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APPLICATION OF THE QUANTUM CLOCK OF SALECKER AND WIGNER TO THE "TUNNELING TIME PROBLEM"

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Peres used Salecker and Wigner's prescription for a quantum clock in a theoretical study of time-of-flight determination of the velocity of a free particle. In the present paper, the quantum clock is applied to a closely related problem, determination of the average time spent inside a one-dimensional potential barrier $V(z)\Theta(z)\Theta(d-z)$ by initially free "incident" electrons of energy E. For an opaque rectangular barrier straightforward application of the approach leads to a "clocked" result that differs by orders of magnitude from the result postulated by Büttiker for the average "intrinsic" dwell time $\tau_D(0,d;E)$. It is shown that this difference can be eliminated by appropriate choice of initial state for the ensemble of identical clocks and by applying to their average behaviour when coupled to tunneling particles the calibration determined for the corresponding ensemble of freely running clocks. The difference that persists for more transparent barriers is attributed, following Peres, to perturbation of the clock and/or particle dynamics during the measurement process. It is most serious in the limit $d\to 0$ where the ratio of clocked to intrinsic times peaks at a value of about 1.6. Because of the nonlinear relation between "actual" time (i.e. the parameter t in the Schrödinger equation) and uncalibrated clock time it does not seem possible to decompose the mean dwell time for an opaque barrier as "measured" by the quantum clock approach considered here into individual components associated with transmitted and reflected particles. This is consistent with the point of view that the "tunneling time", which refers only to the transmitted particles, is not a meaningful concept within conventional interpretations of quantum mechanics.

1. INTRODUCTION

In a theoretical paper written over 35 years ago Salecker and Wigner[1] devised a microscopic clock for the purpose of measuring distances between space-time events. Peres[2] applied their design for a quantum clock to several fundamental problems including a time-of-flight measurement of the velocity of a free nonrelativistic particle of energy E (the relativistic case was treated subsequently by Davies[3] using both the Klein-Gordon and Dirac equations). In the present paper the quantum clock is applied to the infamous "tunneling time problem"[4-10].

In section 2 the characteristic times of interest in this paper are defined and the author's point of view regarding their meaningfulness stated. Section 3 contains the mathematical description of the quantum clock and sketches its application by Peres to the determination of the time spent by a *free* particle of energy E in the one dimensional interval $0\le z\le d$. It is then shown that when an opaque rectangular barrier $V_0\Theta(z)\Theta(d-z)$ is introduced in the interval of interest straightforward application of Peres' approach cannot possibly satisfy his accuracy criterion. In section 4, a more detailed study of the free particle ($V_0=0$) case reveals two ways to improve the accuracy of the quantum clock. These are applied to the finite barrier case in section 5. A brief discussion follows in section 6.

2. THE "TUNNELING TIME PROBLEM"

To define the quantities of interest in the "tunneling time problem" consider on ensemble of scattering experiments in each of which a particle with the same initial wave function $\psi(z,t=0)$ is incident normally from the left on the potential barrier $V(\vec{r})=V(z)\Theta(z)\Theta(d-z)$ which varies only in the z direction. The mean transmission time $\tau_T(0,d)$ is defined as the average time spent in the region $0\le z\le d$ subsequent to t=0 by those particles that are ultimately transmitted; the corresponding mean reflection time $\tau_R(0,d)$ is the average time spent in that region by those that are ultimately reflected. Finally, the mean dwell time $\tau_D(0,d)$ is the average time spent in the interval $0\le z\le d$ by the particles irrespective of whether they are eventually transmitted or reflected. The so-called "tunneling time" or "traversal time" is the quantity $\tau_T(0,d)$.

Immediately prior to beginning the present investigation the author[11] supported the point of view[12] that *within conventional interpretations* of quantum mechanics the quantities τ_T and τ_R are not meaningful concepts essentially because, unless the transmission probability $|T|^2$ is either 0 or 1, it is impossible to decompose the probability density $|\psi(z,t)|^2$ inside the barrier into "to be transmitted" and "to be reflected" components. The analysis of this paper does not bend the rules of orthodox quantum mechanics and the results are consistent with the above point of view. On the other hand, the author also believes that *within Bohm's causal interpretation*[13-16] of quantum mechanics the quantities τ_T and τ_R are meaningful concepts because the above-mentioned decomposition is possible[11,17].

The expression

$$\tau_D(0,d) = \int_0^\infty dt \int_0^d dz |\psi(z,t)|^2 \qquad (1)$$

for the mean dwell time has been derived within both conventional[18] and causal[17] interpretations and is accepted by most but not all[10] workers in the field. For the special case of the stationary-state scattering problem considered by Peres[2], and in the remainder of this paper, (1) becomes[18,5]

$$\tau_D(0,d;E) = \frac{1}{j_{inc}} \int_0^d dz |\psi_K(z)|^2, \tag{2}$$

the expression originally postulated by Büttiker[19]. In (2), $E \equiv \hbar^2 K^2/2M$ is the energy of the electron, j_{inc} is the incident probability current density and $\psi_K(z)\exp[-iEt/\hbar]$ is the stationary-state wavefunction.

