C. RICHARD LEAVENS

THE "TUNNELING-TIME PROBLEM" FOR ELECTRONS

I. INTRODUCTION

The very concept of the motion of an electron during the time interval between its preparation in a given state and the subsequent "collapse" of the time-evolved state during a measurement is widely regarded as a meaningless one within conventional quantum mechanics. Despite this, a large number of theoretical papers, including several reviews (e.g., Hauge and Støvneng 1989; Landauer and Martin 1994), have been devoted to various characteristic times associated with the motion of a particle interacting with a potential barrier. Most of the approaches involve, at least implicitly, some relatively straightforward extension of the fundamental postulates of standard quantum theory. These will be referred to as "conventional" approaches to distinguish them from those based on alternatives to quantum theory, such as Bohmian mechanics. In this paper, the approach based on Bohm's theory is compared to several "conventional" methods that do not involve such radical departures from orthodoxy. In section 2, the characteristic times of interest are defined in words and the underlying reason for the difficulty in translating these words into unique, universally accepted mathematical expressions discussed. In Section 3, Bohmian mechanics is applied to the derivation of expressions for the mean dwell, transmission, reflection and arrival times. The systematic projector approach of Brouard *et al.* (1994) is considered in Section 4 where it is shown that none of the infinite number of possibilities for the mean transmission time that are generated by this method can be equal to the (unique) result from Bohmian mechanics. In Section 5, "conventional" probability current approaches are criticized from the point of view of Bohm's theory. In Section 6, the quantum clock of Salecker and Wigner (1958) is applied to the calculation of the mean and mean-square dwell time. The results for the free particle case (no barrier) are particularly illuminating. Concluding remarks are made in Section 7.

2. DEFINING THE PROBLEM

The often asked question "How long does it take to tunnel?" is at the same time both too vague (What precisely is meant by "it"?) and too specific (Why restrict one's attention to tunneling, i.e. propagation through a classically forbidden region?). The characteristic times of interest in this paper will be precisely defined (at least from the point of view of Bohmian mechanics) and no restriction will be made to the tunneling regime.

J. T. Cushing et al. (eds.), Bohmian Mechanics and Quantum Theory: An Appraisal, 111–129.

Consider an ensemble of a very large number of identically prepared single-particle one-dimensional scattering experiments. In each, a Schrödinger electron with the same initial wave function $\psi(z, t = 0)$ is incident from the left on the potential barrier $U(z) = V(z)\Theta(z)\Theta(d - z)$ assumed to be zero outside the range $0 \leq z \leq d$ [$\Theta(z) = 1$ for $z > 0$ and 0 for $z < 0$]. The mean transmission (reflection) time $\tau_{T(R)}(z_1, z_2)$ is defined as the average time spent in the region $z_1 \leq z \leq z_2$ subsequent to $t = 0$ by those electrons that are ultimately transmitted (reflected). The mean dwell time $\tau_D(z_1, z_2)$ is the average time spent by electrons between z_1 and z_2 irrespective of their ultimate fate. It is assumed throughout that the initial wave packet is normalized to unity and is localized far enough to the left of the barrier that the integrated probability density $|\psi(z, 0)|^2$ from z = Minimum[z_1, 0] to ∞ is negligibly small compared to the transmission probability $|T|^2$, usually by a factor of 10^{-4} or smaller.

The above three characteristic times are intrinsic quantities in the sense that there is no reference to a measuring apparatus in their definitions (other than the implied particle detector well outside the region of interest to determine whether or not an incident electron is eventually transmitted). Because of the position-momentum uncertainty relation it is clear that these intrinsic quantities cannot in general be measured directly by placing stop-watch type particle detectors at z_1 and z_2 (Cushing 1995a). The difficulty with calculating them using the standard formulation of quantum mechanics stems from the fact that there are no universally accepted hermitian operators $\hat{t}_X(z_1, z_2)$ associated with the time spent by a particle of type $X(= T, R$ or $D)$ in the region $[z_1, z_2]$.[1] As regards the mean arrival time $\tau_A(b)$ at the point $z = b$, Allcock (1969) has proven that there does not exist a complete set of measurement eigenstates associated with the time of arrival of a particle at a spatial point, not even for a free particle.

A frequently heard objection to the notion of transmission and reflection times is that they imply the existence of microscopically well-defined particle trajectories, a concept which is expressly forbidden in orthodox quantum mechanics on the grounds that it is impossible, even in principle, to observe such trajectories because of the position-momentum uncertainty relation. Such trajectories, although unobservable,[2] are of central importance in Bohm's causal alternative to quantum mechanics (Bohm 1952a,b; Bohm, Hiley and Kaloyerou 1987; Bell 1987a; Bohm and Hiley 1993; Holland 1993; Dürr *et al.* 1992a; Valentini 1991a,b; Albert 1994; Cushing 1994). In the next section, Bohm's theory is used to derive expressions for the characteristic times introduced above.

3. BOHM TRAJECTORY APPROACH

In Bohmian mechanics, tailored to the problem of interest here, it is postulated that an electron is an actual particle that is always accompanied by a field which probes the potential $U(z)$ and guides the particle's motion accordingly so that it has a well defined position $z(t)$ and velocity $v(t)$ at each instant of time. It is further postulated that the guiding field is the solution $\psi(z, t)$ of the time-dependent Schrödinger equation and that the particle's equation of motion is $v(t) \equiv dz(t)/dt = v(z, t)\,|_{z = z(t)}$ where the velocity field $v(z, t)$ is given by

$$(1) \qquad v(z, t) = \frac{J(z, t)}{|\psi(z, t)|^2},$$

with $J(z, t) \equiv (\hbar/m)\mathrm{Im}[\psi^*(z, t)\partial\psi(z, t)/\partial z]$ the probability current density. It follows from these postulates that $|\psi(z, t)|^2\, dz$ is, as assumed by Bohm, the probability of the electron *being* between z and $z + dz$ at time t (Dürr *et al.* 1992a; Valentini 1991a).

Now, given the initial wave function $\psi(z, t = 0)$ and position $z^{(0)} \equiv z(t = 0)$ of an electron, its subsequent motion is uniquely determined by simultaneous integration of the time-dependent Schrödinger equation for $\psi(z, t)$ and the equation of motion for $z(t)$ to obtain the Bohm trajectory $z(z^{(0)}, t)$. Uncertainty enters only through the probability distribution $|\psi(z^{(0)}, 0)|^2$ for the unknown initial position $z^{(0)}$ of the particle. Hence, the ensemble average of a particle property f, say, can be obtained by determining the value $f(z^{(0)})$ of the property for a particle following the trajectory $z(z^{(0)}, t)$ and then averaging over all $z^{(0)}$:

$$(2) \qquad \langle f(z^{(0)})\rangle_{\mathrm{Bohm}} \equiv \int_{-\infty}^{\infty} dz^{(0)} |\psi(z^{(0)}, 0)|^2 f(z^{(0)}).$$

This simple prescription is now applied to the various characteristic times defined above.

