ARISTOTLE'S LOGIC

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I. Introduction

Experiences are but the seeds of our knowledge. What we "know" or "understand" is the product of interpreting those experiences—of interrelating, interpolating, extrapolating, or, more generally, of constructing inferences on those experiences. It follows that, to the extent that we revere reality, we would like our inferences to be rational, or at least to have some means of recognizing when they are irrational. Herein lies the cultural importance of systems of logic. Systems of logic provide axiomatic, content-independent bases for assessing the validity of our inferences.

For reasons of both its history and its apparent simplicity, it has been the Aristotelian syllogistic logic, more than any other, to which we have turned to evaluate human reasoning. The verdict, upheld over thousands

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of years, is that for the untrained mind the inclination to follow the logic is at best slight. In Erickson's words, "For *most* syllogisms, the modal response is incorrect" (p. 41, 1978). Moreover, as all of us who have taught or taken courses in logic can attest, it is often only with considerable difficulty that the untrained mind can be trained to follow the logic of the syllogism.

From such observations, one might conclude that people are fundamentally irrational. Indeed, it has been argued that from a psychological perspective, the logic is artificial and irrelevant, that thought responds not to such formalities but to knowledge-based pragmatic constraints. More recently, however, psychologists have tended to endorse a different position. People's errors, they have argued, stem not from any difficulties with the logic per se, but from other processes that are necessarily involved in interpreting and solving the syllogisms. It is, for example, well documented that people suffer interpretive biases due to mismatches between the formal and everyday significance of the logic's language. Moreover, it has been argued that the operations required for encoding and interrelating the premise information are sufficiently complex that they may critically tax the reasoner's processing capacity such that would-be-sound deductions are distorted or aborted.

Neither of these classes of excuses is very satisfying. The logic does, after all, delineate the conclusions that may and may not be legitimately drawn from an argument. If reason were truly impervious to the laws of logic, then no one could have proper justification to say so. Similarly, it is of little comfort to assert that our thought processes are fundamentally obedient to the laws of logic while qualifying that, for reasons of confusion or capacity, our thoughts are not.

The latter tack is all the more unsettling when we consider that Aristotle intended the syllogisms to represent the most elementary of possible arguments. The basic propositions of the logic correspond to the simplest dyadic relations: They assert nothing more than that all or some of one class is or is not related to all or some of another. Furthermore, as the syllogisms consist of but three such propositions, two premises and a conclusion, they correspond to the simplest implicative chains. If, as Aristotle argued [*Analytica Priora (AP)* I,23, I,25],[1] every valid argument consists, at core, of nothing more than a chain of these simple syllogisms, and if we do not possess the interpretive wherewithal to follow the syllogisms when

[1]Reference to Aristotle's work will include the title of the treatise followed by the number of the relevant chapter or, in the case of the *Analytica Priora*, the relevant book and chapter. For direct quotes, page and column citations are also included. The titles of the treatises are abbreviated in subsequent mention.

presented one by one, what are we to expect of our capacity for more natural complex arguments?

Moreover, confusion about Aristotelian logic is not the sole prerogative of the casual reasoner. Consider the very basic question of how many of the pairs of simple propositions yield valid conclusions. According to Wason and Johnson-Laird (1972), the answer is 27; according to Erickson (1974), Langer (1953), and Lemmon (1965), it is 24; according to Guyote and Sternberg (1981), Mates (1972), Prior (1973), and Revlis (1975a), it is 19; and according to the *McGraw-Hill Encyclopedia of Science and Technology* (1960), it is 15. Could it be that after thousands of years of trying to train ourselves to analyze the arguments validly, we still don't know how many valid forms there are? If we don't know how many, we can't possibly know which.

The fact of these discrepancies raises the possibility that the apparent irreconcilability of logic and thought may derive in part from some fault with the logic or at least its presentation. It is to this possibility that the present article is addressed.

The body of the article is divided into three major sections. Section II presents a review of the logic with the purpose of establishing the sources and significance of the differences over its interpretation. The purpose of Section III is to review prominent theories of the psychology of syllogistic reasoning and to extract from them the major classes of difficulty that beset the human reasoner. Finally, in Section IV, these difficulties will be examined against our clarified understanding of the essentials of the logic. The goal will be to discern which of the difficulties must indeed reflect weaknesses in our logical dispositions and which might be alleviated through changes in the presentation of the logic.

II. How Many Syllogisms Are Valid?

A. A Review of the Logic

The building blocks of the logic are simple propositions. Not all sentences are propositions, Aristotle explained, "only such are propositions as have in them either truth or falsity" [*De Interpretatione* (*DI*) 4,17a]. Moreover, of propositions there are only two kinds: simple and composite. A simple proposition is "that which asserts or denies something [a predicate] of something [a subject]" (*DI* 5,17a). A composite proposition is nothing more than a compounding of simple propositions.

Aristotle further recognized that, besides being either positive or negative, the character of simple propositions may differ in quantity, as the intended scope of the predicate may be either *universal*, applying to all instances subtended by the subject, or *particular*, applying to but some subset of those. There thus result just four basic propositional frames, as shown in Table I. Through tradition, the universal and particular affirmative frames are labeled *A* and *I*, respectively, from the mnemonic *A*FF*I*RMO, and expressed as "All (subject) are (predicate)" and "Some (subject) are (predicate)." Analogously, the two negative frames are designated *E* and *O*, from the mnemonic N*E*G*O*, and expressed as "No (subject) are (predicate)" and "Some (subject) are not (predicate)."

A syllogism was introduced by Aristotle as "discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so" (*AP* I,1,24b). So described, "syllogism" would seem to be a synonym for "valid argument." As Aristotle had it, however, most valid arguments are more complex than syllogisms. More precisely, most valid arguments consist of *chains* of syllogisms. Syllogisms include only those minimal arguments that logically establish a new relationship between just two terms by means of a relationship that each is known to hold with some third or intermediate term (*AP* I,23).

Formally, then, a syllogism must consist of two premises: one connecting each of the terms in question to the intermediate term (*AP* I,25). Inasmuch as the intermediate term must occur as either the subject or the predicate of each premise, a syllogism may be configured in any of four ways, as shown in Table II. Notably, Aristotle recognized only the first three of these syllogistic figures. Whether he omitted the fourth from his system through insight or oversight has been a matter of much controversy.

Accepting all four figures for the time being and allowing that each of their premises can be filled out with any of the four propositional frames, the syllogism is seen to admit just 64 different pairs of premises. The force of the theory derives from the claim that to specify which of these few pairs of premises yield valid conclusions would be to establish an axiomatic system that was sufficient for evaluating the validity of any argument.

1. The Principal Problem

But why has it been so difficult to establish which of the 64 pairs of premises yield valid conclusions? The principal reason is that, whereas the

TABLE I

THE FOUR TYPES OF CATEGORICAL PROPOSITIONS

A	(Universal affirmative):	All (subject) are (predicate)
I	(Particular affirmative):	Some (subject) are (predicate)
O	(Particular negative):	Some (subject) are not (predicate)
E	(Universal negative):	No (subject) are (predicate)

TABLE II

THE FOUR POSSIBLE COMBINATIONS OF SUBJECT–PREDICATE ARRANGEMENTS OF TWO END TERMS (*A* AND *C*) AND A MIDDLE TERM (*B*) ACROSS THE TWO PREMISES OF A SYLLOGISTIC ARGUMENT

	Figure 1	Figure 2	Figure 3	Figure 4
Premise 1:	*B–A*	*A–B*	*B–A*	*A–B*
Premise 2:	*C–B*	*C–B*	*B–C*	*B–C*
	(Conclusion)	(Conclusion)	(Conclusion)	(Conclusion)

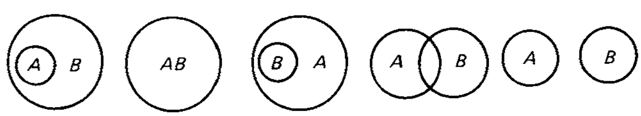
conclusions that follow necessarily from a pair of premises depend on the minimal relationships that can be established between its terms, those relationships bear an ambiguous correspondence to the premises. For example, whether the premise "All *A* are *B*" is seen to entail that "All *B* are *A*" depends on whether *A* is understood to be equivalent or subordinate to *B*. Similarly, "Some *A* are *B*" may or may not be seen to entail that "Some *A* are not *B*" or that "Some *B* are not *A*."

Euler, while conducting a correspondence course with a German princess, recognized this problem and invented a set of five diagrams to help her out (Woodworth, 1938). (This is believed to be the earliest documented evidence that it is not only the "common man" who had had difficulty with the logic.) These five diagrams represent all possible relationships that may obtain between two sets. They are shown, together with their consistent propositions, in Table III. A glance at this table reveals the source of the confusion. Only one of the propositions, "No *A* are *B*," is uniquely as-

TABLE III

THE CORRESPONDENCE BETWEEN THE FOUR CATEGORICAL PROPOSITIONS AND THE FIVE RELATIONS THAT MAY OBTAIN BETWEEN TWO SETS

Euler Diagrams



Propositions						
A:	All *A* are *B*	+	+			
I:	Some *A* are *B*	+	+	+	+	
O:	Some *A* are not *B*			+	+	+
E:	No *A* are *B*					+

sociated with just one of the five set relations, and none of the five set relations is uniquely associated with just one of the propositions.

Not surprisingly, a failure to consider the complete range of set relations that is consistent with a given premise has been found to be a common source of error in syllogistic reasoning (Ceraso & Provitera, 1971; Erickson, 1974, 1978; Neimark & Chapman, 1975). Having done so, however, an even less tractable problem arises in the requirement that, in order to deduce the conclusion of a syllogism, the reasoner must consider all combinations of set relations that are consistent with its *combined* premises. To gain some insight into the difficulty of this task, you are challenged to deduce the conclusion to the premises "Some *B* are *A* and no *C* are *B*." To help you out, I have depicted the set relations corresponding to the combined premises in Fig. 1. All you have to do is study the diagrams and determine whether a valid conclusion about the relation between *A* and *C* exists and, if so, what it is.

It is apparently this problem of generating combined interpretations of the premises that, above all others, has plagued students of the syllogism. It has proved to be a potent factor in psychological studies of syllogistic reasoning (Erickson, 1974; Johnson-Laird & Steedman, 1978; Guyote & Sternberg, 1981), and it seems that even Aristotle fell victim to it as he explicitly denied the validity of several syllogisms only to resurrect them in a (possibly postdated) addendum to his text. In this vein, Johnson-Laird (1982) has noted,

> Whenever I have presented a reasoning problem informally, I have noticed the difficulties that people get themselves into if they use Euler circles. The problem is that there is no simple algorithm for using them that one can learn like one learns, say, the algorithm for long multiplication. Merely drawing circles does not guarantee that all their

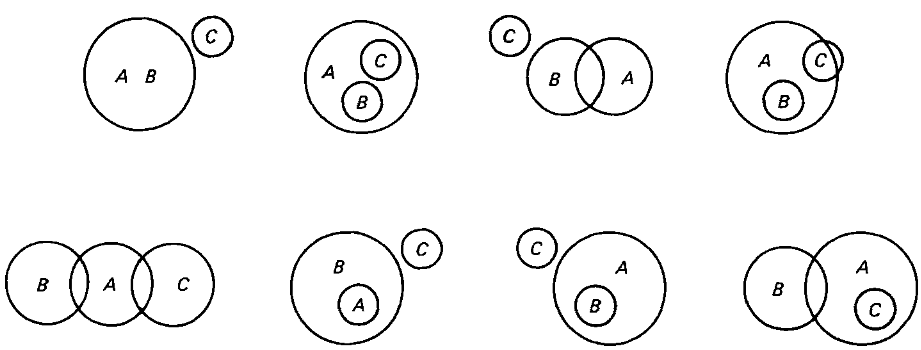


Fig. 1. Set relations corresponding to the premises "Some *B* are *A* and no *C* are *B*." By studying these diagrams, can you determine the conclusion?

possible combinations will be considered exhaustively. The same problem applies to the notation that I have invented for depicting the structure of mental models; if there were a simple algorithm, then doubtless most of us would have mastered it when we first learned to reason. (p. 26)

2. *A Solution to the Problem*

But a straightforward solution to this problem can be invented. Indeed, it follows directly from the Euler diagrams. Specifically, instead of searching directly for the conclusions that may follow each pair of premises, we may begin instead by enumerating those that follow from each pair of Euler's set relations.

Table IV shows all such combinations for the first figure of the syllogism. The first two columns of the table are headed by the Euler diagrams corresponding to the premise "All *B* are *A*," whereas the first two rows are headed by the diagrams corresponding to the premise "All *C* are *B*." Similarly, the first four columns and rows correspond respectively to the premises "Some *B* are *A*" and "Some *C* are *B*"; the last three columns and rows correspond respectively to the premises "Some *B* are not *A*" and "Some *C* are not *B*"; and finally, the premises "No *B* are *A*" and "No *C* are *B*" are represented respectively by the last column and row.

The conclusions that follow from each pair of diagrams are given in that cell of the table that represents their intersection. Particular conclusions that follow only as subalterns or, effectively, understatements of universals, are shown in parentheses. All other conclusions represent the strongest necessary statements that can be made about the relationship of *C* to *A* and, conversely, *A* to *C*. Where a cell contains only a dashed line, no necessary conclusion exists; more specifically, each such cell admits all eight possible conclusions, that is, A, E, I, and O propositions about the relationship of *A* to *C* as well as that of *C* to *A*.

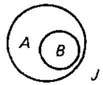
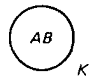
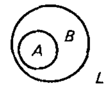
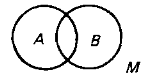
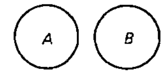
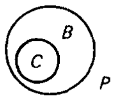
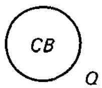
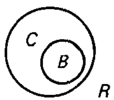
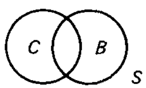
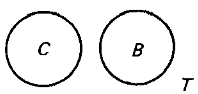
The conclusions to these pairs of set relations should be relatively transparent. For safety's sake, however, the rationale for each is presented below:

J,P. *All C are A*—This is necessarily true because *C* is a proper subset of *B* and *B* is a proper subset of *A*.
(*Some C are A*)—This is the subaltern of "All *C* are *A*": If "*All C* are *A*," it is necessarily true that "Some *C* are *A*."
Some A are C—Because *C* is a subset of *A*, some *A* are necessarily *C*.
Some A are not C—Because *C* is a *proper* subset, that is, is not coextensional with *A*, there must be some *A* that are *not C*

J,Q. *All C are A*—*C* is equivalent to *B*, and all of *B* are included in *A*.
(*Some C are A*)—This is the subaltern of "All *C* are *A*."
Some A are C—Because *B* and *C* are equivalent, those *A* which are *B* must also be *C*.

