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HW5
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1 1

$$\nabla F(x) = 2(x - u)$$
$$\nabla^2 F(x) = 2I$$

A diagonal matrix is positive semidefinite, thus it is convex.

2 2

The function is a sum of terms, thus each gradient element is only a function of p_i. This means all basic operations are vectorizable.

$$\nabla F(p) = -(p * 1/p + \ln(p)) = -(1 + \ln(p))$$
$$\nabla^2 F(p) = diag(-1/p)$$

The hessian is a diagonal matrix of negative values (since p ranges from 0 to 1). Thus, entropy must be concave.

3 3

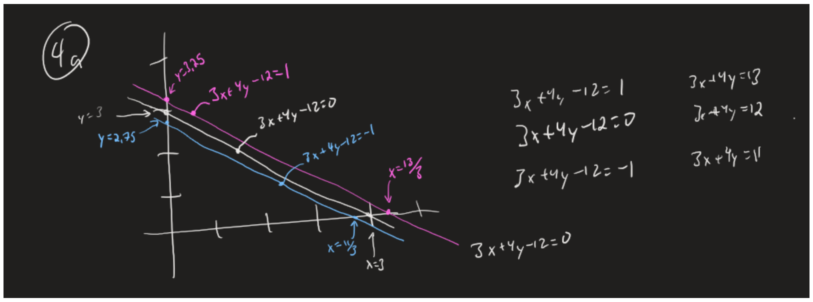
In the perceptron learning algorithm, b is updated by the label everytime an update is performed. Thus,

$$b_0 = 0, b_{final} = 0 + (-1) * p + (1) * q$$

$$b_{final} = q - p$$

4 4

4.1 a



4.2 b

4.3 c

$\gamma = \frac{1}{\|w\|} = \frac{1}{5}$

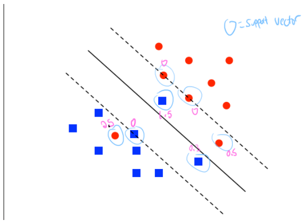
4.4 d

classification $= \text{sign}(w \cdot x + b)$
$= \text{sign}(3 * 2 + 4 * 2 - 12) = \text{sign}(2) = +1$

5 5

5.1 a

5.2 b



If the C parameter increased, we would be punished more for using slack. Thus, the margin would grow, since the magnitude of slack is in terms of the margin size (same absolute error / larger margin = smaller slack).

6 6

6.1 a

Possibly false. The α_i are incremented every time a point is classified wrong, and the point can be classified wrong multiple times.

6.2 b

Necessarily true. Each update increments one entry in the α vector. Thus, the sum of entries must equal the number of updates.

6.3 c

Necessarily true. If we wanted to maximize the number of nonzero entries in the α vector, we can update a different entry each time. This leads to at most k indices having non-zero entries.

6.4 d

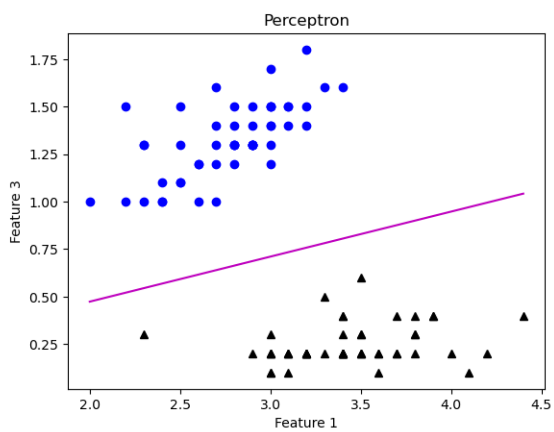
Necessarily true. If a perceptron converges, it means that there are no more possible updates / no misclassified points and the binary classifier is perfectly modeled. This means that there must exist a linear decision boundary - hence the data must be linearly separable.

7 7

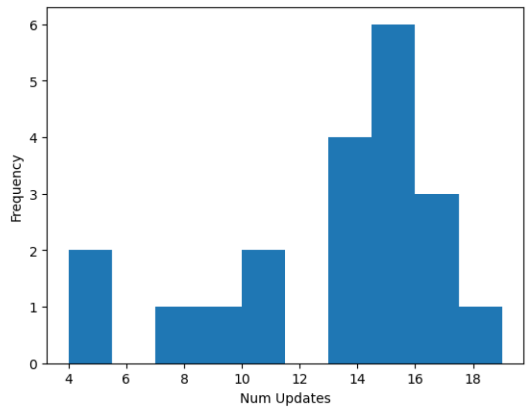
7.1 a

7.2 b

7.3 c



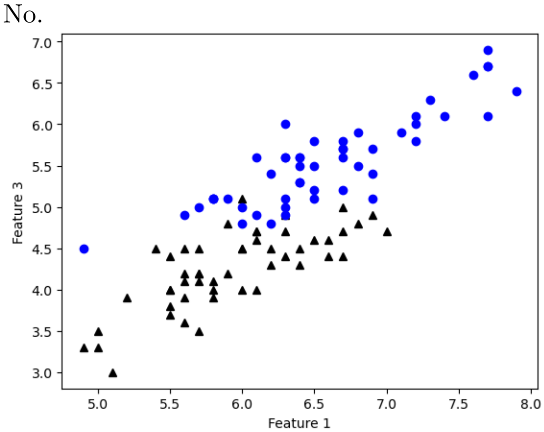
7.4 d



8 8

8.1 a

No.

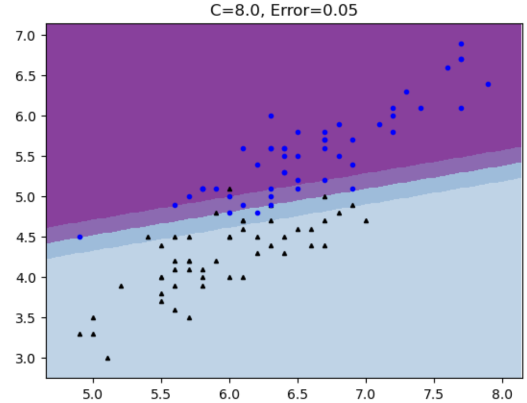


8.2 b

C	Error	Num Support Vectors
0.125	0.07	52
0.25	0.06	45
0.5	0.06	38
1.0	0.07	31
2.0	0.06	24
4.0	0.07	21
8.0	0.05	19
16.0	0.07	16
32.0	0.06	15
64.0	0.05	14
128.0	0.05	14
256.0	0.05	14
512.0	0.05	14

8.3 c

Best value: C=8



In [96]:

```python
import numpy as np
```

Q7

In [97]:

```python
# (7a)
# First Function
def perceptron(w, b, x):
  return np.sign(np.dot(w.flatten(), x.flatten()) + b)

def has_converged(w, b, X, l):
  return np.allclose(np.sign(X @ w + b).flatten(), l.flatten())

MAX_ITERS = 72

# Second Function
def train(X, l):
  # data: n x d, labels: n
  n, d = X.shape
  assert(l.shape == (n,))

  w = np.zeros((d, 1))
  b = 0

  num_updates = 0

  for i in range(MAX_ITERS):
    permutation = np.random.choice(range(n), n, replace=False)
    X_perm = X[permutation]
    l_perm = l[permutation]

    for j in range(n):
      x_j = X_perm[[j],:]
      l_j = l_perm[j]
      if perceptron(w, b, x_j) != l_j:
        num_updates += 1
        w = w + l_j * x_j.reshape((-1, 1))
        b += l_j

    if has_converged(w, b, X_perm, l_perm):
      break

  return w, b, num_updates
```

In [98]:

