Proceedings of the 31st Chinese Control Conference
July 25-27, 2012, Hefei, China

On Parameter Design for Predictive Control with Adaptive Disturbance Model

Wenbo WANG, Jun ZHAO, Zuhua XU
State Key Laborarory of Industrial Control Technology，Zhejiang University，Hangzhou 310027, China
E-mail: wbwang@iipc.zju.edu.cn, jzhao@iipc.zju.edu.cn, xuzh@iipc.zju.edu.cn

Abstract: The step model widely used to estimate the unmeasured output disturbance in MPC at present has limited disturbance rejection performance. Adaptive disturbance model can estimate the disturbance dynamics better and improve the ability of disturbance rejection. Parameter design of the controller has great impact on the control performance. The disturbance rejection strategy of disturbance adaptation predictive control (DMCA) is analyzed in the paper, as well as the effects of controller parameters on system dynamic performance, robustness and disturbance rejection ability. In addition, design methods for parameters such as disturbance prediction horizon, orders of time series model and filter factor for output error are researched and then experience guidelines for the parameter design are summarized. Simulation results show that DMCA can decrease the integral of absolute value of error criterion for the controlled variable by 45% than Dynamic Matrix Control (DMC). The optimization design of controller parameters improves DMCA's ability of predicting and rejecting disturbance further.

Key Words: Model predictive control, Adaptive disturbance model, Disturbance rejection, Parameter design

1 Introduction

Model predictive control (MPC) refers to a class of computer control algorithms that predict the future response of a plant by an explicit process model. MPC technology has been applied to a wide variety of areas including chemicals, automotive and aerospace applications. Along with the increasing complexity of industry systems, quantities of complex and time-varying disturbances affect the control system performance more and more seriously. In DMC and IDCOM, a constant output step disturbance model is used to achieve offset-free control [1]. This method is simple to implement in practice but less effective in rejecting unmeasured disturbances which are time-varying.

To improve MPC's disturbance rejection ability, Morari suggested using ramp model to represent the output disturbance [2]; Wellons presented a generalized analytical predictor to estimate the effect of disturbance by a first-order or second order transfer function [3]. However, these fixed disturbance models have limited performance when disturbance is time-varying. Ohshima et al. proposed an MPC algorithm with adaptive disturbance prediction [4], the dynamics of the unmeasured disturbance is identified by an auto-regressive time series model and the algorithm is effective in a fatty acid distillation column control. Xu Zuhua et al. proposed a disturbance adaptation MPC (DMCA), the unmeasured disturbance is estimated by an ARMA model and a novel MIPLR algorithm is used to identify the parameters for ARMA model [5]. Application in a distillation column shows that DMCA has better disturbance rejection performance.

Parameter design has great impact on MPC's closed-loop performance [6]. Cost function can be freely chosen in MPC and different parameter selection can lead to entirely different control performance. As there is no analytic relation between controller parameters and the robustness, disturbance rejection ability of the controller, so the parameters are usually tuned by trial and error simulations [7]. Some new tuning strategies have been proposed [8-10], but they are generally subject to certain conditions, so trial and error simulations are still important and have wide applications in engineering. DMCA is developed on the basis of DMC and added with the function of disturbance model identification and disturbance prediction, so there are more parameters to be designed. Parameter design for DMCA is researched and then the effects of controller parameters on the disturbance rejection performance are summarized in the paper.

2 Introduction of DMCA

Structure of DMCA control system is showed in Figure 1, $y_{sp}(k)$ is the setpoint of controlled variable, $d(k)$ is the unmeasured disturbance added to the output and $y(k)$ is the plant output, $y_m(k)$ is model prediction value.

