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Model predictive control with adaptive disturbance prediction and its application to fatty acid distillation column control

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This paper describes the results of a joint university–industry study to control a fatty acid distillation sequence, which is plagued with severe disturbance problems. In order to solve the disturbance problem, a model predictive control algorithm is modified in terms of disturbance prediction. Assuming that the dynamics of the unmeasured disturbances is generated by an auto-regressive form, the dynamics of the disturbance can be adaptively identified by using time series data of prediction errors and inputs. Using an identified disturbance model with a process model, future outputs are predicted. Control actions are determined so that the predicted output is as close to the target value as possible. This modified model predictive control aglorithm is applied to a ratio control scheme for three distillation columns. The control system developed has been in use sucessfully for more than six years to produce commercial products.

Keywords: predictive control; adaptive inferential control, disturbance prediction

In a plant at the Wakayama factory of Kao Corporation, a mixture of saturated fatty acids is separated into four products, from which soap and detergents are made. The plant consists of an evaporator and three distillation columns. The composition of the mixture tends to vary. When feed composition and/or feed flow rate changes at a distillation column, the reflux flow rate and/or boil-up flow rate is manipulated at the column to maintain both distillate and bottom product compositions at given setpoints. Because the liquid holdup at the bottom is usually kept constant by manipulating the flow rate of the bottom product, the manipulation of boil-up rate and/or reflux flow rate produces fluctuation of flow rates for both the distillate and the bottom product. Three distillation columns are linked in series in this plant. Therefore, the composition change in the feed to the first column causes disturbances in the feed flow rate to the second column and propagates to the third. In order to achieve the highest productivity possible while maintaining the composition of all four products at desired values in the plant, it is essential to design a control system that can reject the undesirable influence of the disturbance propagation on product composition.

To suppress unmeasured disturbances and improve the regulatory performance, several different approaches have been suggested. Inferential control[1] is one of the control schemes that can counteract the effect of unmeasured disturbance on the controlled variable. It uses measurement of secondary process outputs to infer the effect of the unmeasured disturbance. The performance of the inferential control scheme depends heavily on the accuracy of the primary process model. Furthermore, unmeasured disturbance often varies naturally with respect to time, and seems to change its dynamic characteristics during operation. These changes may degrade the performance of an inferential control system design with a predetermined model. To overcome the time-varying nature of the system in the disturbance rejection problem, Ohshima *et al.*[2] and Shen *et al.*[3] independently proposed an adaptive inferential control algorithm based on the approach of model predictive control. They assumed that the dynamics of unmeasured disturbances was generated by an auto-regressive form. They identified the dynamics adaptively by using time series data of prediction errors. Then, the identified disturbance dynamics and output prediction algorithm were incorporated in a model predictive control scheme to improve the accuracy of output prediction.

In this paper, the disturbance prediction scheme used in the previous work[2] is further extended in the frame of model predictive control and the model predictive control with disturbance prediction is applied to the above-mentioned fatty acid distillation sequence. Experimental data from the plant are employed to illustrate the utility of the proposed model predictive control for composition control in the plant.

Outline of the fatty acid distillation columns

The structure of the fatty acid distillation sequence is illustrated in *Figure 1*. The plant consists of an evaporator and three packed-bed type distillation columns. The three columns are operated at less than atmospheric pressure. A mixture of saturated fatty acids, with components ranging from C_6 to C_{18} fatty acids, is fed to the first distillation column. The C_6, C_8 and C_{10} components are distilled at the first column. The bottom flow is introduced to the evaporator, where the fatty acid is vaporized and separated from impurities such as esters. The vaporized fatty acid is fed to the second distillation column, where C_{12} is distilled. At the second column, the top product is not allowed to drop below 98% C_{12} concentration. The bottom product of the second column is fed to the third one, which separates the C_{14} component from C_{16} and C_{18}. The third distillation column must be operated to keep the concentration of C_{14} in the distillate higher than 98%.

Problems in composition control of the system

The feed material to the first distillation column, a mixture of saturated fatty acids, is produced by a reactor as illustrated in *Figure 2*. Two kinds of unsaturated fatty acid oil, coconut oil and palm oil, are hydrogenated by the batch reactor. The batches produced by the reactor are temporarily stored in an intermediate tank in order to supply the saturated fatty acid continuously to the distillation system. The hydrogenating operation is carried out by alternating use of the two kinds of oil from time to time. Because the compositions of the two kinds of oil are different, as listed in *Table 1*, the concentration of fatty acids in the intermediate tank changes according to the oil being used. As a result, the feed composition to the first distillation column tends to drift high and low. In response to the disturbance in the feed composition, reflux flow rate and/or boil-up flow rate are usually manipulated to retain the given specifications for product composition. If the column-base level is controlled by manipulating the bottom-product rate, the manipulation of boil-up rate and/or reflux flow rate produces a fluctuation of flow rates for both the distillate and the bottom product. These changes in the feed composition and disturbance propagation cause a crucial problem in composition control of the plant.

