Optimizing the Internet

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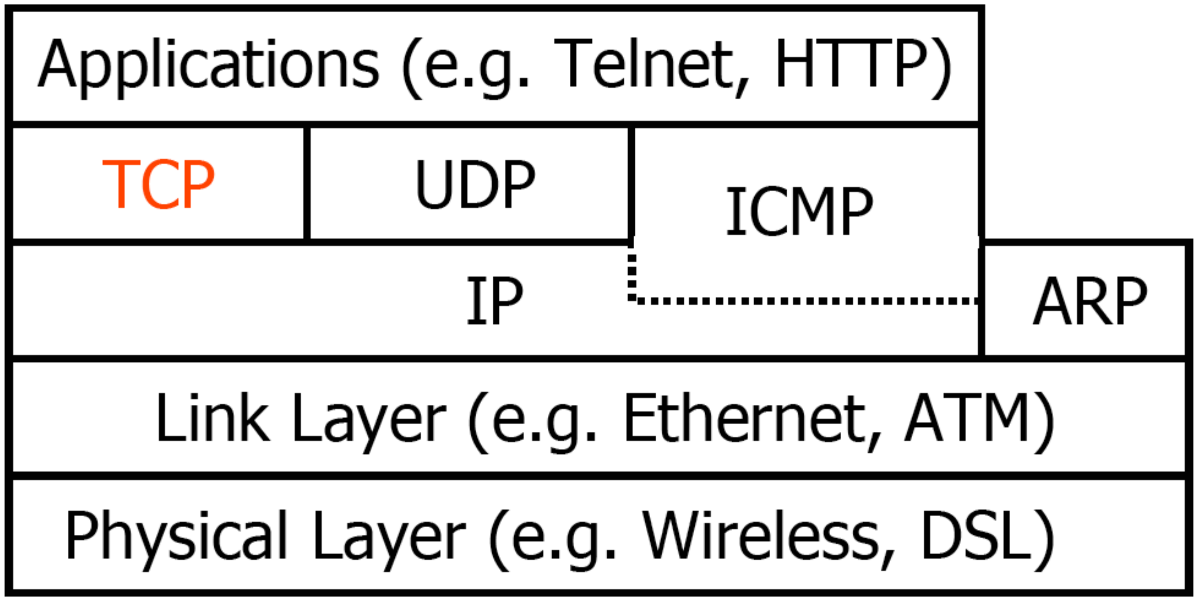
Outline of Lecture

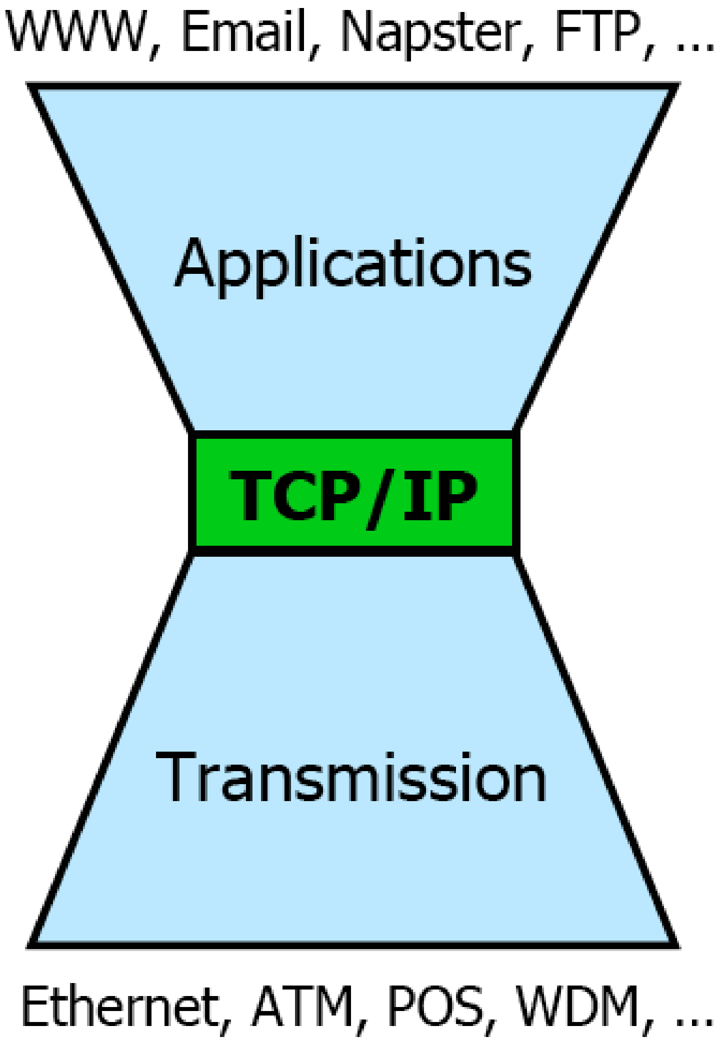
- Introduction to TCP/IP
- Overview of TCP congestion control
- Steady-state analysis via convex optimization
- Dynamic analysis via convex optimization
- Summary

TCP/IP Protocol Stack

- TCP/IP protocol stack (OSI model):
  - Application layer: ftp, http, etc.
  - Transport layer: adds reliability to network layer. The predominant protocol is TCP (also UDP).
  - Network layer: protocols for routing, IP (Internet Protocol)
  - Data link layer: frames over the link + error correction
  - Physical layer: bits and waveforms



Success of TCP/IP



- TCP/IP has been extremely succesful because it is simple and robust:
  - robust against failure
  - robust against technological evolution
  - provides a service to applications (doesn't tell applications what to do)

TCP

- TCP (Transmission Control Protocol) was introduced in the 1970s for file transfer in the Internet.