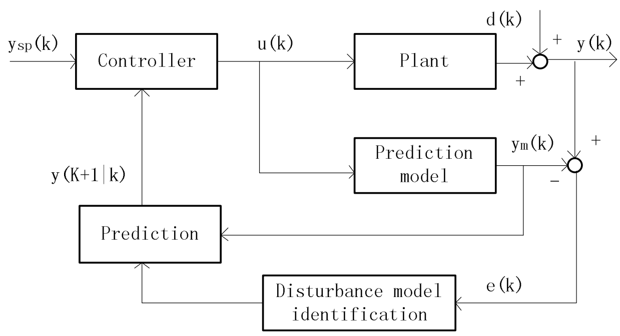

Fig. 1: Structure of DMCA

Denote output error $e(k)$ as the difference of $y(k)$ and $y_m(k)$:

$$\begin{aligned} e(k) &= y(k) - y_m(k \mid k-1) \\ &= G(z)u(k) + d(k) - G_m(z)u(k) \end{aligned} \tag{1}$$

[*]This work is supported by National Natural Science Foundation (NNSF) of China under Grant 60934007.

where $G(z)$ is the plant and $G_m(z)$ is the prediction model.

In DMC strategy, output error $e(k)$ is used to correct the prediction output for the entire prediction horizon H_p:

$$e(k+i \mid k) = e(k),\ i = 1, \cdots, H_p \tag{2}$$

The DMC correction strategy showed above lacks of prediction for the dynamics of the disturbance and has limited disturbance rejection ability. In fact, if the prediction model $G_m(z)$ is good enough, the following formula holds approximately:

$$e(k) \approx d(k); \Delta e(k) \approx \Delta d(k) \tag{3}$$

where $\Delta e(k)$ is the differential signal of $e(k)$.

In DMCA, the disturbance model is identified through $e(k)$. Most disturbances in chemical process are persistent in nature and 'non-stationary', hence $e(k)$ can be described by filtered integrated white noise [11], and this implies that $\Delta e(k)$ is stationary random sequence and can be described by an ARMA (m, n) model:

$$\begin{aligned} &\Delta e(k) + a_1 \Delta e(k-1) + \cdots + a_m \Delta e(k-m) \\ &= \Delta w(k) + c_1 \Delta w(k-1) + \cdots + c_n \Delta w(k-n) \end{aligned} \tag{4}$$

where $\Delta w(k)$ is zero mean white noise, m, n are orders of ARMA model. Parameters a, c can be recursively identified by MIPLR algorithm which makes the disturbance model more precise [5]. Then, by the proper selection of state variables, ARMA model can be transformed into state-space model [12], which is suitable for predicting the disturbance in the future moment.

Actual industrial processes may contain high-frequency noise that will cause unnecessary drastic change of control action. To reduce the impact of high-frequency noise, signal $\Delta e(k)$ can be low-pass filtered before used to predict the disturbance in the ARMA model. A simple low-pass filter is described by

$$\Delta e_f(k) = a\Delta e(k) + (1-a)\Delta e_f(k-1) \tag{5}$$

where $\Delta e_f(k)$ is the signal after filter, a is the filter gain and $0 < a < 1$.

The same quadratic performance index is adopted by both DMCA and DMC, and the receding horizon control strategy is also the same.

The unconstrained MPC is linear and the closed-loop structure can be given. Sensitivity function $S(z)$ is the closed-loop transfer function between $d(k)$ and $y(k)$ showed in fig.1. Bandwidth can be denoted as the frequency when the amplitude curve of the sensitivity function passes through -3dB from the bottom up for the first time. Bandwidth and robustness of MPC controller can be studied by the amplitude curve [13, 14]. More details about DMCA's closed-loop structure can be found in [15].

3 Parameter Design and Simulations for DMCA

In addition to the parameters of conventional MPC such as sampling period, prediction horizon, control horizon and weight coefficients, DMCA has some new parameters related to disturbance model and disturbance prediction, such as orders of the ARMA model, disturbance prediction horizon and filter gain. Only after optimization design of these parameters the DMCA controller can have best control performance. Besides, MPC algorithms must go through the parameter tuning part before implementation, which is also an important part of simulations.

3.1 Simulation object

With a given plant and the unmeasured disturbance $d(k)$, DMCA can obtain the disturbance model by recursive identification and then reject the disturbance. So the plant and the disturbance data is introduced first, then the parameter design methods are given combined with the examples.

