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TRANSMISSION, REFLECTION AND DWELL TIMES WITHIN BOHM'S CAUSAL INTERPRETATION OF QUANTUM MECHANICS

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Although many approaches have been developed within the orthodox interpretation of quantum mechanics for calculating the mean times τ_T and τ_R spent by transmitted and reflected particles inside a one-dimensional barrier, a completely satisfactory solution remains elusive. In this paper it is argued that Bohm's causal or trajectory interpretation provides a simple, well-defined, and unambiguous method for calculating transmission, reflection and dwell times with physically reasonable properties.

In recent years there has been considerable interest[1-3], both fundamental and technological, in the characteristic times[4] τ_T, τ_R and τ_D associated with the tunneling of electrons through one-dimensional barriers. For the idealized case of electrons of precisely defined wavevector $\vec{k} = (0,0,k>0)$ and energy $E \equiv \hbar^2 k^2/2m$ incident from the left on the barrier $V(\vec{r}) = V(z)\Theta(z)\Theta(d-z)$ the transmission and reflection times $\tau_T(k;a,b)$ and $\tau_R(k;a,b)$ are defined[4] as the average times spent in the region $a \le z \le b$ [5] by transmitted and reflected electrons, respectively; the dwell time $\tau_D(k;a,b)$ is the average time spent between a and b irrespective of whether the incident electron is ultimately transmitted or reflected. The quantity $\tau_T(k;a=0,b=d)$ is often called the traversal time or, if $\mathrm{Max}[V(z)]>E$, the tunneling time. Literal interpretation of these definitions requires that in the absence of inelastic scattering and absorption

$$\tau_D(k;a,b) = |T(k)|^2\tau_T(k;a,b) + |R(k)|^2\tau_R(k;a,b) \qquad (1)$$

where $|T(k)|^2$ and $|R(k)|^2$ are the transmission and reflection probabilities for the barrier.[3] Another condition that must be satisfied by each of the characteristic times is

$$\tau_X(k;a,c) = \tau_X(k;a,b) + \tau_X(k;b,c) \quad (X = T,R,D) \qquad (2)$$

with $a \le b \le c$.[3] (It should be noted that criteria (1) and (2) are not universally accepted as valid restrictions on the characteristic times.[2])

In quantum mechanics time is regarded not as an observable represented by a Hermitian operator but rather as a parameter or 'c-number'.[6] Hence, there is no automatic prescription within the formulation of the orthodox interpretation for writing down expressions for the characteristic times τ_T, τ_R and τ_D (in fact, a not uncommon point of view is that τ_T and τ_R are meaningless quantities). Accordingly, over the years many, often ingenious and usually controversial, approaches for calculating these times have been suggested. Although there is no consensus on the transmission and reflection times it is now widely agreed[1-3] that the dwell time is a meaningful quantity given exactly by the average number of electrons in the region of interest $a \le z \le b$ divided by the average number incident on the barrier per unit time[4], i.e.

$$\tau_D(k;a,b) = \frac{1}{j_{inc}(k)}\int_a^b |\Psi_k(z)|^2\,dz \quad (k>0) \qquad (3)$$

where $\Psi_k(z)$ is the steady-state scattering solution of the time-independent Schrödinger equation and $j_{inc}(k) = \hbar k/m$ is the incident probability current density. The dwell time defined by (3) is obviously a real, non-negative quantity that satisfies Equation (2). On the other hand, to the author's knowledge, it is only the 'Larmor precession' transmission and reflection times of Baz'[7] and Rybachenko[8] that satisfy Equations (1) and (2) and are real. These times are given by[3]

$$\tau_T^{(1)}(k;a,b) = -\hbar\left.\frac{\partial\phi_T(E,V,\Delta V)}{\partial\Delta V}\right|_{\Delta V=0} \qquad (4a)$$

$$\tau_R^{(1)}(k;a,b) = -\hbar\left.\frac{\partial\phi_R(E,V,\Delta V)}{\partial\Delta V}\right|_{\Delta V=0} \qquad (4b)$$

where $T(E,V,\Delta V) \equiv |T(E,V,\Delta V)|\exp(i\phi_T(E,V,\Delta V))$ and $R(E,V,\Delta V) \equiv |R(E,V,\Delta V)|\exp(i\phi_R(E,V,\Delta V))$ are the transmission and reflection probability amplitudes for the auxiliary barrier

