BIMBINGAN BELAJAR
"BLESSING"
"Teaching with Integrity, Making History through Knowledge"
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Luas Daerah (Penggunaan Integral)

1. Hitung luas daerah yang dibatasi oleh parabola y = x^2 – 4x + 4; sumbu x dan sumbu y.

2. Hitung luas daerah yang dibatasi oleh parabola y = x^2, y = 4x + 4; x = 2 dan sumbu y.

3. Luas daerah di kuadran 4 yang dibatasi oleh kurva y = $2Sn^2$2x – 2; sumbu x dan sumbu y adalah ...

4. Luas daerah yang dibatasi oleh kurva x = y^3 – 3 dan garis y = $\frac{1}{3}$x = ...

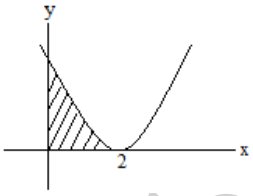
5. Luas daerah yang dibatasi oleh kurva x = y^2 – y, garis x = 3 dan garis y = x adalah ...

6. Jika diketahui garis singgung parabola y = $3x^2$ + ax + 1; pada titik x = -2 membentuk sudut terhadap sumbu x sebesar arc tg 6; luas daerah yang dibatasi oleh garis lurus y = -9x – 59 dan parabola tersebut adalah ...

7. Sebuah garis lurus l melalui titik (0,0) memotong kurva y^2 = x pada sebuah titik dengan absis = t. Jika luas antara kurva dengan garis l adalah $\frac{\sqrt{3}}{3}$ satuan luas. Tentukan nilai t.

8. Hitung luas daerah yang dibatasi oleh $x^2 + y^2 = 2$, y = x^2 dan y ≥ 0.

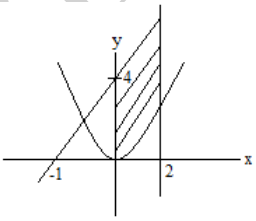
Jawaban

1. 

L = $\int_0^2 (x-2)^2 dx$

= $\frac{1}{3}(x-2)^3\,{}_0^2$

= = - $\frac{1}{3}(-2)^3 = \frac{8}{3}$

2. 

L = $\int_0^2 4x + 4 - x^2 dx$

= $\left(2x^2 + 4x - \frac{1}{3}x^3\right)_0^2$

= 8 + 8 - $\frac{8}{3} = \frac{40}{3}$

3. $y = 2Sn^2 2x - 2$

$y = -2(1 - y = 2Sn^2 2x)$

$y = -2Cos^2 2x$

$y = -2\left(\frac{1+\cos 4x}{2}\right)$

$y = -1 - \cos 4x\ dx$

$L = \int_{270^\circ}^{360^\circ} -1 - cos4x\ dx$

$= \left(-x - \frac{1}{4}Sn4x\right)_{\frac{3\lambda}{2}}^{2\lambda}$

$= (-2\lambda - 0) - (-\frac{3\lambda}{2} - 0)$

$= \left|-\frac{1}{2}\lambda\right|$

$= \frac{1}{2}\lambda$

4. $x = y^3 - y$

$y = \frac{1}{3}x$

$x = 3y$

$3y = y^3 - y$

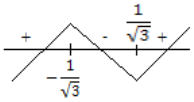
$y^3 - 4y = 0$

$y(y^2 - 4) = 0$

$y(y - 2)(y + 2) = 0$

$y = 0 \mid y = -2 \mid y = 2$

$x = 3y^2 - 1$

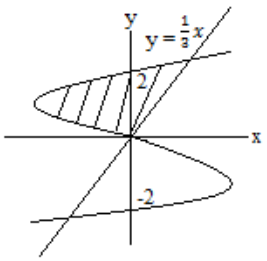


$x = \left(-\frac{1}{\sqrt{3}}\right)^3 + \frac{1}{\sqrt{3}}$

$= -\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}}$

$= \frac{2}{3\sqrt{3}}$

$y = \frac{1}{3}x$

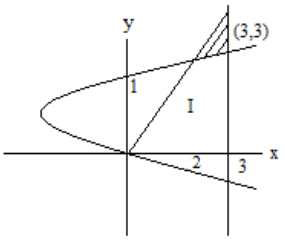


$L = 2\int_0^2 3y - (y^3 - y)dy$

$= 2\left(2y^2 - \frac{1}{4}y^4\right)_0^2$

$= 2(8 - 4) = 2 \cdot 4$

$= 8$

5. 