For a particle that is at $z = z^{(0)}$ at $t = 0$ the time that it spends thereafter in the region $[z_1, z_2]$ is given by the classical stop-watch expression

$$(3) \qquad t(z_1, z_2; z^{(0)}) = \int_0^{\infty} dt \int_{z_1}^{z_2} dz\ \delta[z - z(z^{(0)}, t)].$$

The mean dwell time is then given by

$$(4) \qquad \begin{aligned} \tau_D(z_1, z_2) &= \langle t(z_1, z_2; z^{(0)})\rangle_{\mathrm{Bohm}} = \int_{-\infty}^{\infty} dz^{(0)} |\psi(z^{(0)}, 0)|^2\, t(z_1, z_2; z^{(0)}) \\ &= \int_0^{\infty} dt \int_{z_1}^{z_2} dz \int_{-\infty}^{\infty} dz^{(0)} |\psi(z^{(0)}, 0)|^2\, \delta[z - z(z^{(0)}, t)]. \end{aligned}$$

The innermost integral is just the probability distribution of particle positions at time t, i.e. $|\psi(z, t)|^2$, leading immediately to the widely accepted expression

$$(5) \qquad \tau_D(z_1, z_2) = \int_0^{\infty} dt \int_{z_1}^{z_2} dz\ |\psi(z, t)|^2.$$

An important property of Bohm trajectories $z(z^{(0)}, t)$ with different $z^{(0)}$ (but the same $\psi(z,0)$!) is that they do not intersect each other: if $z_i^{(0)} \neq z_j^{(0)}$, then $z(z_i^{(0)}, t) \neq z(z_j^{(0)}, t)$ for any t (Leavens and Aers 1993, 111; Holland 1993, 85). This fact allows the probability density $|\psi(z, t)|^2$ in

(5) to be decomposed into components $|\psi(z,t)|^2\Theta[z - z_c(t)]$ and $|\psi(z,t)|^2\Theta[z_c(t) - z]$ associated with transmission and reflection, respectively. The bifurcation curve $z_c(t)$ separating transmitted trajectories from reflected ones is the trajectory $z(z_c^{(0)}, t)$ given implicitly by

$$|T|^2 = \int_{z_c(t)}^{\infty} dz\, |\psi(z, t)|^2 \tag{6}$$

where $|T|^2$ is the transmission probability (the reflection probability, $1 - |T|^2$, is denoted by $|R|^2$). Inserting $1 \equiv \Theta[z - z_c(t)] + \Theta[z_c(t) - z]$ into the integrand of (5) immediately gives

$$\tau_D(z_1, z_2) = |T|^2\, \tau_T(z_1, z_2) + |R|^2\, \tau_R(z_1, z_2), \tag{7}$$

$$\tau_T(z_1, z_2) = \frac{1}{|T|^2} \int_0^{\infty} dt \int_{z_1}^{z_2} dz |\psi(z, t)|^2 \Theta[z - z_c(t)], \tag{8a}$$

$$\tau_R(z_1, z_2) = \frac{1}{|R|^2} \int_0^{\infty} dt \int_{z_1}^{z_2} dz |\psi(z, t)|^2 \Theta[z_c(t) - z]. \tag{8b}$$

Within Bohmian mechanics, the average characteristic times $\tau_{X=T,R,D}(z_1, z_2)$ are uniquely defined and each is obviously a real, non-negative, additive[3] quantity. Including the effect of some proposed "measurement" device on the particle trajectories could very well lead to significant departures of the calculated characteristic times from the above intrinsic values but cannot lead to complex-valued or negative results. Hence, if the pointer positions of an ensemble of such devices indicate a negative result for one or more of the characteristic times τ_X then, *from the point of view of Bohmian mechanics*, the devices cannot be properly timing either the intrinsic or perturbed motion of the particles.

Probability distributions $P_X[t_X(z_1, z_2)]$ for the various times can be computed by a straightforward histogram method involving the calculation of a very large number of trajectories $z(z^{(0)}, t)$ and for each adding the weight $|\psi(z^{(0)}, 0)|^2$ to the channel determined by $t_X(z_1, z_2; z^{(0)})$. Several examples are given by Leavens and Aers (1993, 120–132). A much more efficient method, based on the probability current density $J(z, t)$, has been developed (McKinnon and Leavens 1995).

Since transmitted electrons are associated with the leading $|T|^2$ part of $|\psi(z, t)|^2$ it is clear that (8a) with $(z_1, z_2) = (0, d)$ will not in general agree with the mean transmission time $\tau_T(0, d)$ derived on the assumption that the peak (or centroid) of the incident wave packet evolves into the peak (or centroid) of the transmitted wave packet (Hauge, Falck and Fjeldly 1987). As is well-known (Hartman 1962, 3431) the latter approach can lead to superluminal (i.e. $d/\tau_T(0, d) > c$) and even negative values of $\tau_T(0, d)$.

For stationary-state scattering of "incident" electrons with precisely defined wave number $k > 0$ and energy $E \equiv \hbar^2k^2/2m$, expression (5) for the mean dwell time becomes (Leavens and Aers 1990, 60)

(9) $$\tau_D(k;\, z_1,\, z_2) = \frac{1}{J_{k,\,i}} \int_{z_1}^{z_2} dz |\psi_k(z)|^2,$$

where the stationary-state wave function $\psi_k(z)\exp(-iEt/\hbar)$ is normalized so that the incident probability current density $J_{k,i}$ associated with the plane wave $\exp(ikz)$ is $\hbar k/m$. Spiller *et al.* (1990) postulated that for $|T(k)|^2 > 0$ the corresponding mean transmission time is given by

(10) $$\tau_T(k;\, z_1,\, z_2) = \int_{z_1}^{z_2} \frac{dz}{v_k(z)},$$

where $v_k(z) \equiv J_k \,/\, |\psi_k(z)|^2$ with $J_k \equiv |T(k)|^2 J_{k,i}$ the stationary-state probability current density. Since $v_k(z)$ is independent of $z^{(0)}$ it follows that all the transmitted trajectories are parallel to each other and hence that $P_T[t_T(k;\, z_1,\, z_2)] = \delta[t_T(k;\, z_1,\, z_2) - \tau_T(k;\, z_1,\, z_2)]$ for $z_1 > -\infty$. It follows immediately from (9) and (10) that $\tau_T(k;\, z_1,\, z_2) = |T(k)|^{-2}\tau_D(k;\, z_1,\, z_2)$ and then, from (7), that $|R(k)|^2\tau_R(k;\, z_1,\, z_2) = 0$. Hence for $0 < |R(k)|^2 < 1$ it follows that $\tau_R(k;\, z_1,\, z_2) = 0$ and that $P_R[t_R(k;\, z_1,\, z_2)] = \delta[t_R(k;\, z_1,\, z_2)]$ for $z_1 > -\infty$. That, in the stationary-state limit, reflected electrons never enter the region $[z_1,\, z_2]$ can be counter-intuitive if one's intuition is based on pictures of the time evolution of wave packets. The author accordingly tends to regard the result as a peculiarity not of Bohmian mechanics but of the stationary-state limit itself, namely the assumed coherence of a single-particle state over all of space and time from $t = -\infty$.

For an example of the behaviour of $\tau_{X\,=\,T,\,R,\,D}(0,\, d)$, consider an initial gaussian wave function

(11) $$\psi(z,\, t = 0) = [2\pi(\Delta z)^2]^{-1/4} \exp\left[-\left(\frac{z - z_0}{2\Delta z}\right)^2 + ik_0 z\right],$$

with $E^{(0)} \equiv \hbar^2 k_0^2/2m = 5eV$ and $z_0 \cong -4.70\Delta z$ incident on a rectangular barrier $V_0\Theta(z)\Theta(d - z)$ with $V_0 = 2E_0$ and $d = 2.5\text{Å}$. Figure 1 shows the dependence of $\tau_D(0,\, d)$, $|T|^2\tau_T(0,\, d)$ and $|R|^2\tau_R(0,\, d)$ on the spread in wave number $\Delta k = 1/2\Delta z$ (Leavens *et al.* 1995). The parameters were chosen to illustrate the approach to the effective stationary-state regime $\Delta k/k_0 \lesssim 2^{1/2}e^{-2k_0 d}$ (for $E_0 = V_0/2$ and $|T(k_0)|^2 \ll 1$) in which $|R|^2\tau_R(0,\, d) \ll \tau_D(0,\, d)$ and $\tau_T(k_0;\, 0,\, d)$ is a good approximation to $\tau_T(0,\, d)$.

