

## Model Predictive Control: Theory, Computation, and Design 2nd Edition

### Errata for Second Edition, First Printing

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June 3, 2020

- Page 52, Figure 1.6. Change  $L_x$  and  $L_d$  to  $\tilde{L}_x$  and  $\tilde{L}_d$ , respectively, in the figure, and add the following sentence to the caption, "For simplicity we show the steady-state Kalman predictor form of the state estimator where  $\hat{x} := \hat{x}(k | k-1)$  and  $\tilde{L}_x := AL_x + B_d L_d$  and  $\tilde{L}_d := L_d$ ." Thanks to Pratyush Kumar and Travis Arnold of UW for pointing out this erratum.
- Page 93, Equation (2.1). Add  $f: \mathbb{X} \times \mathbb{U} \rightarrow \mathbb{X}$  and that sets  $\mathbb{X}$  and  $\mathbb{U}$  are assumed closed.
- Page 97, ninth line from top, change (2.7) to (2.5).
- Page 97, tenth line from bottom. Change

In by far the majority of applications the control is constrained. Nevertheless, it is of theoretical interest to consider the case when the optimal control problem has no constraints on the control.

to

In by far the majority of applications the set of controls  $\mathbb{U}$  is bounded. Nevertheless, it is of theoretical interest to consider the case when  $\mathbb{U}$  is not bounded; e.g., when the optimal control problem has no constraints on the control.

Thanks to Doug Allan of UW for his help in propagating this change through the rest of Section 2.4.2, page 120.

- Page 97, fourth line from bottom. Change from:

$$\tilde{\mathcal{U}}_N^c(x) := \{\mathbf{u} \mid V_N(x, \mathbf{u}) \leq c\}$$

to:

$$\tilde{\mathcal{U}}_N^c(x) := \{\mathbf{u} \in \mathcal{U}_N(x) \mid V_N(x, \mathbf{u}) \leq c\}$$

- Page 98, Assumption 2.3, lines 1-2. Change "If there are control constraints,  $\mathcal{U}_N(x)$  is defined ..." to "The set  $\mathcal{U}_N(x)$  is defined ..."
- Page 98, Assumption 2.3, line 4. Change "If there are no control constraints ( $\mathbb{Z} = \mathbb{X} \times \mathbb{R}^m$ )," to "If  $\mathbb{U}$  is unbounded."
- Page 98, first line of part (b) of proof. Change "If there are control constraints," to "If  $\mathbb{U}$  is bounded."
- Page 98, fifth line in part (b) of proof. Change

If instead there are no control constraints, that  $\tilde{\mathcal{U}}_N^c(x)$  is closed follows from the fact that  $V_N(\cdot)$  is continuous.

to

If instead  $\mathbb{U}$  is unbounded, the set  $\tilde{\mathcal{U}}_N^c := \{\mathbf{u} \mid V_N(x, \mathbf{u}) \leq c\}$  for  $c \in \mathbb{R}_{>0}$  is closed for all  $c$  and  $x$  because  $V_N(\cdot)$  is continuous;  $\tilde{\mathcal{U}}_N^c(x)$  is the intersection of this set with  $\mathcal{U}_N(x)$ , just shown to be closed. So  $\tilde{\mathcal{U}}_N^c(x)$  is the intersection of closed sets and is closed.