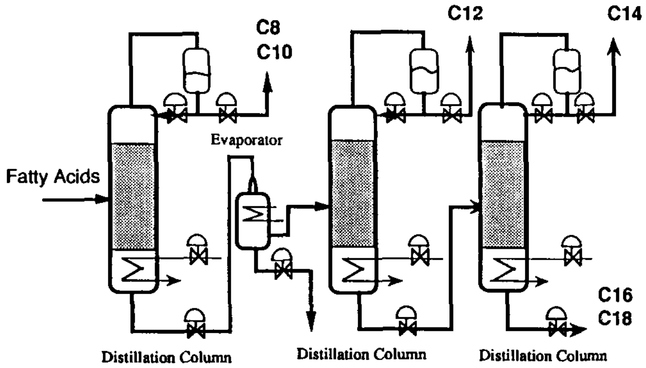

Figure 1 Fatty acid distillation columns

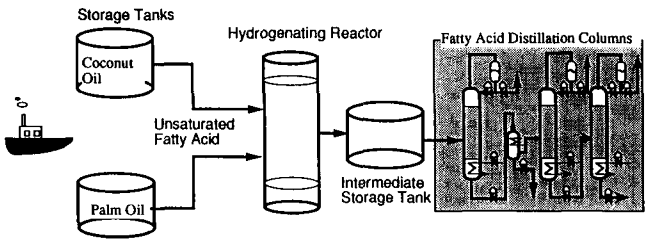

Figure 2 Hydrogenating plant

Furthermore, there is a measurement problem. On-line composition analysers have been improved in recent years and their improvement increases the feasibility of directly measuring the composition of the column product stream. However, the analyser is still expensive and takes a long time to give the analysis results, especially in cases where the specimen consists of multiple components. Because the products in this plant are multiple components of fatty acids, the product composition cannot be measured on-line and it is manually sampled and analysed by gas chromatography in the laboratory. Therefore, it is necessary to find state variables that can be used to infer the product composition.

Disturbance estimation

Auto-regressive model for unmeasured disturbance

The disturbances entering the process are persistent in nature. That is, the feed to the first column tends to vary its composition high and low. Furthermore, the disturbances in the feed pass through some parts of the dynamics of the distillation column, and continue to affect the bottom products. Therefore, a disturbance as

Table 1 Composition of oils

Composition	Coconut oil (%)	Palm oil (%)
C_6	0.898	0.96
C_8	11.279	6.00
C_{10}	7.748	4.64
C_{12}	49.034	49.96
C_{14}	16.071	14.87
C_{16}	7.075	7.79
C_{18}	7.257	14.51
Esters	0.637	1.27

it appears in the output measurement has some dynamics and exhibits a dependence on its prior values.

In order to consider such a dependence of the disturbance on its prior values, the dynamics of an unmeasured disturbance is expressed in an auto-regressive (AR) form[13] as follows.

$$\Delta v(t) = c_1\Delta v(t-1) + c_2\Delta v(t-2) + \ldots + c_q\Delta v(t-q) + w(t) \quad (1)$$

$$\Delta v(t) = z^{-1}C(z^{-1})\Delta v(t) + w(t) \quad (2)$$

where $v(t)$ denotes unmeasured disturbance at time t, and $w(t)$ expresses white noise with zero-mean value. z^{-1} is the delay operator defined by $z^{-1}v(t) = v(t-1)$. Δ is the differencing operator, i.e. $(1 - z^{-1})$. $C(z^{-1})$ is a polynomial in the delay operator z^{-1}, i.e.:

$$C(z^{-1}) := c_1 + c_2z^{-1} + \ldots c_qz^{-q+1} \quad (3)$$

Clarke *et al.*[4] stated that disturbances encountered in practice are nonstationary. There are two principal disturbances: random steps occurring at random times and Brownian motion.

MacGregor[5] also pointed out that the disturbances that we are usually faced with in process control do not have a fixed mean or level, but rather are drifting or nonstationary. He mentioned that such disturbances can usually be made stationary by taking differences between successive values and that the differenced data can be expressed by the stationary models. The model used here is one of the stationary models of the differenced disturbances.

The output $y(t)$ of the process is usually a function of both past input actions and disturbance $v(t)$ as illustrated in *Figure 3a*. That is, the output of the process is generated by the form:

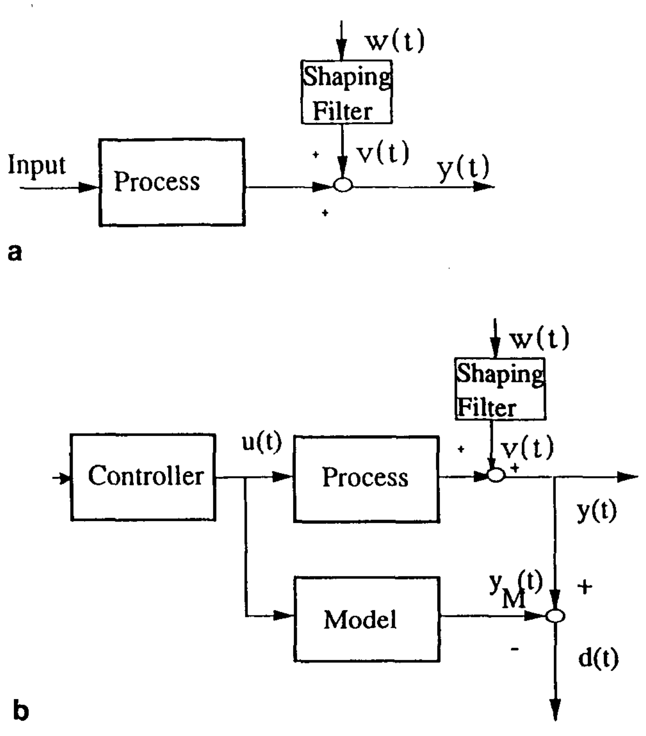

Figure 3 **a**, Disturbance in a process; **b**, prediction error signal in open-loop system

$$y(t) = z^{-1}H(z^{-1})u(t) + v(t) \quad (4)$$

where $H(z^{-1})$ is the polynomial of z^{-1} which represents the process.

Let $\tilde{H}(z^{-1})$ be a process model, by which the effects of past input actions on the output are calculated as follows:

$$y_M(t) = z^{-1}\tilde{H}(z^{-1})u(t) \quad (5)$$

where $\tilde{H}(z^{-1})$ is a polynomial defined by:

$$\tilde{H}(z^{-1}) := \tilde{h}_1 + \tilde{h}_2z^{-1} + \ldots + \tilde{h}_sz^{-s+1} \quad (6)$$

$v(t)$ is inferred by subtracting the model output $y_M(t)$ from the output $y(t)$ as illustrated in *Figure 3b*. The signal $y(t) - y_M(t)$ is called the prediction error and hereafter referred to as $d(t)$. $d(t)$ is a function of both $v(t)$ and input actions as expressed by Equation (7).

$$d(t) = z^{-1}H(z^{-1})u(t) + v(t) - z^{-1}\tilde{H}(z^{-1})u(t) \quad (7)$$

Equation (7) can be rewritten as:

$$\Delta d(t) = z^{-1}C(z^{-1})\Delta d(t) + z^{-1}D(z^{-1})\Delta u(t) + w(t) \quad (8)$$

where

$$z^{-1}D(z^{-1}) := \{1 - z^{-1}C(z^{-1})\}\{z^{-1}H(z^{-1}) - z^{-1}\tilde{H}(z^{-1})\}$$
$$= z^{-1}\{d_1 + d_2z^{-1} + d_3z^{-2} +, \ldots, + d_sz^{-s+1}\}$$

When and if the model is perfect, the disturbance $v(t)$ can be directly estimated from $d(t)$, that is, $d(t) = v(t)$.When the process model is not perfect, $d(t)$ is not equal to $v(t)$.

The coefficients of Equation (8) are then determined from time-series data of $\{\Delta d(t)\}$ and $\{\Delta u(t)\}$ by solving the following least squares problem:

$$\min \sum_{k=0}^{N} \varepsilon(t-k)^2 \quad (9)$$

subject to

$$\varepsilon(t-k) = \Delta d(t-k) - [\Delta d(t-k-1), \Delta d(t-k-2), \ldots, \Delta d(t-k-q), \Delta u(t-k-1), \ldots, \Delta u(t-k-s)]\tilde{\theta} \quad (k = 0,1, \ldots, N) \quad (10)$$

where

$$\tilde{\theta} := [\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_q, \tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_s]^T$$

$z^{-1}H(z^{-1})$ and $z^{-1}\tilde{H}(z^{-1})$ have at least one time delay. Then $z^{-1}D(z^{-1})$ has at least one time delay, z^{-1}. $z^{-1}C(z^{-1})$ also has at least one time delay. Therefore, even when the system is closed by a feedback loop, in other words, even when the input $u(t)$ is determined by the form of a linear combination of $\{d(t-i) \quad i = 0,1,2, \ldots\}$, it is possible to obtain the unbiased estimator $\tilde{\theta}$ by the least squares method.

As described above, the dynamics of a disturbance that is generated by auto-regressive form can be estimated from the prediction errors and previous input signals.

Once the dynamics of the disturbance is identified, the estimated dynamics can be utilized for increasing the accuracy of the output prediction in predictive control.

Algorithm developed

As Morari[6,12] and several other researchers have reported, model predictive control (MPC) has been acknowledged as a powerful process control scheme and has been applied to many industrial processes[7,11]. However, the performance that can be achieved by conventional MPC is not always adequate in resolving the disturbance rejection problem. The reason for this poor performance is that MPC assumes step-like disturbances in the output prediction and it does not account for disturbances that tend to drift high and low. The output prediction performed in conventional MPC is:

$$y_P(t+j/t) = y_M(t+j/t) + y(t) - y_M(t/t) \quad j = 1,2,\ldots, \tag{11}$$

where $y_M(t+j/t)$ is the output value at time $t+j$ that is calculated by the model at time t, $y(t)$ is the process output observed at time t, and $y(t) - y_M(t/t)$ is the prediction error at time t, which is the same signal referred to as $d(t)$.

