Deduction in Aristotle's *Topics* and *Prior Analytics*

Marko Malink (NYU)

(Draft, October 2014)

Abstract: It is widely agreed that Aristotle's *Prior Analytics*, but not the *Topics*, marks the beginning of formal logic. However, it is not clear exactly why this is so. What are the distinctive features in virtue of which Aristotle's discussion of deductions (*syllogismoi*) counts as formal logic in the one treatise but not in the other? In order to answer this question, I argue that in the *Prior Analytics* – unlike in the *Topics* – Aristotle is concerned to make fully explicit all the premisses that are necessary to derive the conclusion in a deduction.

1. Introduction

Aristotle's *Prior Analytics* is generally thought to mark the beginning of formal logic.[1] His discussion of deductions (*syllogismoi*) in this treatise initiated the discipline today known as formal logic. Of course, there are a number of treatises in which Aristotle discusses deductions. A notable example is the *Topics*, which was written before the *Prior Analytics*.[2] The two treatises even share the same definition of what a deduction is (to be found in their first chapters, at 100a25–7 and 24b18–20). Nevertheless, Aristotle's discussion in the *Topics* is

[1] See, for example, Cornford 1935: 264, Russell 1946: 219, Ross 1949: 29, Bocheński 1956: 74, Allen 2001: 13, Ebert & Nortmann 2007: 106–7, Striker 2009: xi.

[2] There is general agreement that the *Topics* was written before the *Prior Analytics*; see, e.g., Brandis 1835: 252–9, Ross 1939: 251–2, Bocheński 1956: 49–51, Kneale & Kneale 1962: 23–4, Brunschwig 1967: lxxxvi–lxxxix, Corcoran 1974: 88, Barnes 1981: 43–8, Allen 1995: 177–9, 2001: 21, 2007: 88–92, Primavesi 1996: 60, Rapp 2000: 20, and Striker 2009: xii.

commonly regarded as not being formal in the relevant way.[3] The traditional view is that formal logic begins not with the *Topics* but with the *Prior Analytics*.

If the traditional view is correct, it is not clear *why* it is correct. On what grounds does Aristotle's discussion of deductions count as formal in the one treatise but not in the other? What exactly are the distinctive features in virtue of which the *Prior Analytics* is a treatise of formal logic? Although these questions go to the heart of Aristotle's logical theory, they have not been sufficiently addressed in the literature, or so I will argue. Accordingly, my aim in this paper is to provide an answer to those questions, thereby elucidating and vindicating the traditional view. I hope that this will contribute to a better understanding both of what is involved in Aristotle's transition from the *Topics* to the *Prior Analytics*, and of why this transition is a major step in the origins of formal logic.

Before embarking on this task, let us survey a few negative answers to the above questions. First, the syllogistic presented by Aristotle in the *Prior Analytics* is not formal in the sense of being symbolic.[4] Aristotle does not employ a symbolic language containing any artificial symbols. Instead, he uses ordinary (if somewhat contrived) Greek consisting exclusively of letters and words familiar to every reader of Greek.

Second, the syllogistic is not formal in the sense of being formalized. As Benjamin Morison has shown, Aristotle does not employ a formalized language in the *Prior Analytics*.[5] For whether or not a given argument counts as a deduction (*syllogismos*) in the *Prior Analytics* cannot be judged solely by attending to the linguistic expressions involved without taking into account their meaning. This is because Aristotle does not introduce a canonical way of expressing the premisses and conclusions of his deductions, but allows for a variety of interchangeable expressions such as 'A belongs to all B', 'A is predicated of all B', 'B is in A as a whole', 'A follows all B', and so on. Aristotle does not specify a closed list of canonical expressions to be used in deductions. Any expression is admissible as long as it has the same

[3] Thus, the discussion of deductions in the *Topics* is sometimes characterized as 'informal' (Burnyeat 1994: 31) or 'pre-formal' (Woods & Irvine 2004: 34 and 38).

[4] Barnes 2007: 274.

[5] Morison 2012: 172–3 and 186–7; *pace* Smiley 1982–3: 1.

meaning as the expressions just mentioned.[6] By contrast, the Stoics employed a limited set of canonical expressions in their syllogistic. If in a deduction one of their canonical expressions is replaced by a non-canonical expression, the resulting argument will fail to be a deduction even if the expressions have the same meaning.[7] Thus, the Stoic syllogistic is formalized in that whether or not an argument counts as a deduction can be ascertained from the linguistic expressions alone without attending to their meaning, by determining whether the argument contains a suitable arrangement of canonical expressions. Aristotle's syllogistic, however, is not so formalized.[8] This is what Alexander of Aphrodisias has in mind when he writes that for Aristotle, unlike for the Stoics, 'a deduction has its being not in the words but in what is signified by the words'.[9] Łukasiewicz puts the point succinctly:

> Aristotelian logic is formal without being formalistic, whereas the logic of the Stoics is both formal and formalistic. (Łukasiewicz 1957: 15)

If Aristotle's syllogistic is not formalistic or formalized, what is the distinctive sense in which it is formal? One might point out that it is formal in that it is completely general and topic-neutral, applicable to any subject matter whatsoever.[10] This is certainly true. But it is equally true for the *Topics*. The dialectical method presented in the *Topics* and the dialectical deductions involved in it are no less general and topic-neutral than the kinds of deduction

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[6] See Brunschwig 1969: 3–5.

[7] Frede 1974a: 5–6 and 13, Barnes 2007: 314–21.

[8] For this difference between Aristotle and the Stoics, see Łukasiewicz 1957: 15–19, Frede 1974a: 13–15, 1974b: 198–201, Morison 2012: 186–7.

[9] οὐκ ἐν ταῖς λέξεσιν ὁ συλλογισμὸς τὸ εἶναι ἔχει ἀλλ' ἐν τοῖς σημαινομένοις, Alexander, *in Pr. An.* 372.29–30; similarly, 373.16–17. Elsewhere he writes: 'They [the Stoics] do not call such arguments deductions since they attend to expression and language (εἰς τὴν φωνὴν καὶ τὴν λέξιν βλέποντες), whereas Aristotle, where the same object is signified, looks to what is signified and not to the expressions.' (*in Pr. An.* 84.15–17)

[10] This is one of the senses in which logic is sometimes said to be 'formal'; see, e.g., Dutilh Novaes 2011: 314–16.

studied in the *Prior Analytics*.[11] In this sense, the treatment of deduction in the *Topics* is formal just as the one in the *Prior Analytics*.[12] Thus, generality and topic-neutrality cannot serve as an answer to the questions raised above.

Finally, it is often thought that the distinctive kind of formality exhibited by the *Prior Analytics* is to be attributed to Aristotle's use in this treatise of schematic letters such as 'A' and 'B' in place of concrete terms such as 'man', 'animal', and 'walking'.[13] For example, Jan Łukasiewicz (1957: 7–8 n. 1), who calls these letters variables, maintains that 'the introduction of variables into logic is one of Aristotle's greatest inventions. ... by using variables Aristotle became the inventor of formal logic'. Similarly, Gisela Striker (2009: xii) writes that 'the crucial innovation ... that makes syllogistic a formal system is the introduction of letters as placeholders for the terms'.

Clearly, Aristotle's schematic letters are a very useful and important piece of notation. Nevertheless, it is doubtful whether they are the crucial feature in virtue of which the *Prior Analytics* marks the beginning of formal logic. To see this, suppose for a moment that Aristotle used these letters in formulating some of his *topoi* in the *Topics*. For instance, consider the following bit of a *topos* from the fourth book.

> Of those of which the species is predicated, the genus must be predicated as well. (*Top.* 4. 1 121a25–6)

Aristotle could have rewritten this sentence as follows:

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[11] Alexander, *in Top.* 3.25–4.10 and 5.4–13, Smith 1993: 338–9, Code 1999: 45. Aristotle holds that dialectic is 'concerned with things which are, in a way, common for all to know, not for any separate science' (*Rhet.*, 1. 1, 1354a1–3; similarly *Post. An.* 1. 11, 77a31).

[12] Accordingly, commentators have pointed out that Aristotle's account of dialectical deductions in the *Topics* can be said to be formal in virtue of its topic-neutrality and in virtue of the fact that its *topoi* provide general argument forms applicable to a large number of particular cases (Allen 1995: 15–16 and 69–72, Smith 1997: xxiv and xxvi, Wagner & Rapp 2004: 8, Wagner 2011: 356).

[13] For example, Cornford 1935: 264–5, Ross 1949: 29, Łukasiewicz 1957: 7–8 and 13–14, Striker 2009: xii.

> If A is a genus of B, B is a species of A, and B is predicated of C, then A is predicated of C.

If Aristotle had used this alternative notation in some places in the *Topics*, should we then consider the *Topics* as the beginning of formal logic? The answer, it seems to me, is negative, at least if by formal logic is meant a certain kind of logical theory rather than a form of notation used to express that theory. For the *topoi* expressed by the alternative notation would be exactly the same as the ones expressed by Aristotle's original formulation.

Conversely, schematic letters are not essential to Aristotle's project in the *Prior Analytics* either. For Aristotle is able to express his syllogistic moods without schematic letters by way of circumscription. For example, when introducing the moods Barbara and Celarent in *Prior Analytics* 1. 4, he first presents them by way of circumscription and only then by means of schematic letters:

> When three terms are so related to one another that the last is in the middle as in a whole and the middle either is or is not in the first as in a whole, it is necessary that there is a syllogism of the extremes… For if A is predicated of all B and B of all C, it is necessary that A is predicated of all C. (*Pr. An.* 1. 4, 25b32–9)

The circumscription of Barbara and Celarent in the first sentence of this passage is not entirely precise. But Aristotle could easily supply the missing details to make it fully precise. In fact, Aristotle could use circumscriptions instead of schematic representations everywhere throughout the *Analytics*.[14] The presentation would be lengthier and less perspicuous, but in principle nothing prevents Aristotle from doing so. Thus he could present his whole syllogistic without using any schematic letters. If he did so, should we then deny the *Prior Analytics* the status of formal logic? Again, the answer, I think, is negative. For Aristotle's theory of syllogistic moods in the three figures would be exactly the same, regardless of what notation is used to present it.

[14] Barnes 2007: 286–92 and 358.

In sum, the distinctive kind of formality exhibited by the *Prior Analytics* but not by the *Topics* cannot be attributed to its use of schematic letters. Nor can it be attributed to its being symbolic, formalized, general, or topic-neutral. The purpose of this paper is to give a more adequate account of the kind of formality in question. I begin by examining Aristotle's approach in the two treatises to indeterminate premisses, that is, to premisses which lack quantifying expressions such as 'all' and 'some' (Sections 2 and 3). In the *Topics*, such quantifying expressions may be omitted in a deduction as long as they are tacitly understood by the interlocutors. In the *Prior Analytics*, by contrast, they may not be omitted if they are relevant to an argument's counting as a deduction; for Aristotle requires that everything relevant be made explicit by *some* linguistic expression. In the syllogistic, nothing of relevance must be left to implicit understanding between speaker and hearer. In this respect, the syllogistic is akin to modern systems of formal logic such as, for example, Gottlob Frege's Begriffsschrift. Furthermore, the syllogistic is akin to modern systems of formal logic in that it aims at gapless deductions in which no premiss is missing (Section 4). Unlike the *Topics*, the *Prior Analytics* provides a criterion for determining when no premiss is missing in a given argument (Section 5). This will put us in a position to identify four essential features of formal logic that are present in the *Prior Analytics* but not in the *Topics*, such that it is correct to say that the former but not the latter marks the beginning of formal logic (Section 6).