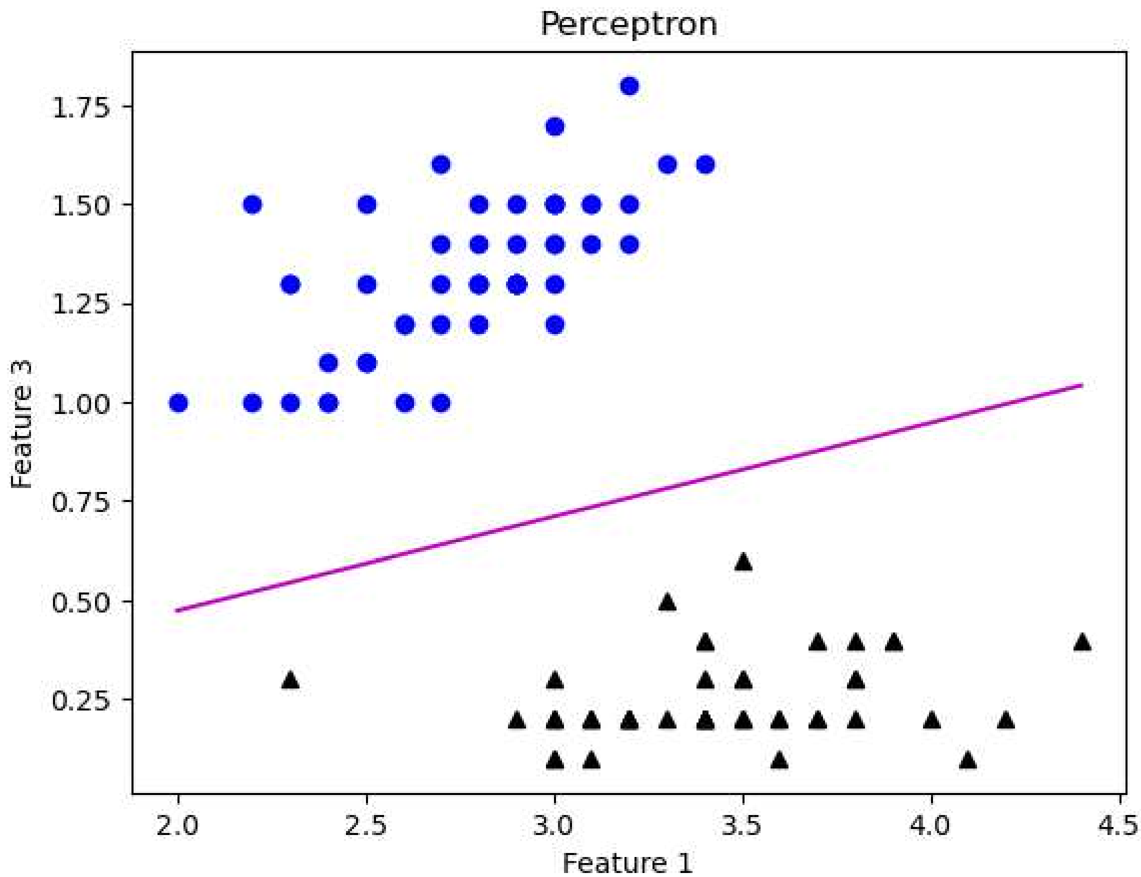
```python
# (7b, 7c)

from sklearn import datasets
iris = datasets.load_iris()
X0 = iris.data
y0 = iris.target

print(X0.shape, y0.shape)

rows = np.array(np.nonzero(y0 <= 1)[0])

X_1 = X0[rows.reshape((-1, 1)),(1,3)]
```

```
y_1 = np.where(y0[rows] == 0, -1, 1)

print(X_1.shape, y_1.shape)
```

```
(150, 4) (150,)
(100, 2) (100,)
```

In [99]:

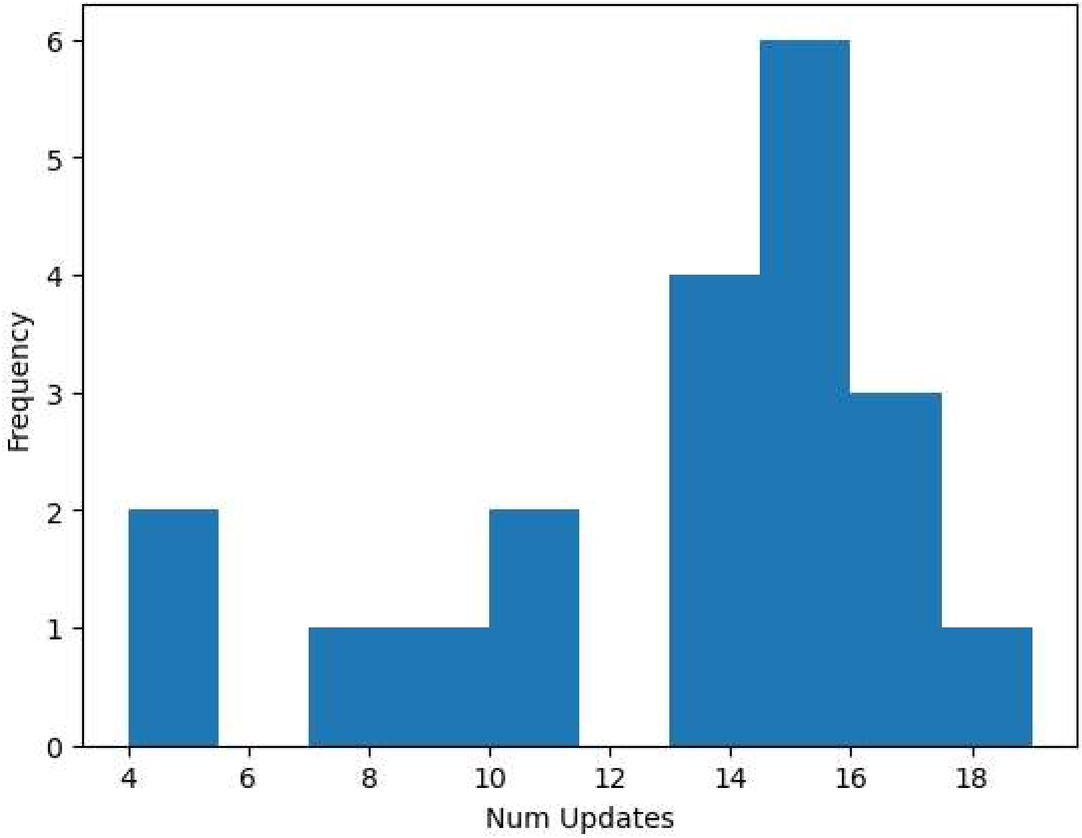
```
import matplotlib.pyplot as plt

# (7c)
np.random.seed(123)
w, b, num_updates = train(X_1, y_1)

def plot(w, b, X, l, draw_line = True, title = "Perceptron"):
  w1, w2 = w.flatten()
  x_min, x_max = np.min(X[:,0]), np.max(X[:,0])
  line_x = np.linspace(x_min, x_max, 100)
  # w1x + w2y + b = 0
  # y = -w1/w2 x - b / w2
  line_y = - w1 / w2 * line_x - b / w2


  plt.plot(X[l==-1,0], X[l==-1,1], "^k")
  plt.plot(X[l==1,0], X[l==1,1], "ob")
  if draw_line:
    plt.plot(line_x, line_y, "-m")
  plt.xlabel("Feature 1")
  plt.ylabel("Feature 3")
  plt.title(title)

plot(w, b, X_1, y_1)
```