This form of output prediction assumes that the same prediction error as $d(t)$ occurs during the prediction horizon. However, this assumption is not adequate for cases in which the disturbance has some dynamics. In the case that disturbance is generated with an identifiable dynamics, the accuracy of output prediction can be improved by utilizing the estimated dynamics in the equation of output prediction.

When the generation mechanisms of the disturbance and setpoint signal are known it is possible for MPC to realize an offset-free control for the disturbance as well as the setpoint change by using a prediction filter even though a plant–model mismatch exists[8]. That is, the one-step-ahead output prediction is performed by the form:

$$y_P(t+1/t) = y_M(t+1/t) + d_P(t+1/t) \tag{12}$$

$d_P(t+1/t)$ is the correction term of the output prediction at time t:

$$d_P(t+1/t) := F(z^{-1})\{y(t) - y_M(t/t)\} \tag{13}$$

If $(1 - F(z^{-1})z^{-1})$ has the mode of disturbance $v(t)$, i.e. $(1 - C(z^{-1})z^{-1})(1 - z^{-1})$, then MPC with this filter realizes an offset-free controller by which the mean value of the error signal becomes zero even for the stochastic disturbance $v(t)$.

In practice, it might be impossible to identify perfectly the generation mechanism of the disturbance, because data available to be used for estimation are finite, the dimension of the dynamics of $v(t)$ is unknown and the dynamics may have a time-varying nature. Therefore, it is necessary to carry out adaptive estimation or recursive estimation. That is, the estimation of $\{c_j\}$ and $\{d_j\}$ is performed at every sampling moment t. The prediction filter in Equation (13) is constructed by using the estimated parameter at time t, $\{\tilde{c}_j(t)\}$.

$$\begin{aligned} F(z^{-1}) := 1 &+ \tilde{c}_1(t) + (\tilde{c}_2(t) - \tilde{c}_1(t))z^{-1} +, \ldots, \\ &+ (\tilde{c}_q(t) - \tilde{c}_{q-1}(t))\, z^{-q+1} - \tilde{c}_q(t) z^{-q} \end{aligned} \tag{14}$$

Furthermore, signals acquired from plants are usually contaminated by noise such as white noise and spike noise. The coefficient parameters $\{\tilde{c}_j(t)\}$ and $\{\tilde{d}_j(t)\}$ may change drastically in response to spike noise. Therefore, from the practical viewpoint it is much better to determine the coefficient parameters of Equation (14) by solving the following least squares problem at every sampling moment, in which weight coefficients are utilized in order to suppress drastic changes in the estimated parameters.

$$\min \sum_{k=0}^{N} \varepsilon(t-k)^2 + \sum_{j=1}^{q} \omega_1 \xi_j(t)^2 + \sum_{j=1}^{s} \omega_2 \eta_j(t)^2 \tag{15}$$

subject to

$$\begin{aligned} \varepsilon(t-k) = \Delta d(t-k) - [&\Delta d(t-k-1), d(t-k-2), \ldots, \\ &\Delta d(t-k-q), \Delta u(t-k-1), \ldots, \\ &\Delta u(t-k-s)]\tilde{\theta} \quad (k = 0,1,\ldots,N) \end{aligned} \tag{16–1}$$

$$\xi_j(t) = \tilde{c}_j(t) - \tilde{c}_j(t-1) \quad (j = 1,\ldots, q) \tag{16–2}$$

$$\eta_j(t) = \tilde{d}_j(t) - \tilde{d}_j(t-1) \quad (j = 1,\ldots, s) \tag{16–3}$$

where

$$\tilde{\theta} := [\tilde{c}_1(t), \tilde{c}_2(t), \ldots, \tilde{c}_q(t), \tilde{d}_1(t), \tilde{d}_2(t), \ldots, \tilde{d}_s(t)]^T$$

ω_1 and ω_2 are the weight coefficients for suppressing drastic changes in the coefficient parameters, $\{\tilde{c}_j(t)\}$, $\{\tilde{d}_j(t)\}$.

This is a standard procedure for estimating the coefficients of the autoregressive integral exogenous model (ARIMA)[13], and the recursive version of the procedure can be easily developed by following the formulation found in several textbooks[14].

