CSC 321: Data Structures

Fall 2012

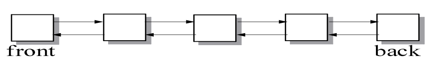
Binary Search Trees

- BST property
- override binary tree methods: add, contains
- search efficiency
- balanced trees: AVL, red-black
- heaps, priority queues, heap sort

1

Searching linked lists

recall: a (linear) linked list only provides sequential access → O(N) searches



it is possible to obtain O(log N) searches using a tree structure

in order to perform binary search efficiently, must be able to

- access the middle element of the list in O(1)
- divide the list into halves in O(1) and recurse

HOW CAN WE GET THIS FUNCTIONALITY FROM A TREE?

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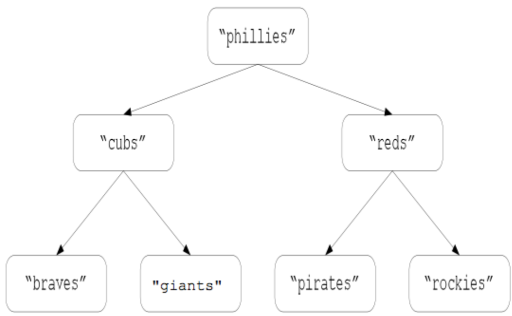
Binary search trees

a *binary search tree* is a binary tree in which, for every node:

- the item stored at the node is ≥ all items stored in its left subtree
- the item stored at the node is < all items stored in its right subtree



in a (balanced) binary search tree:

- middle element = root
- 1st half of list = left subtree
- 2nd half of list = right subtree

furthermore, these properties hold for each subtree

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BinarySearchTree class

can use inheritance to derive BinarySearchTree from BinaryTree

```
public class BinarySearchTree<E extends Comparable<? super E>>
extends BinaryTree<E> {

    public BinarySearchTree() {
        super();
    }

    public void add(E value) {
        // OVERRIDE TO MAINTAIN BINARY SEARCH TREE PROPERTY
    }

    public void CONTAINS(E value) {
        // OVERRIDE TO TAKE ADVANTAGE OF BINARY SEARCH TREE PROPERTY
    }

    public void remove(E value) {
        // DOES THIS NEED TO BE OVERRIDDEN?
    }
}
```

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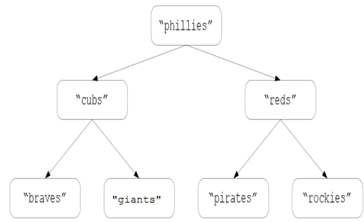
Binary search in BSTs



<u>to search a binary search tree:</u>

1. if the tree is empty, NOT FOUND
2. if desired item is at root, FOUND
3. if desired item < item at root, then recursively search the left subtree
4. if desired item > item at root, then recursively search the right subtree

```
public boolean contains(E value) {
    return this.contains(this.root, value);
}

private boolean contains(TreeNode<E> current, E value) {
    if (current == null) {
        return false;
    }
    else if (value.equals(current.getData())) {
        return true;
    }
    else if (value.compareTo(current.getData()) < 0) {
        return this.contains(current.getLeft(), value);
    }
    else {
        return this.contains(current.getRight(), value);
    }
}
```


Search efficiency

how efficient is search on a BST?

- in the best case?

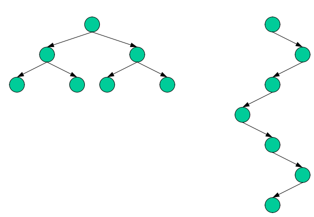
  O(1) if desired item is at the root

- in the worst case?

  O(height of the tree) if item is leaf on the longest path from the root

in order to optimize worst-case behavior, want a (relatively) balanced tree

- otherwise, don't get binary reduction
- e.g., consider two trees, each with 7 nodes



Search efficiency (cont.)