The plant is described as:

$$G(s) = \frac{4.1}{50s+1} e^{-3s} \tag{6}$$

Set the sampling period $T_s = 1s$, 1000 samples of filtered integrated white noise are used as output unmeasured disturbance, and the disturbance is showed in Fig. 2.

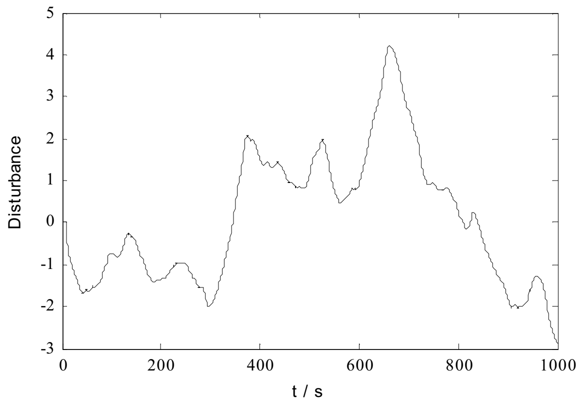

Fig. 2: Unmeasured disturbance on the process output

Integral absolute error (IAE) and integral absolute of the control action increment (IAdU) are adopted as evaluation indices of the controller performance in the simulations.

$$\begin{aligned} IAE &= \sum_{k=1}^{1000} | y(k) - y_{sp}(k) | \\ IAdU &= \sum_{k=1}^{1000} | \Delta u(k) | \end{aligned} \tag{7}$$

3.2 Conventional Parameters Design for DMCA

Conventional parameters refer to the same parameters as usual MPC, including sampling period, modeling horizon, prediction horizon, control horizon and weight coefficients. The design methods for these parameters have been studied in many literatures [7, 16] and can be applied in DMCA. They are briefly summarized as follows.

(1) Sampling period T_s and modeling horizon n

T_s is selected according to the type of the controlled object and its dynamic characteristics, it should meet the Shannon sampling theorem. Dynamic information of the object should be included in the step model completely, so the step response is required to get close to the steady-state value after time nT_s. In order to reject disturbance quickly, small T_s and large n is needed.

(2) Prediction horizon H_p and control horizon H_c

H_p should exceed the time delay of the object's step response, and contains the major part of the dynamic response. Larger H_p makes slower dynamic performance and more robustness for the controller. Larger H_c makes faster dynamic performance and less robustness. These two parameters are generally tuned according to the dynamics of the process, not the disturbance rejection performance [17]. And basically there is condition $H_c \le H_p$.

Select $H_p = 80$, $H_c = 10$, and set the controlled variable's setpoint as 5, CV weight $Q_1 = 1$ and MV increment weight $Q_2 = 0.1$, the simulation results of DMCA and DMC are showed in table 1 and Fig. 3. The results show that the IAE index of DMCA is 45% less than DMC, so the controlled variable of DMCA is closer to the setpoint. However, the IAdU index of DMCA is larger, which means more dramatic changes of control action for DMCA.

Table1: IAE and IAdU for DMCA and DMC

Item	DMCA	DMC
IAE	142.7	258.9
IAdU	46.8	22.8

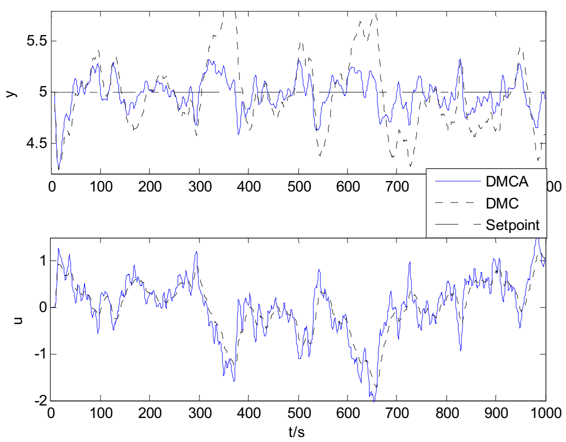

Fig. 3: CVs and MVS for DMCA and DMC

(3) CV weight Q_1 and MV increment weight Q_2

Q_1 indicates different importance of the controlled variable, larger coefficients in Q_1 will make the corresponding CV's control quality better. Q_2 is used to restrict the drastic change of the control action. When Q_2 is small, the control action is strong and may easily lead to overshoot of the controlled variable. When Q_2 is large, the control action may respond slowly.