Now consider the distribution of arrival times at a point $z = b \geq d$ on the far side of the barrier *assuming that there are no reentrant Bohm trajectories through this point.* For $z^{(0)}$ in the range $[z_c^{(0)},\, b]$ let $t(b;\, z^{(0)})$ denote the time at which an electron following the trajectory $z(z^{(0)},\, t)$ reaches $z = b$. Trajectories with $z^{(0)} < z_c^{(0)}$ are reflected ones and, in keeping with the above assumption, never reach $z = b$. For these $t(b;\, z^{(0)})$ is a meaningless quantity (never is not equated with $+\infty$ in our analysis) and trajectories with $z^{(0)} < z_c^{(0)} \equiv z_c(0)$ are excluded from the outset. Those with $z^{(0)} \geq b$ should also be excluded but since the initial state is prepared so that trajectories with $z^{(0)} > b \geq d$ are of negligible importance, this fact will simply be

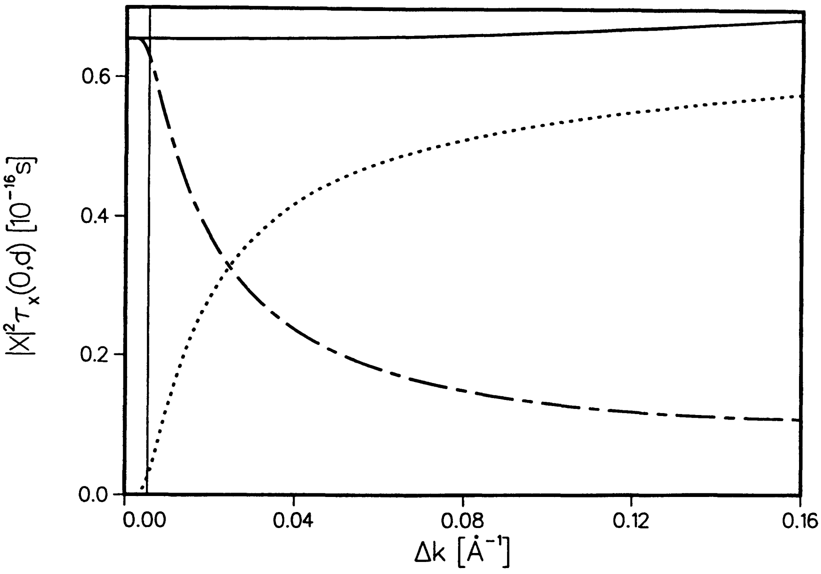

Fig. 1. Dependence on Δk of the transmission (— - —) and reflection (----------) contributions, $|T|^2\tau_T(0, d)$ and $|R|^2\tau_R(0, d)$ respectively, to the mean dwell time (————). The value of Δk below which (10) is expected to be good approximation is indicated by the vertical line. The parameters of the initial wave function and of the scattering potential are given in the text.

ignored. The distribution of arrival times at $z = b$ is then given by

$$(12)\qquad P_A[t(b)] = \left(\int_{z_c^{(0)}}^{\infty} dz^{(0)}|\psi(z^{(0)}, 0)|^2\right)^{-1} \int_{z_c^{(0)}}^{\infty} dz^{(0)}|\psi(z^{(0)}, 0)|^2\delta[t(b) - t(b; z^{(0)})].$$

From (6), the first factor is $|T|^{-2}$. Now

$$(13)\qquad \delta[z(z^{(0)}, t) - b] = \frac{\delta[t - t(b; z^{(0)})]}{|dz(z^{(0)}, t)/dt|} = \frac{\delta[t - t(b; z^{(0)})]}{|v[z(z^{(0)}, t), t]|} .$$

There is only one term because of the assumption that for $z_c^{(0)} \leq z^{(0)} \leq b$ the trajectory $z(z^{(0)}, t)$ passes through $z = b$ once and only once for $t > 0$. Hence,

$$(14)\qquad P_A[t(b)] = \frac{|v[b, t(b)]|}{|T|^2} \int_{z_c^{(0)}}^{\infty} dz^{(0)}|\psi(z^{(0)}, 0)|^2\delta[z(z^{(0)}, t(b)) - b]$$

$$= \frac{v[b, t(b)]\ |\psi[b, t(b)]|^2}{|T|^2} = \frac{J[b, t(b)]}{|T|^2} ,$$

using (1). The absolute value sign has been dropped because $v[b, t(b)]$ can not be negative when there are no reentrant trajectories through

$z = b > d$. When this restrictive assumption is removed one obtains after a more lengthy analysis

$$(15) \qquad P[t(b)] = \frac{|J[b, t(b)]|}{\int_0^\infty \mathrm{d}t\, |J(b, t)|}.$$

The mean arrival time $\tau_A(b)$ is obtained by integrating $t(b)P_A[t(b)]$ over $t(b)$ from 0 to ∞. When (14) is used for $P_A[t(b)]$ one immediately obtains the result derived by Dumont and Marchioro (1993) using "conventional" quantum mechanics. That their result is obtained using (14) rather than (15) suggests that their derivation contains an implicit assumption corresponding to the above one of no reentrant trajectories through $z = b$. Such an assumption can be difficult to articulate without the concept of *actual* particle trajectories. This concept is not an *explicit* ingredient of the "conventional" approaches considered in the next three sections. For each method, the reader should judge for himself whether or not it slips into the interpretation of the mathematics as an implicit assumption.

4. THE SYSTEMATIC PROJECTOR APPROACH

Brouard *et al.* (BSM) (1994) have developed a systematic approach to the determination of a class of possible expressions for τ_T and τ_R. It is based on the following projections:

$$(16) \qquad \hat{D}(z_1, z_2)\psi(z, t) \equiv \Theta(z - z_1)\Theta(z_2 - z)\psi(z, t),$$

$$(17) \qquad \hat{T}\psi(z, t) \equiv \psi_T(z, t), \qquad \hat{R}\psi(z, t) \equiv \psi_R(z, t).$$

The operator $\hat{D}(z_1, z_2)$ associated with the question "Is the particle between z_1 and z_2?" projects from the wave function $\psi(z, t)$ at any time t the part that is "located" in that region. The operator $\hat{T}$ associated with the question "Will the particle ultimately be transmitted?" projects from $\psi(z, t)$ the part which will have only positive wave number components at $t = +\infty$, i.e. the part $\psi_T(z, t)$ associated with transmission. $\hat{R} \equiv \hat{1} - \hat{T}$ projects from $\psi(z, t)$ the part $\psi_R(z, t)$ associated with reflection. For $|T|^2 < 1$, $\psi_T(z, t = 0)$ has two components, one to the left and one to the right of the barrier. For an opaque barrier the latter component completely dominates the former. [Figures 10 and 11 of Leavens (1995b) show snapshots of $\psi_T(z, t)$ and $\psi_R(z, t)$, for the case of an initial gaussian wave packet incident on a rectangular barrier, at a sequence of times t spanning the duration of the scattering process.]

The probability of *finding* that an electron is in the region $z_1 \leq z \leq z_2$ at time t is obtained by integrating $\psi^*(z, t)\, \hat{D}(z_1, z_2)\psi(z, t)$ over all z. This is standard. Integration of the resulting expression over t from 0 to ∞ immediately gives the right-hand-side of (5) for the mean dwell time $\tau_D(z_1, z_2)$. Experimentally, the latter integration would apparently require either strong (i.e. instantaneous, wave function collapsing) $\hat{D}$ measurements on a very large number of ensembles, one for each time t in the summation, or temporally extended weak (i.e. non wave function collapsing) measurements on a single ensemble. Neither approach would likely be widely accepted at the present time as part of conventional quantum mechanics.