- Page 98, lines 7-12 from bottom. Change  $\mathcal{U}_N^c(x)$  to  $\tilde{\mathcal{U}}_N^c(x)$  (five places).
- Page 98, fifth line from bottom. Change  $\tilde{\mathcal{U}}_N(x)$  to  $\tilde{\mathcal{U}}_N^c(x)$ .
- Page 104, Theorem 2.7. Replace "Assumptions 2.2 and 2.3" with "Assumptions 2.2 and 2.3 ( $\mathbb{U}$  bounded)."
- Page 108, first sentence, second paragraph. Change "For all  $j \in \mathbb{I}_{0:N-1}$ , let  $V_j(x, \mathbf{u})$ ,  $\mathcal{U}_j(x)$ ,  $\mathbb{P}_j(x)$ , and  $V_j^0(x)$  be defined, respectively, by (2.3), (2.4), (2.5), and (2.6), with  $N$  replaced by  $j$ ." to:  
"For all  $j \in \mathbb{I}_{0:N-1}$ , let  $V_j(x, \mathbf{u})$ ,  $\mathcal{U}_j(x)$ ,  $\mathbb{Z}_j$ ,  $\mathbb{P}_j(x)$  (and  $V_j^0(x)$ ) be defined, respectively, by (2.3), (2.5), (2.6), and (2.7), with  $N$  replaced by  $j$ ."
- Page 110, Proposition 2.10. Replace "Assumptions 2.2 and 2.3" with "Assumptions 2.2 and 2.3 ( $\mathbb{U}$  bounded)."
- Page 110, Proposition 2.10 (a), first line. Change " $V_j(\cdot)$  is continuous in  $\mathbb{Z}_j$ ," to " $V_j(\cdot)$  is continuous in  $\mathbb{Z}_j$ ."
- Page 110, Proposition 2.10 (b), last sentence. Change "In addition, the set  $X_N$  is positive invariant for  $x^+ = f(x, \kappa_N(x))$ ." to "In addition, the sets  $X_j$  and  $X_{j-1}$  are positive invariant for  $x^+ = f(x, \kappa_j(x))$  for  $j \in \mathbb{I}_{\geq 1}$ ."
- Page 110, proof of Proposition 2.10 (b), last sentence. Change "That  $X_N$  is positive invariant for  $x^+ = f(x, \kappa_N(x))$  follows from (2.10), which shows that  $\kappa_N(\cdot)$  steers every  $x \in X_N$  into  $X_{N-1} \subseteq X_N$ " to "That  $X_j$  is positive invariant for  $x^+ = f(x, \kappa_j(x))$  follows from (2.10), which shows that  $\kappa_j(\cdot)$  steers every  $x \in X_j$  into  $X_{j-1} \subseteq X_j$ . Since  $X_{j-1} \subseteq X_j$ ,  $\kappa_j(\cdot)$  also steers every  $x \in X_{j-1}$  into  $X_{j-1}$ , so  $X_{j-1}$  is positive invariant under control law  $\kappa_j(\cdot)$  as well."
- Page 111, proof of Proposition 2.10 (c), lines 5-10. Replace the following  
this is possible since  $x_i \in X_j$  implies  $x_i \in \mathbb{X} := \{x \in \mathbb{R}^n \mid \mathbb{U}(x) \neq \emptyset\}$ . Since  $\mathbb{Z}_j$  is closed, there exists a subsequence, indexed by  $\mathbb{I}$ , such that  $z_i = (x_i, u_i) \rightarrow \tilde{z} = (\tilde{x}, \tilde{u}) \in \mathbb{Z}_j$  as  $i \rightarrow \infty$ ,  $i \in \mathbb{I}$ . But  $X_j = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U}(x) \text{ such that } f(x, u) \in X_{j-1}\} = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U}(x) \text{ such that } (x, u) \in \mathbb{Z}_j\}$  (see (2.11)). Hence  $\tilde{x} \in X_j$  so that  $X_j$  is closed.

with the following

this is possible since  $x_i \in X_j$  implies  $x_i \in \{\mathbb{X} \mid \mathcal{U}_j(x) \neq \emptyset\}$ . Since  $\mathcal{U}_j(x) \subseteq \mathbb{U}$  and  $\mathbb{U}$  is bounded, by the Bolzano-Weierstrass theorem there exists a subsequence, indexed by  $\mathbb{I}$ , such that  $u_i \rightarrow \bar{u}$  (and  $x_i \rightarrow \bar{x}$ ) as  $i \rightarrow \infty$ ,  $i \in \mathbb{I}$ . The sequence  $(x_i, u_i) \in Z_j$ ,  $i \in \mathbb{I}$  converges, and, since  $Z_j$  is closed,  $(\bar{x}, \bar{u}) \in Z_j$ . Therefore  $f(\bar{x}, \bar{u}) \in X_{j-1}$  and  $\bar{x} \in X_j$  so that  $X_j$  is closed.