2. Premisses and problems

In the first chapter of the *Prior Analytics*, Aristotle defines a premiss as a certain kind of *logos*:

> A premiss (πρότασις) is a *logos* affirming or denying something of something. (*Pr. An.* 1. 1, 24a16–17)

The occurrence of *logos* in this definition is often translated as 'sentence'.[15] Thus, premisses are taken to be linguistic expressions of a certain kind (rather than non-linguistic items signified by linguistic expressions). This is confirmed by Aristotle's discussion of affirmations

[15] For example, Owen 1889: 80, Smith 1989: 1, Striker 2009: 1, and Crivelli 2012: 113.

and denials in *de Interpretatione* 4 and 5.[16] There, Aristotle characterizes affirmations and denials as *logoi*, and makes it clear that these *logoi* are significant utterances (φωνὴ σημαντική, *Int.* 4, 16b26). Thus, affirmations and denials are linguistic expressions endowed with signification.[17] Since the premisses introduced in the first chapter of the *Prior Analytics* are affirmations and denials, they too are such linguistic expressions.[18] They are sentences.

Aristotle proceeds to distinguish three kinds of premisses: universal, particular, and indeterminate ones.

> A premiss ... is either universal or particular or indeterminate (ἀδιόριστος). By universal I mean belonging to all or to none; by particular, belonging to some, or not to some, or not to all; by indeterminate, belonging or not belonging without universality or particularity, as in 'Contraries are studied by the same science' or 'Pleasure is not good'. (*Pr. An.* 1. 1, 24a16–22)

Although this classification is presented as applying to premisses (προτάσεις), it applies equally to conclusions of deductions.[19] The classification concerns the presence or absence of quantifying expressions.[20] Universal premisses contain quantifying expressions such as 'all' and 'no' (as in 'All pleasure is good'). Particular premisses contain expressions such as 'some' and 'not all' (as in 'Some pleasure is good'). Indeterminate premisses differ from universal and

[16] There is good reason to think that Aristotle's definition of πρότασις at 24a16–17 relies on his discussion of affirmations and denials in *de Interpretatione* 4 and 5 (see Alexander *in Pr. An.* 10.13–12.3, Ammonius *in Pr. An.* 15.14–17.10, and Smith 1989: xvii).

[17] Accordingly, the occurrences of *logos* in *de Interpretatione* 4 and 5 are often translated as 'sentence' (e.g., Tredennick 1938: 121, Kneale & Kneale 1962: 45–6, Ackrill 1963: 45–5 and 124–5, Lear 1980: 104, Crivelli 2004: 74).

[18] See Crivelli & Charles 2011: 194, Crivelli 2012: 113–14.

[19] Crivelli & Charles (2011) argue that πρότασις in the *Prior Analytics* is intended to cover premisses and conclusions of deductions alike, and should therefore be translated as 'proposition' rather than 'premiss' (*pace* Smith 1989: 106–7, Striker 2009: 75). However this may be, it is clear that Aristotle's discussion of προτάσεις at 24a16–22 applies not only to premisses but also to conclusions of deductions (Crivelli 2012: 114 n. 5).

[20] Smith 1989: 107.

particular ones in that they do not contain any quantifying expressions.[21] For example, the sentence 'Pleasure is good' is indeterminate. It is indeterminate even if it is actually true that all pleasure is good. Likewise, 'Contraries are studied by the same science' is indeterminate, even if someone utters it with the intention of asserting that all contraries are studied by the same science. Thus, Aristotle's classification is of a linguistic nature, based on the syntactic criterion of whether or not certain expressions are present in a given sentence.

Now, Aristotle presents a similar classification at the beginning of the second book of the *Topics*, as follows:

> Of problems (τῶν προβλημάτων) some are universal, others particular. Universal problems are such as 'All pleasure is good' and 'No pleasure is good'; particular problems are such as 'Some pleasure is good' and 'Some pleasure is not good'. (*Top.* 2. 1, 108b34–109a1)

In this passage, Aristotle is concerned with what he calls 'problems'. These are theses maintained by one of the participants in a dialectical exchange, theses to be refuted or established in the debate.[22] Aristotle distinguishes between universal and particular problems. Unlike in the *Prior Analytics*, he does not mention a third class of indeterminate problems. Moreover, his classification does not seem to rely on the syntactic criteria that govern the classification in the *Prior Analytics*. It is true that Aristotle's examples in the passage just quoted contain quantifying expressions such as 'all' and 'some'. However, in the rest of the second book of the *Topics* Aristotle does not seem to attach any importance to the presence or absence of quantifying expressions. In fact, he virtually never uses such expressions in the

[21] See, e.g., Ammonius *in Pr. An.* 18.15–38, Ross 1949: 289, Łukasiewicz 1957: 4, and Striker 2009: 77.

[22] In *Topics* 1. 4, problems are characterized as questions of the form 'Is it the case that P, or not' (101b32–4). In the present passage, Aristotle departs from this characterization, using the term 'problem' instead to refer to the theses that result from an affirmative or negative answer to those questions (see Alexander *in Top.* 129.16–24 and Slomkowski 1997: 16).

second book, except in the first chapter.[23] For example, consider the following passage from the second chapter:

> Another *topos* is to examine the items to each or to none of which [a predicate] has been said to belong. Look at them species by species . . . For example, if someone has said that opposites are studied by the same science (εἰ τῶν ἀντικειμένων τὴν αὐτὴν ἐπιστήμην ἔφησεν εἶναι[24]), you must examine whether relative opposites, contraries, terms opposed as privation and possession, and contradictory terms are studied by the same science. . . . For if it is shown in any instance that the science is not the same, we shall have demolished the problem. (*Top*. 2. 2, 109b13–24)

This *topos* provides a method for refuting problems in which a predicate is said to belong to *every* (or to *no*) member of a given class of items. In Aristotle's example, the problem is indicated by the sentence 'Opposites are studied by the same science'. This sentence does not contain a quantifying expression; according to the *Prior Analytics*, it is indeterminate. Nervertheless, Aristotle uses it in the present passage to convey the universal claim that *all* opposites are studied by the same science (i.e., that for any given pair of opposites, the two members of the pair are studied by the same science). Likewise, a participant in a dialectical debate might use the sentence to convey this universal claim. The *topos* aims to refute the universal claim by setting out a specific pair of opposites that are not studied by the same science.

Aristotle clearly regards the problems targeted by this *topos* as universal in the sense introduced in chapter 2. 1.[25] Later on he refers to this *topos* by means of the phrase 'as in the

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[23] See Brunschwig 1967: lx.

[24] The translation 'Opposites are studied by the same science' reflects the fact that Aristotle takes this sentence to have a predicative structure in which 'opposites' (τῶν ἀντικειμένων) is the subject term and 'studied by the same science' (ἡ αὐτὴ ἐπιστήμη) is the predicate term; see *Pr. An.* 1. 36, 48b4–9; 2. 26, 69b8–9, Ross 1949: 289, Smith 1989: 107, Barnes 1996: 186 n. 35, and Primavesi 1996: 122–3.

[25]In chapter 2. 1, he introduces a distinction between universal and particular problems. The latter are discussed in chapter 3. 6. This suggests that chapters 2. 2–3. 5 are intended to deal with universal problems (see Alexander

case of universal problems' (καθάπερ ἐν τοῖς καθόλου προβλήμασιν, *Top*. 3. 6, 120a33–4).[26] One of the problems targeted by the *topos* is indicated by the sentence 'Opposites are studied by the same science'. The reason for regarding this problem as universal cannot be the presence of a quantifying expression in the sentence 'Opposites are studied by the same science'. There is no such expression. Rather, the reason seems to be the intended meaning of the sentence as understood by the participants in the dialectical debate. As Gisela Striker puts it, most of 'the theses discussed in the *Topics* . . . are expressed without explicit quantification, but must be understood as being universal'.[27]

The same is true for premisses in the *Topics*. Universal premisses are frequently indicated by sentences that do not contain quantifying expressions. Examples are 'The angry person desires revenge on account of an apparent slight' and 'Someone who has lost knowledge of something has forgotten it'. Aristotle takes each of these two indeterminate sentences to indicate a universal premiss (καθόλου πρότασις), conveying a universal claim about *every* angry person and about *everyone* who has lost knowledge of something.[28]

On the other hand, indeterminate sentences can also be understood as conveying not a universal but a particular claim. For example, as Aristotle explains in *Topics* 2. 4, the

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in Top. 279.12–17). Accordingly, Slomkowski (1997: 134) takes it that 'Aristotle regards the *topoi* in B 2–Γ 5 as containing universal problemata, even though they are not universally quantified by the explicit mention of quantifiers'. Whether or not this is true for all *topoi* in these chapters, the present one at 109b13–24 is clearly one of the best candidates (see Brunschwig 1967: lx, especially n. 2; Primavesi 1996: 103–13, especially 112–13). If this *topos* did not deal with universal problems, it would be difficult to find any *topos* in chapters 2. 2–3. 5 that does.

[26] It is generally agreed that 120a33–4 refers to the *topos* from 2. 2 just quoted (109b13–29); see Pacius 1597a: 625, Waitz 1846: 471, Brunschwig 1967: 78, and Slomkowski 1997: 153 n. 62.

[27] Striker 2009: 194; see also Slomkowski 1997: 134.

[28] The first example is found at *Top*. 8. 1, 156a31–2 (ὁ ὀργιζόμενος ὀρέγεται τιμωρίας διὰ φαινομένην ὀλιγωρίαν); Aristotle describes it as καθόλου πρότασις at 156a28 (see also 156a34). The second example is found at *Top*. 8. 2, 157b12–13 (τὸν ἀποβεβληκότα ἐπιστήμην ἐπιλελῆσθαι); Aristotle describes it as καθόλου at 157b3 (see also 157b10). For further examples of universal premisses formulated without explicit quantification, see Slomkowski 1997: 23. Slomkowski notes that, although 'there is no universal quantifier ('all' or 'no') explicitly stated', '[t]he universal quantifiers are of course implicitly contained in these *protaseis*' (1997: 24; see also 27).

indeterminate sentence 'Animal is winged' can be used to convey that *some*, not *all*, animals are winged. In this case, 'Animal is winged' is true, whereas 'Man is winged' is false:[29]

> It is not necessary that everything that belongs to the genus should also belong to the species; for animal is winged and fourfooted, but man is not (ζῷον μὲν γάρ ἐστι πτηνὸν καὶ τετράπουν, ἄνθρωπος δ' οὔ). But everything that belongs to the species must also belong to the genus; for if man is good, then animal also is good (εἰ γάρ ἐστιν ἄνθρωπος σπουδαῖος, καὶ ζῷόν ἐστι σπουδαῖον[30]). (*Top.* 2. 4, 111a25–9; cf. 111a20–3)

Similarly, 'Animal is not winged' can be used to convey the particular claim that *some* animals are not winged. In this case, 'Animal is winged' and 'Animal is not winged' are both true. This is what Aristotle has in mind in the following passage from the *de Interpretatione*:[31]

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[29] See Alexander *in Top.* 158.31–160.3 and Primavesi 1996: 152–4. Aristotle makes a similar point in the *Categories*: 'If you will call the individual man grammatical it follows that you will call both man and animal [i.e., the species man and the genus animal] grammatical' (*Cat.* 5, 3a4–5; cf. Perin 2007: 135)

[30] This is the Greek text printed by most editors (Bekker 1831, Waitz 1846, Forster 1960, Brunschwig 1967). On the other hand, Strache & Wallies (1923) and Ross (1958) print ἐστί with an orthotone accent on the first syllable (ζῷον μὲν γὰρ ἔστι πτηνὸν and εἰ γὰρ ἔστιν ἄνθρωπος σπουδαῖος, καὶ ζῷον ἔστι σπουδαῖον). Primavesi (1996: 153–4) argues that the latter version should be preferred because it marks an existential reading of ἐστί ('There is a good animal') as opposed to a copula reading ('Animal is good'). However, as Kahn (1973: 420–4) has shown, there is no syntactic or semantic difference between the enclitic and accented forms of ἐστί, but their distribution is largely determined by word order. Contrary to what is sometimes thought, the orthotone accent does not mark an existential reading of ἐστί. The literal translation of both ζῷόν ἐστι σπουδαῖον and ζῷον ἔστι σπουδαῖον is 'Animal is good' (see Kneale & Kneale 1962: 37). The same is true for ἔστιν ἐπιστήμη σπουδαία at *Top.* 2. 4, 111a21–2, and ἔστι λευκὸς ἄνθρωπος at *Int.* 7, 17b9–10. The literal translations of these sentences are 'Knowledge is good' (Kneale & Kneale 1962: 37) and 'Man is pale' (Ackrill 1963: 129, Whitaker 1996: 83–94, Jones 2010: 35–40, Weidemann 2012: 106 n. 5).

