High-gain Observer-based Model Predictive Control for Cross Tracking of Underactuated Autonomous Underwater Vehicles

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Abstract--In this paper, a disturbance observer-based model predictive control system is developed for cross tracking of the underactuated autonomous underwater vehicle under uncertain current disturbances. A high-gain observer is used in the proposed controller to estimate the current velocity, external force and torque. Based on the disturbance estimates, a nonlinear model predictive controller is designed considering the input constraints. The control inputs are solved by optimizing the predicted trajectories of the system under input constraints within a certain time horizon. The simulation results are provided to verify the effectiveness of the proposed control algorithm.

Key words--high-gain observer; model predictive control; cross tracking; current disturbance

I. INTRODUCTION

Cross tracking refers to autonomous underwater vehicles (AUV) with a given initial orientation to reach and follow a reference line which is generated by a geometric reference model and not related to time [1]. It is difficult due to the nonlinear and underactuated properties and input constraints of AUVs.

Indiveri et al. proposed a cross-track controller in the horizontal plane for underactuated AUVs [2]. A time invariant feedback control law is designed based on the inversion method, but the effects of the yaw angle tracking error on the cross tracking error is not analyzed. Do and Pan used the Lyapunov second theorem and inversion technology to investigate the output path tracking problem of underactuated ships under the standard Serret-Frenet coordinate system [3]. They designed a nonlinear velocity observer according to the dynamic perturbation, and analyzed the stability of tracking errors. Woolsey designed a 3-dimensional cross-track controller by using the Lyapunov second theorem, considering the uncertainty in the hydrodynamic damping coefficients, which made the controller robust [4].

The aforementioned works mainly focused on the tracking control of nonlinear, uncertain and underactuated AUVs, but the constraints of actuators were not considered. In this paper, the model predictive control (MPC) algorithm is employed to solve the constrained cross-track control problem for underactuated AUVs well. MPC has the advantages of good control effect, strong robustness, and can effectively overcome the uncertainty, nonlinear and parallel of the process. The most important property is that it can deal with the system constraints explicitly.

MPC has been greatly developed because of its unique advantages. Falcone applied the linear time–varying (LTV) MPC algorithm to solve the vehicle front wheel control and brake problems in which the system constraints were considered to be a linear system model [5]. Oh and Sun used the line-of-sight (LOS) guidance method to establish a model of the way-point tracking problem, designed the navigation and control system of the way-point tracking, and improved the performance of the path tracker by solving the linear constrained optimization problem [6]. But the simulation used a simplified linear model, so the actual system had a certain error. Negenborn employed the linear MPC (LMPC) algorithm and nonlinear MPC (NMPC) algorithm for trajectory tracking control of ships on a horizontal plane [7]. But a simplified linear model was adopted, and the effect of interference was not considered. Li and Sun considered the effects of current disturbance on the yaw angle, and compared the disturbance compensated MPC (DC-MPC) algorithm with the traditional MPC algorithm under constant and sinusoidal disturbances [8]. The effect on the yaw angle was considered, but the position and velocity of the system were not considered.

When there are disturbances outside or errors inside the system, it is necessary to estimate their values in order to eliminate the adverse effects on the system. Khalil and Praly designed a high-gain observer for nonlinear feedback control to estimate the unknown state feedback [9]. Zhang and Zhou used a initial high-gain observer for a class of nonlinear differential-algebraic equation subsystems, and it made observer errors converge to zero exponentially [10]. Li and Kou *et al.* proposed a high-gain observer based on an engine

dynamic model to improve the accuracy of pressure estimation and demonstrated the robustness and accuracy using the Laplace and Lyapunov method [11].

In this paper, the AUV cross tracking control problem with ocean current disturbance is studied, and a high-gain observer-based model predictive control strategy is proposed. By establishing the nonlinear model of AUV, the corresponding high-gain observer is employed to estimate the values of disturbances and the model predictive control with disturbance compensation is used to stabilize the AUV to the desired linear trajectory.

