14.4 Diagonalizing Integer Matrices, Math 102 **Name:** *James Wang*
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Important Facts

- We are only working with free modules
- We recall the commutative diagram from linear algebra

$$\begin{tikzcd} V \arrow[r, "T"] \arrow[d, "\phi_\beta"'] & W \arrow[d, "\phi_\gamma"] \\ \mathbb{R}^n \arrow[r, "A"] & \mathbb{R}^m \end{tikzcd}$$

where $\phi_\beta(v) = [v]_\beta$ and $\phi_\gamma(w) = [w]_\gamma$ are isomorphisms

Theorem 14.4.6
Let A be an integer matrix. There exist products Q and P of elementary integer matrices of appropriate sizes, so that $A' = Q^{-1}AP$ is diagonal, say

$$A' = \begin{bmatrix} \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_k \end{bmatrix} & \\ & 0 \end{bmatrix}$$

where the diagonal entries d_i are positive, and each one divides the next: $d_1|d_2|\cdots|d_k$.

Theorem 14.4.11
Let W be a free abelian group of rank m, and let U be a subgroup of W. Then U is a free abelian group, and its rank is less than or equal to m.

$$\begin{tikzcd} \mathbb{Z}^n \arrow[r, "A"] \arrow[d, two heads, "B"'] & \mathbb{Z}^m \arrow[d, "C"] \\ U \arrow[r, hook, "i"] & W \end{tikzcd}$$

Confusion

- How do we make sense of the sublattices on page 423 without appealing to calculations?