In [100…

```
np.random.seed(1337)
n_trials = 20
updates = []
for i in range(n_trials):
  _, _, n_i = train(X_1, y_1)
  updates.append(n_i)
total_updates = sum(updates)
print(f"Average number of updates: {total_updates}/{n_trials}={total_updates / n_trials
plt.hist(updates)
plt.xlabel("Num Updates")
plt.ylabel("Frequency");
```

```
Average number of updates: 258/20=12.9
```

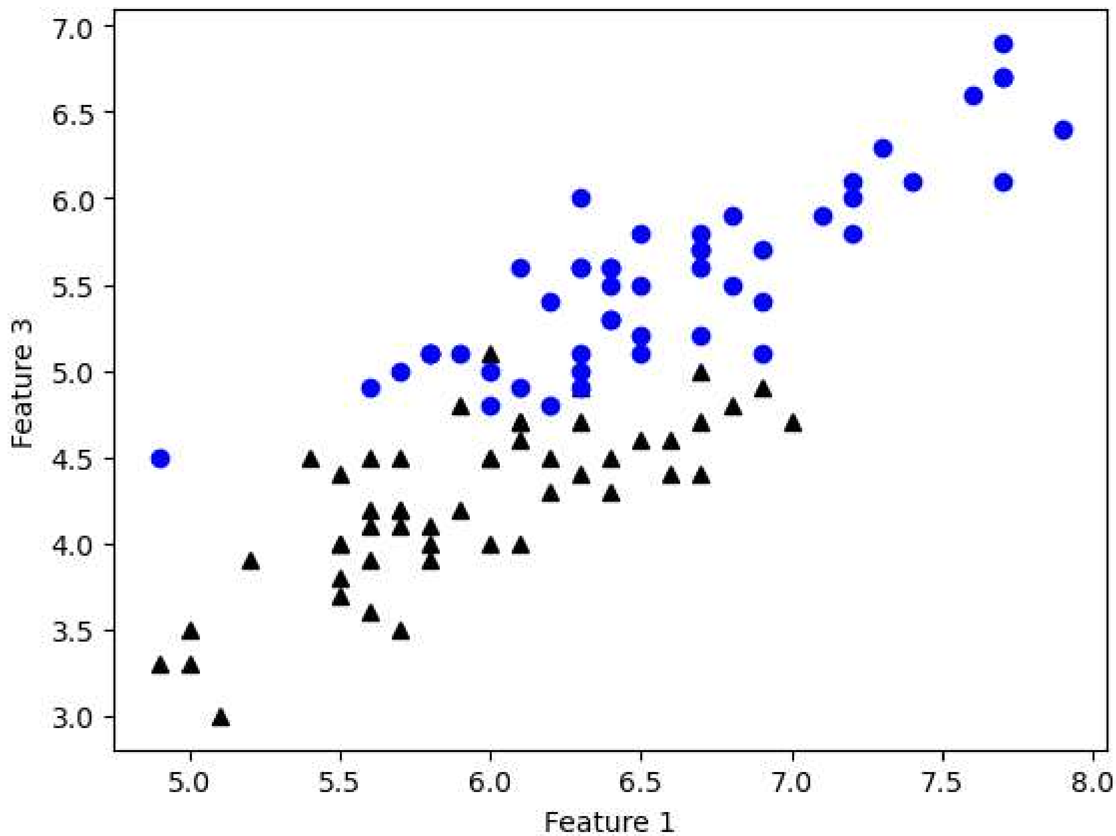


Q8

In [101…

```
rows_2 = np.array(np.nonzero(y0 > 0)[0])
X_2 = X0[rows_2.reshape((-1, 1)),(0, 2)]
y_2 = np.where(y0[rows_2] == 1, -1, 1)

print(X0.shape, y0.shape)
print(X_2.shape, y_2.shape)

plot(np.ones((2, 1)), 1, X_2, y_2, draw_line = False, title = None)

print("(8a) The data is NOT linearly seperable")
```

```
(150, 4) (150,)
(100, 2) (100,)
(8a) The data is NOT linearly seperable
```



In [102…

```
np.c_[np.array([1,2,3]), np.array([4,5,6])]
```

Out[102…

```
array([[1, 4],
       [2, 5],
       [3, 6]])
```

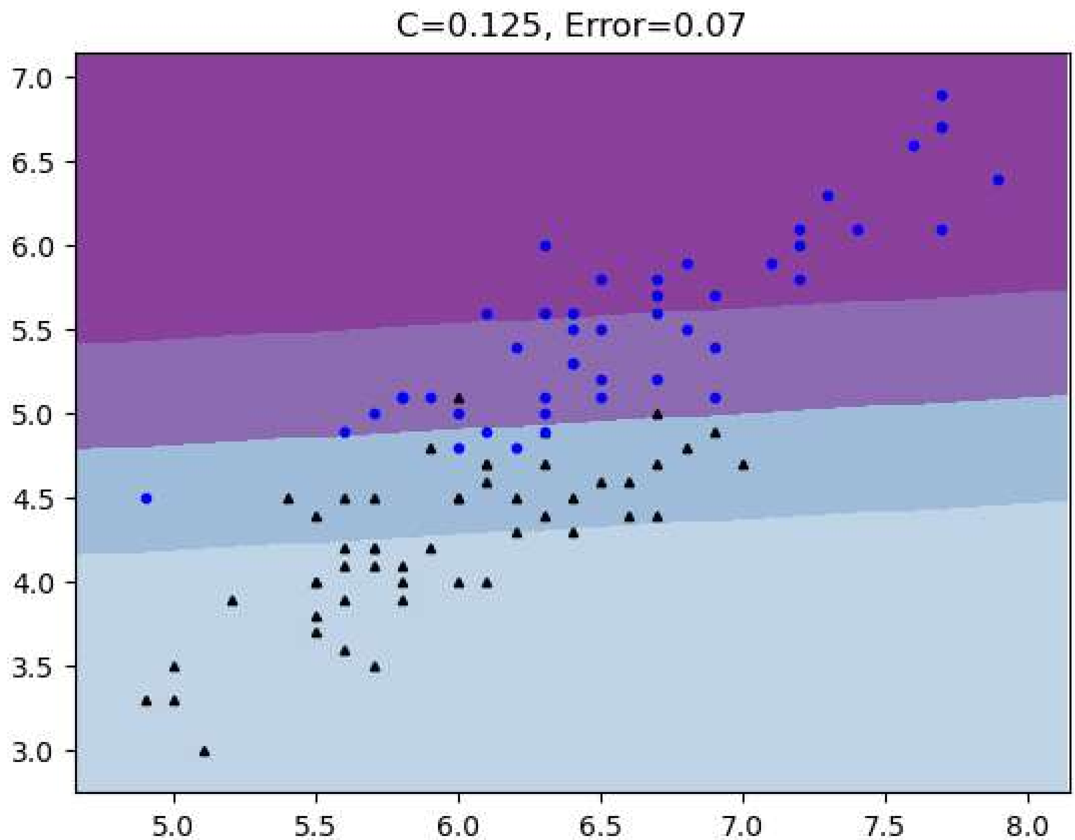
In [103…

```
import sklearn
def full_plot(X, y, svc: sklearn.svm.SVC, C, error):
  plt.plot(X[y==-1,0], X[y==-1,1], "^k", markersize=3)
  plt.plot(X[y==1,0], X[y==1,1], "ob", markersize=3)
  margin = 0.25
  x_min, x_max = np.min(X[:,0]), np.max(X[:,0])
  y_min, y_max = np.min(X[:,1]), np.max(X[:,1])
  delta = 0.01

  xx, yy = np.meshgrid(np.arange(x_min - margin, x_max + margin, delta), np.arange(y_mi
  Z = svc.decision_function(np.c_[xx.flatten(), yy.flatten()])
  for i in range(len(Z)):
    Z[i] = min(Z[i],1.0)
    Z[i] = max(Z[i],-1.0)
    if (Z[i] > 0.0) and (Z[i] < 1.0):
        Z[i] = 0.5
    if (Z[i] < 0.0) and (Z[i] > -1.0):
        Z[i] = -0.5
  Z = Z.reshape(xx.shape)
  plt.pcolormesh(xx, yy, Z, cmap=plt.cm.BuPu, vmin=-2, vmax=2)
  plt.xlim((x_min - margin, x_max + margin))
  plt.ylim((y_min - margin, y_max + margin))
  plt.title(f"C={C}, Error={error}")
  plt.show()
```

