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Analysis and design of chattering-free discrete-time sliding mode control

Jinhui Zhang | **Na Zhang** | **Ganghui Shen** | **Yuanqing Xia**

School of Automation, Beijing Institute of Technology, Beijing, China

Correspondence
Jinhui Zhang, School of Automation, Beijing Institute of Technology, Beijing 100081, China.
Email: zhangjinh@bit.edu.cn

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Summary

In this paper, the disturbance observer–based chattering-free discrete-time sliding mode control (DSMC) approach is proposed for systems with external disturbances. The proposed disturbance observer, which makes full use of the state and input information at the current and last steps, improves the estimation accuracy and achieves accurate compensation for disturbances. Then, with the help of disturbance observer, a new reaching law, which contains not only a nonsmooth term with a dynamically adjusted gain parameter but also a second order difference of the disturbance, is proposed to reduce the range of the quasi-sliding mode band and eliminate chattering. The proposed DSMC approach realizes the active disturbance rejection and strong robustness. Finally, a simulation example is presented to verify the effectiveness of the proposed method.

KEYWORDS

discrete time system, discrete-time sliding mode control (DSMC), disturbance observer, reaching law

1 | INTRODUCTION

Recently, due to the invariance against to parameter perturbations and external disturbance, sliding mode control (SMC),[1] one of the powerful nonlinear control techniques, has acquired tremendous progress in theoretical research and been widely used in some fields, such as industrial automation.[2,3] In general, the design process of SMC can be divided into two steps: the sliding mode surface with desired performance should be designed first, and then, the controller should be organized to drive the system trajectories reaching the pre-defined sliding mode surface from an initial state and slide along it to the origin. With the extensive computer implementations of control algorithms, it is meaningful to extend continuous sliding mode control (CSMC) to discrete sliding mode control (DSMC),[4] and the research on DSMC has became increasingly momentous. Some representative results on DSMC are mentioned.[4-11]

In fact, due to discretization, the high-frequency switching generated by signum function in CSMC cannot be accomplished in DSMC, and thus, many favorable properties of CSMC, such as the distinctive invariance property to the matched disturbance, cannot be guaranteed, and the quasi-sliding mode (QSM) features are often investigated in DSMC. The reaching law–based design approaches are often utilized to design DSMC.[4,12-18] In the work of Gao et al,[4] the quasi-sliding mode band (QSMB) is strictly defined and analyzed, and a reaching law approach is firstly proposed to design DSMC. Recently, some modified reaching laws are also proposed, for example, in the works of Ma et al[15] and Du et al,[16] the signum function in the reaching law defined in the work of Gao et al[4] is replaced by continuous-approximate function and continuous nonsmooth function, respectively. In the work of Ma et al,[18] the gain of the signum function is modified as a dynamical adapting one. Besides, the first-order or second-order difference of the disturbances is also incorporated in the modified reaching law to reduce the width of the QSMB in other works.[15,16,18] It is worth mentioning that, in the aforementioned results, the delay estimation method is always applied, and the disturbance at previous step is used as the estimate of the disturbance at the current step. However, the disturbance may be difficult to measure in some practical systems, which limits the practicability of the existing DSMC approaches.

In this paper, the disturbance observer–based chattering-free DSMC approach will be addressed for system subject to disturbances. The main contributions can be summarized as follows.

(1) By using the system state and control input information at the previous and current steps, a novel discrete-time disturbance observer is proposed, and it can be shown that the disturbance estimation error is in the order of $O(T_s^3)$, where T_s is the sampling period.
(2) By incorporating the second-order difference of the disturbances and a continuous nonsmooth function with dynamically adjusted gain parameter, a novel discrete-time reaching law is proposed, which guarantees that the proposed DSMC is chattering-free and the QSMB is also of the order of $O(T_s^3)$.
(3) Based on the proposed disturbance observer and reaching law, the chattering-free DSMC approach is developed and the active disturbance rejection can be achieved.

Finally, a numerical example is used to verify the effectiveness and the merits of the proposed new design techniques.