3. THE QUANTUM CLOCK ANALYSIS OF PERES

Peres[2] considered a clock with an odd number $N \equiv 2j+1$ of states represented by the alternative sets of orthonormal basis functions

$$u_m(\theta) = (2\pi)^{-1/2} e^{im\theta}, \qquad (m = -j,\ldots,0,\ldots,j) \tag{3}$$

and

$$v_k(\theta) = \frac{1}{N^{1/2}} \sum_{m=-j}^{j} e^{-i\frac{2\pi km}{N}} u_m(\theta) \quad (k = 0,\ldots,j,\ldots,N-1) \tag{4}$$

$$= \frac{1}{(2\pi N)^{1/2}} \sin\left[\frac{N}{2}\left(\theta - \frac{2\pi k}{N}\right)\right] \Big/ \sin\left[\frac{1}{2}\left(\theta - \frac{2\pi k}{N}\right)\right], \tag{5}$$

with $0 \le \theta < 2\pi$. The former are eigenfunctions of the clock Hamiltonian

$$\hat{H}_c \equiv -i\hbar\omega\partial/\partial\theta \qquad (\omega \equiv 2\pi/N\tau) \tag{6}$$

with eigenvalues $m\hbar\omega$. The latter have the property that

$$e^{-i\hat{H}_c\tau/\hbar} v_k(\theta) = v_{k+1(\text{modulo } N)}(\theta) \tag{7}$$

and are eigenfunctions of the clock-time operator

$$\hat{T}_c \equiv \tau \sum_{k'=0}^{N-1} k' \hat{P}_{k'} \quad \left[\text{where } \hat{P}_{k'} v_k(\theta) = \delta_{k',k} v_k(\theta)\right] \tag{8}$$

with eigenvalues $k\tau$. For large N the basis function $v_k(\theta)$ is sharply peaked at $\theta = 2\pi k/N$, corresponding to the time $k\tau$, with an angle uncertainty $\Delta\theta = \pm\pi/N$, corresponding to a time uncertainty $\pm\ \tau/2$. Another important property is that the expectation value of $\hat{H}_c$ in the clock state $v_k(\theta)$ is zero.

To determine the time spent by a free $[V(z)=0]$ particle of energy $E \equiv \hbar^2 K^2/2M$ in the region $0 \le z \le d$ Peres considered a stationary-state scattering experiment with the Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial z^2} - i\hbar\omega\Theta(z)\Theta(d-z)\frac{\partial}{\partial\theta} \ . \tag{9}$$

This describes the situation in which the quantum clock runs only when the spatial coordinate z of the otherwise free particle is in the region of interest. Peres chose

$$\Psi_i(z,\theta) = A_K e^{iKz} v_0(\theta) = A_K e^{iKz} \frac{1}{N^{1/2}} \sum_{m=-j}^{j} u_m(\theta) \tag{10}$$

for the time independent part of the "initial" state $\Psi_i(z,\theta)\exp[-iEt/\hbar]$ of the as yet uncoupled particle-clock system. For large N it is sharply peaked at $\theta=0$. The second term in the Hamiltonian (9) eventually couples the particle and clock coordinates so that the system wavefunction is no longer factorable in z and θ and must be written in the form $\Psi(z,\theta)\exp[-iEt/\hbar]$ with

$$\Psi(z,\theta) = \frac{1}{N^{1/2}} \sum_{m=-j}^{j} \psi_K^{(m)}(z) u_m(\theta) \tag{11}$$

Substituting this in the Schrödinger equation and using $\hat{H}_c u_m(\theta) = m\hbar\omega\, u_m(\theta)$ gives

$$\boxed{\sum_{m=-j}^{j} e^{im\theta}\left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial z^2} + m\hbar\omega\Theta(z)\Theta(d-z) - E\right]\psi_K^{(m)}(z) = 0.} \tag{12}$$

Hence, $\psi_K^{(m)}(z)$ is given by the text book result for the stationary-state wavefunction of a particle of energy $E \equiv \hbar^2 K^2/2M$ in the presence of the rectangular barrier $V_m\Theta(z)\Theta(d-z)$ with $V_m \equiv m\hbar\omega$. Outside the region of interest $0 \le z \le d$ it has the form

$$\psi_K^{(m)}(z) = \begin{cases} A_K[e^{iKz} + R_m(K)e^{-iKz}] & (z \le 0) \ , \\ A_K T_m(K) e^{iKz} & (z \ge d) \ , \end{cases} \tag{13}$$

where A_K is a normalization constant which cancels when expectation values are calculated. Peres considered the case $E >> V_j \equiv j\ \hbar\omega$, the largest barrier induced by interaction with the clock. Then $R_m(K) \cong 0$ and $T_m(K) \equiv |T_m(K)| \exp[i\phi_m^{(T)}(K)] \cong \exp[i(K_m - K)d]$ with $K_m \equiv \hbar^{-1}[2M(E - m\hbar\omega)]^{1/2} \cong K - m\omega/(2E/M)^{1/2}$ $= K - m\omega/(\hbar K/M)$. Hence, the "final" wavefunction for the particle-clock system is $\Psi_f(z \le 0, \theta) \cong 0$ and

$$\Psi_f(z \ge d,\theta) \cong A_K e^{iKz} \frac{1}{N^{1/2}} \sum_{m=-j}^{j} e^{-im\omega d/(\hbar K/M)} u_m(\theta)$$
$$= A_K e^{iKz} v_0(\theta - \omega M d/\hbar K) \tag{14}$$

