1, z_0)$.

   □

2. (Weakly) stronger income density gradient in productive cities.

   Proof. Fix two cities with $c_0 < c_1$. It is sufficient to show that, If $L(i_1, c_0, z_1) > 0$ and the minimum lot size is binding for at least one type in at least one city, then the following hold

   (a) $L(i_1, c_1, z_1) > 0,$ and $L(i_1, c_1, z_0) = 0$ with

   (b) $L(i_0, c_1, z_1) = 0,$ or $L(i_0, c_1, z_0) > 0$

   I prove this. a) $L(i, c_1, z_1) > 0$ and $L(i, c_1, z_0) = 0$ follows from the same argument in the proof of Statement 1.

   It remains to show b). Suppose, for contradiction, that $L(i_0, c_1, z_1) > 0$ and $L(i_0, c_1, z_0) = 0$; that is, high income types strictly outbid low income in the low regulation neighborhood of the productive city. Then $\theta(i_0, c_1, z_1) > \theta(i_0, c_1, z_0)$. However, this immediately implies that $\theta(i_0, c, z_1) > \theta(i_0, c, z_0)$ for every c because i_0 neighborhoods are unregulated and thus the bid rents have the same slopes across income types

[59] Since we assume the minimum lot size is binding for at least one agent *and* not all agents observe the value of a minimal that exceeding income, Lemma 2 implies strict supermodularity.

(Lemma 2). However, this also then implies $\theta(i_1, c, z_1) > \theta(i_1, c, z_0)$ for all c because Lemma 3 ensures that the slope of the bid rent across neighborhoods is nondecreasing in z. Hence, z_0 types are outbid from all neighborhoods and cities, which cannot be an equilibrium.

□

3. The density of housing units is (weakly) decreasing in i.

Proof. It is sufficient to fix a city c and show for any i_0 and i_1 such that $i_0 < i_1$, $\sum_z L(i_1, c, z) \leq \sum_z L(i_0, c, z)$[60]. Since $\theta(i, c, z)$ is decreasing in i (regulation decreases utility when binding, thus maximal rent willing to pay), we must have that the equilibrium rent curve $\theta(i, c) = \max_z \theta(i, c, z)$ is also decreasing in i.

I prove that this is only commesurate with a population that is decreasing in i. Lemma 3 ensures that there are (weakly) relatively more z_1 types in i_1, and, for both z types, housing expenditure per capita is weakly larger in i_1 than i_0 (strict if regulation is binding). Suppose for contradiction that the population is higher in i_1. This *must* imply that total housing expenditure in i_1 is strictly higher than i_0. This implies strictly higher rents in i_1 than i_0 because we assume they have the same production technology for housing, contradicting the fact that $\theta(i, c) = \max_z \theta(i, c, z)$ is decreasing in i.

□

[60]Recall that all neighborhoods have identical land mass.

B.5 Proof of Proposition 2

Before proceeding with the proof, I start with some preliminary definitions. I define $s(i,z) = \frac{I(i)}{wz}$ and $\text{Inc}(i)$ to be the average income in i. Utility of neighborhood i for type z is

$$V(i,z) := \frac{w}{P(i)^\beta}\left[\frac{s(i,z)}{\beta}\right]^\beta \left[\frac{1-s(i,z)}{1-\beta}\right]^{1-\beta} + \Omega(z)\log \text{Inc}(i) + \log \nu(i,z)$$

when $s(i,z) \in (\beta, 1)$ and $\frac{w}{P(i)^\beta} + \Omega(z)\log \text{Inc}(i) + \nu(i,z)$ in an equilibrium where regulation is nonbinding. For each statement, I consider an equilibrium that is non-binding for all income types, but marginal changes would induce binding regulation (the special case where $s(i,z) = \beta$ for some type z). With imperfect labour mobility ($\rho < \infty$), an interior[61] equilibrium in this model can be expressed as

$$e^{V(i,z)} L(i,z)^{-\frac{1}{\rho}} = V(z) \tag{30}$$

for some $V(z)$, and for every type z and neighborhood i. $L(i,z)^{-\frac{1}{\rho}}$ governs the additional selection effect from the idiosyncratic taste shocks over neighborhoods, which unsurprisingly vanishes as the variance of the taste shocks goes to zero ($\rho \to \infty$).

Lastly, recall that I assume $\Omega(z)$ is increasing in z, which is commensurate with the empirical evidence of Section 5. Also, I need to assume $\Omega(z)$ for every z is sufficiently small so as to guarantee equilibrium uniqueness, which in turn allows for comparisons across equilibria of different model parameterizations to make sense[62].

1. Statement 1: For an unregulated equilibrium, the variation in income across neighborhoods is strictly increasing in t.

   Proof.

   Fix any $t \in (0,1)$ and consider how a marginal change in t alters the equilibrium. Totally differentiating (30) for an arbitrary i and z yields an equation governing changes in equilibrium:

   $$\frac{\partial L(i,z)}{L(i,z)} = -\rho\beta\frac{w}{P(i)^\beta}\frac{\partial P(i)}{P(i)} + \rho\Omega(z)\frac{\partial \text{Inc}(i)}{\text{Inc}(i)} + \rho\frac{\partial \nu(i,z)}{\nu(i,z)} - \rho\frac{\partial V(z)}{V(z)} \tag{31}$$

   Define $\Delta\text{Inc} := \frac{\partial \text{Inc}(1)}{\text{Inc}(1)} - \frac{\partial \text{Inc}(0)}{\text{Inc}(0)}$, $\Delta C = -\beta\frac{w}{P(1)^\beta}\frac{\partial P(1)}{P(1)} + \beta\frac{w}{P(0)^\beta}\frac{\partial P(0)}{P(0)}$, and $\Delta\nu(z) := \frac{\partial \nu(1,z)}{\nu(1,z)} - \frac{\partial \nu(0,z)}{\nu(0,z)}$. The objective is to show that $\Delta\text{Inc} > 0$ in equilibrium, which means that neighborhood incomes diverge and thus segregation increases. Equation (31) can be expressed using the following formula by substituting the definition of $\frac{\partial V(z)}{V(z)}$

   $$\frac{\partial L(1,z)}{L(1,z)} = \rho[1-\tilde{f}(1,z)]\left[\Delta C + \Omega(z)\Delta\text{Inc} + \Delta\nu(z)\right] \tag{32}$$

   where $\tilde{f}(i,z)$ is the fraction of type z households who choose i.

[61] An interior equilibrium is one where all types live in all locations where fundamental amenities are strictly positive.

