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Quantum Tunneling Times: A Crucial Test for the Causal Program?

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It is generally believed that Bohm's version of quantum mechanics is observationally equivalent to standard quantum mechanics. A more careful statement is that the two theories will always make the same predictions for any question or problem that is well posed in both interpretations. The transit time of a "particle" between two points in space is not necessarily well defined in standard quantum mechanics, whereas it is in Bohm's theory since there is always a particle following a definite trajectory. For this reason tunneling times (in a scattering configuration) through a potential barrier may be a situation in which Bohm's theory can make a definite prediction when standard quantum mechanics can make none at all. I summarize some of the theoretical and experimental prospects for an unambiguous comparison in the hope that this question will engage the attention of more physicists, especially those experimentalists who now routinely actually do gedanken *experiments.*

1. BACKGROUND

The frontispiece to a special issue of *Annales de l'Institut Henri Poincaré*, dedicated to Professor Vigier to celebrate his forty years of association with the Institute, is a charming photograph of Jean-Pierre Vigier and David Bohm.[1] The two men appear to be engaged in conversation: animated exposition on Vigier's part, more passive reflection on Bohm's. For me, that picture illustrates in dramatically visual terms an essential difference between the two men. This difference is, I believe, also reflected in their styles of doing natural philosophy, as I indicate below.

In his seminal 1952 papers[2] on the quantum-potential version of quantum mechanics, Bohm was able to recast the Schrödinger equation

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into the form of Newton's second law of motion for the dynamical evolution of the position of a particle along its trajectory.[3] There the "guidance condition" giving the velocity $\mathbf{v}$ of the particle is

$$\mathbf{v} = \nabla S \tag{1}$$

where S is the phase of the wave function ψ. Since that time, several attempts have been made to underpin this and similar causal interpretations with models of the requisite physical processes. An early example of this was the work of Bohm and Vigier in which a physically real field ψ (i.e., based on actual physical processes in three-dimensional space), in many ways equivalent to a "fluid" and having irregular and random fluctuations, provided a background medium that acted on particle-like inhomogeneities in this fluid.[4] Their proposal was tentative and *suggestive* of a research program. Thereafter, their research diverged along two quite different paths. Bohm's interests took a metaphysical bent in terms of the wholeness or interrelatedness of all physical phenomena, while Vigier's subsequent work (on a stochastic interpretation of quantum mechanics and on de Broglie's pilot-wave program) concentrated on specific physical models based on the concept of a "covariant aether."[5] Jean-Pierre Vigier has remained basically a physicist in his speculations, always looking for experimental tests that will distinguish some causal version of quantum mechanics from the conventional, or loosely termed "Copenhagen," one. Even though I here discuss possible differences in the predictive power of Bohm's causal quantum theory and of standard quantum mechanics, I hope that Professor Vigier will nonetheless find this topic of some interest and will accept this paper as an expression of gratitude for his enthusiastic and stimulating challenges to the reigning orthodoxy and to the complacency of too many physicists when it comes to foundational issues in quantum theory.

2. QUANTUM TUNNELING TIMES

Since Bohm's theory and standard quantum mechanics have the same set of equations available for calculation, it might seems as though they should be observationally completely equivalent. If the calculation of any observable quantity is well posed in both theories, then both theories will calculate, or produce, the same answer when the (common) formalism is applied. This appears to have been Bohm's own view since, when asked directly in 1986 whether there were any new predictions from his model, Bohm responded: "Not the way it's done. There are no new predictions because it is a new interpretation given to the same theory."[6] John Bell made a similar point, but a bit more circumspectly: "It [the de Broglie–Bohm version of nonrelativistic quantum mechanics] is experimentally equivalent to the usual version insofar as the latter is unambiguous."[7]

Could it be, though, that a certain class of phenomena might correspond to a well posed problem in one theory, but not in the other? That is, might the additional microontology (i.e., particles and definite trajectories) of Bohm's theory issue in a prediction of an observable where Copenhagen would just have no definite prediction to make? These two theories would not each have made definite predictions that disagreed. Rather, one would have made a definite prediction and the other would simply remain agnostic on the question. I suggest that there *may* be a difference for an observable that cannot be represented by a hermitian operator and I assess the likely observational consequences of this.