TABLE IV

COMPLETE EULER MATRIX FOR THE FIRST SYLLOGISTIC FIGURE

		A	A			
		I	I	I	I	
				O	O	O
						E
		 J	 K	 L	 M	 N
I A	 P	All *C* are *A* (Some *C* are *A*) Some *A* are *C* Some *A* are not *C*	All *C* are *A* (Some *C* are *A*) Some *A* are *C* Some *A* are not *C*	—	Some *A* are not *C*	No *C* are *A* (Some *C* are not *A*) No *A* are *C* (Some *A* are not *C*)
I A	 Q	All *C* are *A* (Some *C* are *A*) Some *A* are *C* Some *A* are not *C*	All *C* are *A* (Some *C* are *A*) All *A* are *C* (Some *A* are *C*)	Some *C* are *A* Some *C* are not *A* All *A* are *C* (Some *A* are *C*)	Some *C* are *A* Some *C* are not *A* Some *A* are *C* Some *A* are not *C*	No *C* are *A* (Some *C* are not *A*) No *A* are *C* (Some *A* are not *C*)
I O	 R	Some *C* are *A* Some *A* are *C*	Some *C* are *A* Some *C* are not *A* All *A* are *C* (Some *A* are *C*)	Some *C* are *A* Some *C* are not *A* All *A* are *C* (Some *A* are *C*)	Some *C* are *A* Some *C* are not *A* Some *A* are *C*	Some *C* are not *A*
I O	 S	Some *C* are *A* Some *A* are *C* Some *A* are not *C*	Some *C* are *A* Some *C* are not *A* Some *A* are *C* Some *A* are not *C*	Some *C* are not *A*	—	Some *C* are not *A*
O E	 T	Some *A* are not *C*	No *C* are *A* (Some *C* are not *A*) No *A* are *C* (Some *A* are not *C*)	No *C* are *A* (Some *C* are not *A*) No *A* are *C* (Some *A* are not *C*)	Some *A* are not *C*	—

Some A are not C—Because *B* is a proper subset of *A*, some *A* are not *B*. Because *B* and *C* are equivalent, those *A* which are not *B* cannot be *C*.

J,R. *Some C are A*—Because all *B* are contained in *A*, those *C* which are *B* must be contained in *A*.

Some A are C—Because all *B* are contained in *C*, those *A* which are *B* must be contained in *C*.

J,S. *Some C are A*—Because all *B* are contained in *A*, those *C* which overlap with *B* must be contained in *A*.

Some A are C—Because all *B* are *A*, those *A* which are *B* must overlap with *C*.

Some A are not C—Because all *B* are *A*, those *B* that do not overlap with *C* must correspond to *A* that are not *C*.

J,T. *Some A are not C*—Because *B* and *C* are disjoint, those *A* that are *B* cannot be *C*.

K,P. *All C are A*—Because all *C* are *B* and *B* is equivalent to *A*, it follows that all *C* must be *A*.

(*Some C are A*)—This is the subaltern of "All *C* are *A*."

Some A are C—*A* and *B* are equivalent. Therefore, if *C* is a subset of *B*, it must also be a subset of *A*.

Some A are not C—Because *C* is a *proper* subset of *B*, there must be some *B* or, equivalently, some *A*, that are not *C*.

K,Q. *All C are A*—Both *A* and *C* are equivalent to *B* and therefore to each other.

(*Some C are A*)—This is the subaltern of "All *C* are *A*."

All A are C—Both *A* and *C* are equivalent to *B* and, therefore, to each other.

(*Some A are C*)—This is the subaltern of "All *A* are *C*."

K,R. *Some C are A*—Because *B* is a subset of *C*, some *C* must be *B*. Because *A* and *B* are equivalent, those *C* that are *B* must also be *A*.

Some C are not A—Because *B* is a proper subset of *C*, there must be some *C* that extend beyond *B* and, therefore, beyond *B*'s equivalent, *A*.

All A are C—*A* is equivalent to *B*, and *B* is a proper subset of *C*.

(*Some A are C*)—This is the subaltern of "All *A* are *C*."

K,S. *Some C are A*— Because *A* and *B* are equivalent, *A* must overlap with *C* to the same extent as *B* does.

Some C are not A—Because *A* and *B* are equivalent, *A* must fail to overlap with *C* to the same extent as *B* does.

Some A are C—Because *A* and *B* are equivalent, *A* must overlap with *C* to the same extent as *B* does.

Some A are not C—Because *A* and *B* are equivalent, *A* must fail to overlap with *C* to the same extent as *B* does.

K,T. *No C are A*— Because *A* and *B* are equivalent, *A* must be disjoint with *C* just as *B* is.

(*Some C are not A*)—This is the subaltern of "No *C* are *A*."

No A are C—Because *A* and *B* are equivalent, *A* must be disjoint with *C* just as *B* is.

(*Some A are not C*)—This is the subaltern of "No *A* are *C*."

L,P. *No valid conclusion*—*A* and *C* might be related by any of the eight possible conclusions.

L,Q. *Some C are A*—Because *B* and *C* are equivalent and *B* contains *A*, *C* must also contain *A*.

Some C are not A—Because *A* is a proper subset of *B*, it must also be of *C*.

All A are C—Because *B* and *C* are equivalent and all *A* are *B*, all *A* must be *C*.

(*Some A are C*)—This is the subaltern of "All *A* are *C*."

L,R. *Some C are A*—If *A* is a subset of *B* and *B* is a subset of *C*, then *A* must be a subset of *C*.
Some C are not A—*A* must be a proper subset of *C*.
All A are C—*A* must be a subset of *C*.
(*Some A are C*)—This is the subaltern of "All *A* are *C*."

L,S. *Some C are not A*—Because all *A* are contained within *B* but some *C* is distinct from *B*, at least some *C* must not be *A*.

L,T. *No C are A*—Because all *A* are contained within *B* and *B* and *C* are disjoint, no *C* can be *A*.
(*Some C are not A*)—This is the subaltern of "No *C* are *A*."
No A are C—Because all *A* are contained within *B* and *B* and *C* are disjoint, no *A* can be *C*.
(*Some A are not C*)—This is the subaltern of "No *A* are *C*."

M,P. *Some A are not C*—Because all *C* are contained within *B* but some *A* are distinct from *B*, at least some *A* must not be *C*.

M,Q. *Some C are A*—Because *B* and *C* are equivalent, *C* must overlap with *A* to the same extent that *B* does.
Some C are not A—Because *B* and *C* are equivalent, *C* must fail to overlap with *A* to the same extent that *B* does.
Some A are C—Because *B* and *C* are equivalent, *A* must overlap with *C* just as with *B*.
Some A are not C—Because *B* and *C* are equivalent, *A* must fail to overlap with *C* just as with *B*.

M,R. *Some C are A*—Because *C* includes *B*, *C* must overlap with *A* at least to the extent that *B* does.
Some C are not A—Because all *B* are *C*, those *B* that are not *A* must correspond to *C* that are not *A*.
Some A are C—At least those *A* that overlap with *B* must be *C*.

M,S. *No valid conclusion*—*A* and *C* might be related by any of the eight possible conclusions.

M,T. *Some A are not C*—At least those *A* that are *B* must not be *C*.

N,P. *No C are A*—Because all *C* are contained in *B* and *B* is disjoint from *A*, no *C* can be *A*.
(*Some C are not A*)—This is the subaltern of "No *C* are *A*."
No A are C—*A* is disjoint from *B* and must therefore be disjoint from *B*'s subset, *C*.
(*Some A are not C*)—This is the subaltern of "No *A* are *C*."

N,Q. *No C are A*—If *B* and *C* are equivalent, then *C* must be disjoint from *A* just as *B* is.
(*Some C are not A*)—This is the subaltern of "No *C* are *A*."
No A are C—If *B* and *C* are equivalent, then *C* must be disjoint from *A* just as *B* is.
(*Some A are not C*)—This is the subaltern of "No *A* are *C*."

N,R. *Some C are not A*—At least those *C* that are *B* cannot be *A*.

N,S. *Some C are not A*—At least those *C* that are *B* cannot be *A*.

N,T. *No valid conclusion*—*A* and *C* might be related by any of the eight possible conclusions.

Returning to the syllogisms, a conclusion will follow necessarily from a pair of premises if and only if it follows necessarily from every pair of set relations entailed by those premises. Thus, to determine which of the premise pairs of the first syllogistic figure do indeed yield valid conclusions, one need only collapse across corresponding pairs of Euler diagrams and identify the common conclusions.

The 16 premise pairs of the first syllogistic figure are shown in Table V. Again, premises relating the *A* and *B* terms are listed across the top of the table, and those relating *B* and *C* are listed down the side. At least one valid conclusion is shown to follow from 6 of the premise pairs: AA, AI, AE, IE, EA, and EI. For each of these pairs of premises, the conclusions cited in Table V can be seen to occur in every one of the pertinent cells of Table IV. (Ignore the superscripts on these conclusions for now; their significance will be explained in the following section.) As an example, consider the premise pair AI: "All *B* are *A*" and "Some *C* are *B*." The first of these premises is represented by the Euler diagrams heading the first two columns (J and K) of Table IV; the second premise is represented by the first four rows (P, Q, R, and S) of Table IV. An examination of the eight cells comprising the intersection of these columns and rows reveals that only two conclusions appear in every one. These, then, are the conclusions to the AI argument cited in Table V: "Some *C* are *A*" and "Some *A* are *C*."

The 10 remaining cells of Table V are empty, indicating that no valid conclusion follows from the corresponding pairs of premises. An examination of Table IV confirms that for none of these pairs of premises do the underlying set relations yield any consistent conclusion. Moreover, for 9 of these 10 premise pairs, the related portion of Table IV contains at least one empty or inconclusive cell: If a pair of premises proves inconclusive under any of its interpretations, no valid conclusion can be said to exist. The tenth pair of inconclusive premise is AO. Looking back at Table IV, we see that each of the six set combinations underlying this premise pair does yield some necessary conclusion. Looking across these six cells, however, we see that the conclusions in one cell are, in general, contradicted by the conclusions in another; for example, "Some *C* are *A*" is contradicted by "No *A* are *C*" and "All *A* are *C*" by "Some *A* are not *C*." The only conclusion that is not explicitly contradicted is "Some *C* are not *A*"; however, the necessity of this conclusion is nullified by the fact that both it and its contradictory, "All *C* are *A*," are possible but unnecessary conclusions for cell J,R.

The combined set relations for the second through fourth syllogistic figures are presented in Tables VI, VII, and VIII. For each of these tables, the first two columns and rows correspond to the A premises, the first four to the I premises, the last three to the O premises, and the last alone to the E premises. The justification for the conclusions in these tables is not presented as it was for the first figure, but readers are invited to verify them

TABLE V

FIRST FIGURE PREMISE PAIRS AND CONCLUSIONS AS DERIVED FROM THE CORRESPONDING EULER MATRIX (TABLE IV)

	First premise			
Second premise	A: All B are A	I: Some B are A	O: Some B are not A	E: No B are A
A: All C are B	All C are $A^{A1, P1, P2, C, B}$ (Some C are A)^{C} Some A are $C^{A3, P1}$	—	—	No C are $A^{A1, P1, P2, C, B}$ (Some C are not A)^{C} No A are $C^{A3, P1}$
I: Some C are B	Some C are $A^{A1, P1, P2, C, B}$ Some A are $C^{A3, P1}$	—	—	(Some A are not C) Some C are not $A^{A1, P1, P2, C, B}$
O: Some C are not B	—	—	—	—
E: No C are B	Some A are not $C^{A2, P1}$	Some A are not $C^{A2, P1}$	—	—

[a]Superscripts denote sources endorsing conclusions as follows: A1, *Analytica Priora* I,4–6; A2, *Analytica Priora* I,7; A3, *Analytica Priora* II,1; P1, first mnemonic poem; P2, second mnemonic poem; C, contemporary authorities; B, Boolean authorities.

on their own. When experiencing difficulty in affirming or denying any given conclusion, a good strategy is that of testing its contradictory. For example, uncertainty about whether "All A must be C" can be removed by asking whether it is possible or impossible that "Some A are not C." If any proposition and its contradictory are both possible, neither is necessary. In general, A and O premises with matching subject and predicate contradict each other, as do I and E premises.

It is the compellingness of the individual conclusions in these tables that provides the primary test of their composite accuracy. Two checks on this accuracy, however, can be had from the inherent structure of the tables. The first check derives from the fact that all of the Euler tables (Tables IV, VI, VII, and VIII) should be identical except that their rows and columns have been permuted. To facilitate this check, the labels for the Euler diagrams in Tables VI–VIII have been carried over from Table IV, for example, the same diagram heads column J in all four tables.

The second check derives from the fact that the diagrams heading the rows of the tables are essentially identical to those heading the columns except that the set C has replaced the set A. Because of this, solutions must be reflected in literal converse, that is, with the subject and predicate terms interchanged across corresponding cells of the table. This reflection is most easily seen in the second and third figures (Tables VI and VII) because comparable diagrams occur in the same order across the rows and columns. For each of the second and third figures, the conclusions in the second column of the first row are precisely the converse of those in the second row of the first column, those in the third column of the first row are the converse of those in the third row of the first column, and so on, such that the only unique cells in the matrix are those which fall along the diagonal or, equivalently, those for which comparable diagrams are mapped against each other. Although, because of differences in the horizontal and vertical orderings of the diagrams, this reciprocity is less obvious for the first and fourth figures (Tables IV and VIII), it is nonetheless present. The tables for the first and fourth figures, moreover, permit a check on *each other*: They are essentially identical tables except that the rows of one correspond to the columns of the other, and vice versa, but with the subject and predicate terms interchanged.

In Tables IX–XI, the set relations for the second, third, and fourth figures have been collapsed into syllogisms through the same procedure as was explained for the first figure: A conclusion is cited as valid for a given premise pair if and only if it follows necessarily from all relevant pairs of Euler relations.