which, for large N, is sharply peaked at $\theta = \omega Md/\hbar K$ corresponding to a time-of-flight $\tau_T(0,d)$ through the region $0 \le z \le d$ of $Md/\hbar K$. This gives the expected result $\hbar K/M$ for the intrinsic (i.e., unperturbed by measurement) velocity of a free particle of energy $E \equiv \hbar^2 K^2/2M$. Combining the condition, $j\hbar\omega << E$, for negligible perturbation with the condition, $\tau_T(0,d) >> \tau$, for high resolution gives Peres' criterion $d >> 2\pi/K \equiv \lambda$ for accurate measurement of the intrinsic free particle velocity. For a free particle the intrinsic velocity must be independent of z by symmetry. Hence, for K>0, one can in principle always make the flight path d large enough that the measured and intrinsic velocities agree. This flexibility does not exist if one is interested in, say, the mean dwell time $\tau_D(0,d;E)$ for a particular rectangular barrier $V_0\theta(z)\theta(d-z)$ with $V_0>E$ and d fixed. To apply the quantum clock to this case it is necessary to add the barrier potential energy to the Hamiltonian (9) with the result that the T_m and R_m of (13) become the transmission and reflection probability amplitudes for the rectangular barriers $V_m\Theta(z)\Theta(d-z)$ with $V_m \equiv V_0+m\hbar\omega$ ($m=-j,\dots,0,\dots,j$). For the special case of an opaque ($\kappa_0 d>>1$) rectangular barrier, Büttiker's expression (2) for the mean intrinsic dwell time gives

$$\tau_D(0,d;E) = \frac{\hbar}{V_0}\left(\frac{E}{V_0-E}\right)^{1/2} \qquad (\kappa_0 d>>1) \tag{15}$$

where $\kappa_0 \equiv \hbar^{-1}[2M(V_0-E)]^{1/2}$ is the inverse tunneling length. A necessary condition for negligible perturbation by the clock of the scattering process of interest is $j\hbar\omega << V_0-E$, i.e. $\tau^{-1}<<(2j+1)(V_0-E)/2\pi j\hbar$. Hence, using (15), $\tau_D(0,d;E)/\tau<<(2+j^{-1})[(V_0-E)E]^{1/2}/2\pi V_0<1/4$ must hold. This is completely incompatible with the criterion $\tau_D(0,d;E)>>\tau$ for good time resolution. Hence, the mean dwell time for an opaque rectangular barrier as "measured" by the quantum clock could be grossly different from the underlying intrinsic quantity which is assumed to be measurable, in principle, to arbitrary accuracy by some (unspecified) ideal technique. In the next two sections it will be shown how the quantum clock approach can be modified to approach the ideal one very much more closely.

4. A MORE DETAILED ANALYSIS OF THE FREE PARTICLE CASE

In this section, the restriction $d>>\lambda$ is dropped in order to study the convergence with increasing E and d of the measured and intrinsic values of the mean dwell time $\tau_D(0,d;E)$ for free particles.

It is clear that the coupling term in the Hamiltonian (9) does not distinguish between "to be transmitted" and "to be reflected" particles, assuming for the sake of discussion that these phrases have meaning. Hence, the expectation value of the clock-time operator (8) in the final state $\Psi_f(z,\theta)$ gives the measured or clocked (c) value of the mean dwell time, i.e.

$$\tau_D^c(0,d;E) = \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dz\, \Psi_f^*(z,\theta)\hat{T}_c\Psi_f(z,\theta) \Big/ \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dz\, \Psi_f^*(z,\theta)\Psi_f(z,\theta) \ . \tag{16}$$

Substitution of (11) into (16) keeping only the reflected and transmitted components of (13) for $\psi_K^{(m)}(z)$, application of

$$\begin{aligned}
\hat{T}_c u_m(\theta) &= \hat{T}_c \frac{1}{N^{1/2}} \sum_{k=0}^{N-1} e^{i\frac{2\pi km}{N}} v_k(\theta) \\
&= \frac{1}{N^{1/2}} \sum_{k=0}^{N-1} e^{i\frac{2\pi km}{N}} \tau \sum_{k'=0}^{N-1} k' \hat{P}_{k'} v_k(\theta) \quad \text{[from (8)]} \\
&= \frac{1}{N^{1/2}} \sum_{k=0}^{N-1} e^{i\frac{2\pi km}{N}} \tau k\, v_k(\theta) \\
&= \frac{1}{N} \sum_{k=0}^{N-1} e^{i\frac{2\pi km}{N}} \tau k \sum_{m'=-j}^{j} e^{-i\frac{2\pi km'}{N}} u_{m'}(\theta) \ ,
\end{aligned}$$

the orthonormality of the $u_m(\theta)$s, and the fact that

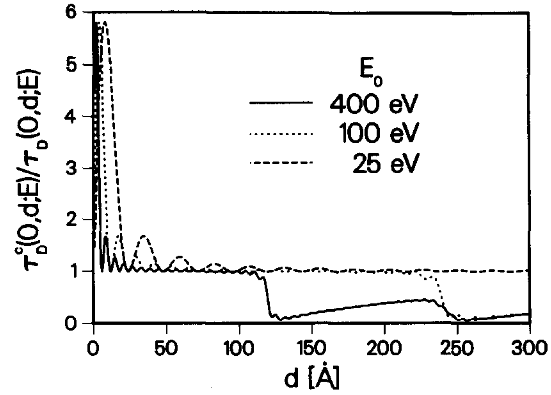

Fig. 1 Ratio, $\tau_D^c(0,d;E)/\tau_D(0,d;E)$, of "clocked" to "intrinsic" mean dwell times for the region [0,d] traversed by a free particle of energy E = 400 eV(————), 100 eV (··············), and 25 eV (– – – – – – –). The parameters of the quantum clock are j=10 and $\alpha \equiv j\hbar\omega/E=0.1$. The initial state of each of the N_c quantum clocks in the ensemble is $v_0(\theta)$.