II. PROBLEM FORMULATION

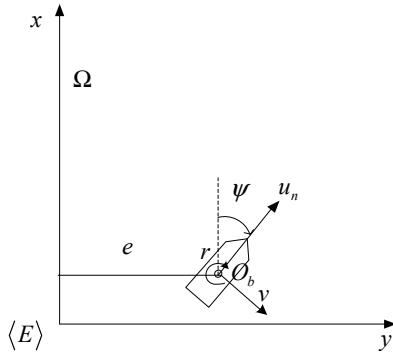The nonlinear model for cross tracking of an underactuated AUV is depicted in Figure 1 [7].

Fig. 1   system modeling

The coordinate system is established regarding the desired straight line as x axis, perpendicular to the direction as y axis. O_b is the buoyant center of AUV, u_n is the forward velocity, e is the distance between the desired line and AUV, ψ is the angle between the AUV and the scheduled trajectory. In the vehicle coordinate system, the dynamics of surge is neglected, and the surge velocity is assumed as a constant. The torque N generated by the rudder is as an input. The nonlinear model is formulated as follows:

$$\begin{aligned}
\dot{e} &= u_n \sin\psi + v\cos\psi + \omega_1 \\
\dot{\psi} &= r \\
\dot{v} &= \left[d_{22}v - m_{11}u_n r\right]/m_{22} + \omega_3 \\
\dot{r} &= \left[-m_{22}u_n v + m_{11}u_n v + d_{33}r + N\right]/m_{33} + \omega_4
\end{aligned} \tag{1}$$

where $m_{11} = m - X_{\dot{u}}$, $m_{22} = m - Y_{\dot{v}}$ and $m_{33} = m - Z_{\dot{\omega}}$ are the inertia factors that contain the added mass, $d_{22} = Y_v$ and $d_{33} = N_r$ are damping coefficients, ω_1 is the sway velocity interference, ω_3 is the sway force interference, ω_4 is the yaw moment disturbance.

The system model can be expressed in a compact form as:

$$\begin{aligned}
\dot{x}(t) &= f\left(x(t), u(t)\right) + \omega \\
\dot{\omega} &= 0 \\
y(t) &= x(t)
\end{aligned} \tag{2}$$

where $x = \begin{bmatrix} e & \psi & v & r \end{bmatrix}^T$, $u = N$.

The cross tracking control target is to actuate the AUV to move towards the given course with the forward velocity u_n and meet the accuracy requirement from any initial states.

III. HIGH-GAIN OBSERVER-BASED MODEL PREDICTIVE CROSS TRACKING CONTROL

A. Control Architecture

The nonlinear MPC close-loop control scheme with a high-gain disturbance observer is shown in Figure 2:
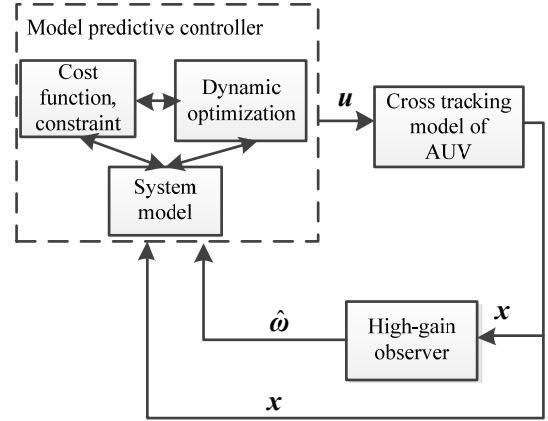

Fig. 2   the Close-loop system with disturbance observer

The architecture in Figure 2 describes the main components of an autonomous vehicle guidance system: the cross-track model, the high-gain observer, and the model predictive controller.

The model predictive controller is composed of three modules: predictive model, moving horizon optimization, and feedback control. The controller calculates the optimal control based on the given constraints and the performance requirements and implements the current control by using the future dynamic behavior of process model predictive system under certain controls, and corrects the future predicted dynamic behavior by measuring the current information.