- TCP protocol:
  - end-to-end control
  - session initiation and termination
  - in-order recovery of packets
  - flow control/congestion control
  - ...

- Why congestion control?

TCP (II)

- Initially TCP had little to control congestion in the network:
  - If several users started transferring files over a bottleneck link exceeding its capacity, then the packets had to be dropped. This resulted in retransmission of lost packets, making things worse.
  - This phenomenon is known as congestion collapse and occured many times in the mid-1980s.

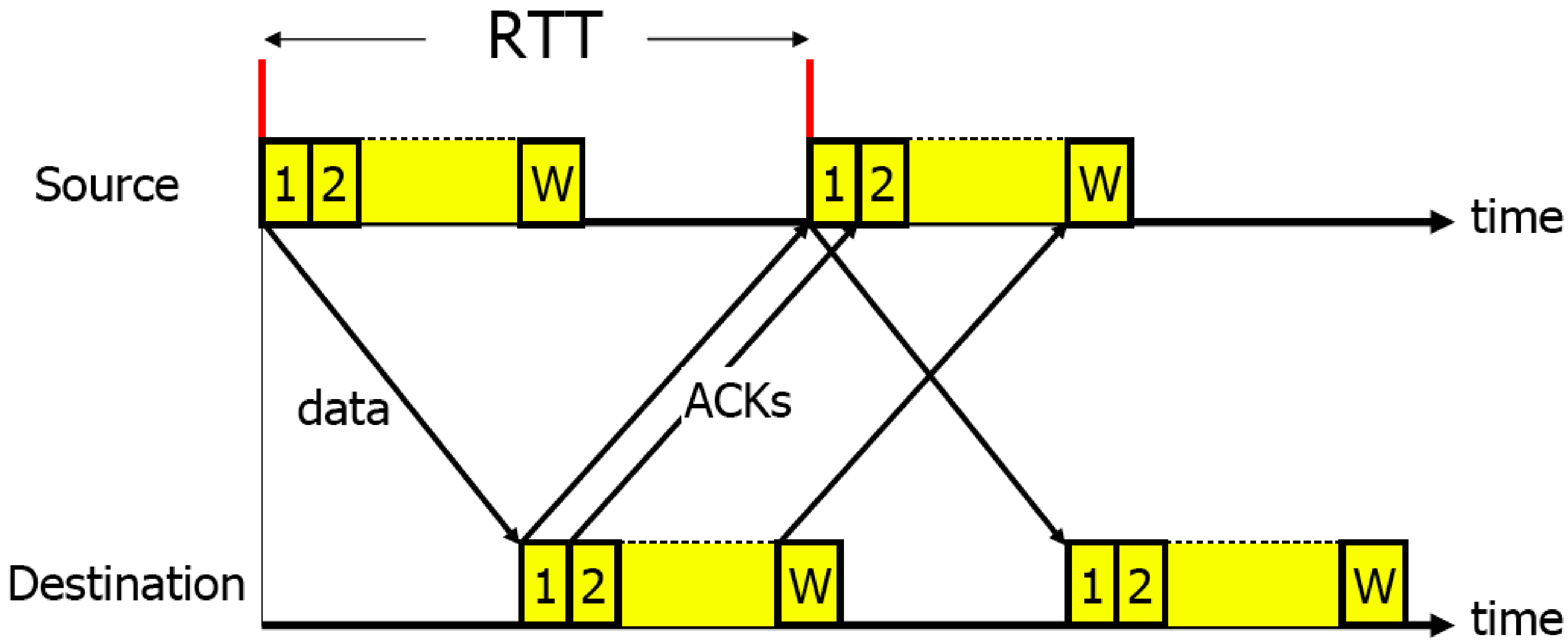
- In Oct. 1986, the Internet had its first collapse: throughput dropped by a factor of 1000 from 32 kbps to 10 bps.

- In 1988, Jacobson proposed a congestion control mechanism for TCP, which has been a remarkably successful algorithm for the growth of the Internet.

TCP Congestion Control

- TCP congestion control:
  - window-based end-to-end flow control, where the destination sends ACK for correctly received packets and the source updates the window size (which is proportional to allowed transmission rate).
  - several instances of TCP congestion control distributively dissolve congestion in bottleneck links by reducing window sizes.
  - sources update window sizes and links update (sometimes implicitly) congestion measures that are fed back to sources using the link.

Window Flow Control



- W packets per Round-Trip Time (RTT).

- Lost packet detected by missing ACK.