In standard quantum mechanics the time t is simply a *parameter*, not an *operator*, and there is, in general, no well-defined meaning to the transit time of an individual particle between two points (because, of course, the particle aspect of an object need not be the appropriate description under all circumstances). To find the average value of an observable one usually computes the expectation value of the corresponding hermitian operator. This cannot be done to find an average value for transit times since time cannot be represented by a hermitian operator.[8] For Bohm's theory, by contrast, the transit time of a particle between any two points is conceptually well defined.

As an illustration of the type of idealized experiment one might do to probe these conceptual differences, consider a beam of particles incident upon a rectangular potential barrier of height V_0 and thickness d.[9] Take the beam to be incident from the left and let detectors 1 and 2 be placed, respectively, at positions x_1 (to the left of the barrier) and x_2 (to its right). Notice that this problem of tunneling in a scattering configuration is importantly different from tunneling out of a well (e.g., the classic case of tunneling of a α-particle out of a nucleus) because of a constraint, Eq. (4) below, that must be satisfied in this case. In the decay of a metastable state, there is just *one* time, the average lifetime τ, to be computed from the parameters of the well. It is the sum rule of Eq. (4) that causes the difficulty for the candidates for the various τs in standard quantum mechanics. In our suggested arrangement, some of the particles in the beam will be transmitted through the barrier and some will be reflected. As an idealized type of *gedanken* experiment, suppose that, particle-by-particle (i.e., event-by-event), we knew the time $t_j^{(1)}$ at which the jth particle passed x_1 (i.e., the time at which detector 1, assumed perfectly efficient, fired) and then the time $t_j^{(2)}$ at which it passed detector 2 (if it is transmitted), or $t_j^{(3)}$ at which it (again) passed detector 1 (if it is reflected).[10] With these as data, we could then compute the time $\tau_j = t_j^{(2)} - t_j^{(1)}$ (or $t_j^{(3)} - t_j^{(1)}$) each particle spent in the region (x_1, x_2) and then the *dwell time* τ_D, which is the average time spent by the particles ($j = 1, 2, 3, \ldots, N$) between points x_1 and x_2. Similarly, one can define a *reflection time* τ_R, which is the average time spent in (x_1, x_2) by the *reflected* particles, and a *transmission time* τ_T. If we grant, for discussion's sake at the moment, that these τs are accessible experimentally (i.e., that the various t_js can be inferred), then we can ask a quantum theory to calculate these quantities. Much thought and effort has gone into how one might coherently calculate these τs in standard quantum mechanics, but there appear to be *no* candidates that are totally consistent.[11] The upshot of such work is that: "[It] leaves open the question of the length of time a transmitted [reflected] particle spends in the barrier region. It is not clear that a generally valid answer to this question exists."[12] This is not totally surprising since, for standard quantum mechanics, there is no particle concept available in the barrier region.

Bohm's theory can be, and has been, used to calculate these various τs.[13] It remains unclear whether or not a confrontation is possible in this area. I return to this question in Section 3. If it should turn out that the predictions of Bohmian mechanics for the τs can be compared with experiment and are found to be in error, then that would count as a refuting instance, a failure for Bohm's program. Copenhagen would be neither supported nor refuted there, since it is unable to make any unambiguous predictions. Bohm's program is clearly at more risk here (i.e., falsifiable). Or, it could happen that Bohm's predictions cannot be compared with experiment (perhaps for in-principle or technical reasons) and a case of evidential underdetermination would still remain.

For the thought experiment outlined above, we could use the values of the τ_js to find the *dwell time* τ_D, which is defined as the average time spent by the particles ($j = 1, 2, 3, \ldots, N$) between points x_1 and x_2, as

$$\tau_D = \frac{1}{N} \sum_{j=1}^{N} \tau_j \tag{2}$$

Similarly, if N_R particles are reflected and N_T transmitted (where $N_R + N_T = N$), then one could calculate a *reflection time* τ_R, which is the average time spent by the *reflected* particles in (x_1, x_2), and a transmission time τ_T as

$$\tau_R = \frac{1}{N_R} \sum_{\{N_R\}} \tau_j, \qquad \tau_T = \frac{1}{N_T} \sum_{\{N_T\}} \tau_j \tag{3}$$