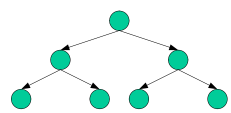
we showed that N nodes can be stored in a binary tree of height $\lceil \log_2(N+1) \rceil$

so, in a balanced binary search tree, searching is O(log N)

N nodes → height of $\lceil \log2(N+1) \rceil$ → in worst case, have to traverse $\lceil \log2(N+1) \rceil$ nodes

what about the average-case efficiency of searching a binary search tree?

- assume that a search for each item in the tree is equally likely
- take the cost of searching for each item and average those costs



costs of search

1

2 + 2

3 + 3 + 3 + 3

→ 17/7 → 2.42

define the *weight* of a tree to be the sum of all node depths (root = 1, …)

average cost of searching a BST = weight of tree / number of nodes in tree

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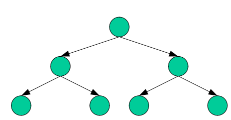
Search efficiency (cont.)



costs of search

1

2 + 2

3 + 3 + 3 + 3

→ 17/7 → 2.42

~log N

costs of search

1
+ 2
+ 3
+ 4
+ 5
+ 6
+ 7

→ 28/7 → 4.00

~N/2

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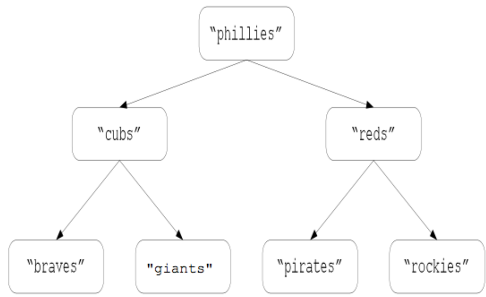
Inserting an item



inserting into a BST

1. traverse edges as in a search
2. when you reach a leaf, add the new node below it

```
public void add(E value) {
    this.root = this.add(this.root, value);
}

private TreeNode<E> add(TreeNode<E> current, E value) {
    if (current == null) {
        return new TreeNode<E>(value, null, null);
    }

    if (value.compareTo(current.getData()) <= 0) {
        current.setLeft(this.add(current.getLeft(), value));
    }
    else {
        current.setRight(this.add(current.getRight(), value));
    }
    return current;
}
```

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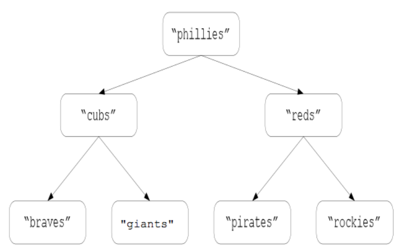
Removing an item



recall BinaryTree remove

1. find node (as in search)
2. if a leaf, simply remove it
3. if no left subtree, reroute parent pointer to right subtree
4. otherwise, replace current value with a leaf value from the left subtree (and remove the leaf node)

CLAIM: as long as you select the rightmost (i.e., maximum) value in the left subtree, this remove algorithm maintains the BST property

WHY?

so, no need to override remove

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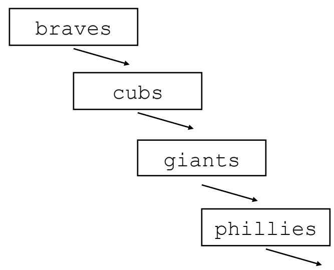
Maintaining balance

PROBLEM: random insertions (and removals) do not guarantee balance

- e.g., suppose you started with an empty tree & added words in alphabetical order
  braves, cubs, giants, phillies, pirates, reds, rockies, …



with repeated insertions/removals, can degenerate so that height is O(N)

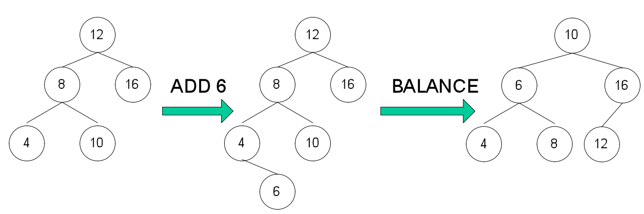
- specialized algorithms exist to maintain balance & ensure O(log N) height
- or take your chances