[62] For Ω sufficiently small, the equilibrium is likely stable because the disutility of a marginal increase in rents for small changes in population dominates any endogenous amenity response that could occur. Obviously, this is not in line with the empirical evidence on unstable neighborhood preferences, e.g. (Davis et al., 2023). I ignore these concerns here.

I use this formula extensively for the proof. Define $\Delta Y_1 = \frac{\partial Y(1)}{Y(1)} = \frac{\partial(\sum_z zL(1,z))}{\sum_z zL(1,z)}$, the total income growth in location 1. Moreover, since preferences are Cobb-Douglas, we know that $\frac{\partial P(i)}{P(i)} = \frac{1}{1+\epsilon}\frac{\partial Y(i)}{Y(i)}$ and so

$$\Delta C = -\frac{\beta}{1+\epsilon}\frac{w}{P(1)^\beta}\frac{\partial Y(1)}{Y(1)} + \frac{\beta}{1+\epsilon}\frac{w}{P(0)^\beta}\frac{\partial Y(0)}{Y(0)} = \tilde{c}\Delta Y_1$$

where $\tilde{c} = -\frac{\beta}{1+\epsilon}\frac{w}{P(1)^\beta} - \frac{\beta}{1+\epsilon}\frac{w}{P(0)^\beta}\frac{Y(1)}{Y(0)}$ and exploiting the fact that $\partial Y(1) = -\partial Y(0)$ (total income is exogenous because wages are exogenous). We know that $\tilde{c} < 0$.

On the other hand, the using the definition of ΔI, we have that $\Delta\text{Inc} = \sum_z [o(1,z) - f(1,z)]\frac{\partial L(1,z)}{L(1,z)} - \sum_z [o(0,z) - f(0,z)]\frac{\partial L(0,z)}{L(0,z)}$, where $o(i,z)$ is the share of income in i payed to z households, and $f(i,z)$ is the fraction of households in i who are type z. Using (32), this can be greatly simplified

$$\Delta\text{Inc} = \rho\sum_z \phi_z[\Delta C + \Omega(z)\Delta\text{Inc} + \Delta\nu(z)]$$

where $\phi_z = [o(1,z) - f(1,z)]\tilde{f}(0,z) + [o(0,z) - f(0,z)]\tilde{f}(1,z)$. Finally, this means (after some algebra and substituting in the above expression for ΔC)

$$\Delta\text{Inc} = \rho\sum_z \phi_z^I[\tilde{c}\Delta_1 Y + \Delta\nu(z)] \tag{33}$$

where $\phi_{iz}^I = \frac{\phi_z}{1-\rho\sum_z \phi_z\Omega(z)}$. Assuming $\Omega(z)$ is sufficiently small for every z, one can easily show that $1 - \rho\sum_z \phi_z\Omega(z) > 0$ for any possible allocation of types in each neighborhood, which we assume thus far.

Finally, we know that, by the definition of $\Delta_1 Y$,

$$\Delta_1 Y = \sum_z o(1,z)\frac{\partial L(1,z)}{L(1,z)}$$

and hence,

$$\Delta_1 Y = \rho\sum_z \phi_z^L[\tilde{c}\Delta_1 Y + \Omega(z)\Delta\text{Inc} + \Delta\nu(z)] \tag{34}$$

with $\phi_z^L := o(1,z)(1-\tilde{f}(1,z))$. Suppose that $\Delta\nu(z_h) = \frac{1}{1-t}\frac{1}{\phi_{z_h}^L} > 0$ and $\Delta\nu(z_l) = -\frac{1}{1-t}\frac{1}{\phi_{z_l}^L} < 0$ and $\Delta\nu(z_m) = 0$, which defines the parameterization functions $\kappa_z(t)$ defining how fundamental amenities vary by t[63]. Substituting in this parameterization into the equation above implies that

$$\Delta_1 Y = \rho\sum_z \phi_z^L[\tilde{c}\Delta_1 Y + \Omega(z)\Delta\text{Inc}] \tag{35}$$

That is, output in $i = 1$ (and thus $i = 0$) is not a function of changing fundamental amenities, which greatly simplifies the analysis. Equations (33) and (35) define a

[63]This parameterization of fundamental amenities is consistent with $\kappa_z(0) = 0$ and $\kappa_z(1) = 1$ for every z. The functions that achieves this are $\kappa_z(t) = e^{-\int_t^1 \frac{1}{\phi_z^L(1-\tilde{t})}\frac{1}{\tilde{t}}d\tilde{t}}$, which depend on equilibrium allocations as a function of t and is continuous for $t \in (0,1)$.

2×2 linear system that can be used to solve for ΔInc in equilibrium. This, in turn, means that

$$\Delta_1 Y = \rho \sum_z \phi_z^{L'} \Omega(z) \Delta \text{Inc}$$

for $\phi_z^{L'} := \frac{\phi_z^L}{1-\rho\tilde{c}\sum_z \phi_z^L}$ which is strictly positive since $\phi_z^L > 0$ for all z and $\tilde{c} < 0$. Finally, substituting this expression for equilibrium output into (33) and simplifying yields a closed form expression for ΔInc in equilibrium

$$\Delta \text{Inc} = \rho \sum_z \frac{\phi_z^I}{\phi^{L''}(z)} \Delta \nu(z) \tag{36}$$

where $\phi^{L''}(z) = 1 - \rho^2 \tilde{c} \sum_z \phi_z^I \sum_{z'} \phi_{z'}^{L'} \Omega(z')$, which is also strictly positive for $\Omega(z)$ small in any equilibrium allocation.

To complete the proof, we need to show that ΔInc > 0. We know that $\phi_{z_h}^I > 0$ since the output share of the highest income types is always larger than their fraction of the population. Moreover, $\phi_{z_l}^I < 0$ for the opposite reason. Since $\Delta\nu(z_h) > 0$, $\Delta\nu(z_h) < 0$ and $\Delta\nu(z_m) = 0$, this is all sufficient to show that ΔInc > 0. Hence, neighborhood incomes diverge strictly in t.

□

2. Statement 2: **Inclusionary Zoning**: There exists a $t^\uparrow \in [0,1]$ such that, for every $t \in (t^\uparrow, 1]$, a small increase in regulation in $i = 1$ increases average income in all locations.

Proof. I start by proving the result for the knife-edge case where $t = 1$, then appealing to continuity arguments of reallocations locally around the $t = 1$ equilibrium. Recall that, at $t = 1$, we have $\nu(1, z_l) = 0$ and $\nu(0, z_h) = 0$, so these types are fixed in their respective neighborhoods. It is then sufficient to show that a marginal change in regulation in $i = 1$ that is *just binding* for z_m types, captured through changes in $I(1)$, causes a reallocation of these households from neighborhood $i = 1$ to $i = 0$. This must increase income in all neighborhoods.