2 | SYSTEM DESCRIPTIONS

Consider the following continuous-time system under external disturbances:

$$\dot{x}(t) = Ax(t) + Bu(t) + B_f f(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}$ is the control input, and $f(t) \in \mathbb{R}^p$ is the external disturbance. A, B, and B_f are constant system matrices with approximate dimensions. The disturbance $f(t)$ is assumed to be smooth and bounded. Let the sampling period be T_s, and $u(t)$ is assumed to be constant during $[kT_s, (k+1)T_s)$, $k \in \mathbb{N}$, the discretized counterpart of continuous-time system (1) can be given as

$$x(k+1) = \Phi x(k) + \Gamma u(k) + d(k), \tag{2}$$

where $\Phi = e^{AT_s}$, $\Gamma = \int_0^{T_s} e^{A\tau} d\tau B$, and

$$d(k) = \int_0^{T_s} e^{A\tau} B_f f\left((k+1)T_s - \tau\right) d\tau.$$

For the disturbance $d(k)$, the following lemma and assumption are necessary.

Lemma 1 (See the work of Abidi et al[19]). *In the discretization from (1) to (2), the bounded and smooth disturbance $d(k)$ possesses the following useful property:*

$$\begin{aligned} d(k) &= \int_0^{T_s} e^{A\tau} B_f f\left((k+1)T_s - \tau\right) d\tau \\ &= \Gamma f(k) + \frac{1}{2}\Gamma v(k) T_s + O\left(T_s^3\right), \end{aligned}$$

where $v(t) = \frac{df(t)}{dt}$, $v(k) = v(kT_s)$, $d(k) - d(k-1) = O(T_s^2)$, and $\delta(k) \triangleq d(k) - 2d(k-1) + d(k-2) = O(T_s^3)$.

Assumption 1. The disturbance $d(k)$ satisfies the two conditions: $\|d(k)\| \le \beta = O(T_s)$ and $\|\delta(k)\| \le \gamma = O(T_s^3)$, where β and γ are two positive constants.

3 | MAIN RESULTS

In this section, we will propose a new disturbance observer–based chattering-free DSMC approach to achieve active disturbance rejection and strong robustness.

3.1 | Disturbance observer

The following discrete-time disturbance observer is proposed to estimate the real disturbance $d(k)$:

$$\begin{aligned} v(k+1) &= \Lambda \hat{d}(k) + (\Lambda - 2I_n)\left(\Phi x(k) + \Gamma u(k)\right) \\ &\quad + \left(\Phi x(k-1) + \Gamma u(k-1)\right) \\ \hat{d}(k) &= v(k) - (\Lambda - 2I_n)x(k) - x(k-1), \end{aligned} \tag{3}$$

where $\hat{d}(k) \in \mathbb{R}^n$ is the disturbance estimate, $v(k) \in \mathbb{R}^n$ is the internal auxiliary variable vector of the observer, and $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ with $|\lambda_i| < 1$, $i = 1, 2, \ldots, n$, is the observer gain matrix chosen by designer. Here, the initial values of disturbance observer (3) are $u(k) = 0$ and $x(k) = x(0)$ for $k \leq 0$.

Define the disturbance estimation error as $\tilde{d}(k) = d(k) - \hat{d}(k)$ and suppose $\hat{d}(0) = 0$. The follow lemma shows that the disturbance estimation error is bounded.

Lemma 2. *The disturbance estimation error $\tilde{d}_i(k)$, $i = 1, 2, \ldots, n$, will converge to a bounded region $\mathcal{B}_i \triangleq \left\{ \tilde{d}_i(k) \middle| |\tilde{d}_i(k)| \leq \frac{\gamma}{1-|\lambda_i|} \right\}$.*

Proof. It follows from (3) that

$$\begin{aligned} \hat{d}(k+1) &= \Lambda \hat{d}(k) + (\Lambda - 2I_n)\left(\Phi x(k) + \Gamma u(k)\right) + \left(\Phi x(k-1) + \Gamma u(k-1)\right) \\ &\quad - (\Lambda - 2I_n)x(k+1) - x(k) \\ &= \Lambda \hat{d}(k) - (\Lambda - 2I_n)d(k) - d(k-1) \\ &= -\Lambda \tilde{d}(k) + 2d(k) - d(k-1), \end{aligned}$$

which implies that the disturbance estimation error $\tilde{d}(k)$ satisfies that

$$\tilde{d}(k+1) = \Lambda \tilde{d}(k) + \delta(k). \tag{4}$$

Note that $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, and it is easy to obtain that

$$\begin{aligned} \tilde{d}_i(k) &= \lambda_i \tilde{d}_i(k-1) + \delta_i(k-1) \\ &= \lambda_i^k \tilde{d}_i(0) + \sum_{j=0}^{k-1} \lambda_i^j \delta_i(k-j-1) \\ &\leq |\lambda_i|^k \left|\tilde{d}_i(0)\right| + \sum_{j=0}^{k-1} |\lambda_i|^j \, |\delta_i(k-j-1)| \\ &\leq \beta |\lambda_i|^k + \gamma \sum_{j=0}^{k-1} |\lambda_i|^j \\ &\leq \beta |\lambda_i|^k + \gamma \left(\frac{1}{1-|\lambda_i|} - |\lambda_i|^k \right) \\ &= (\beta - \gamma)|\lambda_i|^k + \frac{\gamma}{1-|\lambda_i|}. \end{aligned} \tag{5}$$

