

2nd Question

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

for x,y and x',y'

1,2	2,2
2,1	-1,4
3,1	-4,4

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{aligned} a_{11} + 2a_{12} + a_{13} &= 2 \\ a_{21} + 2a_{22} + a_{23} &= 2 \\ 2a_{11} + a_{12} + a_{13} &= -1 \\ 2a_{21} + a_{22} + a_{23} &= 4 \\ 3a_{11} + a_{12} + a_{13} &= -4 \\ 3a_{21} + a_{22} + a_{23} &= 4 \end{aligned}$$

So from here we can get two matrices which we can use to get the values of unknowns in matrix A

$$\begin{aligned} a_{11} + 2a_{12} + a_{13} &= 2 & 1 \ 2 \ 1 \mid 2 \\ 2a_{11} + a_{12} + a_{13} &= -1 & \rightarrow 2 \ 1 \ 1 \mid -1 \\ 3a_{11} + a_{12} + a_{13} &= -4 & 3 \ 1 \ 1 \mid -4 \end{aligned}$$

$$\begin{aligned} a_{21} + 2a_{22} + a_{23} &= 2 & 1 \ 2 \ 1 \mid 2 \\ 2a_{21} + a_{22} + a_{23} &= 4 & \rightarrow 2 \ 1 \ 1 \mid 4 \\ 3a_{21} + a_{22} + a_{23} &= 4 & 3 \ 1 \ 1 \mid 4 \end{aligned}$$

Using Gaussian Elimination

$$\begin{aligned} a_{11} &= -3, a_{12} = 0, a_{13} = 5 \\ a_{21} &= 0, a_{22} = -2, a_{23} = 6 \end{aligned}$$

so the resulting matrix is

$$A = \begin{pmatrix} -3 & 0 & 5 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{pmatrix} \text{ so } AZ = I, \quad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad Z = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \quad AZ = I.$$

$$\begin{pmatrix} -3 & 0 & 5 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

system of Equations after $AZ=I$

$$-3a + 5g = 1, a = -1/3$$

$$-2d + 6g = 0 \quad d = 0$$

$$g = 0$$

$$-3b + 5h = 0, b = 0$$

$$-2e + 6h = 1 \quad e = -1/2$$

$$h = 0$$

$$-3c + 5i = 0, c = 5/3$$

$$-2f + 6i = 0 \quad f = 3$$

$$i = 1$$

$$Z = \begin{pmatrix} -1/3 & 0 & 5/3 \\ 0 & -1/2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

So the transformation matrix is Z .