Evolution of TCP Congestion Control

- With the growth of the Internet over the past decade by several orders of magnitude, there was a need to develop more scalable mechanisms for Internet congestion control (good behavior unaffected by number of nodes, capacities of links, RTT, etc.)

- TCP congestion control algorithms for the Internet were devised based on heuristics, common sense, and trial-and-error in the 1980s-1990s. There was no solid understanding of the convergence and stability properties of the algorithms.

- Mathematical modeling of congestion control based on convex optimization initiated by Kelly and Low in the mid-1990s.

Cronology of TPC Congestion Control

- Tahoe (Jacobson 1988): slow start, congestion avoidance, fast retransmit.
- Reno (Jacobson 1990): fast recovery.
- Vegas (Brakmo & Peterson 1994): new congestion avoidance.
- RED (Floyd & Jacobson 1993): probabilistic making.
- REM (Athuraliya & Low 2000): clear buffer, match rater.
- etc.

NUM Perspective

- Optimization-theoretic perspective of TCP congestion control:
  - a TCP congestion control algorithm carries out a distributed algorithm to solve an implicit global convex optimization problem, a Network Utility Maximization (NUM)
  - source rates are primal variables updated at the sources
  - congestion measures are dual variables (prices) updated at the links.

Overview of TCP Congestion Control

- TCP uses a window-based flow control to pace the transmission of packets.

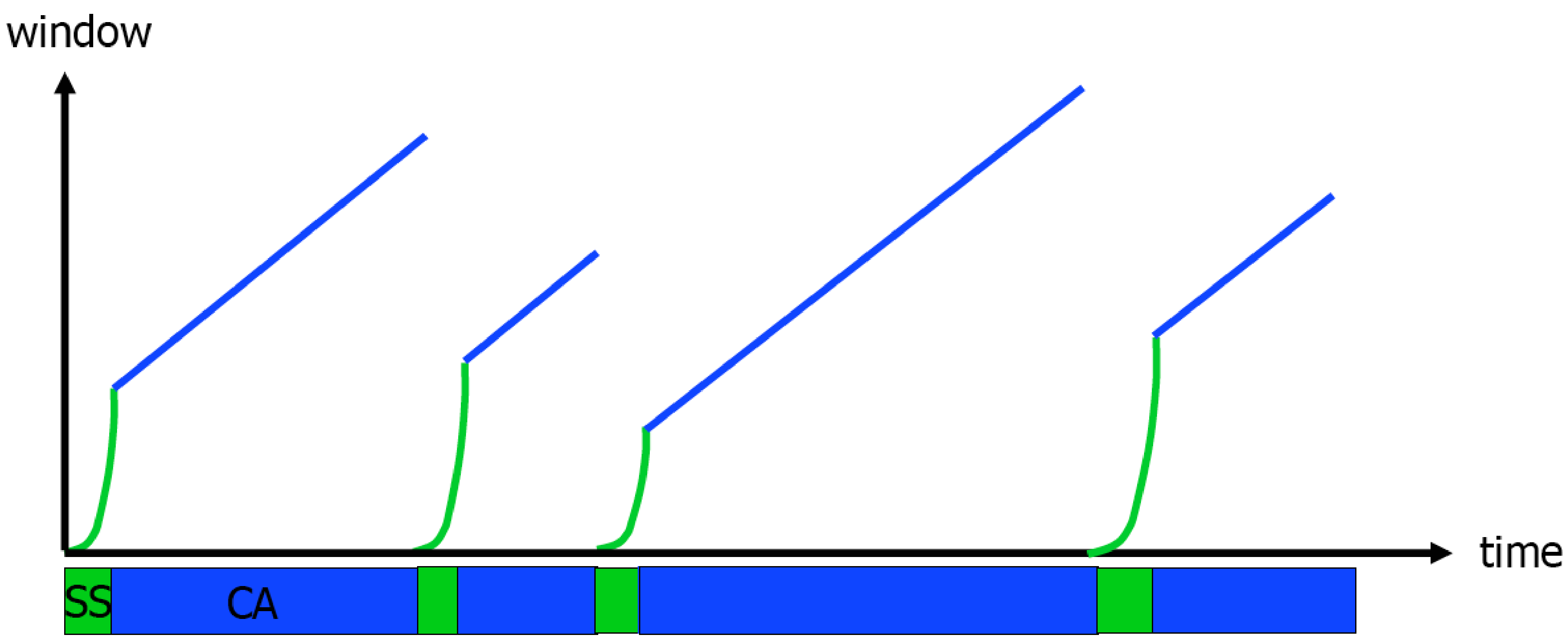
- Each source maintains a "window size" variable that limits the maximum number of packets that can be unacknowledged.

- Two features:

  1. the algorithm is "self-clocking": TCP automatically slows down the source when the network becomes congested.
  2. the window size variable determines the source rate: one window is worth the packets sent every roundtrip time.