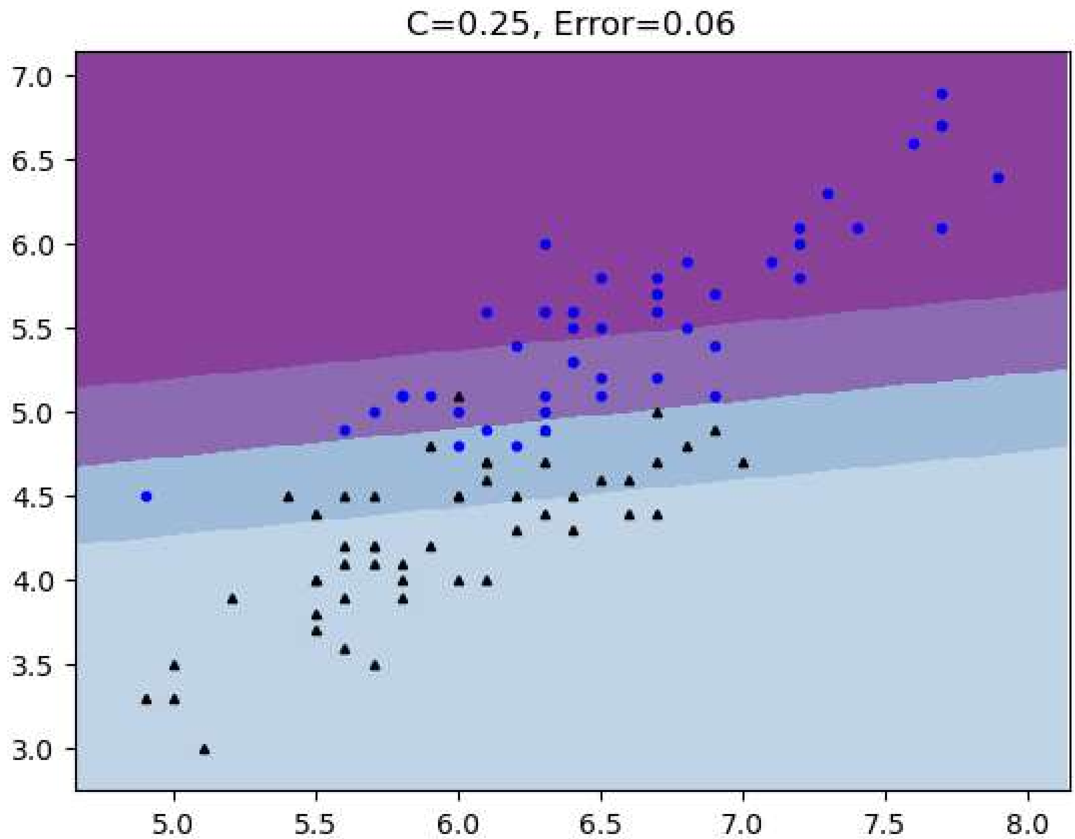
In [104…

```python
import sklearn

Cs = np.exp2(np.arange(-3, 10))
data = []
for c in Cs:
  print(f"=== {c} ===")
  svm = sklearn.svm.SVC(kernel="linear", C=c)
  svc = svm.fit(X=X_2, y=y_2)

  wrong = np.mean(np.not_equal(svm.predict(X_2), y_2))
  print(f"\tError rate: {wrong}")
  full_plot(X_2, y_2, svc, c, wrong)
  num_sv = len(svc.support_)

  data.append((c, wrong, num_sv))
```