$$\tilde{V}(z) \equiv V(z) + \Delta V\Theta(z-a)\Theta(b-z) \quad ,$$

with ΔV independent of z. Unfortunately, the Larmor precession transmission time $\tau_T^{(1)}(k;0,d)$ for the rectangular barrier $V(\vec{r}) = V_0\Theta(z)\Theta(d-z)$ has the undesirable property[4] that it has negligible dependence on the width d of the barrier for $d \gg \kappa^{-1}$, the characteristic tunneling length, where $\kappa \equiv [2m(V_0-E)]^{1/2}/\hbar$. The original Larmor clock analysis[7,8] was significantly modified by Büttiker[4], and then further extended by Leavens and Aers[5], to include the 'spin rotation' times

$$\tau_T^{(2)}(k;a,b) = -\hbar\left.\frac{\partial \ell n|T(E,V,\Delta V)|}{\partial\Delta V}\right|_{\Delta V=0} \quad , \qquad (5a)$$

$$\tau_R^{(2)}(k;a,b)=-\hbar\frac{\partial \ell n|R(E,V,\Delta V)|}{\partial\Delta V}\Big|_{\Delta V=0} \quad . \tag{5b}$$

For the opaque ($\kappa d >> 1$) rectangular barrier the physically reasonable result $\tau_T^{(2)}(k;0,d)\propto d$ is obtained.[4] However, $\tau_T^{(2)}(k;0,d)$ can be negative for $E > V_0$. Büttiker[4] and Büttiker and Landauer[9] defined the transmission time $\tau_T(k;a,b)$ as $[\tau_T^{(1)}(k;a,b)^2+\tau_T^{(2)}(k;a,b)^2]^{1/2}$ (and similarly for τ_R) and in this way interpolated between the "deep tunneling" regime where $\tau_T^{(2)}$ gives reasonable results and the "far above barrier" regime where $\tau_T^{(1)}$ approaches the expected free-particle result. Unfortunately, τ_T and τ_R thus defined do not satisfy either (1) or (2) and it is possible to have $\tau_T(k;a,b+\Delta b)<\tau_T(k;a,b)$ with $\Delta b>0$ for fixed d.[10]

The Feynman path integral technique of Sokolovski and Baskin[11] gives Büttiker's result, Equation (3), for the dwell time but leads to the complex transmission and reflection times $\tau_{T(R)}(k;a,b)=\tau_{T(R)}^{(1)}(k;a,b)-i\tau_{T(R)}^{(2)}(k;a,b)$ with $\tau_{T(R)}^{(1)}$ and $\tau_{T(R)}^{(2)}$ given by Equations (4) and (5) respectively. These complex times satisfy criteria (1) and (2) exactly but their physical meaning and significance is not clear because they are not real except in special circumstances (perfect transmission for real τ_T and perfect reflection for real τ_R). Moreover, Sokolovski and Baskin claim that their results are not unique.

The above sampling of recent results hopefully illustrates the widespread view that the "tunneling time problem" has not been satisfactorily resolved. All these results were derived within the orthodox interpretation of quantum mechanics and it seems fair to ask whether or not better headway might be made using an alternative interpretation. Nasser[12] recently applied Nelson's stochastic interpretation[13] to the problem but the present author[14] has shown that Nasser's ansatz for the transmission time can lead to results that are not at all consistent with Equation (3) for the dwell time. In the remainder of this paper the characteristic times τ_T, τ_R and τ_D will be discussed within Bohm's causal or trajectory interpretation of quantum mechanics.[15-19] It is well known that Bohm's interpretation leads to exactly the same results as the usual one for all measurable quantities.[15-19] However, this claim was made only in the context of observables represented by Hermitian operators. Since time is not an observable in this sense, but rather a parameter, there remains the possibility that Bohm's interpretation which does not require an observer can lead to different results from those obtained, or even obtainable, within the orthodox interpretation. The results should be identical, however, for the special cases of perfect transmission or reflection where τ_T or τ_R, respectively, is equal to the dwell time τ_D given by Equation (3) which is not in doubt.