Across all four figures (Tables V, IX–XI), there are 27 pairs of premises that yield valid conclusions. Counting the different conclusions that these

TABLE VI

Complete Euler Matrix for the Second Syllogistic Figure

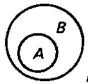
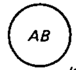
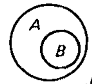
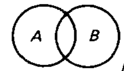
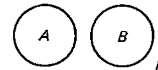
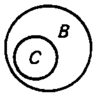
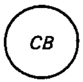
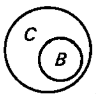
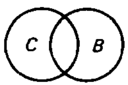
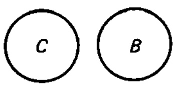
		A	A			
		I	I	I	I	
				O	O	O
						E
		 L	 K	 J	 M	 N
I A	 P	—	All C are A (Some C are A) Some A are C Some A are not C	All C are A (Some C are A) Some A are C Some A are not C	Some A are not C	No C are A (Some C are not A) No A are C (Some A are not C)
I A	 Q	Some C are A Some C are not A All A are C (Some A are C)	All C are A (Some C are A) All A are C (Some A are C)	All C are A (Some C are A) Some A are C Some A are not C	Some C are A Some C are not A Some A are C Some A are not C	No C are A (Some C are not A) No A are C (Some A are not C)
I O	 R	Some C are A Some C are not A All A are C (Some A are C)	Some C are A Some C are not A All A are C (Some A are C)	Some C are A Some C are A	Some C are A Some C are not A Some A are C	Some C are not A
I O	 S	Some C are not A	Some C are A Some C are not A Some A are C Some A are not C	Some C are A Some A are C Some A are not C	—	Some C are not A
O E	 T	No C are A (Some C are not A) No A are C (Some A are not C)	No C are A (Some C are not A) No A are C (Some A are not C)	Some A are not C	Some A are not C	—

TABLE VII

Complete Euler Matrix for the Third Syllogistic Figure

A

I

O

E

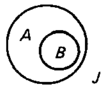
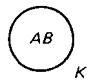
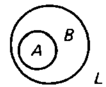
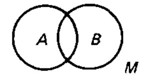
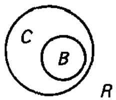
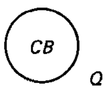
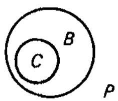
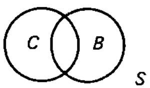
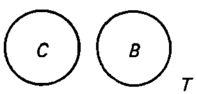
					
I A 	Some C are A Some A are C	Some C are A Some C are not A All A are C (Some A are C)	Some C are A Some C are not A All A are C (Some A are C)	Some C are A Some C are not A Some A are C	Some C are not A
	All C are A (Some C are A) Some A are C Some A are not C	All C are A (Some C are A) All A are C (Some A are C)	Some C are A (Some C are not A) All A are C (Some A are C)	Some C are A Some C are not A Some A are C Some A are not C	No C are A (Some C are not A) No A are C (Some A are not C)
O 	All C are A (Some C are A) Some A are C Some A are not C	All C are A (Some C are A) Some A are C Some A are not C	—	Some A are not C	No C are A (Some C are not A) No A are C (Some A are not C)
	Some C are A Some A are C Some A are not C	Some C are A Some C are not A Some A are C Some A are not C	Some C are not A	—	Some C are not A
E 	Some A are not C	No C are A (Some C are not A) No A are C (Some A are not C)	No C are A (Some C are not A) No A are C (Some A are not C)	Some A are not C	—

TABLE VIII

COMPLETE EULER MATRIX FOR THE FOURTH SYLLOGISTIC FIGURE

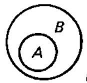
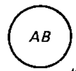
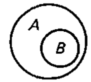
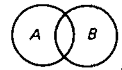
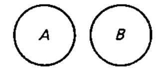
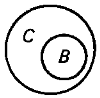
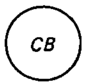
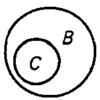
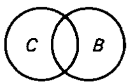
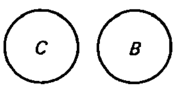
		 L	 K	 J	 M	 N
		A I	A I	I O	I O	O E
I A	 R	Some C are A Some C are not A All A are C Some A are C	Some C are A Some C are not A All A are C (Some A are C)	Some C are A Some A are C	Some C are A Some C are not A Some A are C	Some C are not A
I A	 Q	Some C are A Some C are not A All A are C (Some A are C)	All C are A (Some C are A) All A are C (Some A are C)	All C are A (Some C are A) Some A are C Some A are not C	Some C are A Some C are not A Some A are C Some A are not C	No C are A (Some C are not A) No A are C (Some A are not C)
I O	 P	—	All C are A (Some C are A) Some A are C Some A are not C	All C are A (Some C are A) Some A are C Some A are not C	Some A are not C	No C are A (Some C are not A) No A are C (Some A are not C)
I O	 S	Some C are not A	Some C are A Some C are not A Some A are C Some A are not C	Some C are A Some A are C Some A are not C	—	Some C are not A
O E	 T	No C are A (Some C are not A) No A are C (Some A are not C)	No C are A (Some C are not A) No A are C (Some A are not C)	Some A are not C	Some A are not C	—

TABLE IX

SECOND FIGURE PREMISE PAIRS AND CONCLUSIONS AS DERIVED FROM THE CORRESPONDING EULER MATRIX (TABLE VI)

	First premise[a]			
Second premise	A: All *A* are *B*	I: Some *A* are *B*	O: Some *A* are not *B*	E: No *A* are *B*
A: All *C* are *B*	—	—	Some *A* are not *C*	No *C* are *A*[A1, P1, P2, C, B] (Some *C* are not *A*)[C] No *A* are *C*[A3] (Some *A* are not *C*)
I: Some *C* are *B*	—	—	—	Some *C* are not *A*[A1, P1, P2, C, B]
O: Some *C* are not *B*	Some *C* are not *A*[A1, P1, P2, C, B]	—	—	—
E: No *C* are *B*	No *C* are *A*[A1, P1, P2, C, B] (Some *C* are not *A*)[C] No *A* are *C*[A3] (Some *A* are not *C*)	Some *A* are not *C*[A2]	—	—

[a] Superscripts denote sources endorsing conclusions as follows: A1, *Analytica Priora* I,4–6; A2, *Analytica Priora* I,7; A3, *Analytica Priora* II,1; P1, first mnemonic poem; P2, second mnemonic poem; C, contemporary authorities; B, Boolean authorities.

TABLE X

Third Figure Premise Pairs and Conclusions as Derived from the Corresponding Euler Matrix (Table VII)

Second premise	First premise[a] A: All *B* are *A*	I: Some *B* are *A*	O: Some *B* are not *A*	E: No *B* are *A*
A: All *B* are *C*	Some *C* are *A*[A1, P1, P2, C, B] Some *A* are *C*[A3]	Some *C* are *A*[A1, P1, P2, C, B] Some *A* are *C*[A3]	Some *C* are not *A*[A1, P1, P2, C, B]	Some *C* are not *A*[A1, P1, P2, C, B]
I: Some *B* are *C*	Some *C* are *A*[A1, P1, P2, C, B] Some *A* are *C*[A3]	—	—	Some *C* are not *A*[A1, P1, P2, C, B]
O: Some *B* are not *C*	Some *A* are not *C*	—	—	—
E: No *B* are *C*	Some *A* are not *C*[A2]	Some *A* are not *C*[A2]	—	—

[a] Superscripts denote sources endorsing conclusions as follows: A1, *Analytica Priora* I,4–6; A2, *Analytica Priora* I,7; A3, *Analytica Priora* II,1; P1, first mnemonic poem; P2, second mnemonic poem; C, contemporary authorities; B, Boolean authorities.

TABLE XI

FOURTH FIGURE PREMISE PAIRS AND CONCLUSIONS AS DERIVED FROM THE CORRESPONDING EULER MATRIX (TABLE VIII)

Second premise	First premise[a]			
	A: All *A* are *B*	I: Some *A* are *B*	O: Some *A* are not *B*	E: No *A* are *B*
A: All *B* are *C*	Some *C* are *A*[P2, C] All *A* are *C* (Some *A* are *C*)	Some *C* are *A*[P2, C, B] Some *A* are *C*	—	Some *C* are not *A*[P2, C]
I: Some *B* are *C*	—	—	—	Some *C* are not *A*[P2, C, B]
O: Some *B* are not *C*	—	—	—	—
E: No *B* are *C*	No *C* are *A*[P2, C, B] (Some *C* are not *A*[C] No *A* are *C* (Some *A* are not *C*)	Some *A* are not *C*	—	—

[a] Superscripts denote sources endorsing conclusions as follows: P2, second mnemonic poem; C, contemporary authorities; B, Boolean authorities.

premises yield, there are a total of 48 valid syllogisms, or 12 in each figure. It is also worth noting at this point that like the Euler matrices from which they are derived, each of the second and third figures (Tables IX and X) exhibits a diagonal symmetry but with terms interchanged. The first and fourth figures (Tables V and XI) do not exhibit such internal symmetry but bear an analogous relation to each other. If the *A* and *C* terms of its premises and conclusions are interchanged, the fourth figure is precisely a diagonal reflection of the first.

The Euler matrices developed in this section are thus worthwhile in several ways. First, they provide a relatively transparent and manageable method for evaluating the validity of the arguments. Second, the redundancy among the matrices provides both means for checking their accuracy and, perhaps more importantly, insight into the logical relationship among the four syllogistic figures. Finally, the matrices clearly reveal the underlying simplicity of the logic. Note that if the redundancies that exist within and between tables are factored out, the entire logic of the syllogism—the whole interpretive morass with which we began—reduces to the diagonal half of just one of the matrices, that is, to just 15 pairs of Euler diagrams.

B. Aristotle's Solution

Aristotle, you will recall, recognized only three of the four traditional syllogistic figures. In the beginning of the *Analytica Priora,* he introduces each of these figures in separate chapters, enumerating their valid moods. These moods are presented in Aristotle's (translated) words below. For ease of reference, I have prefixed each with the number of its figure and the letters designating the types of propositions corresponding to its first premise, second premise, and conclusion, in that order.

1-AAA: If *A* is predicated of all *B*, and *B* of all *C*, *A* must be predicated by all *C*. (*AP* I,4,25b)

1-EAE: If *A* is predicated of no *B*, and *B* of all *C*, it is necessary that no *C* will be *A*. (*AP* I,4,26a)

1-AII: Let all *B* be *A* and some *C* be *B*. Then "if predicated of all" means what was said above, it is necessary that some *C* is *A*. (*AP* I,4,26a)

1-EIO:. And if no *B* is *A* but some *C* is *B*, it is necessary that some *C* is not *A*. (*AP* I,4,26a)

2-EAE: Let *M* be predicated of no *N*, but of all *O* since, then the negative relation is convertible, *N* will belong to no *M*: but *M* was assumed to belong to all *O*: consequently *N* will belong to no *O*. (*AP* I,5,27a)

2-AEE: Again, if *M* belongs to all *N* but to no *O*, then *N* will belong to no *O*. For if *M* belongs to no *O*, *O* belongs to no *M*: but *M* (as was said) belongs to all *N*; *O* then will belong to no *N*: for the first figure has again been formed. But since the negative relation is convertible, *N* will belong to no *O*. (*AP* I,5,27a)

2-EIO: For if *M* belongs to no *N*, but to some *O*, it is necessary that *N* does not belong to some *O*. For since the negative statement is convertible, *N* will belong to no *M*: but *M* was admitted to belong to some *O*: therefore *N* will not belong to some *O* for the result is reached by means of the first figure. (*AP* I,5,27a)

2-AOO: Again, if *M* belongs to all *N* but not to some *O*, it is necessary that *N* does not belong to some *O*: for if *N* belongs to all *O* and *M* is predicated also of all *N*, it is necessary for *M* to belong to all *O*. But we assumed not to belong to some *O*. And if *M* belongs to all *N* but not to all *O*, we shall conclude that *N* does not belong to all *O*: the proofs the same as the above. (*AP* I,5,27a)

3-AAI: If [the terms] are universal, whenever both *P* and *R* belong to all *S*, it follows that *P* will necessarily belong to some *R*. For since the affirmative statement is convertible, *S* will belong to some *R*: consequently so that since *P* belongs to all *S*, and *S* to some *R*, *P* must belong to some *R*: for a syllogism in the first figure is produced. It is possible to demonstrate this also *per impossible* and by exposition. For if both *P* and *R* belong to all *S*, should one of the *S*'s, e.g., *N*, be taken, both *P* and *R* will belong to this, and thus *P* will belong to some *R*. (*AP* I,6,28a)

3-EAO: If *R* belongs to all *S*, but *P* to no *S*, there will be a syllogism to prove that *P* will not belong to some *R*. This may be demonstrated in the same way as before by converting the premise RS. It might be proved also *per impossible*, as in the former cases. (*AP* I,6,28a)

3-IAI: For if *R* belongs to all *S*, *P* to some *S*, *P* must belong to some *R*. For since the affirmative statement is convertible *S* will belong to some *P*: consequently since *R* belongs to all *S*, and *S* to some *P*, *R* must also belong to some *P*: therefore *P* must belong to some *R*. (*AP* I,6,28b)

3-AII: Again if *R* belongs to some *S*, and *P* to all *S*, *P* must belong to some *R*. This may be demonstrated in the same way as the preceding. And it is possible to demonstrate also *per impossible* and by exposition, as in the former cases. (*AP* I,6,28b)

3-OAO: But if one term is affirmative, the other negative, and if the affirmative is universal, a syllogism will be possible whenever the minor term is affirmative. For if *R* belongs to all *S*, but *P* does not belong to some *S*, it is necessary that *P* does not belong to some *R*. For if *P* belongs to all *R*, and *R* belongs to all *S*, then *P* will belong to all *S*: but we assumed that it did not. Proof is possible also without reduction *ad impossible*, if one of the *S*s be taken to which *P* does not belong. (*AP* I,6,28b)

3-EIO: But if the negative term is universal, whenever the major is negative and the minor affirmative there will be a syllogism. For if *P* belongs to no *S*, and *R* belongs to some *S*, *P* will not belong to some *R*: for we shall have the first figure again, if the premise RS is converted. (*AP* I,6,28b)

The above stand as the sum total of valid syllogisms offered in Aristotle's initial exposition of the syllogism. Surely for this reason, Aristotle is very often cited as having recognized but 14 valid moods.

