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Venn Diagrams for Plurative Syllogisms

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Plurative Propositions and Syllogisms

BY A "plurative syllogism" we understand a two-premise argument in which, in addition to the familiar categorical propositions of the types *A, E, I,* and *O,* there may also figure *plurative propositions*[1] of these four types:

U: Most S is P
W: Most S is not P
U' (not-U) : Half-or-more S is not P
W' (not-W) : Half-or-more S is P

We construe "Most X is Y" to assert that *more X's are Y than are* Y^* (where Y^* is the set-complement of Y). Since this can be put in the form "the car-

AUTHORS' NOTE: This paper presents details of a finding previously announced by its senior author in an abstract entitled "Plurality Quantification," *Journal of Symbolic Logic,* 27: 372–74 (1962). We take pleasure in acknowledging helpful suggestions by Nuel D. Belnap, Jr.

dinality of the set XY is greater than that of XY^*," plurative propositions can also be construed for infinite classes. The set A-E-I-O-U-W-U'-W' is closed under negation.

An example of a typical plurative syllogism—and in fact a valid one—is the AUU-1 syllogism:

All M is P
Most S is M
———————
Most S is P

This example illustrates how the concepts of mood and figure are readily extended to plurative syllogisms.

Venn-Style Diagrams

The reader will be assumed to be familiar with the well-known device of Venn diagrams for testing the validity of classical syllogisms. We shall now develop means for extending this device to plurative syllogisms involving U and W premises (not—be it noted— U' and W' premises).[2]

The new item of machinery will be an arrow connecting two line-segments, with that toward which the arrow points to be called the *head* of the arrow, and that from which it points to be called the *vane* of the arrow. This notation is to be superimposed upon the usual machinery of Venn diagrams using circles to represent the extensions of syllogistic terms, with stars and shading to indicate the non-emptiness or emptiness (respectively) of regions. The function of such an arrow is *to indicate that the region comprising all the sectors into which the vane falls is of greater cardinality than the region comprising all the sectors into which the head falls*. Thus the plurative proposition "Most S is P" is to be represented by Diagrams I and II.

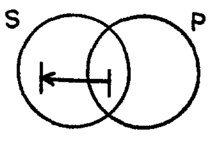

Diagram I

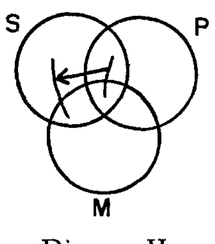

Diagram II

Special Rules for the Arrows

The arrow-notation presented in the preceding section is subject to five rules:

(R1) The vane of an arrow may always be extended.

(R2) The head of an arrow may always be contracted.
(R3) The vane of an arrow may always be contracted out of a shaded region.
(R4) The head of an arrow may always be extended into a shaded region.
(R5) An arrow may always be drawn from a starred region into a shaded one.

The appropriateness and validity of these rules should be obvious. (They are not, however, complete in allowing for all valid arrow-operations. For example it would be appropriate to permit both head and vane of an arrow to be extended into an (the same) adjacent region into which neither falls.)

We shall need to make use also of a rule to the following effect:

(R6) In diagramming the premises of a plurative syllogism, *if* (1) both the heads of the two arrows overlap in one region, and (2) both the vanes overlap in one region, and (3) the head of each arrow overlaps in one region with the vane of the other, *then* a star of non-emptiness can be placed in the vane-overlap region.

This rule is justified on the basis of R2 and R3 by a *reductio ad absurdum* argument. The situation envisaged in the rule is exemplified by Diagram III. Assume now that the region of vane overlap is empty (shaded in). Then by rule (R3) we obtain Diagram IV. But now by rule (R2) we obtain Diagram V, which is absurd, given the intended meaning of the arrows.

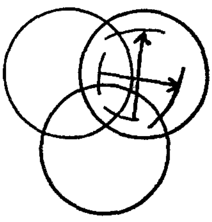

Diagram III

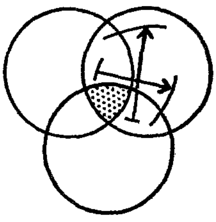

Diagram IV

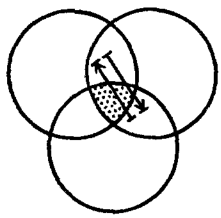

Diagram V

We shall also stipulate a convention: In diagramming plurative syllogisms, the diagramming of the categorical premises should be done before that of the plurative premises. This convention is merely an expedient device for avoiding messy diagrams, and is not rendered necessary by any logical considerations.