[31] See Whitaker 1996: 91–4, Weidemann 2002: 206–7, and Jones 2010: 42–5.

> It is true to say at the same time that man is pale and that man is not pale, or that man is noble and man is not noble (ἅμα γὰρ ἀληθές ἐστιν εἰπεῖν ὅτι ἔστιν ἄνθρωπος λευκὸς καὶ ὅτι οὐκ ἔστιν ἄνθρωπος λευκός, καὶ ἔστιν ἄνθρωπος καλὸς καὶ οὐκ ἔστιν ἄνθρωπος καλός). (*Int*. 7, 17b30–3)

One and the same indeterminate sentence can on one occasion be used to convey a universal claim and on another occasion to convey a particular claim.[32] In the former case, the problem indicated by the sentence counts as universal; in the latter case, it does not count as universal but presumably as particular. Thus, the distinction between universal and particular problems introduced in *Topics* 2. 1 is not based on the syntactic criterion of containing quantifying expressions. Rather, it is based on semantic, perhaps also pragmatic, criteria pertaining to what a sentence is understood to mean by the interlocutors on a given occasion of use.[33]

This also helps explain why Aristotle does not mention indeterminate problems in *Topics* 2. 1. Indeterminacy, as defined in *Prior Analytics* 1. 1, applies to sentences and is defined by a syntactic criterion that is not relevant to the *Topics*' semantic classification. Introducing a class of sentences lacking quantifying expressions would cut across the *Topics*' twofold classification of problems into universal and particular ones. In the *Topics*, problems indicated by indeterminate sentences may be classified as universal or as particular, depending on their intended meaning on a given occasion. As a result, there is no need for a third class of indeterminate problems.[34] Accordingly, Aristotle does not mention indeterminate problems or premisses in chapter 2. 1 or anywhere else in the *Topics*.

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[32] Kneale & Kneale 1962: 37.

[33] This suggests that, in the *Topics*, premisses and problems are not sentences like in the *Prior Analytics*, but should be identified with the intended meaning of a sentence on a given occasion of use. For our purposes, however, it is not necessary to discuss these issues further. Let us simply say that in the *Topics* a sentence *indicates* a premiss or problem, leaving open the precise nature of these premisses and problems.

[34] This is not to say that necessarily *every* indeterminate sentence, on *every* occasion of use, indicates either a universal or a particular problem (or premiss) in the *Topics*. It might be thought that indeterminate sentences can sometimes be used to indicate not a universal or particular problem, but an indeterminate one. For example,

There is one passage, in *Topics* 3. 6, in which Aristotle refers to an 'indeterminate problem' (ἀδιόριστον πρόβλημα, 120a6). However, in this passage Aristotle uses the term 'indeterminate' in a different sense to pick out a certain subclass of particular problems (chapter 3. 6 as a whole is devoted to particular problems). Specifically, he uses it to pick out those particular problems that do not determine whether or not the corresponding universal problem is true.[35] For example, the particular problem indicated by 'Some pleasure is good' is indeterminate in this sense because it does not determine whether or not the corresponding universal problem indicated by 'All pleasure is good' is true. On the other hand, the particular problem indicated by 'Some pleasure is good and some pleasure is not good' is not indeterminate in this sense, because it excludes the truth of the universal problem (120a20–4). Thus, the indeterminate problems with which Aristotle is concerned in chapter 3. 6 are of a different kind from the indeterminate premisses introduced in *Prior Analytics* 1. 1.[36]

'Pleasure is good' might be thought to indicate an indeterminate problem when it is understood by the interlocutors as conveying neither the universal claim that all pleasure is good nor the particular claim that some pleasure is good, but the indeterminate claim that pleasure is good. However, Aristotle never mentions such a semantic kind of indeterminacy in the *Topics*. Nor is it clear whether he would accept it. While he accepts syntactic indeterminacy of linguistic expressions in the *Prior Analytics*, he may have reasons to be sceptical about semantic indeterminacy of intended meanings. Even if he accepted such indeterminate problems, they would be less prominent than indeterminate sentences in the *Prior Analytics*. For most (if not all) indeterminate sentences will indicate a universal or particular problem on a given occasion of use.

[35] See Alexander *in Top*. 288.12–289.31, Maier 1900: 79–80 n. 1, and Brunschwig 1967: lix–lxi and 163–4, 1968: 16–18.

[36] Aristotle's discussion of indeterminate problems in 3. 6 begins as follows: 'On the one hand, if the problem is indeterminate, there is only one way of demolishing it, for example, if someone has said that pleasure is good or is not good (οἷον εἰ ἔφησεν ἡδονὴν ἀγαθὸν εἶναι ἢ μὴ ἀγαθὸν), and has added nothing by way of determination.' (120a6–8) Brunschwig argues that τινά should be inserted after ἔφησεν, so that the clause reads 'if someone has said that *some* pleasure is good or is not good' (Brunschwig 1967: 163–4, 1968: 16–18; followed by Crivelli 2004: 245 n. 21). According to Brunschwig, the insertion is called for because it makes clear that the passage is concerned with indeterminacy in the alternative sense just described. However, even if the passage involves the syntactically indeterminate sentence 'Pleasure is good', 'indeterminate' at 120a6 may still be used in the alternative sense. For the problem indicated by 'Pleasure is good' may be indeterminate in the alternative sense just as the problem indicated by 'Some pleasure is good' (Pacius 1597b: 392, Maier 1900: 79–80 n. 1). Moreover,

The alternative sense of 'indeterminate' just described is also found in some chapters of the *Prior Analytics*.[37] The term 'indeterminate' is ambiguous in the *Prior Analytics* between the syntactic sense defined in chapter 1. 1 and the alternative sense employed in *Topics* 3. 6. In the *Topics*, however, Aristotle only employs this latter sense. The linguistic sense and the syntactic criteria underlying it are absent from, and foreign to, the *Topics*. In the next section, we will see how these differences affect Aristotle's treatment of deduction in the two works.

3. Indeterminate premisses in deductions

In the *Prior Analytics*, Aristotle takes indeterminate sentences to be similar, perhaps equivalent, in logical force to the corresponding particular ones.[38] As a result, he does not endorse any first-figure schemata which have an indeterminate major premiss, just as he does not endorse any first-figure schemata which have a particular major premiss.[39] Accordingly,

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there is strong independent evidence that 'indeterminate' at 120a6 should be understood in the alternative sense. The phrase 'On the one hand, if the problem is indeterminate' (Ἀδιορίστου μὲν οὖν ὄντος τοῦ προβλήματος) at 120a6 corresponds to the phrase 'On the other hand, if the thesis is determinate' (διωρισμένης δὲ τῆς θέσεως οὔσης) at 120a20–1. The latter phrase clearly does not pick out the class of sentences that fail to be indeterminate in the syntactic sense introduced in *Prior Analytics* 1. 1. For, as is clear from 120a21–31, the phrase does not apply to sentences such as 'Some pleasure is good'. Instead, the phrase at 120a20–1 picks out the class of problems that fail to be indeterminate in the alternative sense (see 120a21–31). Thus, it is natural to take the corresponding phrase at 120a6 to pick out the class of problems that are indeterminate in the alternative sense. Also, Aristotle's examples at 120a8–20 are indeterminate in the alternative sense, but not in the syntactic sense.

[37] *Pr. An.* 1. 4, 26b14–16; 1. 5, 27b20–22, 27b28; 1. 6, 28b28–30, 29a6, 1. 15, 35b11; see Alexander *in Pr. An.* 66.2–18, 67.3–7, 88.6–8, 88.31–33, 105.22–26, Waitz 1844: 383, Maier 1896: 162–3, Brunschwig 1969: 13 and 19, Crivelli 2004: 245 n. 21, and Striker 2009: 98–9.

[38] It is generally thought that Aristotle took indeterminate sentences to be equivalent to particular ones; see Alexander *in Pr. An.* 30.29–31, 49.15, 62.24, 111.30–112.2, 267.2, Philoponus *in Pr. An.* 79.4–5, 252.35, Philoponus *in Post. An.* 296.10–11, Waitz 1844: 369, Bocheński 1951: 43, Kneale & Kneale 1962: 55, Ackrill 1963: 129, Thom 1981: 19, and Barnes 1990: 87 and 2007: 141. It should be noted, however, that Aristotle does not explicitly assert the equivalence (Barnes 2002: 107), and that there is room for scepticism (see Whitaker 1996: 86–7).

[39] *Pr. An.* 1. 4, 26a30–9 and 26b21–5; see Alexander *in Pr. An.* 51.30–1.

Aristotle rejects concrete first-figure arguments that have an indeterminate major premiss. For example, at the beginning of chapter 1. 24 he writes:

> Moreover, in every deduction one of the terms must be affirmative, and universality must be present. . . . For let it be proposed to show that musical pleasure is good. Then, if someone should claim that pleasure is good without adding 'all', there will not be a deduction.[40] (*Pr. An.* 1. 24, 41b6–11)

In this passage, Aristotle considers someone who wants to deduce the conclusion 'Musical pleasure is good'. She states the premiss 'Pleasure is good' without adding a universally quantifying expression such as 'all' (μὴ προσθεὶς τὸ πᾶσαν). Thus, she puts forward the following argument:

A1	Pleasure is good.	(major premiss)
	Musical pleasure is pleasure.	(minor premiss)
	Therefore, musical pleasure is good.	(conclusion)

Although this argument fits the pattern of the first figure, Aristotle contends that it fails to be a deduction because the major premiss lacks a quantifying expression. Thus, he denies that A1 is a deduction on the grounds that its major premiss is indeterminate.[41] The implication is that

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[40] ἔτι τε ἐν ἅπαντι δεῖ κατηγορικόν τινα τῶν ὅρων εἶναι καὶ τὸ καθόλου ὑπάρχειν· . . . κείσθω γὰρ τὴν μουσικὴν ἡδονὴν εἶναι σπουδαίαν. εἰ μὲν οὖν ἀξιώσειεν ἡδονὴν εἶναι σπουδαίαν μὴ προσθεὶς τὸ πᾶσαν, οὐκ ἔσται συλλογισμός. I follow Tredennick (1938: 323) in translating τὸ καθόλου in this passage as 'universality' (just as ἄνευ τοῦ καθόλου at 41b7 and at 1. 1, 24a20, is translated as 'without universality'; cf. n. 41 below).

[41] See Alexander *in Pr. An.* 266.32–267.5, Philoponus *in Pr. An.* 252.31–5, Pacius 1597b: 155–6, Waitz 1844: 434, Mendell 1998: 185, and Striker 2009: 179. It seems clear that Aristotle regards the major premiss of A1 as indeterminate in the sense defined in chapter 1. 1 (24a19–22). For one thing, this major premiss is very similar to one of his examples of indeterminate premisses in 1. 1, 'Pleasure is not good' (24a21–2). Moreover, Aristotle characterizes the major premiss of A1 as being 'without universality' (ἄνευ τοῦ καθόλου, 41b7); the same phrase (ἄνευ τοῦ καθόλου) is used in 1. 1 to characterize indeterminate premisses (24a20).

if the indeterminate major premiss is replaced by the universal affirmative sentence 'All pleasure is good', the resulting argument will count as a deduction.[42]

Given that A1 does not count as a deduction in the *Prior Analytics*, A2 will not count as a deduction either since its major premiss is indeterminate:

A2	Opposites are studied by the same science.	(major premiss)
	Contraries are opposites.	(minor premiss)
	Therefore, contraries are studied by the same science.	(conclusion)

In the *Topics*, by contrast, Aristotle accepts A2 as a deduction:

> If you wish to secure an admission that contraries are studied by the same science, you should make the claim not for contraries but for opposites; for, if this is granted, it will then be deduced (συλλογιεῖται) that contraries are studied by the same science, since contraries are opposites. (*Top.* 8. 1, 155b30–4)

In this passage, Aristotle states that the conclusion of A2 is deduced (συλλογιεῖται) from the two premisses. He goes on to describe A2 as a case of establishing a conclusion 'through a deduction' (διὰ συλλογισμοῦ and συλλογισμῷ, 155b35 and b37). Thus he regards A2 as a deduction even though the major premiss lacks a quantifying expression.[43] Presumably, he regards it as a deduction because the major premiss is understood as conveying the universal claim that all opposites are studied by the same science. In this case, as we have seen, the premiss counts as universal in the *Topics*. In the *Prior Analytics*, by contrast, the premiss does not count as universal but as indeterminate, and A2 does not count as a deduction.