$x = y^2 - y$

$y = x$

$x = x^2 - x$

$x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0 \mid x = 2$

$x = y^2 - y$

$x = \left(y - \frac{1}{2}\right)^2 - \frac{1}{4}$

$\left(y - \frac{1}{2}\right)^2 = x + \frac{1}{4}$

$Y = \frac{1}{2} + \sqrt{x + \frac{1}{4}}$

$LI = L\Delta + \int_2^3 parabola\ dx$

$= \frac{2 \cdot 2}{2} + \int_2^3 \frac{1}{2} + \sqrt{x + \frac{1}{4}}\ dx$

$= 2 + \int_2^3 \frac{1}{2} + \sqrt{\frac{4x+1}{4}}\ dx$

$= 2 + \int_2^3 \frac{1 + \sqrt{4x+1}}{2}\ dx$

$= 2 + \left(\frac{1}{2}x + \frac{1}{12}(4x+1)^{\frac{3}{2}}\right)_2^3$

$= 2 + \left(\frac{3}{2} + \frac{1}{12} \cdot 13^{\frac{3}{2}}\right) - \left(1 + \frac{1}{12} \cdot 27\right)$

$= \frac{5}{2} - \frac{9}{4} + \frac{1}{12} \cdot 13^{\frac{3}{2}}$

$= \frac{1}{4} + \frac{1}{12} \cdot 13^{\frac{3}{2}}$

$L\Delta\ Besar = \frac{3\ x\ 3}{2} = \frac{9}{2}$

$L\ yang\ diarsir = \frac{9}{2} - \left(\frac{1}{4} + \frac{1}{12} \cdot 13^{\frac{3}{2}}\right)$

$= \frac{17}{4} - \frac{13^{\frac{3}{2}}}{12}$

6. $y = 3x^2 + ax + 1$

$y' = m = 6x + a$

$y' = -12 + a$

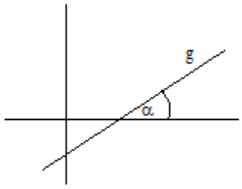
$m = -12 + a$

Titik singgung x = -2

$y = 12 - 2a + 1$

$y = 12 - 2a$

(-2,13 – 2a)



Tg α = 6

m = tg α

-12 + a = 6

A = 18

Titik singgung = (-2,13 – 36)

= (-2, -23)

y = $3x^2$ + 18x + 1

y = -9x – 59

y = y

$3x^2$ + 18x + 1 = -9x – 59

$3x^2$ + 27x + 60 = 0

x^2 + 9x + 20 = 0

(x + 5)(x + 4) = 0

x = -4 | x = -5

L = $\int_{-4}^{-5} -9x - 59 - (3x^2 + 18x + 1)$

= $\frac{1}{2}$

7. y = ax

y^2 = x

y = $\sqrt{x}$

$\sqrt{x}$ = ax

$x = a^2x^2$

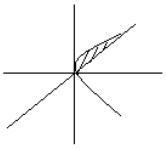
x = t

t = a^2t^2

$a^2 . t$ = 1

$a^2 = \frac{1}{t}$

a = $\frac{1}{\sqrt{t}}$



L = $\int_0^t \sqrt{x}$ - $\frac{1}{\sqrt{t}}$ x = $\frac{\sqrt{3}}{3}$

= $\left(\frac{2}{3}.x^{\frac{3}{2}} - \frac{1}{2\sqrt{t}}.x^2\right)^t = \frac{\sqrt{3}}{3}$

$\frac{2}{3} . t^{\frac{3}{2}} - \frac{1}{2\sqrt{t}} . t^2 = \frac{\sqrt{3}}{3}$

$\frac{2}{3} . t^{\frac{3}{2}} - \frac{1}{2} . t^{\frac{3}{2}} = \frac{\sqrt{3}}{3}$

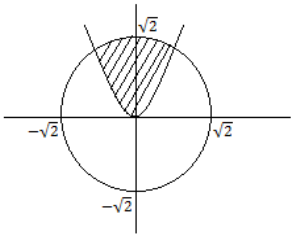
$\frac{1}{6} t^{\frac{3}{2}} = \frac{\sqrt{3}}{3}$

$t^{\frac{3}{2}} = 2\sqrt{3}$

$t = 1\frac{1}{3}\sqrt{3}$

8. $x^2 + y^2 = 2$

$y = x^2$



$x^2 + y^2 = 2$

$x^2 + x^4 = 2$

$x^4 + x^2 - 2 = 0$

$(x^2 + 2)(x^2 - 1) = 0$

$x^2 = 1$

x = 1 | x = -1

$L = 2\int_0^1 \sqrt{2 - x^2} - x^2 \, dx$

$x = \sqrt{2}$ Sn α

$dx = \sqrt{2}$ cos α

$\int \sqrt{2 - x^2} \, dx$

$\int \sqrt{2 - 2Sn^2\alpha} . \sqrt{2} \cos\alpha \, dx$

$\int_0^{\frac{\lambda}{4}} 2 \cos^2\alpha \, dx$

$\int_0^{\frac{\lambda}{4}} 1 + cos2\alpha \, dx$

$= \left(\alpha + \frac{1}{2} Sn2\alpha\right)_0^{\frac{\lambda}{4}}$

$= \frac{\lambda}{4} + \frac{1}{2}$

$L = 2\int_0^1 \sqrt{2 - x^2} - \int_0^1 2x^2 dx$

$= 2\left(\frac{\lambda}{4} + \frac{1}{2}\right) - \frac{2}{3}$

$= \frac{1}{3} + \frac{1}{2}\lambda$