TCP Tahoe

- TCP Tahoe (Jacobson 1988):



SS: Slow Start
CA: Congestion Avoidance

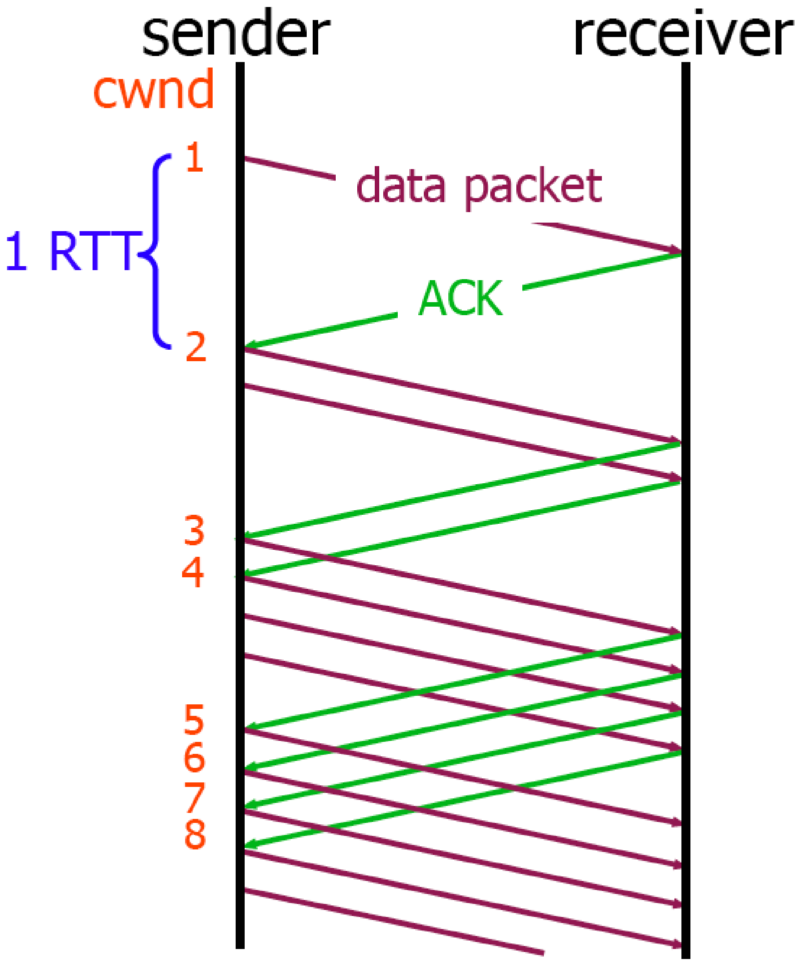
TCP Tahoe: Algorithm

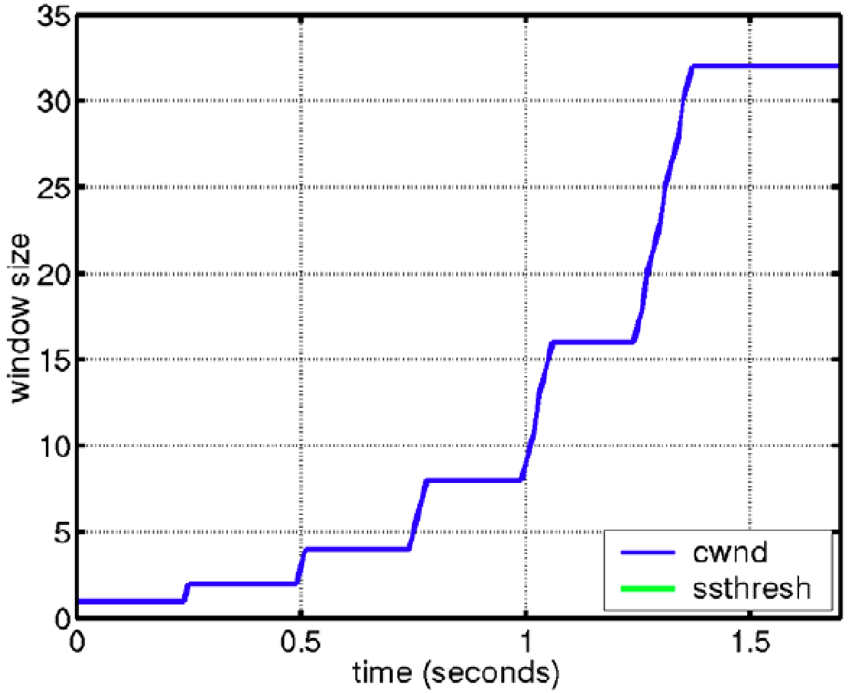
- TCP Tahoe (relies on packet loss):

  Slow-start phase: start with a window of cwnd $= 1$, increase the window size by 1 for every ACK received cwnd $=$ cwnd $+ 1$ (this doubles the window every RTT: exponential growth). If window reaches a threshold, cwnd $\geq$ ssthresh, enter next phase.

  Congestion avoidance phase: increase the window by its reciprocal, cwnd $=$ cwnd$+1/$cwnd, for every ACK received (this increases the window by one every RTT: linear growth).