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[42] The resulting argument will have a universal affirmative major premiss, and an indeterminate affirmative minor premiss and conclusion. Aristotle accepts such arguments as deductions. He can be taken to assert their validity at *Pr. An.* 1. 4, 26a28–30, and 1. 7, 29a27–9; see Alexander *in Pr. An.* 51.24–30. Given that indeterminate sentences are equivalent to the corresponding particular ones (see n. 38), the validity of such arguments follows from the validity of Darii.

[43] See Slomkowski 1997: 26–7.

In order to gain a better understanding of Aristotle's treatment of indeterminate premisses, it is helpful to consider his discussion of indeterminate sentences in *de Interpretatione* 7. There, Aristotle calls them sentences 'stating of a universal not universally' (17b6–12). They state something 'of a universal' because, in the *de Interpretatione*, Aristotle requires that their subject be a general term such as 'man' as opposed to a singular term such as 'Callias'. They state it 'not universally' because they lack any quantifying expressions. Aristotle compares them with universal sentences, which contain universally quantifying expressions. Universal affirmative sentences are contrary to the corresponding universal negative ones; for example, 'Every man is pale' is contrary to 'No man is pale' (17b3–6).[44] This is not the case for indeterminate sentences:

> But when one states something of a universal but not universally, the sentences are not contrary, though what is being conveyed may be contrary (τὰ μέντοι δηλούμενα ἔστιν εἶναι ἐναντία); examples of what I mean by 'stating of a universal not universally' are 'Man is pale' and 'Man is not pale'. (*Int*. 7, 17b6–10)

Indeterminate affirmative sentences are not contrary to the corresponding indeterminate negative ones. 'Man is pale' is not contrary to 'Man is not pale'. Aristotle adds, however, that 'what is being conveyed may be contrary'. What he seems to have in mind is that an indeterminate sentence, as Ackrill puts it, 'may on occasion be intended universally' (Ackrill 1963: 129). A speaker may use 'Man is pale' and 'Man is not pale' to convey the same thing she would convey by using the corresponding universal sentences 'Every man is pale' and 'No man is pale', respectively. When a pair of indeterminate sentences is used in this way, what is conveyed by them (τὰ δηλούμενα) is contrary.[45] Still, the sentences themselves, unlike the

[44] 'Every man is pale' and 'No man is pale' are a pair of ἐναντίαι ἀποφάνσεις (17b4–5). In the *de Interpretatione*, ἀποφάνσεις are taken to be λόγοι, that is, sentences (see Kneale & Kneale 1962: 45–6, Ackrill 1963: 124–5, Lear 1980: 104, Crivelli 2004: 86–7; cf. n. 17 above). Thus, Aristotle holds that the sentence 'Every man is pale' is contrary to the sentence 'No man is pale'.

[45] See Jones 2010: 42–5; cf. Whitaker 1996: 84–5.

universal sentences, are not contrary. After all, as we have seen, indeterminate sentences may also be used to convey the same thing as particular ones. When used in this latter way, 'Man is pale' and 'Man is not pale' are both true (17b29–34). As Aristotle points out, saying that both of these sentences are true

> might seem absurd at first sight, because 'Man is not pale' looks as if it signifies (σημαίνειν) also at the same time that no man is pale. This, however, does not signify the same (οὔτε ταὐτὸν σημαίνει), nor does it necessarily hold at the same time. (*Int*. 7, 17b34–7)

Someone might think that 'Man is not pale' signifies the same thing as 'No man is pale'. But this, Aristotle insists, is not correct. The two sentences do not signify (σημαίνειν) the same, even when a speaker uses them to convey the same thing. What *the sentences* mean considered in themselves is not the same; what *a speaker* means by uttering them on a given occasion may be the same.[46] In other words, their literal meaning is not the same; but their intended meaning, or speaker meaning, on a given occasion of use may be the same.

In the *Topics*, Aristotle regards A2 as a deduction on the grounds that the indeterminate sentence indicating its major premiss, 'Opposites are studied by the same science', is used to convey a universal claim. In doing so, he attends to the speaker meaning of the sentence rather than to its literal meaning. Now, the *Topics* is a treatise on dialectic. Dialectic, for Aristotle, is 'directed at someone else' (πρός ἕτερον; *Top.* 8. 1, 155b10 and b27). That is to say, dialectical activity essentially consists in argumentation between two adversaries opposing one another.[47] Whenever one of the interlocutors utters a sentence in the dialectical exchange, her utterance is directed at the opponent. In such a setting, what is important is not so much the literal meaning of the sentence, but its speaker meaning. Speaker meaning often goes beyond literal meaning and is, at least in part, determined by the speaker's intentions. Given that speaker and hearer are interacting cooperatively in their

[46] See Jones 2010: 42–5.

[47] See Bolton 1994: 102–3 and Primavesi 1996: 35–7.

communication, the hearer will typically be able to understand the intended speaker meaning.[48] For example, the hearer will typically understand that the indeterminate sentence 'Dogs are mammals' is intended to convey a universal claim, whereas the indeterminate sentence 'Dogs are running down the street' is intended to convey a particular claim.

In interpreting each other's utterances, both participants in a dialectical exchange tacitly rely on a body of shared knowledge and contextual information, along with general pragmatic principles governing cooperative communication. As a result, the precise linguistic formulation of premisses and conclusions, the presence or absence of certain expressions in them, is not essential in the *Topics*. For example, in order for A2 to count as a deduction, it is important that the major premiss be understood as conveying a universal claim, but the universality need not be directly expressed in the sentence by a quantifying expression. As long as it is tacitly understood by the interlocutors, the major premiss counts as universal and A2 counts as a deduction in virtue of the intended speaker meaning.

In the *Prior Analytics*, too, Aristotle has to take into account the meaning of the expressions occurring in an argument, since his syllogistic is not formalistic (see Section 1 above). However, unlike in the *Topics*, it is not their speaker meaning that is relevant but only their literal meaning. Moreover, it is not the literal meaning of all expressions occuring in the argument that is relevant. By using schematic letters in place of concrete subject and predicate terms, Aristotle makes it clear that the specific meaning of these terms is not relevant to an argument's counting as a deduction.[49] It is only the meaning of some expressions that is

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[48] As Aristotle explains in *Topics* 8. 11 (161a37–b5), dialectical argumentation, unlike eristic or agonistic argumentation, is a cooperative enterprise (see κοινόν ἔργον at 161a37–8; cf. Owen 1968: 106–8). Aristotle writes (161b2–5): 'The person who questions eristically is a poor dialectician (φαύλως διαλέγεται), and so is the answerer who will not grant what is evident or will not understand what it is that the questioner intends to get (μηδ' ἐκδεχόμενος ὅ τί ποτε βούλεται ὁ ἐρωτῶν πυθέσθαι).' According to this passage, someone who refuses to understand the opponent's utterances in the way they are obviously intended is a poor dialectician (Smith 1997: 140; cf. also *Top.* 1. 18, 108a24–6). If the opponent refuses to cooperate and behaves peevishly (δυσκολαίνει), the argumentation is not dialectical but agonistic (*Top.* 8. 11, 161a23–4). Agonistic and eristic arguments are discussed not in the *Topics* but in the *Sophistici Elenchi* (see *Soph. El.* 2, 165b8–11).

[49] See Alexander *in Pr. An.* 53.28–54.2, 379.14–380.27, Philoponus *in Pr. An.* 46.25–47.9.

important, for example, of expressions such as 'is predicated of all', 'belongs to no', and of the various quantifying expressions occurring in them.[50] If an argument's being a deduction depends on the meaning of such an expression, the expression cannot be omitted. Aristotle is not formalistic and does not dictate exactly *which* expressions to use; but he does require that *some* suitable expression be present. For example, if an argument's being a deduction depends on the universality expressed by 'all' in one of its premisses, then this or an equivalent universally quantifying expression (e.g., 'whole') needs to be present in the premiss. Otherwise the argument will not count as a deduction, even if the universality is tacitly understood by the interlocutors.

This is confirmed by the fact that Aristotle is concerned not to omit any quantifying expressions in the assertoric syllogistic (*Prior Analytics* 1. 1–2 and 4–7). Whenever such an expression is required, he makes it explicit. In the whole assertoric syllogistic, there are only two occurrences of the verb 'belong' (ὑπάρχειν) not accompanied by a quantifying expression (1. 6, 28a25 and 28b21).[51] Both of them occur within a special kind of context (in proofs by ecthesis) which does not require the presence of a quantifying expression.[52] Thus, Aristotle is very careful about the formulation of premisses and conclusions, making sure that not a single quantifying expression is omitted in the assertoric syllogistic.[53]

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[50] In the opening sentence of the *Prior Analytics*, Aristotle promises to elucidate the meaning of 'is predicated of all' and 'is predicated of none' (τί λέγομεν τὸ κατὰ παντὸς ἢ μηδενὸς κατηγορεῖσθαι, *Pr. An.* 1. 1, 24a14–15). This elucidation, which is known as the *dictum de omni et de nullo*, is given at the end of the first chapter (24b28–30). Aristotle appeals to this *dictum* in chapter 1. 4 to justify the validity of his perfect first-figure schemata (25b39–40, 26a24, 26a27; see Alexander *in Pr. An.* 54.9–11, 55.1–3, 61.3–5, 69.14–20, Smith 1989: 111, Byrne 1997: 45–6, Ebert & Nortmann 2007: 292 and 302, Barnes 2007: 392–4, and Striker 2009: 83–4). These schemata are used in chapters 1. 5–6 to establish the validity of the various second- and third-figure schemata in the assertoric syllogistic. Thus, the validity of all these schemata (and of the concrete arguments instantiating them) ultimately depends on the meaning of expressions such as 'is predicated of all' and 'is predicated of none'.

[51] Smith 1982: 119.

[52] For varying accounts of why quantifying expressions are not required in these two passages, see Alexander *in Pr. An.* 100.9–14, Smith 1982: 119–20, and Malink 2008: 524–30.

[53] It should be noted that this is not true for the modal syllogistic (*Pr. An.* 1. 3 and 8–22). There, Aristotle occasionally omits quantifying expressions where they should be used (e.g., 1. 10, 30b11, 34b3–4, 1. 15, 34b23, 1.

This exactness on Aristotle's part is significant because his style of writing is generally not pedantic. Aristotle is known for flexibility of expression and frequent use of allusive and elliptical prose. He thinks that 'we must not seek the same degree of exactness (ἀκρίβειαν) in all areas, but only such as fit the subject matter and is proper to the investigation (τῇ μεθόδῳ)' (*NE* 1. 7, 1098a26–9). To seek more exactness than is required in a given context is a waste of time (ἀργόν, *Pol.* 7. 12, 1331b18–19).[54] For example, when Aristotle introduces various kinds of deduction in the first chapter of the *Topics*, he does not deem it necessary to give an exact account of them because such an account is not required for the kind of investigation pursued in the *Topics*:

> Let the above then be a description in outline of the different kinds of deduction (εἴδη τῶν συλλογισμῶν). . . . It is not our intention to give the exact account (τὸν ἀκριβῆ λόγον) of any of them. What we want to do instead is to describe them in outline (τύπῳ), since we deem it fully sufficient, for the purposes of the present investigation (μέθοδον), to be able to recognize each of them in some way or other. (*Top.* 1. 1, 101a18–24)

19, 38b9–10, 1. 22, 40a27; cf. also 1. 9, 30a18–20 and 1. 15, 34a19–21). However, this looseness is harmless. Once Aristotle has made clear the exact mode of expression in the assertoric syllogistic, he can take a more flexible approach in the modal syllogistic, especially since his focus there is on a number of new complexities generated by modal expressions that are not present in the assertoric syllogistic. There is good reason to think that the modal syllogistic is a later insertion added by Aristotle after most of *Prior Analytics* 1 was completed (Bocheński 1956: 50–1, Łukasiewicz 1957: 131 n. 1, Corcoran 1974: 88 and 120, Striker 2009: 108, Ebert & Nortmann 2007: 110). If this is correct, then chapter 1. 24, in which – as we have seen – Aristotle emphasizes that quantifying expressions must not be omitted (41b6–11), originally came just after the assertoric syllogistic, in which Aristotle does not omit any quantifying expressions.

