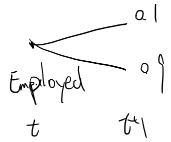
Lecture notes, Sept 24th, 2020

Distribution

$$P = \begin{array}{c|cc} & u & e \\ \hline u & 0.5 & 0.5 \\ e & 0.1 & 0.9 \end{array}$$



$\psi_t = (0 \quad 1)$ raw vec

$\psi_{t+1} = \psi_t \cdot P$

$\psi_t = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\quad \psi_{t+1} = \begin{pmatrix} 0.1 \\ 0.9 \end{pmatrix}$ . . . .

$$\begin{array}{c} 1 \; 2 \; 3 \ldots n \\ \psi_{t+1} \; (\square \qquad\qquad) \end{array}$$

ψ_t given ?

$\Rightarrow \psi_{t+1,1} = \psi_{t,1} \cdot P_{1,1} + \psi_{t,2} \cdot P_{2,1} \cdots + \psi_{t,n} \cdot P_{n,1} = \sum_{i=1}^{n} \psi_{t,i} \cdot P_{i,1} = \psi_t \cdot P_{\cdot,1}$

$P(\psi_{t+1}=1) = P(\psi_{t+1}=1 \mid \psi_t = 1) \cdot P(\psi_t = 1)$

$\underbrace{P(\psi_{t+1}=1 \mid \psi_t = n)}_{P_{1,n}} \cdot \underbrace{P(\psi_t = n)}_{\psi_{t,n}}$

$$\boxed{\psi_{t+1} = \psi_t \cdot P}$$

Two interpretations of distribution from Markov Chain

1. probability evolution of a single agent ✓

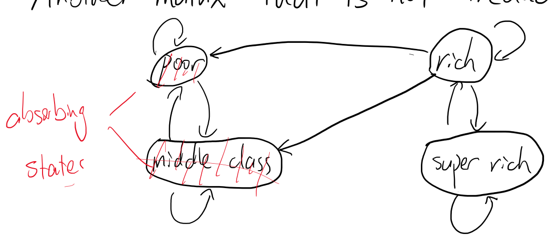
2. cross-sectional distr. of numerous agents : $\psi_t \begin{pmatrix} 0.3 \\ 0.7 \end{pmatrix}$ — 30% of popu. unemployed
   70% employed

$$\psi_{t+1} = (0.3 \quad 0.7) \begin{pmatrix} 0.5 & 0.5 \\ 0.1 & 0.9 \end{pmatrix}$$

$$= (0.22, \quad 0.78) \text{ — at } t+1$$

deterministic! $\leftarrow$ $\boxed{\begin{matrix} 22\% \\ 78\% \end{matrix}}$ unemployed
employed

Another matrix that is not irreducible



Periodic

$\forall \psi_t$, for some m $\psi_{t+m} = \psi_t$

$\psi_{t+m} = \psi_t \cdot P^m \Rightarrow P^m = I$