Consider how z_h types move in response to a marginal increase in regulation in $i = 1$ about an equilibrium that is just about binding for z_m types (i.e. $s(i, z_m) = \beta$). Define $\Delta L = \frac{\partial L(1,z_m)}{L(1,z_m)}$. Using the formula (32),

$$\Delta L = \rho[1 - \tilde{f}(1, z_m)] \left[\Delta C + \Omega(z_m) \Delta \text{Inc}\right]$$

where ΔC retains its definition as the relative log change in consumption values. ΔC must equal

$$\Delta C = \tilde{c} \Delta_1 Y + \frac{w}{P(1)^\beta} \partial D(1, z_m)$$

where $D(1,z) = \left[\frac{s(i,z)}{\beta}\right]^{\beta}\left[\frac{1-s(i,z)}{1-\beta}\right]^{1-\beta}$ is the *distortion factor* due to regulation (which is changed by $I(1)$), as well as $\tilde{c}$ and $\Delta_1 Y$ retaining their definition from the proof of Statement 1. $D(1,z) = 1$ in an equilibrium where regulation is just binding ($s(i,z) = \beta$). The idea is to show $\Delta L < 0$, which will then imply that average incomes in both locations increase.

Using the definition of ΔInc and the fact that $\partial L(i, z_h) = \partial L(i, z_l) = 0$,

$$\Delta\text{Inc} = [o(1,z_m) - f(1,z_m)]\frac{\partial L(1,z_m)}{L(1,z_m)} - [o(0,z_m) - f(0,z_m)]\frac{\partial L(0,z_m)}{L(0,z_m)}$$

which means we can express $\Delta\text{Inc} = \tilde{I}\Delta L$ where $\tilde{I} = o(1,z_m) - f(1,z_m) + [o(0,z_m) - f(0,z_m)]\frac{L(1,z_m)}{L(0,z_m)}$. Moreover, $\Delta_1 Y = o(1,z_m)\Delta L$ by definition. Putting this all together, the change in population in equilibrium can be solved:

$$\Delta L = \rho[1 - \tilde{f}(1,z_m)]\left[\tilde{c}o(1,z_m)\Delta L + \frac{w}{P(1)^{\beta}}\partial D(1,z_m) + \Omega(z_m)\tilde{I}\Delta L\right] \tag{37}$$

This simple linear equation can be solved in ΔL. The sign of ΔL in equilibrium retains the same sign as $\frac{w}{P(1)^{\beta}}\partial D(1,z_m) < 0$ provided $1 - \rho(1 - \tilde{f}(1,z_m))[\tilde{c}o(1,z_m) + \Omega(z_m)\tilde{I}] > 0$, which is guaranteed for $\Omega(z_m)$ small enough. Hence, $\Delta L < 0$. As a consequence, incomes in both locations increase, since $\frac{\partial\text{Inc}(i)}{\text{Inc}(i)} = [o(i,z_m) - f(i,z_m)]\Delta L$, which is positive for $i = 1$ and for $i = 0$ because z_m types are below average income in $i = 1$ and above average in $i = 0$.

Finally, $\frac{\partial\text{Inc}(i)}{\partial I(1)}$ is continuous in t[64] and I established is strictly positive for every i at the $t = 1$ equilibrium. By the Intermediate Value Theorem, there exists a $\tilde{t}(i)$ such that, for all $t(i) \in (\tilde{t}(i), 1]$, $\frac{\partial\text{Inc}_{\tilde{t}(i)}(i)}{\partial I(1)} > 0$. Define $t^{\uparrow} = \max_{i\in\{0,1\}} \tilde{t}(i)$, which proves the statement.

□

3. Statement 3: **Exclusionary Zoning**: There exists a $t_{\downarrow} \in [0,1]$ such that, for every $t \in [0, t^{\downarrow})$, increasing regulation in any neighborhood(s) does not increase average income in all locations. Instead, average income across neighborhoods weighted by the population of z_h types increases.

   Proof. I start by establishing this statement holds exactly at the $t = 0$ equilibrium, and appeal to continuity arguments. I consider a marginal change in $I(i)$ in some neighborhood (without loss of generality assume it is $i = 1$, since both neighborhoods are identical at $t = 0$). This marginal change is around an equilibrium that is just binding for z_l types, i.e. exactly at the point where $s(1, z_l) = \beta$. Using a similar set of equations derived in Statement 1, we can express equilibrium in terms of $\Delta_1 Y$ and ΔInc as follows:

$$\Delta\text{Inc} = \rho\sum_z \phi^I_z[\tilde{c}\Delta_1 Y + \frac{w}{P(1)^{\beta}}\partial D(1,z_l)1_{z=z_l}] \tag{38}$$

[64]By virtue of equilibrium values of Inc(i) being differentiable in exogenous variables, which is a consequence of the differentiability of the equilibrium system and the Implicit function theorem.

where $1_{z=z_l}$ is the indicator function if $z = z_l$, and ϕ_z^I is defined above. Moreover, since each neighborhood is the same in this equilibrium (that is, all types are split evenly across neighborhoods), then it is easy to show that $\sum_z \phi_z^I = 0$. This means that Equation (38) can be expressed as

$$\Delta \text{Inc} = \rho \phi_{z_l}^I \frac{w}{P(1)^\beta} \partial D(1, z_l) > 0$$

since $\partial D(1, z_l) < 0$ and $\phi_{z_l}^I < 0$. Hence, neighborhood incomes are diverging for marginal changes in regulation. We can use this fact to show that $\frac{\partial \text{Inc}(1)}{\text{Inc}(1)} > 0$ (and the opposite in $i = 0$), completing the proof.

$\frac{\partial \text{Inc}(1)}{\text{Inc}(1)}$ can be written as

$$\frac{\partial \text{Inc}(1)}{\text{Inc}(1)} = \rho \sum_z \kappa(z)[\tilde{c}\Delta_1 Y + \Omega(z)\Delta \text{Inc} + \frac{w}{P(1)^\beta} \partial D(1, z_l)]$$

where $\kappa(z) = [o(1, z) - f(1, z)[1 - \tilde{f}(1, z)]$. $\kappa(z)$ also has the property that $\sum_z \kappa(z) = 0$ when neighborhoods are identical. Substituting in the derivation of ΔInc above and simplifying yields

$$\frac{\partial \text{Inc}(1)}{\text{Inc}(1)} = [\rho \sum_z \kappa(z)\Omega(z)][\rho \phi_{z_l}^I \frac{w}{P(1)^\beta} \partial D(1, z_l)] + \rho \kappa(z_l) \frac{w}{P(1)^\beta} \partial D(1, z_l)$$

both terms are strictly positive, since $\kappa(z_h) > 0$ and $\kappa(z_l) < 0$ (along with $\phi_{z_l}^I$), and $\Omega(z)$ is nondecreasing in z (by assumption). Hence, we have that incomes in 1 increase in response to regulation. Moreover, an identical expression can be derived for $\frac{\partial \text{Inc}(0)}{\text{Inc}(0)}$ to show that it is strictly negative. Hence, one neighborhood must increase in income and the other decrease.