19. Page 111, first line of part (d). Change  $X_f$  to  $\mathbb{X}_f$ .
20. Page 111, 11th line from the bottom. Change “ $f^{-1}(\cdot)$  is bounded on bounded sets if  $f(x, u) = Ax + Bu$  and  $A$  is nonsingular or if  $f(\cdot)$  is Lipschitz in  $x$ ,” to “ $f^{-1}(\cdot)$  is bounded on bounded sets if  $\mathbb{U}$  is bounded and either  $f(x, u) = Ax + Bu$  and  $A$  is nonsingular, or  $f_c(x, u)$  is Lipschitz in  $x$ .”
21. Page 114, Assumption 2.14 (a). Change  $\ell(x, u)r$  to  $\ell(x, u)$  in the last inequality.
22. Page 115, Proposition 2.15. Replace “Assumptions 2.2 and 2.3” with “Assumptions 2.2 and 2.3 ( $\mathbb{U}$  bounded).”
23. Page 115, second line before Proposition 2.16. Change  $X_f$  to  $\mathbb{X}_f$ .
24. Page 115, Proposition 2.16. Replace “Assumptions 2.2 and 2.3” with “Assumptions 2.2 and 2.3 ( $\mathbb{U}$  bounded).”
25. Page 116, third line from bottom. Change  $\mathbb{U}^N$  to  $\mathcal{U}_N(x)$ . Thanks to Doug Allan of UW for pointing out this erratum.
26. Page 117, fourth line from top. Delete the sentence, “Since there are no state or terminal constraints, the state sequence  $\tilde{x}$  is clearly feasible if  $u \in \mathbb{U}$ .”
27. Page 117, sixth line from bottom. Delete the repeated phrase, “if, for all  $x \in \mathbb{X}_f$ , there exists a  $u$  such that” before the displayed equation.
28. Page 118, Proposition 2.18. Replace “Assumptions 2.2 and 2.3” with “Assumptions 2.2 and 2.3 ( $\mathbb{U}$  bounded).”
29. Page 118, second displayed equation. Change  $X_f$  to  $\mathbb{X}_f$ .
30. Page 119, 11th line from top. Change the phrase  

The monotonicity property can be used to establish the descent property of  $V_N^0(\cdot)$   
 proved in Theorem 2.19 by noting that

to:

The monotonicity property can also be used to establish the (previously established) descent property of  $V_N^0(\cdot)$  by noting that
31. Page 119, Theorem 2.19. Replace “ $X_N$ ” with “ $X_N$  ( $\tilde{X}_N^c$ , for each  $c \in \mathbb{R}_{>0}$ )” (two places).
32. Page 120. Insert at top of page.

For the proof with  $\mathbb{U}$  unbounded, note that the lower bound and descent condition remain satisfied as before. For the upper bound, if  $\mathbb{X}_f$  contains the origin in its interior, we have that, since  $V_f(\cdot)$  is continuous, for each  $c > 0$  there exists  $0 < \tau \leq c$ , such that  $\text{lev}_\tau V_f$  contains a neighborhood of the origin and is a subset of both  $\mathbb{X}_f$  and  $\tilde{X}_N^c$ . One can then show that  $V_N^0(\cdot) \leq V_f(\cdot)$  for each  $N \geq 0$  on this sublevel set, and therefore  $V_N^0(\cdot)$  is continuous at the origin so that again Proposition B.25 applies, and Assumption 2.17 is satisfied on  $\tilde{X}_N^c$  for each  $c \in \mathbb{R}_{>0}$ .