B. High-gain Disturbance observer

A high-gain observer [9] is designed as follows under the assumption on constant disturbance:

$$\begin{aligned}
\dot{\hat{x}} &= \hat{f}(\hat{x}, u) + h_1(x - \hat{x}) \\
\dot{\hat{\omega}} &= 0 + h_2(x - \hat{x})
\end{aligned} \tag{5}$$

where $\hat{f}(x, u)$ is the nominal value. If f is a known function of (x, u), we can take $\hat{f} = f$. The estimation errors

$$\begin{bmatrix} \tilde{x} \\ \tilde{\omega} \end{bmatrix} = \begin{bmatrix} x - \hat{x} \\ \omega - \hat{\omega} \end{bmatrix} \tag{6}$$

satisfy the equations

116

$$\dot{\tilde{x}} = \delta(x, \tilde{x}, u) + \omega + h_1 \tilde{x}$$
$$\dot{\omega} = -h_2 \tilde{x} \tag{7}$$

where

$$\delta(x, \hat{x}, u) = f(x, u) - f(\hat{x}, u) \tag{8}$$

The following equations are obtained:

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{\omega}} \end{bmatrix} = \begin{bmatrix} -h_1 & 1 \\ -h_2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{\omega} \end{bmatrix} + \begin{bmatrix} \delta \\ 0 \end{bmatrix} \tag{9}$$

The choice of h_1 and h_2 should satisfy the equations above. Because of the actual existence of δ, h_1, h_2 must be designed to eliminate the impact of δ on $\tilde{x}$. The transfer function from δ to $\tilde{x}$ is

$$G_0(s) = \frac{1}{s^2 + h_1 s + h_2} \begin{bmatrix} 1 \\ s + h_1 \end{bmatrix} \tag{10}$$

Only when h_1 and h_2 are both larger than 0, the system is stable. When $G_0(s) = 0$, it will meet the design requirement. But it cannot realize, we must chose the most appropriate values of h_1 and h_2 to make the absolute value of $G_0(s)$ sufficiently small.

The transfer function from δ to $\tilde{x}$ can also be written as follows:

$$G_0(s) = \frac{\dfrac{1}{\sqrt{h_2}}}{\left(\dfrac{s}{\sqrt{h_2}}\right)^2 + \dfrac{h_1}{\sqrt{h_2}}\dfrac{s}{\sqrt{h_2}} + 1} \begin{bmatrix} \dfrac{1}{\sqrt{h_2}} \\ \dfrac{h_1}{\sqrt{h_2}} + \dfrac{s}{\sqrt{h_2}} \end{bmatrix} \tag{11}$$

The value is related to $\dfrac{h_1}{\sqrt{h_2}}$ when h_2 is infinite, and it will meet the design requirement. Define h_1, h_2 as

$$h_1 = \frac{\alpha_1}{\varepsilon}, \quad h_2 = \frac{\alpha_2}{\varepsilon^2} \tag{12}$$

where α_1, α_2 are positive constants. Therefore, by choosing ε small, the high gain observer can be designed more appropriately.

C. Nonlinear Model Predictive Cross-Track Control

In the mathematical simulation of MPC controller, the continuous model cannot be directly used to predict, so it needs to be discretized as follows:

$$x(k+1) = f_d(x(k), u(k))$$
$$y(k) = x(k), \tag{13}$$

where f_d stands for the discretized system function. It is assumed that the control sequence is a constant in the time period of $([(k-1)T_s, kT_s](k = 1, 2, ...N)$. N_p is the prediction horizon, T_s is the sampling interval.

Therefore, based on the current measured or estimated states, future predictions can be calculated interactively as the functions of control sequence $u(k+i)(i = 0, 1, ..., N_p - 1)$. The model predictive cross tracking control can be formulated by a constrained nonlinear programming problem [8]. In step k, the simulation of future steps has a minimum linear tracking error:

$$J(\bar{x}(k), \bar{u}(k)) =$$
$$\sum_{i=1}^{N_p} \left(\bar{y}(k+i|k) - y_{ref}\right)^T Q \left(\bar{y}(k+i|k) - y_{ref}\right) \tag{14}$$
$$+ \sum_{i=0}^{N_p} u(k+i|k)^T R u(k+i|k)$$

