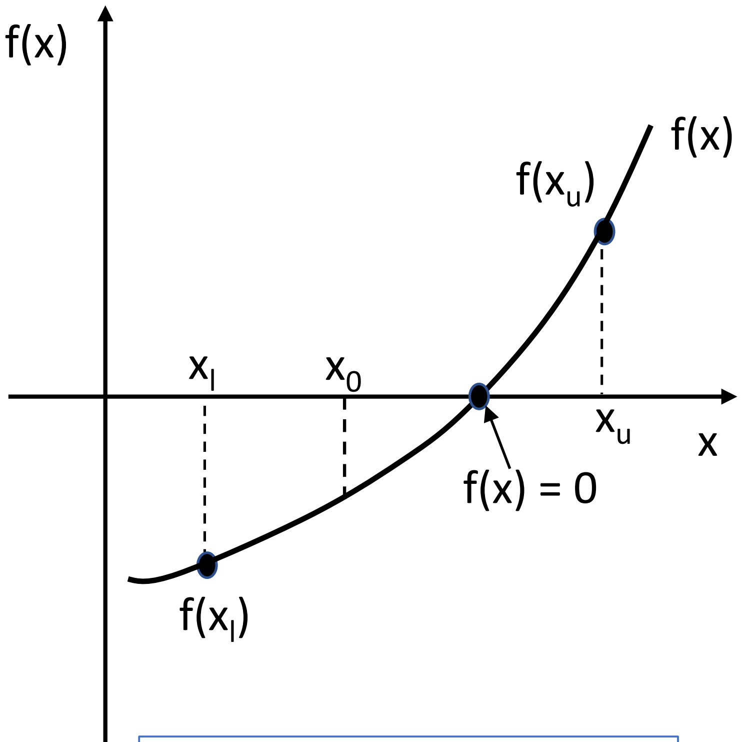
Lecture 13

Methods for Finding Root of Equation

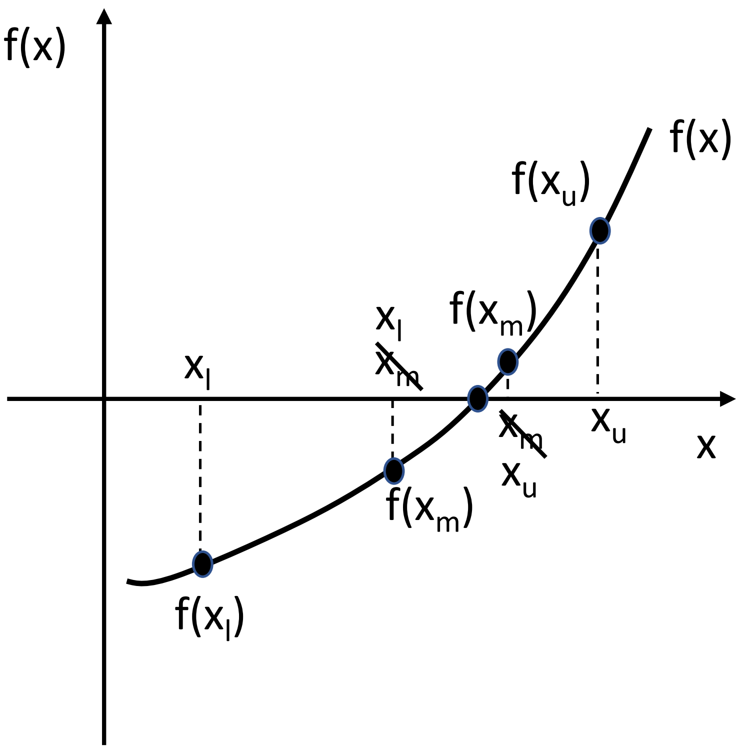
- We are going to find the root of equation f(x)=0
- There are basically two types of methods for finding the root of equation f(x) = 0: bracketing techniques, and open methods.
- Bracketing techniques are based on making two initial guesses that "bracket" the root
- Open method



The x value that makes the function equal to zero is called the root.

Bracketing Techniques

- **Bracketing Techniques**
  - Bisection method



Make sure $f(x_l) * f(x_u) < 0$

- A while loop is needed.
- Initialize x_l , x_u, then calculate the middle $x_m = (x_l + x_u)/2$
- To see which side x_m is on of the root (to see if x_m is on the same side as x_l)