If we map Aristotle's list against Tables V, IX, and X, we find he missed out on two or three conclusive premise pairs per figure. The missing premise pairs were not overlooked by Aristotle in these chapters: They were explicitly rejected. His argument against each is presented below:

1-AEX: But if the first term [*A*] belongs to all the middle [*B*], but the middle to none of the last term [*C*], there will be no syllogism in respect of the extremes; for nothing necessary follows from the terms being so related; for it is possible that the first should belong either to all or to none of the last so that neither a particular nor a universal conclusion is necessary. But if there is no necessary consequence, there cannot be a syllogism by means of these premises. As an example of a universal affirmative relation between the extremes we may take the terms animal, man, horse; of a universal negative relation, the terms animal, man, stone. (*AP* I,4,26a)

1-IEX: Again, if no *C* is *B*, but some *B* is or is not *A*, or not every *B* is *A*, there cannot be a syllogism. Take the terms white, horse, swan: white, horse, raven. (*AP* I,4,26a)

2-OAX: But if *M* is predicated of all *O*, but not of all *N*, there will be no syllogism. Take the terms animal, substance, raven; animal, white, raven. (*AP* I,5,27b)

2-IEX: Nor will there be a conclusion when *M* is predicated of no *O*, but of some *N*. Terms to illustrate a positive relation between the extremes are animal, substance, unit; a negative relation, animal, substance, science. (*AP* I,5,27b)

3-AEX: But if *R* belongs to no *S*, *P* to all *S*, there will be no syllogism. Terms for the positive relation are animal, horse, man: for the negative relation animal, inanimate, man. (*AP* I,6,28a)

3-AOX: But whenever the major is affirmative, no syllogism will be possible, e.g., if *P* belongs to all *S*, and *R* does not belong to some *S*. Terms for the universal affirmative relation are animate, man, animal. For the universal negative relation it is not possible to get terms, if *R* belongs to some *S*, and does not belong to some *S*. For if *P* belongs to all *S*, and *R* to some *S*, then *P* will belong to some *R*: but we assumed that it belongs to no *R*. We must put the matter as before. Since the expression "it does not belong to some" is indefinite, it may be used truly of that also which belongs to none. But if *R* belongs to no *S*, no syllogism is possible, as has been shown. Clearly then no syllogism will be possible here. (*AP* I,6,28b)

3-IEX: But if the negative term is universal, whenever the major is a negative and the minor affirmative there will be a syllogism. For if *P* belongs to no *S*, and *R* belongs to some *S*, *P* will not belong to some *R*: for we shall have the first figure again, if the premise RS is converted. But when the minor is negative, there will be no syllogism. Terms for the positive relation are animal, man, wild: for the negative relation, animal, science, wild—the middle in both being the term wild. (*AP* I,6,28b)

Aristotle achieves most of these refutations by setting up contrary arguments with triads of concrete terms. To follow this part of the discourse, each triad of the terms must be substituted, in the order presented, for the syllogistic variables in alphabetical order. Using the refutation of the first figure premises AE as an example, the terms "animal, man, and horse" and "animal, man, and stone" should be substituted, in turn, for the terms *A, B,* and *C*, respectively. The first triad yields the premise pair, "All men are animals, and no horses are men," thus admitting the conclusion, "All horses are animals." In contrast, the second triad yields the premises "All men are animals, and no stones are men," inviting the conclusion that "No stones are animals." Having thus demonstrated that both A and E propositions *may* follow from the premises, Aristotle concludes that *no* proposition *must* follow from the premises.

If you find this method of disproof somewhat unclear or unsatisfying, be comforted: Authorities on Aristotle have agreed on neither the specifics of its implementation nor its logical acceptability. Patzig (1968), having summarized these points of dispute, argues that although the method may be lacking in formal aesthetics, it is perfectly correct. In its behalf, he quotes Philoponus, an early (sixth century AD) and much admired interpreter of Aristotle: "Ten thousand examples cannot prove a universal proposition, but *one* example is enough to refute it" (p. 183).

Philoponus aside, Aristotle's disproofs do suffer one very serious shortcoming. Although adequate for the task of contrasting conclusions in which the *C* term serves as the subject and the *A* term as the predicate, the procedure, under Aristotle's implementation, is moot with respect to conclusiveness of arguments involving the converse subject-predicate arrangement. If one's purpose is solely that of enumerating concludent premise pairs, without regard to their various conclusions, then this distinction generally makes little difference since, to most of the categorical propositions, there corresponds another with converted subject and predicate terms: "All *C* are *A*" entails "Some *A* are *C*"; "Some *C* are *A*" entails "Some *A* are *C*"; "No *C* are *A*" entails "No *A* are *C*." But this is not true for O propositions: From "Some *C* are not *A*" one may infer with equal uncertainty that all, some, not some, or no *A* are *C*. It follows that where the only necessary conclusion of a premise pair is "Some *A* are not *C*," its concludence cannot be discovered by examining predications of *C* by *A*. Consistent with this point, the concludent premise pairs that Aristotle rejects in chapters 4–6 are precisely those that yield "Some *A* are not *C*" as their sole conclusion.

At some point Aristotle recognized this problem. In the beginning of I,7 of the *Analytica Priora,* in a section purportedly written sometime later than the text that surrounds it (see Lukasiewicz, 1957; Patzig, 1968), Aristotle asserts:

> It is evident also that in all the figures, whenever a proper syllogism does not result, if both the terms are affirmative or negative nothing necessary follows at all, but if one is affirmative, the other negative, and if the negative is stated universally, a syllogism always results relating the minor [as predicate] to the major term, e.g., if *A* belongs to all or some *B*, and *B* belongs to no *C*; for if the premises are converted it is necessary

> that C does not belong to some A. Similarly, also in the other figures: a syllogism always results by means of conversion." (*AP* I,7,29a)

Through this paragraph, he reinstates the previously rejected AE and IE premise pairs for each of the three figures. Through the phrase "a syllogism always results relating the minor to the major term," he is simply pointing out that the conclusion will read "Some A are not C" instead of the standard "Some C are not A." He still ignores the AO pairs of the second and third figures. Perhaps this was an oversight. Alternatively, Patzig (1968) argues that he omitted their mention for the same reason that he forewent explicit statement of the AE and IE arguments in figures 2 and 3: Through valid conversions, each of the arguments can be shown to be redundant with some other, previously proved mood of the same figure.

Later in the *Analytica Priora*, Aristotle additionally notes that any premise pair that yields an A, I, or E conclusion must also yield some conclusion with converted subject and predicate terms:

> Since some syllogisms are universal, others particular, all the universal syllogisms give more than one result, and of particular syllogisms the affirmative yield more than one, the negative yield only the stated conclusion. For all propositions are convertible save only the particular negative: and the conclusion states one definite thing about another definite thing. Consequently all syllogisms have the particular negative yield more than one conclusion, e.g. If A has been proved to belong to all or to some B, then B must belong to some A: and if A has been proved to belong to no B, then B belongs to no A. This is a different conclusion from the former. But if A does not belong to some B, it is not necessary that B should not belong to some A: for it may possibly belong to all A. (*AP* II,1,53a)

In the final analysis, then, Aristotle's count of valid syllogisms for the first three figures is very close to that developed in Section II,A. Of the 19 premise pairs that were found valid herein, Aristotle acknowledged 17, and, as previously mentioned, his omission of the last 2 can be explained on grounds of efficiency. Of the various conclusions to those premises adduced herein, Aristotle explicitly defended all but the subalterns. His neglect of the subalterns can also be explained on grounds of efficiency: Having concluded, for example, that a pair of premises necessarily implies that "All C are A," is there any need or even advantage in separately listing the fact that it also implies "Some C are A"? The greatest discrepancy between the present list of valid syllogisms and Aristotle's lies in his total omission of the fourth figure.

To understand better the relation between the various counts of valid syllogisms, the conclusions in Tables V, IX, X, and XI have been annotated with respect to the sources by which they have been endorsed. The scripts A1, A2, and A3 are owed to Aristotle: A1 marks those 14 syllogisms he defended in his initial exposition of the syllogism (*AP* I,4–6); A2 marks those syllogisms he added on first considering the possibility of conclusions with *A–C* subject-predicate arrangements (*AP* I,7); and A3 marks those syllogisms he acknowledged on considering the valid converses of previously proved conclusions (*AP* II,1).

C. Other Counts

The centuries that have passed since Aristotle have afforded a lot of opportunity for scholars to clarify Aristotle's intentions, and some of these clarifications appear, in retrospect, to reflect outstanding misunderstandings. It has, for example, been authoritatively asserted that only the first figure syllogisms can be valid (Kant, 1762); that only two of the fourth figure syllogisms can be valid (Maier, 1900); that none of the fourth figure syllogisms can be valid (Prantl, 1925); that none of the syllogisms Aristotle raised in I,7 can be valid (Maier, 1900); and that any valid syllogism will be rendered invalid if the order of its two premises is exchanged (Lemmon, 1965; Maier, 1900; Prantl, 1925; Waitz, 1846). At the outset of this section, let me note that I will only consider positions that still enjoy some critical degree of acclaim.

1. Medieval and Renaissance Counts

It may be more to the Medieval and Renaissance philosophers than to Aristotle himself that our present view of the logic is owed. The Medieval logician very obligingly invented a mnemonic poem to help us remember the valid moods of the syllogism. Many of us were admonished at some point in our lives to memorize this poem. Many of us at least can recognize it. Or can we? The poem, as it turns out, shows up in several different versions. Across some of these versions, the differences are only superficial. Across others, however, they are significant. Here are two representative versions:

> Barbara Celarent Darii Ferio Baralipton Celantes Dabitis Fapesmo Frisesomorum; Cesare Campestres Festino Baroco; Darapti Felapton Disamis Datisi Bocardo Ferison. (From Mates, 1972)

> Barbara, Celarent, Darii, Ferio-*que prioris* Cesare, Camestres, Festino, Baroco *secundae Tertia* Darapti, Disamis, Datisi, Felapton Bocardo, Ferison *habet, Quarta insuper addit* Bramantip, Camenes, Dimaris, Fesapo, Fresison. (From Prior, 1973).

Each of the proper names in these poems corresponds to a valid mood; the first three vowels of each give the type of proposition serving, respectively, as the first premise, second premise, and conclusion of the syllogism.

For example, *Barbara* refers to the syllogism *AAA* and *Celarent* to the syllogism *EAE*. For the second, third, and fourth figures, the consonants are significant, too, as they provide instructions as to how the moods can be transformed to valid moods of the first figure. The figures are separated by semicolons in the first poem and by appropriate words in the second. Both of the poems name 19 valid syllogisms, or 5 in excess of Aristotle's original 14. The major difference between them is that, in the first poem, the 5 new moods have been added to the first figure, whereas in the second, they have been added to the fourth figure. In our tables, the moods named by the first and second poem are denoted by P1 and P2, respectively.

The genesis of the five new moods of the first poem is quite easy to uncover. Two of them, *Fapesmo* and *Frisesomorum*, correspond to the once rejected syllogisms of the first figure that Aristotle reinstated in Chapter 7, Book I, of the *Analytica Priora* (A2 in Table V). The other three correspond to the permissible conversions of first figure premises that he raises in Chapter 1, Book II, of the *Analytica Priora* (A3 in Table V).

What is hard to understand about the first poem is why like additions were not made to the second and third figures. Was it because the validity of the AE and IE premises was only explicitly drawn out for the first figure? But Aristotle did follow their proof with "similarly also in the other figures: a syllogism always results by means of conversion" (*AP* I,7,29a). Was it because all of his examples of valid conversions used the letters *A* and *B* as variables? True, in the definitions of the syllogisms, only the first figure moods are expressed with the variables *A* and *B*. But the conclusions of those moods involved only the variables *A* and *C*. Besides, variables are variables and, more than that, Aristotle very clearly states that "*all* [italics mine] syllogisms save the particular negative yield more than one conclusion." In all my reading on the logic, I have never run across an attempt to justify the unbalanced nature of this particular list of moods.

In the second poem, the five additional moods are accorded to the fourth figure. Historians do not agree as to exactly who was responsible for the eventual formalization of the fourth figure. Its inspiration, however, is quite commonly traced to that same section of the *Analytica Priora* in which Aristotle acknowledges the validity of AEO and IEO arguments. Again, the wording of the relevant section is:

> It is evident also that in all the figures, whenever a proper syllogism does not result, if both the terms are affirmative or negative nothing necessary follows at all, but if one is affirmative, the other negative, and if the negative is stated universally, a syllogism always results relating the minor [as predicated] to the major term, e.g., if *A* belongs to all or some *B*, and *B* belongs to no *C*; for if the premises are converted it is necessary that *C* does not belong to some *A*. Similarly, also in the other figures: a syllogism always results by means of conversion. (*AP* I,7,29a)

Thus, through valid conversions, the premises "All (some) *B* are *A*" and "No *C* are *B*" can be transformed to "Some *A* are *B*" and "No *B* are *C*." If the order of the two new premises is reversed (which is permissible because they are linked only by a logical conjunction), the resulting pair is very similar to the previously validated pair, EI (*Ferio*). Indeed, the only difference is that there is an *A* where the *C* should be and vice versa. To accommodate this difference, the conclusion of the premises must also be converted: It must read "Some *A* are not *C*" instead of "Some *C* are not *A*." But Aristotle warned us of this in asserting that the resulting syllogism would relate the "minor term to the major."

On the other hand, as *A* and *C* are nothing more than the names of variables, the logic of the argument is truly indifferent to which of them occurs in which position. If we switch them, calling *A* by *C* and *C* by *A*, then our transformed argument conforms precisely to *Ferio*, conclusion and all. To maintain consistency with our original premise pair, the *A* and *C* variables must be switched in them as well. Thus transposed, the original premise pair becomes "All (some) *B* are *C*" and "No *A* are *B*" and yields the conclusion "Some *C* are not *A*."

The only problem now is that the original but now reconfigured premises no longer fit the mold of the first figure. On the other hand, if they are reordered to read, "No *A* are *B*" and "All (some) *B* are *C*," they are perfectly suited to the fourth figure. Thus we have *Fesapo* and *Fresison*. By beginning with Aristotle's discussion of validly converted conclusions (*AP* II,1) and applying essentially the same logic as was laid out above, the fourth figure syllogisms *Bramantip, Camenes*, and *Dimaris* are similarly had from *Barbara, Celarent,* and *Darii* of the first figure.

Note that any implications of Aristotle's appended section with respect to the second and third figures are effectively ignored in this second poem as they were in the first. I would argue further that the fourth figure moods of this poem are had only through a relatively tortured overinterpretation of what Aristotle actually said. On the other hand, the second poem can be defended over the first in that there is at least a system to its bias: It names all and only all of the premise pairs that produce conclusions having the *C* term as subject and the *A* term as predicate.

2. *The Dominant Contemporary Count*

The number of valid moods most often cited by contemporary authorities on the logic is 24 (e.g., Langer, 1953; Lemmon, 1965; Lukasiewicz, 1957; Prior, 1973). These moods are denoted with a C (for contemporary) in Tables V, IX, X, and XI. They consist precisely of the 19 moods named by our second mnemonic poem plus those 5 that are had by substituting each of the pertinent universal conclusions (e.g., All C are A) with its subaltern (e.g., Some C are A).