Finally, $\frac{\partial \text{Inc}(i)}{\partial I(1)}$ is continuous in t[65] and I established is strictly positive for $i = 1$ and strictly negative for $i = 0$ at the $t = 0$ equilibrium. By the Intermediate Value Theorem, there exists a $\tilde{t}(i)$ for each i such that, for all $t(1) \in [0, \tilde{t}(1))$, $\frac{\partial \text{Inc}_{\tilde{t}(1)}(1)}{\partial I(1)} > 0$ and negative for $t(0) \in [0, \tilde{t}(0))$. Define $t^{\downarrow} = \min_{i \in \{0,1\}} \tilde{t}(i)$, which proves the statement. □

[65]By virtue of equilibrium values of Inc(i) being differentiable in exogenous variables, which is a consequence of the differentiability of the equilibrium system and the Implicit function theorem.

C Appendix: Calibration

C.1 The complete calibration algorithm

In this section, I detail the full algorithm that calibrates the model. I omit details of the calculations of hedonic prices, residualized wages, and other parameters because they are straightforward and explained with detail in Section 4. This calibration starts with preference parameters β and $\bar{A}$, which were calculated in a previously to match aggregate spending on housing for a random sample of $5,000$ block groups.

Algorithm 2 Calibration

(Consumption values and supply parameters)

For all block groups do (for some κ large)

If Partially Regulated[66]

Uniquely solve equation (19) for unregulated prices $P_U(i)$ given all other parameters, including regulated prices $P_R(i)$ and observed value of a minimal lot $I(i)$. Defines consumption value $C(i, z)$ and aggregate housing expenditure for each zone.

Else

Directly calculate $C(i, z)$ and housing expenditure given some housing price $P(i)$ and the value of a minimal lot $I(i)$.

If Fully or Partially Regulated

Calculate $\lambda(i)$ to uniquely solve solve $\lambda(i)P_R(i)^{1+\epsilon(i)}l(i) = I(i)$ for all partially or fully regulated block groups.

Choose $T_{\boldsymbol{o}}(i)$ for each $\boldsymbol{o}$ to clear housing markets by zone. If Fully Regulated, $T_{\boldsymbol{U}}(i) = 0$.

Else

Set $T_U(i)$ to some observed land mass of the block group and $T_R(i) = 0$.

Calculate $\lambda(i)$ to clear housing markets[67].

(Amenities)

For all block groups do

1. Choose amenities to uniquely solve (20) given $C(i, z)$ and populations $L(i, z)$ (up to a normalization) for each type.

---

[67] A neighborhood is partially regulated if a block group is 1) assigned regulation previously, 2) has a positive fraction of housing units in regulated structures observed in the ACS, but this fraction is less than one. Fully regulated neigborhoods are identical to partially regulated neighborhoods except that the fraction of housing units in regulated structures is 1. Unregulated neighborhoods are all neighborhoods not satisfying one of the definitions above.

[67] In an unregulated neighborhood, $T_R(i)$ and $\lambda(i)$ are not separately identifiable parameters, so this allocation has no impact on counterfactual outcomes.

C.2 Calibration when households differ on education

Extending the model to allow for households to differ by education requires some additional calibration methodology, which I detail here.

Wages by Education Since households of differing education levels are not perfect substitutes in production (Equation 15), they require a separate measure of wages. I regress log hourly wages in a set of occupation, sex, race, ancestry, year, quadratic in age and years of education, including MSA fixed effects separately for both college and non-college educated workers. These MSA fixed effects for both education groups form city and education specific wages per unit of effective labour, each normalized to be one in an average city. Productivity $\iota(c, s)$ by education s is then derived to uniquely rationalize employment by education in each city and this city wage under the technology in Equation (15) (after specifying a value of σ).

Local education-type distributions The ACS block group tabulations do not report the joint distribution of households by income z and education status. I impute the joint distribution of neighborhood income and education by calculating the share of college workers by income type at the city level using the ACS microdata sample. I assume that this share by type applies uniformly over all neighborhoods within a given city.

All other calibration methodology when households differ on education is precisely the same as the baseline model where they do not (a type is redefined to be an income-education pair). I also assume that the support of the type distribution and spending shares on housing are the same across education levels within household income types.

C.3 Summary Statistics for key calibrated parameters

Table 6: Summary Statistics for Key Calibrated Parameters, disaggregated by SuperStar city sample

SuperStar city	Aggregate			No			Yes		
Variable	N	Mean	Sd	N	Mean	Sd	N	Mean	Sd
Housing price (Regulated zone)	193805	1	1.1	82851	0.5	0.27	110954	1.4	1.3
Housing price (Unregulated zone)	193805	1.4	1.5	82851	0.73	0.63	110954	1.8	1.8
ln Amenity value (0-25k)	191711	-0.014	0.22	82378	-0.13	0.17	109333	0.074	0.21
ln Amenity value (25-50k)	191711	-0.01	0.21	82378	-0.11	0.16	109333	0.062	0.21
ln Amenity value (50-75k)	191856	-0.013	0.21	82096	-0.11	0.17	109760	0.06	0.21
ln Amenity value (75-100k)	191856	-0.014	0.22	82096	-0.11	0.18	109760	0.059	0.22
ln Amenity value (100-150k)	185915	0.013	0.24	78012	-0.1	0.21	107903	0.097	0.23
ln Amenity value (150-200k)	185915	0.01	0.25	78012	-0.12	0.22	107903	0.1	0.24
ln Amenity value (200k+)	185915	0.0046	0.27	78012	-0.14	0.23	107903	0.11	0.26
Consumption value (0-25k)	193805	0.19	0.11	82851	0.23	0.093	110954	0.17	0.12
Consumption value (25-50k)	193805	0.43	0.11	82851	0.45	0.073	110954	0.42	0.13
Consumption value (50-75k)	193805	0.69	0.11	82851	0.69	0.066	110954	0.69	0.14
Consumption value (75-100k)	193805	0.92	0.11	82851	0.9	0.066	110954	0.94	0.13
Consumption value (100-150k)	193805	1.2	0.11	82851	1.1	0.07	110954	1.2	0.13
Consumption value (150-200k)	193805	1.3	0.11	82851	1.3	0.072	110954	1.4	0.12
Consumption value (200k+)	193805	1.4	0.11	82851	1.4	0.071	110954	1.5	0.1

