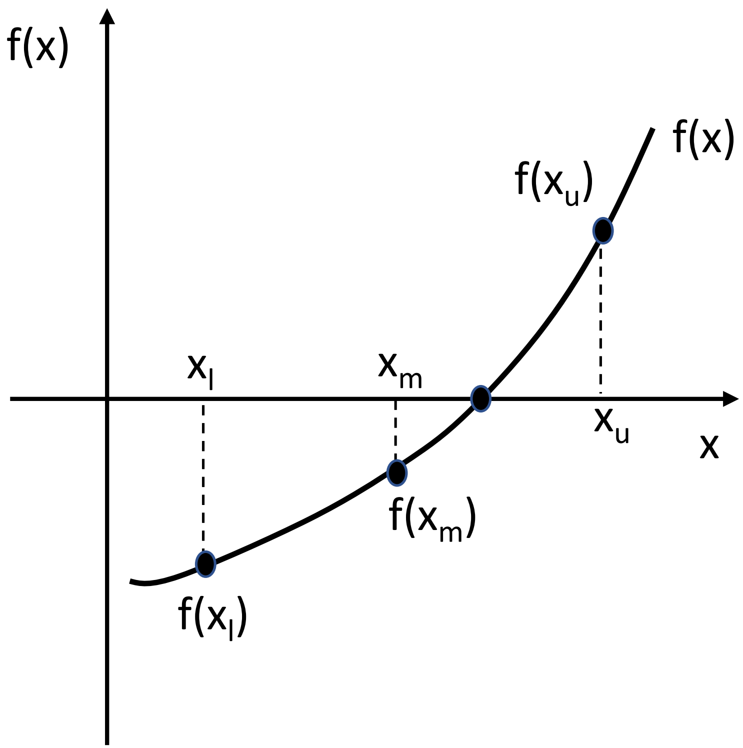
# Lecture 14

# Bracketing Techniques

- **Bracketing Techniques**
  - Bisection method




- A while loop is needed.
- Initialize $x_l$ , $x_u$, then calculate the middle $x_m = (x_l + x_u)/2$
- To see which side $x_m$ is on of the root ( to see if $x_m$ is on the same side as $x_l$)

  when $f(x_l) * f(x_m) > 0$, set $x_l = x_m$

  when $f(x_l) * f(x_m) < 0$, set $x_u = x_m$

  when $f(x_l) * f(x_m) = 0$, $x_m$ is the root
- In the while loop, we have,

  while $\varepsilon_a > \varepsilon_S$

$$\varepsilon_a = \left| \frac{x_n - x_o}{x_n} \right| \times 100\%$$

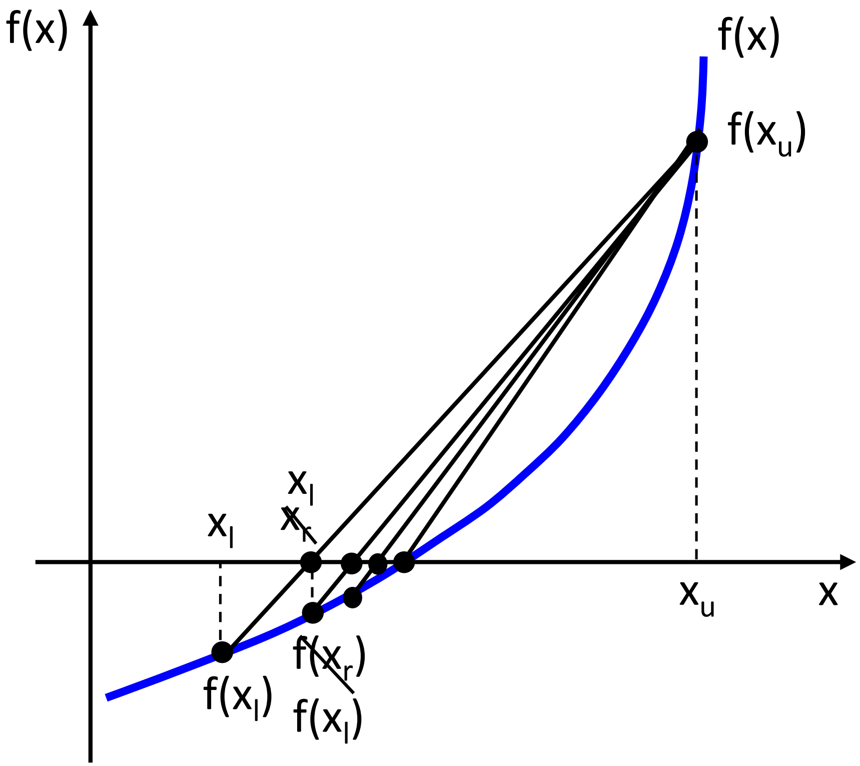
- The result is correct to at least n significant figures if

$$\varepsilon_S = (0.5 \times 10^{(2-n)})\ \%$$

# False-Position Method

- **Bracketing Techniques**

- False-Position Method - If a real root is bounded by $x_l$ and $x_u$ of $f(x)=0$, then we can approximate the solution by doing a linear interpolation between the points $[x_l, f(x_l)]$ and $[x_u, f(x_u)]$ to find the $x_r$ value.