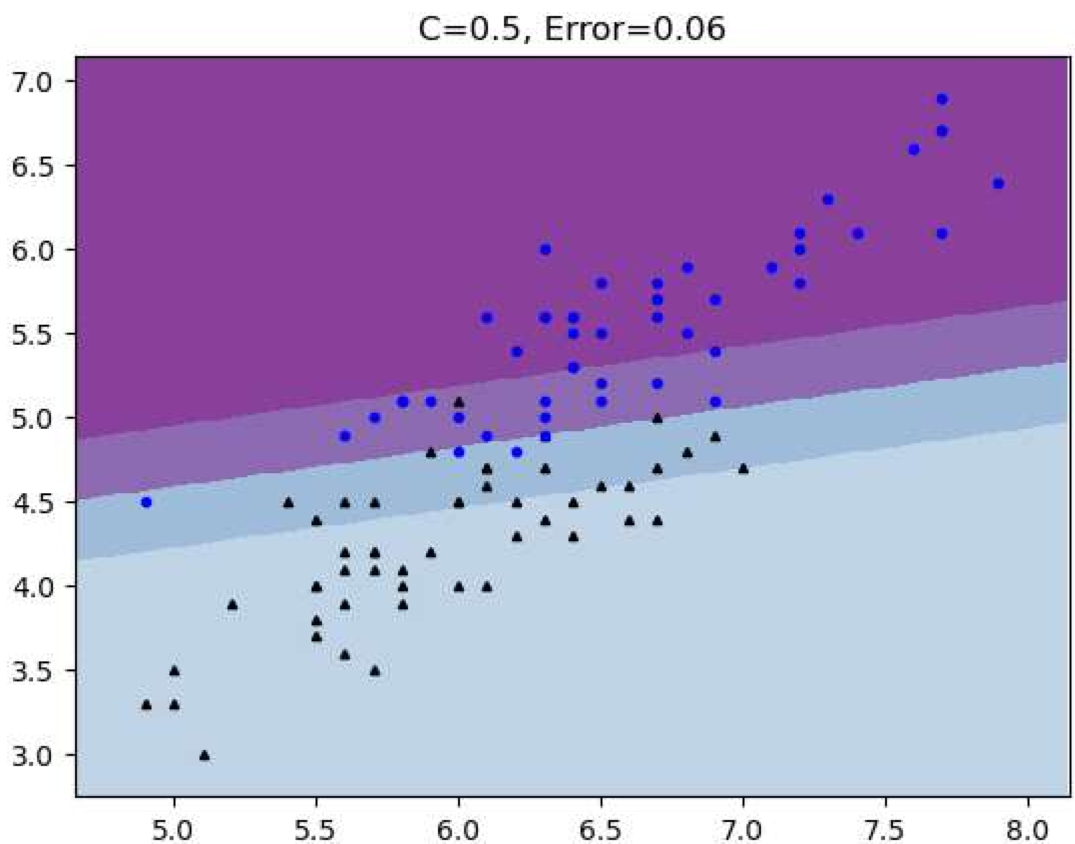
```
=== 0.125 ===
	Error rate: 0.07
```



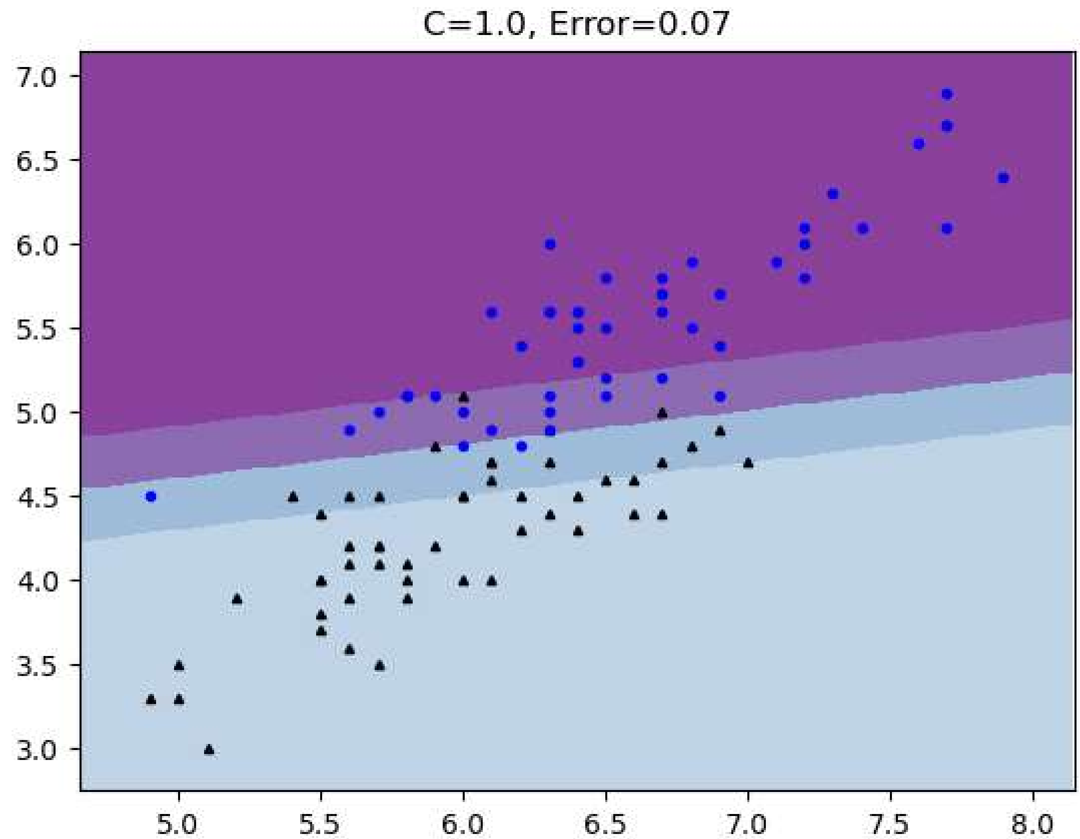

```
=== 0.25 ===
	Error rate: 0.06
```



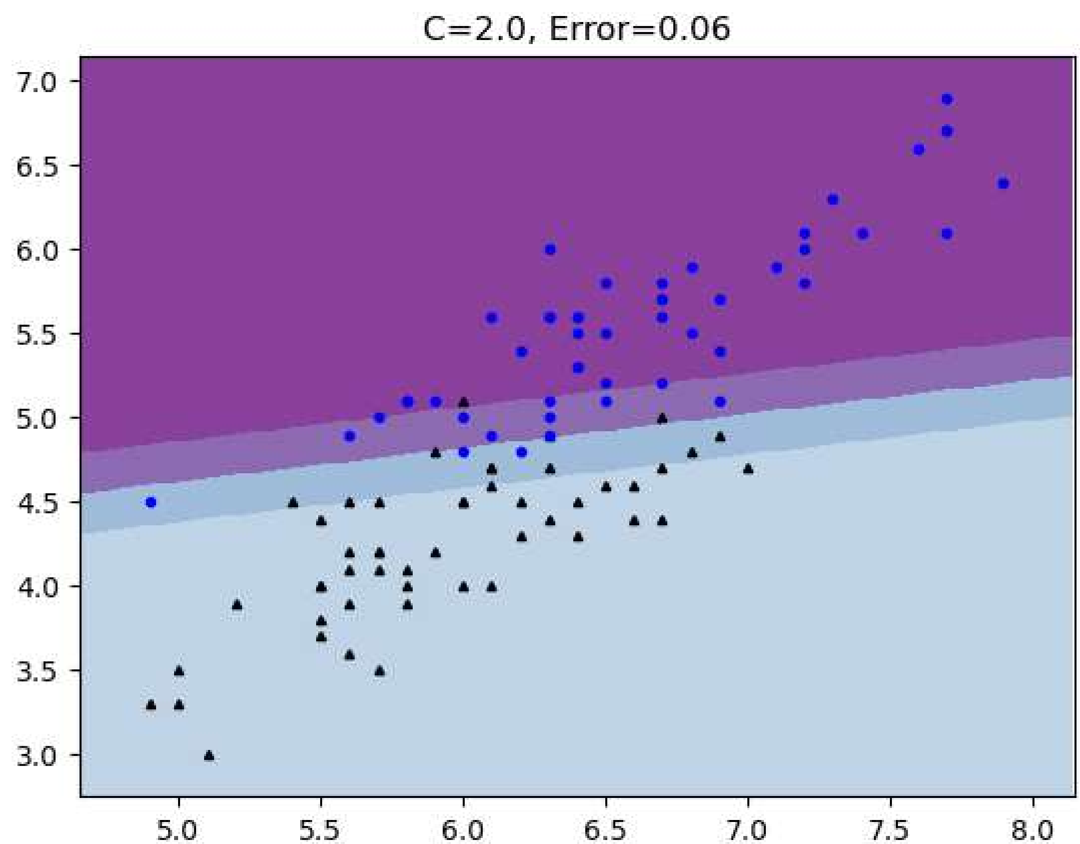

```
=== 0.5 ===
      Error rate: 0.06
```



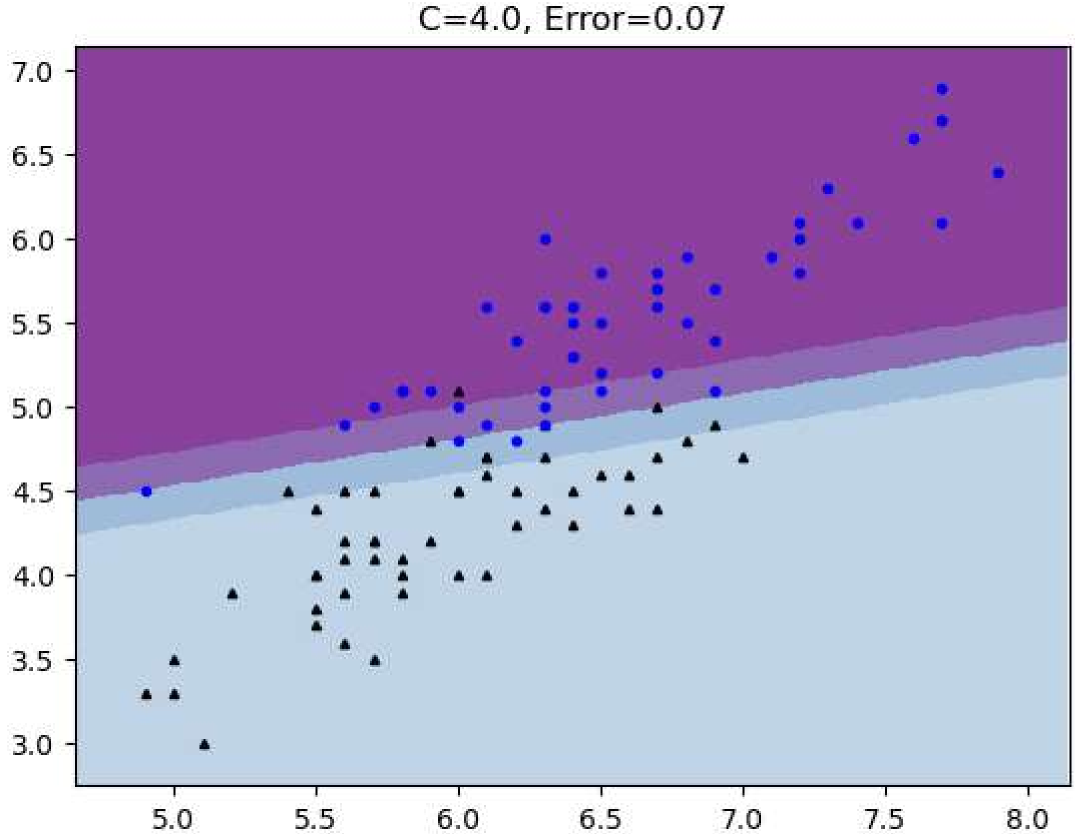

```
=== 1.0 ===
      Error rate: 0.07
```