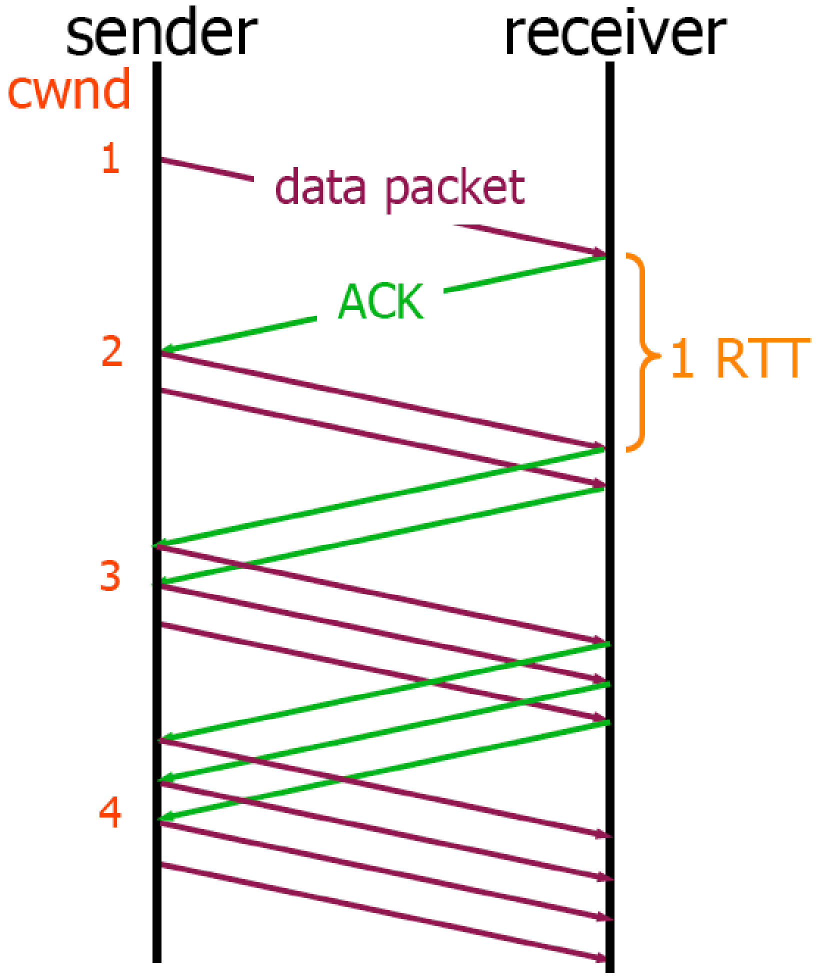
TCP Tahoe: Slow Start

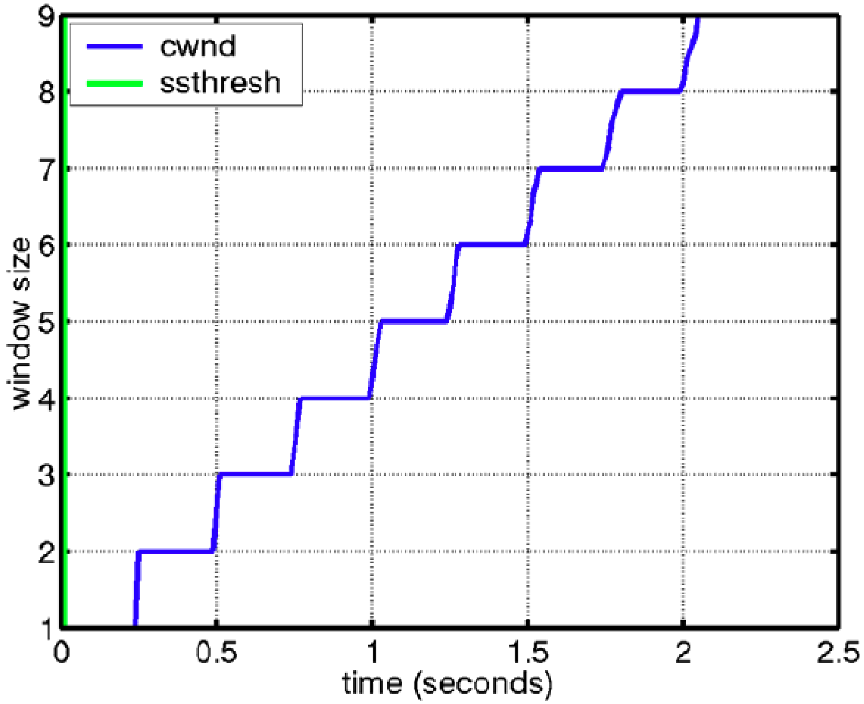




$\text{cwnd} \leftarrow \text{cwnd} + 1$ (for each ACK)