In the one-electron version of Bohm's causal theory a non-relativistic (spinless) electron is a particle the motion of which is determined by an objectively real complex-valued field

$$\Psi(z,t)\equiv R(z,t)\exp[iS(z,t)/\hbar] \qquad \text{(R and S real)}, \tag{6}$$

satisfying the time-dependent Schrödinger equation via the guidance condition[15-19]

$$v(z,t)=\frac{1}{m}\frac{\partial S(z,t)}{\partial z} \tag{7}$$

on the particle's instantaneous velocity. Given $\Psi(z,0)$, if the particle's initial position $z^{(0)}$ is known exactly at $t = 0$ then, just as in classical mechanics, its trajectory $z(z^{(0)},t)$ is uniquely determined by integrating $dz(t)/dt = v(z,t)$. Although such trajectories are smooth they can be quite complicated even for simple systems.[20]

Alternatively, such de Broglie-Bohm trajectories can be obtained by integrating Newton's equation of motion with the usual potential energy $V(z)$ augmented by the "quantum potential" [15-18]

$$Q(z,t)\equiv-\frac{\hbar^2}{2m}\frac{\partial^2 R(z,t)}{\partial z^2}/R(z,t) \quad . \tag{8}$$

$Q(z,t)$ can be large even when $V(z)$ and $R(z,t)\equiv|\Psi(z,t)|$ are both very small. Peculiarities such as this are needed to account for the striking differences between quantum and classical mechanics.[15-19]

In practice, the particle's initial position is not known exactly and uncertainty thus enters Bohm's deterministic theory through the postulated probability distribution for the initial position of the particle: $\rho(z^{(0)},0)dz^{(0)}\equiv|\Psi(z^{(0)},0)|^2\,dz^{(0)}$ is the probability of the particle being between $z^{(0)}$ and $z^{(0)} + dz^{(0)}$ at $t = 0$ even if a position measurement is not made at that instant. For a point-particle that is at $z^{(0)}$ at $t = 0$ the time spent thereafter in the region $a \le z \le b$ is unambiguously given by

$$t(z^{(0)};a,b)=\int_0^\infty dt\,\Theta[z(z^{(0)},t)-a]\Theta[b-z(z^{(0)},t)] \tag{9}$$

where Θ is the unit step function. Taking into account the uncertainty in $z^{(0)}$ immediately gives the dwell time for the wavepacket described by $\Psi(z,t)$:

$$\tau_D(a,b)\equiv\langle t(z^{(0)};a,b)\rangle \tag{10}$$

where

$$\langle f(z^{(0)})\rangle\equiv\int_{-\infty}^{+\infty}dz^{(0)}|\Psi(z^{(0)},0)|^2 f(z^{(0)}) \tag{11}$$

for any function f of $z^{(0)}$.

By introducing unity in the form of an integral over all z of the delta function $\delta[z-z(z^{(0)},t)]$, Equation (10) can be written in the equivalent form

$$\tau_D(a,b)=\int_0^\infty dt\int_a^b dz|\Psi(z,t)|^2 \tag{12}$$

with

$$|\Psi(z,t)|^2\equiv\rho(z,t)\equiv\langle\delta[z-z(z^{(0)},t)]\rangle \quad .$$

Equation (12) was originally derived within the conventional interpretation by Sokolovski and Baskin[11] using a Feynman path integral technique and has been used in turn to derive (3).[3,11] Hence, as required above, the one firm and widely accepted result obtained within the usual interpretation also follows within Bohm's causal interpretation.

In the following discussion of transmission and reflection times for wavepackets it is always assumed that the initial wavepacket described by $\Psi(z,t=0)$ is sufficiently far to the left of the barrier region $0 \le z \le d$ that the probability density $|\Psi(z,t)|^2$ is completely negligible for $z \ge 0$ for all $t \le 0$. It is also assumed that each trajectory $z(z^{(0)},t)$ with non-negligible weight $\rho(z^{(0)},0)$ can be followed (numerically) for a sufficiently long time that it can be labelled with confidence as either transmitted, i.e. $z(z^{(0)},t) > d$ for all $t > t^*(z^{(0)})$, or reflected, i.e. $z(z^{(0)},t) < 0$ for all $t > t^*(z^{(0)})$. In this case

$$\Theta_>(z^{(0)})+\Theta_<(z^{(0)})=1 \qquad (13)$$

where

$$\Theta_>(z^{(0)}) \equiv \Theta[z(z^{(0)},\infty)-d], \quad \Theta_<(z^{(0)}) \equiv \Theta[-z(z^{(0)},\infty)] \quad .$$