Summary statistics for calibrated (and estimated) housing prices, consumption values, and amenity values by income type. These statistics are broken down by the SuperStar and non-SuperStar sample used to construct Facts 1 and 2 (top quartile of density and ACS reported housing prices). They are also reported for the entire sample. Amenity values $b(i,z)$ are normalized to have a mean of 1 (in levels, not logs). Differences in observation counts for amenity values are in block groups with no counts of the corresponding type. Consumption values $C(i,z)$ are rescaled so that they have the same standard deviation as a log Cobb-Douglas index with housing expenditure parameter $\beta = 0.2$.

D Appendix: Estimating $\Omega(z)$

D.1 Econometric model

In this section, I specify a data generating process that justifies the use of the donut instrument to estimate each $\Omega(z)$. Consider a closed city with N neighborhoods; these constitute the observed data. The structural equation for amenity value is, as in Equation (21),

$$\log b(i,z) = \Omega(z) \log \text{Inc}(i) + \beta_1(z) S\big[d_1(i)\big] + \log \nu(i,z)$$

where neighborhood income is determined by the neighborhood choice equation (10) subject to prices clearing housing markets. I specify that fundamental amenities $\log \nu(i,z)$ are a sum of two components:

$$\log \nu(i,z) = \underbrace{\Upsilon(i,z)}_{\text{Unobserved demand factors}} + \underbrace{\xi(i,z)}_{\text{Noise}}$$

where $\xi(i,z)$ is random noise that may be spatially correlated. $\Upsilon(i,z)$ contain unobserved demand factors that are not themselves slopes but can be correlated with slopes, i.e. weather, climate, views and ecetera. Note that if there were no unobserved demand factors[68], an instrument is still required because $\text{Inc}(i)$ will be endogenously determined by the shocks $\xi(i,z)$.

I allow for slopes in the buffer region to be cause unobserved local demand factors (or be correlated through some underlying latent variable). Structurally,

$$\Upsilon(i,z) = \tilde{\gamma}(z) S\big[d_1(i)\big] + \sigma(i,z) \tag{39}$$

for some $\sigma(i,z)$ capturing other demand unobservables orthogonal to slopes in the buffer region d_1. This term may have arbitrary spatial correlation. The identification assumption is that slopes outside the buffer region cannot be correlated with any unobserved demand factors, or

$$S\big[d_2(i)\big] \;\perp\; \sigma(i,z) \tag{40}$$

which in turn requires that $S\big[d_2(i)\big]$ be "excluded" as a demand factor in Equation (39). That is, slopes outside of the buffer cannot provide any additional information to predict unobserved demand factors beyond its ability to predict slopes within the buffer region d_1. Note that this allows for the possibility that slopes have an arbitrary spatial correlation. This identification assumption is equivalent to saying

$$S\big[d_2(i)\big] \;\perp\; \log \tilde{\nu}(i,z) \mid S\big[d_1(i)\big]$$

which is the assumption reported in the paper.

[68]That is, $\Upsilon(i,z) = 0$ with probability 1 for every i and z.

The conditions that determine the relevancy of $S[d_2(i)]$ as an instrument are unmodelled in the paper. The idea behind relevancy stems from the incentive to strategically price housing in space if landowners observe market power (Bayer et al., 2007; Anagol et al., 2021). Slopes of block groups in $d_2(i)$ drive up housing prices in these locations, incentivizing landowners in i to increase prices in response, which in turn drives up neighborhood income via sorting on housing prices.

D.2 First Stage Regression

Table 7: First Stage Regressions pooled across income types.

VARIABLES	(1) ln Income	(2) ln Income	(3) ln Income	(4) ln Income
Slope Control	0.0146***	0.0158***	0.0163***	0.0173***
	(0.0038)	(0.0035)	(0.0032)	(0.0030)
Local Slope Control	0.0086***	0.0099***	0.0124***	0.0125***
	(0.0028)	(0.0023)	(0.0020)	(0.0018)
Slope Donut (0.75-1.25km)	0.0218***	0.0195***	0.0194***	0.0173***
	(0.0035)	(0.0025)	(0.0024)	(0.0023)
Outer Slope Control	-0.0486***	-0.0433***	-0.0406***	-0.0367***
	(0.0109)	(0.0103)	(0.0103)	(0.0097)
Observations	174,926	172,522	172,285	172,285
R^2	0.0128	0.0142	0.0142	0.0143
Specification	IV	IV	IV	IV
Donut	0.75-1.25km	0.75-1.25km	0.75-1.25km	0.75-1.25km
Base Controls	No	Yes	Yes	Yes
Amen/Topo Controls	No	No	Yes	Yes
Density Control	No	No	No	Yes
KP-FStat	38.3	59.3	63.5	55.1
Cluster	MSA	MSA	MSA	MSA

Robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Columns are ordered by income group. "Donut Slope Control" is the average slope within the block group plus a buffer with length equal to d_1. All specifications include MSA fixed effects and standard errors are clustered using a 35km Bartlett kernel. "Local Slope Control" is the average slope within the block group. "Outer slope control" is the average slope from d_2 to 10km. ln Income is instrumented with the average slopes of block groups that have centroids within buffer d_1 and d_2. "Base Controls" include travel time, building age, public transport and bus shares in commuting and CBD distance. "Amen/Topo" controls include various amenities (density of coffee shops, parks, restaurants) and various topographic features (cover of different types of forest such as deciduous or evergreen, wetlands, perennial snow cover). "Density Control" is the within-MSA density ranking of the block group.

D.3 OLS

Table 8: Baseline OLS Specifications by income group.