$|T_m|^2 + |R_m|^2 = 1$ finally leads to

$$\tau_D^c(0,d;E) = \sum_{k=0}^{N-1} k\tau P_k \tag{17}$$

with

$$P_k = \left|\frac{1}{N}\sum_{m=-j}^{j} e^{i\frac{2\pi km}{N}} R_m\right|^2 + \left|\frac{1}{N}\sum_{m=-j}^{j} e^{i\frac{2\pi km}{N}} T_m\right|^2 . \tag{18}$$

It is obvious that $P_k \geq 0$ and easily shown that

$$\sum_{k=0}^{N-1} P_k = 1 \ . \tag{19}$$

Fig. 1 shows the ratio $\tau_D^c(0,d;E)/\tau_D(0,d;E)$ as a function of d for E=400, 100 and 25 eV with the clock parameters $\alpha \equiv j\hbar\omega/E=0.1$ and j=10. A small value of j has been chosen to illustrate the effect of the finite period $2\pi/\omega \equiv N\tau$ of the clock. As expected, the clocked and intrinsic quantities are in very good agreement if d is sufficiently large but not so large that $\tau_D(0,d;E)$ approaches too closely to or exceeds the period of the clock as occurs for the 400 eV curve above d~100 Å and for the 100 eV curve above d~200 Å. From the de Broglie wavelengths λ of 0.6, 1.2 and 2.4 Å for E=400, 100 and 25 eV respectively, it is clear that d must be *very* much greater than λ to obtain good convergence. This is consistent with the fact that the criterion $d>>\lambda$ results from a double inequality of the a>>b>>c type and perhaps should be written $d>>>>\lambda$ to make this clear.

For very small d the weight P_k is large not only for k close to 0 but also for k close to N-1, which should not be too surprising given the cyclic nature of the clock. This suggests that the convergence with increasing d could be improved simply by choosing an intermediate state, say $v_j(\theta)$, rather than $v_0(\theta)$ as the initial clock state. (Correspondingly greater caution must be taken that the period is not too small.) Repeating the above analysis using $v_{k_0}(\theta)$ as the initial state results in the replacement of (17) by

$$\tau_D^c(0,d;E) = \sum_{k=0}^{N-1} (k-k_0)\tau P_{k-k_0} \quad . \tag{20}$$

Focusing on the most slowly convergent of the three curves of Fig. 1, the ratio $\tau_D^c(0,d;E)/\tau_D(0,d;E)$ for E=25 eV is shown in Fig. 2 for $k_0=0$ and $k_0=j$. To make the effect of the finite period of the clock completely negligible j has been increased to 100 for both curves. Convergence is clearly much better for $k_0=j$.

In order to improve the convergence further it is instructive to consider the average behaviour of an ensemble of a very large number N_c of free (i.e. unperturbed) quantum clocks each prepared in the same initial state $v_{k_0}(\theta)$ at t=0. The ensemble-averaged quantum clock time, $t_c^{(0)}(t)$, at time t is obtained by subtracting the expectation value of the clock time operator $\hat{T}_c$ in the initial state $v_{k_0}(\theta)$ from its expectation value in the time-evolved state $\exp(-i\hat{H}_c t/\hbar)\, v_{k_0}(\theta)$ to obtain

$$t_c^{(0)}(t) = \sum_{k=0}^{N-1} k\tau[|c_{k,k_0}(t)|^2 - |c_{k,k_0}(0)|^2] \tag{21}$$

where

$$c_{k,k_0}(t) \equiv \int_0^{2\pi} d\theta\, v_k^*(\theta) e^{-i\hat{H}_c t/\hbar} v_{k_0}(\theta)$$

$$= \frac{1}{N} \frac{\sin[\pi(k-k_0-t/\tau)]}{\sin[\frac{\pi}{N}(k-k_0-t/\tau)]} \quad , \tag{22}$$

using (4), $\hat{H}_c u_m(\theta)=m\hbar\omega\, u_m(\theta)$, the orthonormality of the $u_m(\theta)$ basis functions, and the well-known expression for the sum of a finite geometric series. Noting that $c_{k,k_0}(0)=\delta_{k,k_0}$ finally gives

$$t_c^{(0)}(t) = \frac{1}{N^2} \sum_{k=0}^{N-1} \frac{k\tau \sin^2[\pi(k-k_0-t/\tau)]}{\sin^2[\frac{\pi}{N}(k-k_0-t/\tau)]} - k_0\tau \quad . \tag{23}$$