Since the reflection and transmission coefficients are defined, respectively, as $R = N_R/N$ and $T = N_T/N$, it follows as an identity that

$$\tau_D = R\tau_R + T\tau_T \tag{4}$$

A popular heuristic argument, within the framework of standard quantum mechanics, leads to the result[14]

$$\tau_D(x_1, x_2) = \int_0^\infty dt \int_{x_1}^{x_2} dx\, |\psi(x, t)|^2 = \int \frac{dk}{2\pi}\, |\varphi(k)|^2\, \tau(k; x_1, x_2) \tag{5a}$$

$$\tau_D(k; x_1, x_2) \equiv \frac{1}{v(k)} \int_{x_1}^{x_2} dx\, |\psi(k, x)|^2 \tag{5b}$$

where $\varphi(k)$ is the Fourier decomposition of the incident wave packet. Even if one accepts this candidate for τ_D, there appear to be *no* consistent candidates for τ_R and τ_T.[15]

However, for Bohm's theory the motion of a particle is completely deterministic so that once its initial position x_0 has been given [corresponding to $p_0 = \partial S(x)/\partial x|_{x_0}$ of the guidance condition of Eq. (1)], the time τ spent between x_1 and x_2 is determined [i.e., $\tau = \tau(x_0)$]. These initial positions are distributed according to $|\psi(x_0)|^2$. The various τs can (in principle) be computed as[16]

$$\begin{gathered} \tau_D = \int \tau(x_0)\, |\psi(x_0)|^2\, dx_0 \\ \tau_R = \frac{1}{R} \int_{\{R\}} \tau(x_0)\, |\psi(x_0)|^2\, dx_0, \qquad \tau_T = \frac{1}{T} \int_{\{T\}} \tau(x_0)\, |\psi(x_0)|^2\, dx_0 \end{gathered} \tag{6}$$

while R and T are simply

$$R = \int_{\{R\}} |\psi(x_0)|^2\, dx_0, \qquad T = \int_{\{T\}} |\psi(x_0)|^2\, dx_0 \tag{7}$$

Here $\{R\}$ and $\{T\}$ are, respectively, those subsets of initial particle positions that lie on trajectories that will be reflected and on those that will be transmitted. These definitions satisfy Eq. (4) identically. The scattering of a particle by a barrier has been studied numerically in Bohmian mechanics and these $\tau(x_0)$ can be found (at least numerically).[17] The average τs appear to be sensitive functions of the overall width of the packet, but not necessarily of its detailed structure.[18]

3. EXPERIMENTAL POSSIBILITIES

Now that we have seen that these various tunneling times can be consistently calculated by Bohm's theory, we naturally ask whether or not there are any plausible experimental situations in which we could actually determine those τs to compare with the theoretical predictions. Most of the experimental data that might be related to one type of tunneling time or another come from tunneling through semiconducting barriers, such as those provided by Josephson junctions, by solid-state heterostructures, or by scanning tunneling microscopes (STM).[19] However, it is unclear just what the physical significance is of these "times" that have been extracted from such data and how they might be related to the τs defined above. Typically, a cutoff frequency is measured for a current that depends upon the frequency of an impressed field, and the characteristic time associated with this phenomenon is identified with the tunneling time τ_T.[20] The chain of inference is indirect and the identification not wholly convincing.[21] Also, the quantum tunneling (or escape) time for certain macroscopic variables has been measured[22] and some have claimed[23] that this time is unambiguously related to barrier traversal times. However, this interpretation depends upon the formal analogy that the Josephson junction "can be modeled as a particle of coordinate δ [which is in reality, though, the phase difference between the layers of the junction] moving in a one-dimensional tilted washboard potential."[24] It is precisely because this variable δ actually represents a collective phenomenon for an entire circuit that the scale for its change is long enough to make it measurable.[25] Finally there have also been quantum optics experiments in which interference effects are used to infer the "delay time" of a photon in passing through an optical barrier.[26] While these optical results are very interesting in their own right, it is by no means evident just what they have to do with tunneling times of a particle through a barrier. The argument used is based on a formal analogy of the wave equation for steady-state electromagnetic waves and the time-independent Schrödinger equation.[27] If one wants to test unambiguously the predictions of Bohm's theory, then it is important to appreciate that there is no particle-like photon in Bohm's theory.[28] Any photon–particle analogy made to relate optical delay times to particle tunneling times would be doubly suspect in this case. Therefore, it appears worthwhile to consider the possibility of an actual scattering-configuration experiment with massive objects, such as electrons. None of my observations are made by way of disparaging any of these previous theoretical or experimental efforts, each of which is interesting in its own right. My point is that none of these experiments have been designed to measure directly the quantum tunneling time of a *particle* through a barrier in a configuration that allows a clean test of the predictions of Bohmian mechanics. Of course, one need not have any interest in such a project, but I do find it worth addressing this question.