The transmission and reflection probabilities for the wavepacket are obviously given by

$$|T|^2=\langle\Theta_>(z^{(0)})\rangle \quad ; \quad |R|^2=\langle\Theta_<(z^{(0)})\rangle \quad . \qquad (14)$$

Inserting (13) into (10) immediately gives

$$\tau_D(a,b)=|T|^2\,\tau_T(a,b)+|R|^2\,\tau_R(a,b) \qquad (15)$$

where

$$\tau_T(a,b) \equiv \langle\Theta_>(z^{(0)})\,t(z^{(0)};a,b)\rangle/\langle\Theta_>(z^{(0)})\rangle \qquad (16a)$$

is the mean time spent in the region $a \le z \le b$ by an electron that is ultimately transmitted and

$$\tau_R(a,b) \equiv \langle\Theta_<(z^{(0)})\,t(z^{(0)};a,b)\rangle\,/\,\langle\Theta_<(z^{(0)})\rangle \qquad (16b)$$

is the mean time spent in the region $a \le z \le b$ by an electron that is ultimately reflected. Now, in the limit that the width Δk of the Fourier transform $\phi(k)$ of the initial wavepacket $\Psi(z,t=0)$ is negligible compared to the average wavenumber k_0 of $\phi(k)$ the above characteristic times can be labelled by k_0, i.e. $\tau_D(a,b)$ becomes $\tau_D(k_0;a,b)$ etc., and (14) becomes criterion (1). It is obvious from the definitions (10) or (12) and (16) of the characteristic times that criterion (2) is also satisfied (for arbitrary Δk). Thus, at least in principle, Bohm's interpretation of quantum mechanics leads in a simple, direct way to real, non-negative characteristic times τ_D, τ_T and τ_R that are consistent with each other, i.e. satisfy $\tau_D=|T|^2\,\tau_T+|R|^2\,\tau_R$, and are additive in the sense $\tau_X(a,c)=\tau_X(a,b)+\tau_X(b,c)$ with $a \le b \le c$ and X = T, R and D. Since a precisely defined electron trajectory is a meaningless concept at the microscopic level in the usual interpretation it is possible that the approach suggested here for calculating τ_T and τ_R can lead to new results when $0 < |T|^2 < 1$ that are unobtainable within the usual interpretation. This is not the case, of course, for the infinite barrier ($|T|^2 = 0$) considered below because it essentially involves only the dwell time τ_D given by Equation (12), an exact result within both interpretations. However, in order to illustrate the Bohm trajectory approach τ_D is calculated via Equations (9) to (11) rather than directly with (12) which does not explicitly involve trajectories.

As a simple, non-trivial application of the suggested approach we now calculate the mean time $\tau_D(a,0) \equiv \langle t(z^{(0)};a,0)\rangle$ and root-mean-square time $\langle t(z^{(0)};a,0)^2\rangle^{1/2}$ spent in the region $a \le z \le 0$ in front of the infinite potential barrier $V(z)=\infty\Theta(z)$ for a statistical ensemble of incident electrons described at $t = 0$ by the minimum-uncertainty-product wavefunction

$$\Psi_{inc}(z,t=0)=\frac{1}{[2\pi(\Delta z)^2]^{1/4}}\exp\left[-\left(\frac{z-z_0}{2\Delta z}\right)^2+ik_0z\right] . \qquad (17)$$