Using $c_{k,k_0}(t=n\tau)=\delta_{k-k_0-n,\ell N}$ with n and ℓ integer shows that the ensemble of free quantum clocks works perfectly whenever the "actual" time t, i.e. the parameter t in the time evolution operator $\exp(-i\hat{H}_c t/\hbar)$, is a non-negative (not too large) integral multiple n of τ:

$$t_c^{(0)}(t=n\tau) = t \quad (n=0, 1, ..., N-1-k_0) \quad . \tag{24}$$

This theoretical result, with $k_0=0$, is the basis of Peres' analysis. The remainder of this paper considers the use of the full theoretical result (23) for the ensemble of freely running quantum clocks to calibrate the result (20) for the corresponding ensemble of quantum "stop watches". The basic idea is to choose the clock parameters so that $t_c^{(0)}(t)$ is monotonically increasing for the temporal range of interest so that it can be inverted to give $t[t_c^{(0)}]$ which is then used to calibrate the result (20) to obtain the corresponding "calibrated clock" (cc) result

$$\tau_D^{cc}(0,d;E) \equiv t[\tau_D^c(0,d;E)] \quad . \tag{25}$$

The parameter τ then loses its meaning as the time resolution of the clock. Since the author's primary interest is the finite barrier case the calibration procedure for the free particle case has been implemented only for the E=25 eV case of Fig. 2 for the range 0≤d≤100 Å and has not been optimized. Fig. 3 shows $t[t_c^{(0)}]$ for $k_0=j$ over the range $0 \le t_c^{(0)} \le 5\tau$ needed for the calibration . Very high

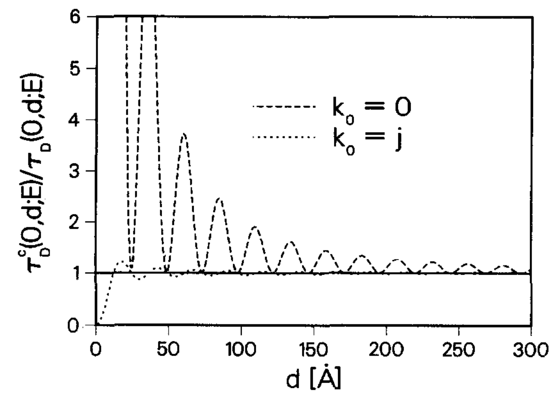

Fig. 2 Improvement in the convergence of the clocked and intrinsic mean free particle dwell times, $\tau_D^c(0,d;E)$ and $\tau_D(0,d;E)$ respectively, with increasing d obtained by choosing the initial state of the quantum clock to be $v_j(\theta)$ (---------------) rather than $v_0(\theta)$ (– – – – – – –). The energy E of the particle is 25 eV and the parameters of the quantum clock are j=100 and $\alpha=j\hbar\omega/E=0.1$.

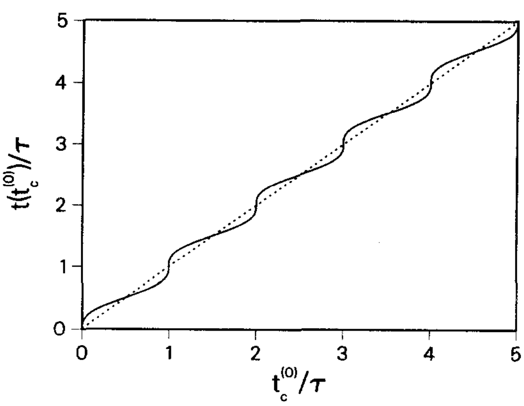

Fig. 3 Relation between the "actual" time t and the mean time $t_c^{(0)}$ given by an ensemble of a very large number N_c of free quantum clocks each with j=10 and each prepared in the initial state $v_j(\theta)$. The quantum clock time $t_c^{(0)}$ agrees exactly with the actual time t at the intersections of the solid curve with the dashed reference curve $t=t_c^{(0)}$.

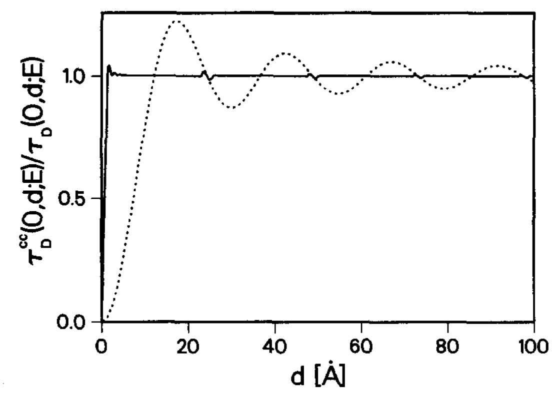

Fig. 4 Comparison of the performance of a calibrated quantum clock (————————) and the uncalibrated one (---------------) of Fig. 2. For both clocks k_0=j=100 and α=0.1. The energy E of the particle is again 25 eV.

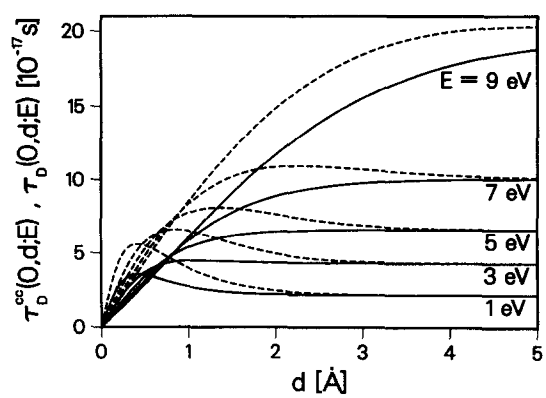

Fig. 6 Comparison of the intrinsic (——————) and calibrated clock (– – – – – – –) mean dwell times, $\tau_D(0,d;E)$ and $\tau_D^{cc}(0,d;E)$ respectively, for electrons of energy E initially incident on a rectangular barrier $V(z)=V_0\Theta(z)\Theta(d-z)$ of height V_0=10 eV and width d, with 0≤d≤5 Å and E=1, 3, 5, 7 and 9 eV. The two curves for E=9 eV do not converge until d≅10 Å. The clock parameters are k_0=j=1 and $\hbar\omega$=0.01E. The uncalibrated clock results $\tau_D^{c}(0,d;E)$ are smaller then $\tau_D(0,d;E)$ by several orders of magnitude and consequently are not shown.