It is obvious that $|\lambda_i|^k \to 0$ as $k \to \infty$ because $|\lambda_i| < 1$, $i = 1, 2, \ldots, n$. Thus, $\tilde{d}_i(k)$ will converge to $\mathcal{B}_i$ finally. □

Remark 1. It is easily seen from (3) that only the states and control input information are used to determine the disturbance estimate $\hat{d}(k)$, which guarantees the practicability of (3). To improve accuracy of the disturbance observer, in disturbance observer (3), the delayed state $x(k-1)$ and $u(k-1)$ are also utilized.

Remark 2. It follows from Lemma 1, $\delta_i(k)$ is of the order $O(T_s^3)$, and thus, if $|\lambda_i|$ is chosen such that $|\lambda_i| < 1$, $i = 1, 2, \dots, n$, the disturbance estimation error $\tilde{d}_i(k)$ can be finally achieved to the accuracy of $O(T_s^3)$. Moreover, the bound of the region $\mathcal{B}_i$ is closely related to λ_i, $i = 1, 2, \dots, n$.

3.2 | Sliding surface

The sliding mode surface is designed as

$$s(k) = Kx(k), \tag{6}$$

where $K \in \mathbb{R}^{1\times n}$ is a constant matrix and is chosen such that $K\Gamma$ is invertible, and the matrix $\Phi - \Gamma(K\Gamma)^{-1}K\Phi$ has one zero pole and $(n-1)$ poles inside the unit disk in the complex z-plane. The objective is to force the states toward and steer them to stay on the sliding mode surface $s(k) = 0$ at every sampling instant.

According to the work of Du et al,[16] the following assumption is made.

Assumption 2. For later expression, define the expression $\zeta(k) \triangleq K(d(k) - 2d(k-1) + d(k-2))$. There is a positive constant δ^* such that $\|\zeta(k)\| \leq \delta^*$, $k = 0, 1, 2 \dots$, and the positive constant δ^* is in the order of $O(T_s^3)$.

3.3 | Reaching law

To reduce the width of the QSMB and avoid the chattering phenomenon, the following reaching law is proposed:

$$s(k+1) = (1 - q_1T_s)s(k) - q_2T_s \tanh\left(|s(k)|^{1-\alpha}\right) \mathrm{sig}^{\alpha}(s(k)) + \zeta(k), \tag{7}$$

where $q_1 > 0$ and $q_2 > 0$ are two tunable parameters satisfying that

$$2 > 2q_1T_s + q_2T_s > 1 \tag{8}$$

$$1 - q_1T_s - q_2T_s > 0, \tag{9}$$

and $\mathrm{sig}^{\alpha}(s(k)) = |s(k)|^{\alpha}\mathrm{sgn}(s(k))$ with $0 < \alpha < 1$ is a continuous nonsmooth function. For the sake of convenience, we define the following notations:

$$\rho \triangleq \frac{\delta^*}{1 - q_1T_s - q_2T_s}, \quad \mu \triangleq \frac{1 - q_1T_s}{1 - q_1T_s - q_2T_s}\delta^* + \delta^*.$$

It is easy to verify that $\mu < \rho$. Before proceeding further, we need the following lemma.

Lemma 3. *The inequality* $\tanh(|x|) \leq |x|$ *holds for any* $x \in \mathbb{R}$.

Proof. Because $\tanh(-x) = -\tanh(x)$ holds for $x < 0$, we only discuss the situation of $x > 0$.

