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HW5
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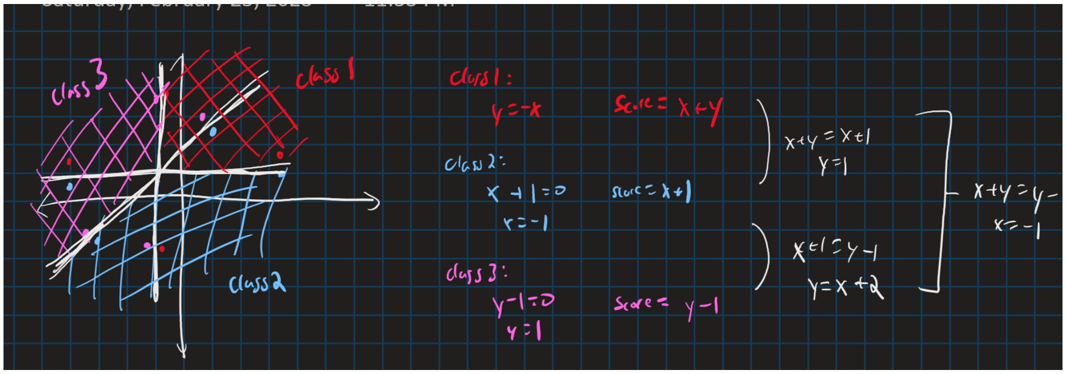
1 1



2 2

$x_1 + 3x_1x_2 - 6x_2^2 - 8 = 0$
$= (x_1, x_2, x_1^2, x_2^2, x_1x_2) \cdot (1, 0, 0, -6, 3) - 8 = 0$
$w = (1, 0, 0, -6, 3)$

3 3

3.1 a

Yes. If the data is linearly separable with just order-1 terms, it can learn a weight vector $\Phi(x)$ that has 0 in which all the terms with degree > 1 are 0.

3.2 b

It will not necessarily return a vector with all quadratic terms being 0. It is possible that the data has some wiggle room where some terms can be nonzero, but still be linearly separable.

4 4

4.1 a

$\alpha = (0, 2, 2, 1)$

4.2 b

$b = 2 * 1 + 2 * -1 + -1 = -1$

5 5

5.1 a

Dimension of α is the number of training points, or 9*4=36.

5.2 b

Number of entries in α that are > 0 is the number of points that used updates. There are only 6 points that used updates (the larger dots)

5.3 c

None are. The constraints of kernel SVM are that all α_i are non-negative.

6 6

7 7

8 8

8.1 a

8.2 b

C	Train Err	Test Err
0.01	0.1544	0.1579
0.1	0.1359	0.1429
1.0	0.1204	0.1219
10	0.1331	0.1326
100	0.1500	0.1626

8.3 c

C	Train Err	Test Err
1.0	0.0137	0.0244

n_support=9158

Q6

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
```

In [2]:

```
data0 = np.loadtxt("data0.txt")
X0 = data0[:,:2]
y0 = data0[:,-1]
print(X0.shape, y0.shape)
```

```
(48, 2) (48,)
```

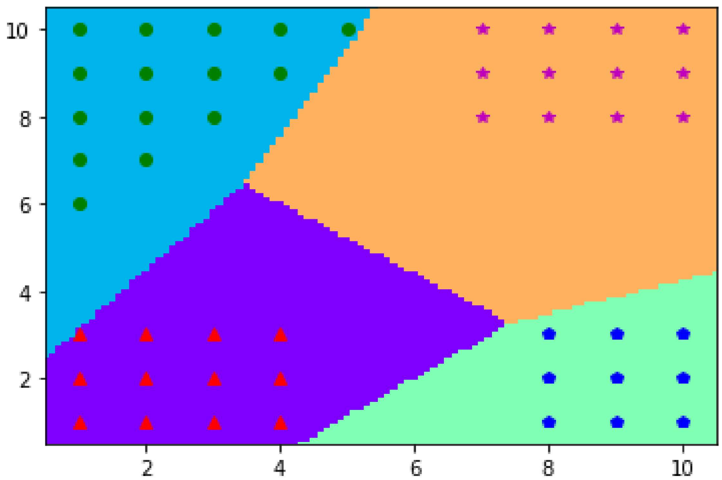
In [3]:

```
def converged(w, b, X, y):
  predictions = np.argmax((X @ w) + b.reshape((1, -1)), axis=1)
  return np.allclose(predictions, y)
def perceptron(X, y):
  num_classes = len(np.unique(y))
  n = X.shape[0]
  w = np.zeros((X.shape[1], num_classes))
  b = np.zeros(num_classes)

  perm = np.random.choice(range(n), n, replace=False)
  X_perm = X[perm]
  y_perm = y[perm]
  num_iterations = 0
  max_iterations = 999
  while not converged(w, b, X, y) and num_iterations < max_iterations:
    for i in range(n):
      Xi = X_perm[i]
      yi = int(y_perm[i])
      y_pred = int(np.argmax(Xi.reshape((1, -1)) @ w + b))
      if yi != y_pred:
        w[:,yi] += Xi
        b[yi] += 1
        w[:,y_pred] -= Xi
        b[y_pred] -= 1
    num_iterations += 1
  return w, b
def plot0(X, y, w, b):
  min_x1, max_x1 = np.min(X[:,0]) - 0.5, np.max(X[:,0]) + 0.5
  min_x2, max_x2 = np.min(X[:,1]) - 0.5, np.max(X[:,1]) + 0.5
  delta = 0.1

  plt.xlim(min_x1, max_x1)
  plt.ylim(min_x2, max_x2)

  xx, yy = np.meshgrid(np.arange(min_x1, max_x1 + delta, delta), np.arange(min_x2, max_
  pairs = np.c_[xx.flatten(), yy.flatten()]
  zz = np.zeros(len(pairs))
  for i, pair in enumerate(pairs):
    pair2 = pair.reshape((1, -1))
    zz[i] = np.argmax((pair2 @ w ).flatten() + b)
  zz = zz.reshape(xx.shape)
  styles = ["^r", "og", "pb", "*m"]
  for i in range(4):
```

```python
    Xi, yi = X[y==i], y[y==i]
    plt.plot(Xi[:,0], Xi[:,1], styles[i])
  plt.pcolormesh(xx, yy, zz, cmap="rainbow", vmin=0, vmax=4)
  plt.show()
```

In [4]:

```python
np.random.seed(123)

w, b = perceptron(X0, y0)
plot0(X0, y0, w, b)
```



Q7

In [5]:

```python
data1 = np.loadtxt("data1.txt")
data2 = np.loadtxt("data2.txt")

X1, y1 = data1[:,:-1], data1[:,-1]
X2, y2 = data2[:,:-1], data2[:,-1]
print(X1.shape, X2.shape, y1.shape, y2.shape)
```

```
(76, 2) (60, 2) (76,) (60,)
```

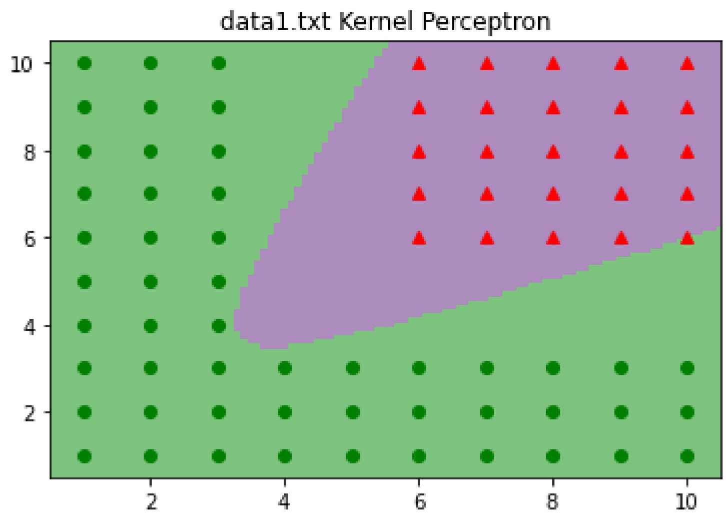
In [6]:

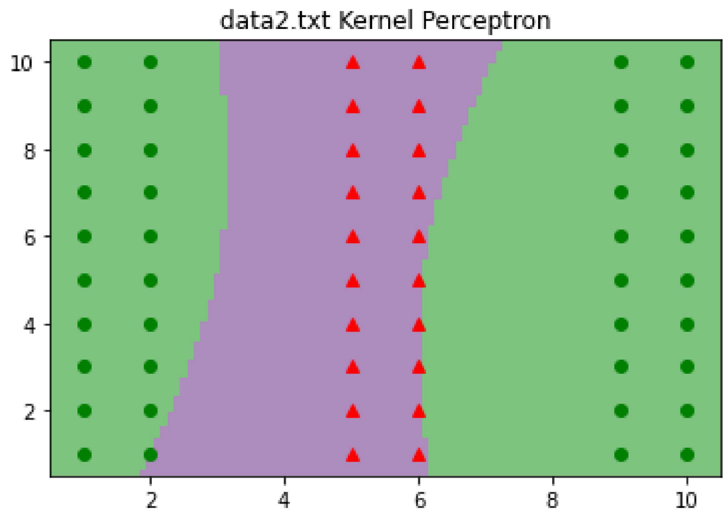
```python
def kernel_classify(b, X, x, y, alpha):
  terms = alpha.flatten() * y.flatten()
  dots = np.square(1 + X @ x.reshape((-1, 1))).flatten()
  w_dot_phi = np.dot(terms, dots)
  decision = w_dot_phi + b
  return np.sign(decision)