Both Q_1 and Q_2 will affect the sensitivity function $S(z)$. The amplitude curves of $S(z)$ with different weights are showed in Fig. 4.

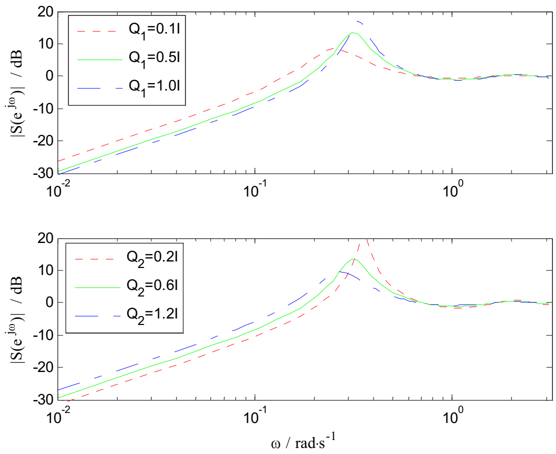

Fig. 4: Amplitude curves of sensitivity function

From the first part of Fig.4 we can know that larger CV weight Q_1 will make the peak value of the sensitivity function get larger, which means worse robustness. Besides, the bandwidth (the frequency when the amplitude curve passes through -3dB from the bottom up for the first time) will get larger, which means faster response speed for the DMCA controller. From the second part of Fig.4 we can know that larger MV increment weight Q_2 will make the peak value get smaller, which means better robustness. Besides, the bandwidth will get smaller, which means slower response speed for the controller.

3.3 Disturbance Model Parameters Design for DMCA

There are some new parameters related to disturbance model, such as orders of the ARMA model, disturbance prediction horizon and filter gain. Tuning of these parameters is very important to the accuracy of the disturbance model and the disturbance prediction. Their tuning strategy and effects on DMCA's disturbance rejection performance will be introduced in this part.

(1) Orders of the disturbance model ARMA (m, n)

As can be seen from formula (4), the higher order of ARMA model, the more parameters to be identified, which needs more time for operation and will not be able to respond quickly for the fast varying disturbance. What's more, as the ARMA model contains history data $\Delta e(k-m),\cdots,\Delta e(k-1)$ high order model contains more old dynamics about disturbance, which will reduce the prediction accuracy of disturbance dynamics in future time. As a result, low order ARMA model is preferred, which will not make the model too complex.

For the plant described in 3.1, simulation results with different orders of ARMA model are showed in table 2 and Fig. 5.

Table2: Relations between order of ARMA and IAE, IAdU

Item	IAE	IAdU

ARMA(1,1)	181.5	49.9
ARMA(2,1)	172.3	48.3
ARMA(2,2)	164.9	46.7
ARMA(3,2)	156.5	46.4
ARMA(3,3)	142.7	46.8
ARMA(4,3)	145.3	46.1
ARMA(4,4)	140.5	45.9
ARMA(5,5)	141.9	47.5
ARMA(6,6)	143.4	47.8
ARMA(7,7)	147.2	49.3
ARMA(8,8)	148.8	51.9