33. Page 122, Proposition 2.24. Replace “ $X_N$ ” with “ $X_N$  ( $\tilde{X}_N^c$ , for each  $c \in \mathbb{R}_{>0}$ )” (two places).
34. Page 123, 6th line from bottom. Change  $x(i) \in \mathbb{X}(k)$  to  $x(i) \in \mathbb{X}(i)$  and  $u(k) \in \mathbb{U}(i)$  to  $u(i) \in \mathbb{U}(i)$ . Thanks to Robin Strässer of Universität Stuttgart for pointing out this and the next several errata on pages 124–130.
35. Page 124, Assumption 2.25. Change  $(x, u) \mapsto V_f(x, u, i)$  to  $x \mapsto V_f(x, i)$ .
36. Page 129, Equation (2.21). Change  $V_N^0(f(x, \kappa_N(x, i)))$  to  $V_N^0(f(x, \kappa_N(x, i), i + 1))$ .
37. Page 130, fifth line below Table 2.1. Change  $x^+ = f(x, \kappa_N(x, i))$  to  $x^+ = f(x, \kappa_N(x, i), i)$ .
38. Page 130, 12th line from bottom. Change  $x+$  to  $x^+$ .
39. Page 132, 8th line of Section 2.5.1. Change  $C'Q_y C$  to  $x' C' Q_y C x$ . Thanks to Professor James Gibson of George Mason University for pointing out this erratum.
40. Page 136, second displayed equation. Change  $A'$  to  $A$ . Thanks to Professor James Gibson of George Mason University for pointing out this erratum.
41. Page 141, first line of quoted material. Change  $X_f$  to  $\mathbb{X}_f$ .
42. Page 142, third line. Change  $x' Q(i) x(i)$  to  $x' Q(i) x$ .
43. Page 142, line 17. Change  $x+$  to  $x^+$ .
44. Page 146, ninth line from top. Replace  $i \in \mathbb{I}_{i:N-1}$  with  $j \in \mathbb{I}_{i:N-1}$ . Thanks to Danylo Malyuta of U. Washington for pointing out this erratum.
45. Page 201, tenth line from bottom. Change  $z(k)$  to  $\bar{x}(k)$ .
46. Page 211, sixth line. Change  $f(x, \kappa_f(x))$  to  $\tilde{f}(x, \kappa_f(x))$ .
47. Page 212, fourth line from bottom. Change  $f(R_c, U, W)$  to  $f(R_c, \mathbb{U}, W)$ .
48. Page 213, ninth line. Change invariant to invariance.
49. Page 216, (3.17). Change  $V_N(x, \mu)$  to  $V_N(x, \boldsymbol{\mu})$ .
50. Page 216, 17th line from bottom. Change  $\mu_{N-1}$  to  $\mu_{N-1}(\cdot)$ .
51. Page 217, third line. Change  $f(x, u)$  to  $f(x, u, \mathbb{W})$ .
52. Page 221, last equation, change inequality to equality.
53. Page 222, first line. Change  $\ell(x, \kappa_N(x), w(0))$  to  $\ell(x, \kappa_N(x), w(0))$ .

54. Page 222, lines 10, 12, and 15. Change subscript  $0, N - 2$  to  $0 : N - 2$ .
55. Page 223, fifth line from bottom. Change “can done” to “can be done.”
56. Page 227, 13th line from bottom. Change  $S_K(i)$  to  $S(i)$ . Thanks to Roushan Rezvani of Linköping University for pointing out this erratum.
57. Page 229, fifth displayed equation. Change term  $K(x - \bar{x})$  to  $BK(x - \bar{x})$ .
58. Page 230, tenth line. Change  $\kappa_N(\bar{x})$  to  $\bar{\kappa}_N(\bar{x})$ .
59. Page 230, 12th line from bottom. Change  $S_K(\infty \times \{0\})$  to  $S_K(\infty) \times \{0\}$ .
60. Page 232, fourth line from bottom. Change  $\mathbb{P}_N(\bar{x})$  to  $\bar{\mathbb{P}}_N(\bar{x})$ .
61. Page 233, fifth line from bottom. Change  $\mathbb{I} \geq N$  to  $\mathbb{I}_{\geq N}$ .
62. Page 234, 15th line from bottom. Change  $\mathbb{W}^i$  to  $A_K^i \mathbb{W}$ .
63. Page 234, last line. Replace last equation with  $\bar{\kappa}_N^*(x) := \bar{\kappa}_N(\bar{x}^*(x)) + K(x - \bar{x}^*(x))$ .
64. Page 237, 12th line from bottom. Change  $x$  to  $x_0$ .
65. Page 238, 14th line. Change  $\bar{u}^T Q \bar{u}$  to  $\bar{u}^T R \bar{u}$ .
66. Page 238, third line from bottom. Change  $z$  to  $\bar{x}$ .
67. Page 239, eighth line from bottom. Change  $\mathbf{x}$  to  $\bar{\mathbf{x}}$ .
68. Page 241, fifth line from bottom. Change  $V_N^0(x, t)^+$  to  $V_N^0((x, t)^+)$ .
69. Page 249, 14th line. Change  $\geq 1 - \varepsilon$  to  $\geq 1 - \varepsilon\}$ .
70. Page 250, eighth line from bottom. Change  $(1/2)x'Qx + u'Ru$  to  $(1/2)(x'Qx + u'Ru)$ .
71. Page 251, 12th line from bottom. Change  $\bar{x} \in \mathbb{X}_f$  to  $\bar{x}(N) \in \mathbb{X}_f$ .
72. Page 251, third line from bottom. Change  $\bar{\mathbb{X}}_1 \subset \bar{\mathbb{X}}$  to  $\bar{\mathbb{X}}_1 \subset \mathbb{X}$ .
73. Page 252, first line. Change  $\mathbb{P}_N(\bar{x})$  to  $\bar{\mathbb{P}}_N(\bar{x})$ .
74. Page 252, second line. Change  $\bar{\mathbb{X}}_1 \subset \bar{\mathbb{X}}$  to  $\bar{\mathbb{X}}_1 \subset \mathbb{X}$ .
75. Page 253, first displayed equation. Change  $f^0$  to  $f$ .
76. Page 254, seventh line from bottom. Change  $\mathbb{X}$  to  $\bar{\mathbb{X}}_1$ .
77. Page 274, Definition 4.8. Change the phrase