VARIABLES	(1) ln Amenity (Low)	(2) ln Amenity (Med)	(3) ln Amenity (High)
ln Income	-0.0447***	0.0525***	0.1669***
	(0.0034)	(0.0047)	(0.0049)
Slope Control	0.0033***	0.0041***	0.0025***
	(0.0012)	(0.0014)	(0.0009)
Local Slope Control	0.0014***	0.0012**	-0.0005
	(0.0004)	(0.0005)	(0.0006)
Outer Slope Control	-0.0004	-0.0033	-0.0009
	(0.0028)	(0.0032)	(0.0035)
Observations	170,951	171,045	165,317
R^2	0.0354	0.0463	0.3244
Specification	OLS	OLS	OLS
Donut	0.75-1.25km	0.75-1.25km	0.75-1.25km
Base Controls	Yes	Yes	Yes
Amen/Topo Controls	No	No	No
Density Control	No	No	No

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Columns are ordered by income group. ”Donut Slope Control” is the average slope within the block group plus a buffer with length equal to d_1. All specifications include MSA fixed effects and standard errors are clustered using a 35km Bartlett kernel. ”Outer slope control” is the average slope from d_2 to 10km. $\ln$ Income is instrumented with the average slopes of block groups that have centroids within buffer d_1 and d_2. ”Local Slope Control” is the average slope within the block group. ”Slope Donut” is the instrument. ”Base Controls” include travel time, building age, public transport and bus shares in commuting and CBD distance. ”Amen/Topo” controls include various amenities (density of coffee shops, parks, restaurants) and various topographic features (cover of different types of forest such as deciduous or evergreen, wetlands, perennial snow cover). ”Density Control” is the within-MSA density ranking of the block group.

D.4 Robustness

Various controls Table 9 compares estimates pooled over each income type with various controls. Column (1) is the OLS estimate controlling only for block group land mass. Column (2) introduces the IV estimate with the same set of controls as (1), showing downward bias. Column (3) adds and additional a set of "base" controls: commuting time, median building age, share of public and bus transport in commuting, and CBD distance rankings. Including these controls increases the estimate, which I confirm is entirely driven by non-bus public transportation use. Column (4) adds additional topographic characteristics (such as forest cover) and observed amenities (density of coffee shops, bars, parks) from NaNDA with little changes to the estimate. This is the pooled version of the baseline IV estimates in Table 1. Lastly, Column (5) additionally controls for housing unit density and this causes a large increase in estimates. This is not surprising given the negative relationship between income and density within cities (Fact 1). I prefer not to use this estimate as density and amenity value are very similar equilibrium objects, and so density is likely a bad control.

Table 9: Pooled IV Specification with various controls

VARIABLES	(1) ln Amenity	(2) ln Amenity	(3) ln Amenity	(4) ln Amenity	(5) ln Amenity
ln Income	0.0568***	0.2021***	0.2287***	0.2450***	0.3260***
	(0.0029)	(0.0301)	(0.0318)	(0.0317)	(0.0414)
Slope Control	0.0031***	-0.0014*	-0.0018**	-0.0017**	-0.0041***
	(0.0010)	(0.0008)	(0.0008)	(0.0008)	(0.0010)
Local Slope Control	0.0016***	0.0006	-0.0008	-0.0012*	-0.0015*
	(0.0005)	(0.0007)	(0.0006)	(0.0007)	(0.0008)
Outer Slope Control	-0.0052	0.0009	0.0051	0.0053*	0.0063*
	(0.0038)	(0.0039)	(0.0033)	(0.0031)	(0.0036)
Observations	174,926	174,926	172,522	172,285	172,285
R^2	0.0999	-0.4049	-0.4866	-0.5995	-1.1917
Specification	OLS	IV	IV	IV	IV
Donut	.	0.75-1.25km	0.75-1.25km	0.75-1.25km	0.75-1.25km
Base Controls	No	No	Yes	Yes	Yes
Amen/Topo Controls	No	No	No	Yes	Yes
Density Control	No	No	No	No	Yes
FStat Bart c 35 km	.	55.7	65.3	69.2	57.7

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Pooled IV Specification with various controls, using the average amenity across low, medium and high income groups. "Donut Slope Control" is the average slope within the block group plus a buffer with length equal to d_1. All specifications include MSA fixed effects and standard errors are clustered using a 35km Bartlett kernel. "Local Slope Control" is the average slope within the block group. "Outer slope control" is the average slope from d_2 to 10km. ln Income is instrumented with the average slopes of block groups that have centroids within buffer d_1 and d_2. "Base Controls" include travel time, building age, public transport and bus shares in commuting and CBD distance. "Amen/Topo" controls include various amenities (density of coffee shops, parks, restaurants) and various topographic features (cover of different types of forest such as deciduous or evergreen, wetlands, perennial snow cover). "Density Control" is the within-MSA density ranking of the block group.

Different donuts The distances used for the donut instrument in Table 1 may seem small. To test how estimates vary by donut size, I compare various pooled estimates in Table 10. Each column is ordered by increasing radii, and I find that larger radii correspond to significantly larger estimates in some cases. This is further evidence that the instrument is correcting for downward bias that is not a product of measurement error. Column (3) corresponds to the baseline pooled estimate. Estimates in Column (2) are considerably smaller than in (3). Column (4) is the farthest donut that maintains decent instrument relevancy, with similar estimates to (3). Donuts beyond column (4) are very weak instruments with either implausibly large or negative point estimates. These don't fit with reasonable priors.

Additional checks The calibrated amenity values used in estimation depend on the calibration of consumption values $C(i, z)$ and the choice of within-city migration elasticities ρ. The estimates are robust to a host of alternative calibrations. First, results remain virtually unchanged for high and low values of ρ based on confidence intervals in Baum-Snow and Han (2021). Second, controlling for the hedonic price index derived from Equation (18) only changes estimates by a few percentage points. Third, estimation results are similar when preferences are assumed Cobb-Douglas rather than Stone-Geary.

Table 10: Pooled IV Specification with various donuts

VARIABLES	(1) ln Amenity	(2) ln Amenity	(3) ln Amenity	(4) ln Amenity	(5) ln Amenity	(6) ln Amenity	(7) ln Amenity
ln Income	0.1574***	0.1848***	0.2287***	0.2483***	0.5978*	-0.0636	-0.1893
	(0.0205)	(0.0243)	(0.0318)	(0.0491)	(0.3577)	(0.0874)	(0.2866)
Slope Control	0.0004	-0.0008	-0.0018**	-0.0015	-0.0048	0.0048***	0.0037**
	(0.0006)	(0.0006)	(0.0008)	(0.0012)	(0.0050)	(0.0014)	(0.0017)
Local Slope Control	-0.0003	-0.0001	-0.0008	-0.0014*	-0.0078	0.0032*	0.0069
	(0.0006)	(0.0005)	(0.0006)	(0.0008)	(0.0058)	(0.0018)	(0.0067)
Outer Slope Control	0.0025	0.0034	0.0051	0.0056	0.0222	-0.0083**	-0.0130
	(0.0029)	(0.0031)	(0.0033)	(0.0037)	(0.0165)	(0.0041)	(0.0105)
Observations	175,080	170,704	172,522	172,855	186,945	189,590	188,145
R^2	-0.0465	-0.1876	-0.4866	-0.6514	-6.4638	-0.0791	-1.0593
Specification	IV	IV	IV	IV	IV	IV	IV
Donut	0.2-1km	0.5-1km	0.75-1.25km	1-1.5km	1.5-3km	3-6km	6-8km
Base Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Amen/Topo Controls	No	No	No	No	No	No	No
Density Control	No	No	No	No	No	No	No
FStat Bart c 35 km	118.8	95.0	65.3	31.1	2.3	12.6	2.2