$x_r$ is an approximate of the root.

If $x_r$ is on the same side as $x_l$, the $x_r$ will be defined as the new $x_l$. Otherwise, $x_r$ will be defined as the new $x_u$.

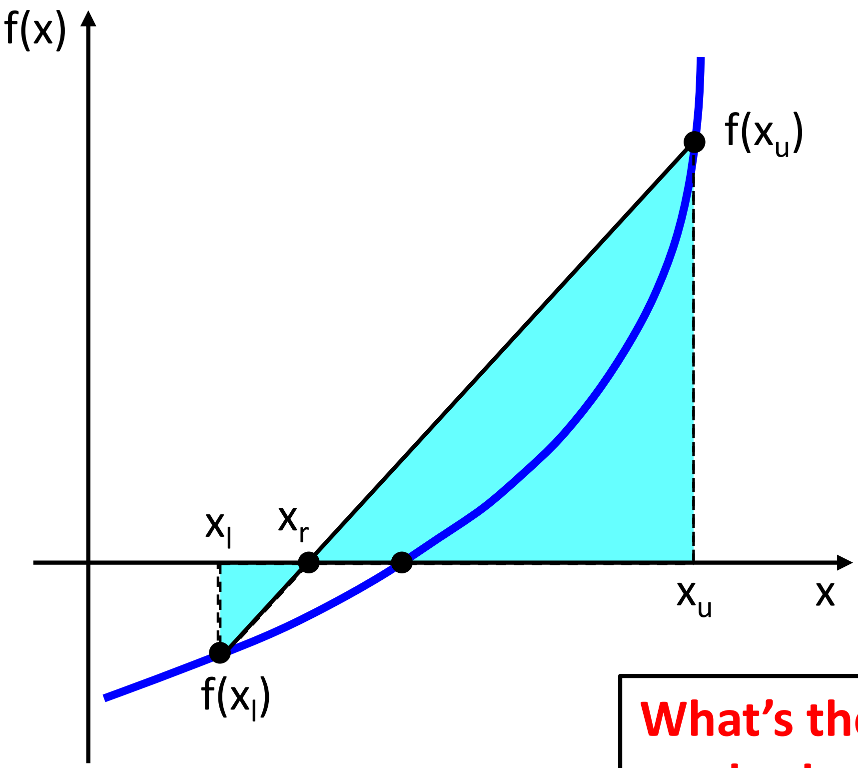
How to determine the side of $x_r$ ? (examine $f(x_r)*f(x_l)$)

The key step is to find $x_r$ for any given $x_l$ and $x_u$

**Make sure $f(x_l) * f(x_u)<0$**

# False-Position Method

- If a real root is bounded by $x_l$ and $x_u$ of $f(x)=0$, then we can approximate the solution by doing a linear interpolation between the points $[x_l, f(x_l)]$ and $[x_u, f(x_u)]$ to find the $x_r$ value.




Using similar triangles we get:

