# THE PROBLEM OF CLOCK SYNCHRONIZATION: A RELATIVISTIC APPROACH

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**Abstract.** The problem of synchronization of the Earth-based clocks has been discussed in the framework of General Relativity Theory. The synchronization is considered as the transformation of the observers' proper time scales to the coordinate time scale of local inertial geocentric reference system, which is single for all the observers. The formulas for the relativistic corrections occurring in some methods of Earth-based clock synchronization (transported clock, duplex communication via geostationary satellite and meteor-burst link, LASSO experiments) have been derived enabling one to attain the accuracy of 0.1 ns.

**Key words:** general relativity, clock synchronization

## 1. Introduction: The Notion of Synchronization in General Relativity

Realization of a high accuracy time scale is well-known to be a very important problem for investigations in the field of geodynamics, radio astrometry (VLBI), for high accuracy navigation on the Earth and in space. The accuracy of up-to-date atomic clocks is $10^{-14} - 10^{-15}$ and improves by about an order of magnitude every seven years [Allan, Ashby, 1986]. In not so distant future the frequency standards will apparently be developed enabling one to attain the accuracy of $10^{-17} - 10^{-18}$ [Matisson, 1989]. However, the accuracy of clocks is not sufficient for the realization of a time scale. It is indispensable to solve the problem of high accuracy synchronization of clocks which realize the single time scale. Besides its direct intention, the synchronization of time standards may improve the total accuracy of time scale with respect to the accuracy of a separate clock [Allan, 1981; Allan, Ashby, 1986]. Indeed, the synchronization allows one to remove in part systematic differences in clock rates. The random differences can be diminished by the usual methods of the mathematical statistics.

The present accuracy of clocks synchronization may achieve 1 ns. This value is by 2–3 orders of magnitude smaller than the main relativistic effects. The numerous experiments which deal with the transportation and synchronization of high accuracy atomic clocks [Vessot, 1979; Alley, 1983; Allan, Ashby, 1986] completely confirm predictions of the General Relativity Theory (GRT). The significant magnitude of the relativistic effects in the process of remote clocks synchronization and realization of single self-consistent time scale results in the necessity to analyze these effects consequently in the framework of GRT and with high level of accuracy.

The notion of synchronization is closely connected with the notion of simultaneity. Indeed, synchronized clocks must simultaneously produce the same time markers.

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Newtonian mechanics and Newtonian theory of gravitation postulate the existence of absolute and independent from each other space and time. The notion of simultaneity has absolute meaning as well. Independently of used reference system (RS) two events can be either simultaneous or not. One can believe that in Newtonian theory two events are supposed to be simultaneous if these events correspond to one and the same value of absolute Newtonian time.

In the Special Relativity Theory (SRT) the situation changes drastically. The refusal of the notions of absolute time and space, the difference of the time rates in different inertial reference systems result in the notion of simultaneity losing its absolute (and unique) meaning.

For analysis of any process in the framework of GRT one must introduce certain four-dimensional RS, which consists of one time coordinate (so called coordinate time of the RS) and three space coordinates. Special Relativity Theory as having been formulated by Einstein deals only with so called inertial reference systems. Not going into unnecessary details one can say that any RS, in which bodies not affected by forces move linearly and uniformly, is called inertial RS. As the matter of fact, the following definition of simultaneity has been adopted in SRT [Born, 1962; Møller, 1972]. Two events fixed in some RS by the values of their coordinates $(t_1, x_1, y_1, z_1)$ and $(t_2, x_2, y_2, z_2)$ are considered to be simultaneous with respect to this RS, if the values of time coordinate corresponding to them are equal:$t_1 = t_2$. In the following this definition of simultaneity (and corresponding definition of synchronization) we shall call coordinate simultaneity (and coordinate synchronization). From the definition of coordinate simultaneity one can see that the events, which are simultaneous in one inertial RS, may turn out to be non-simultaneous in another inertial RS (see, for example, [Møller, 1972]). Thus, the notion of simultaneity in SRT becomes relative, i.e. dependent on coordinate system being employed.

For synchronization of clocks, which are at rest relative to an inertial RS, Einstein has suggested the following procedure. Let us consider two clocks $a$ and $b$. At the moment when the reading of the clock $a$ is $\tau_{a1}$ the station $a$ emits a signal, which arrives at station $b$ at the moment $\tau_{b0}$ according to the clock $b$, and then is reflected and returns to station $a$ at the time moment $\tau_{a2}$. The moment $\tau_{a0} = \frac{1}{2}(\tau_{a1} + \tau_{a2}) = \tau_{a1} + \frac{1}{2}(\tau_{a2} - \tau_{a1})$ according to the clock $a$ is considered to be simultaneous with the moment $\tau_{b0}$ according to the clock $b$. This procedure is often called Einsteinian synchronization. Sometimes one can meet another definitions of this procedure [Born, 1962; Fock, 1957]. But all the definitions are equivalent to the above-formulated one. Einsteinian procedure is no more than practically convenient algorithm enabling one to achieve coordinate synchronization of clocks, which are at rest relative to inertial RS. The basis of this algorithm is the equality of light velocities when propagating from the clock $a$ to the clock $b$ and back or, to put it another way, the isotropy of light velocity in inertial RS. This equality is a consequence of the SRT postulates and the condition of resting of the clocks in certain inertial RS. Einstein has emphasized [Einstein, 1953] that synchronization could be carried out in any reasonable way. But the simplicity of equations of light propagation in any inertial RS as well as maximal possible speed of the transmission of information make this procedure preferable. It should be noted once again that the Einsteinian procedure has been originally intended *only* for synchronization of clocks, which are *at rest* relative to an *inertial* RS. In our opinion, any extension of this procedure to more complicate situations is absolutely incorrect.

The present accuracy of observations does not permit us to confine ourselves to the approximation of SRT. Therefore, we must appeal to the General Relativity Theory. It should be noted that in transported clock synchronization relativistic effects due to the gravitational field of the Earth are of the same order of magnitude as the effects of special relativity. Thus, the approximation of special relativity has no independent interest as applied to the problem of Earth-based clock synchronization.

It is well-known that in infinitesimal domains of space-time special relativity turns out to be a good approximation of general one. In agreement with this fact, the definition of simultaneity in SRT can be adopted as the definition of simultaneity of two infinitesimally close events in the framework of GRT. So, to synchronize two infinitesimally close clocks one can resort to the above-mentioned Einstein procedure. However, attempts to achieve Einsteinian synchronization of the clocks in finite region of space-time encounter, in general case, principal difficulties. Indeed, consequently synchronizing a number of infinitesimally close clocks situated along a curve by using Einstein procedure we can synchronize clocks separated by any finite distance. But the result of such synchronization turns out to be dependent on the chosen curve, i.e. on the method of synchronization. Similar effects as applied to Earth-based clocks have been discussed in [Cohen, Moses, 1977; Cohen, Moses, Rosenblum, 1983a,b, 1984]. The absence of transitivity is clear manifestation of the above-mentioned ambiguity of the Einstein synchronization in the framework of GRT: if clock $a$ is synchronized with clock $b$ and $b$ in turn with $c$ then $a$ is not necessarily synchronized with $c$. Analogous effects result in impossibility of consistent synchronization of Earth-based clock by means of the Einstein procedure (the main cause is the non-inertiality of RS which is connected rigidly with the surface of the rotating Earth).

Extension of the concept of coordinate simultaneity and coordinate synchronization on the general relativity is the way out of the situation. In the framework of GRT as well as in special relativity we must introduce four-dimensional RS. However, in general relativity all reference systems are completely equivalent. Reference system in GRT is no more than a method of mapping of four-dimensional Riemanian space onto four-dimensional Euclidean one. Formally speaking, the definition of coordinate simultaneity in GRT totally coincides with the analogous definition in SRT (see above). This definition enables one to avoid any ambiguities and vagueness of Einstein synchronization in the framework of GRT and to introduce a single self-consistent time scale in quite different space-time regions and with any reasonable level of accuracy. The choice of RS to be used for coordinate synchronization is arbitrary. But we have no cause to worry. Given the clocks synchronized with respect to coordinate time of one RS we can easily synchronize them with respect to coordinate time of another RS.

The detailed discussion of the concept of coordinate synchronization of Earth-based clocks is contained in [Ashby, Allan, 1979; Allan, Ashby, 1986]. A number of issues have been clarified in [Ashby, Allan, 1984; Podlaha, 1984; Skalafuris, 1985; Borisova et al., 1988; Huang et al., 1989]. In our opinion this concept needs to be further developed. The RS which have been used in above-mentioned papers have been constructed accounting for gravitational field of the Earth only. The precision of synchronization claimed by the authors is 1 ns. This precision is not always sufficient at present. On the other hand, the methods of the papers in question do not allow to improve the precision up to the level of 0.1 ns. Beside this, some new technical methods of synchronization have been developed in the last years [Gubanov et al., 1989; Kascheev and Bondar, 1989]. These methods do not appear to have been considered from the relativistic point of view. All these points lead to the necessity to re-consider and to make more precise relativistic algorithms of synchronization.

In the present paper the usual notations will be used: $G$ is Newtonian constant of gravitation; $c$ is locally measured light velocity; the Greek indices $\alpha, \beta, \gamma, \ldots$ take values $0, 1, 2, 3$; the small Roman indices $i, j, k, \ldots$ run from 1 to 3; repeated index imply summation irrespective of the places of this repeated index; the italic capitals $A, B, C, \ldots$ number the gravitating bodies of the Solar system; $\delta_{ij} = \left[ \begin{array}{l} 1, i = j \\ 0, i \neq j \end{array} \right.$ is the Kronecker symbol; the underlined quantities $\underline{x}, \underline{w}, \ldots$ are three-dimensional vectors; the absolute values of these vectors calculated by means of usual Euclidean metric $\delta_{ij}$ are designated by the same letters as the original vectors but without underlining: $x, w, \ldots$; the signature of the metric is $+ - - -$.