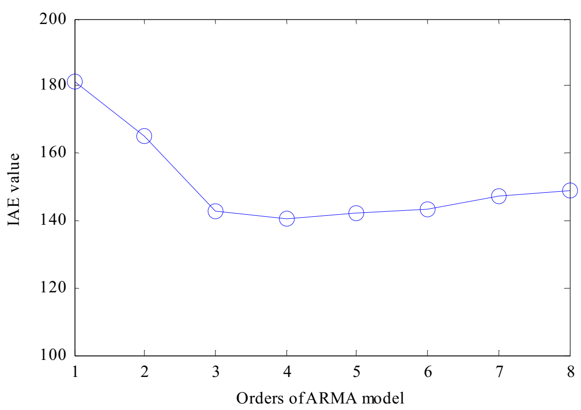

Fig. 5: Relations between order of ARMA and IAE

It can be seen that IAEs are too large when first or second order ARMA models are selected, the reason is that these models are too simple to describe the disturbance dynamics. When the order gets three or above, IAE will get better. However, IAdU will also become larger and make the control action change drastically at the same time. So, these facts taken together, third or fourth order model are selected usually.

20 steps ahead prediction of disturbance by ARMA (3, 3) is showed in Fig. 6. The left-most curve in the figure denotes the 20 steps ahead (609~628 seconds) prediction of disturbance at 608 second, the first circled point in the curve denotes the real disturbance value at 608 second and each solid point afterwards denotes the prediction value for the next moment. By this curve we know that the unmeasured disturbance will get larger in the future, which coincide with the real disturbance value showed in Fig. 2. The second to eighth curves from left to right in Fig. 6 denote the 20 steps ahead disturbance predictions at 609~615 seconds in turn(To avoid curves aliasing, the space between each curve is enlarged in the figure, so the horizontal ordinate of the curves and the time are not very consistent). Comparing the prediction values at different time and the true at some moment, we can know that the disturbance prediction by ARMA model is quite accurate when disturbance prediction horizon is not too large.

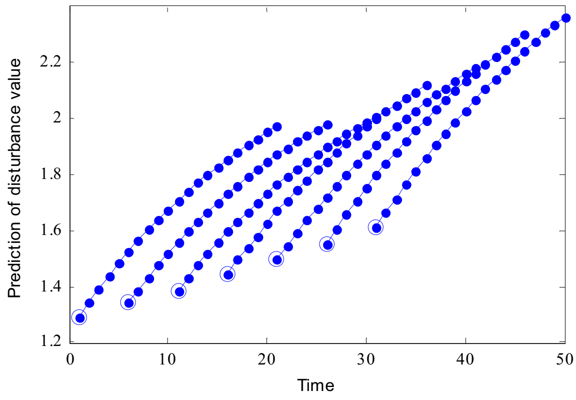

Fig. 6: Disturbance prediction by the ARMA model

(2) Disturbance prediction horizon H_d

The ARMA model identified by MIPLR algorithm can be can be transformed into state-space model which is suitable for multi-step ahead prediction of the disturbance. However, disturbances in actual industrial processes are complex and time-varying, so long-term estimation is clearly not accurate enough. Besides, ARMA model's long-term prediction ability is not very good. Usually H_d is set smaller than the prediction horizon H_p and then the disturbance is assumed to be constant between time H_d+1 and H_p.

$$\begin{aligned} &d(k+H_d+i\,|\,k)=d(k+H_d\,|\,k) \\ &i=1,\cdots,H_p-H_d \end{aligned} \tag{8}$$

Simulation results with ARMA (3, 3) model and different H_d are showed in table 3 and Fig. 7.

Table3: Relations between Hd and IAE, IAdU

Hd	IAE	IAdU	Hd	IAE	IAdU
1	245.9	26.2	11	116.7	58.6
2	220.5	30.2	12	116.1	60.1
3	197.6	34.5	13	116.1	61.4
4	176.9	38.8	14	116.5	62.6
5	159.3	42.6	15	117.1	63.7
6	142.7	46.8	16	117.8	64.1
7	133.9	49.4	17	118.8	64.5
8	126.0	52.8	18	119.7	64.8
9	120.9	55.6	19	120.8	65.0
10	118.2	57.8	20	120.9	65.1