Define $g(x) = x - \tanh(x)$, and it is easy to obtain that

$$\begin{aligned}\frac{\mathrm{d}g(x)}{\mathrm{d}x} &= 1 - \frac{(\mathrm{e}^x + \mathrm{e}^{-x})^2 - (\mathrm{e}^x - \mathrm{e}^{-x})^2}{(\mathrm{e}^x + \mathrm{e}^{-x})^2} \\ &= \tanh^2(x) > 0,\end{aligned}$$

which is together with the fact $g(0) = 0$. It implies that $g(x) > 0$ always holds for $x > 0$, and thus, $\tanh(|x|) \leq |x|$ holds for any $x \in \mathbb{R}$. □

Theorem 1. *For the reaching law (7) and any initial state* $s(0)$ *satisfying* $|s(0)| > \rho$, $s(k)$ *will converge to the following QSMB* Ω *within at most* M^* *steps:*

$$\Omega \triangleq \{s(k) \,|\, |s(k)| \leq \mu\} \tag{10}$$

and $M^* = \lceil m^* \rceil + 2$ *with*

$$m^* \triangleq \frac{s^2(0) - \rho^2}{(q_2 T_s \tanh(\rho^{1-\alpha})\rho^\alpha)^2}, \tag{11}$$

and $\lceil m^* \rceil$ *denotes the floor operation, ie, the maximal integer bounded below the real number* m^**, and once* $s(k)$ *enter the region* Ω*, it cannot escape from the region.*

Proof. The proof is divided into two steps.

Step I: We will prove that, if $|s(k)| \geq \rho$, $s(k)$ will converge to the QSMB Ω within finite steps from any initial state $s(0) > \rho$. Consider the Lyapunov function $V(k) = \frac{1}{2}s^2(k)$ and define $\Delta s(k) = s(k+1) - s(k)$. In consideration of (7), it can be verified that

$$\begin{aligned}\Delta V(k) &= \frac{1}{2}s^2(k+1) - \frac{1}{2}s^2(k)\\ &= s(k+1)\Delta s(k) - \frac{1}{2}(\Delta s(k))^2\\ &= -\left((1-q_1T_s)s(k) - q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)\mathrm{sig}^\alpha\left(s(k)\right) + \zeta(k)\right)\\ &\quad\times\left(q_1T_ss(k) + q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)\mathrm{sig}^\alpha\left(s(k)\right) - \zeta(k)\right) - \frac{1}{2}\Delta s(k)^2.\end{aligned}$$

If $s(k) \geq \rho$, it is easy to prove that

$$\begin{aligned}&(1-q_1T_s)s(k) - q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)\mathrm{sig}^\alpha\left(s(k)\right) + \zeta(k)\\ &\quad\geq (1-q_1T_s)s(k) - q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^\alpha - \delta^*\\ &\quad\geq (1-q_1T_s)s(k) - q_2T_s|s(k)|^{1-\alpha}|s(k)|^\alpha - \delta^*\\ &\quad= (1-q_1T_s-q_2T_s)s(k) - \delta^* \geq 0.\end{aligned}$$

In light of (8), the following inequality also holds:

$$\begin{aligned}&q_1T_ss(k) + q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)\mathrm{sig}^\alpha\left(s(k)\right) - \zeta(k)\\ &\quad\geq q_1T_ss(k) + q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^\alpha - \delta^*\\ &\quad\geq q_1T_s\frac{\delta^*}{1-q_1T_s-q_2T_s} + q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^\alpha - \delta^*\\ &\quad\geq q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^\alpha > 0.\end{aligned}$$

If $s(k) \leq -\rho$, we can show that the following inequality holds if (9) is satisfied. That is,

$$\begin{aligned}&(1-q_1T_s)s(k) - q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)\mathrm{sig}^\alpha\left(s(k)\right) + \zeta(k)\\ &\quad\leq (1-q_1T_s)s(k) + q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^\alpha + \delta^*\\ &\quad\leq (1-q_1T_s)s(k) + q_2T_s|s(k)|^{1-\alpha}|s(k)|^\alpha + \delta^*\\ &\quad= (1-q_1T_s-q_2T_s)s(k) + \delta^* \leq 0.\end{aligned}$$

Moreover, in view of (8), it is easily derived that

$$\begin{aligned}&q_1T_ss(k) + q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)\mathrm{sig}^\alpha\left(s(k)\right) - \zeta(k)\\ &\quad\leq q_1T_ss(k) - q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^\alpha + \delta^*\\ &\quad\leq -q_1T_s\frac{\delta^*}{1-q_1T_s-q_2T_s} - q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^\alpha + \delta^*\\ &\quad\leq -q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^\alpha < 0.\end{aligned}$$