## 2. Coordinate Synchronization in General Case

In the framework of GRT the properties of a reference system are completely described by the components of its metric tensor. Let us choose certain RS. We denote coordinate time, spatial coordinates and metric tensor of the chosen RS by $t = x^0/c$, $x^i$, $g_{\alpha\beta}(t, \underline{x})$ respectively. The metric tensor $g_{\alpha\beta}(t, \underline{x})$ depends on the coordinates of the centers-of-mass of gravitating bodies $\underline{x}_A(t), \underline{x}_B(t), \underline{x}_C(t), \ldots$, which are functions of time. These functions are to be determined from observations of different nature. When observing, an observer use the readings of his own clock, which measure proper time on his world line. The world line itself must be determined from observations.

Let the observer $a$ move along the trajectory $\underline{x}_a(t)$. The proper time of this observer $\tau_a$ is related with coordinate time $t$ by the well-known expression (a dot denotes the derivative with respect to $t$):

$$\dot{\tau}_a(t, \dot{\underline{x}}_a(t), \underline{x}_a(t)) = \Big(g_{00}(t, \underline{x}_a(t)) + \frac{2}{c}\, g_{0i}(t, \underline{x}_a(t))\dot{x}_a^i(t) + \\ + \frac{1}{c^2}\, g_{ij}(t, \underline{x}_a(t))\dot{x}_a^i(t)\dot{x}_a^j(t)\Big)^{1/2}. \qquad (2.1)$$

To establish one-to-one transformation between the coordinate time $t$ and the readings of clock $\tau_a$ it is not sufficient to use only formula (2.1). One must set up an initial values for the differential equation (2.1). Some moment $t_{a0}$ of the coordinate time must be prescribed to the moment $\tau_{a0}$ of the proper time (or vice verse). Using the equation (2.1) and the condition $\tau_{a0}(t_{a0}) = \tau_{a0}$ we can transform the readings of clocks $\tau_a$ to the moments of coordinate time scale $t$. In the present paper we consider two coordinate time scales $t$ and $t'$ to be different even if they differ only by a constant term: $t = t' + \text{const}$. It should be noted that the definition of a scale $t$ includes some equation of the form $\tau_{a0}(t_{a0}) = \tau_{a0}$. Having processed the observations, which refer to the derived scale $t$, we can obtain the coordinates of the observer himself $\underline{x}_a(t)$ and other objects $\underline{x}_A(t), \ldots$

Let us consider another observer $b$. His clock measures proper time $\tau_b$ on the world line $\underline{x}_b(t)$, which generally speaking does not coincide with the world line of the observer $a$. The proper time $\tau_b$ is connected with the coordinate time $t$ by the equation analogous to (2.1) with some initial values $\tau_{b0}(t_{b0}) = \tau_{b0}$.

To synchronize clocks $a$ and $b$ is to find the initial values $\tau_{b0}(t_{b0}) = \tau_{b0}$ for clock $b$ enabling one to transform to the same coordinate time scale $t$, to which we have decided to transform from the readings of clock $a$. For this purpose we must mark the reading of clock $b$ corresponding to some event on the world line of the observer $b$, whose coordinate time $t_{b0}$ we can calculate using the coordinates of the observers $\underline{x}_a(t), \underline{x}_b(t)$ and perhaps other objects. The above-mentioned event on the world line of clock $b$ we will call 'synchronizing' event.

Generally speaking, the observers $a$ and $b$ have totally equal rights. Postulating some equation of the form $\tau_{b0}(t'_{b0}) = \tau_{b0}$, the observer $b$ can transform from the readings of his clock $\tau_b$ to some coordinate time scale $t'$, which differs from the scale $t$ of the observer $a$ only by a constant term: $t = t' + t_{ab}$. Observing different celestial objects, calculating the moments of scale $t'$ corresponding to these observations, and performing the reduction of the observations, the observer $b$ can determine his own coordinates $\underline{x}_b(t')$, and coordinates of other celestial objects $\underline{x}_A(t), \underline{x}_B(t), \ldots$ in the form of functions of the time scale $t'$. Comparison of the results of observations and calculations of the observers $a$ and $b$ permits to obtain the difference $t_{ab}$ of the coordinate time scales $t$ and $t'$, that is to synchronize the clocks. Moreover, the observer $b$ may synchronize his clock with the clock $a$ by observing celestial bodies and comparing the results of these observations with $\underline{x}_A(t), \underline{x}_B(t), \ldots$ and $\underline{x}_b(t)$. In this case some natural event (an observation of occultation of a star by a planet, etc.) may be considered as 'synchronizing' event.

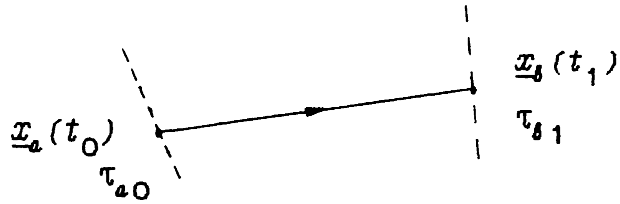


Fig. 1. One-way synchronization.

In principle the above-mentioned considerations allow one to synchronize clocks without some special procedure like signal exchange. Something similar happens during the processing of VLBI observations, which (besides other results) enables one to synchronize clocks at the stations [Counselman et al., 1977; Clark et al., 1979]. The only difference is the following. When treating VLBI observations, the joint reduction of observations made by both observers is performed accounting for the possible lack of clock synchronization.

More simple, fast and reliable manner of synchronization is artificial creating of 'synchronizing' event on the world line of the observer $b$. Such methods of clock synchronization are often called 'autonomous' or 'independent' methods. The principal feature of all autonomous methods of synchronization lies in the fact that the coordinates $\underline{x}_b(t)$ of the observer $b$ must be known *a priori*. These coordinates must be calculated on the basis of observations performed by the observer $a$ in his coordinate time scale $t$. Let us consider some simplest methods of creating of 'synchronizing' event.

At some moment $\tau_{a0}$ of proper time scale, which is supposed to correspond to the moment $t_0$ of the coordinate time, the observer $a$ emits electromagnetic signal (Fig. 1). This signal reaches the observer $b$ at the moment $t_1$ of coordinate time. On the basis of coordinates of the observers $\underline{x}_a(t), \underline{x}_b(t)$, and the light propagation laws in the chosen RS we can calculate the moment $t_1$.

Indeed,

$$c(t_1 - t_0) = |\underline{x}_b(t_1) - \underline{x}_a(t_0)| + \Delta\left(\underline{x}_a(t_0), \underline{x}_b(t_1)\right), \qquad (2.2)$$

$\Delta\left(\underline{x}_0, \underline{x}_1\right)$ being relativistic gravitational time dilation (Shapiro effect) on the path from the point $\underline{x}_0$ to $\underline{x}_1$. Solving the equation (2.2) by the iterations

$$\begin{aligned} c(t_1^{(0)} - t_0) &= |\underline{x}_b(t_0) - \underline{x}_a(t_0)| + \Delta(\underline{x}_a(t_0), \underline{x}_b(t_0)), \\ c(t_1^{(n)} - t_0) &= |\underline{x}_b(t_1^{(n-1)}) - \underline{x}_a(t_0)| + \Delta(\underline{x}_a(t_0), \underline{x}_b(t_1^{(n-1)})), \\ n &= 1, 2, \ldots \end{aligned} \qquad (2.3)$$

one can find moment $t_1 = \lim_{n\to\infty} t_1^{(n)}$.

On the other hand, the signal arrives at the observer $b$ at the moment $\tau_{b1}$ relative to his proper time scale. Thus, we can prescribe the calculated moment $t_1$ to the observed moment $\tau_{b1}$ and, thereby, synchronize observers' clocks.

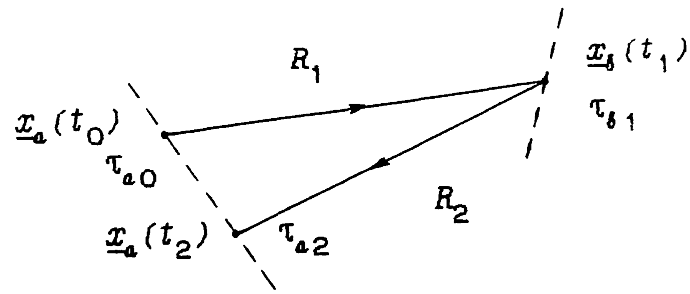


Fig. 2. Two-way synchronization.

This procedure, which is correct in principle, has at least one drawback – large inaccuracies of synchronization. Coordinates of observers are always known with some errors $\delta x$, which with the factor $1/c$ directly contribute to the inaccuracy of determination of the moment $t_1$: $\delta t_1 \sim \frac{1}{c}\delta x$.

Using the fact that real observers move with the velocities $v$ which are much less than the light velocity $c$, one can significantly diminish the influence of inaccuracies of the observers' coordinates on the synchronization accuracy. Let the signal receiving at the moment $\tau_{b1}$ is reflected and returns to the observer $a$ at the moment $\tau_{a2}$ (Fig. 2).