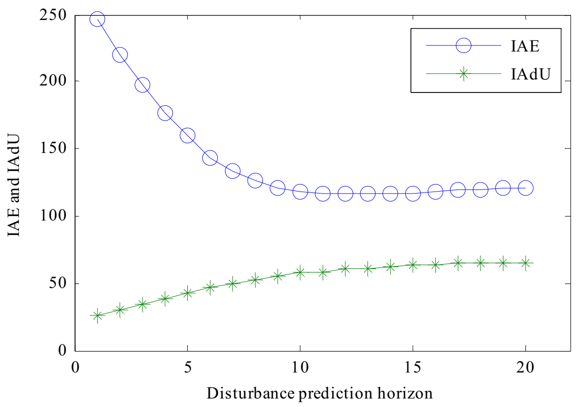

Fig. 7: Relations between Hd and IAE, IAdU

It can be seen that IAE is the largest when H_d is 1, the result is similar to DMC. IAE decreases rapidly when H_d increases. When H_d approaches about 12, the disturbance rejection performance is the best. But further increase of H_d will leads to increase of IAE because of the accumulation of model error. Besides, IAdU increases monotonically with H_d. So the tuning of H_d should take into account both IAE and IAdU, that is, the disturbance rejection effect and smoothness of the control action.

The tuning of H_d also relates to time delay of the plant, H_d should exceed the time delay. Simulation and application experiences give the following approximate formula of H_d

$$H_d \approx \tau + 8 \tag{9}$$

where τ is time delay of the plant. The best disturbance rejection effect is acquired when disturbance prediction horizon is about 8 steps longer than time delay of the plant.

(3) Filter gain a

The control actions change drastically than DMC. To reduce the unnecessary change, an adjustable parameter of filter gain a is introduced to the DMCA algorithm. The smaller a, the faster attenuation characteristic for the filter. Transfer function for the filter described in formula (5) is as following:

$$H(z) = \frac{a}{1-(1-a)z^{-1}}, 0 < a < 1 \tag{10}$$

Set $z = e^{jwT_s}$ and the frequency response of the filter can be get, the amplitude-frequency response curves with different a are showed in Fig. 8.

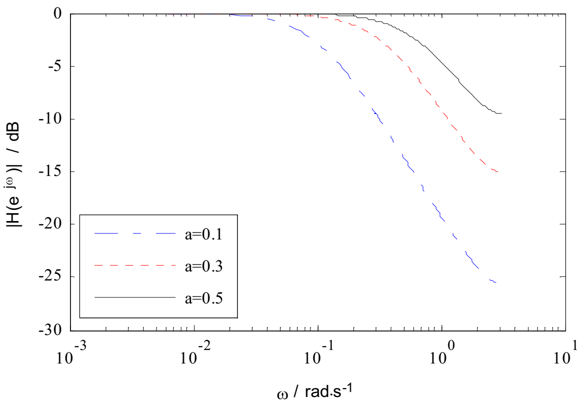

Fig. 8: Amplitude-frequency response curves of the filter

Simulation results with ARMA (3, 3) model, $H_d = 10$ and different a are showed in table 4 and Fig. 9.

Table 4: Relations between filter gain and IAE, IAdU

Filter gain	IAE	IAdU
1	118.2	57.8
0.9	118.6	56.7
0.8	119.2	55.6
0.7	120.7	54.3
0.6	122.5	52.8
0.5	124.4	50.7
0.4	127.0	48.3
0.3	130.4	44.5
0.2	134.6	39.3
0.1	137.3	34.6

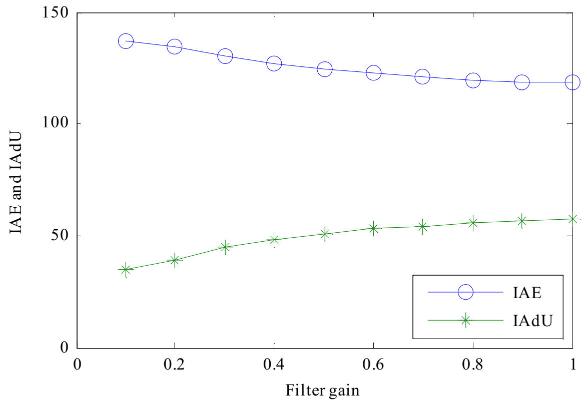

Fig. 9: Relations between filter gain and IAE, IAdU

When filter gain is set to 1, it is equivalent to that the data is not filtered. With the decrease of a, the low-pass filtering effect is enhanced and IAdU gets smaller, so the control action's dramatic changes are weakened. But smaller a will also lead to more phase lag and IAE increase. So usually a is set between 0.2 and 0.6 according to the actual situation.