Therefore, based on the above discussions, it is obvious that, if $|s(k)| \geq \rho$,

$$\begin{aligned}\Delta V(k) &\leq -\frac{1}{2}(\Delta s(k))^2 \\ &\leq -\frac{1}{2}\left(q_2 T_s \tanh(\rho^{1-\alpha})\rho^{\alpha}\right)^2,\end{aligned}$$

and thus,

$$s^2(k+1) - s^2(k) \leq -\left(q_2 T_s \tanh(\rho^{1-\alpha})\rho^{\alpha}\right)^2,$$

the following inequality can be obtained by iterations:

$$s^2(m) \leq s^2(0) - m\left(q_2 T_s \tanh(\rho^{1-\alpha})\rho^{\alpha}\right)^2.$$

Setting $s^2(0) - m(q_2 T_s \tanh(\rho^{1-\alpha})\rho^{\alpha})^2 = \rho^2$ yields that m^* defined in (11). Thus, $|s(k + M^* - 1)| < \rho$ holds after $\lceil m^* \rceil + 1$ steps.

Step II: We will prove the fact that, if $|s(k)| \leq \rho$, then $|s(k+1)| < \mu$. If $0 < s(k) < \rho$, we can demonstrate that

$$\begin{aligned}s(k+1) &= (1 - q_1 T_s)s(k) - q_2 T \tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^{\alpha} + \zeta(k) \\ &< (1 - q_1 T_s)s(k) + \zeta(k) \\ &< \frac{1 - q_1 T_s}{1 - q_1 T_s - q_2 T_s}\delta^* + \delta^* \\ &= \mu\end{aligned}$$

and

$$\begin{aligned}s(k+1) &= (1 - q_1 T_s)s(k) - q_2 T_s \tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^{\alpha} + \zeta(k) \\ &> (1 - q_1 T_s)s(k) - q_2 T_s s(k) + \zeta(k) \\ &> -\delta^* > -\mu\end{aligned}$$

are satisfied. Thus, $|s(k+1)| < \mu$ holds.

On the other hand, if $-\rho < s(k) < 0$, we can also show that

$$\begin{aligned}s(k+1) &= (1 - q_1 T_s)s(k) + q_2 T_s \tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^{\alpha} + \zeta(k) \\ &< (1 - q_1 T_s)s(k) - q_2 T_s s(k) + \zeta(k) \\ &< \delta^* < \mu\end{aligned}$$

and

$$\begin{aligned}s(k+1) &= (1 - q_1 T_s)s(k) + q_2 T_s \tanh\left(|s(k)|^{1-\alpha}\right)|s(k)|^{\alpha} + \zeta(k) \\ &> (1 - q_1 T_s)s(k) + \zeta(k) \\ &> -\frac{1 - q_1 T_s}{1 - q_1 T_s - q_2 T_s}\delta^* - \delta^* \\ &= -\mu\end{aligned}$$

always hold, and it implies $|s(k+1)| < \mu$.

Therefore, we can conclude that, if $|s(k)| \leq \rho$, $|s(k+1)| < \mu$ also holds, which implies that once $s(k)$ enter the region Ω within M* steps, it cannot escape from it. □

Remark 3. As is shown in Figure 1, it has been shown that, in Region I, $s(k)$ satisfies the reaching conditions and reaches into Region II in $\lceil m^* \rceil + 1$ steps. Then, there are two situations that may occur for $s(k)$: (1) $s(k)$ gets straight into Region III (eg, $|s(k)| < \mu$), and according to Theorem 1, once $s(k)$ enters Region III, it will never escape it; (2) $s(k)$

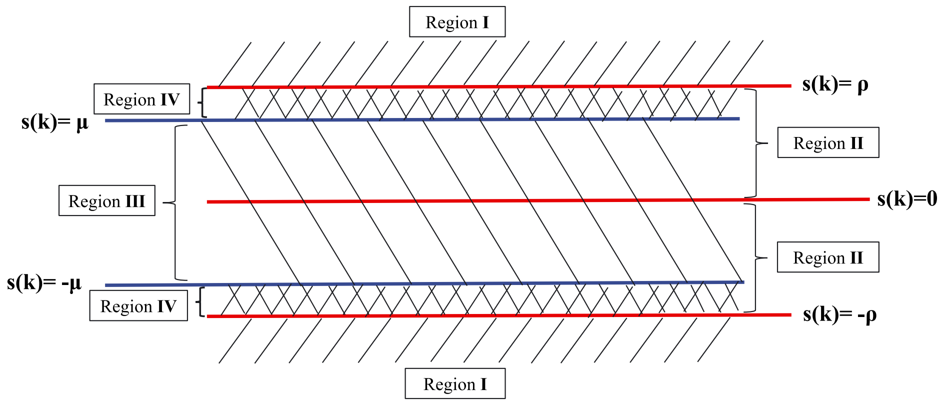

FIGURE 1 Illustration of the relation between ρ and μ [Colour figure can be viewed at wileyonlinelibrary.com]

enters into region IV (eg, $\mu < |s(k)| < \rho$), and it can always access Region III in the next step and no longer escape from it. Therefore, $s(k)$ can enter QSMB Ω within at most $M^* = \lceil m^* \rceil + 2$ steps.