Integrating the equation (2.1) and using the values $\tau_{a0}, t_0, \tau_{a2}$, one can calculate the moment $t_2$ of coordinate time corresponding to the reading $\tau_{a2}$ of the clock $a$:

$$\int_{t_0}^{t_2} \dot{\tau}_a(t, \dot{\underline{x}}_a(t), \underline{x}_a(t))\, dt = \tau_{a2} - \tau_{a0}. \tag{2.4}$$

The moment $t_1$ to be computed is defined by the following set of equations:

$$\begin{aligned} t_1 &= t_0 + \frac{1}{2}(t_2 - t_0) + \delta + \delta_{gr}, \\ \delta &= \frac{1}{2c}(R_1 - R_2), \\ \delta_{gr} &= \frac{1}{2c}\Big(\Delta(\underline{x}_a(t_0), \underline{x}_b(t_1))) - \Delta(\underline{x}_b(t_1), \underline{x}_a(t_2)))\Big), \\ \underline{R}_1 &= \underline{x}_b(t_1) - \underline{x}_a(t_0), \ \underline{R}_2 = \underline{x}_b(t_1) - \underline{x}_a(t_2). \end{aligned} \tag{2.5}$$

In the flat space-time of the special relativity theory the correction $\delta_{gr}$ becomes equal to zero. The correction $\delta$ results from the difference of the path length $R_1$ and $R_2$ corresponding to the light propagation from the observer $a$ to the observer $b$ and back, which is caused by the motion of the observer $a$ relative to the chosen RS. It should be noted that writing down the formula describing the coordinate time interval needed for a photon to propagate from one fixed point to another in the form (2.2), we implicitly impose certain limitations on the employed reference system. Using metric language these limitations may be expressed by the fact that non-Galilean part of the metric $h_{\alpha\beta} = g_{\alpha\beta} - \eta_{\alpha\beta}$ ($\eta_{00} = 1, \eta_{0i} = 0, \eta_{ij} = -\delta_{ij}$ being Minkowski flat space-time metric) is caused only by the gravitational fields of the bodies $A, B, \ldots$ and vanishes when gravitational field is absent. By other words, if we neglect the influence of gravitational field our RS must turn into an inertial RS of the special relativity theory. If such limitations are undesirable for some reasons, in right-hand side of (2.2) one must add the term $\epsilon\left(\underline{x}_a(t_0), \underline{x}_b(t_1)\right)$ reflecting non-gravitational anisotropy of the light velocity in non-inertial RS. In rotating RS such anisotropy leads to the effects similar to the Sagnac effect [Post, 1967; Ashtekar, Magnon, 1975; Saburi, 1976; Landau, Lifshitz, 1975]. In the present paper we will consider only such reference systems which satisfy the limitation in question.

To compute the corrections $\delta$ and $\delta_{gr}$ in (2.5) it is necessary to know the coordinates of the observer $b$ at the moment of signal reflection $\underline{x}_b(t_1)$. Therefore, the moment $t_1$ must be computed by successive iterations:

$$\begin{aligned} t_1^{(0)} &= \frac{1}{2}(t_0 + t_2), \\ t_1^{(n)} &= \frac{1}{2}(t_0 + t_2) + \delta^{(n-1)} + \delta_{gr}^{(n-1)}, \\ n &= 1, 2, \ldots, \end{aligned} \tag{2.6}$$

when computing corrections $\delta^{(i)}$ and $\delta_{gr}^{(i)}$ coordinates of the observer $b$ being evaluated at the moment $t_1^{(i)}$.

In the case when the velocity of an observer is small relative to the light velocity, the relation $|R_1 - R_2| \ll R_1$ becomes true and the correction $\delta$ may be expressed in the form:

$$\begin{aligned} \delta &= \frac{1}{2c}\Big(\underline{n}\,\Delta\underline{x}_a - \frac{1}{2R_1}|\underline{n}\times\Delta\underline{x}_a|^2 - \\ &\quad - \frac{1}{2R_1^2}(\underline{n}\,\Delta\underline{x}_a)|\underline{n}\times\Delta\underline{x}_a|^2 + O(|\Delta\underline{x}_a|^4 R_1^{-3})\Big), \\ \underline{n} &= \underline{R}_1/\,R_1, \\ \Delta\underline{x}_a &= \underline{x}_a(t_2) - \underline{x}_a(t_0) = \dot{\underline{x}}_a(t_0)(t_2 - t_0) + \frac{1}{2}\ddot{\underline{x}}_a(t_0)(t_2 - t_0)^2 + \ldots \end{aligned} \tag{2.7}$$

Thus, the errors of synchronization are caused mostly by the inaccuracies of $\Delta\underline{x}_a$.

So called method of transported (or portable) clock is another method of synchronization (Fig. 3). Let we have another one clock $c$. At the beginning the positions of the clocks $a$ and $c$ coincide. At some moment $t_0$ of coordinate time we

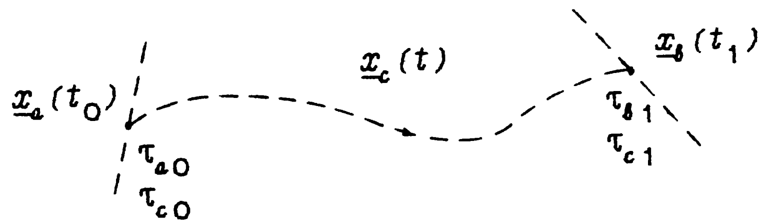


Fig. 3. Clock-transport synchronization.

compare clocks $a$ and $c$, that is we mark their readings $\tau_{a0}$ and $\tau_{c0}$ at this moment. Then clock $c$ is transported along known trajectory $\underline{x}_c(t)$. After that we can compare clock $b$ with clock $c$. Their reading at the moment of comparison is $\tau_{c1}$ and $\tau_{b1}$. The coordinate time $t_1$ corresponding to this moment may be computed on the basis of known values $\tau_{c0}$ and $\tau_{c1}$, trajectory $\underline{x}_c(t)$, and moment $t_0$. Inverting the equation

$$\int_{t_0}^{t_1} \dot{\tau}_c(t, \underline{\dot{x}}_c(t), \underline{x}_c(t))\, dt = \tau_{c1} - \tau_{c0} \tag{2.8}$$

we get the value $t_1$ and, thereby, we find initial value $\tau_b(t_1) = \tau_{b1}$ enabling one to perform the transformation from the proper time scale of the observer $b$ to the coordinate time scale $t$.

In spite of their simplicity, the considered algorithms of synchronization contain all principal, from the view point of relativity theory, aspects of practically employed methods of earth-based clock synchronization.

## 3. Geocentric Reference System: Light Propagation, Relation between Coordinate Time and Proper Time of an Observer

### 3.1. Metric Tensor

According to the basic principle of GRT, an analysis of any process and phenomenon occurring in the neighborhood of the Earth may be performed in any RS. In just the same way, coordinate time of any RS may be considered as the coordinate time scale. Besides the solving of synchronization problem, RS is employed to analyze the motion of bodies and to describe the procedures of observations. The choice of RS is governed mostly by the considerations of convenience.

The widely-accepted versions of a barycentric reference system (BRS) [Fock, 1959; Landau, Lifshitz, 1975; Will, 1981] turn out to be convenient for the analysis of the planets' motion and the light propagation inside the Solar system and outside its boundaries. BRS is valid in the whole Solar system and its neighborhood up to the boundary of the Solar system's near zone, which dimensions are limited by the minimal wavelength of the gravitational waves being emitted by the Solar system. BRS is the relativistic generalization of Newtonian inertial RS, whose origin coincides with the barycenter of the Solar system. Synchronization with respect to the coordinate time of BRS is the only way to establish self-consistent global time scale in the whole Solar system.

Many branches of science which need self-consistent time scale (geodynamics, navigation and so on) deal with the processes bounded in space by the neighborhood of the Earth (say, by geostationary orbit). In the same time, BRS is inconvenient when analyzing processes and phenomenons being located inside the compact subsystem, i.e. inside space domain whose dimensions are small with respect to the distance to the nearest body not contained in the domain involved. Utilization of BRS for an analysis of physical phenomenons being located in the neighborhood of the Earth (the motion of an earth satellite, light propagation etc.) results in some practical difficulties.

Let us suppose that when analyzing observations of an earth satellite on the basis of Newtonian physics we have errors of the order of $\mathcal{A}$. Slightly idealizing the situation we consider that the errors are caused mostly by relativistic effects. If we construct relativistic model of these observations using BRS, the differences between relativistic formulas describing the motion of the satellite, the light propagation etc., and their Newtonian analogs are significantly (by an order of magnitude) greater than $\mathcal{A}$. When calculating immediately observable quantities these large corrections compensate each other to a marked degree and final expressions contain the terms of order of $\mathcal{A}$. Inconveniences are obvious. In order to overcome this difficulty one must construct in the neighborhood of the Earth local inertial RS, in which gravitational field of external bodies can be represented in the form of tidal terms only. Different approaches to the construction of such RS have been developed by a number of authors [Ashby, Bertotti, 1986; Fukushima et al., 1986b; Brumberg, Kopejkin, 1989; Voinov, 1990]. It is RS of this kind that has been used in [Ashby, Allan, 1979; Allan, Ashby, 1986] to describe the algorithms of synchronization.

In this paper we follow the approach which has been developed in [Brumberg, Kopejkin, 1989; Kopejkin, 1988]. The authors has constructed local inertial geocentric RS (GRS) in garmonic gauge. Let us denote coordinate time and spatial coordinates of this RS by $u$ and $w^i$ respectively. Its metrics is defined by the formulas

$$
\begin{aligned}
g_{00}(u,\underline{w}) &= 1 - \frac{2}{c^2}(U_E + U_B) + O(c^{-4}), \\
g_{0i}(u,\underline{w}) &= \frac{4}{c^3}(U_E^i + O(w^2)) + O(c^{-5}), \\
g_{ij}(u,\underline{w}) &= -\delta_{ij}(1 + \frac{2}{c^2}(U_E + U_B)) + O(c^{-4}),
\end{aligned} \tag{3.1}
$$

$$
U_E = \frac{GM_E}{w} + \frac{1}{2}\frac{GI_{ij}^E}{w^3}(-\delta_{ij} + 3\frac{w^i w^j}{w^2}) + \ldots,
$$

$$
\begin{aligned}
U_E^i &= \frac{1}{2} G \epsilon^i{}_{jk} S_E^j \frac{w^k}{w^3} + \ldots, \\
U_B &= \frac{3}{2} \sum_{A \neq E} \Big[ Q_{ij}^A w^i w^j + O(GM_A \frac{w^3}{r_{EA}^4}) \Big], \\
Q_{ij}^A &= \frac{GM_A}{r_{EA}^3} \Big( \frac{r_{EA}^i r_{EA}^j}{r_{EA}^2} - \frac{1}{3}\delta_{ij} \Big)
\end{aligned} \tag{3.2}
$$

where $M_A$ is the mass of the body $A$; $\underline{r}_{EA} = \underline{x}_E - \underline{x}_A$ is the difference between the barycentric positions of the centers of mass of the Earth and the body $A$; $\epsilon^i{}_{jk}$ is the fully antisymmetric Levi-Civita symbol, $\epsilon^1{}_{23} = +1$; $\underline{S}_E$ is the angular momentum vector of the Earth (spin)

$$
\underline{S}_E \approx I^E \underline{\omega}_E, \tag{3.3}
$$

$\underline{\omega}_E$ is the angular velocity of the Earth; $I_E$ is its moment of inertia with respect to the rotation axis; $I_{ij}^E$ is the Earth's quadrupole moments defined by

$$
I_{ij}^E = \int_E \rho_E w^i w^j \, d^3x, \tag{3.4}
$$

$\rho_E$ being the mass density and the integration being performed over all the volume of the body $A$. In the metric (3.1)–(3.2) terms of higher orders of magnitude are omitted.