4 Conclusion

Adaptive disturbance model can estimate the dynamic characteristics of unmeasured disturbances more accurately and the disturbance rejection ability is enhanced by disturbance prediction. Parameter tuning also has great impact on MPC's disturbance rejection performance. Parameter design methods for DMCA controller are researched in the paper, impacts of different parameters on the control performance are summarized by simulations and some experiential criteria are proposed. Simulations show that after optimization design of the parameters, DMCA controller's disturbance rejection performance is improved further.

References.

[1] S. J Qin, T A. Badgwell. A survey of industrial model predictive control technology. *Control Engineering Practice*, 2003, 11(7): 733-764.

[2] M. Morari, J H. Lee, Model predictive control: the good, the bad, and the ugly. *Chemical Process Control—CPC IV*, 1991: 419-444.

[3] M. C. Wellons, T.F. Edgar. The generalized analytical predictor for chemical process control. *Ind. Eng. Chem. Res*, 1987, 26(8): 1523-1536.

[4] M. Ohshima, H. Hashimoto. Model predictive control with adaptive disturbance prediction and its application to fatty acid distillation column control. *Journal of Process Control*, 1995, 5(1): 41-48.

[5] Xu Zuhua, Zhu Yucai, Han Kai. Multi-iteration pseudo-linear regression method and an adaptive disturbance model for MPC. *Journal of Process Control*, 2010, 20(4): 384-395.

[6] R. Shridhar, D J. Cooper. A novel tuning strategy for multivariable model predictive control. *ISA transactions*, 1997, 36(4): 273-280.

[7] Xi Yugeng. *Predictive Control*. Beijing: National Defense Press, 1993.

[8] J. Trierweiler, L. Farina. RPN tuning strategy for model predictive control. *Journal of Process Control*, 2003. 13(7): 591-598.

[9] W. Wojsznis, J W.Gudaz. Practical approach to tuning MPC. *ISA Transactions*, 2003, 42(1): 149-162.

[10] Xu Zuhua, Zhao Jun, Qian Jixin. Robust design method of predictive controller parameter based on min-max ruler. *Journal of Chemical Industry and Engineering (China)*, 2004, 55(4):613-617.

[11] J. Lee, Z. Yu. Tuning of model predictive controllers for robust performance. *Computers & Chemical Engineering*, 1994, 18(1): 15-37.

[12] H. Kai, Zhao Jun, Zhu Yucai. A robust MPC technique with adaptive disturbance model. *Journal of the Chemical Industry and Engineering Society of China*, 2009, 60(7):1730-1738.

[13] J M. Maciejowski. *Predictive Control: with Constraints*. Pearson education, 2002.

[14] J. Fan, G E. Stewart, G A. Dumont. Two-dimensional frequency analysis for unconstrained model predictive control of cross-directional processes. *Automatica*, 2004, 40(11): 1891-1903.

[15] Wang Wenbo, Zhao Jun, Xu Zuhua. Frequency behavior analysis for predictive control with adaptive disturbance model. *Computers and Applied Chemistry*, 2011. 28(12): 1492-1496.

[16] R. Shridhar, D J. Cooper. A tuning strategy for unconstrained multivariable model predictive control. *Industrial & Engineering Chemistry Research*, 1998. 37(10): 4003-4016.

[17] M. Alamir, G. Bornard. Stability of a truncated infinite constrained receding horizon scheme: the general discrete nonlinear case. *Automatica*, 1995. 31(9): 1353-1356.