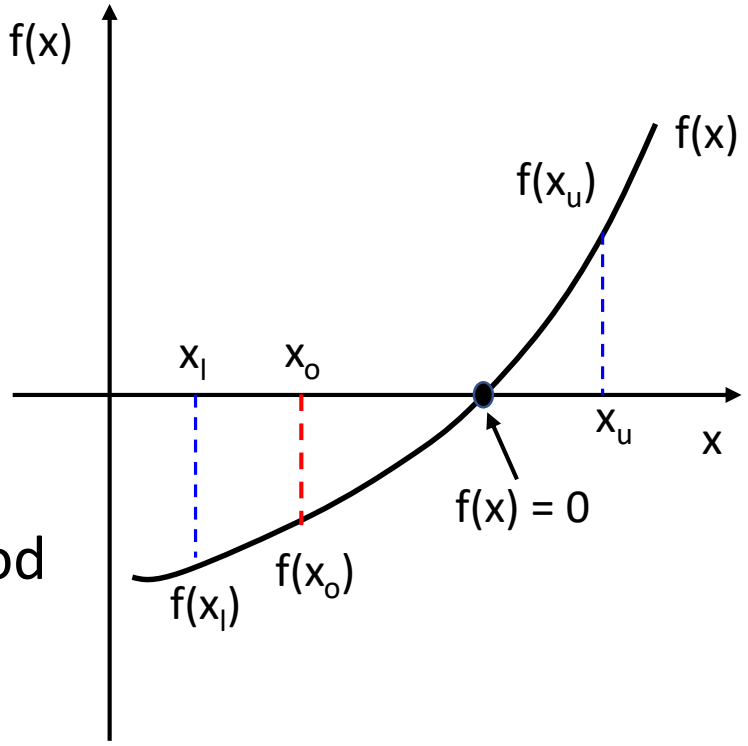
Lecture 17

Methods for Finding Root of Equation

- There are basically two types of methods for find the root of equation f(x) = 0

- Bracketing techniques
  - ✓ Bisection method
  - ✓ False position method

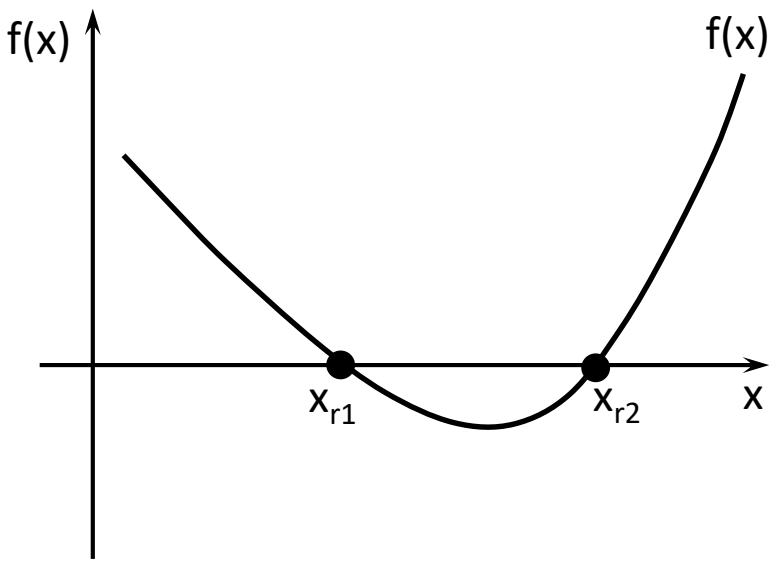
- Open methods
  - ✓ Fixed-point iteration method
  - ✓ Newton-Raphson method