### 3.2. Light Propagation

The light propagation in GRS has been investigated in [Voinov, 1990; Klioner, 1990a,b]. For the problem of synchronization the following formula defining the coordinate time for a photon to propagate from one fixed point $\underline{w}_0$ to another $\underline{w}$ is important

$$
\begin{aligned}
c(u - u_0) &= |\underline{w} - \underline{w}_0| + \Delta, \\
\Delta &= \Delta_E + \Delta_B
\end{aligned} \tag{3.5}
$$

where $\Delta_E$ and $\Delta_B$ are relativistic time delay caused by the gravitational fields of the Earth and external bodies respectively. These quantities are defined by:

$$
\begin{aligned}
\Delta_E &= \frac{2GM_E}{c^2} \ln \frac{w + w_0 + |\underline{w} - \underline{w}_0|}{w + w_0 - |\underline{w} - \underline{w}_0|} + \ldots, \\
\Delta_B &= \sum_{A \neq E} \Big\{ \frac{3}{c} Q_{ij}^A w_0^i w_0^j (u - u_0) + 3\, Q_{ij}^A w_0^i \mu^j (u - u_0)^2 + \\
&\quad + c\, Q_{ij}^A \mu^i \mu^j (u - u_0)^3 \Big\} + \ldots .
\end{aligned} \tag{3.6}
$$

Here $\underline{\mu}$ is the unit direction of propagation of a photon at the moment $u_0$ ( $\underline{\mu} = (\underline{w} - \underline{w}_0)/|\underline{w} - \underline{w}_0| + O(c^{-2})$ ). In relativistic terms $u - u_0$ is equal to $|\underline{w} - \underline{w}_0|/c$. The value of $Q^A_{ij}$ must be calculated at the moment $u_0$.

In the expression for $\Delta_E$ terms caused by the non-sphericity of the Earth and its rotation are omitted. The influence of these factors on the light propagation has been investigated in [Richter, Matzner, 1981; 1982a,b; 1983; Klioner, 1989b]. The results of the papers allow us to conclude that the total effect of the factors in question on $\Delta_E$ is less than 0.05 ps, the effect of the Earth's rotation being by 3 orders of magnitude smaller than the effect of its non-sphericity. This value is much smaller than the present and anticipated accuracy of observations.

If the trajectory of a photon is bounded by the geocentric sphere of the radius of 50000 km, the values of relativistic time delay may be estimated as $|\frac{1}{c}\,\Delta_E| \leq 163$ps, $|\frac{1}{c}\,\Delta_B| \leq 0.01$ps. The only significant effects in $\Delta_B$ are due to the Sun and Moon. Thus, with sufficient accuracy one can suppose that in (3.5) $\Delta = \Delta_E$.

### 3.3. Proper Time and Coordinate Time

The differential relation between proper time $\tau$ of an observer moving along the trajectory $\underline{w}(u)$ and the coordinate time $u$ of GRS has the form

$$
\begin{aligned}
\dot{\tau} &= 1 + f(\underline{w}, \dot{\underline{w}}, u), \\
f(\underline{w}, \dot{\underline{w}}, u) &= -\frac{1}{c^2}\left(\frac{1}{2}\dot{\underline{w}}^2(u) + U_E(u, \underline{w}(u)) + \right. \\
&\left. + U_B(u, \underline{w}(u))\right) + O(c^{-4}),
\end{aligned}
\tag{3.7}
$$

a dot denoting the derivative with respect to $u$: $\dot{\tau} = \frac{d\tau}{du}$.

Let us consider this formula in details. The influence of the Earth's monopole field on $\dot{\tau}$ is maximal at the surface of the Earth (here it has the value $7 \cdot 10^{-10}$) and decrease proportional to $1/w$. The influence of the observer's geocentric velocity may have approximately the same maximal value. For a clock, which is at rest relative to the Earth's surface, this quantity is much smaller $-\frac{1}{2}\,\dot{\underline{w}}^2/c^2 \leq 1.2 \cdot 10^{-12}$. Near the Earth's surface there is significant effect due to quadrupole gravitational field of the Earth:

$$
\begin{aligned}
\dot{\tau}_Q &= -\frac{1}{2c^2}\frac{GI^E_{ij}}{w^3}\left(-\delta_{ij} + 3\frac{w^j w^j}{w^2}\right) \approx \\
&\approx \frac{1}{2}\frac{GM_E}{c^2}\frac{P_E^2}{w^3}J_2^E(3\sin^2\varphi - 1) \leq 8 \cdot 10^{-13}
\end{aligned}
\tag{3.8}
$$

where $P_E$ is the equatorial radius of the Earth, $J_2^E$ is the coefficient of second zonal garmonic of its gravitational field, $\varphi$ is the geographical latitude of the point of observation. The effect of harmonics of higher orders is by approximately 3 orders of magnitude smaller and equal to $10^{-15} - 10^{-16}$.

The influence of external bodies on the rate of the clock of an Earth-based observer can be divided naturally into two components: direct influence, manifesting itself in varying of the potential of external bodies at the point of observation (first of all, due to diurnal rotation of the Earth), and indirect influence, which consists in tidal deformations of the Earth's body and its gravitational field and results in varying of the potential of the Earth itself. Thus, the total effect of the potential of external bodies is

$$\begin{aligned}\dot{\tau}_B &= -\frac{1}{2}(1 + k_2 - h_2) \times \\ &\times \sum_{A \neq E} \Big\{ \frac{GM_A}{c^2} \frac{w^2}{r_{EA}^3}(3\cos^2 \alpha_A(u) - 1) + O\Big(\frac{GM_A}{c^2}\frac{w^3}{r_{EA}^4}\Big)\Big\}. \end{aligned} \tag{3.9}$$

Here $\alpha_A(u)$ is the angle between the vectors $\underline{w}$ and $\underline{r}_{EA}$, $k_2$ and $h_2$ are the Love numbers characterizing the reaction of the Earth's body on the tidal potential of external bodies. For the most models of internal structure of the Earth $1 + k_2 - h_2 \approx 0.7$. The dependence of $\alpha_A$ on time is caused, first of all, by diurnal rotation of the Earth. Each term in (3.9) can be divided into a secular term and two periodic ones – diurnal and semi-diurnal:

$$\begin{aligned}\dot{\tau}_B &= -(1 + k_2 - h_2) \sum_{A \neq E} \frac{GM_A}{c^2} \frac{w^2}{r_{EA}^3} \times \\ &\times \Big[\frac{1}{4}(3\sin^2 \delta_A - 1)(3\sin^2 \varphi - 1) + \\ &+ \frac{3}{4}\cos^2 \delta_A \cos^2 \varphi \cos 2\omega_E(u - u_A) + \\ &+ \frac{3}{4}\sin 2\delta_A \sin 2\varphi \cos \omega_E(u - u_A)\Big], \end{aligned} \tag{3.10}$$

$\delta_A$ being declination of the body $A$, $u_A$ being the moment of upper culmination of the body $A$ at the place of observation. Maximal value of the secular term in (3.10) is less than $2 \cdot 10^{-17}$. Integrating $\dot{\tau}_B$ with respect to $u$, one can find that maximal difference between the scales $\tau$ and $u$ is 0.3 ps and 0.2 ps for the diurnal and semi-diurnal terms respectively. Actually, because of the mutual motion of the Solar system bodies, $\delta_A$ and $u_A$ depend on time as well. This dependence results in the appearance of the complicated long-periodical effects whose investigation is out the scope of our paper.

### 3.4. Units of Measurement

It is convenient for practical use of the coordinate time scale $u$ that its rate be equal to the rate of the proper time of an observer on the geoid. Here, by the geoid we mean the relativistic $u$-geoid [Bjerhammar, 1985; Soffel et al., 1988; Kopejkin, 1991] – two-dimensional surface close to the mean sea level, in any point of which the rate of the proper time of an observer, who is at rest relative to the Earth's surface, with respect to the geocentric time $u$ is constant:

$$\frac{d\tau}{du} = k_T = \text{const.} \tag{3.11}$$

This definition is one of the possible relativistic generalizations of the Newtonian definition of the geoid. From the metric of GRS it is easy to see that the equation (3.11) is equivalent to

$$\begin{aligned} W(\underline{w}, \dot{\underline{w}}, u) &\equiv -c^2 f(\underline{w}, \dot{\underline{w}}, u) = \\ &= \frac{1}{2}\dot{\underline{w}}^2(u) + U_E(\underline{w}, u) + \sum_{A=S,L} \frac{3}{2} Q^A_{ij} w^i w^j + \ldots + O(c^{-2}) = \\ &= W_0 = c^2(1 - k_T). \end{aligned} \tag{3.12}$$

Here it is to be kept in mind that for the points of the Earth's surface $\dot{\underline{w}} = \underline{\omega}_E \times \underline{w}$.

From the theoretical point of view, the terms explicitly written in (3.12) define the surface of $u$-geoid with the accuracy of 1 cm. The effect of missed relativistic terms, which are $O(c^{-2})$, is less than 0.5 cm. Quadrupole tidal field $Q^A_{ij}$ must be taken into account for the Moon ($\sim$ 35 cm) and the Sun ($\sim$ 16 cm) only. Octupole tidal field $Q^A_{ijk}$ [Brumberg, Kopejkin, 1989] being neglected in (3.12) gives significant effect only for the Moon ($\sim$ 0.6 cm). The equation of $u$-geoid (3.12), corresponding to the Newtonian approximation of GRT, coincides with the generally accepted Newtonian definition of the geoid.