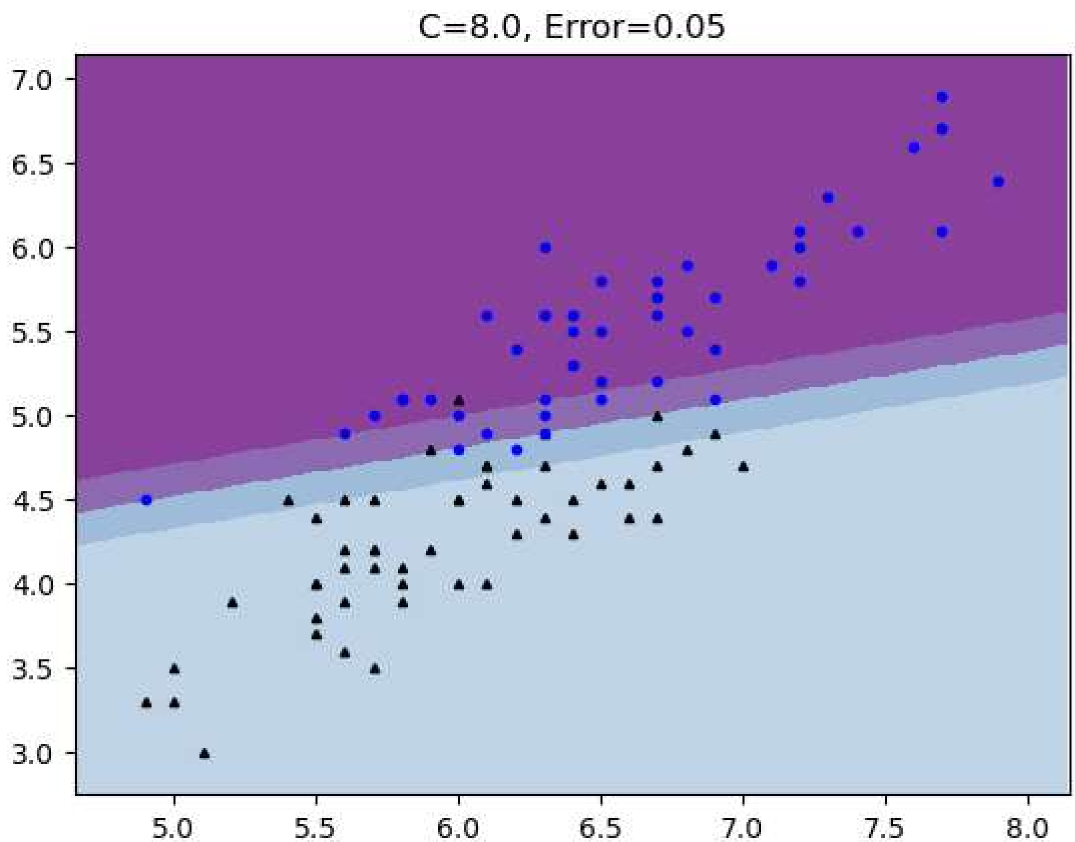

```
=== 2.0 ===
     Error rate: 0.06
```



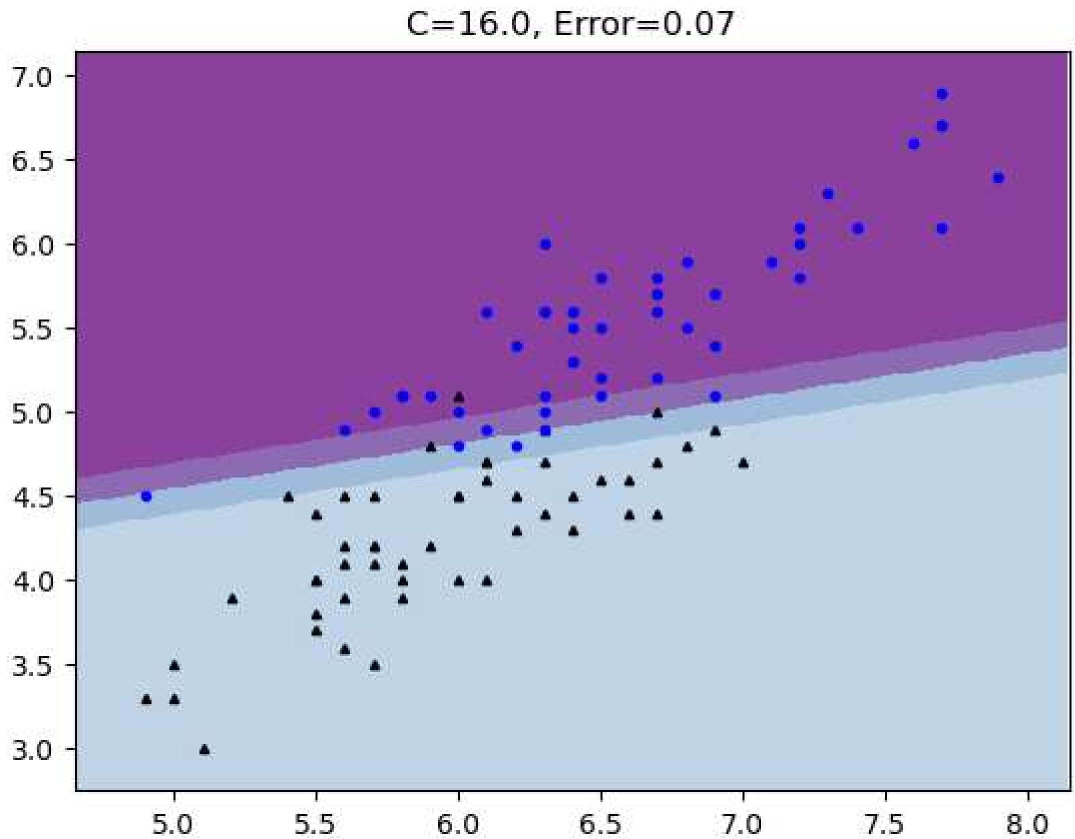

```
=== 4.0 ===
     Error rate: 0.07
```