Validity Testing

Given our arrow-notation and its rules, the process of a Venn-diagrammatic "testing" of plurative syllogisms in which no primed plurative propositions occur is the standard and familiar one of (1) recording in the three-circle diagram the information afforded by the premises of the syllogism, and then (2) verifying by inspection whether the situation claimed by the conclusion in fact obtains. For example, consider the aforementioned *AUU*-1 syllogism, together with its diagram, Diagram VI:

All *M* is *P*
Most *S* is *M*

Most *S* is *P*

Now since the head of the diagram can be contracted and its vane extended, we obtain Diagram VII, which tells us that the conclusion "Most S is P" is indeed warranted.

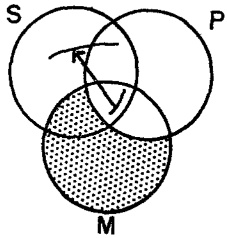

Diagram VI

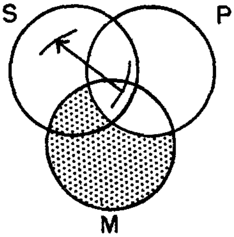

Diagram VII

Reduction of the Primed Cases

Our arrow technique is not designed to accommodate the primed plurative propositions *U′* and *W′*. We shall now show that this is a venial shortcoming from the standpoint of the validity-testing of plurative syllogisms. For it can be established that the following principle obtains:

> Every valid plurative syllogism in which primed propositions occur is either (i) obtained trivially from a valid unprimed plurative by either (a) the strengthening of a premise (*I* to *W′* or O to *U′*) or (b) weakening the conclusion (*A* to *W′* or *E* to *U′*) or (ii) obtained from an unprimed plurative syllogism by antilogism (or both).

To show this, we remark that plurative syllogisms can be grouped into the following three cases:

Case A: *No primes in the premises.*

Now if the conclusion is not primed, we are home. If it *is* primed, then the syllogism can be valid only if this primed conclusion can be obtained by weakening a validly drawn unprimed conclusion (case (i-b) above).

Case B: *Exactly one primed premise.*

B-1: *Unprimed conclusion*

A universal conclusion is now impossible (given the primed premise), so the conclusion must be *I*, *O*, *U*, or *W*. Assume it is *I* or *O*; then it must also be obtainable if the primed premise is weakened (case (i-a) above). So assume the conclusion is *U* or *W*. Then there are four possibilities:

	(1)	(2)	(3)	(4)
Primed Premise	U'	U'	W'	W'
Conclusion	U	W	U	W

In case (1) there can be no valid syllogism, for if there were, we could strengthen the premise U' to *E* and weaken the conclusion *U* to *I* and the resulting syllogism would have to be valid. In case (4) there can be no valid syllogism, for if there were, we could strengthen the premise W' to *A* and weaken the conclusion *W* to *O*. In cases (2) and (3) the absence of a valid conclusion can be shown by considering the exactly-half case.

B-2: *Primed conclusion*

Here antilogism is operative (case (ii) above).

Case C: *Both premises primed.*

That no valid conclusion can be drawn in this case can be shown by taking both premises to represent the exactly-half case.

This completes the demonstration of the principle stated at the outset of this section.

Completeness of the Method

To show the completeness of our diagrammatic testing procedure for unprimed plurative syllogisms (i.e., those involving no primed propositions) two points must be established: (i) Whenever an unprimed plurative syllogism may be claimed to be valid on the basis of diagrammatic test, it is in fact valid. (ii) Whenever an unprimed plurative syllogism is in fact valid, this validity can be exhibited by means of a diagrammatic test. It is not necessary to undertake any further consideration of point (i), which is rendered obvious by the very design of the diagrammatic test. However, point (ii) presents greater difficulty. We shall proceed by cases. Valid unprimed plurative syllogisms may be divided into three groups:

(A) *Trivial* ones, in which either (1) the conclusion of a valid (standard) categorical syllogism is weakened from "all" to "most" (*A* to *U*) or from "no" to "most-not" (*E* to *W*), or (2) a premise is strengthened from "some" to "most" or from "some-not" to "most-not."

(B) *Quasi-trivial* ones, obtained from a valid (standard) categorical syllogism by a change from "all" to "most" or from "some" to "most" uniformly in the conclusion and in the major premise.