Putting $W_0 = 6.263686 \cdot 10^7$ (this constant is part of the system of geophysical constants [Fukushima, 1990]), one obtains

$$k_T = 1 - 6.969290 \cdot 10^{-10}. \tag{3.13}$$

Let us introduce new variables $\tilde{u}$ and $\tilde{\underline{w}}$:

$$\begin{aligned} \tilde{u} &= k_T u, \\ \tilde{\underline{w}} &= k_S \underline{w}. \end{aligned} \tag{3.14}$$

Then for an observer on the geoid

$$\frac{d\tau}{d\tilde{u}} = 1. \tag{3.15}$$

The transformation (3.14) is to be considered as re-definition of the units of measurement of time and length [Fukushima et al., 1986a; Brumberg, Kopejkin, 1990]. One can believe that in (3.14) $u$ and $\underline{w}$ are the numerical values of the coordinates of an event, expressed in the SI units [s] and [m], while $\tilde{u}$ and $\underline{\tilde{w}}$ are the numerical values of the coordinates of the same event, expressed in the units $[s_G]$ and $[m_G]$, which are defined by

$$
\begin{aligned}
&[c_G] = [c]/k_T, \\
&[m_G] = [m]/k_S. \qquad (3.16)
\end{aligned}
$$

According to the definition of the SI units, $1[s_G]$ is the duration of $9192631770/k_T$ periods of radiation corresponding to the transition between two hyperfine levels of the ground state of the caesium 133 atom; $1[m_G]$ is the length of the trajectory passed by the light in vacuum during the time $1/299792458\ k_T/k_S\ [s_G]$. An observer in his practice uses the SI units.

The masses of the bodies and the light velocity expressed in the units $[s_G]$ and $[m_G]$ are connected with the usual SI values by

$$
\begin{aligned}
\widetilde{GM}_A &= k_S^3/k_T^2\, GM_A, \\
\tilde{c} &= k_S/k_T\, c. \qquad (3.17)
\end{aligned}
$$

The choice of the constant $k_S$ is governed mostly by the considerations of convenience. In our opinion, the most acceptable values are the following:

1. $k_S = 1$:
   The unit of length coincides with the SI one; $\widetilde{GM}_A = GM_A/k_T^2$, $\tilde{c} = c/k_T$; This choice of $k_S$ has been considered in [Fukushima et al., 1986a; Kopejkin, 1989].
2. $k_S = k_T^{2/3}$:
   The values of the bodies' masses remain unchanged; $\widetilde{GM}_A = GM_A$, $\tilde{c} = k_T^{-1/3}\, c$.
3. $k_S = k_T$:
   The value of the light velocity does not change; $\widetilde{GM}_A = k_T\, GM_A, \tilde{c} = c$; This appears to be the most appropriate value of $k_S$. This value has been recommended in [Fukushima et al., 1986a; Brumberg, Kopejkin, 1990].

In the new variables $(\tilde{u}, \underline{\tilde{w}})$ the formulas (3.5)–(3.6) have the same functional form as in the variables $(u, \underline{w})$:

$$
\tilde{c}(\tilde{u} - \tilde{u}_0) = |\underline{\tilde{w}} - \underline{\tilde{w}}_0| + \frac{2\widetilde{GM}_E}{\tilde{c}^2} \ln \frac{\tilde{w} + \tilde{w}_0 + |\underline{\tilde{w}} - \underline{\tilde{w}}_0|}{\tilde{w} + \tilde{w}_0 - |\underline{\tilde{w}} - \underline{\tilde{w}}_0|} + \ldots. \qquad (3.18)
$$

Finally, let us obtain the appropriate expression defining the relation between the GRS coordinate time and the proper time of an observer, which is situated at the height $h$ above the geoid and moves with respect to the Earth's surface having the velocity $\underline{\nu}$. According to the definition of the geoid, we have

$$\begin{aligned}
W(\underline{w}_0(1 \, + \, \frac{h}{w_0}),\, \underline{\omega}_E \times \underline{w}_0(1 + \frac{h}{w_0}) + \underline{\nu},\, u_0) &= W(\underline{w}_0,\, \underline{\omega}_E \times \underline{w}_0,\, u_0) + \\
&+ \frac{\partial W}{\partial w^i}(\underline{w}_0,\, \underline{\omega}_E \times \underline{w}_0,\, u_0)\frac{w_0^i}{w_0}h + \\
&+ \frac{1}{2}\frac{\partial^2 W}{\partial w^i \partial w^j}(\underline{w}_0,\, \underline{\omega}_E \times \underline{w}_0,\, u_0)\frac{w_0^i w_0^j}{w_0^2}h^2 + \\
&+ \frac{1}{2}\underline{\nu}^2 + (\underline{\omega}_E \times \underline{w}_0)\underline{\nu} + O(h^3) = \\
&= W_0 - g(\underline{w}_0,\, u_0)h + \frac{1}{2}n(\underline{w}_0,\, u_0)h^2 + \frac{1}{2}\underline{\nu}^2 + (\underline{\omega}_E \times \underline{w}_0)\underline{\nu} + O(h^3)
\end{aligned} \tag{3.19}$$

where $\underline{w}_0$ is the vector, whose absolute value is equal to the height of the geoid in the direction of $\underline{w}_0/w_0$ at the moment $u_0$; $g = g(\underline{w}_0, u_0) \approx g(\varphi)$ is the gravity acceleration at the point of intersection of the observer's radius-vector with the geoid, $n(\underline{w}_0, u_0) \approx$ const being its vertical gradient:

$$\begin{aligned}
g &= 9.78033(1 + 0.00530 \sin^2\varphi) + \ldots \ \mathrm{m/s^2}, \\
n &= 3.086 \cdot 10^{-6} + \ldots \ \mathrm{s^{-2}}.
\end{aligned} \tag{3.20}$$

At the right-hand side of (3.19) we have neglected terms of the order of $O(h^3)$, which appear when expanding the left-hand side in Taylor series. Relative error due to these terms are approximately equal to $2.7 \cdot 10^{-21}\,(h/\,1\ \mathrm{km})^3$. Thus, given the error $\epsilon$ one can use (3.19), if $|h| \le 33(\epsilon/10^{-16})^{1/3}$km. If we neglect in (3.19) the term proportional to $h^2$, the limitation on maximal height above the geoid have the form $|h| \le 2.4(\epsilon/10^{-16})^{1/2}$km. This is not sufficient obviously when transporting a high accuracy clock (if $h = 12$ km, the error becomes equal to $2 \cdot 10^{-15}$).

Accounting for the numerical values of the constants involved one can rewrite (3.19) as

$$\begin{aligned}
\frac{d\tau}{d\tilde{u}} &= 1 + \frac{g}{c^2}h - \frac{n}{2c^2}h^2 - \frac{1}{2c^2}\nu^2 - \frac{1}{c^2}(\underline{\omega}_E \times \underline{w}_0)\underline{\nu} \approx \\
&\approx 1 + 1.08821 \cdot 10^{-16}h + 5.77 \cdot 10^{-19}h \sin^2\varphi - \\
&\quad -1.716 \cdot 10^{-23}h^2 - 5.563 \cdot 10^{-18}\nu^2 - \\
&\quad -5.1750 \cdot 10^{-15}\nu \cos\varphi \cos\theta + \\
&\quad +1.74 \cdot 10^{-17}\nu \cos\varphi \sin^2\varphi \cos\theta + \ldots,
\end{aligned} \tag{3.21}$$

$\varphi$ being the latitude of the point of observation, $\theta$ being the course angle (the angle between the direction of the observer's motion and the direction of the east). The values of $h$ and $\nu$ are to be expressed in m and m/s respectively.

The formula (3.21) has the accuracy $10^{-16}$ provided that $h \leq 15$ km and $\nu \leq 300$ m/s. For a clock which is at rest relative to the Earth's surface, the relation between $\tau$ and $\tilde{u}$ is defined by the first four terms in (3.21).

In the next section we will suppose that all quantities relating to GRS are measured in $[\mathrm{s_G}]$ and $[\mathrm{m_G}]$ and we will miss out the tilde over $\tilde{u}$, $\tilde{\underline{w}}$, $\widetilde{GM_A}$ and $\tilde{c}$.

## 4. Algorithms of Earth-Based Clock Synchronization

In the present section we will consider some methods of synchronization, which are supposed to be used for autonomous synchronization of the clocks at the stations of radiointerferometric networks.

### 4.1. Transported Clock

The problem is to find the value of $t_1$ from the integral (2.8). By the coordinate time $t$ we mean the GRS time scale $u$. Inverting (3.21), one gets

$$\begin{aligned} u_1 - u_0 = \; & \tau_1 - \tau_0 - 1.08821 \cdot 10^{-16} \int_{\tau_0}^{\tau_1} h(\tau)\, d\tau - \\ & -5.77 \cdot 10^{-19} \int_{\tau_0}^{\tau_1} h(\tau) \sin^2 \varphi(\tau)\, d\tau + \\ & +1.716 \cdot 10^{-23} \int_{\tau_0}^{\tau_1} h^2(\tau)\, d\tau + 5.563 \cdot 10^{-18} \int_{\tau_0}^{\tau_1} \nu^2(\tau)\, d\tau + \\ & +5.1750 \cdot 10^{-15} \int_{\tau_0}^{\tau_1} \nu(\tau) \cos\varphi(\tau)\, \cos\theta(\tau)\, d\tau - \\ & -1.74 \cdot 10^{-17} \int_{\tau_0}^{\tau_1} \nu(\tau) \cos\varphi(\tau)\, \sin^2\varphi(\tau)\, \cos\theta(\tau)\, d\tau. \end{aligned} \tag{4.1}$$

The functions $h(\tau)$, $\varphi(\tau)$, $\nu(\tau)$, $\theta(\tau)$ are defined by the trajectory of transported clock. Permissible uncertainties of these functions strongly depend on the particular conditions and duration of clock transport.

Let us consider an example. The clock is transported by jet airplane during 8 hours with the velocity $\nu = 300$ m/s and at the height $h = 12$ km. The required accuracy of determination of $u_1$, that is the accuracy of synchronization to be attained, is supposed to be 1 ns. We consider that the transported clock is accurate enough (in the case under consideration its accuracy should not be worse than $2 \cdot 10^{-14}$). It is easy to prove that under these conditions the above-mentioned functions are to be known with the mean uncertainties

$$\begin{aligned} \sigma_h &\leq 100\,\mathrm{m}, \\ \sigma_\nu &\leq 4\,\mathrm{m/s}, \\ \sigma_\varphi &\leq 10', \\ \sigma_\theta &\leq 10', \end{aligned} \tag{4.2}$$

If the accuracy of synchronization to be attained be 0.1 ns, permissible uncertainties decrease by an order of magnitude.