$$x(k) = x_{measure}(k) + \hat{w}(k) \tag{15}$$

$$x(k+i+1) = f_d(x(k+i), u(k+i)) \tag{16}$$

$$\bar{y}(k+i) = x(k+i) \tag{17}$$

$$y_{ref}(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{18}$$

$$u_{min} \leq u(k+i) \leq u_{max}, i = 0, 1, ..., N_p - 1 \tag{19}$$

where $x(k)$ is the state matrix, $\bar{u}(k)$ is the control input matrix, $\bar{y}(k+i|k)$ stands for the future prediction output vector of step $k+i$ at step k. Q and R are weighting matrices of appropriate dimensions. ~~We~~ obtain a sequence of optimal results $u(k|k), u(k+1|k), ...u(k+N_p-1|k)$ by solving the optimization problem Eqn.(14)–(19), and only the first element $u(k|k)$ is applied to the desired input. At each step, we can complete the MPC algorithm through the following analysis process:

(1) At step k, estimate error $\hat{\omega}(k)$ using the high-gain observer.

(2) Measure the current states $x(k)$ of the system as the inputs of controller and obtain the future predicted outputs $\bar{y}(k+i|k), i = 1, ..., N_p$ as functions of future control inputs $u(k+i|k), i = 0, 1, ..., N_p - 1$.

(3) Solve the nonlinear programming problem Eqn.(14) (using MATLAB function *fmincon*) under the constraints and get the optimal control sequence $u(k+i|k), i = 0, 1, ..., N_p - 1$.

117

(4) The first element of the above optimal solution $u(k|k)$ is the output of the controller, and it will be used as the input of the system model.

(5) At next simulation step $k+1$, go to step (1).

We can see that the cost function of MPC is not defined in the infinite horizon, so it cannot guarantee the minimum value of cost function is monotonically decreasing each step, which means optimality does not imply stability[12]. The theorem of the cross-track system is as follows:

***Theorem** 1*: The model predictive control system for cross tracking is stable when the terminal constraints X_f are close sets, and the terminal cost function $F\left(x\left(k+N_p|k\right)\right)$ is continuous- differential [13].

Proof: The cost function can be extended to the whole time domain:

$$J(k) = \sum_{i=0}^{N_p-1} g\left(x(k+i|k), u(k+i|k)\right) + F\left(x\left(k+N_p|k\right)\right) \tag{20}$$

where $g(x(k), u(k))$ is the stage cost at step k while

$$F\left(x\left(k+N_p|k\right)\right) = \sum_{i=N_p}^{\infty} g\left(x(k+i|k), u(k+i|k)\right)$$ .is its cost function after step $k+N_p$. Obviously, the value of $F\left(x\left(k+N_p|k\right)\right)$ is not implemented in MPC but it affects the stability of the control system. It is assumed that the controller $\Phi(x)$ can ensure the system asymptotically stable when the system enters X_f.

At step k, the optimal control sequence is expressed as

$$U^*(k) = \{u^*(k|k), \dots, u^*(k+N_p-1|k)\} \tag{21}$$

and its resultant optimal state trajectory is expressed as

$$X^*(k) = \{x^*(k+1|k), \dots, x^*(k+N_p|k)\} \tag{22}$$

Then the minimized cost function at step k is

$$J_p^* = \sum_{i=0}^{N_p-1} g\left(x^*(k+i|k), u^*(k+i|k)\right) + F\left(x^*(k+N_p|k)\right) \tag{23}$$

At step $k+1$, the state quantity $x^*(k+1|k)$ is controlled by the input quantity $u^*(k|k)$. Then a feasible solution is

$$U(k+1) = \{u(k+1|k+1), \dots, u(k+N_p|k+1)\} \tag{24}$$

where $u(k+i|k+1) = u^*(k+i|k) \left(1 \le i \le N_p - 1\right)$,

$$u(k+N_p|k+1) = \Phi\left(x^*(k+N_p|k)\right).$$

Then its new state trajectory is

$$X^*(k+1) = \{x^*(k+2|k), \dots, x^*(k+N_p+1|k)\} \tag{25}$$

The cost function is

$$J_p(k+1) = \sum_{i=1}^{N_p} g\left(x^*(k+i|k), u^*(k+i|k)\right) + F\left(x^*(k+N_p+1|k)\right) \tag{26}$$

