16.1 Symmetric Functions, Math 102 **Name:** *James Wang*
Sept 26, 2023

Important Facts

- The symmetric polynomials is a subring of $R[u]$.
- Every permutation in S_n can be written as a product of transpositions.
- The orbit sums and monomials of the elementary symmetric functions are two bases of the symmetric polynomials.

Theorem 16.1.6 Symmetric Functions Theorem
Every symmetric polynomial $g(u_1, \ldots, u_n)$ with coefficients in a ring R can be written in a unique way as a polynomial in the elementary symmetric functions $s_1, \ldots, s_n$.

Lemma 16.1.11
Let g be a symmetric polynomial of degree d in the variables $u_1, \ldots, u_n$, and suppose that $g^\circ = Q(s_1^\circ, \ldots, s_{n-1}^\circ)$. Then $g = Q(s_1, \ldots, s_{n-1}) + s_n h$, where h is a symmetric polynomial in $u_1, \ldots, u_n$ of degree $d - n$.

Corollary 16.1.12
Suppose that a polynomial $f(x) = x^n - a_1 x^{n-1} + \cdots \pm a_n$ has coefficients in a field F, and that it splits completely in an extension field K, with roots $\alpha_1, \ldots, \alpha_n$. Let $g(u_1, \ldots u_n)$ be a symmetric polynomial in $u_1, \ldots, u_n$ with coefficients in F. Then $g(\alpha_1, \ldots, \alpha_n)$ is an element of F.

Proposition 16.1.14
Let $p_1 = p_1(u_1, \ldots, u_n)$ be a polynomial, let $\{p_1, \ldots, p_k\}$ be its orbit for the operation of the symmetric group on the variables, and let $w = w_1, \ldots, w_k$ be another set of variables, where k is the number of polynomials in the orbit of p_1. If $h(w_1, \ldots, w_k)$ is a symmetric polynomial in w, then $h(p_1, \ldots, p_k)$ is a symmetric polynomial in u.

Confusion

- What does the second induction on the degree of a symmetric polynomial look like?