Now, consider the following two resolutions of $\hat{D}$ into two parts, one associated only with transmission and one only with reflection:

$$\hat{D} = \begin{cases} \hat{1}\hat{D} = (\hat{T} + \hat{R})\hat{D} = \hat{T}\hat{D} + \hat{R}\hat{D}, \\ \hat{D}\hat{1} = \hat{D}(\hat{T} + \hat{R}) = \hat{D}\hat{T} + \hat{D}\hat{R}. \end{cases} \tag{18}$$

Since $\hat{T}$ and $\hat{D}$ do not commute with each other there is no *a priori* reason for interpreting either the spatial integral over all z of $\psi^*(z, t)\hat{D}(z_1, z_2)\hat{T}\psi(z, t)$ or of $\psi^*(z, t)\hat{T}\hat{D}(z_1, z_2)\psi(z, t)$ as the probability of finding that a "to be transmitted" electron is in the region $z_1 \leq z \leq z_2$ at time t. BSM acknowledged that the proper response to this dilemma could very well be to regard the concept of transmission time as a meaningless one. However, they adopted the positive response of investigating the properties of the mean transmission times associated with different combinations of $\hat{T}$ and $\hat{D}$ such as $\hat{T}\hat{D}$, $\hat{D}\hat{T}$ and the (hermitian) symmetrized operator $(\hat{T}\hat{D})_{\text{symm}} = (\hat{T}\hat{D} + \hat{D}\hat{T})/2$. Since $\hat{D}$ is a projection operator ($\hat{D}^2 = \hat{D}$) this approach can be continued indefinitely by systematically inserting one or more factors of $\hat{1} \equiv \hat{T} + \hat{R}$ into $\hat{D}^n$ with $n = 1, 2, \ldots$ and leads to an infinite number of resolutions of $\hat{D}$ and an infinite number of possible mean transmission, reflection and interference times:

$$\hat{D} = F(\hat{T}, \hat{D}) + F(\hat{R}, \hat{D}) + G(\hat{T}, \hat{R}, \hat{D}), \tag{19}$$

$$\tau_T^{F(T,D)}(z_1, z_2) = \frac{1}{|T|^2} \int_0^\infty \mathrm{d}t \int_{-\infty}^{\infty} \mathrm{d}z\, \psi^*(z, t) F(\hat{T}, \hat{D}) \psi(z, t), \tag{20a}$$

$$\tau_R^{F(R,D)}(z_1, z_2) = \frac{1}{|R|^2} \int_0^\infty \mathrm{d}t \int_{-\infty}^{\infty} \mathrm{d}z\, \psi^*(z, t) F(\hat{R}, \hat{D}) \psi(z, t), \tag{20b}$$

$$\tau_{T-R}^{G(T,R,D)}(z_1, z_2) = \int_0^\infty \mathrm{d}t \int_{-\infty}^{\infty} \mathrm{d}z\, \psi^*(z, t) G(\hat{T}, \hat{R}, \hat{D}) \psi(z, t), \tag{20c}$$

satisfying

$$\tau_D(z_1, z_2) = |T|^2 \tau_T^{F(T,D)}(z_1, z_2) + |R|^2 \tau_R^{F(R,D)}(z_1, z_2) + \tau_{T-R}^{G(T,R,D)}(z_1, z_2). \tag{21}$$

BSM showed that some of the simplest resolutions of $\hat{D}$ lead to mean transmission and reflection times that are identical to ones obtained by a variety of well-known approaches. For example,

$$\tau_T^{TD}(z_1, z_2) = \frac{1}{|T|^2} \int_0^\infty \mathrm{d}t \int_{z_1}^{z_2} \mathrm{d}z\, \psi_T^*(z, t) \psi(z, t), \tag{22a}$$

$$\tau_R^{RD}(z_1, z_2) = \frac{1}{|R|^2} \int_0^\infty \mathrm{d}t \int_{z_1}^{z_2} \mathrm{d}z\, \psi_R^*(z, t) \psi(z, t), \tag{22b}$$

are equal to the complex-valued mean transmission and reflection times derived by Sokolovski and co-workers (Sokolovski and Baskin 1987; Sokolovski and Connor 1990) using the (appropriately weighted) Feynman paths connecting the space-time points ($z < 0$, $t = 0$) and ($z' > d$, t_∞) for transmission or ($z' < 0$, t_∞) for reflection to decompose the mean dwell time. (The time t_∞ must be large enough that the scattering process is essentially completed.) There is no interference term in this case and the sum rule (7) is satisfied exactly.

BSM also showed that $\tau_T^{(TD)\text{symm}} = \text{Re}[\tau_T^{TD}]$ and $\tau_R^{(RD)\text{symm}} = \text{Re}[\tau_R^{RD}]$ which are identical to the mean transmission and reflection times derived by Rybachenko (1967) using the Larmor precession of an electron's spin in a magnetic field as a clock (Baz' 1967).[4] They are also identical to the results of Iannaccone and Pellegrini (1994) based on a different Feynman path integral approach, of Leavens and McKinnon (1994) using the Salecker–Wigner (1958) clock, and of Steinberg (1995a; 1995b) based on the weak measurement theory of Aharonov and his coworkers (Aharonov *et al.* 1988; Aharonov and Vaidman 1990). Steinberg's work is especially noteworthy for two reasons: (1) Giving the post-selected state $\psi_T(z, t_\infty)$ or $\psi_R(z, t_\infty)$ the same status as the initially prepared state $\psi(z, 0)$ automatically leads within weak measurement theory to the unique choice $F(\hat{T}, \hat{D}) = \hat{T}\hat{D}$, $F(\hat{R}, \hat{D}) = \hat{R}\hat{D}$, $G(\hat{T}, \hat{R}, \hat{D}) = 0$. (2) The theory reveals clearly the physical meaning of the real *and* imaginary parts of (22). The real part of τ_T^{TD} is the average elapsed time indicated by the pointers on those stopwatches belonging to the subensemble for which the scattered electron is transmitted; the imaginary part is proportional to the average momentum shift $\langle \Delta P_{\text{pointer}} \rangle_T$ of these pointers arising from the assumed particle-pointer interaction $\lambda t_\infty^{-1} \Theta(t) \Theta(t_\infty - t) \hat{D}(z_1, z_2) \hat{P}_{\text{pointer}}$. This average momentum shift goes to zero in the limit of vanishingly small precision in the initial position of the pointer.

As a final example, the decomposition $\hat{D} = \hat{1}\hat{D}\hat{1} = (\hat{T} + \hat{R})\hat{D}(\hat{T} + \hat{R}) = \hat{T}\hat{D}\hat{T} + \hat{R}\hat{D}\hat{R} + \hat{T}\hat{D}\hat{R} + \hat{R}\hat{D}\hat{T}$ leads to the mean transmission, reflection and interference times introduced by Muga *et al.* (1992) and van Tiggelen *et al.* (1993).