The relativistic terms, in the order they are written in (4.1), have the values: 37.6 ns, 0.2 ns, 0.07 ns, 14.4 ns, 44.7 ns, 0.06 ns (for the terms which depend on $\varphi$ and $\theta$ the maximal values are indicated). The total effect may amount to 67.9 ns.

In order to compute $u_1$ with given level of accuracy $\sigma_u$, the uncertainties of $h$, $\nu$, $\varphi$ and $\theta$ must satisfy an inequality of the form $a^2\sigma_h^2 + b^2\sigma_\nu^2 + c^2\sigma_\varphi^2 + d^2\sigma_\theta^2 \leq \sigma_u^2$, $a$, $b$, $c$, and$d$ being some numerical coefficients. The expressions (4.2) are one of the possible combinations of the values which satisfy the inequality involved. For example, we can choose $\sigma_h \leq 200\,\mathrm{m}$, $\sigma_\nu \leq 2.5\,\mathrm{m/s}$ and so on. Nevertheless, the expressions (4.2) give correct information about the values of permissible uncertainties. In the following, when considering other methods of synchronization we will indicate partial solutions of analogous inequalities. It should be understood that this is exactly partial solutions.

### 4.2. Duplex Link via Satellite

When synchronizing Earth-based clocks situated far enough from each other, direct exchange of electromagnetic signals (Fig. 2) becomes impossible. In this case one must resort to the help of a relay station. Artificial earth satellites or meteor tracks – the regions of ionized air appearing when a meteor passes through the Earth's atmosphere – can be utilized as a relay station. There are many technical methods of synchronization with the aid of earth satellites. Here we will consider the method which utilize duplex communication link via satellite. High cost of duplex satellite link hinder this method from wide spreading in practice. Nevertheless, the method in question is one of the most precise methods of synchronization.

The scheme of the method is shown in Fig. 4. At the time when the reading of the clock $a$ is $\tau_{a0}$ (this moment corresponds to the moment $u_0$ of the coordinate time) the station $a$ emits a signal, which arrives at the satellite at the moment $u_1$ and then comes to the station $b$ at the moment when the reading of its clock is $\tau_{b2}$ (the coordinate time $u_2$). The station $b$ emits the signal at the time $\tau_{b0}$ ($u_0 + \Delta u$ in the coordinate scale). This signal reaches the satellite at the moment $u_3$ and comes to the station $a$ at the moment $\tau_{a4}$ ($u_4$). We suppose that during the observation the coordinates of the stations $\underline{w}_a(u)$, $\underline{w}_b(u)$ as well as those of the satellite $\underline{w}_s(u)$ are known. The observable quantities are two intervals of the proper time of the clocks $a$ and $b$: $\tau_a = \tau_{a4} - \tau_{a0}$ and $\tau_b = \tau_{b2} - \tau_{b0}$ . It is assumed herewith that the

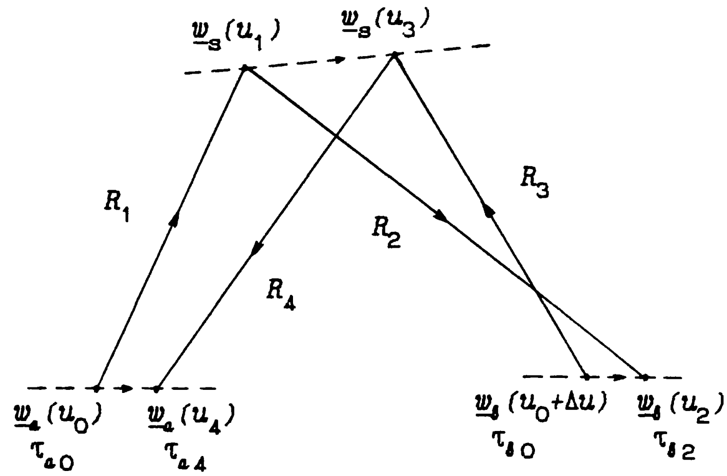


Fig. 4. Duplex link synchronization.

clocks have been initially synchronized to the accuracy of a few tenth of second. Thus, both $\tau_a$ and $\tau_b$ are non-negative. The quantity to be determined is coordinate desynchronization $\Delta u$ of the clocks. Generally speaking, one can employ any satellite. In the following estimations we shall suppose that the satellite in use is geostationary one with arbitrary inclination of the orbit.

Using the formula (4.1) the observables $\tau_a$ and $\tau_b$ can be easily transformed to the corresponding intervals of the coordinate time. Since $\tau_a$ and $\tau_b$ are less than 0.56 s (the time needed for a photon to propagate from one station to another and back), we have

$$\begin{aligned} u_a &= u_4 - u_0 = \tau_a, \\ u_b &= u_2 - u_0 - \Delta u = \tau_b. \end{aligned} \tag{4.3}$$

Here we neglect the quantities of the order of 1 ps. In the following we shall suppose that the atmospheric delay and the delay due to the equipment are properly accounted for (see [Gubanov et al., 1989]).

We can write the following set of equations:

$$\begin{aligned} c(u_1 - u_0) &= R_1 + \Delta_1, & \underline{R}_1 &= \underline{w}_s(u_1) - \underline{w}_a(u_0), \\ c(u_2 - u_1) &= R_2 + \Delta_2, & \underline{R}_2 &= \underline{w}_s(u_1) - \underline{w}_b(u_2), \\ c(u_3 - u_0 - \Delta u) &= R_3 + \Delta_3, & \underline{R}_3 &= \underline{w}_s(u_3) - \underline{w}_b(u_0 + \Delta u), \\ c(u_4 - u_3) &= R_4 + \Delta_4, & \underline{R}_4 &= \underline{w}_s(u_3) - \underline{w}_a(u_4), \end{aligned} \tag{4.4}$$

$\Delta_i$ being gravitational time delay along the path $\underline{R}_i$. Adding the first equation to the second one, then the third equation to the fourth one, and subtracting the results, we can find the expression defining desynchronization to be determined

$$\begin{aligned}
\Delta u &= \frac{1}{2}(u_a - u_b) + \delta + \delta_{gr}, \\
\delta &= \frac{1}{2c}(R_1 + R_2 - R_3 - R_4), \\
\delta_{gr} &= \frac{1}{2c}(\Delta_1 + \Delta_2 - \Delta_3 - \Delta_4).
\end{aligned} \tag{4.5}$$

Although every $\Delta_i$ may amount to 77 ps, the correction $\delta_{gr}$ caused by the gravitational field of the Earth does not exceed 1 ps and can be neglected.

We denote

$$\begin{aligned}
\Delta \underline{w}_s &= \underline{w}_s(u_3) - \underline{w}_s(u_1), \\
\Delta \underline{w}_a &= \underline{w}_a(u_4) - \underline{w}_a(u_0), \\
\Delta \underline{w}_b &= \underline{w}_b(u_2) - \underline{w}_b(u_0 + \Delta u), \\
\underline{n}_a &= \underline{R}_1/R_1,\ \underline{n}_b = \underline{R}_2/R_2.
\end{aligned} \tag{4.6}$$

The values of $|\Delta \underline{w}_a|$ and $|\Delta \underline{w}_b|$ are less than 260 m, and $|\Delta \underline{w}_s| \leq 880$ m. Thus, neglecting terms of the order of 10 ps, we have

$$\delta = \frac{1}{2c}\left(\underline{n}_a \Delta \underline{w}_a - \underline{n}_b \Delta \underline{w}_b - (\underline{n}_a + \underline{n}_b)\,\Delta \underline{w}_s + O(|\Delta \underline{w}|^2/R)\right). \tag{4.7}$$

The vectors of displacement must be computed as

$$\begin{aligned}
\Delta \underline{w}_a &= \underline{\omega}_E \times \underline{w}_a(u_0) u_a + \frac{1}{2}\underline{\omega}_E \times (\underline{\omega}_E \times \underline{w}_a(u_0)) u_a^2, \\
\Delta \underline{w}_b &= \underline{\omega}_E \times \underline{w}_b(u_0 + \Delta u) u_b + \frac{1}{2}\underline{\omega}_E \times (\underline{\omega}_E \times \underline{w}_b(u_0 + \Delta u)) u_b^2, \\
\Delta \underline{w}_s &= \underline{v}_s(u_1) u_s + \frac{1}{2}\underline{a}_s(u_1) u_s^2 = \underline{v}_s(u_1) u_s - \frac{GM_E}{2 w_s^3(u_1)} \underline{w}_s(u_1) u_s^2
\end{aligned} \tag{4.8}$$

where $\underline{v}_s$ and $\underline{a}_s \approx -\frac{1}{2} GM_E w_s^{-3}\ \underline{w}_s$ are the velocity and acceleration of the satellite, $u_s = u_3 - u_1$ is the time interval between the moments of reception of the signals coming from the stations $b$ and $a$. The terms which are quadratic with respect to $u$ may amount to 1 cm in $\Delta \underline{w}_s$, and 0.5 cm in $\Delta \underline{w}_a$ and $\Delta \underline{w}_b$.

Depending on the relative location of the stations and the satellite as well as the initial desynchronization $\Delta u$ of the clocks, the correction $\delta$ may vary from $-430$ ns to +430 ns. It is easy to show that, in order to compute the value of $\delta$ with the accuracy of 0.1 ns, the coordinates of the satellite and stations are to be known with the precision

$$\begin{aligned} \sigma_{ws} &\leq 500\,\mathrm{m}, \\ \sigma_{wa}, \sigma_{wb} &\leq 100\,\mathrm{m}. \end{aligned} \tag{4.9}$$

Let us note that the polar motion can be neglected because its effect on $|\Delta \underline{w}_a|$ and $|\Delta \underline{w}_b|$ is less than 1 mm. The velocity of the satellite $\underline{v}_s$ is to be known with relatively high accuracy:

$$\sigma_{vs} \leq 5\ \mathrm{cm/s}. \tag{4.10}$$

The absence of a clock onboard the satellite does not permit the direct measurements of $u_s$. Nevertheless, this quantity may be computed with sufficient accuracy ($\sim 1\ \mu$s):

$$u_s = \frac{1}{2}(u_a - u_b) + \frac{1}{c}(R_3 - R_1), \tag{4.11}$$

the difference $R_3 - R_1$ being calculated on the basis of coordinates having the accuracy (4.9).