Therefore,

$$\begin{aligned} J_p^*(k) &- J_p(k+1) \\ &= g\left(x^*(k|k), u^*(k|k)\right) + F\left(x^*(k+N_p|k)\right) \\ &\quad - g\left(x^*(k+N_p|k), u^*(k+N_p|k)\right) \\ &\quad - F\left(x^*(k+N_p+1|k)\right) \\ &= g\left(x^*(k|k), u^*(k|k)\right) \ge 0 \end{aligned} \tag{27}$$

That means $J_p(k+1) \le J_p^*(k)$. In addition, $U(k+1)$ is a feasible solution of step $k+1$, $U^*(k+1)$ is the optimal solution. Therefore, we have

$$J_p^*(k+1) \le J_p(k+1) \le J_p^*(k). \tag{28}$$

In summary, the MPC control strategy is asymptotically stable.

Because the control inputs and system states are under constraints, the system may not reach the terminal constraint set X_f within its prediction horizon. Assume that the state $x(k)$ of the step m reaches the terminal constraints X_f, so the cost function after step $k+N_p$ can be written as follows:

$$F\left(x\left(k+N_p|k\right)\right) = \sum_{i=N_p}^{m-1} g\left(x(k+i|k), u(k+i|k)\right) + F\left(x(k+m|k)\right) \tag{29}$$

Then the stability of the proposed system under the constraints X_f can also be shown in Eqn.(29) similarly to Eqn.(20). In conclusion, the stability of the proposed cross-track system is proved.

IV.   SIMULATION STUDIES

In this paper, the model parameters of the REMUS AUV are shown in Table 1:

118

TABLE I.   MODEL PARAMETERS

parameter	value	parameter	value
$m\,(\mathrm{kg})$	30.5	$g\,(\mathrm{N/kg})$	9.81
$Y_{\dot{v}}\,(\mathrm{kg})$	-35.5	$Z_{\dot{w}}\,(\mathrm{kg})$	-35.5
$N_{\dot{r}}\,(\mathrm{kg\cdot m^2/s})$	-188	$X_u\,(\mathrm{kg})$	-0.930
$Y_v\,(\mathrm{kg/s})$	-262		

In order to verify the effectiveness of the high-gain observer, the model predictive control without disturbance estimation and the model predictive control based on the high-gain observer are designed respectively. In the simulation, the parameters are assumed as follows:

The initial state $x0 = \begin{bmatrix} 10 & \pi/6 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$;

The initial value of input $u(0) = 0$;

The forward velocity $u_{\mathrm{n}} = 2.5$ m/s;

The desired output $y = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$;

The input constraint $|N| \leq 6.15 u_n^2 \delta$, $\delta = 15°$;

The constant sea current $\omega = \begin{bmatrix} 0.1 & 0 & 0.4 & 0 \end{bmatrix}^{\mathrm{T}}$ (perpendicular to the desired trajectory direction, with 0.1 m sway velocity, 0.4 N sway force interference and no rotating) .

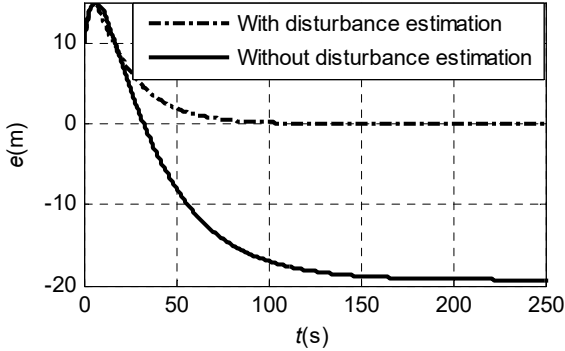The simulation results are reported as follows:
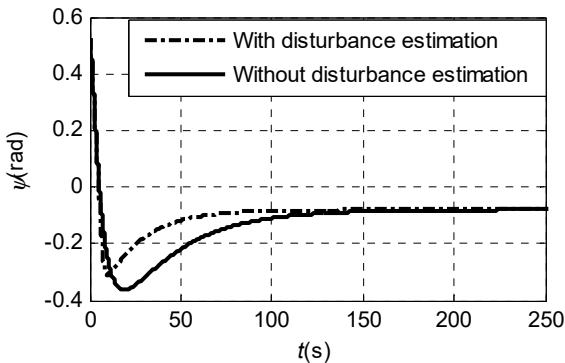
Fig. 3   *cross-track errors of the AUV*

