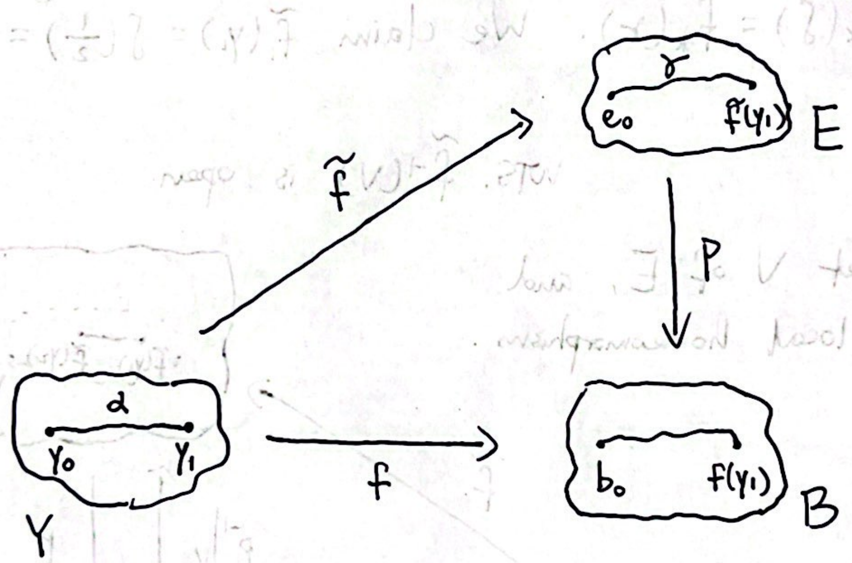
Lemma 79.1:

$$\begin{tikzcd}
 & (E, e_0) \arrow[d, "p"] \\
(Y, y_0) \arrow[ur, "\tilde{f}"] \arrow[r, "f"'] & (B, b_0)
\end{tikzcd}$$

Given a continuous f, when can we lift it to a covering space?

$\tilde{f}$ is a lift:



We define $\tilde{f}(y_1)$ using path lifting by treating it as the end point.

Uniqueness follows from the uniqueness of path liftings.

$$\tilde{f}(y_1) = \gamma(1)$$

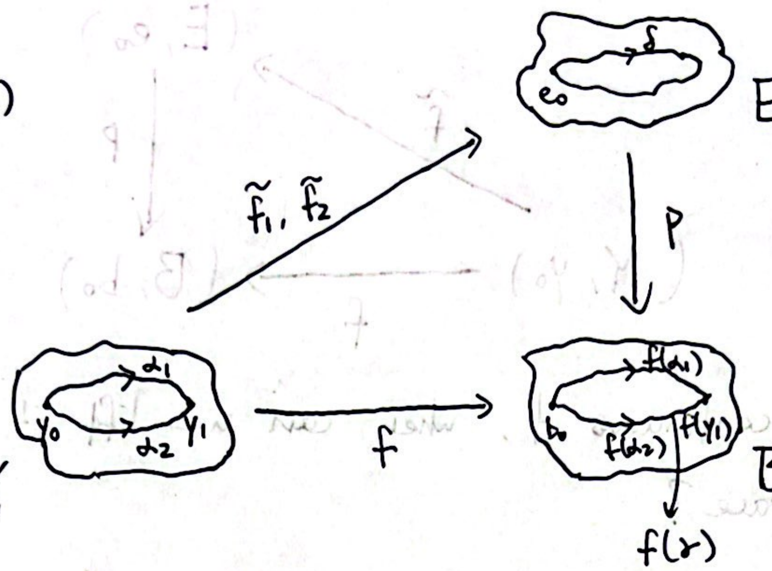
$$p \circ \tilde{f}(y_1) = f(y_1)$$

$\tilde{f}$ is well-defined:

WTS. $\tilde{f}_1(y_1) = \tilde{f}_2(y_1)$

let $\gamma = \alpha_1 * \alpha_2^{-1}$,

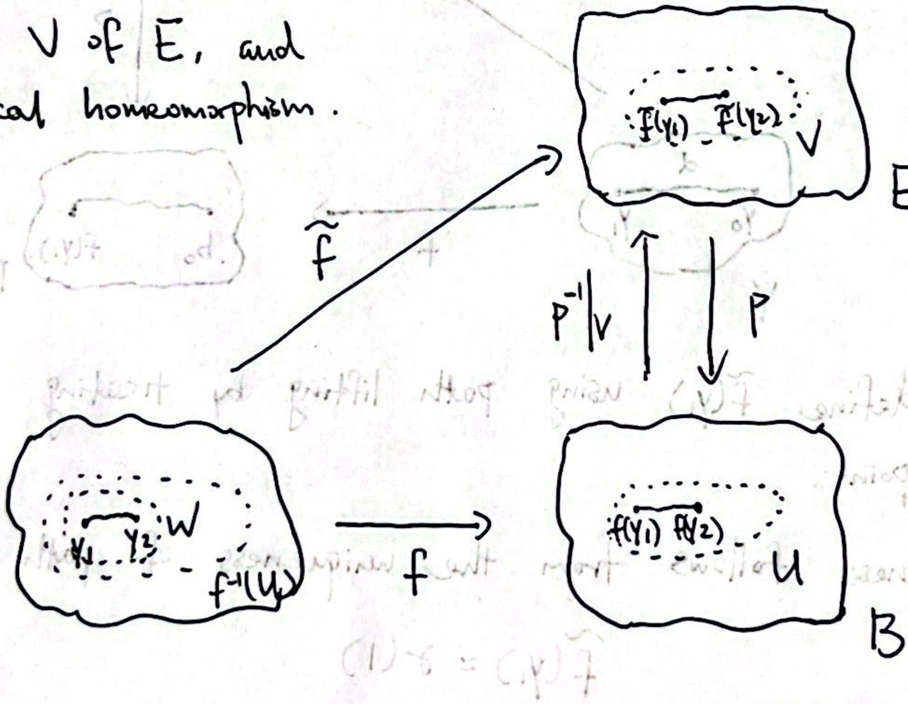
so $\gamma \in \pi_1(Y, y_0)$



Since $f_*(\gamma) \subset p_*(\pi_1(E, e_0))$, there exists $\delta \in \pi_1(E, e_0)$ such that $p_*(\delta) = f_*(\gamma)$. We claim $\tilde{f}_1(y_1) = \delta(\frac{1}{2}) = \tilde{f}_2(y_1)$.

$\tilde{f}$ is continuous: WTS. $\tilde{f}^{-1}(V)$ is open

Take an open set V of E, and let U be its local homeomorphism.



Take $y_1 \in \tilde{f}^{-1}(V)$, so $f(y_1) \in U$ and $y_1 \in f^{-1}(U)$. Since $f^{-1}(U)$ is open, there is a path-connected neighborhood W of y_1 so that $W \in f^{-1}(U)$, which means our definition of $\tilde{f}$ can be used for elements of W.

Note $f(W) \subseteq U$, and $p^{-1}|_V(f(W)) \subseteq V$. Hence $W \subseteq \tilde{f}^{-1}(V)$.