Balancing trees

on average, N random insertions into a BST yields O(log N) height

- however, degenerative cases exist (e.g., if data is close to ordered)

we can ensure logarithmic depth by maintaining balance



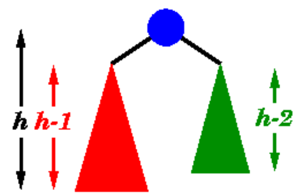
maintaining full balance can be costly

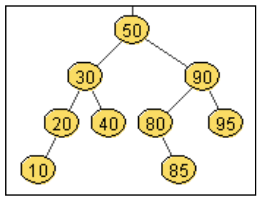
- however, full balance is not needed to ensure O(log N) operations

AVL trees

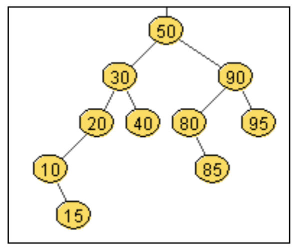
an AVL tree is a binary search tree where

- for every node, the heights of the left and right subtrees differ by at most 1
- first self-balancing binary search tree variant
- named after Adelson-Velskii & Landis (1962)





AVL tree



not an AVL tree – WHY?

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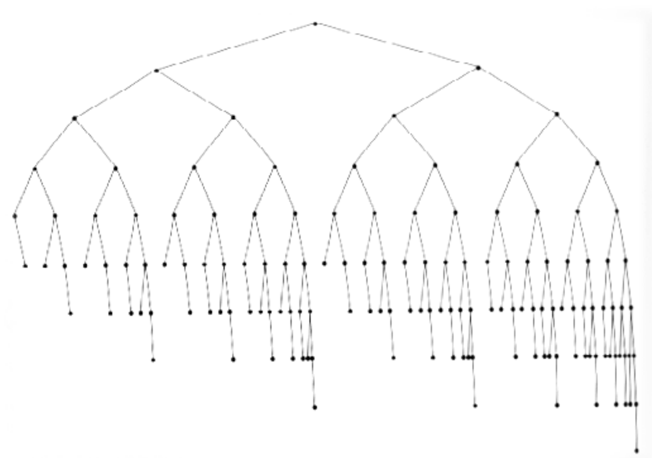
AVL trees and balance

the AVL property is weaker than full balance, but sufficient to ensure logarithmic height

- height of AVL tree with N nodes < 2 log(N+2) → searching is O(log N)



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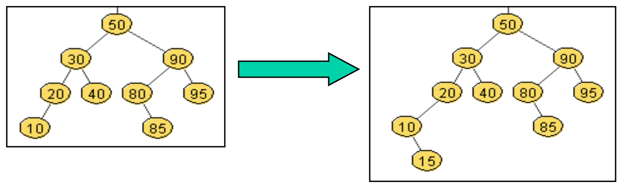
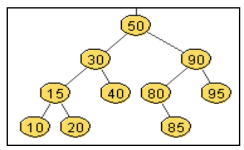
Inserting/removing from AVL tree

when you insert or remove from an AVL tree, imbalances can occur



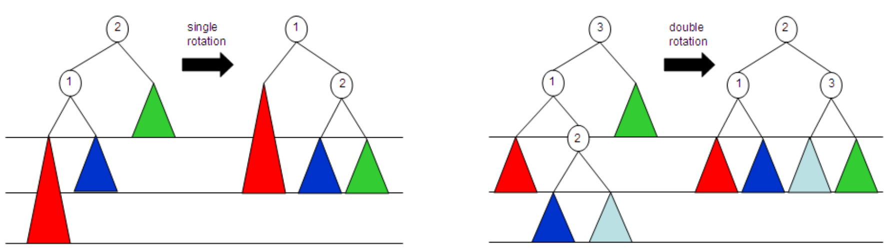
- if an imbalance occurs, must rotate subtrees to retain the AVL property



- see www.site.uottawa.ca/~stan/csi2514/applets/avl/BT.html

AVL tree rotations

there are two possible types of rotations, depending upon the imbalance caused by the insertion/removal



worst case, inserting/removing requires traversing the path back to the root and rotating at each level