The estimate is RGAS if for all  $x_0$  and  $\bar{x}_0$ , and bounded  $(\mathbf{w}, \mathbf{v})$ , there exist functions  $\alpha(\cdot) \in \mathcal{KL}$  and  $\delta_w(\cdot) \in \mathcal{K}$  such that

to

The estimate is RGAS if there exist functions  $\alpha(\cdot) \in \mathcal{KL}$  and  $\delta_w(\cdot) \in \mathcal{K}$  such that for all  $x_0$  and  $\bar{x}_0$ , and bounded  $(\mathbf{w}, \mathbf{v})$

Thanks to Doug Allan of UW for pointing out this erratum.

78. Page 319, Change “There exists  $\delta > 0$ ” to “There exists  $\delta > 0$  and  $\beta(\cdot) \in \mathcal{KL}$  and  $\sigma(\cdot) \in \mathcal{K}$ ”. Thanks to Robin Strässer of Universität Stuttgart for pointing out this and the next several errata on pages 321–322.
79. Page 319, last line; page 322, 2nd line from bottom. Change  $f(\hat{x} + e, \kappa_N(\hat{x}), w)$  to  $f(\hat{x} + e, \kappa_N(\hat{x}), w) - e^+$ .
80. Page 321, 2nd line from bottom. Change  $f(\hat{x} - e, \kappa_N(\hat{x}), w) + e^+$  to  $f(\hat{x} + e, \kappa_N(\hat{x}), w) - e^+$ .
81. Page 322, 3rd line. Change  $x^+$  to  $f(\hat{x}, \kappa_N(\hat{x}), 0)$ .
82. Page 322, 11th line from top; 14th line from top; 19th–21st line from top; 23rd line from top; Change all  $f(\hat{x}, \kappa_N(\hat{x}))$  to  $f(\hat{x}, \kappa_N(\hat{x}), 0)$ .
83. Page 322, 13th line from top. Change  $f(x, u)$  to  $f(x, u, w)$ .
84. Page 322, 14th line from top. Change  $+e^+$  to  $-e^+$ .
85. Page 322, last line. Change  $\text{lev}_\rho \mathcal{X}_N$  to  $\text{lev}_\rho V_N^0$ .
86. Page 329, Exercise 4.7, 3rd line. Change, “let  $y(\cdot)$  be any  $\mathcal{K}$  function,” to “let  $y(\cdot)$  be any  $\mathcal{K}$  function and  $a_i \in \mathbb{R}_{\geq 0}$ ,  $i \in \mathbb{I}_{1:m}$ .”
87. Page 366, Exercise 5.5. Change  $d_H(\mathbb{A} \oplus \mathbb{C}, \mathbb{B} \oplus \mathbb{C}) = d_H(\mathbb{A}, \mathbb{B})$  to  $d_H(\mathbb{A} \oplus \mathbb{C}, \mathbb{B} \oplus \mathbb{C}) \leq d_H(\mathbb{A}, \mathbb{B})$ . Thanks to Dr. Saša V. Raković for pointing out this erratum.
88. Page 366, Exercise 5.5. Delete the phrase, “satisfying  $\mathbb{B} \subseteq \mathbb{A}$ .”
89. Page 366, Exercise 5.6. Delete the phrase, “satisfying  $\mathbb{A} \subseteq \mathbb{B}$ .”
90. Page 366, Exercise 5.8. Replace  $+$  with  $\oplus$  (two places).
91. Page 527, fifth line from bottom. Change “plus  $n$  forward sweeps,” to “plus  $m$  forward sweeps.”