[54] See Anagnostopoulos 1994: 131–40. When some degree of inexactness is appropriate for the purposes of a given investigation, Aristotle tends 'to view exactness as something toilsome and as something that reflects the kind of pettiness or meanness that he elsewhere associates with the behavior of the illiberal person' (Anagnostopoulos 1994: 126).

While the *Topics* does not give an exact account of deduction, the *Prior Analytics* is intended to provide such an account.[55] There, unlike in the *Topics*, Aristotle carefully attends to the exact placement of quantifying expressions. Presumably he would not have done so if he did not deem it necessary. Aristotle does not explain why it is necessary. But the most natural explanation is that, for the purposes of the investigation undertaken in the *Prior Analytics*, everything in the premisses and conclusion that is relevant to the argument's counting as a deduction needs to be made explicit by some linguistic expression, and that nothing of relevance should be left to tacit understanding between speaker and hearer.

If this is Aristotle's rationale, it coincides with basic commitments of modern formal logic as it has developed since the end of the 19th century. For example, Gottlob Frege – one of the founding fathers of modern logic – describes the system introduced in his *Begriffsschrift* as follows:

> All those pecularities of ordinary language that result only from the interaction of speaker and hearer – as when, for example, the speaker takes the expectations of the hearer into account and seeks to put them on the right track even before the sentence is uttered – have nothing that answers to them in my formula language, since in a judgment I consider only that which influences its *possible consequences.* Everything necessary for a correct inference is expressed in full . . .; *nothing is left to guesswork.*[56] (Frege, *Begriffsschrift*, 1879: §3)

This view is widely shared by contemporary logicians and is representative of all systems of formal logic ever since Frege. If I am correct, the passage just quoted applies equally to Aristotle's syllogistic in the *Prior Analytics* (except that the syllogistic should not be described

[55] See Alexander *in Top.* 26.13–19; similarly, Smith 1997: 51.

[56] Similarly, Frege writes in his 1914 essay 'Logic in Mathematics': 'The sentences of our everyday language leave a good deal to guesswork. It is the surrounding circumstances that enable us to make the right guess. The sentence I utter does not always contain everything that is necessary . . . But a language that is intended for scientific employment must not leave anything to guesswork.' (Frege 1979: 213)

as a 'formula language' since it is neither symbolic nor formalistic). To be clear, Aristotle's syllogistic differs fundamentally from modern systems of formal logic such as Frege's Begriffsschrift, and should not be assimilated to them. For example, it differs from them in matters of logical syntax, expressive power, and in the kinds of inferences they approve of.[57] Despite these differences, however, they share a common objective that often goes unnoticed: to abstract from speaker meaning and not to 'leave anything to guesswork' in their respective accounts of deductive inference. As we will see in Sections 4 and 5, they share another important objective in that they aim at gapless deductions in which no premisses are missing.

4. Missing premisses

In the first chapter of the *Topics*, Aristotle defines a deduction as a kind of argument in which a conclusion follows necessarily from given premisses 'through' these premisses:

> A deduction is an argument in which, certain things having been supposed, something different from the things supposed results of necessity through the things supposed (διά τῶν κειμένων). (*Top.* 1. 1, 100a25–7)

This definition is repeated almost verbatim in the first chapter of the *Prior Analytics* (24b18–20).[58] There, the phrase 'through the things supposed' (διά τῶν κειμένων) is replaced by 'in virtue of these things being so' (τῷ ταῦτα εἶναι, 24b20). This latter phrase, Aristotle explains, means that the conclusion results 'because of these things' (διά ταῦτα, 24b20–1). Aristotle uses

[57] In fact, Frege regards it as a distinctive achievement of his *Begriffsschrift* that it departs further from Aristotle's logical syntax than its predecessors (Frege 1979: 15, see also 1879: vii and 2–4; for a discussion of these differences, see Barnes 1996: 175–8 1 and 196–7; 2007: 93–113). Accordingly, it is sometimes thought that 'Post-Aristotelian logic begins only with Frege' (Sluga 1980: 65), and that 'not until the twentieth century was [Aristotelian logic] finally supplanted as a result of the work of Frege and his successors' (Smith 1995: 27; similarly Beaney 1996: 37).

[58] Similar definitions are given in *Soph. El.* 1, 164b27–165a2 and *Rhet.* 1. 2, 1356b16–18.

these three phrases interchangeably and seems to regard them as mutually equivalent.[59] Thus, he gives the same definition of deduction in the *Topics* and the *Prior Analytics*.[60] In both treatises, the definition includes the condition that the conclusion follow through, or because of, or in virtue of the premisses. Let us call this the 'causal condition'.

Nevertheless, there are differences in how the causal condition is put to use in the two treatises. In the *Topics*, the most striking use Aristotle makes of the condition is to exclude arguments which contain superfluous premisses which are not necessary to deduce the conclusion (8. 11, 161b28–30).[61] In the *Prior Analytics*, by contrast, he invokes it to exclude arguments in which premisses are missing. In the first chapter of the *Prior Analytics*, he elucidates the causal condition by stating that it is tantamount to the condition of 'needing no further term from outside in order for the necessity to come about' (24b20–2). Although Aristotle speaks of terms that are missing, it is generally agreed that he has in mind missing premisses.[62] His point is that all premisses necessary to deduce the conclusion must be present in the argument. No premiss must be missing. In the *Topics*, however, Aristotle does not elucidate the causal condition in this way, nor does he invoke the condition to exclude arguments in which premisses are missing (see Section 5 below).

Although the requirement that no premiss be missing in a deduction may seem obvious almost to the point of triviality, it raises a substantive question: How to determine whether or not a premiss is missing in a given argument in order for it to count as a deduction (*syllogismos*)? Aristotle addresses this question in *Prior Analytics* 1. 32. He points out that in some cases it is not easy to recognize whether a premiss is missing. It is especially difficult, he

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[59] See Barnes 1981: 23–4 n. 9, Rapp 2002: 63 and 164. In *Topics* 8. 11 (161b28–30), Aristotle takes the condition expressed by the phrase τῷ ταῦτα εἶναι to be part of the definition of deduction (see also *Rhet.* 1. 2, 1356b17, and *Soph. El.* 6, 168b24). Thus, Aristotle seems to regard this phrase as equivalent to the one used in the definition of deduction in *Topics* 1. 1 (διά τῶν κειμένων).

[60] See Barnes 1981: 23–4 n. 9, Bolton 1994: 116, Primavesi 1996: 59 n. 2, Rapp 2000: 17-20; similarly Striker 2009: 79–81.

[61] See Alexander *in Top.* 2. 2, 13.28–14.2 and 568.18–23, Frede 1974a: 22, Barnes 1980: 168–9, Allen 1995: 189, Smith 1997: 143, Mignucci 2002: 251, Ebert & Nortmann 2007: 227–8, and Striker 2009: 81–2.

[62] Frede 1974a: 22, Ebert & Nortmann 2007: 227, Striker 2009: 81, Keyt 2009: 36, Di Lascio 2014: 274.

explains, in arguments in which the conclusion follows necessarily from the premisses although one or more premisses are missing (47a22–35).[63] Such arguments do not count as deductions; but since the conclusion is a necessary consequence of the premisses, it is easy to be misled into thinking that no premiss is missing in them and that they are deductions. Aristotle gives the following example of such an argument (47a24–8):

A3 A substance is not destroyed by the destruction of what is not a substance.
If the things out of which something is composed are destroyed, then what consists of them must also perish.
Therefore, any part of a substance is a substance.

He comments on this argument as follows:

> When these [i.e., the two premisses of A3] have been assumed, it is necessary that any part of a substance be a substance; yet it has not been deduced through the things assumed, but premisses are missing (οὐ μὴν συλλελόγισται διὰ τῶν εἰλημμένων, ἀλλ' ἐλλείπουσι[64] προτάσεις). (*Pr. An.* 1. 32, 47a26–8)

Although the conclusion follows necessarily from the premisses, A3 fails to be a deduction because one or more premisses are missing in it (47a31–5). Since it is not a deduction, A3

[63] For such arguments, see Alexander *in Pr. An.* 21.28–30, 344.9–345.12, 346.27–8, Philoponus *in Pr. An.* 320.16–322.18, 323.18–27, Frede 1974a: 20–3, Mignucci 2002: 248–56.

[64] In this passage, ἐλλείπειν is often taken to mean 'be missing' (Pacius 1597a: 262, Mueller 2006: 30, Ebert & Nortmann 2007: 78, Striker 2009: 52). For this meaning of ἐλλείπειν, see Bonitz, *Index Arist.* 238b5–11. Accordingly, the phrase ἐλλείπουσι προτάσεις at 47a28 is sometimes translated as 'premisses have been left out' (von Kirchmann 1877: 75, Smith 1989: 51). This is supported by the parallel phrase 'one of the necessary premisses has been left out' (τι τῶν ἀναγκαίων παραλέλειπται) at 47a19.

must violate one of the conditions laid down in Aristotle's definition of deduction. Presumably, it violates the causal condition.[65]

Aristotle does not specify which premiss(es) is (are) missing in A3. Alexander suggests that it is a premiss such as 'A whole is composed of its parts'.[66] In any case, whichever premiss or premisses are missing, Aristotle makes it is clear that in order to turn A3 into a deduction these premisses must be added:

> Sometimes people propose a universal premiss but do not assume the premiss that is contained in it, either in writing or in speech (οὔτε γράφοντες οὔτ' ἐρωτῶντες[67]). Or they . . . instead ask for other useless things. So we must see whether something superfluous has been assumed and whether one of the necessary premisses has been left out (τι τῶν ἀναγκαίων παραλέλειπται); and the one should be posited and the other taken away. (*Pr. An.* 1. 32, 47a14–20)

When people put forward arguments, they sometimes omit premisses that are necessary to deduce the conclusion. The premisses may be tacitly understood by the interlocutors, but they are not made explicit 'either in writing or in speech'. Such arguments violate the causal condition. For the causal condition, as explained in the first chapter of the *Prior Analytics*, requires that all necessary premisses be expressly stated. Or, as Michael Frede puts it, the condition requires that 'all the assumptions on which the inference is based have been made explicit' (Frede 1974a: 22).[68]

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[65] Mignucci 2002: 254–6. The causal condition requires that the conclusion follow 'through the things supposed' (διά τῶν κειμένων, *Top.* 1. 1, 100a26–7; *Soph. El.* 1, 165a2), whereas Aristotle denies that the conclusion of A3 has been deduced 'through the things assumed' (διὰ τῶν εἰλημμένων, *Pr. An.* 1. 32, 47a27–8).

[66] Alexander *in Pr. An.* 347.5–7. For alternative suggestions, see Pacius 1597a: 262, Ebert & Nortmann 2007: 800–5, and Striker 2009: 214. Alexander, Pacius, and Ebert & Nortmann argue that one must not only add a premiss in A3, but also transform the two premisses already present in it.

