

1 1

10 x 10 image, 1 channel = 10 x 10 x 1 = 100 dimensional vector

2 2

$$\sqrt{(1-3)^2 + (2-2)^2 + (3-1)^2} = \sqrt{4+0+4} = 2\sqrt{2}$$

3 3

3.1 a

Correctly Classified 25% of the time, **Error Rate** = 75%.

3.2 b

The most frequent label (label A) should be always returned. This results in the smallest error possible, of 50% (error = 1 - frequency).

4 4

4.1 a

The point (0.5, 0.5) lies in the region enclosed by Class 2. Thus, this point has a label of 2.

4.2 b

$$\begin{aligned}\|(0.5, 0.5) - (1.5, 0.5)\| &= \|(-1, 0)\| = 1 \\ \|(0.5, 1.5) - (1.5, 0.5)\| &= \|(-1, 1)\| = \sqrt{2}\end{aligned}$$

Label will be the label of the nearest neighbor (0.5, 0.5), or Class 2

4.3 c

$$\begin{aligned}\|(0.5, 0.5) - (2, 2)\| &= \|(1.5, 1.5)\| = \sqrt{1.5^2 + 1.5^2} = \sqrt{4.5} \\ \|(0.5, 1.5) - (2, 2)\| &= \|(1.5, .5)\| = \sqrt{2.5}\end{aligned}$$

Label will be the label of the nearest neighbor (0.5, 1.5), or Class 1

4.4 d

Class 3 will never be predicted because there does not exist any points in the training set that are within the boundary enclosed by Class 3

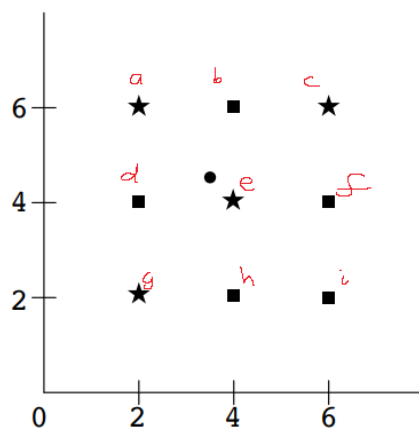
4.5 e

With a training set of just two points, the decision boundary is simply a perpendicular line passing through the midpoint of the two training points. That is, a horizontal line passing through (0.5, 1), or the line $y=1$. This means that all points in the half-square above $y=1$ are classified as class 1, and the half-square below $y=1$ are classified as class 2. All Class 1 and Class 2 test labels are correctly classified, but all Class 3 test points are incorrectly classified (consisting of 50% of the points). Thus, the error rate would be 50%.

5 5

Based on the image, we find the distances to the query point from each point:

$\sqrt{1.5^2 + 1.5^2} = \sqrt{4.5}$	$\sqrt{1.5^2 + 0.5^2} = \sqrt{2.5}$	$\sqrt{1.5^2 + 2.5^2} = \sqrt{8.5}$
$\sqrt{0.5^2 + 1.5^2} = \sqrt{2.5}$	$\sqrt{0.5^2 + 0.5^2} = \sqrt{0.5}$	$\sqrt{0.5^2 + 2.5^2} = \sqrt{6.5}$
$\sqrt{2.5^2 + 1.5^2} = \sqrt{8.5}$	$\sqrt{2.5^2 + 0.5^2} = \sqrt{6.5}$	$\sqrt{2.5^2 + 2.5^2} = \sqrt{12.5}$



Order of closest points: $[\{e\}, \{d, b\}, \{a\}, \{h, f\}, \{c, g\}, \{i\}]$

5.1 a

1-NN: $\{(e, \text{star})\}$. The majority label is **star**.

5.2 b

3-NN: $\{(e, \text{star}), (d, \text{square}), (b, \text{square})\}$. The majority label is **square**.