(C) *Non-trivial* ones, falling into neither of the above groups, of which (as can be verified by checking the possibilities) there are exactly two, namely *UUI*-3 and *WUO*-3:

Most M is P	Most M is not P
Most M is S	Most M is S
Some S is P	Some S is not P

We shall deal with each of these three cases in turn.

Case *A*. It is readily verified that Rules R3 and R4 for arrow-manipulation suffice for the diagrammatic validation of syllogisms of this sort: the diagram of the premises either exhibits or can be transformed to exhibit the diagram for the conclusion.

Case *B*. Here a case-by-case check can serve to show the adequacy of the method. (We omit the necessary but tedious details.)

Case C. This especially interesting case constitutes the *raison d'être* of Rule R6. The two syllogisms of this case (*UUI*-3 and *WUO*-3) can be validated diagrammatically by this rule.

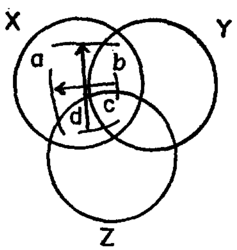

Diagram VIII

Although our diagrammatic technique is (demonstrably) complete for testing the validity of syllogistic inferences, it would have to be supplemented to become capable of accommodating certain asyllogistic arguments. For example, consider the premises "Most Y is X" and "Most Z is X," shown in Diagram VIII. Since $d + c > a + b$ and $b + c > a + d$ we obtain $c > a$, i.e., our premises entail "There are more XYZ's than XY*Z*'s." But this inference is not comprehended within our diagram-technique as formulated above. (We owe this example to P. T. Geach.)

Conclusion

It is readily shown that (if the *A's* are a *proper* subset of the entire domain of discourse, and the cardinality of this domain is *not* specified as some finite number) "Most *A*'s are *B*'s" cannot be defined by means of the *usual* resources of *quantificational logic*, or any other type of quantification for that matter (understanding quantifiers to be defined with respect to the entire domain at issue). Nevertheless, the logic of the plurative propositions "Most S is P" and "Most S is not P" is an extremely simple matter. For example, as we have shown, syllogisms involving such propositions are subject to a validity test using Venn-diagrams (by our suitably elaborated employment of arrows to indicate the comparative size of two regions of the diagram). More generally, the machinery needed for the analysis of such syllogisms is much less than is required for De Morgan's "numerically definite syllogisms."[3]

Consider the two arguments:

All *A*'s are *B*'s	Most *C*'s are *A*'s
———	Most *C*'s are *B*'s
All parts of *A*'s are parts of *B*'s	———
	Some *A*'s are *B*'s

Textbooks often charge that traditional logic is "inadequate" because it cannot accommodate patently valid arguments like the first. But this holds equally true of modern quantificational logic itself, which cannot accommodate the second. Powerful tool though it is, quantificational logic is unequal to certain childishly simple valid arguments, which have featured in the logical literature for over a century (i.e., since the days of De Morgan and Boole).[4] Plurative syllogisms afford an interesting instance of an inferential task in which the powerful machinery of quantificational logic fails us, but to which the humble technique of Venn diagrams proves adequate.

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NOTES

[1] We owe this term to correspondence with P. T. Geach. The treatment of such propositions goes back to the Middle Ages at least, and some discussion of the matter can be found, for example, in Averroes' *Quaesita in libros logicae Aristotelis.*

[2] We could complicate the machinery introduced here by letting a *plain* arrow from region *A* to region *B* mean "There are at least as many *A*'s as *B*'s" and letting a *flagged* (or barred) arrow mean "There are more *A*'s than *B*'s." (A flagged arrow now serves the function of the plain arrow in our text.) Now we can easily diagram U' and W': to "negate" an arrow we reverse its direction and "alter" its flagging (i.e., flag it if unflagged, unflag it if flagged). We can now also accommodate Z-propositions of the form "There are just as many *X*'s as *Y*'s" by an (unflagged) arrow pointing in both opposite directions. (However, it seems that no simple diagrammatic procedure is available for accommodating Z', the negate of Z.)

[3] *Formal Logic* (La Salle, Ill.: Open Court, 1926), Chap. VIII. Compare also Henry A. Finch, "Validity Rules for Proportionally Quantified Syllogisms," *Philosophy of Science*, 24:1–18 (1957).

[4] To be sure, once the quantificational system is elaborated in some way to the point where arithmetic is possible—so that we can *count* the extensions of properties and compare the results of such countings—then this impotence is overcome.