Even leaving aside the interpretative controversy, one can ask about the possibility of determining experimentally tunneling times that would correspond to those predicted by Bohm's theory. Here I consider the feasibility of a type of experiment designed to address just this question. With current technology one is able to fabricate a layered structure whose thickness is of atomic dimensions (i.e., a few to several Å).[29] This creates the possibility of making a barrier whose width d is on the order of 10^{-9} m. In the following discussion I optimistically bracket some difficult technical questions, assume that such a barrier is available, and then consider a few theoretical and practical limitations on a tunneling-time experiment. It is also important to appreciate that event-by-event time measurements, as opposed to measurements of averaged quantities alone (such as currents), are necessary to test these predictions for the τs. Currents alone provide the coefficients R and T of Eq. (4), but not the τs.

Let me now return to the "experiment" discussed in Section 2. An immediate problem is that, even if we can prepare the incoming electron in a given state (represented, say, by a Gaussian packet)[30], a measurement of the arrival time at detector 1 placed at x_1 will "collapse" the wave function. In that case, any subsequent transit time predicted on the basis of the known incident wave function would be quite useless. However, we could remove the detector at position 1 and replace it with a state-preparation device (such as a fast gate) that would allow (on the average) just one electron at a time through. We would then know (conditionally) that *if* there was an electron in this packet, *then* its time of arrival (actually, of preparation) at x_1 (i.e., the $t_j^{(1)}$ of Section 2 above) would be known to within an error related to the width of the packet. Detectors 1 and/or 2 at positions x_1 and x_2 could then be inserted (or turned on). If one of them registered (assuming perfect efficiency for now) at some time ($t_j^{(2)}$ or $t_j^{(3)}$) after the the state had been prepared, then we would know the transit or reflection time for that electron. Of course, if we are willing to assume something like "fair sample" (i.e., that the fraction of electrons actually detected is a statistically fair sample of the entire ensemble of electrons actually prepared by the chopper), then the efficiency of the detectors is not important. We would then use only those data generated by the firing of counter 1 or 2. It is of no importance that we destroy the state upon measurement of this second time, since (this run of) the experiment is then over. We have thus determined the τ_j needed to calculate the τs of Eqs. (2) and (3).

If Δx were the width of the initial packet prepared at x_1, the the time t_1 at which the particle is (conditionally) known to be at x_1 would be

$t_1 \approx 0 \pm \Delta x/v_0$ (v_0 where is the speed of the incident packet). *In principle*, this error could be made as small as we like (for large enough v_0). The distance from x_1 to the left face of the barrier would have to be a few times Δx so that the initial packet's "birth" would have been completed before appreciable interference with the barrier begins. Similarly, the detector at x_2 should be out of the interference region. For a fairly well defined energy, the incident packet must be narrow in k-space, which means a fair spread Δx, usually several times d (the width of the barrier). This means that the distance $(x_2 - x_1)$ would be, say, 50–100d. Since the "free" transit time would then be on the order of $100d/v_0$, the tunneling time through the distance d inside the barrier would have to be nearly an order of magnitude or so greater than d/v_0 for any effect to be detectable. That is a possibility for the τ_Ts predicted in Bohm's theory and I return to this point below. On the basis of these cursory considerations, it does not appear that there is any point of *principle* (such as the Heisenberg uncertainty relations) that would necessarily preclude measuring at least τ_T. This would seem to allow the possibility to generate the experimental data necessary to test the predictions of Bohm's theory (or, indeed, of standard quantum theory if it is able to supply them). However, practical considerations, based on the actual numerical values of Planck's constant, the mass m of the electron, and the size of atomic dimensions (≈ 1 Å) present another class of constraints.