For cases in which the model is perfect, the prediction error signal at time t, i.e. $d(t)$, becomes a function of only its prior values and can be modelled by the AR model. Therefore, when the model is perfect or model mismatch is very small, the coefficient parameters $\{\tilde{c}_j(t)\}$ are estimated by using only prior values of $d(t)$ in the following least squares problem:

$$\min \sum_{k=0}^{N} \varepsilon(t-k)^2 + \sum_{j=1}^{q} \omega_1 \xi_j(t)^2 \tag{17}$$

$$\varepsilon(t-k) = \Delta d(t-k) - [\Delta d(t-k-1), \Delta d(t-k-2), \ldots, \Delta d(t-k-q)]\tilde{\theta} \quad (k = 0, 1, \ldots, N) \tag{18-1}$$

$$\xi_j(t) = \tilde{c}_j(t) - \tilde{c}_j(t-1) \quad (j = 1, \ldots, q) \tag{18-2}$$

where

$$\tilde{\theta} := [\tilde{c}_1(t), \tilde{c}_2(t), \ldots, \tilde{c}_q(t)]^{\mathrm{T}}.$$

Control action is determined in such a way that the output value predicted by Equation (12) is as close as possible to the target value. The target value is given by a reference trajectory. The reference trajectory is formulated to make the future deviation of the output at time $t+1$ from the setpoint $r(t+1)$ α time as small as the present deviation $\{y(t) - r(t)\}$[10]:

$$y_{\mathrm{R}}(t+1) - r(t+1) = \alpha\{y(t) - r(t)\} \quad 0 < \alpha < 1 \tag{19}$$

i.e.

$$y_{\mathrm{R}}(t+1) = \alpha y(t) + (1-\alpha)r(t) + \{r(t+1) - r(t)\} \tag{20}$$

The major advantage of using the reference trajectory instead of using the input suppression factors is that the reference trajectory provides plant operators with ease of use for on-site tuning as well as for changing the controlled response without recalculating the dynamic matrix. As is clear from the following sections, the form of the reference trajectory is determined so as to guarantee an offset-free control to any setpoint changes described by polynomial equations.

The control action is implemented at time t, and held until the next sampling point. At the next sampling point, a new measurement becomes available. The calculation of the coefficient parameters $\{\tilde{c}_j(t)\}$, $\{\tilde{d}_j(t)\}$, output prediction, and determination of control action are repeated at the next sampling point. This procedure is repeated at every sampling point. This is the algorithm of MPC with disturbance prediction.

In the next section, the structure of the proposed control system is analysed from the viewpoints of the internal model principle and minimum variance control, on the assumption that the coefficient parameters $\{\tilde{c}_j(t)\}$ in the prediction filter $F(z^{-1})$ have converged to true values, $\{c_j\}$.

Structure of the modified MPC

Viewpoint of the internal model principle

Let a stable process be expressed by Equation (4). Using Equation (5) as a process model, MPC with disturbance prediction is applied to the process.

The setpoint is changed according to the following equation:

$$r(t+1) = R(z^{-1})r(t) \tag{21}$$

where $R(z^{-1})$ is a polynomial of the delay operator.

For example, the step-like change in the setpoint is described by using $R(z^{-1}) = 1$ with an initial value r_0 in Equation (21). The ramp-like change is described by $R(z^{-1}) = 2 - z^{-1}$ and an initial value.

Under the condition that the unmeasured disturbance $v(t)$ enters the process, the disturbance prediction is performed by Equation (13), and one-step prediction control is carried out. Control action is then determined in the form of:

$$u(t) = \frac{y_{\mathrm{R}}(t+1) - \bar{F}(z^{-1})y(t)}{(1 - \bar{F}(z^{-1})z^{-1})\tilde{H}(z^{-1})} \tag{22}$$

where $\bar{F}(z^{-1})$ denotes the prediction filter whose coefficient parameters are $\{c_j\}$:

$$\bar{F}(z^{-1}) := 1 + c_1 + (c_2 - c_1)z^{-1} + \ldots, + (c_q - c_{q-1})z^{-q+1} - c_q z^{-q} \tag{23}$$

The closed-loop response of the error signal $e(t)$ $(= r(t) - y(t))$ is given by:

$$D_1(z^{-1})e(t) = N_1(z^{-1})r(t) + N_2(z^{-1})v(t) \tag{24}$$

where

$$D_1(z^{-1}) := [1 - \bar{F}(z^{-1})z^{-1}]\tilde{H}(z^{-1}) + [\bar{F}(z^{-1}) - \alpha]\, z^{-1}H(z^{-1}) \tag{25-1}$$

$$N_1(z^{-1}) := [1 - \bar{F}(z^{-1})z^{-1}][\tilde{H}(z^{-1}) - H(z^{-1})] - [1 - R(z^{-1})z^{-1}]H(z^{-1}) \tag{25-2}$$

$$N_2(z^{-1}) := [1 - \bar{F}(z^{-1})z^{-1}]\tilde{H}(z^{-1}) \tag{25-3}$$

The right-hand side of Equation (24) is composed of two terms. The first term shows the effect on the error caused by a change in the setpoint, while the second constitutes the effect on the error caused by a change in disturbance. The structure of the control system is illustrated in *Figure 4*.