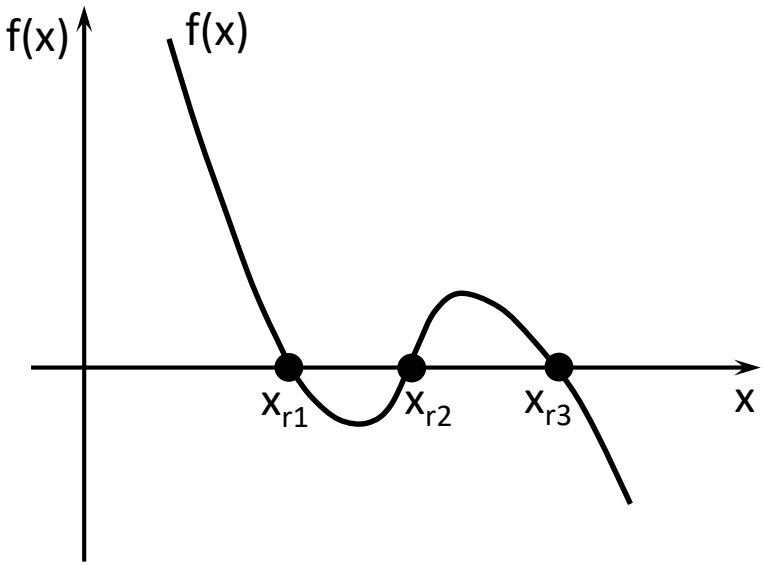
Multiple Roots

- **Equations with Multiple Roots, f(x) = 0**

two roots



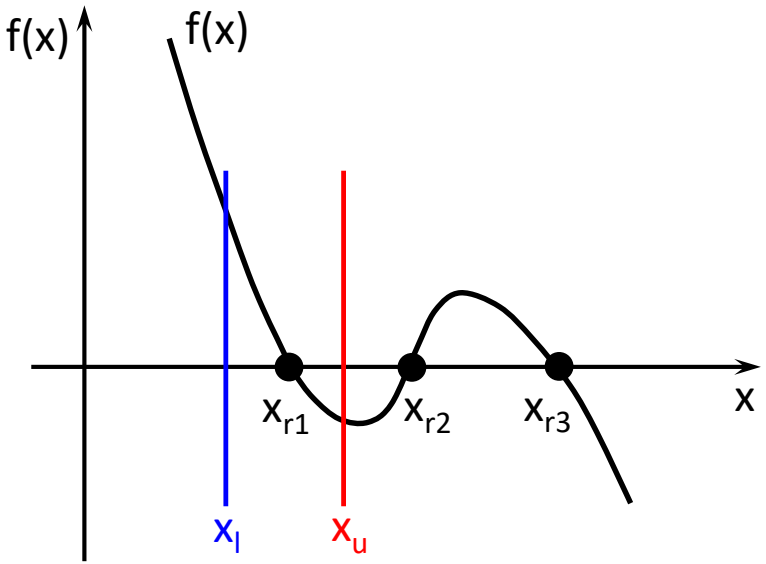
three roots



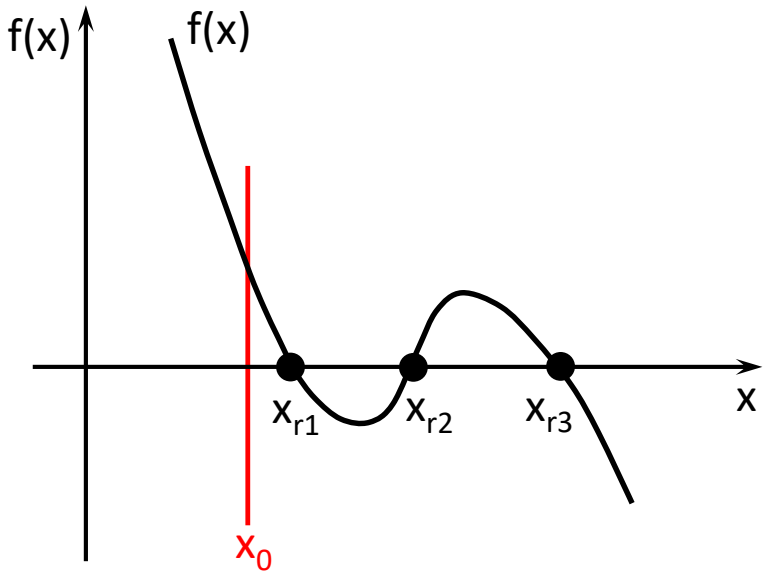
Multiple Roots

- **Equations with Multiple Roots, f(x) = 0**

bracketing techniques

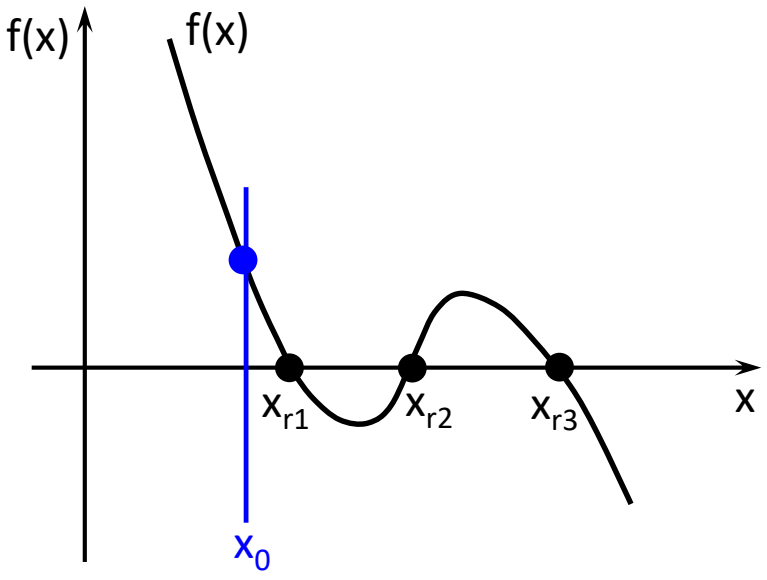


open methods



Multiple Roots

➢ **Equations with Multiple Roots, f(x) = 0**



The general way to find the multiple roots.

- plot the function within a certain x range of interest.
- March from the small x value of the range, and each time increment x with a small amount.
- As soon as we see the function changes its sign, we know we have a root. Then use bracketing techniques or open methods to find the root.
- Then continue that process until we find all the roots we are interested in.

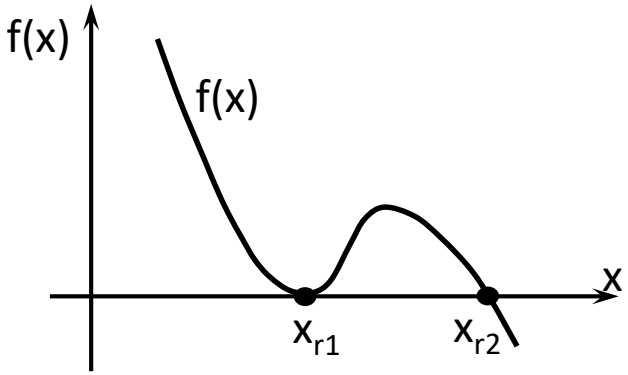
Multiple Roots

➢ **Multiple Roots**

- None of the methods deal with multiple roots efficiently.

➢ **Difficulties**

- Function does not change sign, therefore, cannot use bracketing methods.



fzero Function

➢ **fzero Function**

- MATLAB's fzero is a powerful built-in function to find the root of equations.
- Uses a combination of bisection, secant, and inverse quadratic interpolation methods.
- Syntax:
  - ✓ x = fzero(function,x0)
  - ✓ [x, fx] = fzero(function, x0)

fzero has nothing to do with multiple roots. It is used to find a particular root, according to x0.

'function' is the name of the function being evaluated

'x0' is the initial guess of x

'x' is the location of the root

'fx' is the function value evaluated at the root

fzero Function

➢ **Fzero Function**

- Syntax:
  - ✓ [x, fx, exitflag] = fzero (function, x0) returns a value of exitflag that describes the exit condition of fzero.

    1 => function converged to a solution x.

    -1 => Algorithm was terminated by the out function

    -3 => NaN or inf function value was encountered during search for an interval containing a sign change

    -4 => Complex function value was encountered during search for an interval containing a sign change

    -5 => fzero might have converged to a singular point

    -6 => fzero can not detect a change in sign of the function.

fzero Function

- Example:

  solve the following equation using *fzero* function

$$W - Qc - kV\sqrt{c} = 0$$

  where V = 10^5, Q = 10^5, k = 0.25, and W = 10^6

fzero Function

- Example:

```
clc
clear

V = 1*10^6;
Q = 1*10^5;                          Define constants
k = 0.25;
W = 1*10^6;

fc = @(c) W-Q*c-k*V*sqrt(c);         Define function handle of f(x)

x = [0 : .1 : 10];
plot(x, fc(x), 'LineWidth', 3)       Plot function f(x)
grid

%xo = input('Enter a guess for the root:  ');

%--------method 1-------------------------------------
%xr = fzero(fc,  xo);
%fprintf('The root of f(x) is %.2f\n\n',xr);

%--------method 2-------------------------------------
%xr = fzero(@(c) W-Q*c-k*V*sqrt(c),  xo);
%fprintf('The root of f(x) is %.2f\n\n',xr);

%--------method 3-------------------------------------
%options = optimset('display', 'iter');
%xr = fzero(fc,  xo, options);
%fprintf('The root of f(x) is %.2f\n\n',xr);

%--------method 4-------------------------------------
%[xr, fxr] = fzero(fc,  xo);
%fprintf('The root of f(x) is %.2f\n\n',xr);
%fprintf('The value of f(x) at the root is %.2e\n\n ', fxr)
```

Multiple Roots

- Summary
  - Equations with multiple roots
  - fzero function

- **Homework on Canvas**