numerical accuracy is needed near the points $t_c^{(0)}=n\tau$ where $t[t_c^{(0)}]$ varies extremely rapidly (for large n the inversion cannot be carried out without changing the clock parameters because $t_c^{(0)}(t)$ becomes markedly non-monotonic). Fig. 4 compares $\tau_D^{cc}(0,d;E)$ with $\tau_D^{c}(0,d;E)$ both with k_0=j for the parameters of Fig. 2. The improvement is dramatic. The calibration was carried out using linear interpolation with 10^4 equally spaced values of $t_c^{(0)}$ in the range [0,5τ] of Fig. 3. Comparison with the corresponding results using interpolation with only half of these values of $t_c^{(0)}$ indicates that the glitches (with the exception of the first, near d=0) could be further reduced in amplitude by improving the interpolation procedure. The behaviour near d=0 will be considered in more detail in the next section.

5. THE FINITE BARRIER CASE

This section considers the case in which the potential energy profile $U(z)=V(z)\Theta(z)\Theta(d-z)$ contains a barrier localized within the region 0≤z≤d monitored by the quantum clocks. The only change in the theory of the previous section is that the transmission and reflection probabilities T_m and R_m are now those for the potential barriers $V_m(z)\equiv[V(z)+m\hbar\omega]\Theta(z)\Theta(d-z)$, m=-j, ..., 0, ..., j. For definiteness and to make the calculations as easy as possible, the rectangular barrier $V_0\Theta(z)\Theta(d-z)$ is used for all the calculations. It is instructive, and apparently sufficient, to consider the simplest type of quantum clock, namely the one with k_0=j=1. Fig. 5 is a sketch of the ensemble of N_c clocks for this special case at the beginning and end of the gedanken scattering experiment.

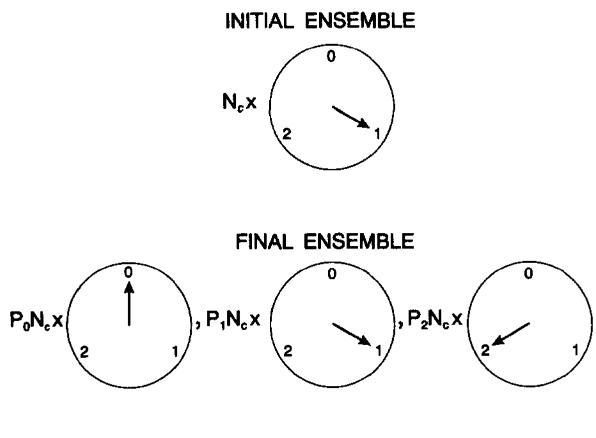

Fig. 5 The initial (i.e., as prepared) and final ensembles of N_c quantum "stop watches" for the case k_0=j=1.

For k_0=j=1 equation (23) for the free quantum clock time $t_c^{(0)}(t)$ becomes, after a little trigonometry,

$$t_c^{(0)}(t) = \frac{16\sqrt{3}}{9}\,\tau\sin^3\left(\frac{\pi t}{3\tau}\right)\cos\left(\frac{\pi t}{3\tau}\right) \tag{26}$$

$$\cong \frac{16\sqrt{3}}{9}\,\tau\left(\frac{\pi t}{3\tau}\right)^3 \quad (t<<\tau) . \tag{27}$$

It is easy to show that (26) is monotonically increasing for the range 0≤t≤τ. For the regime t<<τ which is the only one encountered in the calculations presented below

$$t[t_c^{(0)}] \cong \frac{3}{\pi}\left(\frac{9t_c^{(0)}}{16\sqrt{3}\,\tau}\right)^{1/3}\tau \quad (t<<\tau) . \tag{28}$$

For rectangular barriers of height V_0=10 eV and width d in the range [0,5 Å] Fig. 6 compares $\tau_D^{cc}(0,d;E)$ calculated for k_0=j=1 and α=0.01 with the intrinsic quantity

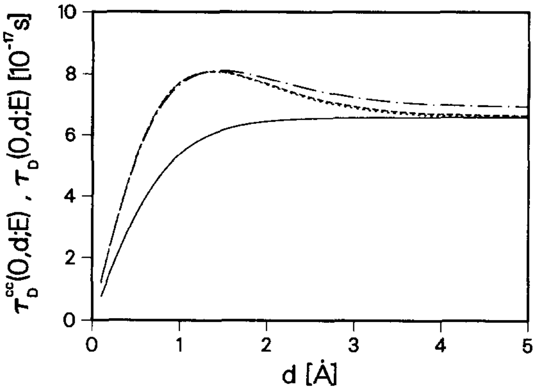

Fig. 7 Dependence of the mean dwell time $\tau_D^{cc}(0,d;E)$ on $\alpha \equiv j\hbar\omega/E$ with j=1 (k_0=j) for electrons of energy E=V_0/2=5 eV. Comparison with the intrinsic mean dwell time (————) are shown as a function of d for α=0.001 and 0.01 (-------------), 0.20 (– – – – – –) and 0.50 (— - — - — - —).