The unmeasured disturbance $v(t)$ is governed by:

$$(1 - C(z^{-1})z^{-1})(1 - z^{-1})v(t) = w(t) \tag{26}$$

In order for the control system to realize offset-free control for a disturbance and change in the setpoint, it becomes clear from the internal model principle that $\bar{F}(z^{-1})$ has to be designed so that $(1 - \bar{F}(z^{-1})z^{-1})$ possesses the same dynamics as that of the disturbances $\{(1 - z^{-1})(1 - C(z^{-1})z^{-1})\}$ and the setpoint $(1 - R(z^{-1})z^{-1})$. Then Equation (24) becomes:

$$D_1(z^{-1})e(t) = \tilde{H}(z^{-1})w(t) \tag{27}$$

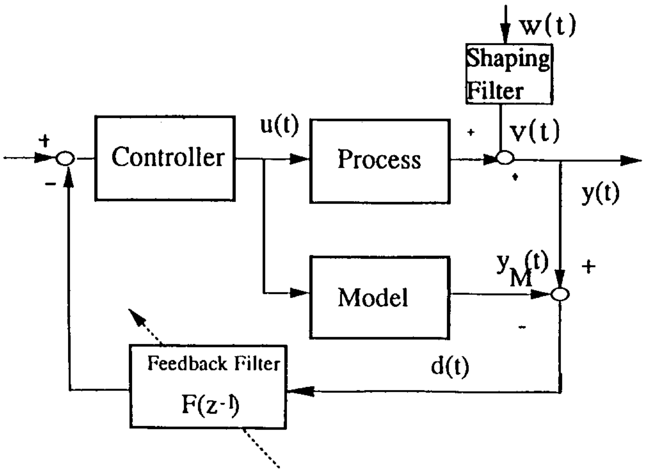

Figure 4 Structure of MPC with disturbance prediction

Since the mean of $w(t)$ is zero, the mean of error signal $e(t)$ becomes zero when $D_1(z^{-1})$ is a stable polynomial.

For example, $(1 - \bar{F}(z^{-1})z^{-1})$ should contain the mode of $(1 - z^{-1})^2$ and $(1 - C(z^{-1})z^{-1})$ in order for the control system to realize an offset-free control for ramp changes in the setpoint and changes in the disturbance $v(t)$.

More detailed discussion on the design of the prediction filter $F(z^{-1})$ was given by Ohno *et al.*[8].

The form of the disturbance prediction proposed by Shen *et al.*[3] and Ohshima *et al.*[2] does not take the same form as Equations (13) and (14). It is given by:

$$d_p(t+1/t) = \sum_{j=1}^{q} v_j(t)d(t-j) \tag{28}$$

where the coefficient parameters $\{v_j\}$ are determined from time series data of prediction error.

Shen pointed out in his paper that $\Sigma_{j=1}^{q} v(t) = 1$ should be held so as to achieve offset-free control in responding to step-changes of the setpoint. He stated that the parameter estimation method may need to be modified using Lagrangian multipliers in order to satisfy the constraint on parameters[3]. However, by performing the disturbance prediction in the form described by Equation (13), $(1 - \bar{F}(z^{-1})z^{-1})$ automatically contains the mode of $(1 - z^{-1})$ and such a complicated estimation method is not needed.

Viewpoint of minimum variance control

In this section the structure of the proposed control system is examined in terms of minimum variance control. Coupled with Equation (1), Equation (4) shows the form of CARIMA:

$$(1 - C(z^{-1})z^{-1})\Delta y(t+1) = (1 - C(z^{-1})z^{-1})H(z^{-1})\Delta u(t) + w(t+1) \tag{29}$$

To derive the one-step-ahead minimum variance predictor $y(t+1)$, consider the solution, $E(z^{-1})$, $G(z^{-1})$ of the next equation:

$$1 = (1 - C(z^{-1})z^{-1})(1 - z^{-1})E(z^{-1}) + z^{-1}G(z^{-1}) \tag{30}$$

where $E(z^{-1})$ and $G(z^{-1})$ are polynomials uniquely determined by the dynamics of disturbance, $1 - C(z^{-1})z^{-1}$.

$E(z^{-1}) = 1$, $G(z^{-1}) = z\{1 - (1 - C(z^{-1})z^{-1})(1 - z^{-1})\}$ is the minimal order unique solution of Equation (30).

By using this solution, we obtain Equation (31), by which the future output value $y(t+1)$ can be predicted by the current and past input and output signals:

$$y(t+1) = G(z^{-1})y(t) + (1 - C(z^{-1})z^{-1})H(z^{-1})\Delta u(t) + w(t+1) \tag{31}$$

Because $w(t+1)$ is unknown at time t, and only the process model can be utilized in the design stage, the minimum variance predictor is:

$$\hat{y}(t+1/t) = G(z^{-1})y(t) + (1 - C(z^{-1})z^{-1})\tilde{H}(z^{-1})\Delta u(t) \tag{32}$$

where $\tilde{H}(z^{-1})$ is the model of $H(z^{-1})$.