def kernel_converge(b, X, y, alpha):
  dot_matrix = np.square(1 + X @ X.T)
  column_terms = (alpha * y).reshape((-1, 1))

  logit_matrix = dot_matrix * column_terms
  predictions = np.sum(logit_matrix, axis=0) + b
  return np.allclose(np.sign(predictions), y)

def kernel_perceptron(X, y):
  n, d = X.shape
  alpha = np.zeros(n)
```

```python
  b = 0

  for i in range(8000):
    perm = np.random.choice(n, n, replace=False)
    for j in perm:
      xj = X[j]
      yj = y[j]
      if kernel_classify(b, X, xj, y, alpha) != yj:
        alpha[j] += 1
        b += yj
    if kernel_converge(b, X, y, alpha):
      return alpha, b, X, y
  raise Exception("Did not converge")


def gen_classify(alpha, b, X, y):
  def clas(x):
    phis = ((1 + X @ x.reshape((-1, 1))) ** 2).flatten()
    decision = np.sum(alpha * y * phis) + b
    return np.sign(decision)
  return clas

def plot1(X, y, classify, title=""):
  min_x1, max_x1 = np.min(X[:,0]) - 0.5, np.max(X[:,0]) + 0.5
  min_x2, max_x2 = np.min(X[:,1]) - 0.5, np.max(X[:,1]) + 0.5
  delta = 0.1

  plt.xlim(min_x1, max_x1)
  plt.ylim(min_x2, max_x2)

  xx, yy = np.meshgrid(np.arange(min_x1, max_x1 + delta, delta), np.arange(min_x2, max_
  pairs = np.c_[xx.flatten(), yy.flatten()]
  zz = np.zeros(len(pairs))
  for i, pair in enumerate(pairs):
    pair2 = pair.reshape((1, -1))
    zz[i] = classify(pair)
  zz = zz.reshape(xx.shape)
  styles = ["^r", "og"]
  for idx, label in enumerate([-1, 1]):
    Xi, yi = X[y==label], y[y==label]
    plt.plot(Xi[:,0], Xi[:,1], styles[idx])
  plt.pcolormesh(xx, yy, zz, cmap=plt.cm.PRGn, vmin=-2, vmax=2)
  if title:
    plt.title(title)
  plt.show()
```

In [7]:

```python
np.random.seed(123)
alpha_1, b_1, X_1, y_1 = kernel_perceptron(X1, y1)
clas_1 = gen_classify(alpha_1, b_1, X_1, y_1)
plot1(X_1, y_1, clas_1, title = "data1.txt Kernel Perceptron")