5.3 c

5-NN: $\{(e, \text{star}), (d, \text{square}), (b, \text{square}), (a, \text{star}), (h/f, \text{square})\}$. The majority label is **square**. Even though the 5th nearest neighbor has two options, the classification outcome is the same since all possible candidate points have the same label (square).

6 6

Each fold is $10,000/4 = 2,500$, but the training sets are $(k-1)$ folds, so each training set is $2500*3=7500$ data values.

7 7

Point	Label	Predicted Label	Correct
Point 1	+	{+}	Yes
Point 2	+	{+}	Yes
Point 3	-	{+}	No
Point 4	+	{-}	No

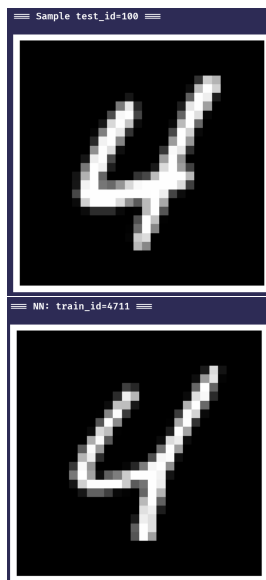
LOOCV error for 1-NN: 50%.

Point	Label	Predicted Label	Correct
Point 1	+	{+, -, +} → +	Yes
Point 2	+	{+, -, +} → +	Yes
Point 3	-	{+, +, +} → +	No
Point 4	+	{-, +, +} → +	Yes

LOOCV error for 3-NN: 25%

8 8

8.1 a

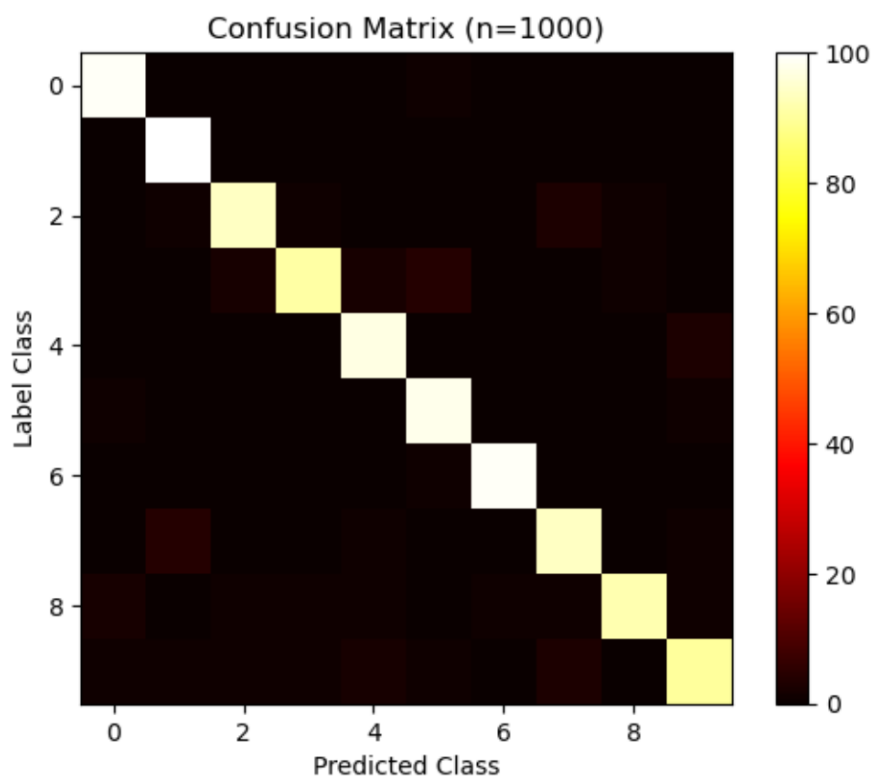


Because both the images have the same label, it is correctly classified.