For the calculations reported here $E_0 \equiv \hbar^2k_0^2/2m$ is 4 eV corresponding to an average wavenumber k_0 for the Fourier transformed wavepacket $\phi(k)$ of very close to 1Å^{-1}. For definiteness, the $t = 0$ centroid z_0 of the incident wavepacket is chosen so that $|\Psi_{inc}(\mathrm{Min}(a),0)|^2 / |\Psi_{inc}(z_0,0)|^2=\exp(-N_0)$, i.e. $z_0=\mathrm{Min}(a)-\sqrt{2N_0}\,\Delta z$, where Min(a) is the minimum value of a of interest. In all cases considered here Min(a) = −10Å, $N_0 = 9$ and $\Delta z = (2\Delta k)^{-1}$ where Δk is the width of $\phi(k)$. The de Broglie-Bohm particle velocity associated with the initial incident part of the wavefunction, $\Psi_{inc}(z,0)$ is $v_{inc}(z,0)=(1/m)\,\partial S_{inc}(z,0)/\partial z=\hbar k_0/m$, independent of z.

It is a straightforward exercise to show that the required solution, $\Psi(z,t) \equiv \Psi_{inc}(z,t) + \Psi_{refl}(z,t)$, of the time-dependent Schrödinger equation is given by

$$\Psi(z,t)=\alpha\exp[\beta_0+\beta_2z^2+i(\gamma_0+\gamma_2z^2)]$$
$$\bullet\,[\exp(\delta z+i\varepsilon z)-\exp(-\delta z-i\varepsilon z)]\Theta(-z) \qquad (18)$$

where

$$\alpha \equiv [2(\Delta z)^2\eta/\pi]^{1/4}\exp[-(\Delta z)^2k_0^2] \quad ,$$

$$\beta_0 \equiv \eta(\Delta z)^2[4(\Delta z)^4k_0^2-z_0^2-2k_0z_0\hbar t/m] \quad ,$$

$$\beta_2 \equiv -\,\eta(\Delta z)^2 \quad ,$$

$$\gamma_0 \equiv -\,\eta[4(\Delta z)^4k_0z_0+2(\Delta z)^4k_0^2\hbar t/m-z_0^2\hbar t/2m] \quad ,$$

$$\gamma_2 \equiv \eta(\hbar t/2m) \quad ,$$

$$\delta \equiv 2\eta(\Delta z)^2(z_0+k_0\hbar t/m) \quad ,$$

$$\varepsilon \equiv \eta[4(\Delta z)^4k_0-z_0\hbar t/m] \quad ,$$

$$\eta \equiv [4(\Delta z)^4+(\hbar t/m)^2]^{-1} \quad .$$

It should be noted that $\Psi(z,t)$ has a string of nodes at the points $z_n \equiv n\pi/k_0$ $(n = 1, 2, \ldots)$ at the instant $t = t_0 \equiv |z_0|/(\hbar k_0/m)$.

After casting $\Psi(z,t)$ into the form $R(z,t)\exp(iS(z,t)/\hbar)$ it is easy to obtain the following expressions for the quantum potential and particle velocity:

$$Q(z,t)=\frac{\hbar^2}{2m}\left\{-2\beta_2\left[1+2\beta_2 z^2+2\left(\frac{\delta\sinh 2\delta z+\varepsilon\sin 2\varepsilon z}{\cosh 2\delta z-\cos 2\varepsilon z}\right)z\right]\right.$$
$$-2\frac{[\delta^2\cosh 2\delta z+\varepsilon^2\cos 2\varepsilon z]}{\cosh 2\delta z-\cos 2\varepsilon z}$$
$$\left.+\frac{[\delta\sinh 2\delta z+\varepsilon\sin 2\varepsilon z]^2}{[\cosh 2\delta z-\cos 2\varepsilon z]^2}\right\}, \qquad (19)$$

$$v(z,t)=\frac{\hbar}{m}\left\{2\gamma_2 z+\left[\frac{\varepsilon\tanh\delta z-\delta\sin\varepsilon z\cos\varepsilon z\,\mathrm{sech}^2\delta z}{\sin^2\varepsilon z+\tanh^2\delta z\cos^2\varepsilon z}\right]\right\}. \qquad (20)$$

At $t=0$ the expression for the particle velocity becomes

$$v(z,t=0)=\frac{\hbar}{m}\left\{\frac{k_0\tanh(z_0z/2(\Delta z)^2)-(z_0/4(\Delta z)^2)\sin 2k_0z\,\mathrm{sech}^2(z_0z/2(\Delta z)^2)}{\sin^2 k_0z+\tanh^2(z_0z/2(\Delta z)^2)\cos^2 k_0z}\right\}.$$