In contemporary works, the need to justify the exclusion of syllogisms whose conclusions involve the A term as subject and the C term as predicate is finessed through definition. Prior's (1973) example is typical:

> This [a categorical syllogism] is an inference involving three categorical propositions—two as premisses and one as conclusion—and with three terms . . . distributed as follows: the predicate of the conclusion, called the "major term" appears in one of the premisses, called in consequence the "major premiss" . . . the subject of the conclusion, called the "minor term," appears in the other premiss, called in consequence the "minor premiss"; and the third term, called the "middle term," appears in both premisses, but is not in the conclusion at all. These characteristics suffice to define "categorical syllogism." (pp. 110–111)

To be sure, there is much material in the *Analytica Priora* to support this constraint, not the least of which is that, in his initial exposition of the syllogism, Aristotle examines the validity of C–A conclusions only. But there is also much to suggest that his initial preoccupation with C–A conclusions was fostered by a combination of rhetorical consistency and logical oversight rather than knowing conviction. Patzig (1968), having combed through *Analytica Priora* quite thoroughly, marshals a very convincing collection of evidence that the order of the terms in a conclusion was irrelevant to Aristotle's concept of a syllogism.

3. *The Boolean Count*

There is one other number that is cited with sufficient prominence to deserve note: 15. As this number arises in Boolean treatments of the logic, the pertinent moods are marked with a B in Tables V, IX, X, and XI. The rationale for 15 valid moods derives from the observation that a particular proposition, by virtue of the definition of "some," implicitly asserts that there exists at least one entity corresponding to its subject term. Particular propositions will therefore be false whenever no such entity exists. In contrast, universal propositions may be true regardless of whether their terms are empty. By implication, given true universal premises, the logic can never in itself guarantee the necessity of a particular conclusion. Such modes can be valid only if appropriate assumptions about the existence of their terms are added.

Because the existence of the terms generally *is* assumed in psychological studies of the logic, the so-called existential fallacy is of little concern in the present context. It is, however, of interest that the count of 15 valid syllogisms is had by deleting those 9 moods with universal premises and particular conclusions from the 24 moods so often cited by other contemporary logicians (C in Tables V, IX, X, and XI). Thus, the Booleans, too, admit the fourth figure, but restrict consideration to those syllogisms whose conclusions involve *C–A* subject-predicate arrangements.

D. Assessing the Alternatives

The 24 moods that are generally recognized as valid by contemporary logicians stand as exactly half of the 48 that were shown to be valid earlier in this article. More specifically, they are exactly that half of the syllogisms for which the *C* term serves as subject of the conclusion and the *A* term as predicate.

The issue of whether the order of the terms in the conclusion should be part of the definition of syllogism is not merely one of formal meticulousness. It relates to the very purpose of the theory. Under the dominant contemporary interpretation, the theory amounts to a specification of which all possible pairs of the categorical premises or, equivalently, all possible pairs of minimally qualified and quantified dyadic relations between *A* and *B* and *C* and *B* will allow one to infer whether *A* can be attributed to *C*. In contrast, with *no* requirements as to the order of the terms in the conclusion, the theory becomes a specification of which of all possible pairs of minimally qualified and quantified dyadic relations between *A* and *B* and *C* and *B* will permit *any* valid inference to be drawn about the relationship between *A* and *C*. I submit that it is people's appreciation of the latter that has been of primary interest to psychologists. Moreover, I strongly suspect that the latter was the closer to Aristotle's primary interest, and this may also relate to the status of the controversial fourth figure.

The traditional fourth figure was essentially ignored by Aristotle. Why? In I,23 of the *Analytica Priora*, he asserts that in order to relate *A* to *B* syllogistically,

> we must take something in relation to both and this is possible in three ways (either by predicating *A* of *C* and *C* of *B*, or *C* of both, or both of *C*) and these are the figures of which we have spoken, it is clear that every syllogism must be made in one or the other of these figures. (*AP* I,23,41a)

Thus, Aristotle was not only very definite in his insistence that the logic be based on exactly three figures, but expressed it in a way that makes the absence of the fourth glaringly obvious. On the other hand, as we have seen, the validity of all five of the traditionally endorsed premise pairs of the fourth figure are defended, at least indirectly, by Aristotle in I,7 and II,1.

How are we to reconcile these passages? If Aristotle recognized the individual moods of the fourth figure, why did he reject the figure as a whole? The argument endorsed by Lukasiewicz (1957) is that Aristotle only came to recognize the fourth figure some time after most of the *Analytica Priora* had been written. To correct for his error, Aristotle then inserted the relevant passages of I,7 and II,1. To be sure, the fourth figure is developed with less rigor than the other three. But, Lukasiewicz continues, "Aristotle did not have time to draw up systematically all the new discoveries he had made, and left the continuation of his work to his pupil Theophrastus" (p. 27). (In view of this argument, it is worth noting that Theophrastus accorded the new moods of I,7 and II,1 to the first figure, not the fourth.)

In contrast with Lukasiewicz, Patzig (1968) argues that Aristotle appended the relevant passages of I,7 and II,1 to the text to correct, not for an overlooked fourth figure, but for the overlooked possibility of A-C conclusions. It is Patzig's belief that Aristotle purposefully ignored the fourth figure because, within the definitional system that he had set up, there was no graceful way to distinguish it clearly from the first. "The price of this course," Patzig reflects, "is that his three figures do not contain all of the syllogisms which he admits to be valid—thus confuting the assertion of [the above passage] that all valid syllogisms belong to one of the three figures" (p. 109).

I am inclined to agree with the essence but not the tone of Patzig's argument. I believe Aristotle excluded the fourth figure, not because he found it difficult to articulate a definition for it that was both clear and distinct from that of the first, but because he found it to be *logically* indistinguishable from the first.

A glance back at Tables V and XI proves the logical equivalence of these two figures. The two tables are identical except that the rows of one appear as the columns of the other. There are also two apparent differences between them that result from the exchange of their rows and columns. First, where there are Cs in one of the tables, there are As in the other. But, again, A and C are only labels for variables; whether A is called by C and vice versa is of no substantive difference in itself. Second, for equivalent syllogisms, the order of the premises is reversed across figures such that EAE of the first figure corresponds to AEE of the second EIO of the first to IEO of the fourth, and so on. But, as discussed previously, the order of the premises of a syllogism is immaterial to its logic.

If their variables are replaced with real-world terms, the equivalence of the two figures becomes even more obvious. As examples, consider the following pairs:

(1)	Figure 1 AAA	All mammals are animals All horses are mammals ——— All horses are animals
	Figure 4 AAA	All horses are mammals All mammals are animals ——— All horses are animals
(2)	Figure 1 EAE	No mammals are insects All horses are mammals ——— No horses are insects
	Figure 4 AEE	All horses are mammals No mammals are insects ——— No horses are insects

The point is that, except for the order of the premises, the two syllogisms of each pair are identical.

It might be objected that I contrived the similarity of the above pairs of syllogisms in that what I substituted for the *A* term in one, I substituted for the *C* term in the other. This construal of the terms is necessary, however, in order that the premises of each argument be true—the sin qua non of the syllogism. Moreover, the way in which Aristotle originally distinguished between the terms depended on neither the letters by which they were called, nor the premises in which they occurred, nor their order in the conclusion. Rather, it depended on the relative status of the sets to which the terms referred: "I call that the major in which the middle is contained and that term minor which comes under the middle" (*AP* I,4,26a). By this functional definition, the substitutions in the above arguments are for matched terms; for example, "horse" consistently serves as the term which, by formal or syntactic constraints, must be the minor term in each of the syllogisms.

The notion that Aristotle would have taken this sort of equivalence between the first and fourth figure as sufficient grounds for admitting but one of them to the system finds support in the thematic structure of his text. In I,4–6, Aristotle does indeed delineate all possible arguments in each of the three figures. But the valid syllogisms of the first figure are treated differently from those of the second and third. Specifically, the four moods of the first figure that are presented as valid in I,4, are presented as such without justification. Instead of proving their validity, Aristotle asserts that they are "perfect" where a "perfect" syllogism has been defined as one "which needs nothing other than what has been stated to make plain what necessarily follows" (*AP* I,2,24b). Each of the valid moods of the second and third figure are, in contrast, proved to be such. In particular, their proofs consist in demonstrations that each of them can, through some series of valid conversions, be derived from one of the valid moods of the first figure. In keeping with this, Aristotle asserted over and over again that the valid second and third figure syllogisms are *not* perfect—they can be *made* perfect only by means of the first figure and certain supplementary statements or operations.

The suggestion is that Aristotle's purpose in these chapters is only incidentally one of enumerating concludent combinations of categorical premises. His primary purpose seems instead to be one of reducing valid argumentation to the minimal necessary system. This goal seems even more apparent in I,7. As discussed previously, the chapter begins with the reinstatement of the previously rejected AE and IE premise pairs. Again, each of these premise pairs is defended by relating it, through conversion, to some previously validated mood. Next, Aristotle reminds us that it is only by reduction to the first figure that syllogisms of the second and third figures are proved valid. He then demonstrates that the particular moods of the first figure, *Darii* and *Ferio*, can be reduced to the universal moods, *Barbara* and *Celarent*. The conclusion that follows, and that Aristotle spends the rest of the chapter emphasizing, is that "*all* syllogisms may be reduced to the universal syllogisms in the first figure" (*AP* I,7,29b).

The remainder of Book I of the *Analytica Priora* is essentially addressed to the issues of how to recognize syllogisms of the different figures in any of the various costumes they may take on; how to discriminate true syllogisms from their various fraudulent cousins; and how to analyze or construct extended arguments through syllogistic chains. Again, Aristotle is firm throughout in his reference to exactly three figures. Moreover, he treats the three in a way that is consistent with the hypothesis that he perceived the fourth figure as logically equivalent with the first. For example:

> If then the middle term is a predicate and subject of predication, or if it is a predicate, and something else is denied of it, we shall have the first figure: if it both is a predicate and is denied of something, the middle figure: if other things are predicated of it, or one is denied, the other predicated, the last figure . . . we shall recognize the figure by the position of the middle term. (*AP* I,32,47b).

If it was indeed Aristotle's intention to exclude the fourth figure from the formal logic, then the additional moods cited in I,7 and II,1 of the *Analytica Priora* must have been meant for the first three figures. By implication, it must not have been Aristotle's intention to exclude arguments with *A–C* subject–predicate structures from the logic. We are thus back to the position that Aristotle's reckoning of the valid moods of the first three figures was very close to that adduced herein (see Section II,B).

At a more general level, the foregoing discussion suggests that to criticize Aristotle for failing to articulate as many syllogisms as the system would admit is misdirected. His intention would seem to have been, instead, to collapse the system to as few syllogisms as it would demand. Across Book I of the *Analytica Priora*, Aristotle argued (1) that any valid argument can be reduced to a chain of syllogisms; (2) that any syllogism can be reduced to one of the "perfect" syllogisms of the first figure; and (3) that any first figure syllogism can, in turn, be reduced to one of the two universal, first figure syllogisms. In context, the syllogistic system seems little more than an intermediate step in the effort to isolate the essential logic of argumentation.

III. Theories of the Psychology of Syllogistic Reasoning

In the section to follow, we will examine theories of the psychology of syllogistic reasoning. The goal will be to extract from them some common set of factors that might explain people's difficulty with the arguments. Afterward, we will return to the logic as Aristotle developed it and reconsider the issue of whether the logic was poorly designed for humans or humans were poorly designed for logic.

A. The Perfect Syllogisms

Aristotle suggested that people should, in general, be naturally competent with the four "perfect," first figure syllogisms, *Barbara, Celarent, Darii,* and *Ferio*. In Philoponus's words:

> A perfect syllogism is one the conclusion of which everybody is able to draw; as if someone said "the just is beautiful; the beautiful is good" for here anyone can understand "therefore the just is good." An imperfect syllogism is one the conclusion of which a logician can draw, e.g., "every man is substance; every man is animal." The conclusion of this is: "Therefore some substance is animal". . . . For perfect syllogisms both have necessity and evidently have it; imperfect syllogisms, such as all those of the second and third figures, have necessity but do not have it evidently: they need a logician to take the necessity which comes from the premisses but is not evident, and lead it into the light by means of conversions. (Cited in Patzig, 1968, p. 73)

The careful reader may have noticed that the "perfect" example provided by Philoponus corresponds to a syllogism of the traditional fourth figure, not the first. This is consistent with Aristotle's text. In formal presentations of the syllogisms, Aristotle generally expressed the relations between terms, not with the simple copula, but with such phrases as "belongs to" and "is predicated of." For example, his formal description of *Barbara* was "If *A* is predicated of all *B* and *B* of all *C, A* must be predicated of all *C*" (*AP* I,4,25b). When Aristotle did connect the terms of first figure syllogisms with the simple copula, he very often transposed the order of their premises as well, so that, like Philoponus's example, they fit the mold of the fourth figure, for example, "If planets do not twinkle and what does not twinkle is near, then the planets must be near" (*Analytica Posteriora* 78a).

Aside from adding support to our hypothesis that Aristotle perceived the first and fourth figures as logically indistinguishable, such examples provide an explanation for his contention that the logic of the first figure syllogisms should be especially natural or apparent. Specifically, in Aristotle's renditions of first figure syllogisms, the "middle" or repeated term generally occurs in the middle of the two premises, sandwiched between the two outer terms. In this way, the order of the terms in these syllogisms directly supports notice and coordination of any transitive relations between them. [See Kneale & Kneale (1965) and Patzig (1968) for a defense of this argument.]

Psychological studies have invariably been based on the traditional rather than the Aristotelian syllogistic. In traditional presentations of the logic, the terms are linked with the copula but the premises are not transposed. Within the traditional framework, therefore, it is not the first figure but the fourth that exhibits the "perfect" syntactic chaining of terms. In particular, it is those traditional fourth figure moods with nontraditional *A–C* conclusions that should be most accessible to the naive reasoner.

In keeping with this, in a study in which subjects were asked to generate conclusions for all 64 pairs of premises, Johnson-Laird and Steedman (1978) found that concludent premises of the fourth figure evoked valid responses slightly more often than did concludent premises of the other figures, including the first (see also Johnson-Laird, 1982, for replications of this effect). And, as would be predicted, the fourth figure conclusions given were strongly biased toward *A–C* subject–predicate structures.