On the other hand, output prediction performed in the modified model predictive control (MPC with disturbance prediction) is written as:

$$\begin{aligned} y_P(t+1/t) &= y_M(t+1/t) + F(z^{-1})(y(t) - y_M(t/t)) \\ &= (1 - F(z^{-1})z^{-1})y_M(t+1/t) + F(z^{-1})y(t) \end{aligned} \tag{33}$$

When the coefficient parameters $\{\tilde{c}_j(t)\}$ have converged to true values $\{c_j\}$, $(1 - F(z^{-1})z^{-1})$ consists of $(1 - z^{-1})$ and $(1 - C(z^{-1})z^{-1})$:

$$(1 - F(z^{-1})z^{-1}) = (1-z^{-1})(1 - C(z^{-1})z^{-1}) \tag{34}$$

Substituting Equation (34) into (33), we obtain

$$y_P(t+1/t) = (1 - z^{-1})(1 - C(z^{-1})z^{-1})y_M(t+1/t) + \{1 - (1 - z^{-1})(1 - C(z^{-1})z^{-1})\}z\, y(t) \tag{35}$$

Using the relation, $G(z^{-1}) = z\{1 - (1 - C(z^{-1})z^{-1})(1 - z^{-1})\}$ and Equation (5), Equation (35) is rewritten as:

$$y_P(t+1/t) = G(z^{-1})y(t) + (1 - C(z^{-1})z^{-1})\tilde{H}(z^{-1})(1 - z^{-1})u(t) \tag{36}$$

Comparing Equation (36) with Equation (32), it becomes clear that the output prediction performed in the modified MPC is a minimum variance predictor. Therefore, the controller that determines control action satisfying $y_R(t+1) = y_P(t+1/t)$ is a minimum variance controller.

Application of modified MPC to fatty acid distillation columns

Indirect composition measurement

As mentioned in the previous section, the objective of the control system for the fatty acid plant is to regulate the composition in the distillate at a desired value while suppressing the influence of disturbances in the feed composition.

Measurement of the composition is carried out in the laboratory and generally takes a half to one hour to analyse a sample. Consequently, an indirect composition estimation is necessary to construct a composition control system in this plant. The most commonly used composition estimation technique is the measurement of temperature in the column[9]. By plottng the temperature profiles for different steady states, we select measuring points: one for the rectifying section and another for the stripping section, where the deviation in temperature is the largest in each section. *Figure 5* shows a linear relationship between C_{14} composition in the distillate and the temperature at the measuring point at the second column. For each column, the temperature at the measuring point in the rectifying section has a strong linear relation to the composition of the key component in the distillate. The temperature at the measuring point in the stripping section has a linear relation to the composition of bottom product, but the linear relationship does not hold when the oil species are changed. Therefore, the bottom composition is left uncontrolled and the boiling-up rate (V) is kept constant. Then, a one-point composition control system is realized at each column.

Ratio control

To regulate the composition of key components in the distillates, the temperature reading at the measuring point in each rectifying section is adjusted to take into account the pressure of the column, and the pressure-

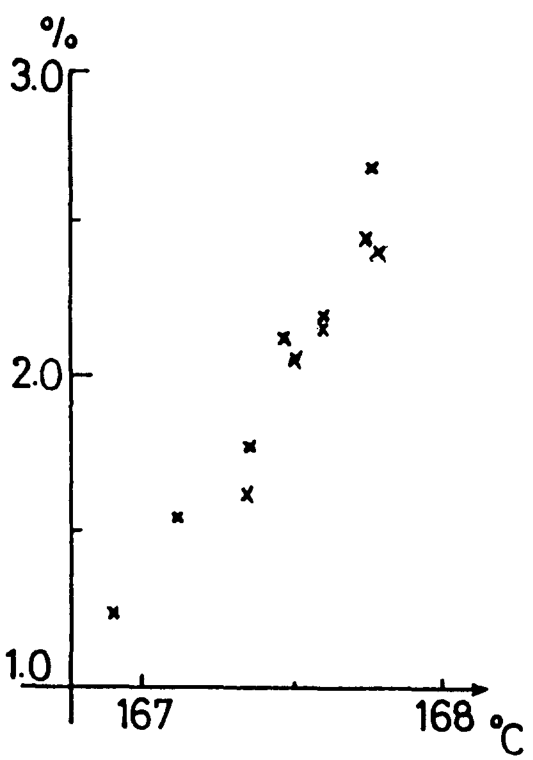

Figure 5 Relationship between C14 and temperature at a selected position

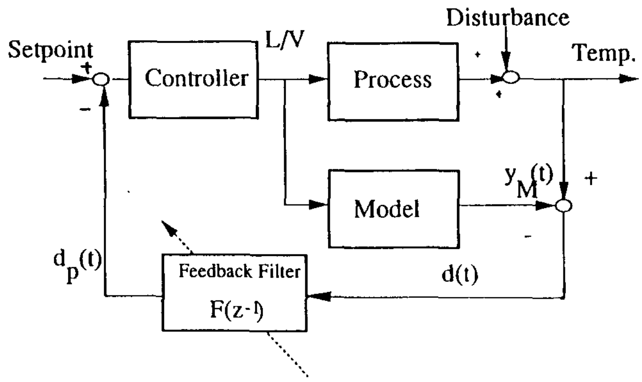

Figure 6 Structure of the control system in each distillation column

compensated temperature is controlled by manipulating the ratio of reflux flow rate to boiling-up rate.