Remark 4. In reaching law (7), the hyperbolic tangent function term $\tanh(|s(k)|^{1-\alpha})$ is introduced to scale the gain of the nonsmooth term $\mathrm{sig}^{\alpha}(s(k))$. More specifically, on the one hand, if the state trajectory is far from the sliding surface, $\tanh(|s(k)|^{1-\alpha})$ tends to 1, and thus, the reaching speed of reaching law (7) is almost same as the reaching law proposed in the work of Du et al.[16] On the other hand, when the system state is near to the sliding mode surface, $\mathrm{sig}^{\alpha}(s(k))$ may result in the undesired chattering, while $\tanh(|s(k)|^{1-\alpha}) \approx |s(k)|^{1-\alpha}$, the reaching law (7) reduces as $s(k+1) = (1 - q_1T_s - q_2T_s)s(k) + \zeta(k)$, which ensures that the chattering phenomenon is avoided and convergence of the reaching law (7) is still guaranteed.

Remark 5. In the work of Du et al,[16] the QSMB is proposed as $\bar{\Omega} = \{s(k) : |s(k)| \le \bar{\rho}\}$ where $\bar{\rho}$ is defined as

$$\begin{aligned}\bar{\rho} &= \psi(\alpha) \cdot \max\left\{\left(\frac{\bar{\delta}^*}{q_2T_s}\right)^{\frac{1}{\alpha}}, \left(\frac{q_2T_s}{1-q_1T_s}\right)^{\frac{1}{1-\alpha}}\right\} \\ &= \psi(\alpha) \cdot \max\left\{(O(T_s))^{\frac{1}{\alpha}}, (O(T_s))^{\frac{1}{1-\alpha}}\right\},\end{aligned}$$

where T_s is sampling time, $0 < \alpha < 1$, $\bar{\delta}^* = O(T_s^2)$, and $\psi(\alpha) = 1 + \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}$. It should be pointed out that: (1) the bound $\bar{\rho}$ is explicitly related to the value of α, and when $\alpha = \frac{1}{2}$, the smallest width of $\bar{\Omega}$ can be determined as $\bar{\rho} = O(T_s^2)$; (2) the bound $\bar{\rho}$ may be independent of the disturbance estimation error bound $\bar{\delta}^*$ owing to the maximum operation. In this paper, the QSMB is determined as (10), and it is obvious that ρ is only dependent on the disturbance estimation error bound δ^* for given q_1, q_2, and T_s, which indicates that the more accurate the disturbance estimation error is, the smaller bound of QSMB is. Besides, the precision of QSMB Ω is not affected by the value of α, and thus, more freedom is available for the choice of α to achieve better control performance.

3.4 | Chattering-free sliding mode controller

With the proposed reaching law, the chattering-free sliding mode controller can be designed. Moreover, the following theorem can be concluded.

Theorem 2. *For the discrete time system (2) with disturbance observer (3) and sliding mode surface (6), the following sliding mode controller can be designed by applying the reaching law (7):*

$$u(k) = -(K\Gamma)^{-1}\left[q_1T_ss(k) + q_2T_s\tanh\left(|s(k)|^{1-\alpha}\right)\mathrm{sig}^{\alpha}\left(s(k)\right) - s(k) + K\Phi x(k) + K\left(2\hat{d}(k-1) - \hat{d}(k-2)\right)\right]. \tag{12}$$

Proof. The controller (12) can be obtained by comparing system (2) with reaching law (7) immediately. □

Remark 6. In the work of Du et al,[16] the nonsmooth function $\text{sig}^{\alpha}(s(k))$ is incorporated in the reaching law and the controller, and the undesired chattering occurs if $s(k)$ is small enough. It is worth mentioning that the proposed controller (12) is chattering-free owing to the $\text{sgn}(s(k))$ or $\text{sig}^{\alpha}(s(k))$ is replaced by $\tanh(|s(k)|^{1-\alpha})\text{sig}^{\alpha}(s(k))$.