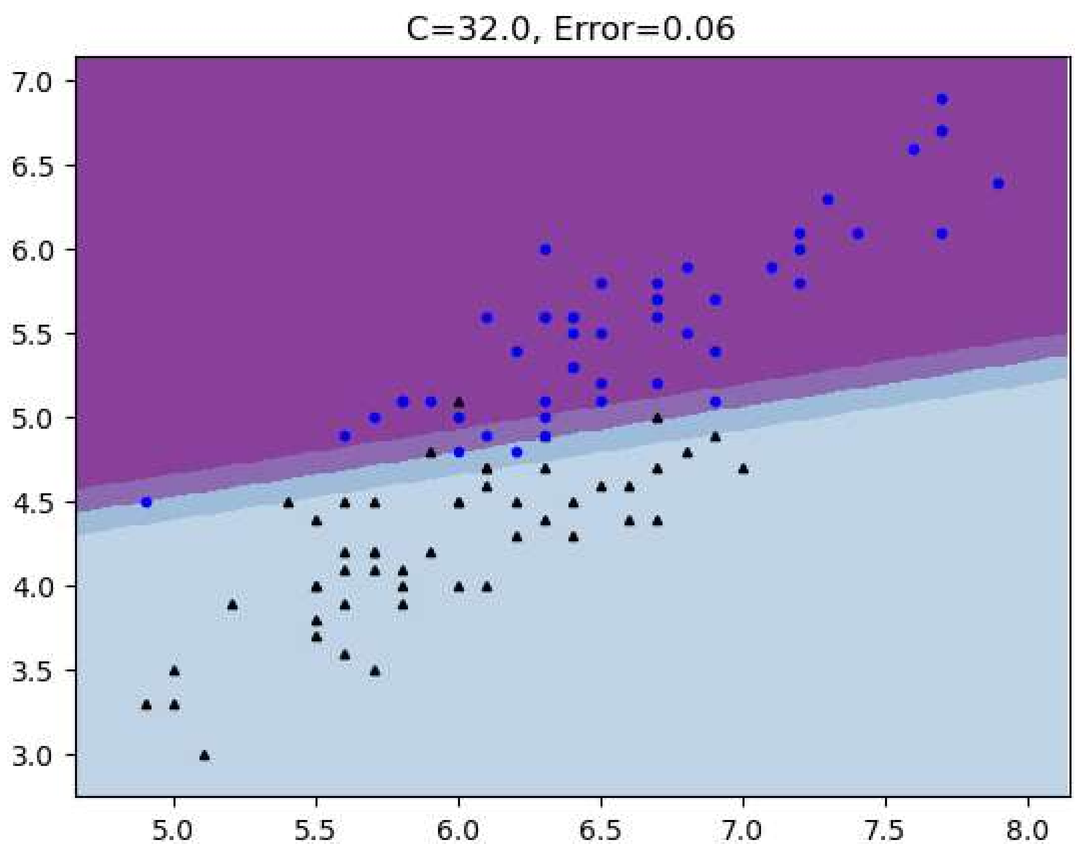

```
=== 8.0 ===
      Error rate: 0.05
```



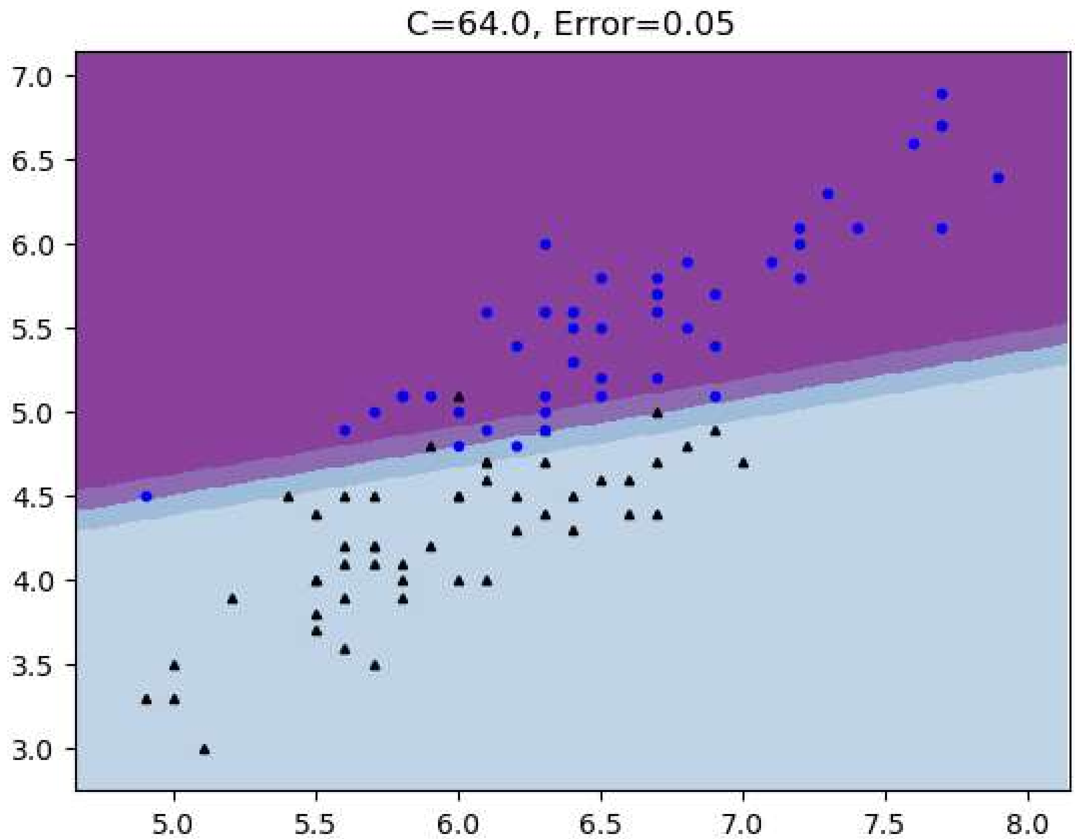

```
=== 16.0 ===
      Error rate: 0.07
```



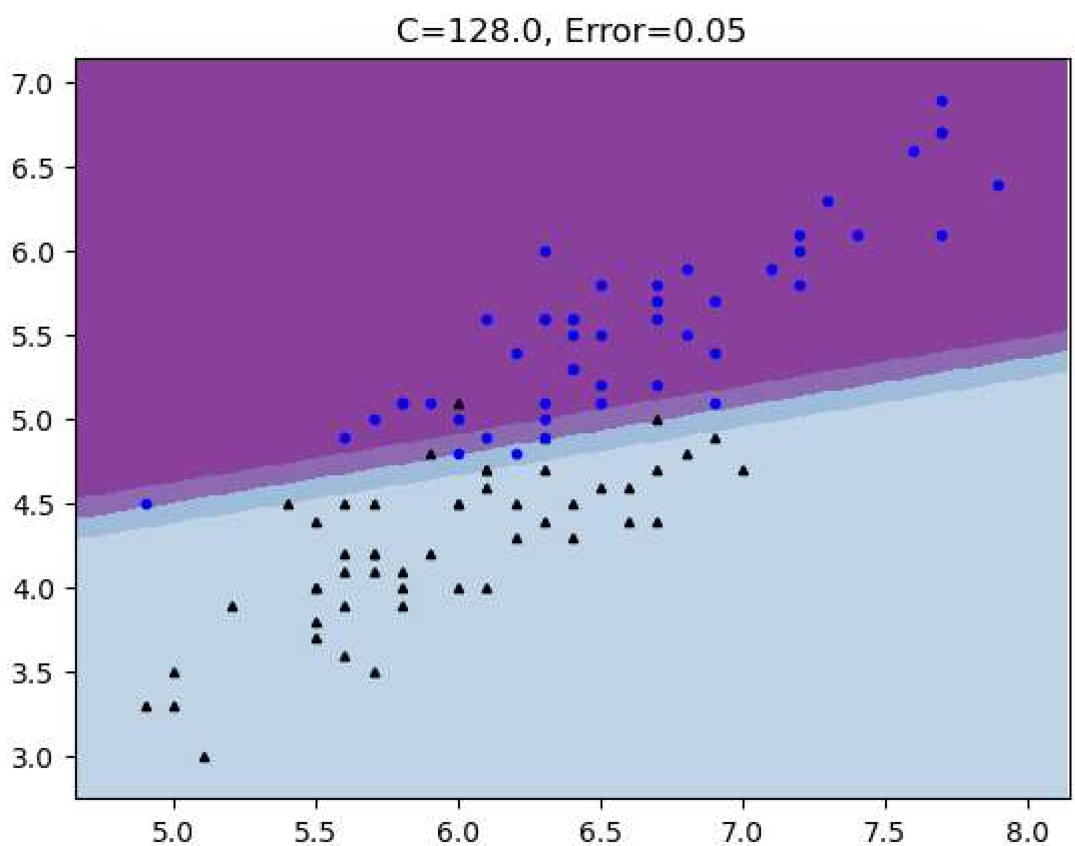

```
=== 32.0 ===
      Error rate: 0.06
```