$\tau_D(0,d;E)$ evaluated using Büttiker's analytic result [equation (2.20b) of ref. 19] for E=1, 3, 5, 7 and 9 eV. The agreement is excellent for opaque ($\kappa_0 d$>>1) barriers but deteriorates steadily as the barriers become more and more transparent with $\tau_D^{cc}(0,d;E)/\tau_D(0,d;E)$ peaking at a value of about 1.6 at d=0. The improvement over the corresponding uncalibrated clock results is enormous. But, no further improvement has been obtained: neither increasing j from 1 to 10 (with k_0=j) nor decreasing $\alpha \equiv j\hbar\omega/E$ from 0.01 to 0.001 leads to discernible changes in the curves of Fig. 6. In fact, as shown in Fig. 7 for E=5 eV, α must be increased from 0.01 by an order of magnitude to obtain a noticeable change in $\tau_D^{cc}(0,d;E)$. The behaviour for small d is particularly insensitive to changes in the clock parameters. Fig. 8 shows the small d behaviour of $\tau_D(0,d;E)$ and

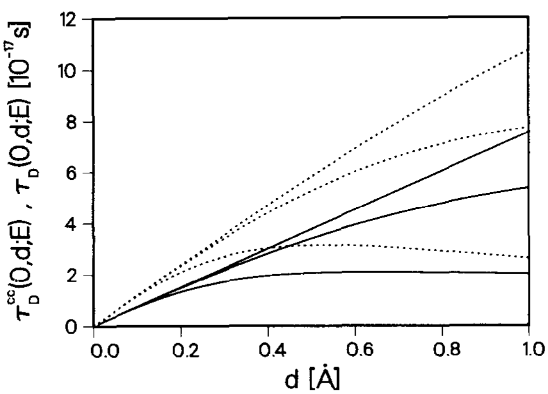

Fig. 8 Intrinsic (————) and calibrated clock (---------------) mean dwell times, $\tau_D(0,d;E)$ and $\tau_D^{cc}(0,d;E)$ for electrons of energy E=5 eV initially incident on a thin rectangular barrier $V_0\Theta(z)\Theta(d-z)$ of height V_0=0, 10 and 20 eV (top to bottom respectively).

$\tau_D^{cc}(0,d;E)$ for V_0=0, 10 and 20 eV. In the limit d→0 the reciprocals of the slopes $\partial\tau_D(0,d;E)/\partial d$ of the three curves for $\tau_D(0,d;E)$ converge to the value $\hbar K/M$, the free particle velocity, as expected. The clocked value of the free particle velocity obtained from the slope of $\tau_D^{cc}(0,d;E)$ in the limit d→0 is independent of V_0 but smaller than $\hbar K/M$ by about 37%. The momentum-position uncertainty relation $\Delta p\Delta z \geq \hbar/2$ would be violated with $\Delta p \sim 0.37\hbar K$ and $\Delta z \to 0$ but it does not apply here because the clocked result for the free particle velocity at z=0 (in the limit d→0) does not pertain to a particular measured instant of time. If one arbitrarily, out of curiosity, uses the minimum uncertainty relation to convert the difference between clocked and intrinsic velocities for d→0 into a distance one obtains $\Delta z \cong 0.2\lambda$. The significance, if any, of this distance is not at all clear to the author. It might be illuminating to express the above numerically determined numbers (1.6, etc.) for the limit d→0 by exact results involving the appropriate fundamental constants. Unfortunately, since $\tau_D^{cc} \approx (\tau_D^c)^{1/3} \propto d$ and $\tau_D^c << \tau \propto \omega^{-1}$, this would involve expansion of T_m, R_m, etc. to at least third order in $\hbar\omega$ and d.

The V_0=0 results in Fig. 8 make contact with the free particle results of the previous section. Although the modifications made there to the quantum clock approach allow accurate determination of the intrinsic free particle velocity for flight paths much smaller than indicated by the original criterion d>>>>λ, it is clear that d still cannot be reduced to zero without introducing a significant departure of the clocked result from the expected one.

For both the free particle and finite barrier cases it would be incorrect in general to attribute the entire difference between $\tau_D^c(0,d;E)$ and $\tau_D(0,d;E)$ to perturbation of the particle and/or clock dynamics by the measurement process because that difference can be very significantly reduced, even eliminated, by using the calibration determined for the corresponding ensemble of freely running clocks. Once that has been accomplished it seems not unreasonable to attribute the residual difference, i.e. the difference between $\tau_D^{cc}(0,d;E)$ and $\tau_D(0,d;E)$, to such perturbations. That they are relatively ineffective for opaque barriers can be made plausible by considering the limiting case (d=∞) in which $|R_m|$ is exactly equal to unity and $|T_m|$ is exactly equal to zero for all m. In this limit

$$P_k = \left| \frac{1}{N} \sum_{m=-j}^{j} e^{i\left[\frac{2\pi mk}{N} + \phi_m^{(R)}(K)\right]} \right|^2 \qquad (|R_m|^2 = 1) \ . \tag{29}$$

Now assume that $j\hbar\omega$ is sufficiently small that replacing $\phi_m^{(R)}(K)$ by $\phi_0^{(R)}(K) + m\hbar\omega(\partial\phi_0^{(R)}(K)/\partial\overline{V})$, with $\overline{V}$ the average barrier height, is a sufficiently accurate approximation. The first term, $\phi_0^{(R)}(K)$, does not contribute to P_k and, from the work of Sokolovski and Baskin[18], $-\hbar(\partial\phi_0^{(R)}(K)/\partial\overline{V})$ is exactly equal to $\tau_D(0,\infty;E)$. Substituting this into (29) and then the resulting expression for P_k into (20) eventually gives

$$\tau_D^c(0,\infty;E) = t_c^{(0)}[\tau_D(0,\infty;E)] \qquad (|R_m|^2 = 1) \ . \tag{30}$$