Remark 7. In other works,[15,16,18,19] $d(k-1)$ is used as the disturbance estimate $\hat{d}(k)$ in the controller. However, in some practical situations, the disturbances are difficult or even impossible to be measured physically by sensors. Instead, the disturbance estimates, generated by disturbance (3), are utilized in controller (12), which makes it possible for the practical implementation of the proposed control approach.

4 | NUMERICAL EXAMPLE

Consider the high-precision position control for a piezomotor-driven linear stage system.[16] The dynamical equation is

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= \frac{k_f}{m}u(t) - \frac{k_v}{m}x_2(t) + f(t), \end{aligned} \tag{13}$$

where $x_1(t)$ is the linear displacement, $x_2(t)$ is the linear velocity, $u(t)$ is the voltage input, and $f(t)$ is an unknown disturbance, and the parameters are selected as $m = 1$, $k_v = 144$, $k_f = 6$, and the unknown disturbance is given by $f(t) = 1 + 2.2\sin(0.5\pi t)$ for $t > 0$. The initial condition is $x(0) = [-1, 0]^T$. The sampling period is selected as $T_s = 1$ ms. The sliding surface parameter K is chosen as $K = [0.5, 0.5]$.

In what follows, a comparative simulation is performed between the DSMC approaches proposed in this paper and in the work of Du et al.[16] The sliding mode controller proposed in the work of Du et al[16] is

$$u(k) = -(K\Gamma)^{-1}\left[q_1 T_s s(k) + q_2 T_s \text{sig}^{\alpha}(s(k)) - s(k) + K\Phi x(k) + K\hat{d}(k)\right], \tag{14}$$

where $\hat{d}(k)$ is obtained by

$$\hat{d}(k) = d(k-1). \tag{15}$$

The simulation results are shown in Figures 2 and 3. It is obviously that, by applying the proposed DSMC approach, the chattering phenomenon is avoided thoroughly. By the way, it can be seen from Figure 4, the proposed disturbance observer also outperform the common disturbance estimation approach in other works.[15,16,18,19]

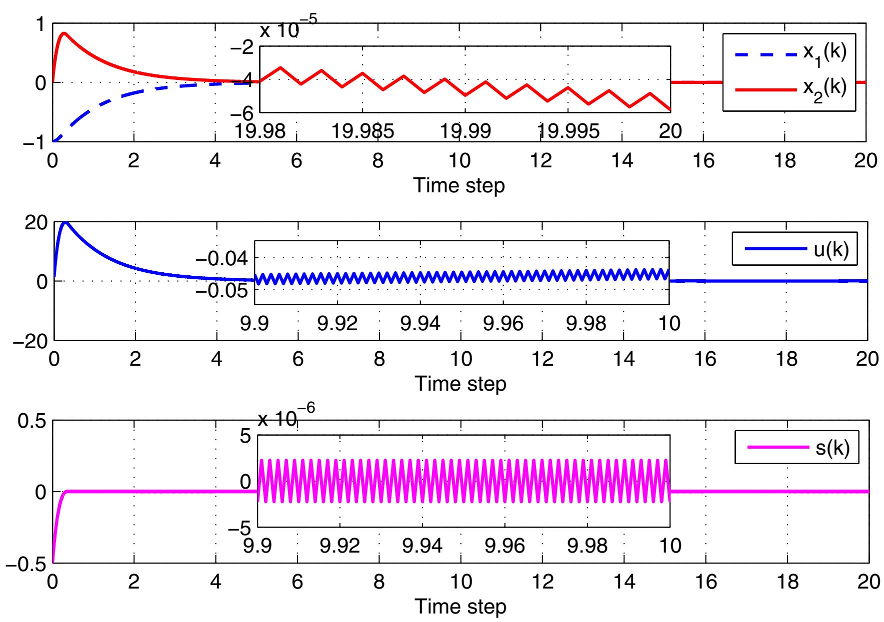

FIGURE 2 Simulation results by applying controller (14) in the work of Du et al[16] [Colour figure can be viewed at wileyonlinelibrary.com]