TCP Tahoe: Congestion Avoidance

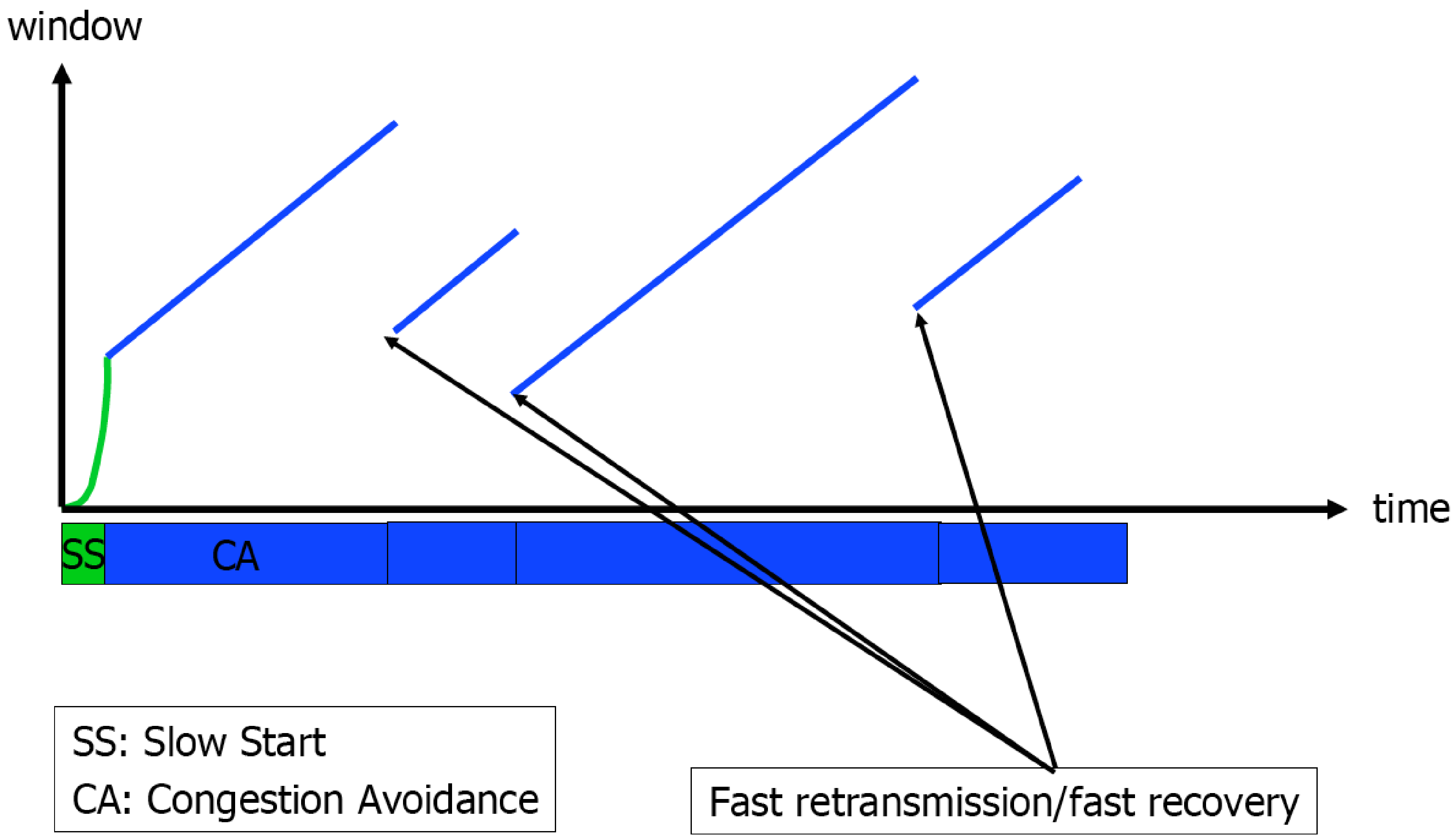




$\text{cwnd} \leftarrow \text{cwnd} + 1$ (for each cwnd ACKS)

TCP Reno

- TCP Reno (Jacobson 1990):