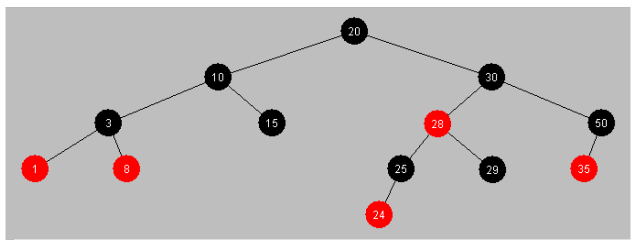
- each rotation is a constant amount of work ➔ inserting/removing is O(log N)

Red-black trees

a red-black tree is a binary search tree in which each node is assigned a color (either red or black) such that

1. *the root is black*
2. *a red node never has a red child*
3. *every path from root to leaf has the same number of black nodes*

- add & remove preserve these properties (complex, but still O(log N))
- red-black properties ensure that tree height < 2 log(N+1) → O(log N) search



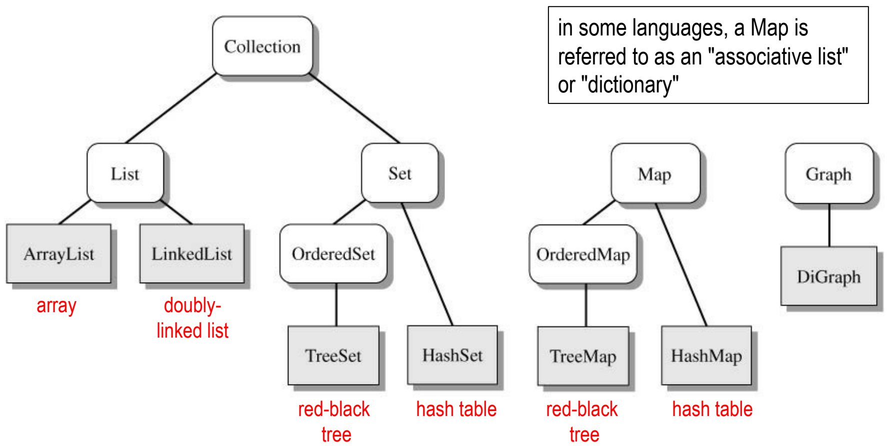
see a demo at gauss.ececs.uc.edu/RedBlack/redblack.html

Java Collection classes

recall the Java Collection Framework

- defined using interfaces abstract classes, and inheritance

in some languages, a Map is referred to as an "associative list" or "dictionary"



Sets

java.util.Set interface: an unordered collection of items, with no duplicates

```
public interface Set<E> extends Collection<E> {
    boolean add(E o);              // adds o to this Set
    boolean remove(Object o);      // removes o from this Set
    boolean contains(Object o);    // returns true if o in this Set
    boolean isEmpty();             // returns true if empty Set
    int size();                    // returns number of elements
    void clear();                  // removes all elements
    Iterator<E> iterator();        // returns iterator
    . . .
}
```

implemented by TreeSet and TreeMap classes

TreeSet implementation

- ✓ implemented using a red-black tree; items stored in the nodes (must be Comparable)
- ✓ provides O(log N) add, remove, and contains (guaranteed)
- ✓ iteration over a TreeSet accesses the items in order (based on compareTo)

HashSet implementation

- ✓ HashSet utlizes a hash table data structure LATER
- ✓ HashSet provides O(1) add, remove, and contains (on average, but can degrade)