An important and useful property of the mean transmission and reflection times generated by the systematic approach of BSM (1994) is expressed by

$$\text{(23a)} \quad \tau_T^{F(T,D)}(z_1, z_2) = \frac{1}{|T|^2} \int_0^\infty \frac{dk}{2\pi} |\varnothing(k)|^2 |T(k)|^2 \tau_T^{F(T,D)}(k; z_1, z_2),$$

$$\text{(23b)} \quad \tau_R^{F(R,D)}(z_1, z_2) = \frac{1}{|R|^2} \int_0^\infty \frac{dk}{2\pi} |\varnothing(k)|^2 |R(k)|^2 \tau_R^{F(R,D)}(k; z_1, z_2),$$

where $\varnothing(k)$ is the Fourier transform of $\psi(z, 0)$, $|T(k)|^2$ and $|R(k)|^2$ are the transmission and reflection probabilities, and $\tau_T^{F(T,D)}(k; z_1, z_2)$ and $\tau_R^{F(R,D)}(k; z_1, z_2)$ the mean transmission and reflection times for the stationary-state scattering problem. The derivation of (23a) and (23b) is critically dependent on the fact that the integrands of (22a) and (22b) are bilinear in $\psi(z, t)$. This is not the case for the integrands of (8a) and (8b) for the corresponding Bohm trajectory results where $\psi(z, t)$ enters non-bilinearly because $z_c(t)$ defined by (6) is itself an implicit functional of $\psi(z, t)$. Hence, equations of the form (23a) and (23b) do not apply to the average Bohm trajectory transmission and reflection times.[5] This means that there is no conceivable choice of F for which $\tau^F_{T(R)}$ is equal to expression (8a(b)) for $\tau_{T(R)}$ for all initial wave packets and potential barriers. This has been confirmed (Leavens 1995a, 91) by an explicit counter example to the conjecture that such a choice exists.

Now consider the special case $F(T, D) = (TD)_{\text{symm}}$ and $F(R, D) = (RD)_{\text{symm}}$ for the stationary-state scattering of electrons by a rectangular barrier. It is well-known (Rybachenko 1967) that $\tau_T^{(TD)\text{symm}}(k; 0, d)$ is essentially independent of d in the opaque barrier regime $\kappa d \gg 1$, with $\kappa \equiv [2m(V_0 - E)]^{1/2}/\hbar$, where $|T(k)|^2 \propto \exp(-2\kappa d)$. Calculations of $\tau_T^{(TD)\text{symm}}(k; 0, z)$ for such a barrier imply that transmitted electrons move through the interior ($\kappa^{-1} \lesssim z \lesssim d - \kappa^{-1}$) of the barrier exceedingly quickly and that superluminal speeds occur at the center $z = d/2$ for quite ordinary barrier parameters (e.g. $d \gtrsim 6\text{Å}$ for $V_0 = 2E = 10eV$) (Leavens and Aers 1990, 66).[6] An even more surprising result (Leavens and McKinnon 1994, 16) is obtained by considering reflected electrons in the region $[z_1 = d, z_2 = z]$ on the far side of the barrier. For the special case $V_0 = 2E$ it is easy to show that $\tau_R^{(RD)\text{symm}}(k; d, z) \leq 0$. There is no doubt that the stop-watch pointers for the reflected electron subensemble would, on average, be displaced backwards if the corresponding weak $\hat{D}$ measurement were to be carried out on each system. However, in the author's opinion this should not be interpreted to mean that a reflected electron, either as an actual particle or even as encoded information, can spend a negative amount of time in the region $[d, z]$.

For the non-stationary case, Steinberg (1995b) has suggested that the complex-valued quantities $\psi_T^*(z, t)\psi(z, t)$ and $\psi_R^*(z, t)\psi(z, t)$ appearing in (22) as a result of the decomposition $|\psi|^2 = (\psi_T + \psi_R)^*\psi$ be considered as "elements of reality". For $|T|^2 < 1$, both $\psi_T^*(z, 0)\psi(z, 0)$ and $\psi_R^*(z, 0)\psi(z, 0)$ are very much more strongly localized on the left side of the barrier than are $\psi_T(z, 0)$ and $\psi_R(z, 0)$. Figure 2 shows three snapshots of $\text{Re}[\psi_R^*(z, t)\psi(z, t)]$ for z inside and immediately to the right of a thin rectangular barrier with $d = 1.5\text{Å}$ and $V_0 = 2E_0 = 10\ eV$ (the centroid and width of the initial incident gaussian are $z_0 = -38.9\text{Å}$ and $\Delta z = 6.25\text{Å}$). The oscillations about zero for $z \geq d$ are of decreasing amplitude with the first one negative. This is the origin of the negative values of $\tau_R^{(RD)\text{symm}}(d, z) = \text{Re}[\tau_R^{RD}(d, z)]$ obtained from (22b).

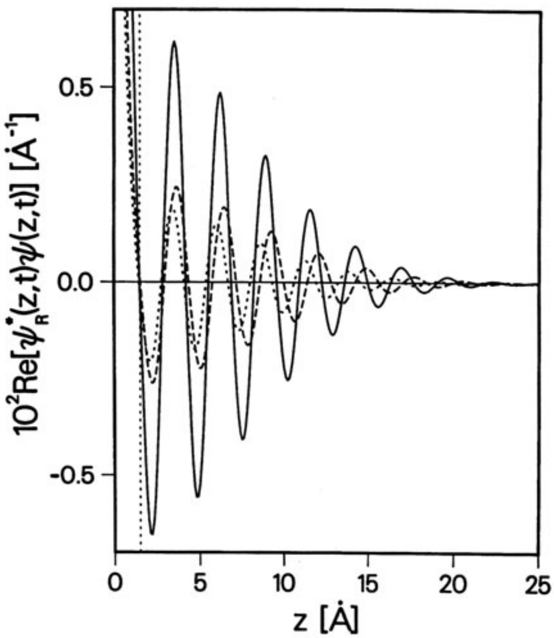

Fig. 2. The "element of reality" $\mathrm{Re}[\psi_R^*(z, t)\psi(z, t)]$ for $z > 0$ and $t = nt_0/4$ [$t_0 \equiv -z_0/(\hbar k_0/m)$] with $n = 3$ (----------), 4 (———) and 5 (———). The parameters are given in the text.

5. "CONVENTIONAL' PROBABILITY CURRENT DENSITY APPROACH

It has been postulated (Olkhovsky and Recami 1992; Muga, Brouard and Sala 1992) that the mean transmission and reflection times for the barrier region can be expressed in terms of the probability current density $J(z, t)$ as

$$\text{(24a)}\quad \tau_T^J(0, d) = \frac{\int_0^\infty dt\, t\, J_+(d, t)}{\int_0^\infty dt\, J_+(d, t)} - \frac{\int_0^\infty dt\, t\, J_+(0, t)}{\int_0^\infty dt\, J_+(0, t)},$$

$$\text{(24b)}\quad \tau_R^J(0, d) = \frac{\int_0^\infty dt\, t\, J_-(0, t)}{\int_0^\infty dt\, J_-(0, t)} - \frac{\int_0^\infty dt\, t\, J_+(0, t)}{\int_0^\infty dt\, J_+(0, t)}.$$

The first term on the right side of (24a) is interpreted as the mean exit time of particles from the barrier through its right edge at $z = d$ and the first term on the right side of (24b) as the mean exit time through its left edge at $z = 0$. The second term on the right side of both expressions is interpreted as the mean time at which incident particles enter the barrier.

The right-going and left-going components $J_+(z, t)$ and $J_-(z, t)$ respectively, of the probability current density are not uniquely defined within conventional quantum mechanics and many would accordingly regard them as ill-defined or meaningless quantities. Olkhovsky and Recami (1992) postulated that

$$(25) \qquad J_\pm(z, t) = J(z, t)\, \Theta\, [\pm J(z, t)],$$

while Muga *et al.* (1992) assumed that

$$(26) \qquad J_+(z, t) = \int_0^\infty \mathrm{d}p \left(\frac{p}{m}\right) f_W(z, p, t),$$

$$J_-(z, t) = \int_{-\infty}^0 \mathrm{d}p \left(\frac{p}{m}\right) f_W(z, p, t),$$

where $p \equiv \hbar k/m$ is free particle momentum and $f_W(z, p, t)$ is the Wigner function. In both cases $J(z, t) = J_+(z, t) + J_-(z, t)$ with no interference term.