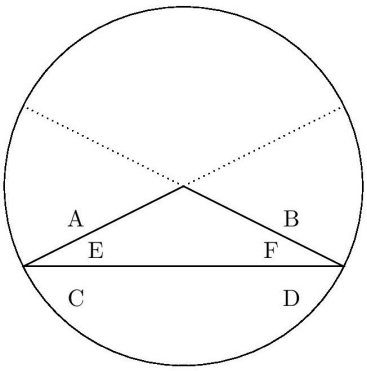
[67] I follow Smith (1989: 51) and Ebert & Nortmann (2007: 78) in rendering ἐρωτῶντες as 'in speech'.

[68] Likewise, Keyt (2009: 36) takes the causal condition to require that 'the conclusion of a syllogism must follow from explicit, rather than tacit or suppressed, premisses'. Similarly, Mignucci 2002: 250–2 and Striker 2009: 81.

When Aristotle says that people sometimes fail to make premisses explicit 'in speech' (ἐρωτῶντες), he appears to be referring to dialectical debates between two interlocutors. It is less clear what he is referring to when he says that they sometimes fail to do so 'in writing' (γράφοντες). Pacius (1597b: 185) thinks that Aristotle primarily has in mind mathematical texts. As Striker (2009: 214) points out, this is supported by Aristotle's discussion of a mathematical example in *Prior Analytics* 1. 24. In this chapter, as we have seen, Aristotle argues that every deduction must contain a universal premiss. He first illustrates this claim by A1, which fails to be a deduction because the major premiss is not universal but indeterminate (41b6–11). He then proceeds to illustrate the claim by a series of mathematical arguments which will not count as deductions unless they contain an explicit universal premiss:

> This becomes more evident in geometrical proofs, for example, the proof that the base angles of an isosceles triangle are equal. Let the lines A and B be drawn to the center. Then, if one should assume that angle AC is equal to angle BD without asserting generally that the angles of semicircles are equal, and again if one should assume that C is equal to D without making the additional assumption that this holds for every angle of a segment, and if one should then, lastly, assume that, the whole angles being equal and the angles subtracted being equal, the remaining angles E and F are equal – he will beg the question [and hence fail to give a deduction[69]], unless he assumes [generally] that equal things remain when equals are subtracted from equals. (*Pr. An.* 1. 24, 41b13–22)

[69] While it is not entirely clear why failure to assume the appropriate universal premiss will result in begging the question (*petitio principii*), it is clear that, for Aristotle, such arguments will fail to be deductions (Philoponus *in Pr. An.* 254.12–23). In Aristotle's view, an argument that begs the question is not a deduction since his definition of deduction requires that the conclusion be distinct from any of the premisses (ἕτερόν τι τῶν κειμένων, *Pr. An.* 1. 1, 24b19; *Top.* 1. 1, 100a25–6); see *Soph. El.* 5, 167a25–6; 6, 168b25–6; Alexander *in Pr. An.* 18.12–20.29, Ammonius *in Pr. An.* 27.34–28.20, Frede 1974a: 20–21, Bolton 1994: 110–12, Barnes 2007: 487–90, Striker 2009: 80, Crivelli 2012: 125, and Di Lascio 2014: 274.



In this passage, Aristotle discusses a proof to the effect that the base angles of an isosceles triangle, E and F, are equal. Both angles are parts of the larger angles AC and BD, respectively. These are 'mixed' angles formed by the lines A and B and the curve of the circle. The base angles result from substracting the mixed angles C and D from the larger mixed angles. Thus, Aristotle infers the equality of the base angles from the equality of AC and BD and the equality of C and D.[70] He does so by means of three deductions which can be represented as follows:[71]

A4 All angles of semicircles are equal.
AC and BD are angles of semicircles.
Therefore, AC and BD are equal.

A5 All angles of the same segment of a circle are equal.
C and D are angles of the same segment of a circle.
Therefore, C and D are equal.

A6 All remainders of subtracting equals from equals are equal.
E and F are remainders of subtracting equals from equals.

[70] For the details of this proof, see Alexander *in Pr. An.* 268.6–269.15, Philoponus *in Pr. An.* 253.28–254.12, and Ross 1949: 374–6. While mixed angles play only a marginal role in Euclid, they were probably more prominent in pre-Euclidean geometry (see Heath 1921: 338–9 and 381–2; Mueller 1981: 187).

[71] See McKirahan 1992: 157–8 and Ebert & Nortmann 2007: 749–50.

Therefore, E and F are equal.

Aristotle takes these deductions to be instances of a valid schema in the first figure.[72] Each of them has a universal affirmative major premiss. If this premiss is omitted, the resulting arguments will not be deductions.[73] Thus, Aristotle denies that A7–9 are deductions:

A7 AC and BD are angles of semicircles.
Therefore, AC and BD are equal.

A8 C and D are angles of the same segment of a circle.
Therefore, C and D are equal.

A9 E and F are remainders of subtracting equals from equals.
Therefore, E and F are equal.

The major premisses that are missing in A7 and A8 are theorems of geometry. The one missing in A9 is an axiom (that is, an unproved principle common to more than one science).[74] It is one of the common notions listed by Euclid at the beginning of the *Elements*. As such, the missing premisses are true of necessity. Accordingly, the conclusions of A7–9 follow necessarily from the remaining premiss. Yet, for Aristotle, A7–9 are not deductions just as A3 is not a deduction. If someone puts forward, in writing or speech, an argument such as A9, she has failed to produce a deduction. For example, consider the following argument put forward by Euclid in his proof that the base angles of an isosceles triangle are equal:

[72] Specifically, they can be viewed as instances of the schema described in n. 42 above.

[73] See Philoponus *in Pr. An.* 254.12–23 and n. 69 above.

[74] *Post. An.* 1. 2, 72a14–18; 1. 10, 76a37–41, 76b20–1; 1. 11, 77a29–31. See Barnes 1994: 99–100 and 138–9.

> Since the whole angle ABG was proved equal to the angle ACF, and in these the angle CBG is equal to the angle BCF, the remaining angle ABC is equal to the remaining angle ACB. (Euclid, *Elements* 1. 5, 32–5)

This argument is parallel to A9. The first two clauses in Euclid's text correspond to the premiss of A9, the last clause corresponds to the conclusion. Euclid does not mention a universal premiss to the effect that all remainders of subtracting equals from equals are equal. Of course, he mentioned it earlier as one of his common notions and is tacitly relying on it here. Nevertheless, he does not express it in writing in the passage just quoted. Thus, Aristotle will insist that Euclid has failed to produce a deduction. Since proofs are a kind of deduction, Euclid has failed to give a proof of his theorem.

A similar criticism of Euclid is articulated by Gottlob Frege. In his essay 'On the scientific justification of a Begriffsschrift' (1882), Frege writes:

> Even such a conscientious and rigorous writer as Euclid often makes tacit use of premisses to be found neither among his axioms and postulates nor among the premisses of the particular theorem being proved. For instance, in proving the 19th theorem of the first book of the *Elements* . . ., he makes tacit use of the statements:
>
> (1) If a line is not larger than another line, then it is equal to the other or smaller than it.
>
> (2) If an angle is equal to another angle, then it is not larger than the other.
>
> (3) If an angle is smaller than another angle, then it is not larger than the other.
>
> (Frege 1964: 157)

Frege is referring to passages such as the following:

> AC is not smaller than AB. And it was proved that it is not equal either. Therefore AC is larger than AB. (Euclid, *Elements* 1. 19, 13–15)

Euclid's argument in this passage tacitly relies on a missing premiss, namely, Frege's (1). Thus, the argument is deficient in much the same way as the earlier argument corresponding to A9.[75] In both Aristotle's and Frege's view, some of Euclid's arguments are deficient because he fails to make explicit all the premisses on which they rely. His arguments have gaps in that certain premisses are missing in them. In order to detect and avoid such deficiencies, Frege introduces his Begriffsschrift:

> It was important to keep the chain of inferences free of gaps. In attempting to comply with this requirement in the strictest possible way I found the inadequacy of language to be an obstacle. . . . This deficiency led me to the idea of the present Begriffsschrift. Its first purpose, therefore, is to provide us with the most reliable test of the validity of a chain of inferences and to point out every assumption that tries to sneak in unnoticed, so that its origin can be investigated.[76] (Frege, *Begriffsschrift*, 1879: iv)

Frege's Begriffsschrift is primarily a tool for detecting gaps, or missing premisses, in deductive arguments. For example, if Euclid's argument quoted above is translated into Begriffsschrift, it becomes clear that a premiss is missing in it.[77] Similarly, Aristotle uses his syllogistic as a tool for detecting missing premisses. When Aristotle denies in *Prior Analytics* 1. 24 that arguments such as A7–9 are deductions even though the conclusion follows necessarily from the premiss(es), he does so on the basis of his syllogistic; for none of these arguments contains a universal premiss, whereas the syllogistic requires that every deduction contain a universal premiss. Thus, James Allen writes:

> Aristotle conceived the categorical syllogistic as, among other things, a way of bringing out and making explicit the often unstated premisses because of which the conclusion

[75] Except that in the present argument, the missing premiss is not mentioned by Euclid as one of his common notions.

[76] Similarly, Frege 1884: §91 and 1893: vi–vii.

[77] See Weiner 1990: 52–3 and 67; 2010: 43–4.

> of a syllogism follows of necessity. . . . The analysis of arguments with the aid of the categorical syllogistic uncovers assumptions on which they depend that often go unnoticed and unsaid. (Allen 2001: 24)

Aristotle's syllogistic provides a standard for determining whether or not premisses are missing in a given deductive argument. As we will see in the next section, this is a distinctive achievement of the *Prior Analytics* that sets it apart from the *Topics*.

5. Gapless deductions

While the issue of missing premisses has a prominent place in the *Prior Analytics*, it is largely absent from the *Topics*. In the eighth book of the *Topics*, Aristotle occasionally refers to 'necessary premisses' (ἀναγκαῖαι προτάσεις), by which he means premisses that are necessary to deduce the conclusion in an argument.[78] If not all of them are present, a premiss is missing. However, Aristotle does not, in the *Topics*, undertake to determine which premisses are the necessary ones in a given argument. Nor does he criticize arguments just because they do not contain all the necessary premisses. He only criticizes arguments if they do not contain those necessary premisses that are less reputable than the conclusion or inferior to the premisses which are present in the argument:

> The third criticism of an argument applies if a deduction does come about (γίνοιτο) with certain premisses added, but these are inferior (χείρω) to the ones asked for and less reputable (ἔνδοξα) than the conclusion. (*Top.* 8. 11, 161b26–8)

What Aristotle does not say in this passage, or anywhere else in the *Topics*, is that arguments are open to criticism if one of their necessary premisses is missing. The implication is that it is acceptable to omit necessary premisses as long as they are more reputable than the conclusion

[78] Aristotle characterizes necessary premisses as those 'through which the deduction comes about' (δι' ὧν ὁ συλλογισμὸς γίνεται, *Top.* 8. 1, 155b19–20 and 29). He mentions them a few times later on (e.g., 8. 1, 155b36, 156a10, 157a12; 8. 11, 161b29–30), but does not discuss them in any detail.

or superior to the other premisses.[79] This does not mean that arguments in which such premisses are omitted count as deductions. On the contrary, Aristotle's use of the verb 'come about' (γίνοιτο) in the passage just quoted suggests that they do not count as deductions. Crucially, however, Aristotle does not feel the need to criticize such arguments. For the purposes of the *Topics*, they are perfectly acceptable. For example, if the major premisses of A4–6 are more reputable than the conclusion, they may be omitted. Thus, A7–9 will be acceptable arguments even though they are not deductions.

This may be part of the reason why, in the *Topics*, Aristotle does not attempt to determine the set of premisses that are necessary in a given argument in order for it to count as a deduction. More important, it is doubtful whether he would be in a position to do so in the *Topics*. There is no single, obvious way to determine that set of premisses. For example, it would not be unreasonable for someone to maintain that as soon as the conclusion follows necessarily from the premiss(es), the argument counts as a deduction and no further premiss is needed. This would mean that, contrary to Aristotle's view, A3 and A7–9 are deductions.

Conversely, someone might contend that premisses are missing even in arguments that Aristotle regards as deductions. For example, consider the following straightforward argument in Barbara:

A10 All animals are mortal.
All humans are animals.
Therefore, all humans are mortal.