Let me now turn to some numerical estimates. If we are interested in discriminating between the predictions of Bohm's theory and those of standard quantum mechanics and consider measuring τ_T, then we must avoid any regime in which $T \approx 1$ since, for total transmission, we see from Eq. (4) that then $\tau_T \approx \tau_D$. Both Bohm's theory and standard quantum theory have Eqs. (5) available to compute τ_D so that there would be no difference here. That leaves two different classes of cases: $T \approx R \approx 1/2$ and $T \ll 1$.

If we want an appreciable part of the packet to penetrate the barrier, then we can easily estimate the order of magnitude of the relevant physical parameters required. Since, inside the barrier, the probability amplitude decays as $e^{-d/\lambda}$, where λ is the de Broglie wavelength, we require that $d/\lambda \approx 1$ or

$$\frac{\sqrt{2m(V_0 - E)}}{h} d \approx 1 \tag{8}$$

To be able to discriminate between the incident energy E and the barrier height V_0, we require that $V_0 - E$ and V_0 (or E) be comparable, say $V_0 - E \approx (1/2) V_0$. Then Eq. (8) becomes[31]

$$\frac{m}{\hbar^2} V_0 d^2 \approx 1 \tag{9}$$

Given the actual values of m (for an electron, the least massive particle available to us for such experiments), $\hbar$ and $d \approx 1$–10 Å, one easily sees that $V_0 \approx 0.10$–1.0 eV.[32] This corresponds to an incident beam velocity of about 10^5–10^6 m/s. The difficulty, however, is that the characteristic transit times are of the order of $t \approx d/v_0 \approx 10^{-16}$–$10^{-14}$ s. Even an extremely optimistic assessment of current or immediately foreseeable experimental technique for *direct* time measurements would place picosecond resolutions (that is, those on the order of 10^{-12} s) at the border of the doable. (In principle, however, there seems to be no reason that one could not do better, say femtoseconds, 10^{-15} s). Furthermore, numerical calculations[33] for Bohmian trajectories for parameters in this range ($d \approx 5$ Å, $V_0 \approx 10$ eV) yield values for τ_T in the range of 10^{-15} s so that the total measured transit times over the distance $(x_2 - x_1)$ would scarcely differ with the barrier in compared to the situation with the barrier removed (since $50d/v_0 \approx 10^{-14}$ s). The effect sought would not be detectable.

So, what about the regime in which $T \ll 1$? Here one would lose a lot of intensity, but the times τ_T can then be made to approach 10^{-14} s. Such calculations[34] (for $V_0 = 10$ eV, $E \approx 5$–10 eV, $d = 5$ Å, $\Delta k \approx 0.02$ Å^{-1}, or $\Delta x \approx 5d$) correspond to $T \approx 10^{-4}$–10^{-5}. Even though this does, at great cost to the detection rate, produce a value of τ_T *comparable* to $(x_2 - x_1)/v_0$, it does not circumvent the problem that the background time scale is on the order of 10^{-15} s. The prospects for a direct experimental determination of these tunneling times appear at best marginal at present.

I appreciate that the entire "experimental" procedure outlined here is extremely naïve and that the negative conclusions may be mainly a function of the way I have severely circumscribed the problem. However, if this question is perceived as being of sufficient interest, then those modern experimentalists who have made their reputations by routinely actually performing what had previously been considered mere *gedanken* experiments may, yet again, be motivated to do the impossible.[35]