The last term of (24a) is identical to the last term of (24b). Olkhovsky and Recami and also Muga *et al.* considered this a strong point in favour of the approach because it reflects the fact that within conventional quantum mechanics one cannot separate $J_+(0, t)$ into "to be transmitted" and "to be reflected" components $[J_+(0, t)]_T$ and $[J_+(0, t)]_R$. Olkhovsky *et al.* (1994) argue that this non-separability arises from quantum interference terms in $J_+(0, t)$. [Straightforward substitution of the wave function decomposition $\psi = \psi_T + \psi_R$ of the projector approach into the expression for J leads to $J = J_T + J_R + J_{T-R}$ with a $T - R$ interference term J_{T-R} that is nonzero in general.] However, the last term of (24a) and (24b), being a quotient of two integrals involving $J_+(0, t)$, is of the form $[A_T + A_R + A_{T-R}]/[B_T + B_R + B_{T-R}]$ and hence is in general non-separable even if the interference terms A_{T-R} and B_{T-R} are zero. This means that (24a) and (24b) do not apply in general even to the corresponding classical scattering problem as is clearly shown by the simple example constructed by Delgado *et al.* (1995).

It is instructive to look at (24a) and (24b) from the point of view of Bohmian mechanics. Although innumerable possibilities for the decomposition $J = J_+ + J_-$ have been suggested (Leavens 1995a, 92), Bohm's theory allows only one of these, namely the decomposition (25) postulated by Olkhovsky and Recami. This follows readily from the fact that within Bohmian mechanics only a single particle trajectory contributes to $J(z, t) \equiv |\psi(z, t)|^2\, v(z, t)$ at each space-time point (z, t) because of the non-intersection property. Moreover, the existence of deterministic non-intersecting trajectories also leads to a unique decomposition of $J(z,t)$ into components associated only with transmission or reflection:

$$(27) \qquad J(z, t) \equiv [J(z, t)]_T + [J(z, t)]_R$$

with

$$(28) \qquad [J(z, t)]_T = J(z, t)\Theta[z - z_c(t)], \qquad [J(z, t)]_R = J(z, t)\Theta[z_c(t) - z].$$

Now, the right-hand-side of expression (5) for the dwell time can be written in terms of $J(z, t)$ by multiplying the continuity equation $\partial|\psi(z, t)|^2/\partial t + \partial J(z, t)/\partial z = 0$ by t and then integrating over t from 0 to infinity and over z from z_1 to z_2 to obtain (Jaworsky and Wardlaw 1988)

$$(29) \qquad \tau_D(z_1, z_2) = \int_0^\infty \mathrm{d}t \; t[J(z_2, t) - J(z_1, t)].$$

Substitution of (27) into (29) immediately gives the Bohm trajectory mean transmission and reflection times in the form[7]

$$(30a) \qquad \tau_T(z_1, z_2) = \frac{1}{|T|^2} \int_0^\infty \mathrm{d}t \; t\{[J(z_2, t)]_T - [J(z_1, t)]_T\},$$

$$(30b) \qquad \tau_R(z_1, z_2) = \frac{1}{|R|^2} \int_0^\infty \mathrm{d}t \; t\{[J(z_2, t)]_R - [J(z_1, t)]_R\}.$$

Within Bohmian mechanics the mean time at which "to be transmitted" electrons enter the barrier is not equal to the mean time at which "to be reflected" electrons do. For example, for the case in which both transmitted and some reflected trajectories enter the barrier (i.e. for $\Delta k/k_0$ not too small), the simplest situation involves two times t_T and t_R with $t_T < t_R$ such that only transmitted trajectories enter the barrier for $0 \le t \le t_T$, only reflected trajectories enter for $t_T \le t \le t_R$, and none enter for $t > t_R$. Clearly, for this case the mean time at which a transmitted particle enters the barrier is less than the mean time of entrance for all particles that enter the barrier. This means that the subtracted term in (24a) is too large and can lead to anomalously small and even negative values of $\tau_T^J(0, d)$ (Leavens 1993; Delgado *et al.* 1995).

6. THE QUANTUM CLOCK OF SALECKER AND WIGNER

The quantum clock devised by Salecker and Wigner (1958) was applied by Peres (1980) to a time-of-flight measurement of the velocity of a free nonrelativistic particle of well-defined energy E. The relativistic version was treated subsequently by Davies (1986). Leavens and McKinnon (1994) applied the quantum clock approach, suitably modified, to a calculation of the characteristic times $\tau_X(k; z_1, z_2)$ for both nonrelativistic and relativistic electrons. In this section this work is reviewed and then extended in an attempt to calculate the mean-square dwell time.

Following Peres, consider a quantum clock with an odd number $N \equiv 2j + 1$ of states which are conveniently represented by the alternative sets of orthonormal basis functions

$$(31) \qquad u_m(\theta) = (2\pi)^{-1/2} e^{im\theta} \qquad (m = -j, \ldots, 0, \ldots, j)$$

and

$$(32) \qquad v_n(\theta) = N^{-1/2} \sum_{m=-j}^{j} e^{-i2\pi nm/N} u_m(\theta) \qquad (n = 0, \ldots, j, \ldots, N-1)$$

with $0 \leq \theta < 2\pi$. The $u_m(\theta)$ are eigenfunctions of the clock Hamiltonian

$$(33) \qquad \hat{H}_c \equiv -i\hbar\omega \frac{\partial}{\partial\theta} \qquad \left(\omega \equiv \frac{2\pi}{N\tau}\right)$$

with eigenvalues $m\hbar\omega$ while the $v_n(\theta)$ are eigenfunctions of the clock-time operator

$$(34) \qquad \hat{T}_c \equiv \sum_{n'=0}^{N-1} n'\tau\hat{P}_{n'} \qquad [\text{where } \hat{P}_{n'}v_n(\theta) \equiv \delta_{n',n}v_n(\theta)],$$

with eigenvalues $n\tau$. The expectation value of $\hat{H}_c$ in the state $v_n(\theta)$ is zero. For large N the basis function $v_n(\theta)$ is sharply peaked at $\theta = 2\pi n/N$ with an uncertainty of $\pm\pi/N$, corresponding to the time $n\tau$ with an uncertainty of $\pm\tau/2$. In addition, the $v_n(\theta)$ have the important digital clock-like property

$$(35) \qquad e^{-i\hat{H}_c\tau/\hbar}v_n(\theta) = v_{n+1(\text{modulo } N)}(\theta).$$

To determine the time spent by a free $[V(z) = 0]$ particle of well-defined energy $E \equiv \hbar^2k^2/2m$ in the region $[z_1, z_2]$, Peres considered a stationary-state scattering experiment described by the Hamiltonian

$$(36) \qquad \hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \Theta(z - z_1)\Theta(z_2 - z)\hat{H}_c.$$

The associated classical picture is of a stop-watch that runs only when the otherwise free particle is in the region of interest. Solution of the Schrödinger equation for the "final" state $\psi_f(z, \theta)\exp[-iEt_\infty/\hbar]$ that "evolves" from the assumed "initial" state $\psi_i(z, \theta) = A_k\exp(ikz)v_0(\theta)$ and calculation of the expectation value of the elapsed time, $[\langle\psi_f^*(z, \theta)\hat{T}_c\psi_f(z, \theta)\rangle - \langle\psi_i^*(z, \theta)\hat{T}_c\psi_i(z, \theta)\rangle]$, recorded by the ensemble of quantum clocks gives the expected result $\tau_T(z_1, z_2) = (z_2 - z_1)/(\hbar k/m)$ provided that the clocked region $[z_1, z_2]$ is very much longer than the de Broglie wavelength $\lambda = 2\pi/k$ of the free particle. This criterion is obtained by demanding that the conditions for negligible perturbation of the particle by the clock, $E \gg j\hbar\omega$, and for good time resolution, $\tau_T(z_1, z_2) \gg \tau$, are met simultaneously.