Hence, for this special case, calibration using (25) gives $\tau_D^{cc}(0,\infty;E) = \tau_D(0,\infty;E)$. Since $\tau_D(0,d;E)$ for a rectangular

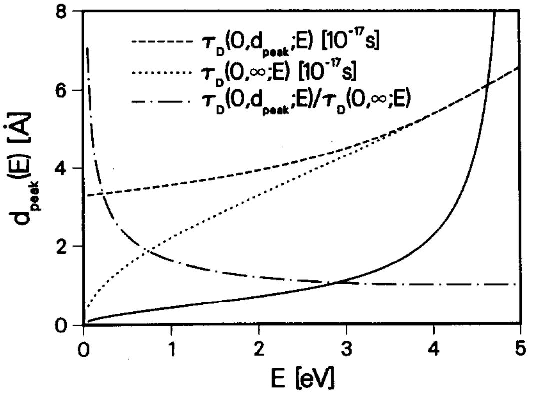

Fig. 9 For fixed electron energy E and rectangular barrier height V_0 the intrinsic dwell time $\tau_D(0,d;E)$ peaks as a function of barrier width d at $d=d_{peak}$. The dependences on E of d_{peak} (————), $\tau_D(0,d_{peak};E)$ (– – – – – –), $\tau_D(0,\infty;E)$ (------------), and $\tau_D(0,d_{peak};E)/\tau_D(0,\infty;E)$ (— - — - —) are shown for V_0=10 eV.

barrier with $\kappa_0 d>>1$ is finite and essentially independent of d the relative difference between $\tau_D^{cc}(0,d;E)$ and $\tau_D(0,d;E)$ can be made as small as desired by making d sufficiently large. For a rectangular barrier with $\kappa_0 d<<1$ a similar calculation shows that the absolute difference between $\tau_D^{cc}(0,d;E)$ and $\tau_D(0,d;E)$ goes to zero as $d\to 0$. However, because $\tau_D(0,d;E)$ itself also goes to zero it does not follow that the relative difference can be made as small as desired simply by making d sufficiently small. In fact, as mentioned above, it is actually largest for d=0.

Returning to Fig. 6, the dependence of $\tau_D(0,d;E)$ on d for $E<V_0/2$ is worthy of note because it peaks at a finite value, d_{peak}, of d. That such a peak should exist follows readily from the opaque and transparent barrier limits of Büttiker's expression[19],

$$\tau_D(0,d;E) \cong \frac{2MK}{\hbar\kappa_0(K^2+\kappa_0^2)} \qquad (\kappa_0 d>>1)\ , \qquad (31)$$

$$\tau_D(0,d;E) \cong \frac{Md}{\hbar K} \qquad (\kappa_0 d<<1)\ , \qquad (32)$$

and the fact that the presence of the barrier does not have a dramatic effect until d becomes comparable to the tunneling length κ_0^{-1}. Define $\tilde{d}$ as the value of d at which linear extrapolations of (31) and (32) outside their ranges of validity intersect, i.e. $\tilde{d}=(2E/V_0)\kappa_0^{-1}$. If $\tilde{d}\gtrsim\kappa_0^{-1}$, i.e. $E\gtrsim V_0/2$, then $\tau_D(0,d;E)$ can bend over to the limiting value (31) without developing a peak; if $E\lesssim V_0/2$ a peak should develop. This is confirmed by Fig. 9 which shows the dependence of d_{peak}, $\tau_D(0,d_{peak};E)$, $\tau_D(0,\infty;E)$ and $\tau_D(0,d_{peak};E)/\tau_D(0,\infty;E)$ on E for a rectangular barrier of height V_0=10 eV.

6. DISCUSSION

In this paper it has been shown how the quantum clock approach of Salecker-Wigner and Peres can be modified to obtain "measured" results for the mean dwell time of a rectangular barrier that are not very different from the results predicted by Büttiker for the underlying intrinsic quantity $\tau_D(0,d;E)$. For the opaque barrier the agreement is excellent. However, the characteristic time that has been of most interest in the field, namely the mean transmission time $\tau_T(0,d;E)$, has been largely ignored in this paper because the quantum clock itself does not distinguish between "to be transmitted" and "to be reflected" particles. Now, it should be possible to include in each member of the ensemble a distant detector that would indicate whether or not the scattered particle is transmitted without disrupting the localized particle-clock interaction. Moreover, from (18), it is seen that the distribution P_k of clock-time eigenvalues $k\tau$ for the uncalibrated quantum clock is a sum of two terms, one involving only the transmission probability amplitudes T_m and the other only the reflection probability amplitudes R_m. Unfortunately, only T_0 and R_0 describe the system of actual interest and, in addition, the uncalibrated clock result for the mean dwell time is grossly different from the intrinsic value. Both of these difficulties can be overcome by applying the calibration procedure to an ensemble of quantum clocks with sufficiently small maximum perburbation energy $j\hbar\omega$. In the process, however, the decomposition of the mean dwell time into two additive components associated only with transmission and reflection respectively is destroyed because the calibrated result for the dwell time is proportional to the cube root of the uncalibrated result. This is consistent with the point of view that it is only the dwell time, which does not distinguish between transmitted and reflected particles, that is a meaningful concept in conventional interpretations of quantum mechanics.

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