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REFERENCES

1. N. Delruelle and A. Kypriadinis, eds., *Ann. Inst. Henri Poincaré/Phys. Théor.* **49** (3) (1988).
2. D. Bohm, *Phys. Rev.* **85**, 166, 180 (1952).
3. For recent presentations of this program, see D. Bohm and B. J. Hiley, *The Undivided Universe* (Routledge, Chapman & Hall, London, 1993); P. R. Holland, *The Quantum Theory of Motion* (Cambridge University Press, Cambridge, 1993); J. T. Cushing, *Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony* (University of Chicago Press, Chicago, 1994).
4. D. Bohm and J.-P. Vigier, *Phys. Rev.* **96**, 208 (1954).
5. D. Bohm, *Causality and Chance in Modern Physics* (University of Pennsylvania Press, Philadelphia, 1957); *Wholeness and the Implicate Order* (Routledge & Kegan Paul, London, 1980); D. Bohm, B. J. Hiley, and P. N. Kaloyerou, *Phys. Rep.* **144**, 321 (1987); J.-P. Vigier, *Astron. Nachr.* **303**, 55 (1982), and in E. I. Bitsakis and C. A. Nicolaides, eds., *The Concept of Probability* (Kluwer, Dordrecht, 1989), pp. 133–140.
6. D. Home, *Science Today* **20** (11), 25 (1986), p. 49. I thank Dr. Home for bringing this interview to my attention.
7. J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1980), p. 111.
8. W. Pauli, in *Handbuch der Physik* **24**, Pt. 1, 2nd edn. (Springer, Berlin, 1933), p. 140, gave an argument that time cannot be an operator T conjugate to the Hamiltonian H (energy operator) satisfying the commutation relation $[T, H] = i\hbar$. A standard argument shows that the existence of such a time operator would imply that the energy spectrum of H would always be continuous and could never be bounded from below. But that is contrary to observation.
9. For further discussion of the theoretical issues involved, see, for instance, E. H. Hauge and J. A. Støvneng, *Rev. Mod. Phys.* **61**, 917 (1989), and C. R. Leavens and G. C. Aers, in *Scanning Tunneling Microscopy III*, R. Wiesendanger and H.-J. Güntherodt, eds. (Springer Berlin, 1993), pp. 105–140.
10. In Section 3 I return to the question of how one might determine these times without causing a "collapse" of the wave function at position x_1.
11. Specifically, a consistent set of dwell, transmission, and reflection times must be real (as opposed to complex), positive quantities and satisfy the constraint of Eq. (4). The question of the ability or inability of standard quantum mechanics to define an acceptable set of τs is a controversial one and I do not pretend to resolve it here. See, for example, Hauge and Støvneng, Ref. 9; Leavens and Aers, Ref. 9; V. S. Olkhovsky and E. Recami, *Phys. Rep.* **214**, 339 (1992); D. Sokolovski and J. N. L. Connor, *Phys. Rev. A* **47**, 4677 (1993); S. Gull, A. Lasenby, and C. Doran, *Found. Phys.* **23**, 1329 (1993); R. Landauer and Th. Martin, *Rev. Mod. Phys.* **66**, 217 (1994); S. Brouard, R. Sala, and J. G. Muga, *Phys. Rev. A* **49**, 4312 (1994); C. R. Leavens, "Bohm trajectory and Feynman path approaches to the "tunneling time problem," *Found. Phys.*, to be published.
12. Hauge and Støvneng, Ref. 9, p. 917. They discuss this question of various tunneling times only within the framework of the standard interpretation of quantum mechanics and do not consider Bohm's interpretation.
13. See, for example, Leavens and Aers, Ref. 9, and references therein.
14. Hauge and Støvneng, Ref. 9, pp. 934–935.
15. Hauge and Støvneng, Ref. 9. See also M. Daumer, D. Dürr, S. Goldstein, and N. Zanghi, "Scattering and the role of operators in Bohmian mechanics," to be published.