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Dictionary revisited

note: our Dictionary class could have been implemented using a Set

- Strings are Comparable, so could use either implementation
- TreeSet has the advantage that iterating over the Set elements gives them in order (here, alphabetical order)

```
import java.util.Set;
import java.util.TreeSet;
import java.util.Scanner;
import java.io.File;

public class Dictionary {
    private Set<String> words;

    public Dictionary() {
        this.words = new TreeSet<String>();
    }

    public Dictionary(String filename) {
        this();
        try {
            Scanner infile = new Scanner(new File(filename));
            while (infile.hasNext()) {
                String nextWord = infile.next();
                this.add(nextWord);
            }
        }
        catch (java.io.FileNotFoundException e) {
            System.out.println("FILE NOT FOUND");
        }
    }

    public void add(String newWord) {
        this.words.add(newWord.toLowerCase());
    }

    public void remove(String oldWord) {
        this.words.remove(oldWord.toLowerCase());
    }

    public boolean contains(String testWord) {
        return this.words.contains(testWord.toLowerCase());
    }
}
```

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Maps

java.util.Map interface: a collection of key → value mappings

```
public interface Map<K, V> {
    boolean put(K key, V value);    // adds key→value to Map
    V remove(Object key);           // removes key→? entry from Map
    V get(Object key);              // returns true if o in this Set
    boolean containsKey(Object key);     // returns true if key is stored
    boolean containsValue(Object value); // returns true if value is stored
    boolean isEmpty();              // returns true if empty Set
    int size();                     // returns number of elements
    void clear();                   // removes all elements
    Set<K> keySet();                // returns set of all keys
    . . .
}
```

implemented by TreeMap and HashMap classes

TreeMap implementation

- ✓ utilizes a red-black tree to store key/value pairs; ordered by the (Comparable) keys
- ✓ provides O(log N) put, get, and containsKey (guaranteed)
- ✓ keySet() returns a TreeSet, so iteration over the keySet accesses the key in order

HashMap implementation

- ✓ HashSet utlizes a HashSet to store key/value pairs LATER
- ✓ HashSet provides O(1) put, get, and containsKey (on average, but can degrade)

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Word frequencies

a variant of Dictionary is WordFreq

- stores words & their frequencies (number of times they occur)
- can represent the word→counter pairs in a Map
- again, could utilize either Map implementation
- since TreeMap is used, showAll displays words + counts in alphabetical order

```
import java.util.Map;
import java.util.TreeMap;
import java.util.Scanner;
import java.io.File;

public class WordFreq {
    private Map<String, Integer> words;

    public WordFreq() {
        words = new TreeMap<String, Integer>();
    }

    public WordFreq(String filename) {
        this();
        try {
            Scanner infile = new Scanner(new File(filename));
            while (infile.hasNext()) {
                String nextWord = infile.next();
                this.add(nextWord);
            }
        }
        catch (java.io.FileNotFoundException e) {
            System.out.println("FILE NOT FOUND");
        }
    }

    public void add(String newWord) {
        String cleanWord = newWord.toLowerCase();
        if (words.containsKey(cleanWord)) {
            words.put(cleanWord, words.get(cleanWord)+1);
        }
        else {
            words.put(cleanWord, 1);
        }
    }

    public void showAll() {
        for (String str : words.keySet()) {
            System.out.println(str + ": " + words.get(str));
        }
    }
}
```

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Other tree structures

a *heap* is a common tree structure that:

- can efficiently implement a priority queue (a list of items that are accessed based on some ranking or priority as opposed to FIFO/LIFO)
- can also be used to implement another O(N log N) sort

motivation: many real-world applications involve optimal scheduling

- choosing the next in line at the deli
- prioritizing a list of chores
- balancing transmission of multiple signals over limited bandwidth
- selecting a job from a printer queue
- multiprogramming/multitasking

all these applications require

- storing a collection of prioritizable items, and
- selecting and/or removing the highest priority item

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Priority queue

priority queue is the ADT that encapsulates these 3 operations:

✓ *add item (with a given priority)*
✓ *find highest priority item*
✓ *remove highest priority item*

e.g., assume printer jobs are given a priority 1-5, with 1 being the most urgent

a priority queue can be implemented in a variety of ways

- unsorted list

job1	job 2	job 3	job 4	job 5
3	4	1	4	2

  efficiency of add? efficiency of find? efficiency of remove?

