Problem Set 1
ECON 407, Fall 2020

September 22, 2020

Note: Please **DON'T** use any existing packages to create Markov chains and run the simulations, etc. Roll your own routines.

1 Desperate Unemployment

Assume that a worker can be in 3 states: Unemployed (U), Employed (E), and Desperate (D). The probabilities of transitioning between these three states are described as following,

1. An Unemployed worker finds a job (becomes Employed) with probability α;
2. An Unemployed worker becomes Desperate with probability β;
3. An Employed worker loses a job with probability γ (becomes Unemployed). Otherwise, she/he stays Employed. The Employed workers never become Desperate directly;
4. A Desperate worker has probability ϵ of receiving a job opportunity (becomes Employed) coming out of nowhere. A Desperate worker also has probability δ to become Unemployed – to gain some courage and to try to find job starting from the next period.

We assume the following parameters.

$$
\begin{aligned}
\alpha &= 0.4 \\
\beta &= 0.1 \\
\gamma &= 0.2 \\
\epsilon &= 0.001 \\
\delta &= 0.1
\end{aligned}
$$

1.1 Markov Matrix

1. Define a transition matrix (denoted as P) for the Markov chain of the employment status. Be explicit of the ordering of the states and the non-zero elements.
2. Assume that a worker is Unemployed (U) at period 0. Make a figure that shows the probability that the worker will be Employed (E) in period t for all $t = 1, 2, \ldots, 40$.

3. Assume that a worker is Desperate (D) at period 0. What's the probability that the worker is Employed (E) at period 4?

4. What's the stationary distribution of U, E and D?

1.2 Worker and Income Simulation

Take a currently Unemployed worker. For all $t = 1, \ldots, T$, we define the fraction of time spent in the employment state as

$$\bar{X}_t \equiv \frac{1}{t} \sum_{\tau=1}^{t} \mathbf{1}\{X_\tau = E\}$$

where

$$\mathbf{1}\{X_t = E\} = \begin{cases} 1 & X_t = E \\ 0 & \text{otherwise} \end{cases}$$

is an indicator function.

1. Use the transition matrix from the previous part to simulate a path of $T = 1000$ states for the worker (note: the worker is currently Unemployed)

2. Plot the proportion of time spent employed for the worker for each $t = 1, \ldots T$, i.e., $\{\bar{X}_t\}_{t=1}^{T}$. What's the connection between $\{\bar{X}_t\}_{t=1}^{T}$ and the stationary distribution?

3. Assume a worker can receive $w_E = 1$ when Employed, and $w_U = w_D = 0.2$ when Unemployed or Desperate. Compute the average expected income as

$$\bar{W}_t \equiv \frac{1}{t} \sum_{\tau=1}^{t} \left[w_E \cdot \mathbf{1}\{X_\tau = E\} + w_U \cdot \mathbf{1}\{X_\tau = U\} + w_D \cdot \mathbf{1}\{X_\tau = D\} \right]$$

With the same state simulation as in the last two parts, plot $\{\bar{W}_t\}_{t=1}^{T}$. Any observations?

2 COVID-19: Time Varying Markov Matrix

COVID-19 provides everyone a chance to learn some basic epidemiology. One of the simplest compartmental models is called The SIR Model. S (Susceptible), I (Infectious), and R (Recovered), are exactly three Markov states. Unlike the cases that we studied in class that the Markov matrix is constant across time, here we might have a time-varying Markov matrix.

This model is a *different, discrete-time version* of the ODEs that you can find online. We describe the transitions in words:

- Denote the fraction of S- ,I- and R-type of total population as $\tilde{S}, \tilde{I}, \tilde{R}$ respectively.
- The probability that S-type becomes an I-type is $\beta \times \tilde{I}$. S-type remains Susceptible otherwise.
- The probability that I-type becomes an R-type is γ. I-type remains Infectious otherwise.
- You stay R-type forever once you Recovered.

1. Write down the Markov transition matrix at time t where the fraction of different types of the total population are $\tilde{S}_t, \tilde{I}_t, \tilde{R}_t$. i.e., What's the amount of $\tilde{S}_{t+1}, \tilde{I}_{t+1}, \tilde{R}_{t+1}$ given $\tilde{S}_t, \tilde{I}_t, \tilde{R}_t$.?

2. Assume $\tilde{S}_0 = 0.98, \tilde{I}_0 = 0.02, \tilde{R}_0 = 0$, and $\beta = 0.3, \gamma = 0.15$. Make a figure that simulates the model for 40 periods, i.e., plots $\tilde{S}_t, \tilde{I}_t, \tilde{R}_t$ for $t = 0, 1, 2, \ldots, 40$.

3. Describe the results you get from the simulation. What do the three lines look like? When do the maximum and minimum appear of all these three time series?

4. (Comparative statics) Now we hold all the other parameters the same, but assume $\beta = 0.4$ instead. We define $R_0 = \frac{\beta}{\gamma}$ (R_0 is known as "basic reproduction ratio"). Compare the results you get with what you have from part 2. What conclusion can you draw?