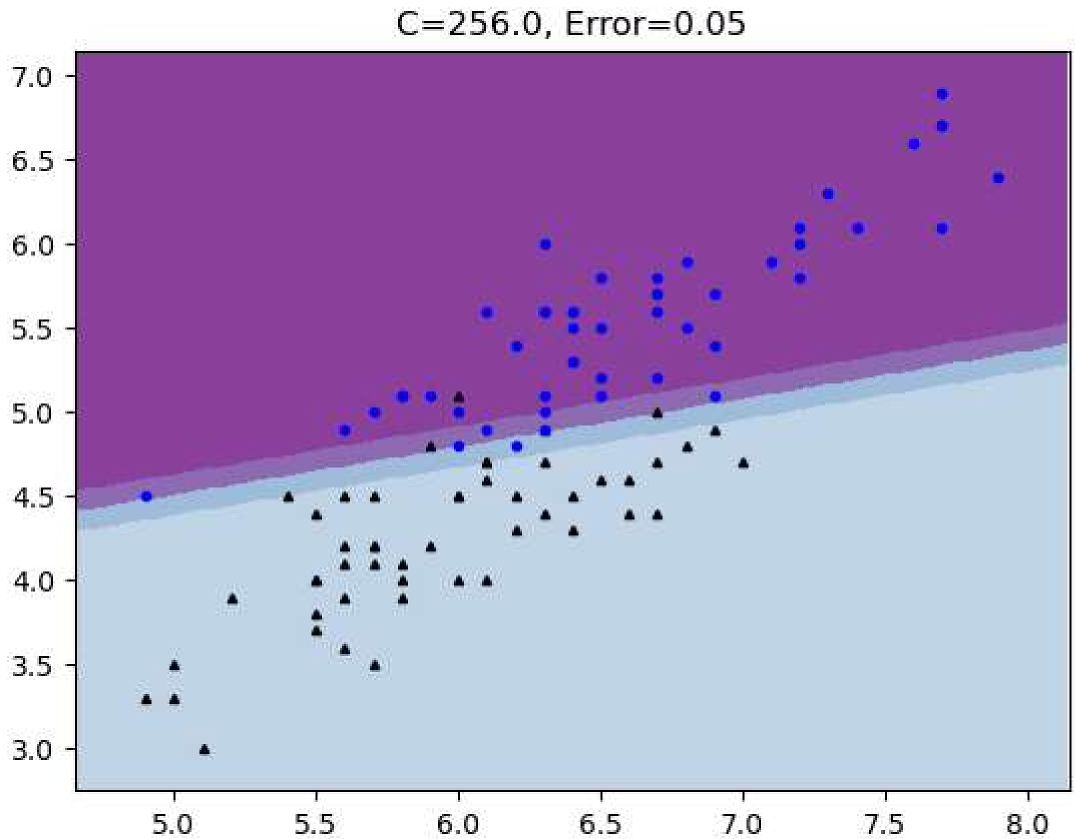

```
=== 64.0 ===
      Error rate: 0.05
```



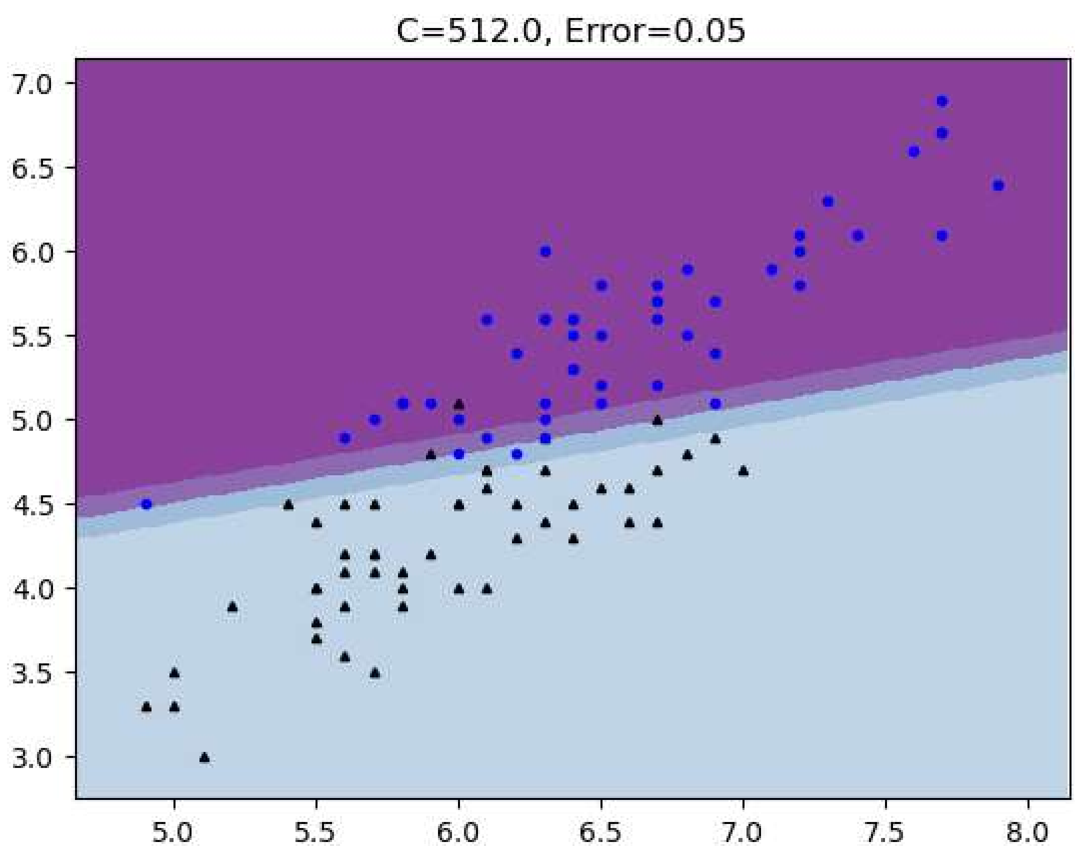

```
=== 128.0 ===
      Error rate: 0.05
```




```
=== 256.0 ===
      Error rate: 0.05
```




```
=== 512.0 ===
        Error rate: 0.05
```



In [105…

```
for v in data:
  print(str(v)[1:-1].replace(',', ' &') + "\\\\\\hline")
```

```
0.125 & 0.07 & 52\\\hline
0.25 & 0.06 & 45\\\hline
0.5 & 0.06 & 38\\\hline
1.0 & 0.07 & 31\\\hline
2.0 & 0.06 & 24\\\hline
4.0 & 0.07 & 21\\\hline
8.0 & 0.05 & 19\\\hline
16.0 & 0.07 & 16\\\hline
32.0 & 0.06 & 15\\\hline
64.0 & 0.05 & 14\\\hline
128.0 & 0.05 & 14\\\hline
256.0 & 0.05 & 14\\\hline
512.0 & 0.05 & 14\\\hline
```