For Aristotle, this is a deduction. No premiss is missing. However, some ancient theorists denied that A10 is a deduction and insisted that a premise is missing in it, namely a premiss

[79] Alexander *in Top.* 568.13–18. Similarly, Aristotle states that arguments are subject to criticism if 'the conclusion does not come about either when some premisses are taken away or some premisses are added' (*Top.* 8. 11, 161b22–4). Again, this implies that an argument in which premisses are missing is not subject to criticism as long as it can be turned into a deduction by adding suitable premisses; see Allen 1995: 189 and Rapp 2000: 27-32.

such as 'If all animals are mortal and all humans are animal, then all humans are mortal'. We know that this was the view of theorists whom Alexander calls 'moderns' (νεώτεροι) and who may well have been Stoics.[80]

Alternatively, one might point out that Aristotle takes the validity of Barbara to depend on the *dictum de omni* (that is, his explanation of the meaning of phrases such as 'is predicated of all').[81] Accordingly, the validity of A10 depends on a version of the *dictum de omni* explaining the meaning of the construction 'All ... are ___'. Based on this, one might argue that A10 lacks a premiss, and that a suitable version of the *dictum de omni* needs to be added as an extra premiss in order for it to count as a deduction.

Or, one might argue that A10 lacks a premiss to the effect that the middle term, 'animals', has the same meaning in both premisses. After all, if the middle term were ambiguous and did not have the same meaning in the two premisses, then A10 would fail to be a deduction. For example, consider the following fallacious argument:

A11 All circles are geometrical figures. (TRUE)
Homer's poem is a circle. (TRUE)
Therefore, Homer's poem is a geometrical figure. (FALSE)

The premisses of this argument are understood in such a way that they are both true; the conclusion is false. As Aristotle sees it, this is because the middle term, 'circle', does not have the same meaning in the two premisses. He holds that due to this ambiguity A11 is not a deduction but a fallacy (παραλογισμός).[82] At the same time, he states in the *Sophistici Elenchi* that A11 can be turned into a deduction by adding a premiss to the effect that the middle term

[80] Alexander *in Pr. An.* 262.28–29, 345.13–18, 390.16–18; see Mueller 1969: 179–80, Frede 1974a: 2–5 and 10, Barnes 1990: 73–5 and 114–16.

[81] *Pr. An.* 1. 4, 25b37–40; see n. 50 above.

[82] *Post. An.* 1. 12, 77b27–33, *Soph. El.* 10, 171a5–11; see Pacius 1597a: 810, Poste 1866: 124, von Kirchmann 1883: 21, Barnes 1994: 149–50, Dorion 1995: 275.

has the same meaning in both premisses.[83] Even though this premiss is false, it would turn A11 into a valid deductive argument. Accordingly, if the middle term of an argument in Barbara such as A10 did not have the same meaning in both premisses, the argument would fail to be a deduction just as A11. Thus, the validity of arguments such as A10 depends on the assumption that the middle term is not used ambiguously. One might argue, then, that in order for such arguments to count as deductions this assumption needs to be made explicit as an additional premiss.

Of course, Aristotle would reject such a view. Still, it is not obvious how he would justify rejecting it. The above considerations illustrate the difficulty of determining, in a principled way, whether or not a premiss is missing in a given argument. Aristotle's definition of deduction in itself does not provide a determinate standard by which to decide this question. In the *Topics*, Aristotle does not supply such a standard.[84] But in the *Prior Analytics* he does. When he elucidates the causal condition in *Prior Analytics* 1. 1, he emphasizes the need to make explicit all the premisses necessary for the deduction (24b20–2).[85] The syllogistic presented in chapters 1. 1–22 supplies a clear and precise standard for deciding which premisses are necessary. Based on this standard, Aristotle is in a position to assert, for instance, that no premiss is missing in A10, but that premisses are missing in A3 and A7–9.[86] This is not to say that he would be in a good position ultimately to explain *why* no premiss is missing in A10 and *why* the moderns are wrong in thinking otherwise. But, unlike in the *Topics*, he can now justifiably assert *that* no premiss is missing. As Burnyeat points out, Aristotle introduces his syllogistic

[83] *Soph. El.* 8, 170a12–19; 22, 178a24–8; see Fait 2007: xxii–xxv, 2013: 243–50.

[84] See Allen 1995: 189–90, 2001: 53 and 69.

[85] See nn. 62 and 68 above.

[86] Moreover, Aristotle is in a position to determine the precise number of premises that should be present in a deduction given the number of terms occurring in it: in every deduction, he argues, the number of premisses is one less than the number of terms (*Pr. An.* 1. 25, 42b1–16). In the *Topics*, by contrast, Aristotle is not able to specify the number of premises that should be present in a deduction.

> not as a further contribution to the definition of *sullogismos*, which the *Prior Analytics* repeats from the *Topics*, but as a way of testing when an argument is valid in the sense thereby defined and when it is not. (Burnyeat 1994: 15)

Aristotle takes his syllogistic to be applicable to all deductions. In his view, every deduction 'comes about through the three figures'.[87] By this he means that every deduction conforms, or contains a part that conforms, to the valid schemata in his three figures. Thus, Aristotle holds 'that syllogistic is a universal test of formal or deductive validity' (Burnyeat 1982: 201), in much the same way that Frege conceived of the Begriffsschrift as 'the most reliable test of the validity of a chain of inferences' (Frege 1879: iv).

Now, Aristotle acknowledges a class of what he calls deductions from a hypothesis (συλλογισμοί ἐξ ὑποθέσεως), which cannot be completely analyzed as instances of his schemata in the three figures. They always contain a part that can be so analyzed, but they also contain a part that resists such an analysis.[88] It is disputed whether Aristotle took these deductions from a hypothesis to be genuine deductions satisfying his official definition of deduction.[89] If he did, then his test is not complete in that it does not capture all deductions. But whether or not it is complete, the test is clearly sound with respect to his definition of deduction: whenever an argument instantiates one of the schemata in the three figures (or a series of these schemata), then no premiss is missing and the argument satisfies his definition deduction.

According to Aristotle's test, no premiss is missing in instances of Barbara such as A10. In particular, there is no need for an additional premiss to the effect that the middle term is not used ambiguously. In general, questions of ambiguity are not discussed in the *Prior*

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[87] *Pr. An.* 1. 23, 40b20–22; 41b1–5; 1. 25, 42a30–1; 1. 28, 44b6–8; 1. 29, 45b36–46a2. At 2. 23, 68b9–13, he states that this is also true for dialectical deductions (see Allen 2001: 21; cf. *Pr. An.* 1. 29–30, 45b36–46a10, and Primavesi 1996: 60).

[88] *Pr. An.* 1. 23, 41a21–b5; 1. 44, 50a25–32 and 50b2–3. See Lear 1980: 34–5 and 39–41, Striker 1998: 210–17, and Speca 2001: 13–14.

[89] Some commentators deny that he took them to be genuine deductions (Barnes 1997: 164–6 and Bobzien 2002: 371 n. 32, Ebrey), whereas others affirm it (Lear 1980: 41–2, Slomkowski 1997: 128–9, Crivelli 2011: 179–81).

Analytics. On the other hand, they are prominent in the *Topics* and *Sophistici Elenchi*. Homonymy is one of the thirteen sources of fallacy identified in the *Sophistici Elenchi* (*Soph. El.* 4, 165b23–166a6). In the *Topics*, the ability to detect ambiguities is listed as one of the four 'tools by means of which we may be well equipped with deductions' (*Top.* 1. 13, 105a21–5; see also *Top.* 1. 15). The tool is useful, among other things, for resisting fallacies such as A11 (*Top.* 1. 18, 108a26–9). In other words, it is useful for determining whether or not a given argument is a genuine deduction. Given this, it is noticeable how little attention Aristotle pays to the issue of ambiguity in the *Prior Analytics*. When he asserts the validity of schemata such as Barbara and Darii, he does not warn the reader that they may fail if the terms involved are ambiguous. Still, he must be aware that ambiguity poses a threat to the validity of his schemata. Presumably he thinks that fallacious arguments due to ambiguity do not qualify as instances of his valid schemata. For example, the following argument must not qualify as an instance of Darii:

A12 All circles are geometrical figures. (TRUE)
Some poem is a circle. (TRUE)
Therefore, some poem is a geometrical figure. (FALSE)

If this argument were an instance of Darii, then Darii would be invalid; for any schema that has an instance with true premisses and a false conclusion is invalid. Thus, Aristotle ought to be able to exclude from his syllogistic troublesome ambiguities such as the one in A12. He must ensure that the terms (ὅροι) for which his schematic letters are placeholders are not ambiguous.[90] Arguably, Aristotle is in a good position to ensure this without resorting to

[90] Thus, Atherton (1993: 409) holds that Aristotle's syllogistic 'has no proper place for ambiguity'. She argues that, in the *Prior Analytics*, 'Aristotle sees himself as working with words and their complexes ..., but seems to have assumed total elimination of ambiguities at the level of the logical "object-language"' (Atherton 1993: 458). Likewise, Harris (2009: 99) argues that in the *Prior Analytics* 'the elimination of homonymy has to be a practical proposal if the syllogism has to stand on its own two feet. Nowhere does Aristotle attempt to face up to this. Instead, he tacitly assumes that the requirement can somehow be met.' As I will argue shortly, Aristotle is in a better position than Harris suggests to justify the elimination of homonymous terms from the syllogistic.

arbitrary stipulation. This can be seen as follows. Susanne Bobzien has shown that, according to the *De interpretatione*, a declarative sentence that contains an ambiguous term is not one but more than one affirmation or denial (*Int.* 8, 18a18–26), just as a question sentence that contains an ambiguous term is not one but more than one question (*Soph. El.* 17, 175b39–176a18).[91] At the same time, Aristotle requires in the *De interpretatione* that every genuine affirmation and denial be one and not more than one affirmation and denial (that is, a sentence in which one thing is affirmed or denied of one thing).[92] Consequently, a sentence that contains an ambiguous term does not count as a genuine affirmation or denial. Now, in the first chapter of the *Prior Analytics*, the premisses of deductions are defined to be genuine affirmations and denials.[93] The same is true for the conclusions of deductions.[94] Thus, the premisses and conclusions of deductions cannot contain ambiguous terms; for otherwise they would not be genuine, unitary affirmations and denials.

This also explains why Aristotle did not deem it necessary to discuss ambiguity or homonymy in the *Prior Analytics*. By characterizing the premisses of deductions as genuine affirmations and denials in the first chapter of the *Prior Analytics* he makes it clear from the beginning that ambiguous terms have no place in the syllogistic.[95] Of course, Aristotle realizes that ambiguous terms abound in ordinary language. But, as he explains in *Metaphysics* Γ 4, ambiguous terms can be eliminated from a language by introducing a new word for each of

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[91] Bobzien 2005: 258–64 and 2007: 301–12.

[92] *Int.* 8, 18a12–26; 10 19b5–7; 11 20b12–26.

[93] *Pr. An.* 1. 1, 24a16–17; see nn. 16 and 18 above. As a result, every premise of a deduction is required to be one and not more than one affirmation, denial, or question. This is confirmed by *Soph. El.* 6, 169a6–18, where Aristotle regards this latter requirement as part of the definition deduction (Malink 2014: 157–8). On this account, it follows from Aristotle's definition of deduction that the premisses of a deduction cannot contain ambiguous terms.

[94] See n. 19 above.