TCP Reno: Algorithm

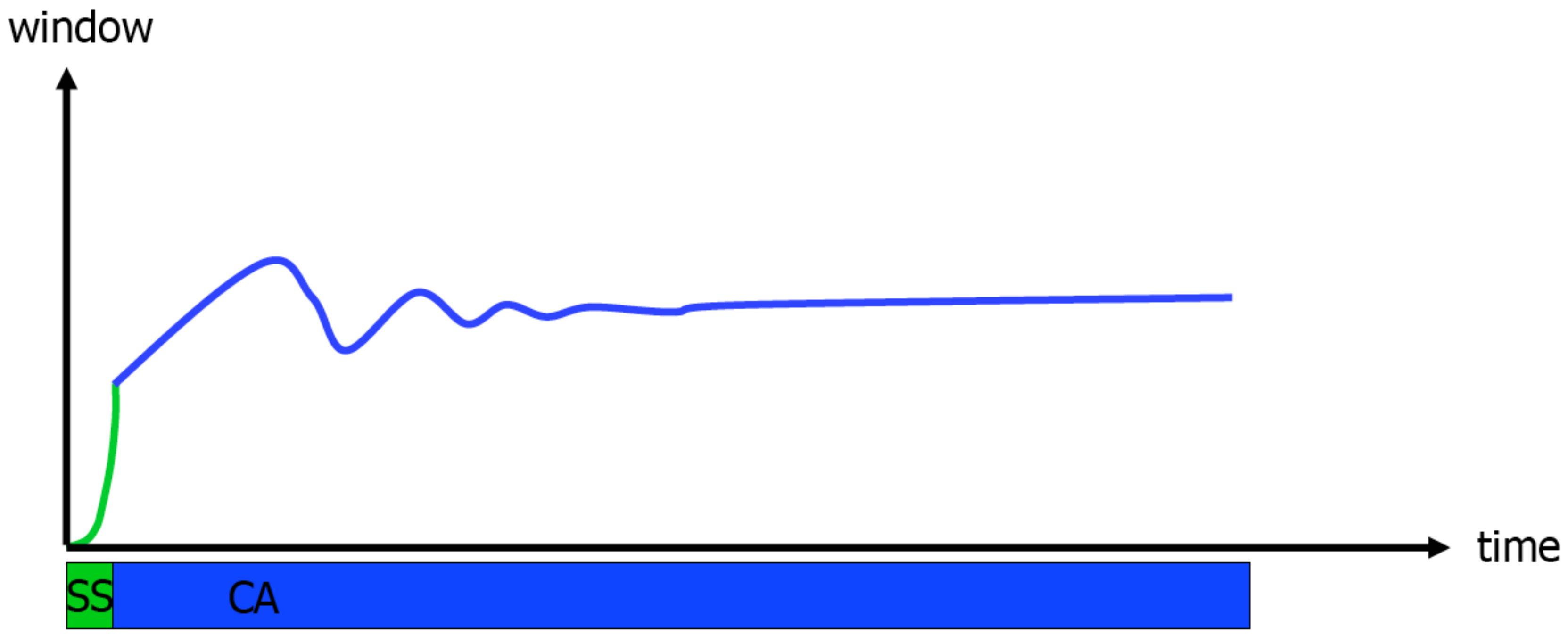
- TCP Reno (relies on packet loss):

  Slow-start phase: start with a window of 1, increase the window size by 1 for every ACK received (this doubles the window every RTT: exponential growth). If window reaches a threshold enter next phase. If a packet loss is detected, halve threshold and set window to 1.

  Congestion avoidance phase: increase the window by its reciprocal for every ACK received (this increases the window by one every RTT: linear growth). If a packet loss is detected, halve threshold, set window to some given value, and enter slow-start phase again.

TCP Vegas

- TCP Vegas (Brakmo & Peterson 1994)



- Converges, no retransmission
- … provided buffer is large enough

TCP Vegas: Algorithm

- Window update for TCP Vegas (relies on queuing delay) in the congestion avoidance phase:

$$w_s(t+1) = \begin{cases} w_s(t) + \frac{1}{D_s(t)} & \text{if } \frac{w_s(t)}{d_s} - \frac{w_s(t)}{D_s(t)} < \alpha_s \\ w_s(t) - \frac{1}{D_s(t)} & \text{if } \frac{w_s(t)}{d_s} - \frac{w_s(t)}{D_s(t)} > \alpha_s \\ w_s(t) & \text{else} \end{cases}$$

where

  – $D_s(t)$ is the RTT (Round Trip Time)
  – d_s is the propagation time (minimum $D_s(t)$)
  – $w_s(t)/d_s$ is the expected rate
  – $w_s(t)/D_s(t)$ is the actual rate

TCP Vegas: Algorithm (II)

- Observe that
  - the difference $\frac{w_s(t)}{d_s} - \frac{w_s(t)}{D_s(t)}$ should be kept between α_s and β_s (for simplicity we will assume $\alpha_s = \beta_s$)
  - $w_s(t) - d_s x_s(t) =$ total backlog buffered in the path of s ($x_s(t) = w_s(t)/D_s(t)$).

- Therefore, we can think of a source incrementing/decrementing its window according to whether the total backlog is smaller/larger than $\alpha_s d_s$.

TCP Vegas: Algorithm (II)

- We will now show that:

1. Steady-state analysis: the objective of TCP Vegas is to maximize the aggregate source utility subject to capacity constraints of network resources.

2. Dynamic analysis: the TCP Vegas algorithm is a dual method to solve the maximization problem.

Steady-State Analysis (I)

- When the algorithm converges, the equilibrium windows $w_s^\star$ and the associated equilibrium round-trip times $D_s^\star$ satisfy:

$$\frac{w_s^\star}{d_s} - \frac{w_s^\star}{D_s^\star} = \alpha_s \qquad \text{for all } s.$$

- Consider now the following NUM:

$$\begin{array}{ll} \underset{\mathbf{x}\geq\mathbf{0}}{\text{maximize}} & \sum_s U_s\left(x_s\right) \\ \text{subject to} & \sum_{s:l\in\mathcal{L}(s)} x_s \leq c_l \quad \forall l \end{array}$$

with utilities: $U_s\left(x_s\right) = \alpha_s d_s \log x_s$.

Steady-State Analysis (II)

- Since the NUM problem is convex, we know that a rate vector $\mathbf{x}^\star$ is optimal if and only if we can find Lagrange multipliers so that the KKT conditions are satisfied.