$$\frac{-f(x_l)}{x_r - x_l} = \frac{f(x_u)}{x_u - x_r}$$

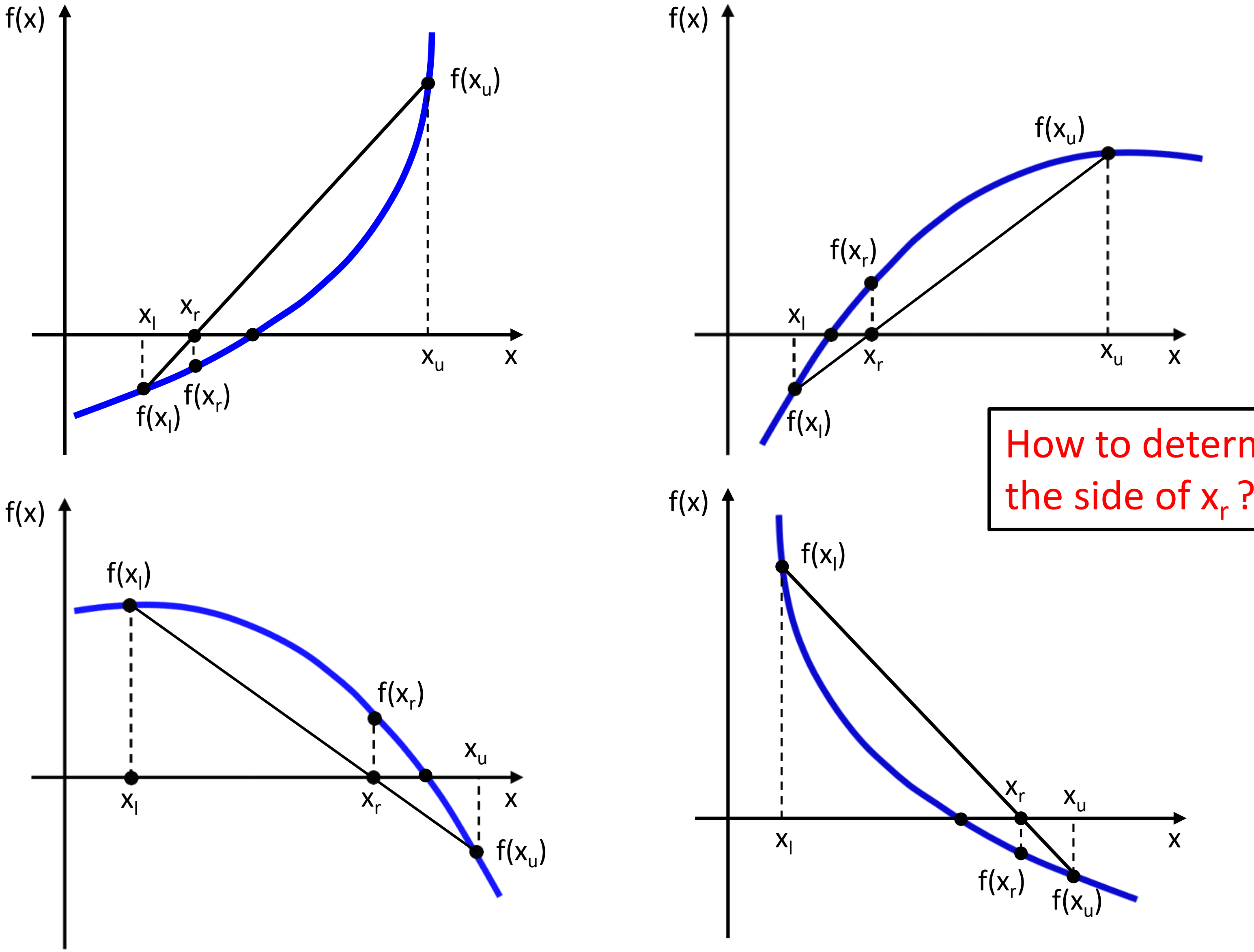
Which can be solved to get

$$x_r = \frac{x_l f_u - x_u f_l}{f_u - f_l}$$

where $f_u = f(x_u)$ and $f_l = f(x_l)$

**What's the difference between this method and the bisection method?**

# False-Position Method




How to determine the side of $x_r$ ?

# False-Position Method

1. Find a pair of values of x, $x_l$ and $x_u$ bracketing the root, that means $f(x_l) * f(x_u) < 0$.
2. Do linear fit between $(x_l, f(x_l))$ and $(x_u, f(x_u))$
3. Estimate the value of the root $x_r$ and evaluate $f(x_r)$.
3. Use the new point $x_r$ to replace one of the original points, keeping the two points on opposite sides of the root.

   a. If $f(x_l) * f(x_r) < 0$, $x_u = x_r$

   b. If $f(x_l) * f(x_r) > 0$, $x_l = x_r$

   c. If $f(x_l) * f(x_r) = 0$, then root is $x_r$ and terminate.

4. Compare $\varepsilon_s$ with $\varepsilon_a$

   An approximate percentage relative error can be calculated by the equation

$$\left|\frac{x_r^{new} - x_r^{old}}{x_r^{new}}\right| \times 100\%$$

5. If $\varepsilon_a < \varepsilon_s$ , stop. Otherwise go to step 2

# False-Position Method

- **Example**

  $f(x) = e^{-x} - x^3$ . find the x that make $f(x) = 0$

Same example as that in Lecture 13

- 
```
>> f = @(x) exp(-x) – x.^3
f =
    @(x) exp(-x)-x.^3
```

```
>> x = linspace(0,2,100);
>> plot(x, f(x))
>> grid
```

In order to define xl and xu, we plot the function first.

# False-Position Method

Program using bisection method

```
clc
clear

f = @(x) exp(-x) - x.^3;

xl = 0;
xu =2;
n = 4; % number of sig. figures
es = 0.5*10^(2-n) ;

ea = 1;
xo = 10;
cnt = 0;
while ea > es
     xm = (xl+xu)/2;
     if f(xl)*f(xm) > 0
        xl = xm;
     elseif f(xl)*f(xm) < 0
        xu = xm;
     else
        xo = xm;
     end
      ea = 100*(abs(xm-xo)/xm);
      xo = xm;
      cnt = cnt +1;
   end
fprintf('The root of f(x) is: %.4f\n\n', xm)

fprintf('It took %d iterations to converge. \n\n', cnt)
```

```
xm = (xl*f(xu) – xu*f(xl)) / (f(xu) – f(xl));
```

- $x_r = \frac{x_l f_u - x_u f_l}{f_u - f_l}$
- $f(x) = e^{-x} - x^3$

# False-Position Method

- Summary
  - False-Position Method

- Homework on Canvas