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Pooled IV Specification with various donuts, using the average amenity across low, medium and high income groups. "Donut Slope Control" is the average slope within the block group plus a buffer with length equal to d_1. All specifications include MSA fixed effects and standard errors are clustered using a 35km Bartlett kernel. "Local Slope Control" is the average slope within the block group. ln Income is instrumented with the average slopes of block groups that have centroids within buffer d_1 and d_2. "Base Controls" include travel time, building age, public transport and bus shares in commuting and CBD distance. "Amen/Topo" controls include various amenities (density of coffee shops, parks, restaurants) and various topographic features (cover of different types of forest such as deciduous or evergreen, wetlands, perennial snow cover). "Density Control" is the within-MSA density ranking of the block group.

Table 11: Placebo tests, pooled and by income type

VARIABLES	(1) ln Fund. Amenity	(2) ln Fund. Amenity	(3) ln Fund. Amenity (low)	(4) ln Fund. Amenity (med)	(5) ln Fund. Amenity (high)
ln Income (2020)	0.0253	-0.0881***	0.0478	-0.0187	0.0405
	(0.0365)	(0.0028)	(0.0418)	(0.0441)	(0.0333)
Slope Control	-0.0006	0.0027***	-0.0020**	-0.0009	0.0003
	(0.0008)	(0.0010)	(0.0010)	(0.0010)	(0.0008)
Local Slope Control	-0.0008	0.0002	0.0003	-0.0014*	-0.0013**
	(0.0006)	(0.0004)	(0.0008)	(0.0007)	(0.0006)
Outer Slope Control	0.0081***	0.0040	0.0112***	0.0065*	0.0052**
	(0.0031)	(0.0026)	(0.0038)	(0.0034)	(0.0025)
Observations	172,937	172,937	172,679	172,174	162,566
R^2	0.2112	0.3506	0.1128	0.2844	0.1807
Specification	IV	OLS	IV	IV	IV
Donut	0.75-1.25km	0.75-1.25km	0.75-1.25km	0.75-1.25km	0.75-1.25km
Base Controls	Yes	Yes	Yes	Yes	Yes
Amen/Topo Controls	No	No	No	No	No
Density Control	No	No	No	No	No
FStat Bart c 35 km	61.5		61.0	60.4	58.8

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Placebo tests. Column (1) reports the estimate of a pooled specification of (23). Column (2) estimates (23) with OLS under the assumption that the pooled baseline estimate of $\Omega(z)$ is identified. The remaining columns estimate (23) with IV disaggregated by low, medium and high income types. All specifications include MSA fixed effects and standard errors are clustered using a 35km Bartlett kernel. "Local Slope Control" is the average slope within the block group. ln Income is instrumented with the average slopes of block groups that have centroids within buffer d_1 and d_2. "Base Controls" include travel time, building age, public transport and bus shares in commuting and CBD distance. "Amen/Topo" controls include various amenities (density of coffee shops, parks, restaurants) and various topographic features (cover of different types of forest such as deciduous or evergreen, wetlands, perennial snow cover). "Density Control" is the within-MSA density ranking of the block group.

E Appendix: Counterfactuals

E.1 Algorithm for computing counterfactuals

In this section, I detail the algorithm used to compute counterfactual outcomes. There are two main sets of state variables in this algorithm: 1) Populations (by neighborhood, zone and type) and 2) Spending shares on housing (by neighborhood, zone and type) that are iterated on to solve for an equilibrium both across neighborhoods and for housing markets within neighborhoods.

Algorithm 3 Counterfactuals

Initialize error on populations (with tolerance ϵ_L) and spending shares (tolerance ϵ_S)

While Population Error $> \epsilon_L$ or Spending Share Error $> \epsilon_S$ do

For each neighborhood i, zone $\boldsymbol{o}$

1. Solve for housing prices $P_{\boldsymbol{o}}(i)$ to clear markets given current neighborhood-zone allocation $L_{\boldsymbol{o}}(i,z)$ and spending shares $\beta_{\boldsymbol{o}}(i,z)$
2. Update wages, amenities and/or productivity given current neighborhood allocation (when applicable)
3. Given wages, amenities, productivity and housing prices from above, do for each z:
   (a) Calculate what the desired neighborhood allocation $\tilde{L}_{\boldsymbol{o}}(i,z)$ would be (Equation 10)
   (b) Calculate what the desired spending share on housing $\tilde{\beta}_{\boldsymbol{o}}(i,z)$ would be (using the solution to Equation 6)

Update Population Error $= \max_{i,\boldsymbol{o},z} |\tilde{L}_{\boldsymbol{o}}(i,z) - L_{\boldsymbol{o}}(i,z)|$ and Spending Share Error $= \max_{i,\boldsymbol{o},z} |\tilde{\beta}_{\boldsymbol{o}}(i,z) - \beta_{\boldsymbol{o}}(i,z)|$

Set $L_{\boldsymbol{o}}(i,z) = L_{\boldsymbol{o}}(i,z) + \kappa[\tilde{L}_{\boldsymbol{o}}(i,z) - L_{\boldsymbol{o}}(i,z)]$ and $\beta_{\boldsymbol{o}}(i,z) = \beta_{\boldsymbol{o}}(i,z) + \kappa[\tilde{\beta}_{\boldsymbol{o}}(i,z) - \beta_{\boldsymbol{o}}(i,z)]$ for some parameter $\kappa < 1$.

E.2 Weighing renters and landowners welfare: continued

In this appendix, I provide details on the justification of the welfare formula for homeowners:

$$\big[1 - s(z)\big]\text{CompVar}(z) + s(z)\text{LandValueChange}$$

Where CompVar(z) is the renter's compensating variation associated with deregulation, LandValueChange is the loss in aggregate land values, and $s(z)$ are weights that represent the share of total income from imputed rents and income from rental properties by type. I also detail how weights $s(z)$ are constructed.