If $|z| \gg 2(\Delta z)^2/|z_0|$ then $v(z,t=0)=\hbar k_0/m$ to a good approximation, reflecting the fact that for sufficiently large $|z|$ at $t=0$ the interference between incident and reflected components of the wavepacket is of negligible importance. On the other hand, if $|z| \ll 2(\Delta z)^2/|z_0|$ and $(2k_0)^{-1}$ then $v(z,t=0) = (\hbar k_0/m)(z_0/3(\Delta z)^2)z$ to a good approximation, reflecting the vital importance of such interference in this regime. Between these large and small $|z|$ regimes the oscillatory terms in $v(z,t=0)$ may be important.

Numerical integration of $dz(t)/dt = v(z,t)$ subject to the initial condition $z(t=0) = z^{(0)}$ yields the particle trajectory $z(z^{(0)},t)$. A selection of such de Broglie-Bohm trajectories calculated using Equation (20) for the particle velocity are shown in Fig. 1 for $\Delta k = 0.04$ Å^{-1} (i.e. $\Delta z = 12.5$ Å) and for the 15 Å region immediately in front of the infinite barrier. The bending of the trajectories away from the nodes that appear at the instant $t = t_0$ at the points z_n is quite striking as is the bunching of trajectories just in front of the infinite barrier. When $t = t_0$, the time taken for a free particle of velocity $\hbar k_0/m$ to travel a distance $|z_0|$, $\delta = 0$ and $v(z,t_0) = 2\hbar\gamma_2 z/m < 0$. This means that all the trajectories must turn around before $t = t_0$. In Fig. 1 the parameters are such that $|v(z,t_0)|/(\hbar k_0/m) \ll 1$ and the turning points occur very close to t_0 so that the trajectories are approximately symmetric about $t = t_0$. For much larger values of Δk the trajectories are quite asymmetric.

For each particle trajectory $z(z^{(0)},t)$ it is easy to calculate via (9) the time $t(z^{(0)};a,0)$ spent by the particle in the region $a \le z \le 0$. The corresponding average time $\langle t(z^{(0)};a,0)\rangle \equiv \tau_D(a,0)$ and mean-square-time $\langle t(z^{(0)};a,0)^2\rangle$ for the ensemble of particles described by $\rho(z^{(0)},0)$ are obtained by integrating $\rho(z^{(0)},0)\,t(z^{(0)};a,0)$ and $\rho(z^{(0)},0)\,t(z^{(0)};a,0)^2$ over all $z^{(0)}$ with

$$\rho(z^{(0)},0)\equiv|\Psi(z^{(0)},0)|^2=\frac{\Theta(-z^{(0)})}{(\pi/2)^{1/2}\Delta z}[\cosh(z_0z^{(0)}/(\Delta z)^2)-\cos 2k_0z^{(0)}]\exp\left[-\frac{(z_0^2+z^{(0)2})}{2(\Delta z)^2}\right].$$

The results for $\langle t(z^{(0)};a,0)\rangle$ with -10 Å $\le a \le 0$ are shown in Fig. 2 for $\Delta k = 0.08$ and 0.02 Å^{-1}. The stationary-state, plane-wave ($\Delta k = 0$) dwell time $\tau_D(k_0;a,0)$ defined by Equation (3) and the equivalent (because $|R|^2 = 1$) Larmor precession reflection time $\tau_R(k_0;a,0)$ defined by (4b) are easily calculated with the result

$$\tau_D(k_0;a,0)=\tau_R(k_0;a,0)=\frac{2|a|}{(\hbar k_0/m)}\left(1-\frac{\sin 2k_0a}{2k_0a}\right). \qquad (21)$$

While the mean time $\langle t(z^{(0)};a,0)\rangle$ for $\Delta k = 0.08$ Å^{-1} shows significant departures from (21) for $a \lesssim 2$ Å the result for $\Delta k = 0.02$ Å^{-1} is barely distinguishable from (21) over the entire 10 Å range shown in the figure. It is encouraging that the $\Delta k = 0$ limit is numerically accessible via the Bohm trajectory technique.