Let us consider the particular case when the satellite moves in the equatorial plane. In this case $\underline{v}_s = \underline{\omega}_E \times \underline{w}_s$ and combining (4.7) and (4.8) we can write

$$\begin{aligned} \delta &= \frac{1}{2c}\underline{\omega}_E \times \underline{w}_s \left\{ \frac{\underline{w}_a}{R_1}(u_s - u_a) + \frac{\underline{w}_b}{R_2}(u_s + u_b) \right\} + \\ &\quad + O(u^2_{a,b,s}) + O(|\Delta w|^2/R) \approx \frac{2\omega_E}{c^2} A, \\ \underline{w}_s &= \underline{w}_s(u_1),\ \underline{w}_a = \underline{w}_a(u_0),\ \underline{w}_b = \underline{w}_b(u_0 + \Delta u) \end{aligned} \tag{4.12}$$

where $A = -\frac{1}{2\omega_E}\underline{\omega}_E \times \underline{w}_s(\underline{w}_a - \underline{w}_b)$ is the area of the quadrangle whose vertexes are the center of mass of the Earth, and the projections of the stations and satellite onto the equatorial plane. The value $A$ is positive when the station $a$ is situated westward with respect to the stations $b$, and negative otherwise. The formula (4.12) have the accuracy of order of 1 ns.

The scheme under consideration (Fig. 4) contains some other variants of the clock synchronization via duplex satellite link (in particular, the modification when the station $b$ reflects the signal coming from the station $a$, and does not emit another signal).

## 4.3. LASSO

The method LASSO (LAser Synchronization from Stationary Orbit) [Ashby, Allan, 1979; Serene, 1981, 1988] is one of the most promising methods of Earth-based clock synchronization. The potential accuracy of this method exceeds 0.1 ns. The

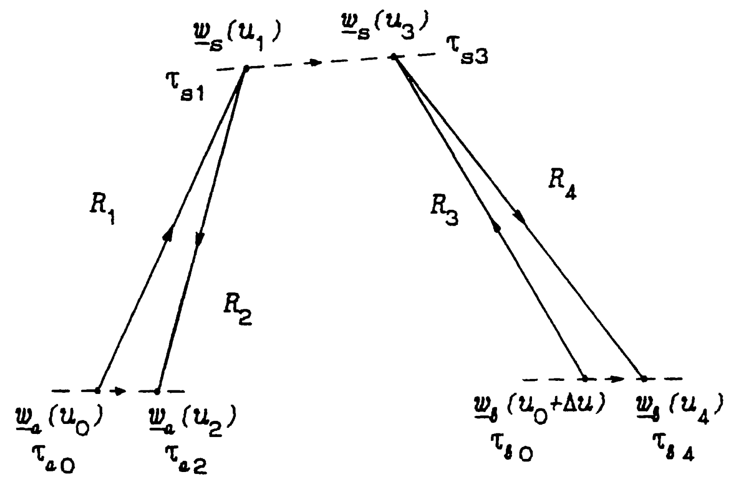


Fig. 5. LASSO.

utilization of laser pulses diminishes the influence of the Earth's atmosphere on the uncertainties of synchronization. The drawbacks of LASSO are obvious – high cost of the equipment and dependence on the weather conditions. The satellite employed in LASSO must have special onboard equipment – laser corner reflectors equipped with the fast photo diodes and precise interval timer.

The scheme of LASSO is shown in Fig. 5. This scheme is quite similar to the scheme of the duplex method considered in the previous subsection. At the moment when the reading of the clock $a$ is $\tau_{a0}$ ($u_0$ of the coordinate time) the station $a$ emits a laser pulse which reaches the satellite at the moment $u_1$ (the reading of the satellite clock is $\tau_{s1}$), makes the onboard timer start, and is reflected backward to the station $a$ where is received at the moment $\tau_{a2}$ ($u_2$ in the coordinate time scale). Just the same procedure is repeated from the station $b$. The pulse coming from the station $b$ stops the onboard timer. The readings of the timer are sent to the observer via radio link. The observable quantities are three intervals of the proper times of the stations $a$ and $b$, and the onboard clock: $\tau_a = \tau_{a2} - \tau_{a0}, \tau_b = \tau_{b2} - \tau_{b0}, \tau_s = \tau_{s3} - \tau_{s1}$. The intervals $\tau_a$ and $\tau_b$ are less than 0.28 s (the geostationary satellites SIRIO-2, METEOSAT-P2 are employed in LASSO [Serene, 1981, 1988]). Therefore, with the sufficient accuracy

$$\begin{aligned} u_a &= u_2 - u_0 = \tau_a, \\ u_b &= u_4 - u_0 - \Delta u = \tau_b. \end{aligned} \tag{4.13}$$

Generally speaking, the interval $\tau_s$ is arbitrary and must be transformed to the interval of coordinate time on the basis of (3.7) and (3.2). However, if $\tau_s \leq 1$ s and the satellite is geostationary, we can write

$$u_s = u_3 - u_1 = (1 - 5.4 \cdot 10^{-10})\tau_s. \tag{4.14}$$

From the expressions which are analogous to (4.4) one can easily obtain the expression for the desynchronization to be determined

$$
\begin{aligned}
\Delta u &= \frac{1}{2}(u_a - u_b) + u_s + \delta + \delta_{gr}, \\
\delta &= \frac{1}{2c}(R_1 - R_2 - R_3 + R_4), \\
\delta_{gr} &= \frac{1}{2c}(\Delta_1 - \Delta_2 - \Delta_3 + \Delta_4)
\end{aligned}
\tag{4.15}
$$

where

$$
\begin{aligned}
\underline{R}_1 &= \underline{w}_s(u_1) - \underline{w}_a(u_0), \\
\underline{R}_2 &= \underline{w}_s(u_1) - \underline{w}_a(u_2), \\
\underline{R}_3 &= \underline{w}_s(u_3) - \underline{w}_b(u_0 + \Delta u), \\
\underline{R}_4 &= \underline{w}_s(u_3) - \underline{w}_b(u_4),
\end{aligned}
\tag{4.16}
$$

$\Delta_i$ being the gravitational time delay along the path $\underline{R}_i$. Similarly to the duplex method, $\delta_{gr} < 1$ ns and can be neglected. The correction $\delta$ may be written as

$$
\begin{aligned}
\delta &= \frac{1}{2c}\left(\underline{n}_a \Delta \underline{w}_a - \underline{n}_b \Delta \underline{w}_b + O(|\Delta \underline{w}|^2/R)\right), \\
\Delta \underline{w}_a &= \underline{w}_a(u_2) - \underline{w}_a(u_0) = \underline{\omega}_E \times \underline{w}_a(u_0) u_a + \ldots, \\
\Delta \underline{w}_b &= \underline{w}_b(u_4) - \underline{w}_b(u_0 + \Delta u) = \underline{\omega}_E \times \underline{w}_b(u_0 + \Delta u) u_b + \ldots, \\
\underline{n}_a &= \underline{R}_1/R_1, \ \underline{n}_b = \underline{R}_3/R_3.
\end{aligned}
\tag{4.17}
$$

In the case of LASSO $|\Delta \underline{w}_a|$ and $|\Delta \underline{w}_b|$ do not exceed 130 m. It is easy to see that in order to compute the value of $\delta$ with the accuracy of 0.1 ns we must know the coordinates of the satellite and stations with the accuracies

$$
\begin{aligned}
\sigma_{ws} &\leq 4600\,\mathrm{m}, \\
\sigma_{wa}, \sigma_{wb} &\leq \ \ 500\,\mathrm{m}.
\end{aligned}
\tag{4.18}
$$

Relativistic correction (4.17) depends on the motion of the satellite only indirectly. Therefore, the velocity of the satellite may not be known with high accuracy.

### 4.4. Synchronization via Meteor-Burst Link

In spite of not so large maximal distance between the clocks to be synchronized ($\leq$ 2300 km) and interrupted character of the link (the existence of a suitable meteor track being unamenable to control is needed), this method is widely employed, because of low cost of the equipment and relatively high accuracy of synchronization [Kascheev, Bondar, 1989]. The method in question does not appear to have been considered from the relativistic point of view. The possible explanation is the following. The equipment already developed enables one to attain the accuracy of the order of 40 ns. In the same time, the relativistic correction in this method may amount to 10 ns only. The high potential accuracy of the method ($\sim$ 1 ns) makes us to analyze consequently the relativistic effects.

There are many technical modifications of the synchronization method via meteor-burst link [Kascheev, Bondar, 1989]. We will consider only one modification, which is quite similar to the synchronization method via duplex satellite link (Fig. 4). The utilization of a meteor track instead of a satellite for the retransmission of a signal is the only difference between the methods. The expressions (4.3)–(4.8) and (4.11) remain to be correct in the case being considered in the present subsection.

The characteristic feature of the meteor-burst link method of the clock synchronization consists in the fact that the location of the meteor track (that is, the location of the relay station) is not known in principle. The location of the retransmission point $\underline{w}_s$ must satisfy two conditions. First, this point must be observable from both two stations $a$ and $b$. Second, its height above the Earth's surface is from 80 km to 105 km (just at these heights the most of meteor tracks which can be employed for meteor-burst link forms). The reflection of radio waves from meteor tracks is subjected to the usual law of reflection known from the geometrical optics. According to this law the meteor track is to be tangent to the ellipse which passes through the point of retransmission and whose focuses coincide with the stations.