The idea behind this welfare formula is simple. CompVar(z) is a measure of how much a renter's labour income would have to fall to have welfare comparable to pre-deregulation levels. I think about this as an appreciation of a "labour income asset" in a portfolio, so this appreciation should be weighed by the share of labour income to total income from all sources. Similarly, land value depreciation should be weighed by the share of total income from land.

I outline a strategy to calculate $s(z)$, share of total income from land, for each type. I assume that income from land is proportional to rental expenditure in the baseline equilibrium for all households of that same type. This is because I do not observe the ownership structure of the rental stock, and cannot use it to assign housing wealth for each type of household. Aggregate spending on housing services in the model closely fits the data for each type because I choose preference parameters β and $\bar{A}$ to target it. Finally, I calculate $s(z)$ as

$$s(z) = \frac{\text{RentalIncome}(z)}{z + \text{RentalIncome}(z)}$$

Where RentalIncome(z) is the expenditure on rents per capita by type in the baseline equilibrium. This yields the following $s(z)$ weights: 0.52 for 0-25k, 0.32 for 25-50k, 0.23 for 50-75k, 0.19 for 75-100k, 0.16 for 100-150k, 0.14 for 150-200k and 0.1 for 200k+. Weights are decreasing in income, which reflects the fact that the share of housing to total wealth is declining in income for homeowners (Greaney, 2023).

E.3 Shapely value decomposition

This appendix briefly explains the Shapely decomposition to isolate the roles of changing consumption and amenity values performed in Section 6. Consider any the two equilibrium outcome vectors $C_t(z)$ (consumption value) and $b_t(z)$ (amenity value), where $t = 0$ denotes pre-counterfactual and $t = 1$ counterfactual outcomes, and vector components represent individual neighborhoods. Let $\boldsymbol{W}(C_t, b_t)$ be the renter welfare measure from Equation (11) evaluated at some vector of consumption values C_t and amenity values b_t.

I define an *effect* of changing consumption as

$$\tilde{\boldsymbol{W}}_{C,t}(z) = \log \boldsymbol{W}(C_1(z), b_t(z)) - \log \boldsymbol{W}(C_0(z), b_t(z)) \tag{41}$$

and similarly an *effect* of changing amenities as

$$\tilde{\boldsymbol{W}}_{b,t}(z) = \log \boldsymbol{W}(C_t(z), b_1(z)) - \log \boldsymbol{W}(C_t(z), b_1(z))$$

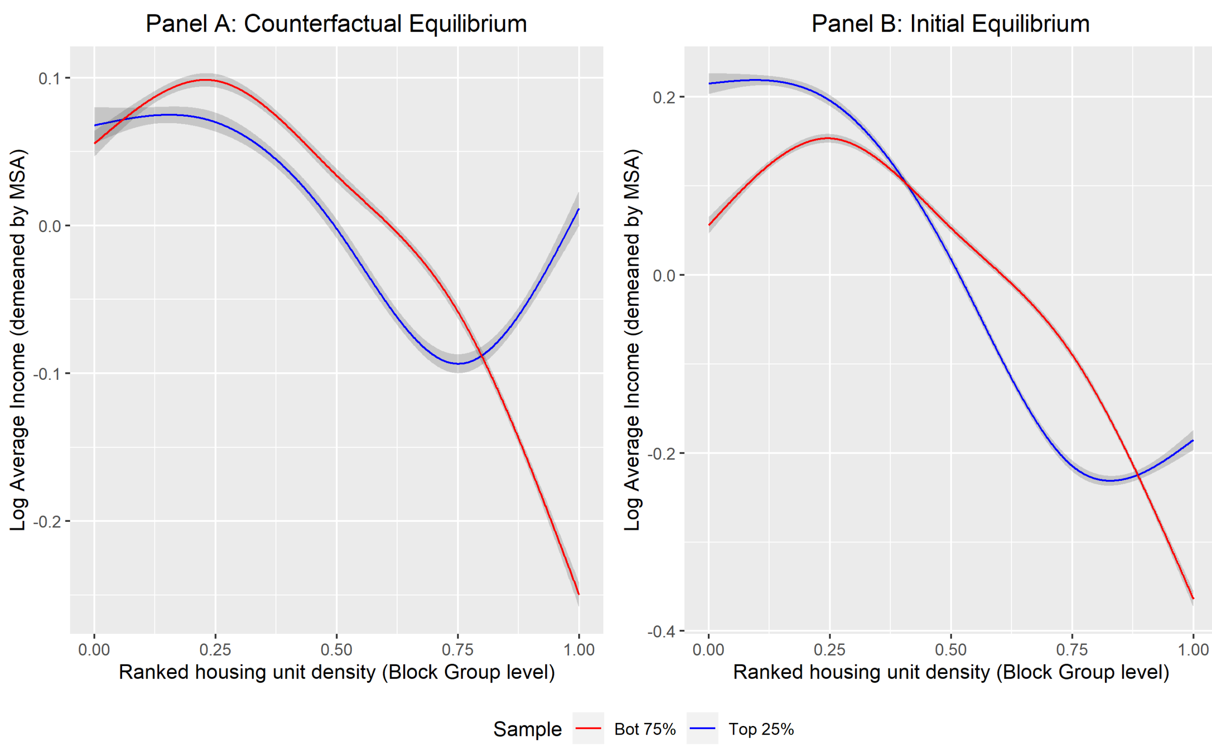
for some $t \in \{0, 1\}$. A *Shapely effect* $S_C(z)$ for the consumption is the unweighted average of consumption effects over t. That is,

$$S_C(z) = \frac{1}{2} \sum_{t \in \{0,1\}} \tilde{\boldsymbol{W}}_{C,t}(z) \tag{42}$$

with an analogous definition for amenities. By definition, $S_C(z)+S_b(z) = \log \boldsymbol{W}(C_1(z), b_1(z)) - \log \boldsymbol{W}(C_0(z), b_0(z))$, which is the total welfare effect of a counterfactual. In Table 5 (and other tables) I report this Shapely decomposition with the total effect $S_C(z)+S_b(z)$ rescaled to match the compensating variation associated with the counterfactual.

E.4 Supplementary figures

Figure 16: Income-Density Gradients in baseline and counterfactual.



Panel B corresponds to the same estimates of Panel A of Figure 1. Panel A compares the income density gradient across ”superstar” sample cities and ”non-superstar” cities that is generated in an equilibrium without minimum lot sizes. The gentrification of high density neighborhoods is apparent. Differences in the income density gradient across samples disappear when transitioning from the initial equilibrium to the counterfactual equilibrium.