8.2 b

```
[[ 99.  0.  0.  0.  0.  1.  0.  0.  0.  0.]
 [  0. 100.  0.  0.  0.  0.  0.  0.  0.  0.]
 [  0.  1. 94.  1.  0.  0.  0.  3.  1.  0.]
 [  0.  0.  2. 91.  2.  4.  0.  0.  1.  0.]
 [  0.  0.  0.  0. 97.  0.  0.  0.  0.  3.]
 [  1.  0.  0.  0.  0. 98.  0.  0.  0.  1.]
 [  0.  0.  0.  0.  0.  1. 99.  0.  0.  0.]
 [  0.  4.  0.  0.  1.  0.  0. 94.  0.  1.]
 [  2.  0.  1.  1.  1.  0.  1.  1. 92.  1.]
 [  1.  1.  1.  1.  2.  1.  0.  3.  0. 90.]]
```

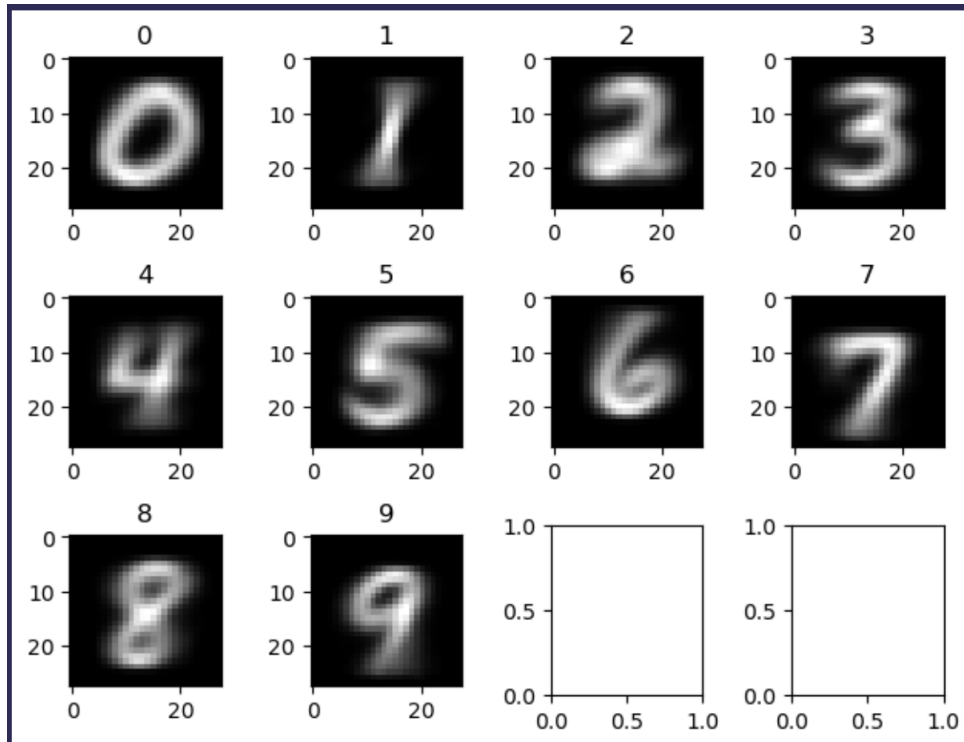
<matplotlib.colorbar.Colorbar at 0x2daa16f33a0>



Digit	Error	Digit	Error
0	1/100	1	0/100
2	6/100	3	9/100
4	3/100	5	2/100
6	1/100	7	6/100
8	8/100	9	10/100

Digit 9 was the most mis-classified, digit 1 was the least mis-classified.

8.3 c



9 9

9.1 a

L1 Norm Error: 0.2167

L2 Norm Error: 0.2333

9.2 b

L1 Norm:

	NO	DH	SL
NO	14	0	2
DH	9	9	0
SL	1	1	24

L2 Norm:

	NO	DH	SL
NO	12	1	3
DH	9	9	0
SL	1	0	25