Now add the scattering potential $V(z)\Theta(z)\Theta(d - z)$ to the Hamiltonian (36) and consider the application of the quantum clock method to the calculation of the characteristic times $\tau_X(k; z_1, z_2)$. First consider the important special case of the mean dwell time $\tau_D(k; 0, d)$ for an opaque ($\kappa d \gg 1$) rectangular barrier of height V_0. A necessary condition for negligible perturbation of the particle-barrier scattering process of interest by the particle-clock interaction is $V_0 - E \gg j\hbar\omega$ ($|T|^2$ for the undisturbed system depends exponentially on $(V_0 - E)^{1/2}$). Writing this in terms of the expected result $\tau_D(k; 0, d) = \hbar V_0^{-1}E^{1/2}(V_0 - E)^{-1/2}$ obtained from (9) assuming $\kappa d \gg 1$, one obtains after a little algebra $\tau_D(k; 0, d) \ll \tau/4$. This condition is completely incompatible with the criterion $\tau_D(k; 0, d) \gg \tau$ for good time resolution. Leavens and McKinnon (1994) by-passed the latter condition by using the actual elapsed time $t - t_i$ to calibrate the mean elapsed time $[\langle\varphi^*(\theta, t)\hat{T}_c\varphi(\theta, t)\rangle - \langle\varphi^*(\theta, t_i)\hat{T}_c\varphi(\theta, t_i)\rangle]$ calculated for an ensemble of *freely running* quantum clocks $[\hat{H} = \hat{H}_c]$, each prepared in the same initial state $\varphi(\theta, t_i) \equiv \exp(-i\hat{H}_c t_i/\hbar)v_0(\theta)$. This calibration is then applied to the quantity $[\langle\psi_f^*(z, \theta)\hat{T}_c\psi_f(z, \theta)\rangle - \langle\psi_i^*(z, \theta)\hat{T}_c\psi_i(z, \theta)\rangle]$, with $\psi_i(z, \theta) = A_k\exp(ikz)\varphi(\theta, t_i)$, calculated for the clocked particle-(arbitrary) barrier scattering process of interest. Since $\tau \equiv 2\pi/N\omega$ has now lost its meaning as the time resolution of the (calibrated) clock there is no advantage to having $N\omega$ very large. In particular, one can choose the smallest reasonable value for N, i.e. $N = 3$, and let the corresponding particle-clock interaction energy $\hbar\omega$ become arbitrarily small. For $\tau < t_i < 2\tau$ this procedure gives precisely the expected result (9) for the mean dwell time $\tau_D(k; z_1, z_2)$.[8] With the use of a distant ideal particle detector, the complete set of clock readings can be sorted into two subsets, one for transmitted and one for reflected particles, without perturbing the scattering process under investigation. The corresponding (calibrated) quantum clock results for the mean transmission and reflection times $\tau_T(k; z_1, z_2)$ and $\tau_R(k; z_1, z_2)$ are identical to the weak measurement results of Steinberg (1995a; 1995b). Leavens and McKinnon (1994, 17) extended their (calibrated) Salecker–Wigner clock analysis to Dirac electrons but found that the superluminal speeds and negative reflection times discussed in Section 4 persisted. Within Bohmian mechanics, on the other hand, the natural extension of the equation of motion (1) to Dirac electrons (Holland 1993, 503–518) gives a particle velocity that cannot exceed c in magnitude.

For an ensemble of freely running $N = 3$ quantum clocks each prepared in the state $\varphi(\theta, t_i = \tau) = v_1(\theta)$ it is straightforward to show that

$$(37) \qquad \langle\varphi^*(\theta, t)\,(\hat{T}_c - \tau)^2\varphi(\theta, t)\rangle \cong (8\pi^2/27)\,(t - \tau)^2, \qquad (|t - \tau| \ll \tau).$$

If one interprets the expectation value $\mathrm{limit}_{\omega\to 0}\,\langle\psi_f^*(z, \theta)(\hat{T}_c - \tau)^2\psi_f(z, \theta)\rangle$ multiplied by the calibration factor $(8\pi^2/27)^{-1}$ from (37) as the mean-square dwell time $\overline{t_D^2(k; z_1, z_2)}$ then one obtains precisely the Feynman path integral result derived by Iannaccone and Pellegrini (1994, 16551) and suggested by Schulman and Ziolkowski (1989, 270–277).[9] For the special case of a symmetrical $[V(d/2 - z') = V(d/2 + z')]$ opaque barrier one obtains $\overline{t_D^2(k; z_1, z_2)} = [\overline{t_D(k; z_1, z_2)}]^2 \equiv [\tau_D(k; 0, d)]^2$ to an excellent approximation. Unless one invokes negative probabilities, this leads to the surprising conclusion that virtually every reflected electron in the ensemble spends almost exactly the same amount of time inside such a barrier. Now consider the special case of a *free* particle $[V(z) = 0]$. In this limit one obtains $\overline{t_D^2(k; z_1, z_2)} = [\overline{t_D(k; z_1, z_2)}]^2\ \{1 + \sin^2[k(z_2 - z_1)]/[k(z_2 - z_1)]^2\}$ where $\overline{t_D(k; z_1, z_2)} = m(z_2 - z_1)/\hbar k$. This means that for arbitrary z_1 every electron in the ensemble spends precisely the same length of time $m(z_2 - z_1)/\hbar k$ in the region $[z_1, z_2]$ if and only if $k(z_2 - z_1) = \infty$ or $n\pi$ with $n = 1, 2, 3, \ldots$. Because of these bizarre results, the author cannot accept the Salecker–

Wigner clock result for the mean-square dwell time as the second moment of the assumed distribution of intrinsic dwell times. This conclusion is completely consistent with the often repeated claim of Bell (1987a), Bohm and Hiley (1993) and Dürr *et al.* (1996a) that, with the exception of position, one does not in general measure pre-existing values of particle properties.

The first two moments of the dwell time derived by Iannaccone and Pellegrini (1994) are identical to the real parts of the corresponding complex-valued moments derived by Sokolovski and Connor (1991). If this correspondence between the results of the two path integral methods extends to higher moments, then one encounters obvious inconsistencies. For example, for an opaque symmetrical barrier not only is $\overline{t_D^3(k;\, 0, d)}$ not close to $[\overline{t_D(k;\, 0, d)}]^3$, as implied by the results for the first two moments, but it can be negative! Similarly, for the free particle case $\overline{t_D^3(k;\, z_1, z_2)}$ should be equal to $[m(z_2 - z_1)/\hbar k]^3$ whenever $k(z_2 - z_1) = n\pi$ with $n = 1, 2, 3, \ldots$ but this is found not to be the case.

For completeness, it should be recalled that the Bohm trajectory result for the distribution of intrinsic dwell times in the stationary-state limit is

$$P_D[t_D(k;\, z_1, z_2)] = |T(k)|^2\delta\{t_D(k;\, z_1, z_2) - \tau_T(k;\, z_1, z_2)] + |R(k)|^2\delta\{t_D(k;\, z_1, z_2) - \tau_R(k;\, z_1, z_2)], \tag{38}$$

for $z_1 > -\infty$ and *arbitrary* barrier potential $V(z)$. For $|T(k)|^2 > 0$, $\tau_R(k;\, z_1, z_2) = 0$.