On the other hand, as compared with all valid moods, the four "perfect" fourth figure moods (AAA, IAI, AEE, and IEO) did not, by any means, stand out as being especially easy. Furthermore, Johnson-Laird and Steedman's subjects were significantly more prone to generate conclusions even to *inconcludent* premise pairs of the fourth figure than to inconcludent pairs of the others. It thus seems that although the chaining of the terms in the fourth figure may indeed boost the availability of a conclusion, there is no evidence that it does the same for its underlying logic. Together these findings imply that, with respect to the psychological transparency of an argument, there must be other factors that are at least as important as the order of the terms.

B. The Atmosphere Hypothesis

Proper interpretation of a syllogism depends upon precise analysis of the interrelations that may hold between its terms, but this, Woodworth (1938) pointed out, sometimes involves more work than the reasoner is willing or able to invest. In such cases, Woodworth and Sells (1935) hypothesized, reasoners frequently base their responses on the "atmosphere" of quantity and quality set by the premises. Specifically, when both premises are affirmative, reasoners will tend toward an affirmative response. When both premises are universal, reasoners will tend toward a universal response. When either premise is particular or negative, a particular or negative response will be more likely. Woodworth (1938) suggested that the mechanism governing the atmosphere effect is one and the same as that which prompts us toward such grammatical errors as "The laboratory equipment in these situations *were* in many instances . . ." (p. 817). That is to say that the effect was intended, as it sounds, to reflect nothing more profound than linguistic gloss.

At least part of the reason that the atmosphere hypothesis has received so much attention in the psychological literature is that many would like so much to disprove it. Not surprisingly, many perceive the hypothesis as a dismaying attack on human rationality (c.f. Ceraso & Provitera, 1971; Wason & Johnson-Laird, 1972). But, regardless of the authors' positions, the data and the hypothesis have, on balance, persisted in agreeing with each other quite well (e.g., Begg & Denny, 1969; Ceraso & Provitera, 1971; Chapman & Chapman, 1959; Revlis, 1975b; Simpson & Johnson, 1966; Guyote & Sternberg, 1981). Could this be pure coincidence? Or is it the case, as the effect suggests, that human argumentation is driven as much by rhyme as by reason?

An escape from this dilemma can be found through the Euler tables (Tables IV, VI, VII, and VIII). An examination of these tables reveals that for every pair of premises except EE, regardless of the figure, there is at least one legitimate interpretation that leads necessarily to a conclusion predicted by the atmosphere hypothesis. More precisely, as shown in the last column of Table XII, for every pair of premises except EE, the *majority* of legitimate interpretations—82% on average—lead necessarily to either the *A–C* or the *C–A* conclusion that is favored by their atmosphere. What this means is that the now well-documented atmosphere effect may not be the product of linguistic whimsy at all: it may instead be the product of solid deductive reasoning, albeit on but a fraction of the appropriate representations of the premise information. At the very least, Table XII makes clear that, once having been raised through whatever process, the probability with which an invalid conclusion that is consistent with the atmosphere hypothesis can be properly rejected will depend on the completeness with which the reasoner has encoded the premises of the argument.

But neither is the complete encoding of a pair of premises enough to guarantee the demise of such a conclusion. The right-most column of Table XIII shows that for every legitimate interpretation of every pair of premises, at least one of the two conclusions favored by the atmosphere effect is *possible* or *may* follow. The other tabulations in Table XIII show that either atmosphere conclusion by itself may follow from every interpretation of the majority of pertinent premise pairs as well as from the majority of interpretations of every pertinent premise pair. The significance of these counts is that, even if all interpretations of a pair of premises have been recognized, the probability of dismissing an invalid atmosphere conclusion will depend additionally on the rigor of the reasoner's hypothesis-testing procedure. In particular, people are known to have a strong tendency toward examining available information only for support and not for falsification of their working hypotheses (for a review, see Wason & Johnson-Laird, 1972). To the extent that this tendency is operative in syllogistic reasoning tasks, Table XIII shows that it would generally support any bias toward conclusions predicted by the atmosphere hypothesis.

Taken together, Tables XII and XIII suggest that the atmosphere effect may stem, not from syntactic set as originally hypothesized, but from the process involved in the interpretation of the ambiguous quantifiers and the verification of candidate conclusions. If reasoners concentrate on just one of the possible interpretations of a pair of premises for purposes of deducing and ascertaining the necessity of a trial conclusion, Table XII shows that they may well end up with a proposition that is consistent with the atmosphere hypothesis. If, before committing themselves to such a trial solution, reasoners proceed to check its compatibility, but not its necessity, against other interpretations of the premises, Table XIII shows that they may well fail to reject the conclusion even if it is invalid. Furthermore, in multiple-choice or true–false tasks, as have been used in most psychological studies of the logic, the process of deciding upon a response might consist only in the second of these steps. That is, instead of going to the bother of independently generating trial conclusions from the premises, the reasoner might simply work from the alternatives provided, checking the compatibility or possibility of each against the various interpretations of the premises. Such a shortcut would render conclusions predicted by the atmosphere hypothesis all the more likely.

On the other hand, syntactic explanations of the atmosphere effect have been resuscitated of late. Specifically, it is argued that such syntactic bias is a consequence of the fact that the syllogisms are presented, and therefore

TABLE XII

NECESSARY CONCLUSIONS THAT ARE CONSISTENT WITH THE ATMOSPHERE HYPOTHESIS

			Number of relevant interpretations for which favored conclusion is necessary										
	Conclusion favored by	Total number of relevant	Figure 1		Figure 2		Figure 3		Figure 4		Percentage all figures		
Premises	atmosphere	interpretations	*A–C*	*C–A*	*A–C*	*C–A*	*A–C*	*C–A*	*A–C*	*C–A*	*A–C*	*C–A*	*A–C* or *C–A*
AA	A	4	1	4[a]	2	2	2	2	4[a]	1	56	56	88
AI	I	8	8[a]	8[a]	6	6	8[a]	8[a]	6	6	88	88	88
AO	O	6	4	3	3	6[a]	6[a]	2	4	4	71	63	92
AE	E	2	1	1	2[a]	2[a]	1	1	2[a]	2[a]	75	75	75
IA	I	8	6	6	6	6	8[a]	8[a]	8[a]	8[a]	88	88	88
II	I	16	12	12	12	12	12	12	12	12	75	75	75
IO	O	12	6	7	6	7	9	4	9	4	63	46	83
IE	O	4	4[a]	2	4[a]	2	4[a]	2	4[a]	2	100	50	100
OA	O	6	4	4	6[a]	3	2	6[a]	3	4	63	71	92
OI	O	12	4	9	7	6	4	9	7	6	46	63	83
OO	O	9	2	6	3	3	4	4	6	2	42	42	72
OE	O	3	2	1	2	0	2	1	2	0	67	17	67
EA	E	2	2[a]	2[a]	2[a]	2[a]	1	1	1	1	75	75	75
EI	O	4	2	4[a]	2	4[a]	2	4[a]	2	4[a]	50	100	100
EO	O	3	0	2	0	2	1	2	1	2	17	67	67
EE	E	1	0	0	0	0	0	0	0	0	0	0	0
		100	58	71	63	63	66	66	71	58	64.5	64.5	82

[a] Valid conclusions.

TABLE XIII

POSSIBLE CONCLUSIONS THAT ARE CONSISTENT WITH THE ATMOSPHERE HYPOTHESIS

Premises	Conclusion favored by atmosphere	Total number of relevant interpretations	Number of relevant interpretations for which favored conclusions is possible										
			Figure 1		Figure 2		Figure 3		Figure 4		Percentage all figures		
			A–C	*C–A*	*A–C*	*C–A*	*A–C*	*C–A*	*A–C*	*C–A*	*A–C*	*C–A*	*A–C* or *C–A*
AA	A	4	1	4	3	3	3	3	4	1	69	69	100
AI	I	8	8	8	8	8	8	8	8	8	100	100	100
AO	O	6	5	6	4	6	6	4	6	5	88	88	100
AE	E	2	2	2	2	2	2	2	2	2	100	100	100
IA	I	8	8	8	8	8	8	8	8	8	100	100	100
II	I	16	16	16	16	16	16	16	16	16	100	100	100
IO	O	12	10	12	10	12	12	10	12	10	92	92	100
IE	O	4	4	4	4	4	4	4	4	4	100	100	100
OA	O	6	5	6	6	4	4	6	6	5	88	88	100
OI	O	12	10	12	12	10	10	12	12	10	92	92	100
OO	O	9	8	9	9	9	9	9	9	8	97	97	100
OE	O	3	3	3	3	3	3	3	3	3	100	100	100
EA	E	2	2	2	2	2	2	2	2	2	100	100	100
EI	O	4	4	4	4	4	4	4	4	4	100	100	100
EO	O	3	3	3	3	3	3	3	3	3	100	100	100
EE	E	1	1	1	1	1	1	1	1	1	100	100	100
		100	90	100	95	95	95	95	100	90	95	95	100

tend to be treated, as linguistic information rather than logical formalisms. In treating the syllogisms as efforts after linguistic communication, the reasoner tends to ascribe a certain degree of rhetorical coherence to them which, as logical formalisms, they do not warrant.

In particular, Begg and Harris (1982) have found that people strive to establish some consistency amid the relationships expressed within the arguments. In their experiment, subjects were given pairs of inconcludent premises and asked to fill in or instantiate their terms so that they would be true in the real world. Instead of interpreting the two premises independently, subjects were found to treat them as a package, imposing the same relationship on each. Thus, the likelihood was that if one premise was instantiated so as to express intersection, so too, if possible, would be the other; if one premise was instantiated so as to express subordination, so too, if possible, would be the other; and so on. Inasmuch as greater ambiguity is equivalent to greater interpretive freedom, it is not surprising that this tendency was especially evident for particular premises. Furthermore, although the subjects were not asked to express conclusions to the pairs of premises, their instantiations were such that the implicit relationship between the A and C terms was highly likely to match the relationship imposed on at least one of the two premises.

In short, Begg and Harris's (1982) subjects behaved in close compliance with the atmosphere hypothesis. Given the choice, people evidently prefer to instantiate the terms of a syllogism such that the argument as a whole does not merely consist of a series of disjointed propositions, but instead, within the available degrees of freedom, attains a coherent, interarticulated rhetorical structure. Given the strength of their own disposition toward this end, it should not be surprising for them to suppose the same of others and, in particular, of the syllogism's producer. Thus, within Begg and Harris's framework, the atmosphere effect is seen as the *result* of efforts after consistent syntactic structure rather than vice versa.

To summarize, although the atmosphere hypothesis, as originally presented, might easily be seen to impute a distasteful degree of irrationality to the reasoner, the effect it predicts might alternatively result through certain categories of entirely rational, if imperfect, reasoning strategies. The first of these categories may be seen as a response to the inherent ambiguity of the categorical propositions: by choice or default the reasoner may focus attention on the implications of but some subset of the permissible interpretations of the premises. The second of these categories reflects a common weakness of the human reasoner but may be aggravated by the heavy processing load that is required for syllogistic reasoning and by the format of the typical experimental task: The reasoner may attempt to verify the generality of a tentative conclusion by checking only its compatibility but not its necessity against alternate interpretations of the premises. The third category of strategy through which the atmosphere effect might be produced consists in treating the syllogisms as efforts after linguistic communication rather than logical formalisms. Within this category, the reasoner will be prone toward overinterpreting the premises such that the information they are seen to offer approximates the completeness and coherence that is to be expected of cooperative discourse.

C. The Conversion Hypothesis

Woodworth (1938) reported a very simple experiment of Eidens (1929) in which subjects were asked to answer questions of the form. "What can you say about *P*, given that all *S* are *P*?" The questions were varied so as to probe all four types of categorical propositions. The important result, Woodworth summarized, was that most of the eleven subjects, all of them highly educated adults, answered with the converse for all of the propositions, including A and O.

Woodworth took this result in stride, pointing out that the atmosphere phenomenon should make it easy to accept illicit converses. Later, however, Chapman and Chapman (1959) suggested that the tendency toward illicit conversion was inspired in and of itself and was better viewed as a cause than a consequence of the atmosphere effect.

Most recently, Revlin (Revlin & Leirer, 1978; Revlis, 1975a,b) has adopted Chapman and Chapman's conversion hypothesis as a central assumption of his theory of the psychology of syllogistic reasoning. According to Revlin, in the course of interpreting quantified relations between two categories, there exists an operation that treats the relation as symmetric. Through this operation, the reasoner automatically encodes both the given relation and its converse. If the original proposition was meaningful, but the converse is silly (e.g., "all ducks are birds" versus "All birds are ducks"), then the converse is dropped. Otherwise, it becomes the priority representation in the reasoner's meaning stack. Combining this version of the conversion hypothesis with the assumption that, at least in multiple-choice tasks, people are biased against responding that no valid conclusion exists, Revlin has succeeded in predicting his subjects' performance quite well.

Despite the successes of Revlin's model, the assumption that the encoding of a categorical premise obligatorily includes its conversion is suspect. First, when the variables of the syllogistic formulae are replaced with meaningful terms, errors of conversion become rare. As mentioned above, Revlin holds that this is not evidence that conversion has not occurred, but only that an additional semantic evaluation process also occurs. In contrast, Chapman and Chapman (1959) suggested that, at least for A propositions, it might be the very presence of the variable terms that provokes conversion. Specifically, they suggested that, with letters for terms, the syllogisms take on a mathematics-like formality, and that the subjects, accordingly, interpret the copula, "are," as meaning "equal to" and, therefore, as being convertible.

Detracting further from Revlin's thesis is the fact that when adults have been asked to interpret the categorical propositions in terms of the Euler diagrams, blatant conversion errors have been nonexistent (Begg & Harris, 1982; Neimark & Chapman, 1975). That is, propositions of the form "All *A* are *B*" were never construed to mean that *B* was a subset of *A* (diagram J of our Euler tables); similarly, "Some *A* are not *B*" was never construed to mean that *A* was a subset of *B* (diagram L). Both of these errors might be expected if Revlin's strong version of the conversion hypothesis were taken at face value. Notably, Neimark and Chapman (1975) found that, very occasionally, their youngest subjects (12 year olds) did commit these errors, and under the load of solving complete syllogisms, it is entirely possible that adults would too. Still, it seems clear that people do not automatically convert the premises in the course of encoding them.