For perfect reflection ($|R|^2 = 1$) the Sokolovski-Baskin[11] reflection time $\tau_R(k_0;a,0)$ is real and identical to the Larmor precession result (4b). Hence, for the special case

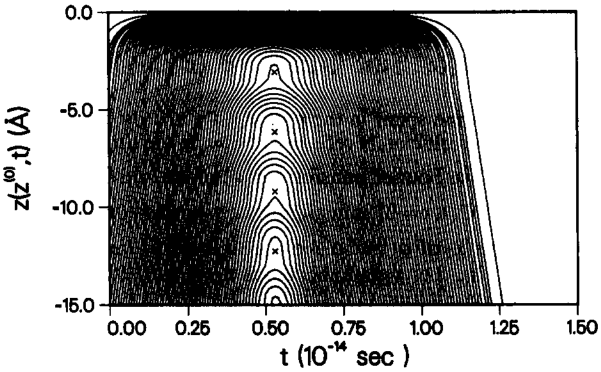

Fig. 1 De Broglie-Bohm trajectories $z(z^{(0)},t)$ for an initial Gaussian wavepacket incident on the infinite potential barrier $V(z) = \infty\Theta(z)$. The starting points $z^{(0)} \equiv z(z^{(0)}, t=0)$ for the trajectories shown are $z^{(0)} \equiv \ell$ Å ($\ell = 1,2,\ldots$). The wavepacket parameters are $E_0 = 4$ eV and $\Delta k = 0.04$ Å^{-1} (i.e. $\Delta z = 12.50$ Å and $z_0 = -63.03$ Å). The space-time points (z_n,t_0) at which $\Psi(z,t)$ is zero are indicated for $n = 1,2,3$ and 4.

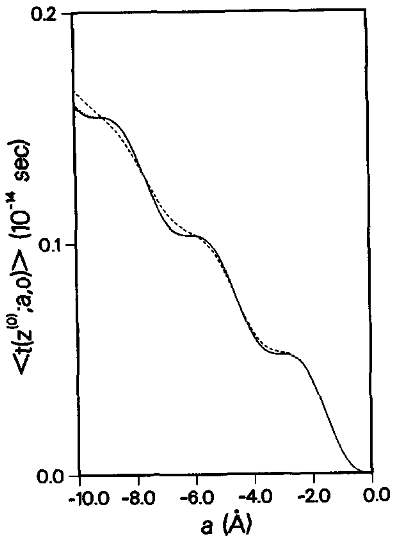

Fig. 2 The average time $<t(z^{(0)};a,0)>$ spent in the region $a \le z \le 0$ just in front of the infinite barrier $V(z) = \infty\Theta(z)$ by a statistical ensemble of electrons described by the initial incident gaussian wavepacket having $E_0 = 4$ eV, $z_0 = \mathrm{Min}(a) - \sqrt{2N_0}\,\Delta z$ where $\mathrm{Min}(a) = -10$ Å, $N_0 = 9$ and $\Delta z \equiv (2\Delta k)^{-1}$ with $\Delta k = 0.08$ and 0.02 Å^{-1} (-------- and ········ respectively). The dwell time for the corresponding plane-wave (Δk=0) stationary-state scattering approach is included for comparison (————).

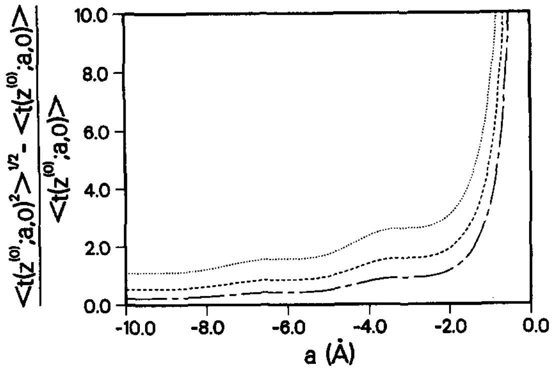

Fig. 3 $[<t(a,0)^2>^{1/2} - <t(a,0)>]/<t(a,0)>$ for $\Delta k = 0.08$, 0.04 and 0.02 Å^{-1} (— - — -, --------, and ·········· respectively). The other wavepacket parameters are the same as for Fig. 2.