16. R. C. Leavens, *Solid State Commun.* **74**, 923 (1990), has shown that Bohm's theory leads directly to Eqs. (5) and that the corresponding expressions for τ_T and τ_R satisfy Eq. (4). Leavens and Aers, Ref. 9 [Eqs. (6.14)–(6.21)], give explicit expressions for calculating these τs without actually having to compute the trajectories themselves.
17. C. Dewdney, G. Horton, M. M. Lam, Z. Malik, and M. Schmidt, *Found. Phys.* **22**, 1217 (1992), show graphically the time evolution of $x(t)$ for a collection of incident particles. (Their calculations correspond to values of the parameters for which $T = 0.5$.) Leavens, Ref. 16 and *Solid State Commun.* **76**, 253 (1990), **85**, 115 (1993), has done numerical calculations for the various τs. A more direct and efficient way to calculate these times is given in W. R. McKinnon and C. R. Leavens, "Distributions of delay times and transmission times in Bohm's causal interpretation of quantum mechanics" (*Phys. Rev. A*, to be published), who calculate not only the mean transmission time τ_T, but also the *distribution* of these transmission times (cf. Leavens and Aers, Ref. 9, pp. 120–123).
18. There is indication of extreme sensitivity to the overall *size* of the wave packet [Leavens (1990), Ref. 17, p. 261]. However, the τs themselves are given as *weighted* averages over the $|\phi(k)|^2$ (cf. Leavens and Aers, Ref. 9, pp. 112 and 114) so that they depend upon the integrated value of this distribution, rather than directly upon its local variations in k-space.
19. See, for example, P. Guéret, E. Marclay, and H. Meier, *Appl. Phys. Lett.* **53**, 1617 (1988); J. M. Martinis *et al.*, *Physica B* **152**, 159 (188); H. Q. Nguyen *et al.*, *IEEE Trans. Electron. Devices* **36**, 2671 (1989); P. Guéret, in J. M. Chamberlain, L. Eaves, and J.-C. Portal (eds.), *Electronic Properties of Multilayers and Low-Dimensional Semiconductor Structures* (Plenum Press, New York, 1990), pp. 317–329. M. Baublitz, *Phys. Rev. A* **47**, 2423 (1993), has recently suggested using the mean energy of tunneling electrons from similar experiments to compare the predictions of various versions of stochastic (*not* Bohmian) quantum mechanics with standard quantum mechanics.
20. Hauge and Støvneng, Ref. 9, pp. 921–923; M. Bütttiker, in J. M. Chamberlain *et al.*, Ref. 19, pp. 297–315, especially pp. 298–299.
21. For example, C. R. Leavens, and G. C. Aers, *Sold State Commun.* **78**, 1015 (1991), have shown that the mean transmission time extracted from the cutoff frequency of the transmission coefficient for a time-modulated barrier is *not* a reliable indicator of the actual barrier transmission time for a static barrier.
22. D. Esteve *et al.*, *Phys. Scr.* **T29**, 121 (1989).
23. R. Landauer, *Nature (London)* **341**, 567 (1989); Landauer and Martin, Ref. 11.
24. Esteve *et al.*, Ref. 22, p. 121.
25. Landauer, Ref. 23, p. 568.
26. A. M. Steinberg, P. G. Kwiat, and R. Y. Chaio, *Phys. Rev. Lett.* **71**, 7087 (1993); R. Y. Chiao, P. G. Kwiat, and A. M. Steinberg, *Sci. Am.* **269** (2), 52 (1993) and references therein.
27. A. Enders and G. Nimtz, *Phys. Rev. E* **48**, 632 (1993).
28. D. Bohm, Ref. 2, pp. 189–193.
29. See, for example, D. Heitmann, pp. 151–173, and M. van Rossum *et al.*, pp. 229–242, in Chamberlain *et al.*, Ref. 19; J. B. Barner and S. T. Ruggiero, *Phys. Rev. Lett.* **59**, 807 (1987).
30. It is in principle possible to prepare any specified initial wave function we want, as shown by W. E. Lamb, *Phys. Today* **22** (4), 23 (1969).
31. An exact treatment of the rectangular barrier [e.g., A. Messiah, *Quantum Mechanics*, Vol. I (North-Holland, Amsterdam, 1965, p. 97] leads to $(2m/h^2)\, V_0 d^2 \approx 1$ for appreciable barrier penetration for $E \leqslant V_0$.

32. An exact calculation for a rectangular barrier leads to a transmission coefficient of $T = 0.5$ when $(m/h^2)\, V_0 d^2 \approx 1$, which is consistent with the previous note and with my crude estimate here.
33. Leavens and Aers, Ref. 9.
34. Leavens and Aers, Ref. 9, pp. 118–119; Gull *et al.*, Ref. 11, especially pp. 1341-1343.
35. Because of space limitations, I have not addressed the important question of the predictive equivalence of Bohmian mechanics and standard quantum mechanics for the correlations of counter outputs in an *actual* (as opposed to an *ideal*) experiment. Even then though, "Bohm" and "Copenhagen" differ on the *meaning* given to these correlations.