The algorithm of modified model predictive control is applied to the ratio control. The structure of the developed control system in each distillation column is depicted in *Figure 6*.

In order to develop the step response model $z^{-1}\tilde{H}(z^{-1})$ for each column, open-loop step response tests are performed by keeping other columns under control and not changing the feed oil. Using the identified step response model, a model predictive controller with disturbance prediction is developed. The parameters N, q and s in the disturbance prediction are determined through trial and error.

Figures 7 and *8* show the behaviour of the controlled temperature of the third column. *Figure 7* presents the closed-loop response when the modified model predictive control is performed, and *Figure 8* shows the closed-loop response when a conventional PID scheme, which is tuned by trial and error, is applied to the ratio control. It can be seen that the closed-loop response of PID shows fairly large oscillation. On the other hand, this oscillation is damped by performing disturbance prediction. As a result, product impurity is controlled satisfactorily. *Figures 9* and *10* show plant data on the top product impurity at the second column. The impurity measured by the gas chromatograph in the laboratory is plotted by a dotted curve. The broken line

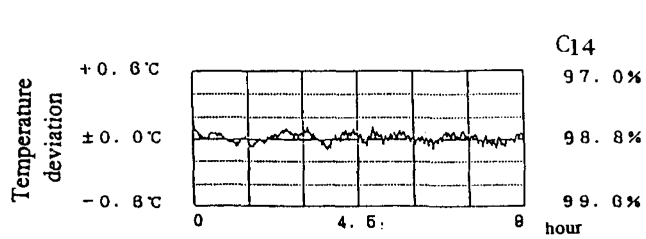

Figure 7 Temperature response under MPC with disturbance prediction

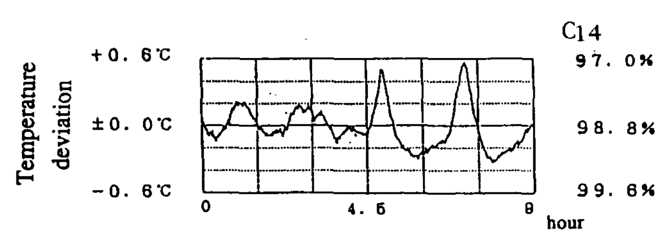

Figure 8 Temperature response under PI control

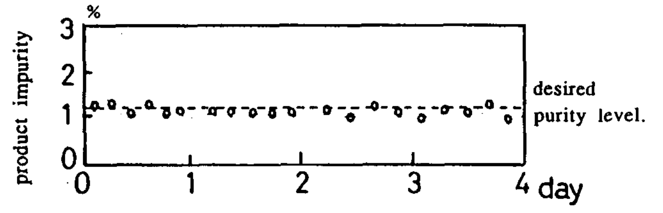

Figure 9 Product impurity controlled by MPC with disturbance prediction

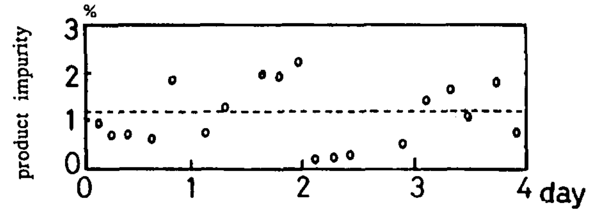

Figure 10 Manually adjusted PID controlled product impurity

denotes the desired purity level. The result when the modified MPC is applied is illustrated in *Figure 9*. *Figure 10* shows time series data of impurity before the modified MPC was installed. As shown in the figure, the proposed control scheme decreases the rate of off-specification and releases plant operators from the burden of frequent retuning of the PID controller. This control system has been in use successfully for more than six years at the fatty acid plant in Kao Corporation to produce commercial products.

Conclusions

An on-line parameter estimation algorithm for the dynamics of disturbance and the predictive control algorithm have been incorporated to improve regulatory performance of the composition control in a fatty acid distillation column sequence.

The structure of the proposed control system was analysed from the viewpoint of the internal model principle and minimum variance control. From theoretical analysis, it becomes clear that the prediction filter $F(z^{-1})$ has to be designed so as to possess the same mode as that of disturbance and that of the setpoint in order to realize an offset-free control. It is also clear that the control system realizes minimum variance control for the disturbance.

The current one-step-ahead prediction algorithm cannot handle non-minimum phase and time delay systems. In order to deal with such systems, it is necessary to extend the prediction function to multistep ahead prediction. In the mathematical formulation, the extension can be easily attained by modifying both output prediction and disturbance prediction forms to the multistep ahead prediction forms. However, in a practical sense, it can be easily imagined that the prediction horizon is extended, the prediction error of the disturbance becomes large and control performance worse. Therefore, the application of the multi-step version of the proposed algorithm to non-minimum phase or time-delay systems requires much more sophisticated tuning than that experienced in the fatty acid column control.

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