To find $\cos(n\theta)$ do $(\cos\theta + i\sin\theta)^n$

To find $\cos^n\theta$ do $\left(z + \frac{1}{z}\right)^n$

To simplify $\frac{z}{k + be^{ki\theta}}$, $\times$ by $\frac{(k+be^{-ki\theta})}{(k+be^{-ki\theta})}$

To simplify $\frac{z}{k + ke^{ki\theta}}$, $\times$ by $\frac{e^{-\frac{ki\theta}{2}}}{e^{-\frac{ki\theta}{2}}}$

For roots of complex numbers

> To remember: $\cdot$ $-\pi < \theta_{arg} \le \pi$
>
> $\cdot$ leave z^n as long as possible and do all the mod-arg and $2k\pi$ stuff to the $a + bi$ part

$$z^n = a + bi$$

1. write in mod-arg form

$$z^n = R(\cos\alpha + i\sin\alpha)$$

2. add $2k\pi$

$$z^n = R(\cos(\alpha + 2k\pi) + i\sin(\alpha + 2\pi k))$$

3. put z in mod-arg form also

$$z^n = (r(\cos\theta + i\sin\theta))^n = R(\cos(\alpha + 2k\pi) + i\sin(\alpha + 2k\pi))$$

4. expand out and equate

$$r^n(\cos n\theta + i\sin n\theta) = R(\cos(\alpha + 2k\pi) + i\sin(\alpha + 2k\pi))$$

$$r^n = R \qquad n\theta = \alpha + 2k\pi$$

$$r = \sqrt[n]{R}$$

Eg 1. Solve $z^4 = 2 + 2i\sqrt{3}$

$$z^4 = 2 + 2i\sqrt{3} \longrightarrow \text{mod} = \sqrt{2^2 + (2\sqrt{3})^2} = 4$$

$$\text{argument} = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

$$z^4 = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$= 4\left(\cos\left(\frac{\pi}{3} + 2k\pi\right) + i\sin\left(\frac{\pi}{3} + 2k\pi\right)\right)$$

$$(r(\cos\theta + i\sin\theta))^4 = 4\left(\cos\left(\frac{\pi}{3} + 2k\pi\right) + i\sin\left(\frac{\pi}{3} + 2k\pi\right)\right) \Rightarrow r^4(\cos 4\theta + i\sin 4\theta) = 4\left(\cos\left(\frac{\pi}{3} + 2k\pi\right) + i\sin\left(\frac{\pi}{3} + 2k\pi\right)\right)$$

$$r^4 = 4 \qquad 4\theta = \frac{\pi}{3} + 2k\pi$$

$$r = \sqrt{2}$$

$$4\theta_0 = \frac{\pi}{3} \qquad = \frac{\pi}{12}$$

$$4\theta_1 = \frac{\pi}{3} + 2\pi = \frac{7}{3}\pi \qquad = \frac{7\pi}{12}$$

$$4\theta_{-1} = \frac{\pi}{3} - 2\pi = -\frac{5}{3}\pi \qquad = \frac{-5\pi}{12}$$

$$4\theta_2 = \frac{\pi}{3} + 4\pi = \frac{13}{3}\pi \qquad = \frac{13\pi}{12} = -\frac{11}{12}\pi$$

should be in the interval $[-\pi, \pi]$

$$\cancel{z = \sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$$

$$\cancel{= \sqrt{2}\left(\cos\frac{7\pi}{3} + i\sin\frac{7\pi}{3}\right)}$$

$$\cancel{= \sqrt{2}\left(\cos\frac{-5\pi}{3} + i\sin\frac{-5\pi}{3}\right)}$$

$$\cancel{= \sqrt{2}\left(\cos\frac{13}{3}\pi + i\sin\frac{13}{3}\pi\right)}$$

$$z = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

$$\sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$$

$$\sqrt{2}\left(\cos\frac{-5\pi}{12} + i\sin\frac{-5\pi}{12}\right)$$

$$\cancel{\sqrt{2}\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right)}$$

$$\sqrt{2}\left(\cos\frac{-11\pi}{12} + i\sin\frac{-11\pi}{12}\right)$$

Eg. 2 Solve $z^3 + 4\sqrt{2} + 4i\sqrt{2} = 0$

$$z^3 = -4\sqrt{2} - 4i\sqrt{2}$$

$\hookrightarrow$ mod $= \sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2} = 8$

arg $= -\tan^{-1} \frac{4\sqrt{2}}{4\sqrt{2}} = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3}{4}\pi$

$$z^3 = 8\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$$

$$= 8\left(\cos\left(\frac{-3\pi}{4} + 2k\pi\right) + i\sin\left(\frac{-3\pi}{4} + 2k\pi\right)\right)$$

$$z^3 = \left(r(\cos\theta + i\sin\theta)\right)^3 = 8\left(\cos\left(\frac{-3\pi}{4} + 2k\pi\right) + i\sin\left(\frac{-3\pi}{4} + 2k\pi\right)\right)$$

$$r^3(\cos 3\theta + i\sin 3\theta) = 8\left(\cos\left(\frac{-3\pi}{4} + 2k\pi\right) + i\sin\left(\frac{-3\pi}{4} + 2k\pi\right)\right)$$

$$r^3 = 8 \qquad 3\theta = \frac{-3\pi}{4} + 2k\pi$$

$$r = 2 \qquad \theta_0 = -\frac{\pi}{4}$$

$$\theta_1 = \frac{-\frac{3\pi}{4} + 2\pi}{3} = \frac{5}{12}\pi$$

$$\theta_2 = \frac{-\frac{3\pi}{4} - 2\pi}{3} = -\frac{11}{12}\pi$$

$$z = 2\left(\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4}\right),\ 2\left(\cos\frac{5\pi}{12} + i\sin\frac{5}{12}\pi\right),\ 2\left(\cos\frac{-11}{12}\pi + i\sin\frac{-11}{12}\pi\right)$$