The observation that led Chapman and Chapman (1959) to suggest the conversion hypotheses was that those invalid moods which were most consistently misjudged as valid would in fact have been valid had they been presented in a different figure. Inasmuch as it is precisely the order of the terms within premises that distinguishes one figure from another, such errors could be fully reconciled by the conversion of one or both of the arguments' premises. The difficulty with this explanation is, of course, that willy-nilly conversion of the premises will not work. Transforming a particular mood from invalid to valid by changing its figure depends on converting the exact right combination of premises. Within Revlin's theory, reasoners are driven toward this combination by force of their bias against responding "no valid conclusion": Through a systematic process, they discard the converted version of the first and then the second premise until a concludent combination is found.

An alternative and more plausible explanation for apparent conversion errors was suggested (but not pursued) by both Chapman and Chapman (1959) and Revlin (1978; Revlis, 1975a). Specifically, such errors would be expected if, instead of recognizing all possible interpretations of the premises, people tended to encode only the symmetrical version of each type. Thus, A propositions would be represented by identity relation (diagram K or Q in the Euler tables), I and O propositions by intersecting sets (diagram M or S), and E propositions by nonintersecting sets (diagram N or T). An examination of Tables IV, VI, VII, and VIII proves that, if people did so, the apparent concludence of any given argument would be wholly dependent on its mood and independent of its figure—for example, AAA arguments would always involve cell K,Q and, therefore, would always seem as solid in the second figure as they would in any other. Furthermore, unlike the automatic conversion hypothesis, the notion that people may selectively attend to the symmetrical representations of the categorical propositions has, as will be discussed in the next section, received relatively strong empirical support.

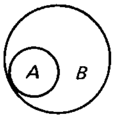
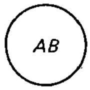
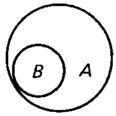
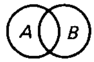
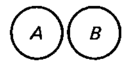
D. The Set Analysis Theory

Virtually no article on the psychology of the syllogism has failed to note that at least part of the reasoner's difficulty with the arguments may derive from the ambiguity of the categorical propositions. Ceraso and Provitera (1971) suggested that this might indeed be the principle source of difficulty. To test their hypothesis, they compared people's performance on two different sets of syllogisms. The premises for the first set of syllogisms consisted of the traditional categorical propositions. For the second set, each premise consisted of an unambiguous description of exactly one of the Euler relations, for example, the subset–superset relation was expressed as "Whenever I have a yellow block, it is striped, but there are some striped blocks which are not yellow" (p. 403). As predicted, the subjects' performance was considerably better with the modified syllogisms. Moreover, they responded to the traditional syllogisms as though they had been presented with the modified syllogisms. The only other major category of error in Ceraso and Provitera's data was directly related to the number of distinct ways in which the two premise relations could be combined.

Following Ceraso and Provitera's lead, Erickson (1974) theorized that people might generally interpret the categorical propositions in terms of just one of their underlying relations. His assumptions about the probabilities with which any one of these relations would be chosen are given in Table XIV. Using these values, Erickson tested the predictions of his theory against the performance of subjects with valid syllogisms in a conclusion-production task. The fit proved to be excellent. Interestingly, it was even better under the assumption that subjects examine but one of the possible combinations of their chosen Euler relations ($r = .97$) than under the assumption that they examine all possible combinations of the pair of relations ($r = .84$). Through subsequent experiments, Erickson has developed estimates of the probabilities with which, for given pairs of Euler relations, each of the various combined interpretations will be chosen. With these estimates, the theory has also proved to be a good predictor of people's performance on invalid syllogisms.

TABLE XIV

ERICKSON'S (1974) ESTIMATES OF THE PROBABILITIES WITH WHICH EACH OF THE SET RELATIONS WILL BE USED TO REPRESENT EACH OF THE CATEGORICAL PROPOSITIONS

Propositions					
All *A* are *B*	.25	.75	0	0	0
Some *A* are *B*	0	0	.25	.75	0
Some *A* are not *B*	0	0	0	1	0
No *A* are *B*	0	0	0	0	1

Implicit in the values in Table XIV are the suggestions, first, that people are indeed somewhat biased toward the symmetrical representation of each of the propositions and, second, that there are indeed certain legitimate interpretations of the particular propositions which people are biased against. According to Erickson, these values were based on nothing more than a rough retrofit from his conclusion-production data. Nevertheless, they are quite consistent with results from studies in which subjects were directly requested to map the propositions onto the Euler diagrams (Begg & Harris, 1982; Neimark & Chapman, 1975). The only significant exception is that the latter investigators found that, like affirmative particular propositions, negative particulars readily admit the appropriate subset–superset relation as well as the intersection relation.

Begg and Harris (1982) have argued that the motive behind people's incomplete mappings of the particular propositions is, once again, the tendency to treat the syllogisms as instances of normal linguistic offerings. Applying Grice's (1967) conversational maxims, the reasoner can expect that the information to be conveyed has been expressed as succinctly but completely and unambiguously as possible given his or her own known interpretative needs and biases. In particular, the reasoner may expect that "some" is intended to mean "some but not all." If the communicator meant for the possibility of "all" to be understood, it should have been specified; if the communicator meant to be vague, the quantifier should have been omitted all together.

Finally, reasoners' willingness to make do with the first joint interpretation of the premises that they generate, may be taken as another instance of their reluctance or inability to recognize bona fide challenges to their working hypotheses.

E. The Analogical Theory

Recently, Johnson-Laird and Steedman (1978) proposed a very appealing model of human syllogistic reasoning. Within their model, people represent the classes corresponding to the terms of the arguments by imagining some arbitrary number of their exemplars. The relations between classes are then encoded as positive or negative pointers from the exemplars corresponding to the subject of a proposition to the exemplars corresponding to the predicate. To capture the multiple meanings of the propositions, the representations may include optional exemplars that are accordingly connected or not. For example, the proposition "All *A* are *B*" would be represented by a set of *a*s which point, one-to-one, to a set of *b*s; the possibility that not all *B* are *A* would be recognized by including some optional *b*s that are not connected to *a*s. Similarly, "Some *A* are *B*" would be represented by some number of *a*s pointing, one-to-one, to some number of *b*s and an optional set of unconnected *a*s and *b*s. The negative relations are not as gracefully configured, but the idea is the same. The second premise is then represented by relating some set of *c*s to the same set of *b*s. In this way, the process of combining the two premises is merely a concomitant of encoding them.

A conclusion to the syllogism is formulated by examining the paths that can be constructed from the *a*s to the *c*s via the *b*s. If the *a*s and *c*s can only be connected via positive pointers, the reasoner is expected to conclude that some *A* are *C*. If all of the *a*s can be thus connected, the reasoner is expected to conclude that all *A* are *C*. The conditions for the negative relations are similar.

Because the terms of the arguments are represented as sets of discrete elements, there is rarely but one way in which the paths between the *a*s and the *c*s might be configured. As an example, consider the premises "All *A* are *B*" and "All *C* are *B*." If all of the *a*s and the *c*s point to the *same* subset of *b*s, the evident conclusion will be "All *A* are *C*"; if all of the *a*s point to *different* *b*s from any of the *c*s, the evident conclusion will be "No *A* are *C*"; if the linked *b*s overlap, the evident conclusion will be "Some *A* are *C*." To ascertain the validity of their conclusions, therefore, reasoners must test them to see whether there is any alternate arrangement or reading of the pointers that would render them false.

The very nice aspect of Johnson-Laird and Steedman's model is that, within it, the dominant categories of reasoning errors are the products of very plausible hazards of the representational process. Failures to recognize all possible interpretations of the premises are produced by failures to register or properly interconnect all conceivable optional elements. The atmosphere effect will be produced by a bias toward linking the *a*s and the *c*s to the same subset of *b*s. The common reluctance to seek falsifying evidence for one's working conclusion would be equivalent to a reluctance or difficulty in radically altering one's established representation of the premise information.

The model also handles and even elaborates the effect of the order of the terms that was anticipated by Aristotle in his distinction between perfect and imperfect syllogisms. Recall that, within the model, the links between the terms of a proposition are unidirectional—they are schematized by pointers extending from the exemplars of its subject to the exemplars of its predicate. The underlying assumption is that the relation should be far more difficult to access in the reverse direction.

From this assumption, the general advantage of the first and fourth figure syllogisms follows readily. In the representations for both of these figures, one end term should point *to* the middle term while the other should point *from* the middle term. Thus, a natural path is set up from one end to the other. The assumption also leads to the prediction that the concludence of those first figure syllogisms that were initially overlooked by Aristotle and disallowed within the traditional framework should be especially difficult to establish. The reason is that the conclusion can only be established by traversing the paths through the terms in the unnatural direction, from predicates to subjects. For the same reason, the converse syllogisms of the fourth figure should be equally elusive. These effects are in fact strongly expressed in the performance of Johnson-Laird and Steedman's subjects.

The explanation for this subject-to-predicate effect is to be found, once again, in the linguistic predispositions evoked by the arguments. In normal language, the subject of a sentence, or more generally, the information presented earlier in a discourse, sets the topic or context within which later information is to be interpreted. To the extent that reasoners treat the syllogisms as normal language, they will carry certain expectations about the coherence or relations in topic or emphasis that ought to hold among the premises and the conclusion they beg. As a consequence, reasoners are likely to display a certain deference toward using whichever of the end terms has served as a subject in the premises, as the subject or topic of their conclusions. According to Sanford and Garrod (1981), this bias is not merely a question of linguistic cooperation, but a product of the way in which the information is processed in memory. Under this view, the reasoner is virtually obliged to maintain the topic of the argument.

F. The Transitive Chain Model

The transitive chain model was recently proposed by Guyote and Sternberg (1981) and is mentioned here primarily for the sake of completeness. At the level of premise representation, the model resembles Erickson's

(1974, 1978) in that it is assumed that what people understand from the categorical premises is exactly that information that is schematized in the Euler diagrams. Instead of representing the information with the Euler diagrams as Erickson has, however, Guyote and Sternberg restate it in a propositional form of their own invention. As an example, the Euler diagram showing that *B* is a proper set of *A* would, within Guyote and Sternberg's model, be represented as:

$$B \rightarrow A \quad \left| \quad \begin{array}{l} a_1 \rightarrow B \\ a_2 \rightarrow -B \end{array} \right.$$

Letting upper-case letters refer to whole classes and lower-case letters to parts of classes, this representation is to mean that all of class *B* belongs to *A* whereas at least one member of class *A* belongs to *B* and at least one does not. Again, that is exactly what is conveyed by the corresponding Euler diagram. Inasmuch as both Erickson and Guyote and Sternberg assert that the representations they use in explicating their respective theories should correspond to reasoners' mental representations, not literally, but only by analogy, the difference in their representational schemes is, in itself, of no theoretical significance.

Guyote and Sternberg's premise combination process resembles that of Johnson-Laird and Steedman's (1978) in that it consists in building paths from representations of one of the end terms to the representations of the other via their common associations with the middle term. It is in this process, however, that Guyote and Sternberg's choice of representational scheme makes a difference. First, within Johnson-Laird and Steedman's model, *A–C* versus *C–A* conclusions are had by traversing the same representation of the premises in opposite directions. In contrast, within Guyote and Sternberg's model, the availability of both *A–C* and *C–A* conclusions depends upon constructing and processing two separate interpretive chains: *ABBC* and *CBBA*. Oddly enough, although Guyote and Sternberg assume that people generally do process the relations between premises in both directions, the multiple-choice task through which their model is tested includes only *A–C* responses. Second, within Johnson-Laird and Steedman's model, the classes are consistently represented by collections of tokens, such that alternate interpretations of the individual premises can be captured within the same representation and alternate interpretations of the combined premises are had by varying the paths from one end of the representation to the other. As an explanatory device, Johnson-Laird and Steedman's model has the additional asset that the various relations that can be constructed between the *A* and *C* terms are open to concrete manipulation and visual inspection by the reader. In contrast, within Guyote and Sternberg's model, each interpretation of each premise gets its own, separate mental representation, and complete representation of the combined premises requires the construction of a separate *ABBC* and *CBBA* chain for every different pair of interpretations of the individual premises. Furthermore, because the representational scheme only admits symbols for whole classes or their exceptional elements, the relations that may hold between the two end terms of a chain are generally not transparent; their specification is instead supposed to depend upon the application of the appropriate interpretive rules. Unfortunately, in order to respond to all combinations of symbols—positive and negative, generic and token—the list of interpretive rules is necessarily quite long and complicated. (Notably, Guyote and Sternberg have provided a complete list of requisite rules in neither the body nor the appendix of their text.) Finally, like the cells in our Euler tables, any given premise chain may admit up to four different possible relations between *A* and *C*, and these too, once generated, are presumably encoded as separate propositions. In short, as compared to Johnson-Laird and Steedman's representational system, Guyote and Sternberg's provides a relatively economical means of encoding individual interpretations of the premises. But it is both less economical and less supportive for the tasks of encoding multiple interpretations of a premise and of combining the pairs of interpretations in the quest for potential conclusions.

After Guyote and Sternberg's theoretical reasoners are finished combining interpretations of the premises, they are left with some set of propositions representing possible relations between *A* and *C*. Their task at this point is to formulate a conclusion that is consistent with all such propositions. If they can think of only one consistent conclusion, they will look through the multiple-choice list, responding when they find it. (Again, the choices included only *A*–*C* responses despite the fact that the problems included several for which only *C*–*A* conclusions are valid.) If the reasoners think of more than one consistent conclusion, they choose between them on the basis of a modified version of the atmosphere principle. On the other hand, if the initial components of their candidate relations do not match, that is, if some of the propositions start with lower-case and others with upper-case letters, then with a probability of *c*, reasoners simply declare themselves confused and respond that the argument is indeterminate. At this point it is worth recalling that the propositions representing the various relations between classes in Guyote and Sternberg's system are isomorphic with the Euler diagrams. If Guyote and Sternberg had chosen to use the diagrams in the place of their system of upper- and lower-case letters, they might have found no need to posit this confusion factor.

Guyote and Sternberg offer that the major difference between Erickson's (1974) complete combination model and their own centers on his assumption that people generally encode just one interpretation of each premise. Guyote and Sternberg assume, in contrast, that people always encode all interpretations of each premise but that, for reasons of memory or processing capacity, they can combine and consider the consequences of at most four pairs of interpretations. Indeed, by modeling their assumptions mathematically and estimating the parameters from their subjects' performance, Guyote and Sternberg eventually conclude that, as often as not, people in fact combine only one pair of premise interpretations.

Note that if people generally work with just one interpretation of the premises, the contention that they recognize more than one is of no practical significance. Guyote and Sternberg further assume that the pairs of interpretations with which people will choose to work will be chosen in order of their representational complexity. It just happens that, for any given premise, the representationally simplest interpretation corresponds to one of the symmetrical Euler diagrams, that is, it is precisely the interpretation that is favored within Erickson's models.