- sorted list (sorted by priority)

job4	job 2	job 1	job 5	job 3
4	4	3	2	1

  efficiency of add? efficiency of find? efficiency of remove?

- others?

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java.util.PriorityQueue

Java provides a `PriorityQueue` class

```
public class PriorityQueue<E extends Comparable<? super E>> {
    /** Constructs an empty priority queue
     */
    public PriorityQueue<E>() { … }

    /** Adds an item to the priority queue (ordered based on compareTo)
     *    @param newItem the item to be added
     *    @return true if the items was added successfully
     */
    public boolean add(E newItem) { … }

    /** Accesses the smallest item from the priority queue (based on compareTo)
     *    @return the smallest item
     */
    public E peek() { … }

    /** Accesses and removes the smallest item (based on compareTo)
     *    @return the smallest item
     */
    public E remove() { … }

    public int size() { … }
    public void clear() { … }
    . . .
}
```

the underlying data structure is a special kind of binary tree called a *heap*

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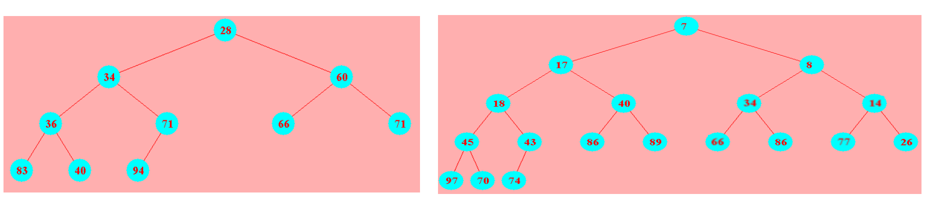
Heaps

a *complete tree* is a tree in which

- all leaves are on the same level or else on 2 adjacent levels
- all leaves at the lowest level are as far left as possible

a *heap* is complete binary tree in which

- for every node, the value stored is ≤ the values stored in both subtrees
  (technically, this is a min-heap -- can also define a max-heap where the value is ≥)



since complete, a heap has minimal height = $\lfloor \log_2 N \rfloor + 1$

- can insert in O(height) = O(log N), but searching is O(N)
- not good for general storage, but perfect for implementing priority queues
  can access min value in O(1), remove min value in O(height) = O(log N)

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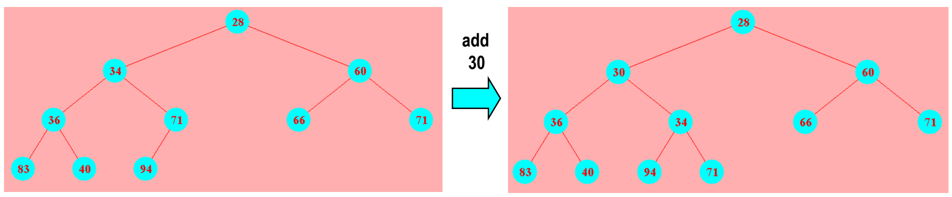
Inserting into a heap

to insert into a heap

- place new item in next open leaf position
- if new value is smaller than parent, then swap nodes
- continue up toward the root, swapping with parent, until smaller parent found

see http://www.cosc.canterbury.ac.nz/people/mukundan/dsal/MinHeapAppl.html



note: insertion maintains completeness and the heap property

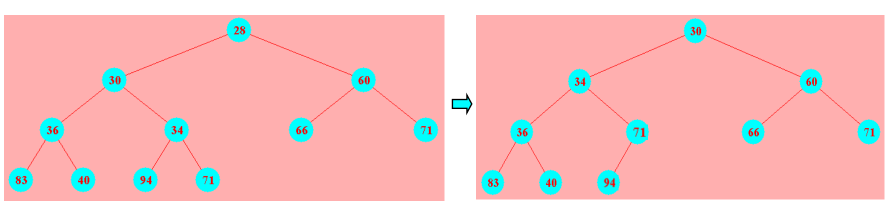
- worst case, if add smallest value, will have to swap all the way up to the root
- but only nodes on the path are swapped → O(height) = O(log N) swaps

