Lecture notes, Sept 22nd, 2020

Markov matrix

P_{ij}

$\forall i \quad \sum_{j=1}^{n} P_{ij} = 1$

state i $\rightarrow$ state j

each row characterizes one state

P is Markov matrix $\rightarrow$ P^k is Markov matrix

$\forall i,j, \quad P_{ij} \geq 0$

$\forall i \quad \sum_j P_{ij} = 1$

Induction

WANT: $\forall k = 1, 2, \ldots$

P^k is a Markov matrix

1. $P^1 = P$ is a Markov matrix
2. Want to prove: if $Q_{n\times n}$ is a Markov matrix then $Q \cdot P$ is a Markov matrix

(a) $\forall i,j \quad Q_{ij} \geq 0, \; P_{ij} \geq 0 \Rightarrow (QP)_{ij} \geq 0$ ✓

(b) $\forall i \quad \sum Q_{ij} = 1 \quad \sum P_{ij} = 1$ ✓

$$QP_{ij} = \sum_k Q_{ik} \cdot P_{kj}$$

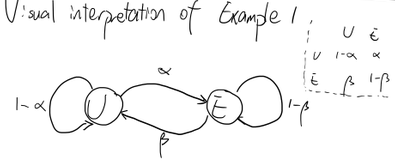
$$\sum_j (QP)_{ij} = \sum_j \sum_k Q_{ik} \cdot P_{kj}$$

$$= \sum_k \sum_j Q_{ik} P_{kj} = \sum_k Q_{ik} \left(\sum_j P_{kj} \right)$$

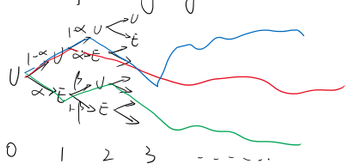
$$= \sum_k Q_{ik} \cdot 1$$

$$= \sum_k Q_{ik} = 1 \quad \square$$

Visual interpretation of Example 1:



simulation of a single agent

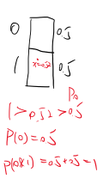


period 0 1 2 3 ------

Transition Example

state 1

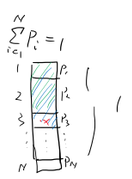
1	P_1
2	P_2
3	⋮
⋮	⋮
N	P_N



$1 > p_2 \geq 0.5$

$P(0) = 0.5$

$P(0 \text{ or } 1) = 0.5 + 0.5 = 1$



Draw $x \sim U_{(0,1)}$

if $x < P_1$

go to state 1

if $P_1 < x < P_1 + P_2$

go to state 2

if $P_1 + P_2 < x < P_1 + P_2 + P_3$

state 3

⋮

if $P_1 + P_2 \cdots P_{N-1} < x$

state N