IV. The Reasoner versus the Logic

From the preceding review of the psychology of the syllogism, we may adduce three major classes of difficulty that may beset the human reasoner: (1) inappropriate application of language comprehension heuristics; (2) incomplete processing of the premise information; and (3) the tendency to verify rather than validate working hypotheses. But difficulties in processing the syllogisms are not necessarily indicative of incompetencies with the logic that the syllogisms are intended to convey.

As was argued in Section II,D, Aristotle developed the syllogisms only as an intermediate form in his analysis of discourse. Although through history they have often been taken as the laws of logic, that was not the original intent. The syllogisms were intended only as expressions of those laws. Their import, therefore, lies not in their individual forms, but in their collective content, and that might be expressed in many other ways. In this section we will consider the extent to which people's apparent difficulties with the logic might be owed to some fault in design or presentation of the syllogisms.

A. Inappropriate Application of Language Comprehension Heuristics

Among the recurrent findings of studies on the psychology of syllogistic reasoning is that people tend to construe the premises and construct their conclusions so as to lend the arguments a degree of coherence and completeness that they do not literally possess. This, it is argued, is a consequence of the fact that the syllogisms are presented, and therefore tend to be treated as linguistic information rather than logical formalisms.

Analyses of natural language make clear that its content scarcely alludes to its meaning. Instead, accurate, efficient linguistic communication depends upon a cooperative contract between speakers or writers and their audience. Specifically, of the information that is relevant to their message, speakers or writers are to specify only as much as they think their audience might not otherwise presume or supply on their own. Within the limits thus defined, their contribution must be complete, coherent, and unambiguous. This means, of course, that in order for the message to appear complete, coherent, and unambiguous to its recipient, she or he in turn must readily supply whatever information and clarification is required to make it so.

If the syllogisms are perceived as efforts after linguistic communication, then reasoners will naturally overinterpret them. On the assumption that the information to be conveyed is expressed as succinctly but completely and unambiguously as possible given her or his own known interpretive needs and biases, the reasoner may rightfully support that "some" is intended to mean "some but not all" (see, e.g., Begg & Harris, 1982; Chapman & Chapman, 1959); if the possibility of "all" is meant to be understood, then why specify "some"? The reason dates back to Aristotle: He was concerned that, without the "some," indefinite statements were too often interpreted as *universals*. In the effort to achieve an effective compromise, psychologists have generally prefaced their experiments with the instruction that "some" is to be interpreted as "at least one and possibly all." This has been found to help (see Frase, 1966), but not a lot. It thus seems that people's difficulty in processing the ambiguity of the propositions is largely independent of how they are expressed.

As language users, reasoners further expect the arguments to reflect a certain degree of structural coherence. They thus strive to maintain whichever of the end terms has served as a subject of the premises, as the subject or topic of their conclusion (Johnson-Laird & Steedman, 1978), and, within the interpretive freedom afforded by the ambiguity of the premises, to impose consistency on the relation expressed by the separate propositions (Begg & Harris, 1982).

In contrast to the problem with particular and indefinite propositions, people's inclination to impose order on the topical and relational structure of the arguments can only be blamed on the presentation of the logic. Aristotle, after all, invented the syllogisms as a means of enabling people to extract the logically necessary information from discourse and thus to loose themselves from the interpretive acquiescence that language invites. To the extent that people nevertheless perceive and treat the syllogisms as discourse, the system cannot serve its purpose.

How could this problem be remedied? One possibility might be to make the system less seductively language-like. One might recast it on terms of, say, propositional logic. The major drawback to this solution is that the logic would remain relatively inaccessible except to reasoners with special training.

One might alternatively rid the arguments of their linguistic character by having people translate them into appropriate combination of Euler diagrams. The Euler matrices (Tables IV, VI, VII, and VIII) yield a relatively simple set of rules for identifying inconcludent premise pairs. Specifically, whenever *both* premises can be mapped onto (1) the overlap relation (diagrams M and S), (2) the exclusion relation (diagrams N and T), or (3) the relation wherein the middle term is a proper superset of the other (diagrams L and P), no valid conclusion will exist. The reason is that, in each of these three cases, the middle term, *B*, totally fails to mediate the relation between *A* and *C*—it provides absolutely no information or constraints on how *A* and *C* might relate to each other. The only inconcludent premise pairs that are not covered by these three rules are AO of the first figure and OA of the fourth. The includence of these premise pairs derives from the fact that from the O propositions *B* may be either a proper subset of or entirely distinct from the related end term: When combined with the A proposition, which asserts that *B* is a subset of the other end term, these two possibilities lead to conclusions that contradict each other and thereby rule out all others. The principal disadvantage to the Euler diagrams is that no such simple set of rules can be formulated for determining the permissible conclusions to valid syllogisms, and the alternate route of deducing the common conclusions to all pertinent pairs of diagrams would seem difficult to manage in one's head—although, as will be discussed later, it can be markedly simplified.

As yet another approach, contemporary logicians offer a set of rules through which one can evaluate the concludence of the arguments without even considering their meaning (see, e.g., Copi, 1961; Lemmon, 1965):

1. In a valid syllogism, no term can be distributed in the conclusion unless it is distributed in the premises.
2. The middle term of a valid syllogism must be distributed at least once.
3. If one of the premises of a syllogism is negative, its conclusion must be negative.
4. There is no valid syllogism with two negative premises.

Through the (occasionally chained and complex) application of these rules, one can identify all of the inconcludent premise pairs and generate permissible conclusions to all of the others that are recognized within the contemporary syllogistic system. My only problems with these rules, aside from the fact that they are not consistently easy to use, are (1) they do not help me understand the logic behind the deductions they prescribe, and (2) they do not work once the possibility of A–C conclusions is admitted.

As a final possibility, a solution might be had by changing the manner in which the syllogisms are presented. Instead of presenting them, as Aristotle did, as skeletal arguments into which discourse could be translated, one might present them as a set of rules through which discourse could be evaluated. For example, the essential logic of the syllogism could be developed through queries such as, "If all you know is that at least some *A* are *B*, what might you find out about the relation between *B* and *C* that would permit you to infer something definite about the way in which *A* and *C* are related?" The only possibilities are that all *B* are *C* or that no *B* are *C*. The complete set of such concludent pairs of dyadic relations is given in Table XV. This set is sufficient to identify all of the concludent premise pairs of the categorical syllogisms. If one is interested not just in identifying concludent pairs but, further, in the strongest conclusions that can be had, the relations provided in Table XVI will suffice. The relations in Table XVI are sufficient to identify all of the valid categorical syllogisms, conclusions and all.

The systems presented in Tables XV and XVI are vastly simpler than the traditional syllogistic system. Instead of 64 different pairs of premises, Table XV includes just 4, and Table XVI (ignoring the redundant pairs) includes just 6. There are two reasons why the lists of premise pairs in Tables XV and XVI are so short. The first is that inconcludent pairs are not enumerated but defined by exclusion. The second is that the figures are partially collapsed because each of t_i and t_j can represent either end term. Such flexible specification of the end terms is intended not only to reduce the requisite number of rules but, further, to eliminate the topic effect.

TABLE XV

MINIMALLY CONCLUDENT PAIRS OF DYADIC RELATIONS[a]

Relation i	Relation j	Conclusion
At least some t_i are m	All m are t_j	At least some t_i are t_j
	No m are t_j	At least some t_i are not t_j
At least some t_i are not m	*All* t_j are m	At least some t_i are not t_j
At least some m are not t_i	All m are t_j	At least some t_j are not t_i

[a] The two terms to be related are represented by t_i and t_j; the mediating term is represented by m.

TABLE XVI

DYADIC RELATIONS AND THEIR STRONGEST CONCLUSIONS[a]

Relation i	Relation j	Conclusions
1. All t_i are m	a. All m are t_j	All t_i are t_j; At least some t_j are t_i
	b. At least some t_j are not m	At least some t_j are not t_i
	c. No t_j are m	No t_j are t_i
2. All m are t_i	a. All t_j are m[b]	All t_j are t_i; At least some t_i are t_j
	b. At least some t_j are m	At least some t_j are t_i
	c. At least some m are not t_j	At least some t_i are not t_j
3. No t_i are m	a. All t_i are m[c]	No t_i are t_j
	b. At least some t_j are m	At least some t_j are not t_i; At least some t_i are not t_j

[a] The two terms to be related are represented by t_i and t_j; the mediating term is represented by m.
[b] This pair is redundant with pair 1a.
[c] This pair is redundant with pair 1c.

B. INCOMPLETE PROCESSING OF THE PREMISE INFORMATION

Incomplete processing of the premise information is surely a major source of error in human syllogistic reasoning. In Section III, it was suggested as sufficient cause for both the atmosphere effect and the apparent phenomenon of illicit converison. In addition, it is centrally assumed in the models of Erickson (1974, 1978), Johnson-Laird and Steedman (1978), and Guyote and Sternberg (1981).

The problem derives in part from people's difficulty in recognizing the alternate interpretations of the individual premises. But having done so, the task of determining their joint implications is a challenge in itself. As is shown by the Euler matrices (Tables IV, VI, VII, and VIII), each premise of a syllogism may be translated into as many as 4 distinct set relations; each pair of premises may be translated into as many as 16 combined set relations; each combined set relation may yield as many as four different conclusions; and of all those conclusions, the only ones that follow validly from the original premises are those which are common to all of their combined interpretations. From this perspective, failures to process the premise information properly might be wholly attributed to limitations of memory or processing capacity.

On the other hand—and this point is both very important and generally overlooked—it is not necessary that the reasoner consider all interpretations of the premises. It is sufficient that he or she consider only those that bracket or capture the extremes of the range of possible relationships that can be established between the end terms. Any conclusion that is compatible with both the strongest and weakest interpretations of the premises will necessarily be compatible with any between them; if no such conclusion exists, the premises are indeterminant.

This bracketing principle substantially reduces the storage and processing requirements of the arguments. In the present context, however, it is perhaps more interesting that it renders the overlap relation entirely superfluous: As the overlap relation is itself the most neutral or intermediate interpretation of the particular propositions, so too are the conclusions it yields. Moreover, the identity relation is proved almost as useless: It can mediate extreme conclusions only when paired with itself. Recall that these two relations correspond respectively to people's preferred interpretations of particular and universal affirmative premises.

C. The Verification Fallacy

Of the three classes of difficulties that were extracted from the theories and data on the psychology of syllogistic reasoning, this one must be owed exclusively to the reasoner. The tendency to "test" one's hypotheses by seeking out only information that fits them, or by interpreting information only so that it does fit them, is an extremely prevalent weakness of the human reasoner. In the preceding review, the verification fallacy was expressly cited for people's willingness to accept possible but unnecessary conclusions, but it may also be responsible for their reluctance or inability to properly recognize the disparate meanings of "some." More generally, the verification fallacy may spring from the very same mechanism as that which drives the necessarily presumptuous art of language comprehension. Ironically, it was precisely this presumptuousness which Aristotle intended for the logic to correct.

V. Summary

The syllogisms, as developed by Aristotle, were intended to represent the simplest of all possible implicative chains and, as such, to provide people with a simple, content-independent system for reducing and evaluating all possible argumentation. Aristotle's system, however, has eluded casual reasoners and has been disputed by scholars since its inception. The purpose of this article was to consider why this has been so: Does it reflect a fundamental irrationality of people or can it instead be ascribed to some flaw in the design or presentation of the logic?

Motivated by the disagreements among scholars as to the number of valid syllogisms that exist, the discussion was first focused on the logic itself. Most significantly, a system was invented for ascertaining which of the 64 possible pairs of premises do indeed yield necessary conclusions. Because of the transparency with which premises are mapped onto conclusions within this system, it should be an asset to students of the syllogism for practical reasons alone. In addition, however, the system was found to allow a number of valuable insights into the underlying structure of the syllogistic logic.

The results of this system were then compared with other analyses found in the literature. Excepting certain differences that were shown to be highly superficial in nature, the results of our new system were found to be in close agreement with Aristotle's text. Furthermore, the alternate counts of the syllogisms were generally seen to derive from trivial excursions from Aristotle's text. Some were traced to overly literal adoptions of parts of his original text, for example, the acknowledgment of exactly and only those valid arguments that he explicitly spelled out or the exclusion of all arguments whose conclusions contain the major (*C*) term as subject. Others were traced to logically inconsequential expansions of the system, for example, the formalization of the fourth figure and the separate recognition of subalternate conclusions. The final count considered, the Boolean count, additionally involved recognition of the existential fallacy which, though logically compelling in itself, may be considered peripheral to the syllogistic system.

An important coda to this discussion was the suggestion that Aristotle did not see the syllogistic system as an end in itself. To the extent that he developed the system, he consistently emphasized its internal redundancy; conversely, he frequently ignored or glossed over forms that offered no new structural or logical insights. For Aristotle, it seems that the syllogisms were but one form of expression, a convenient intermediate step in the effort to identify the basic logic of argumentation.

Consideration was then turned to theories of the psychology of syllogistic reasoning. From them we extracted three major classes of difficulty that beset the human reasoner. The first of these was the distortion of the syllogisms' content through inappropriate application of language comprehension heuristics. Inasmuch as such heuristics are crucial to making sense of natural langauge, one would not want people to remove them from their repertoire in any general way. On the other hand, in the interest of enabling logical analysis of discourse, one would like people to be able to escape from them when appropriate. The recommendation was therefore to recast the logic in a way that was less likely to trigger people's language comprehension routines. As additional desiderata, such a system should be relatively easy to understand and to translate to and from natural language. Two options were suggested: the Euler matrices and the minimal syllogistic frames of Tables XV and XVI.

The second class of difficulties was traced to the multiple possible interpretations of the premises. This class of difficulties was attributed in part to the processing load involved in managing all appropriate interpretations and their combinations. It was shown that this problem could be greatly reduced, however, if people would narrow consideration from the full range of interpretations that are possible to only those extreme interpretations that most rigidly constrain the conclusions.

The third class of difficulties was people's tendency to support rather than challenge their immediate understanding of the arguments. This was seen as a fundamental shortcoming of human reasoning and perhaps as the single most important reason why training with some digestible form of the logic may be considered so important a part of people's education. The syllogisms as developed by Aristotle represent one effort at capturing the logic in such a form. A major point of this article, however, is that they are not the only possible means of so doing. Psychologists interested in assessing people's logical abilities and educators interested in developing their logical abilities might be well advised to pay less attention to the syllogisms per se and more to the logic they were intended to convey.

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