

NUMERICAL ANALYSIS
HOMEWORK 1
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SOLUTIONS

2.0

$$x^4 - 3x^2 - 3 = 0 \quad \text{on } [1, 2] \quad p_0 = 1.$$

$$x = (3x^2 + 3)^{1/4} \text{ converges on } [1, 2]$$

for k such that $0 < k < 1$

$$|g'(x)| < k \quad \text{for all } x \in (a, b).$$

Iterations for the function $x = (3x^2 + 3)^{1/4}$.

Iterations	P_n	$p_n - p_{n-1}$
1	1.56508	0.565085
2	1.79357	0.228488
3	1.88594	0.0923709
4	1.92285	0.0369041
5	1.93751	0.0146597
6	1.94332	0.00580939

Theoretical value of n .

$$g'(x) = 6x(3x^2 + 3)^{-3/4} / 4$$

$$g'(1) = 0.391$$

$$|p_n - p| \leq k^n / (1 - k) \leq |p_1 - p_0| \quad \text{for all } n \geq 1$$

$$k = 0.391$$

$$e \leq k^n / (1 - k), \quad e = 10e-5$$

$$\log(e * (1 - k)) / \log k \leq n$$

$$5.4 \leq n$$

$$n = 6,$$

Theoretical number of Iterations is 6.

3.0

Exercise 4

a. Secant Method

Iterations	P_n	$p_n - p_{n-1}$
1	-0.685073	0.685073
2	-1.25208	0.567003
3	-0.807206	0.444871
4	-0.847784	0.0405782
5	-0.866528	0.0187444
6	-0.865456	0.00107246

$$p_3 = -0.807206$$

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b. Regular falsi Method

Iterations	P_n	$p_n - p_{n-1}$
1	-0.685073	0.685073
2	-0.841355	0.156282
3	-0.862547	0.0211924
4	-0.865123	0.00257597
5	-0.865432	0.000308646

$$p_3 = -0.862547$$

Exercise 5

a. $x^3 - 2x^2 - 5 = 0$, $[1, 4]$

$$p_n = p_{n-1} - f(p_{n-1})/f'(p_{n-1})$$

$$p_n = p_{n-1} - (x^3 - 2x^2 - 5)/(3x^2 - 4x) \quad p_0 = 3.$$

Iterations	p_n	$p_n - p_{n-1}$
1	3.15625	0.84375
2	2.7786	0.377647
3	2.69466	0.0839438
4	2.69066	0.00400313
5	2.69065	8.89279e-06

root is 2.69065 for $p_0 = 3$.

b. $x^3 + 3x^2 - 1 = 0$, $[-3, -2]$

$$p_n = p_{n-1} - f(p_{n-1})/f'(p_{n-1})$$

$$p_n = p_{n-1} - (x^3 + 3x^2 - 1)/(3x^2 + 6x) \quad p_0 = -2.5.$$

Iterations	p_n	$p_n - p_{n-1}$
1	-3.06667	0.566667
2	-2.90088	0.165791
3	-2.87972	0.0211557
4	-2.87939	0.00033458

root is -2.87939 for $p_0 = -2.5$.

c. $x - \cos x = 0$, $[0, \pi/2]$

$$p_n = p_{n-1} - f(p_{n-1})/f'(p_{n-1})$$

$$p_n = p_{n-1} - (x - \cos x)/(1 + \sin(x)) \quad p_0 = 1.$$

Iterations	p_n	$p_n - p_{n-1}$
1	0.750364	0.249636
2	0.739113	0.011251
3	0.739085	2.77575e-05

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root is 0.739085 for $p_0 = 1$.

d. $x - 0.8 - 0.2 \sin x = 0$, $[0, \pi/2]$

$$p_n = p_{n-1} - f(p_{n-1})/f'(p_{n-1})$$

$$p_n = p_{n-1} - (x - 0.8 - 0.2 \sin(x)) / (1 - 0.2 \cos(x)) \quad p_0 = \pi/2 .$$

Iterations	p_n	$p_n - p_{n-1}$
1	0.999909	0.570091
2	0.964452	0.0354568
3	0.964334	0.000118471

root is 0.964334 for $p_0 = \pi/2$.