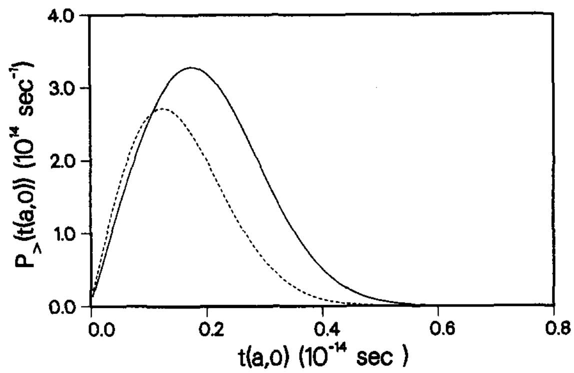

Fig. 4 Dwell time distribution functions $P[t(a,0)] \equiv P_0(a)\,\delta[t(a,0)] + P_>[t(a,0)]$ for $\Delta k = 0.08$ Å^{-1} and $a = -10$ and -5 Å (———— and ---------- respectively). The other parameters are the same as in Figures 2 and 3. Note that it is $P_>[t(a,0)]$, not $P[t(a,0)]$, that is plotted. $P_0 = 0.164$ for $a = -10$ Å and 0.435 for $a = -5$ Å.

of the infinite barrier it is given by (21). Now, it is possible to generalize the Feynman path integral technique of Sokolovski and Baskin to obtain the corresponding mean-square reflection time.[3] This quantity is complex but its real part is exactly equal to the square of $\tau_R(k_0;a,0)$ given by (21).[3] This means that if the imaginary part is discarded as being meaningless then the reflection time distribution function must be a delta function (this is the one-dimensional analogue of the "remarkable result" obtained by Baz' in three dimensions using his Larmor precession approach[21]). It is quite clear from the trajectories shown in Fig. 1 that within the Bohm trajectory approach this will not be the case, at least not for $\Delta k = 0.04$ Å^{-1}. Figure 3 shows that it is most unlikely to be the case in the $\Delta k = 0$ limit considered by Baz' because the relative difference between $< t(z^{(0)};a,0)^2 >^{1/2}$ and $< t(z^{(0)};a,0) >$ increases with decreasing Δk.

The calculation of transmission and reflection time distributions appears intractable within any of the conventional approaches. On the other hand, it presents no additional difficulties for the Bohm trajectory approach. It is clear from Fig. 1 that for $|a|$ not too large $t(z^{(0)};a,0)$ is exactly zero for a significant fraction of the trajectories (i.e. those which turn around before reaching $z = a$). Hence, it is convenient to write the reflection time distribution function in the form

$$P(t(a,0)) = P_0\delta(t(a,0)) + P_>(t(a,0))\ . \qquad (22)$$

($P_>$ is normalized to $1-P_0$). In terms of $P_>(t(a,0))$

$$<t(z^{(0)};a,0)^n> = \int_0^\infty dx\, x^n P_>(x) \qquad (n = 1, 2, \ldots).$$

Figure 4 shows $P_>(t(a,0))$ calculated for $\Delta k = 0.08$ Å^{-1} and $a = -10$ and -5 Å.

The above results for $< t(z^{(0)};a,0)^2 >$ and $P(t(a,0))$ provide some grounds for expecting that the Bohm trajectory approach will not duplicate the unphysical Larmor precession result $\tau_D(k;0,d) \propto d^0$ for an opaque rectangular barrier of width d.

Granted that the calculations reported in this paper were enormously simplified by the availability of $\Psi(z,t)$ and hence $v(z,t)$ in closed form, numerical calculations of reflection and transmission times within the Bohm trajectory approach should be tractable for more interesting barriers. Top priority should be given to determining the dependence of the transmission time $\tau_T(k_0;0,d)$ for an opaque rectangular barrier on its width d. If this dependence should turn out to be reasonable (e.g. $\propto d$) then it will provide concrete evidence in support of Bohm's interpretation of quantum mechanics. Moreover, calculations of τ_T for the rectangular barrier as a function of *d* and *E* should reveal whether or not there is any simple connection with the Larmor precession and spin rotation times $\tau_T^{(1)}$ and $\tau_T^{(2)}$ which are very much easier to calculate.

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