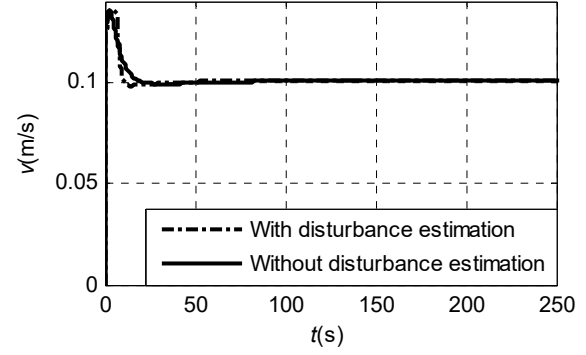Fig. 4   *yaw angles between AUV and desired line*
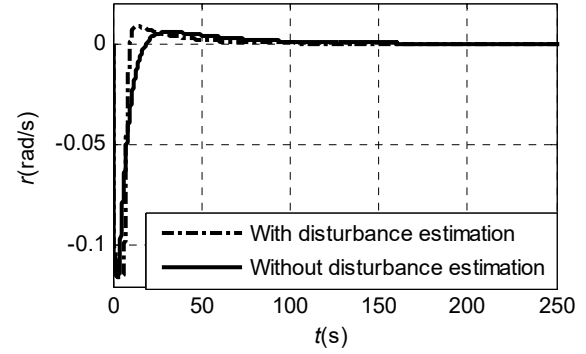
Fig. 5   *sway velocities*
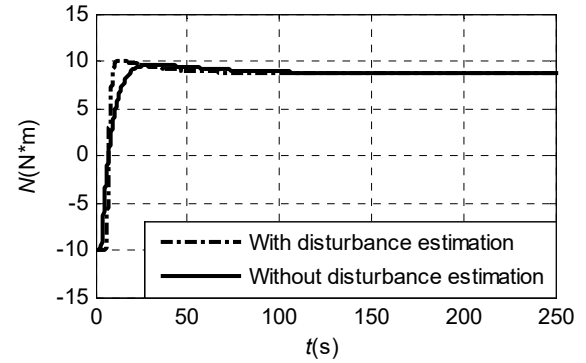
Fig. 6   *yaw rates*

Fig. 7   *control torques*

It can be observed that without the estimation of disturbance, the simulation result is that the AUV runs along the parallel straight line 19 m away from the desired trajectory at the forward velocity 2.5 m/s and the sway velocity 0.1 m/s. It fails to reach the desired path. However, the AUV finally arrives and moves along the desired trajectory with the same velocity under high-gain observer-based MPC. They also have something in common that the yaw angle is -0.08 rad, the yaw rate is 0 and the input torque is 8.7 N*m.

No matter the disturbance observer is used or not, the system can reach the equilibrium state. There is an angle between the direction of forward velocity and the desired line, but the direction of the total velocity is the same as the desired line. The observer estimates the disturbance and the controller eliminates it, so the system can reach the desired states. While the controller without observer cannot gain any information of disturbance, so the impact of disturbance is reflected directly.

In summary, the high-gain observer can estimate the system disturbances exactly, and the high-gain observer-

119

based model predictive controller can ensure the stability and accuracy of the cross tracking control.

V. CONCLUSION

In this paper, a high-gain observer-based model predictive control algorithm is proposed for the cross tracking control system of AUVs. According to the problem of ocean current disturbance, a high-gain observer is designed to estimate the disturbance value. A nonlinear model predictive controller is utilized based on the value of disturbance estimation and the system model. By comparing the simulation results with and without the disturbance observer, the necessity of the high-gain observer is justified, and the effectiveness of the high-gain observer-based model predictive control algorithm is demonstrated.

VI. ACKNOWLEDGMENT

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