  when $f(x_l) * f(x_m) > 0$, set $x_l = x_m$

  when $f(x_l) * f(x_m) < 0$, set $x_u = x_m$

  when $f(x_l) * f(x_m) = 0$, x_m is the root
- In the while loop, we have,

  while $\varepsilon_a > \varepsilon_S$

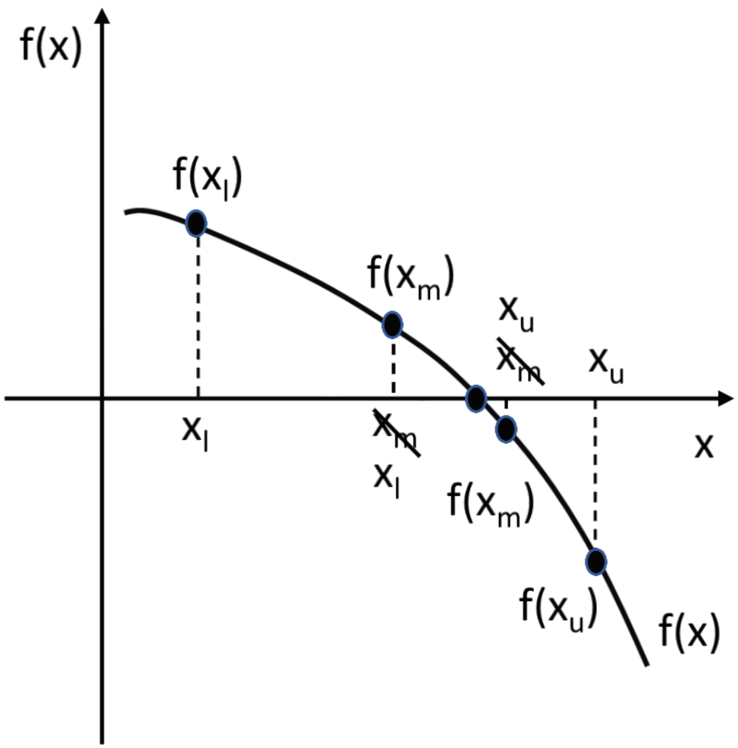
$$\varepsilon_a = \left| \frac{x_n - x_o}{x_n} \right| \times 100\%$$

- The result is correct to at least n significant figures if

$$\varepsilon_S = (0.5 \times 10^{(2-n)})\ \%$$

Bisection Method

- **Bracketing Techniques**
  - Bisection method



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  (to see if x_m is on the same side as x_l)

  when $f(x_l) * f(x_m) > 0$, set $x_l = x_m$

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  when $f(x_l) * f(x_m) = 0$, x_m is the root
- In the while loop, we have,

  while $\varepsilon_a > \varepsilon_s$

$$\varepsilon_a = \left|\frac{x_n - x_o}{x_n}\right| \times 100\%$$

- The criterion the result is correct to at least n significant figures

$$\varepsilon_s = (0.5 \times 10^{(2-n)})\ \%$$

Bisection Method

➢ Bisection Method (Steps)

1. Pick x_l and x_u such that they bound the root of interest, $f(x_l)*f(x_u) <0$.
2. Estimate the root by evaluating $x_{mid} = (x_l+x_u)/2$. (x_{mid} is an approximate of the root)
3. Determine the side of x_{mid}
   a. If $f(x_l) * f(x_{mid})>0$, x_{mid} is on the same side as x_l, then $x_l=x_{mid}$, go to step 4.
   b. If $f(x_l) * f(x_{mid}) < 0$, x_{mid} is on different side with x_l, then $x_u=x_{mid}$ and go to step 4.
   c. If $f(x_l) * f(x_{mid}) = 0$, then root is x_{mid} and terminate.
4. Compare ε_s with ε_a. An approximate percentage relative error can be calculated by the equation

$$\left|\frac{x_{mid}^{new} - x_{mid}^{old}}{x_{mid}^{new}}\right| \times 100\%$$

5. If $\varepsilon_a < \varepsilon_s$ stop. Otherwise go to step 2.

Bisection Method

```
while ea > es
        xm = (xl + xu)/2

        if f(xl) * f(xm) > 0
                xl = xm
        elseif f(xl) * f(xm) < 0
                xu = xm
        else
                xo = xm
        end

        ea = 100 * abs((xm-xo)/xm)
        xo = xm
end
```

Find x_m

If-elseif statements were used to find the side of x_m and define the new x_l or x_u.

x_m is an approximate of the root

Evaluate the relative error

Store x_m in the output variable or old-value variable

This while loop is the same for any problems using Bisection method to solve

Bisection Method

- **Equations and functions**

- Examples
- $f(x) = e^{-x} - x^3 \quad \rightarrow \quad e^{-x} - x^3 = 0$
- $f(x) = x^3 - 3x^2 + 2 \quad \rightarrow \quad x^3 - 3x^2 + 2 = 0$
- $S^2 + S + 3 = \ln(S/10)$ to get the function, move everything to the left-hand side, $S^2 + S + 3 - \ln(S/10) = 0$, the corresponding function is

  $f(S) = S^2 + S + 3 - \ln(S/10)$

  $f(x) = x^2 + x + 3 - \ln(x/10)$

We want to find the x that make $f(x) = 0$

Bisection Method

➢ **Example**

- $f(x) = e^{-x} - x^3$. I want to find the x that make f(x) = 0

- We have no idea what the root is going to be, so we cannot make the initial guesses of x values. Draw a plot of f(x) to see what the root looks like and determine x_l and x_u

```
>> f = @(x) exp(-x) – x.^3
f =
     @(x) exp(-x)-x.^3
>> x = linspace(0,10,100);
>> plot(x, f(x))
>> x = linspace(0,2,100);
>> plot(x, f(x))
>> grid
```

Bisection Method

```
clc
clear

f = @(x) exp(-x) - x.^3;

xl = 0;
xu =2;
n = 4; % number of sig. figures
es = 0.5*10^(2-n) ;

ea = 1;
xo = 10;
cnt = 0;
while ea > es
      xm = (xl+xu)/2;
      if f(xl)*f(xm) > 0
          xl = xm;
      elseif f(xl)*f(xm) < 0
          xu = xm;
      else
          xo = xm;
      end
       ea = 100*(abs(xm-xo)/xm);
       xo = xm;
       cnt = cnt +1;
   end
fprintf('The root of f(x) is: %.4f\n\n', xm)

fprintf('It took %d iterations to converge. \n\n', cnt)
```

Define the function handle

Define the lower and upper value of x

Define es

Initialize ea. It can be any number greater than es

Initialize xo for the estimation of ea in iteration 1

Initialize counter with 0

While loop is used to find the solution. It is largely the same for any problems using Bisection method

Bisection Method

➢ **Summary**

- **Bisection Method**

➢ **Homework on Canvas**