[95] By contrast, the terminology of affirmation and denial is absent from the first book of the *Topics*, including the characterization of premisses and problems in *Topics* 1. 10 and 11. Unlike the *Prior Analytics*, the *Topics* does not seem to rely on the discussion of affirmations and denials in the *De interpretatione.* This fits with the view that the *De interpretatione* was written after the *Topics* but before the *Prior Analytics* (Bocheński 1956: 49–51, similarly Sainati 1968: 203–4).

the various things signified by them.[96] The result is a language that does not contain any ambiguous terms. If I am correct, Aristotle's syllogistic is intended to apply to such a language.[97]

More generally, Aristotle's syllogistic seems to operate on the assumption that the sentences that serve as premisses and conclusions in deductions are not ambiguous.[98] This, I suggest, is another respect in which the syllogistic is akin to modern systems of formal logic such as Frege's Begriffsschrift. In 'On the scientific justification of a Begriffsschrift', Frege writes:[99]

> We need a system of symbols from which every ambiguity is banned, which has a strict logical form from which the content cannot escape. (Frege 1964: 158)

Elsewhere he writes:

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[96] *Metaph.* Γ 4, 1006a34–b11; see Alexander *in Metaph.* 277.9–14 and 277.20–31.

[97] This does not prevent Aristotle in the *Prior Analytics* from using terms which are actually ambiguous. For example, he often uses 'good' and 'white' as examples of syllogistic terms, even though he thinks that they are ambiguous (*Top.* 1. 15, 106a23–32, 106b4–12, 107a3–13, 107a36–b5, 107b13–18). This is harmless as long as he does not use them ambiguously. Aristotle may be assuming that, whenever such terms occur in the syllogistic, they are not ambiguous but signify only one of the things they ordinarily signify.

[98] It is sometimes thought that indeterminate sentences are ambiguous (e.g., Jones 2010: 42–4). Since they are admissible in the syllogistic, Keyt infers that 'Aristotle seems tempted to apply logic to ambiguous statements' (Keyt 2009: 33). However, it is not clear whether Aristotle took indeterminate sentences to be ambiguous in the syllogistic. He does not apply his usual terminology of ambiguity in connection with indeterminate sentences. It is open to him to treat them as having a unitary meaning such that they are true just in case either the corresponding particular sentence or the corresponding universal sentence is true (i.e., just in case the corresponding particular sentence is true; see n. 38). In the modal syllogistic, however, Aristotle does use ambiguous sentences. His expression for possibility (ἐνδέχεσθαι) is ambiguous between two kinds of possibility known as one-sided and two-sided possibility (see πολλαχῶς λέγεται τὸ ἐνδέχεσθαι at *Pr. An.* 1. 3, 25a37–8, and ὁμωνύμως ἐνδέχεσθαι λέγομεν at 1. 13, 32a20–1). Aristotle usually leaves it to the reader to determine which of the two he means in a given context. This is another respect in which the modal syllogistic falls short of the standards of precision found in the assertoric syllogistic (see n. 53 above).

[99] See Weiner 2004: 102; cf. also 2010: 43–4.

> A sign must not be ambiguous. Freedom from ambiguity is the most important requirement for a system of signs which is to be used for scientific purposes. (Frege 1979: 213; see also 227)

The Begriffsschrift is designed to satisfy this requirement, and so are all systems of formal logic after Frege. Like Aristotle's syllogistic, they are formulated in languages that do not contain any ambiguous expressions.

6. Conclusion

We are now in a position to return to our initial question: On what grounds does the *Prior Analytics*, but not the *Topics*, count as the beginning of formal logic? We have identified four distinctive features of the *Prior Analytics* that can help us answer the question. First, in the *Prior Analytics* Aristotle abstracts from the speaker meaning of the sentences involved in a deduction, and only takes into account their literal meaning. Nothing of relevance is left to tacit understanding between speaker and hearer. Every aspect of the meaning that is relevant to an argument's counting as a deduction is made explicit by *some* linguistic expression, even if Aristotle is not formalistic and does not prescribe *which* expression to use (Sections 2 and 3). Second, Aristotle is concerned to make explicit all premises that are necessary to deduce the conclusion in a given argument (Section 4). Third, Aristotle provides a criterion for determining when all the necessary premises have been made explicit. The criterion is sound with respect to his definition of deduction: whenever an argument meets the criterion, no premiss is missing and the argument is a deduction (Section 5). Fourth, deductions are supposed to be formulated in a language that is free from homonymy and ambiguity (Section 5).

These four features are characteristic of the *Prior Analytics* but absent from the *Topics*. At the same time, they are emphasized by Gottlob Frege in his discussion of the Begriffsschrift, and are central to any system of formal logic ever since Frege. It is in virtue of

these four features, I suggest, that the *Prior Analytics* but not the *Topics* is a treatise of formal logic.[100]

Let me add three points of clarification concerning this last claim. First, there is a question about the unity of the four features just listed. While the first three features can be grouped under the heading of 'making everything explicit', the fourth one seems to stand apart from them. A case can be made, however, that the fourth feature too falls under the same heading. For, if ambiguous terms are admitted, the validity of deductions in Barbara requires an additional premiss to the effect that the two occurrences of the middle term have the same meaning.[101] In order for this additional premiss to say what it is supposed to say, it must contain an expression that picks out the two occurrences of the middle term.[102] But if terms are allowed to be ambiguous, there is no guarantee that this expression is not ambiguous, and that it has the right meaning for it pick out the two occurrences of the middle term. Thus, there is a need for yet another premiss connecting the additional premiss to the two original premisses by stating that the expression in question picks out the two occurrences of the middle term – and so on to infinity. In this way, acceptance of ambiguous

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[100] The third feature requires that the criterion be sound with respect to Aristotle's definition of deduction. Someone who, like Alexander's moderns (see n. 80), endorses an alternative conception of deduction with respect to which Aristotle's criterion is not sound might have reason to deny Aristotle's syllogistic the status of formal logic. Accordingly, the fact that the syllogistic today is usually granted this status seems to depend in part on the fact that the syllogistic is (largely) sound with respect to modern systems of formal logic. In the *Begriffsschrift* (1879: §12), Frege explains how to translate Aristotle's four main kinds of assertoric sentences into his own quantifier logic. On this translation, all schemata which Aristotle regards as valid in the assertoric syllogistic are valid in modern quantifier logic, except those that are affected by what is known as the problem of existential import (see, e.g., Beaney 1996: 49–56 and Malink 2013: 34–46). Accordingly, it is sometime thought that parts of Aristotle's assertoric syllogistic fail to be sound since they rely on tacit assumptions of existential import, and that because of this 'even the assertoric syllogistic is not purely formal' (Rini 2011: 28; similarly Oliver & Smiley 2013: 182–3).

[101] See n. 83 and the discussion of A11 in Section 5. Similar premisses will be needed for the major and minor terms.

[102] It might contain two distinct expressions each of which picks out one of the two occurrences, or it might conatain a single expression that picks out both occurrences at the same time.

terms leads into an infinite regress of premisses. As a result, the project of making explicit all the premisses on which a given deduction depends becomes impossible. Aristotle's decision to exclude ambiguous terms from consideration in the syllogistic is a way to block the regress. It may not be the only way to block it, but it is one of the easiest. If this is correct, the fourth feature listed above contributes to the goal of making fully explicit all the premisses that are necessary in order for an argument to count as a deduction, by ensuring that the goal is feasible in the first place. The four features are unified in that they all contribute, in different ways, to this goal.[103]

Second, a clarification concerning the term 'formal' is in order. What, one might ask, is the meaning of this term such that it is correct to say that Aristotle's syllogistic qualifies as 'formal' logic in virtue of those four features? There is a wide variety of senses in which a given kind of logic may be said to be 'formal'.[104] It is beyond the scope of this paper to survey these senses and to discuss whether and how they are related to the above four features. My concern here is not so much with the meaning of the term 'formal'. Rather, my aim is to identify essential features of the discipline that we today call formal logic, and to show that they are present in the *Prior Analytics* but not in the *Topics.* And while there is no consensus on what exactly those essential features are, I hope to have made it plausible that the above four features are to be counted among them.

Finally, one might wonder whether the four features are sufficient to account for the syllogistic's status as formal logic. After all, there are a number of other features that might be

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[103] This does not mean that absolutely *everything* that is relevant to an argument's counting as a deduction is made epxlicit in the premisses. For example, the *dictum de omni et de nullo* is often relevant to an argument's counting as a deduction (n. 50), but is not mentioned in the premisses (see Section 5). Furthermore, it is highly relevant to an argument's counting as a deduction that it satisfies the definition of deduction; yet Aristotle is clear that this definition should not be mentioned in the premisses of a deduction (*Post. An.* 2. 6, 92a11–19; see Philoponus *in Post. An.* 355.10–356.6, Barnes 1994: 212–13, and Fait 2013: 262–3).

[104] For an overview, see MacFarlane 2000: 31–78 and Dutilh Novaes 2011. Some of these senses can be traced back to Aristotle's distinction between matter and form. As is well known, however, this distinction is absent from the *Organon* (Burnyeat 2001: 8). Aristotle does not, in the *Organon*, employ the terminology of 'form' and 'formal' in connection with deductions (see Barnes 1990: 39–40 and 2007: 277–82).

thought to be relevant. For example, the syllogistic is reductive in that the validity of second- and third-figure schemata is derived from that of first-figure schemata by means of conversion rules and proof by *reductio*. According to Striker (1996: 204), the presence of these reductive derivation is 'one of the most important differences between the *Prior Analytics* and the *Topics*'. On the other hand, the above four features would also be compatible with a mere list of valid schemata without any such derivations. Nevertheless, it is doubtful whether the presence of those derivations should be regarded as a fifth feature that helps explain why the syllogistic is a treatise of formal logic. To see this, consider for a moment the structure of a demonstrative science as expounded in the *Posterior Analytics*. Although Aristotle does not seem to regard the syllogistic as a science in the strict sense, it closely resembles a demonstrative science: the first-figure schemata and conversion rules correspond to the unproved principles, the second- and third-figure schemata correspond to the proved theorems, and Aristotle's derivations of the latter from the former in *Prior Analytics* 1. 5-6 correspond to demonstrations in the science.[105] Had Aristotle omitted the derivations, the syllogistic would lack demonstrative structure. But this does not mean that it would not count as a treatise of formal logic. In the development of a science, demonstration and explanation is typically preceded by a preliminary inquiry collecting the relevant facts (*Posterior Analytics* 2. 1, 89b23–31). For example, the *Historia Animalium* presents a collection of zoological facts largely without offering scientific demonstrations and explanations of them, leaving the latter task to treatises such as the *De Partibus Animalium*.[106] But the *Topics* does not stand to the *Prior Analytics* as the *Historia Animalium* stands to the *De Partibus Animalium*; for the *Topics* does not contain a list of second- and third-figure schemata for which the *Prior Analytics* supplies the appropriate derivations by reducing them to first-figure schemata. Instead, I suggest, the crucial difference between the *Topics* and the *Prior Analytics* is captured by the four features identified above. Just as the *Historia Animalium* counts as a zoological treatise even if it does not contain the completed demonstrative science of zoology, any treatise which possesses the four features will count as a treatise of formal logic even if it does not have the

[105] Barnes 2007: 362–9.

[106] See, e.g., Lennox 2001: 123–4 and Leunissen 2010: 79–80.

demonstrative structure of a reductive system. If this is correct, the four features suffice to explain why the *Prior Analytics* is a treatise of formal logic. Had they been present in the *Topics,* then the *Topics* would mark the beginning of formal logic.

The four features are aimed at making fully explicit all premisses that are necessary for a given argument to count as a deduction. By setting up this aim, Aristotle has established one of the central tenets of formal logic. I. M. Bocheński writes:

> The whole business of Formal Logic is to make tacit assumptions of reasoning explicit.[107] To deny the thesis [that in Formal Logic all tacit assumptions must be made explicit] would, consequently, be to contradict the very nature of Formal Logic, not only as it is now, but as it has always been since the time of Aristotle. (Bocheński 1948: 52)

I hope to have shown that Bocheński is right. While the specific account of deduction provided by Aristotle's syllogistic may play only a minor role in contemporary logic, his *Prior Analytics* initiated a project which continues to define the discipline of formal logic today.

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[107] Similarly, John MacFarlane (forthcoming, section 1) describes a view of logic on which 'the role of logic is to help us make explicit everything on which an inference depends. When we have teased out hidden assumptions to the point where we have a formally valid argument, then we know that the process of explicitation has come to an end; we have made all of the assumptions on which the inference depends explicit.'

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