Removing from a heap

to remove the min value (root) of a heap

- replace root with last node on bottom level
- if new root value is greater than either child, swap with smaller child
- continue down toward the leaves, swapping with smaller child, until smallest

see http://www.cosc.canterbury.ac.nz/people/mukundan/dsal/MinHeapAppl.html



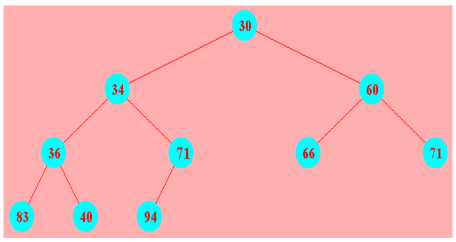
note: removing root maintains completeness and the heap property

- worst case, if last value is largest, will have to swap all the way down to leaf
- but only nodes on the path are swapped → O(height) = O(log N) swaps

Implementing a heap

a heap provides for O(1) find min, O(log N) insertion and min removal

- also has a simple, List-based implementation
- since there are no holes in a heap, can store nodes in an ArrayList, level-by-level



30	34	60	36	71	66	71	83	40	94

- root is at index `0`
- last leaf is at index `size()-1`
- for a node at index `i`, children are at `2*i+1` and `2*i+2`
- to add at next available leaf, simply add at end

`MinHeap` class

```
import java.util.ArrayList;

public class MinHeap<E extends Comparable<? super E>> {
    private ArrayList<E> values;

    public MinHeap() {
        this.values = new ArrayList<E>();
    }

    public E minValue() {
        if (this.values.size() == 0) {
            throw new java.util.NoSuchElementException();
        }
        return this.values.get(0);
    }

    public void add(E newValue) {
        this.values.add(newValue);
        int pos = this.values.size()-1;

        while (pos > 0) {
            if (newValue.compareTo(this.values.get((pos-1)/2)) < 0) {
                this.values.set(pos, this.values.get((pos-1)/2));
                pos = (pos-1)/2;
            }
            else {
                break;
            }
        }
        this.values.set(pos, newValue);
    }

    . . .
```

we can define our own simple min-heap implementation

- `minValue` returns the value at index 0
- `add` places the new value at the next available leaf (i.e., end of list), then moves upward until in position

`MinHeap` class (cont.)

```
. . .

public void remove() {
    E newValue = this.values.remove(this.values.size()-1);
    int pos = 0;

    if (this.values.size() > 0) {
        while (2*pos+1 < this.values.size()) {
            int minChild = 2*pos+1;
            if (2*pos+2 < this.values.size() &&
                  this.values.get(2*pos+2).compareTo(this.values.get(2*pos+1)) < 0) {
                minChild = 2*pos+2;
            }

            if (newValue.compareTo(this.values.get(minChild)) > 0) {
                this.values.set(pos, this.values.get(minChild));
                pos = minChild;
            }
            else {
                break;
            }
        }
        this.values.set(pos, newValue);
    }
}
```

- `remove` removes the last leaf (i.e., last index), copies its value to the root, and then moves downward until in position

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Heap sort

the priority queue nature of heaps suggests an efficient sorting algorithm

- start with the ArrayList to be sorted
- construct a heap out of the elements
- repeatedly, remove min element and put back into the ArrayList

```
public static <E extends Comparable<? super E>>
void heapSort(ArrayList<E> items) {
    MinHeap<E> itemHeap = new MyMinHeap<E>();

    for (int i = 0; i < items.size(); i++) {
        itemHeap.add(items.get(i));
    }

    for (int i = 0; i < items.size(); i++) {
        items.set(i, itemHeap.minValue());
        itemHeap.remove();
    }
}
```

- N items in list, each insertion can require O(log N) swaps to reheapify
  →construct heap in O(N log N)
- N items in heap, each removal can require O(log N) swap to reheapify
  →copy back in O(N log N)

thus, overall efficiency is O(N log N), which is as good as it gets!

- can also implement so that the sorting is done in place, requires no extra storage

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