The uncertainty of the location of the point $\underline{w}_s$ leads to the impossibility of accurate computing of the relativistic correction $\delta$ being defined by (4.7). The gravitational effect $\delta_{gr}$ turns out to be negligible again. Our aim is to calculate 'averaged' value $\overline{\delta}$ of the relativistic correction over all possible locations of the point $\underline{w}_s$ when synchronizing the clocks situated at two given points $\underline{w}_a$ and $\underline{w}_b$. Besides this, we must point out the maximal deviation of $\overline{\delta}$ from the true value $\delta$. We suppose that the motion of the retransmission point relative to GRS is caused only by the rotation of the Earth. We neglect herewith possible additional displacement of the meteor track with respect to the Earth's surface due to the wind.

It is easy to see that in any case $u_a$ and $u_b$ are less than 16 ms, and $|u_s| \leq$ 8 ms. According to our assumption concerning the character of the motion of the retransmission point, $\underline{v}_s = \underline{\omega}_E \times \underline{w}_s$ and the relativistic correction (4.7) is defined by the formula which coincide formally with (4.12)

$$\delta = \frac{1}{2c}\underline{\omega}_E \times \underline{w}_s(u_1) \left\{ \frac{\underline{w}_a(u_0)}{R_1}(u_s - u_a) + \frac{\underline{w}_b(u_0 + \Delta u)}{R_2}(u_s + u_b) \right\} + \ldots \approx$$
$$\approx -\frac{\omega_E \times \underline{w}_s(u_1)}{c^2} \left( \underline{w}_a(u_0) - \underline{w}_b(u_0 + \Delta u) \right) + \ldots, \tag{4.19}$$

In (4.19) the terms which are less than 10 ps are missed out. Depending on the mutual location of the stations and meteor track, the relativistic correction (4.19) changes from $-12.1$ ns to $+12.1$ ns.

Using the mathematical language we can say that our aim is to find the maximal $\delta_{max}(\underline{\omega}_E, \underline{w}_a, \underline{w}_b)$ and minimal $\delta_{min}(\underline{\omega}_E, \underline{w}_a, \underline{w}_b)$ values of the function (4.19) over the region

$$\begin{aligned} &6458\text{ km} \le |\underline{w}_s| \le 6483\text{ km}, \\ &(\underline{w}_s - \underline{w}_a)\underline{w}_a \ge 0, \\ &(\underline{w}_s - \underline{w}_b)\underline{w}_b \ge 0. \end{aligned} \tag{4.20}$$

The quantities $\underline{\omega}_E, \underline{w}_a, \underline{w}_b$ are considered herewith as constant parameters. Then the 'averaged' value of the relativistic correction is defined as the arithmetic mean: $\overline{\delta} = \frac{1}{2}(\delta_{max} + \delta_{min})$. Maximal deviation of the 'averaged' value from the true one is $\Delta\delta = \max|\delta - \overline{\delta}\,| = \frac{1}{2}(\delta_{max} - \delta_{min})$. Another interpretations of 'averaging' of (4.19) are possible. For example, one can integrate $\delta$ over the region (4.20). It is remarkable that the expression (4.19) may be 'averaged' analytically:

$$\begin{aligned} \overline{\delta}(\underline{\omega}_E\ ,\ \underline{w}_a, \underline{w}_b) = {} & \frac{\omega_E\sqrt{2}}{c^2} P_E(P_E + \overline{h}) \times \\ & \times \frac{\cos\varphi_a\ \cos\varphi_b\ \sin(\lambda_a - \lambda_b)}{(1 + \sin\varphi_a\ \sin\varphi_b + \cos\varphi_a\ \cos\varphi_b\ \cos(\lambda_a - \lambda_b))^{1/2}} \end{aligned} \tag{4.21}$$

where $\overline{h} \approx 92.5$ km is the mean height of meteor tracks, $P_E$ is the radius of the Earth, $\varphi_a, \varphi_b, \lambda_a, \lambda_b$ are the geographical latitudes and longitudes of the stations (longitude to the east from Greenwich is negative), $\omega_E$ is the angular velocity of the Earth.

One can show that

$$|\Delta\delta| \le \frac{\omega_E}{c^2}\, b\sqrt{2\,(P_E + h)}\left(P_E + h - P_E^2\left(P_E^2 - b^2/4\right)^{-1/2}\right)^{1/2}, \tag{4.22}$$

$b$ being the distance between the stations. For moderate distance between the stations ($b \sim 1620$ km) the error $\Delta\delta$ may amount to 1.1 ns. For extremely long ($b \sim 2300$ km) and extremely short ($b \sim 0$ km) distances the error $\Delta\delta$ becomes zero: $\Delta\delta \approx 0$. Thus, in spite of the fact that we don't know the position $\underline{w}_s$ of the retransmission point, we can calculate the relativistic correction with the accuracy $\sim 1$ ns. In order to compute $\overline{\delta}$ with the accuracy of 1 ns, the coordinates of the stations must be known with the uncertainties

$$\sigma_{\varphi_{a,b}} = \sigma_{\lambda_{a,b}} \le 25'. \tag{4.23}$$

## 5. Remarks

1. The concept of coordinate synchronization can be successfully applied to different domains of space-time with quite different physical conditions inside them. Barycentric RS (BRS) may by used to define a single time scale in the whole Solar system. The only method of synchronization of the clocks situated in space (onboard space vehicles) appears to be the two-way synchronization (Fig. 2). On the basis of the expressions (2.4)–(2.7), the equations of the light propagation in BRS [Brumberg, 1972; Will, 1981; Brumberg, 1987; Klioner, 1989a,b] as well as the coordinates of the observers and gravitating bodies of the Solar system one can easily obtain the algorithms of synchronization with respect to the BRS coordinate time. The answer to the question what terms in (2.1), (2.7) as well as in the equations of the light propagation we must take into account when computing $\delta$, $\delta_{gr}$ and $t_2$, so that the final inaccuracy of $t_1$ be equal to some given value, strongly depends on the trajectories of the observers and gravitating bodies. This question must be analyzed separately in every particular case.

In the space-time domain where both BRS and GRS are valid we can synchronize our clocks with respect to the coordinate time of either BRS or GRS. Let us suppose that using some method we have synchronized clocks $a$ and $b$ with respect to the coordinate time of GRS. This means that for two events, which have the coordinates $(u_0, \underline{w}_a(u_0))$ and $(u_0 + \Delta u, \underline{w}_b(u_0 + \Delta u))$ relative to GRS, we know the readings $\tau_{a1}$ and $\tau_{b2}$ of the clocks $a$ and $b$ respectively. The same two events have the coordinates $(t_0, \underline{x}_a(t_0))$ and $(t_0 + \Delta t, \underline{x}_b(t_0 + \Delta t))$ relative to BRS, $\Delta t$ being coordinate desynchronization with respect to the coordinate time of BRS. The relation between $\Delta u$ and $\Delta t$ can be easily obtained on the basis of the relativistic coordinate transformation between BRS and GRS. The explicit expressions may be found in [Klioner, 1990b].

2. Another practically important space-time domain where convenient single time scale is needed is the neighborhood of the Earth of the radius $10^6$ km. This region contains the trajectories of high satellites and the Moon. GRS which has been used in the present paper can not be utilized for the above-stated purpose, because the metric of GRS is expressed in the form of the series in powers of $w/r_{EA}$, $w$ being the geocentric distance of the point, $r_{EA}$ being the distance between the mass centers of the Earth and the body $A$. Formally speaking, the domain of definition of GRS is limited by the distance between the Earth and the nearest body, that is by the radius of the lunar orbit. Actually, since the series in powers of $w/r_{EA}$ converge very slowly when $w/r_{EA} \sim 1$, GRS in the form (3.1)–(3.3) may be used in still smaller domain of space, and in order to attain acceptable accuracy one must take into account too many terms.

To overcome this difficulty we must construct local inertial geocentric RS avoiding the expansions of its metric tensor in powers of $w/r_{EA}$ (at least, for $A = L$, that is for the Moon). Such RS has been constructed in [Voinov, 1990; Brumberg, 1991].

3. There are two different, from the theoretical point of view, ways to re-define the scale $u$ so that its mean rate coincide with the mean rate of the proper time of an observer situated on the geoid. In this paper we follow one of them. We introduce units of measurement in GRS different from the SI units. Another way is to consider the expression (3.14) as the relativistic coordinate transformation and to introduce new coordinate system '$\widetilde{\mathrm{GRS}}$'. All quantities defined in '$\widetilde{\mathrm{GRS}}$' are measured in the usual SI units. The components of metric tensor $\tilde{g}_{\alpha\beta}(\tilde{u}, \underline{\tilde{w}})$ of '$\widetilde{\mathrm{GRS}}$' differ from those of GRS only by constant factors:

$$\begin{aligned}
\tilde{g}_{00}(\tilde{u}, \underline{\tilde{w}}) &= \frac{1}{k_T^2} g_{00}(u, \underline{w}), \\
\tilde{g}_{0i}(\tilde{u}, \underline{\tilde{w}}) &= \frac{1}{k_T k_S} g_{0i}(u, \underline{w}), \\
\tilde{g}_{ij}(\tilde{u}, \underline{\tilde{w}}) &= \frac{1}{k_S^2} g_{ij}(u, \underline{w}).
\end{aligned} \tag{5.1}$$

Although these two ways lead to the same final results, they must be distinguished clearly.

4. From the theoretical point of view it is most consequent not to introduce the scale $\tilde{u}$, but to use directly the coordinate time of GRS. This corresponds to the unit values of the scaling factors in (3.14). In the last years this choice becomes more and more popular [Guinot, 1990]. In this case the only correction to be applied to the formulas of the section 4 is the constant factors $k_T^{-1}$ in right-hand sides of (4.3), (4.13) and (4.14) appearing from (3.11).

5. In the present paper we have considered in details relativistic effects in the clock synchronization. Besides the relativistic effects, equipment's delays, tropospheric and ionospheric time delays in the light propagation, the effects of the magnetic field of the Earth and so on influence on the results of the Earth-based clock synchronization. In order to minimize technical errors in practical measurements the results of a number of consequent observations are averaged giving the observables ($u_a$, $u_b$ in (4.5) etc.). Relativistic corrections change in time (although slowly) and every separate observation corresponds to its own value of the correction. Only after all sources of the errors having been accounted for, one can say about real accuracy of synchronization.

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