np.random.seed(999)
alpha_2, b_2, X_2, y_2 = kernel_perceptron(X2, y2)
clas_2 = gen_classify(alpha_2, b_2, X_2, y_2)
plot1(X_2, y_2, clas_2, title = "data2.txt Kernel Perceptron")
```





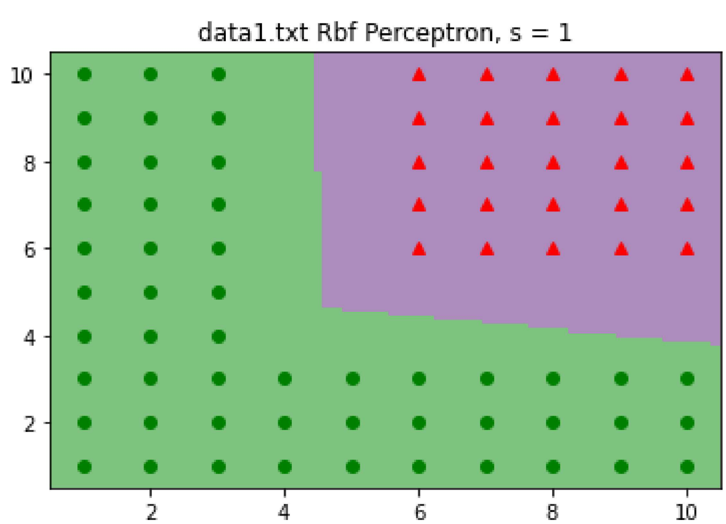
In [8]:

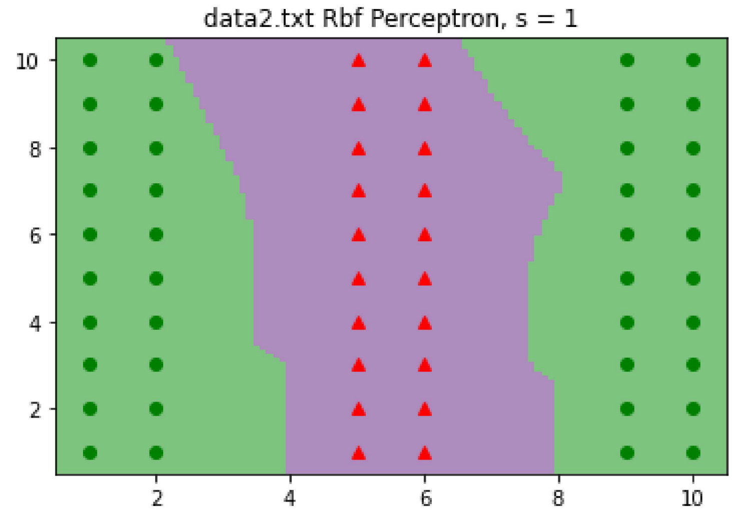
```
def rbf_classify(b, X, x, y, alpha, s):
  terms = alpha.flatten() * y.flatten()
  distances = np.exp(-np.sum(np.square(X - x.reshape((1, -1))), axis = 1) / (s ** 2))
  decision = np.sum(distances * terms) + b
  return np.sign(decision)

def rbf_converge(b, X, y, alpha, s):
  n, d = X.shape
  terms = alpha.flatten() * y.flatten()
  for i in range(n):
    xi = X[i]
    yi = y[i]
    distances = np.exp(-np.sum(np.square(X - xi.reshape((1, -1))), axis = 1) / (s ** 2)
    decision = np.sum(distances * terms) + b
    if not np.sign(decision) == yi:
      return False
  return True

def rbf_perceptron(X, y, s):
  n, d = X.shape
  alpha = np.zeros(n)
  b = 0

  for i in range(8000):
    perm = np.random.choice(n, n, replace=False)
```

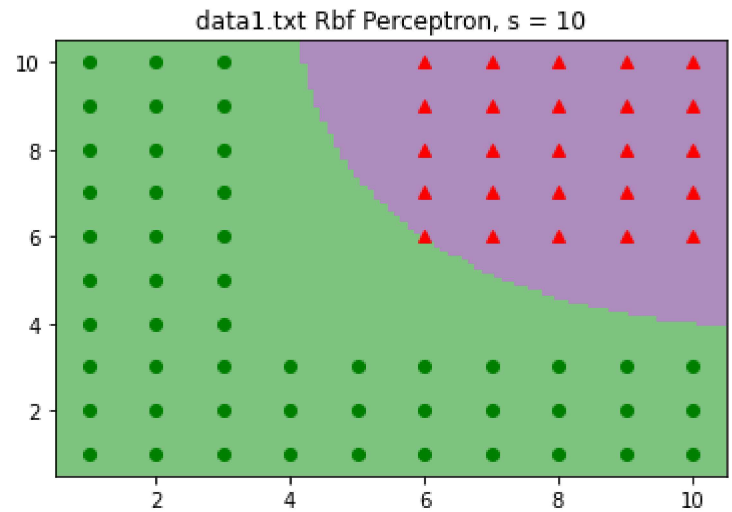
```
    for j in perm:
      xj = X[j]
      yj = y[j]
      if rbf_classify(b, X, xj, y, alpha, s) != yj:
        alpha[j] += 1
        b += yj
    if rbf_converge(b, X, y, alpha, s):
      return alpha, b, X, y
  raise Exception("Did not converge")