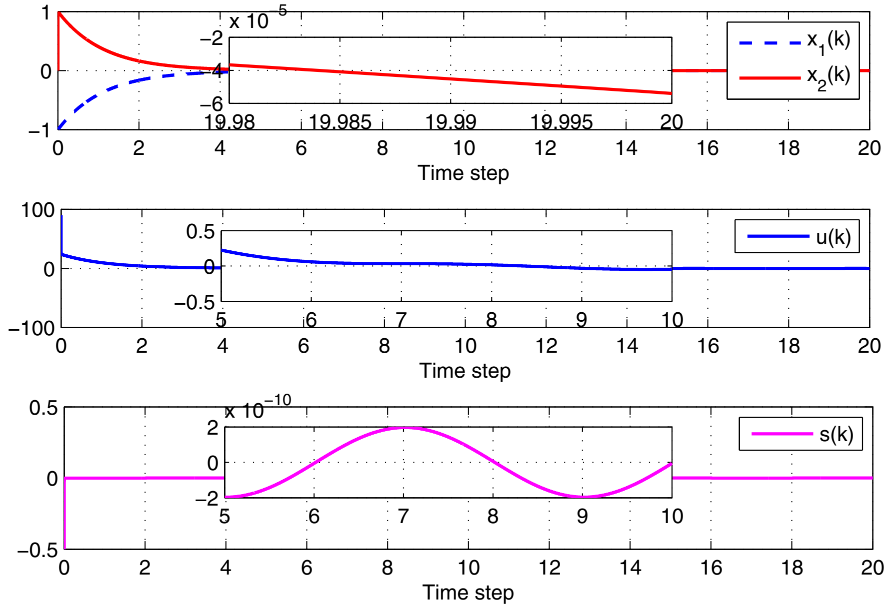

FIGURE 3 Simulation results by applying controller (12) in this paper [Colour figure can be viewed at wileyonlinelibrary.com]

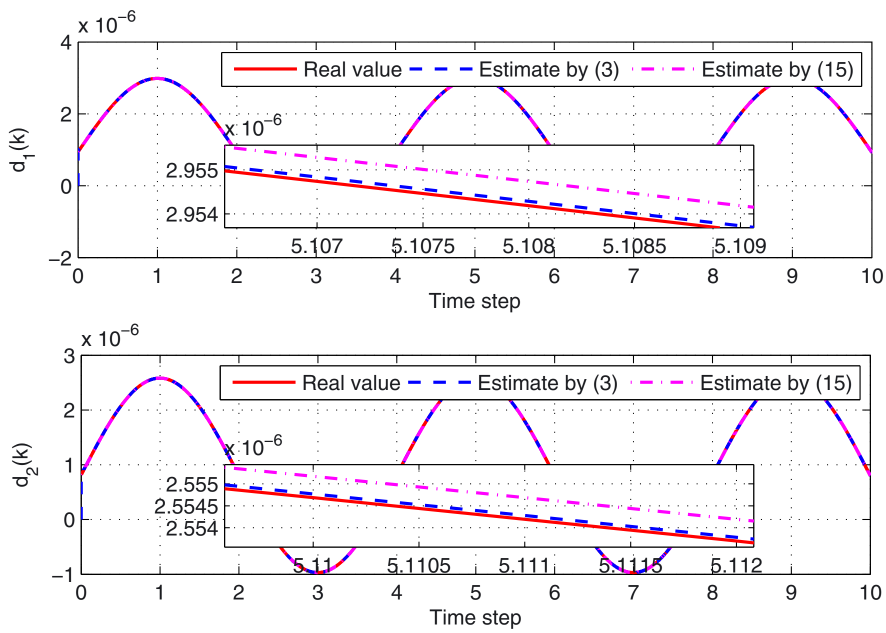

FIGURE 4 Comparative results on disturbance estimation [Colour figure can be viewed at wileyonlinelibrary.com]

5 | CONCLUSIONS

This paper has investigated the disturbance observer–based chattering-free DSMC approach for discrete-time systems under disturbances. By making the utmost of the system states and control inputs, the discrete disturbance observer has been proposed to obtain precise estimation of unknown disturbance, and a new reaching law has also been established by incorporating a nonsmooth function with a dynamically adjusted gain parameter. Then, the chattering-free sliding mode controller has been developed to actively counteract the disturbances. The simulation example is finally given to demonstrate the validity of the established control approaches. In the future works, it is meaningful to consider the output-based disturbance observer design and discrete-time nonlinear disturbance observer design problems.

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ORCID

Jinhui Zhang https://orcid.org/0000-0002-2405-894X
Yuanqing Xia https://orcid.org/0000-0002-5977-4911

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