7. CONCLUDING REMARKS

It is well known that Bohmian mechanics leads to exactly the same experimental predictions as conventional quantum theory whenever the predictions of the latter are unambiguous. The concepts of transmission and reflection times for quantum particles are not meaningful within standard text-book quantum mechanics, strictly applied, or within some modern reinterpretations such as those based on consistent histories (Griffiths 1984; Gell-Mann and Hartle 1990; Omnès 1992; Yamada 1996). Despite this, there is a large and rapidly expanding theoretical literature devoted to calculation of these quantities. Several of the approaches that have been proposed, including the one based on weak measurement theory, are contained in the systematic projector approach which leads in principle, via rather straightforward extension of conventional quantum mechanics, to an unlimited number of possible expressions for the mean transmission and reflection times. None of these "conventional" possibilities for τ_T and τ_R can be equal to the results that emerge uniquely and almost trivially from Bohmian mechanics. An essential difference between the projector and Bohm trajectory approaches is that the former is based on the wave-like decomposition $\psi = \psi_T + \psi_R$ while the latter is based on the particle-like decomposition $|\psi|^2 = [|\psi|^2]_T + [|\psi|^2]_R$. The "conventional" probability current density approach is based solely on the ill-defined $J = J_+ + J_-$ decomposition of J into right-going and left-going components. This approach is conventional only in that no attempt is made to decompose J into transmitted and reflected components J_T and J_R. Although particle-like decompositions $J = J_+ + J_-$ and $J = J_T + J_R$ are both uniquely defined within Bohmian mechanics, only the latter is needed to calculate τ_T and τ_R from J.

Since expression (5) for the mean dwell time is well established, it appears from the above considerations that it is the ambiguity (within "conventional" quantum mechanics) in the decomposition into transmitted and reflected components that is at the heart of the "tunneling time problem". Weak measurement theory removes this ambiguity but leads to expressions for τ_T and τ_R that many, including the author, find unacceptable because they can lead to superluminal mean transmission speeds and negative mean reflection times. The (calibrated) Salecker–Wigner quantum clock approach throws further light on the problem. The calibration procedure allows the particle-clock interaction to be made arbitrarily weak and, perhaps not surprisingly, it gives precisely the weak measurement results for τ_D, τ_T and τ_R. The approach is readily extended to Dirac electrons and shows that the superluminal and negative times mentioned above are not artifacts of using the non-relativistic Schrödinger equation. The approach can also be extended to calculate the mean-square dwell time and gives precisely an expression obtained using Feynman paths. The resulting behaviour is bizarre in both the zero barrier and opaque (symmetric) barrier limits. This behaviour cannot be blamed on the particular transmission-reflection decomposition used because none was, of course, needed. Moreover, since peculiar behaviour is obtained even for free electrons, it seems that the word "tunneling" in "tunneling-time problem" is a red herring. The underlying problem is simply that conventional quantum mechanics is not a theory for electron motion (Holland 1993).

Is it consistent for the author to regard the quantum clock analysis leading to (5) as a proper derivation of the mean dwell time, rather than a mathematical fluke, and simultaneously to reject the corresponding results for the mean transmission and reflection times and the mean-square dwell time as being unphysical? It is consistent if there is something fundamentally different about the latter three quantities. There *is* within Bohmian mechanics: the mean dwell time can be computed without needing to calculate any particle trajectories; the other three quantities cannot [e.g., computation of the mean transmission and reflection times using (8a) and (8b) requires the trajectory $z_c(t)$].

From the above it is concluded that the basic difference between the Bohm trajectory and the "conventional" approaches considered is that Bohmian mechanics consistently treats a "particle" as an actual particle[10] at all times (Dürr *et al.* 1992a, 843), not just at those instants when an ideal (i.e. strong) measurement is made of its position. Aside from those instants, the "conventional" approaches are stuck with waves and the resulting ambiguities associated with timing an extended entity, especially one that spreads and splits into two or more components.[11] In the Bohm trajectory approach it is the presence of an actual particle in a given region that is being clocked. If the advocates of one of the "conventional" approaches considered in this paper should make the same claim, then the two approaches are clearly incompatible. If, on the other hand, it is some wave-like aspect of the electron that is being clocked then there is the possibility that the two approaches are complementary. In this case, it should be stated as clearly as possible just which aspect of the electron, if any, other than its statistical effect on the pointer of a clock, is being considered and whether or not the presence of this aspect in the region of interest is actually being timed. If this time can be negative, then a clear explanation should be given of what a negative time of presence in a region of space actually means. In the author's opinion it is not good enough to pass it off as just another magical feature of quantum mechanics. This is particularly so because a "conventional" approach as defined here necessarily involves some as yet unjustified extension of standard text-book quantum mechanics. If, however, the mean transmission and reflection times tell us nothing more about the electrons than what average shifts they impart to pointers in the respective subensembles, then this involves completely standard but unmagical quantum mechanics. There is now nothing mysterious about negative pointer shifts but does the apparatus really merit being called a clock when it is not actually timing anything?

ACKNOWLEDGEMENTS

The author has benefited from numerous discussions with his NRC colleagues G. C. Aers and W. R. McKinnon and also with J. T. Cushing (Notre Dame), S. Goldstein (Rutgers), G. Iannaccone (Pisa) and J. G. Muga (La Laguna).

National Research Council of Canada

NOTES

1 For the special case of the dwell time ($X = D$) there is widespread agreement on the correct expression for the mean value τ_D and one can construct a hermitian operator that produces this result; but it is not at all clear that the nth power of this operator will produce the correct nth moment of the dwell time for $n > 1$ [Daumer *et al.* 1994].

2 A Bohm trajectory can be retrodicted from a precise measurement of the time of arrival at $z = b$, if $\psi(z, 0)$ is known for all z.

3 $\tau_x(z_1, z_3) = \tau_x(z_1, z_2) + \tau_x(z_2, z_3)$ with $z_1 \leq z_2 \leq z_3$.

4 The quantities $-\mathrm{Im}[\tau_T^{TD}]$ and $-\mathrm{Im}[\tau_R^{RD}]$ are identical to the so-called mean spin-rotation transmission and reflection times introduced by Büttiker (1983); $|\tau_T^{TD}|$ and $|\tau_R^{RD}|$ are equal to the well-known Büttiker–Landauer (1982) expressions for τ_T and τ_R.

[5] This is readily confirmed for the reflection case by recalling that within Bohmian mechanics $\tau_R(k;\, z_1,\, z_2) = 0$ for $0 < |R(k)|^2 < 1$ but $\tau_R(z_1,\, z_2)$ is in general not equal to zero (see Figure 1) as would be the case if (23b) applied to these quantities.
[6] It is readily shown that in the tunneling regime $V_0 > E$, the Bohm trajectory result for $\tau_T(k;\, 0,\, d)$ *exceeds* the free particle ($V_0 = 0$) result $md/\hbar k$ for all $d > 0$.
[7] These expressions were suggested to the author by S. Goldstein.
[8] For $\tau < t_i < 2\tau$, the initial clock state is not an eigenstate of $\hat{T}_c$. Hence, a strong measurement of the initial mean clock time would collapse the wave function for each ensemble member to one of the eigenstates of $\hat{T}_c$, thus aborting the desired initial state. Any such attempt to verify the initial mean clock time must be done on a different (but identically prepared) ensemble from the one used to determine the final mean clock time.
[9] $\overline{t_D^2(k;\, z_1,\, z_2)} = |T(k)|^2\, \overline{t_T^2(k;\, z_1,\, z_2)} + |R(k)|^2\, \overline{t_R^2(k;\, z_1,\, z_2)}$ relates the mean-square dwell time to the mean-square transmission and reflection times actually considered in these references.
[10] In the author's opinion, Feynman paths, unlike Bohm trajectories, are to be interpreted only as useful mathematical constructs, not as actually realizable particle trajectories.
[11] That the Bohm trajectory and projector approaches both lead to (5) for the mean dwell time is consistent with the fact that $|\psi(z,\, t)|^2$ can be interpreted both as a probability distribution for the position of a particle and as the local intensity of a wave.