def rbf_gen_classify(alpha, b, X, y, s):
  def clas(x):
    terms = alpha.flatten() * y.flatten()
    distances = np.exp(-np.sum(np.square(X - x.reshape((1, -1))), axis = 1) / (s ** 2))
    decision = np.sum(distances * terms) + b
    return np.sign(decision)
  return clas
```

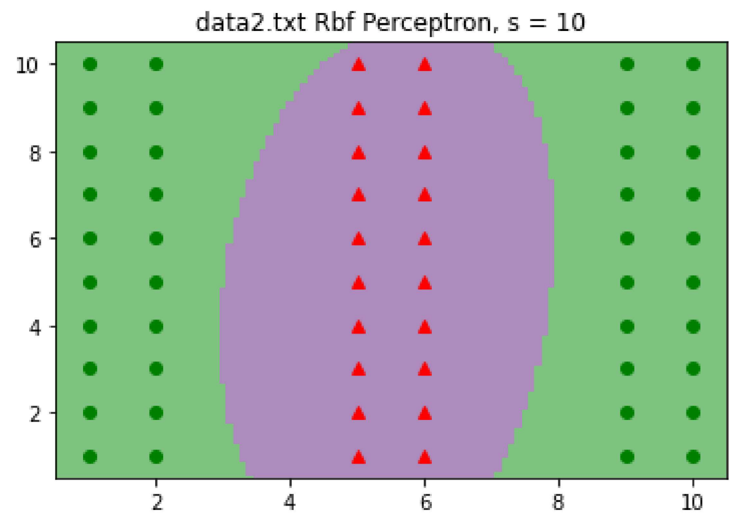
In [9]:

```
s = 1

np.random.seed(123)
alpha_1, b_1, X_1, y_1 = rbf_perceptron(X1, y1, s)
clas_1 = rbf_gen_classify(alpha_1, b_1, X_1, y_1, s)
plot1(X_1, y_1, clas_1, title = f"data1.txt Rbf Perceptron, s = {s}")

np.random.seed(999)
alpha_2, b_2, X_2, y_2 = rbf_perceptron(X2, y2, s)
clas_2 = rbf_gen_classify(alpha_2, b_2, X_2, y_2, s)
plot1(X_2, y_2, clas_2, title = f"data2.txt Rbf Perceptron, s = {s}")

s = 10

np.random.seed(123)
alpha_1, b_1, X_1, y_1 = rbf_perceptron(X1, y1, s)
clas_1 = rbf_gen_classify(alpha_1, b_1, X_1, y_1, s)
plot1(X_1, y_1, clas_1, title = f"data1.txt Rbf Perceptron, s = {s}")

np.random.seed(999)
alpha_2, b_2, X_2, y_2 = rbf_perceptron(X2, y2, s)
clas_2 = rbf_gen_classify(alpha_2, b_2, X_2, y_2, s)
plot1(X_2, y_2, clas_2, title = f"data2.txt Rbf Perceptron, s = {s}")
```









Q8

In [10]:

```
import sklearn.datasets as ds

data = ds.fetch_openml("mnist_784")
```

```
c:\Users\James\anaconda3\envs\mlenv\lib\site-packages\sklearn\datasets\_openml.py:932: F
utureWarning: The default value of `parser` will change from `'liac-arff'` to `'auto'` i
n 1.4. You can set `parser='auto'` to silence this warning. Therefore, an `ImportError`
will be raised from 1.4 if the dataset is dense and pandas is not installed. Note that t
he pandas parser may return different data types. See the Notes Section in fetch_openm
l's API doc for details.
  warn(
```

In [11]:

```
X = data['data'].to_numpy()
y = np.array([int(q) for q in data['target']])

print(X.shape, y.shape)
X_tr, X_te = X[:60000,:], X[60000:,:]
y_tr, y_te = y[:60000], y[60000:]

print(X_tr.shape, X_te.shape, y_tr.shape, y_te.shape)
```

```
(70000, 784) (70000,)
(60000, 784) (10000, 784) (60000,) (10000,)
```

In [12]:

```
from sklearn.svm import LinearSVC

def experiment(svm: LinearSVC, X_tr, X_te, y_tr, y_te):
  reg = svm.fit(X_tr, y_tr)
  pred_tr = reg.predict(X_tr)
  pred_te = reg.predict(X_te)

  err_tr = np.mean(np.not_equal(y_tr.flatten(), pred_tr))
  err_te = np.mean(np.not_equal(y_te.flatten(), pred_te))

  return err_tr, err_te, reg

for C in [0.01, 0.1, 1.0, 10, 100]:
  svm = LinearSVC(loss="hinge", C=C, max_iter=100)
  err_tr, err_te, reg = experiment(svm, X_tr, X_te, y_tr, y_te)
  print(f"C={C}")
  print(f"\t{err_tr=}, {err_te=}")
```

```
c:\Users\James\anaconda3\envs\mlenv\lib\site-packages\sklearn\svm\_base.py:1244: Converg
enceWarning: Liblinear failed to converge, increase the number of iterations.
  warnings.warn(
C=0.01
        err_tr=0.15438333333333334, err_te=0.1579
c:\Users\James\anaconda3\envs\mlenv\lib\site-packages\sklearn\svm\_base.py:1244: Converg
enceWarning: Liblinear failed to converge, increase the number of iterations.
  warnings.warn(
C=0.1
        err_tr=0.13591666666666666, err_te=0.1429
c:\Users\James\anaconda3\envs\mlenv\lib\site-packages\sklearn\svm\_base.py:1244: Converg
enceWarning: Liblinear failed to converge, increase the number of iterations.
  warnings.warn(
C=1.0
        err_tr=0.12038333333333333, err_te=0.1219
c:\Users\James\anaconda3\envs\mlenv\lib\site-packages\sklearn\svm\_base.py:1244: Converg
enceWarning: Liblinear failed to converge, increase the number of iterations.
  warnings.warn(
C=10
        err_tr=0.13311666666666666, err_te=0.1326
```

```
C=100
	err_tr=0.15, err_te=0.1626
c:\Users\James\anaconda3\envs\mlenv\lib\site-packages\sklearn\svm\_base.py:1244: ConvergenceWarning: Liblinear failed to converge, increase the number of iterations.
  warnings.warn(
```

In [14]:

```
from sklearn.svm import SVC

svm = SVC(kernel="poly", degree = 2, C=1.0, max_iter=500)
err_tr, err_te, reg = experiment(svm, X_tr, X_te, y_tr, y_te)
print(f"C={1}")
print(f"\t{err_tr=}, {err_te=}")
print(f"\tnum_sup_vec = {len(reg.support_)=}")
```

```
c:\Users\James\anaconda3\envs\mlenv\lib\site-packages\sklearn\svm\_base.py:299: ConvergenceWarning: Solver terminated early (max_iter=500).  Consider pre-processing your data with StandardScaler or MinMaxScaler.
  warnings.warn(
C=1
	err_tr=0.013966666666666667, err_te=0.0244
	num_sup_vec = len(reg.support_)=9158
```

In [15]:

```
import matplotlib.pyplot as plt
```

In [22]:

```
xx, yy = np.meshgrid(np.arange(-5, 5, 0.01), np.arange(-5, 5, 0.01))
c1 = xx + yy
c2 = xx + 1
c3 = yy - 1
z = np.argmax(np.stack((c1, c2, c3)), axis=0)
print(xx.shape, yy.shape)
print(z.shape)
plt.pcolormesh(xx, yy, z, cmap="rainbow", vmin=0, vmax=2)
plt.axhline(0, color='black')
plt.axvline(0, color='black')
```

```
(1000, 1000) (1000, 1000)
(1000, 1000)
```

Out[22]: <matplotlib.lines.Line2D at 0x22e52dd4580>