- The Lagrangian and its gradient are

$$L(\mathbf{x}, \boldsymbol{\lambda}) \;=\; \sum_s U_s(x_s) + \sum_l \lambda_l \left(c_l - \sum_{s:l \in \mathcal{L}(s)} x_s \right)$$

$$\nabla_{x_s} L = U_s'(x_s) - \sum_{l \in \mathcal{L}(s)} \lambda_l = \frac{\alpha_s d_s}{x_s} - \sum_{l \in \mathcal{L}(s)} \lambda_l$$

Steady-State Analysis (III)

- KKT conditions:

$$\begin{aligned} \sum_s x_s \leq c_l, \qquad \lambda_s \geq 0 \\ \frac{\alpha_s d_s}{x_s} &= \sum_{l \in \mathcal{L}(s)} \lambda_l \\ \lambda_s \left(\sum_s x_s - c_l \right) &= 0 \end{aligned}$$

- Let's verify that, indeed, the equilibrium point satisfies the KKT conditions for some $\boldsymbol{\lambda}$.

Steady-State Analysis (IV)

- The rate at equilibrium is $x_s^\star = \frac{w_s^\star}{D_s^\star}$.

- Let $b_l^\star$ be the equilibrium backlog at link l. The fraction of $b_l^\star$ that belongs to source s is $b_l^\star \frac{x_s^\star}{c_l}$ where c_l is the link capacity.

- Hence, the source s maintains a backlog of $\sum_{l \in \mathcal{L}(s)} b_l^\star \frac{x_s^\star}{c_l}$ along its path in equilibrium.

- On the other hand, recall that the expression for the total backlog buffered along the path of source s is $w_s(t) - d_s x_s(t)$.

Steady-State Analysis (V)

- Thus, we can write

$$w_s^\star - d_s x_s^\star = \sum_{l \in \mathcal{L}(s)} b_l^\star \frac{x_s^\star}{c_l}$$

and then

$$\alpha_s = \frac{w_s^\star}{d_s} - \frac{w_s^\star}{D_s^\star} = \frac{1}{d_s}\left(w_s^\star - d_s x_s^\star\right) = \frac{1}{d_s}\left(\sum_{l \in \mathcal{L}(s)} b_l^\star \frac{x_s^\star}{c_l}\right).$$

- If we now denote $\lambda_l^\star = b_l^\star / c_l$, we can rewrite the above as

$$\frac{\alpha_s d_s}{x_s^\star} = \sum_{l \in \mathcal{L}(s)} \lambda_l^\star$$

which is one of the KKT conditions.

Steady-State Analysis (VI)

- Regarding the primal and dual feasibility KKT conditions, clearly $\lambda_l^\star \geq 0$ (by definition) and $\sum_s x_s^\star \leq c_l$ (otherwise we would be magically transmitting at a rate higher than capacity).

- The only remaining condition to verify is the complementary slackness. Since the backlog $b_l^\star = 0$ at link l if the aggregate rate is strictly less than the capacity (implying $\lambda_l^\star = 0$), we have

$$\lambda_s^\star \left(\sum_s x_s - c_l \right) = 0.$$

Steady-State Analysis (VII)

- Summarizing the steady-state analysis, we have seen that an equilibrium point of TCP Vegas satisfies the KKT conditions and, therefore, is an optimal solution of the following NUM:

$$\begin{array}{ll} \underset{\mathbf{x}\geq\mathbf{0}}{\text{maximize}} & \sum_s U_s\left(x_s\right) \\ \text{subject to} & \sum_{s:l\in\mathcal{L}(s)} x_s \leq c_l \quad \forall l \end{array}$$

  with utilities $U_s\left(x_s\right) = \alpha_s d_s \log x_s$.

- The following analysis will deal not just with the final equilibrium point of the algorithm but with the dynamics of the updates in the TCP Vegas algorithm.

Dynamic Analysis (I)

- Let's now derive the dual algorithm to solve the NUM.

- The Lagrangian is

$$
\begin{aligned}
L(\mathbf{x}, \boldsymbol{\lambda}) &= \sum_s U_s(x_s) + \sum_l \lambda_l \left(c_l - \sum_{s:l \in \mathcal{L}(s)} x_s \right) \\
&= \sum_s \left(U_s(x_s) - x_s \sum_{l \in \mathcal{L}(s)} \lambda_l \right) + \sum_l \lambda_l c_l
\end{aligned}
$$

where $\lambda^s = \sum_{l \in \mathcal{L}(s)} \lambda_l$.

Dynamic Analysis (II)

- The dual function is then

$$g(\boldsymbol{\lambda}) = \max_{\mathbf{x}} L(\mathbf{x}, \boldsymbol{\lambda}) = \sum_s \max_{x_s} \left[U_s(x_s) - x_s \lambda^s\right] + \sum_l \lambda_l c_l$$

- Therefore, the dual method consists of the master problem:

$$\underset{\boldsymbol{\lambda} \geq \mathbf{0}}{\text{minimize}} \quad \sum_s g_s^\star(\lambda^s) + \sum_l \lambda_l c_l$$

and the subproblems

$$g_s^\star(\lambda^s) = \max_{x_s} \quad U_s(x_s) - x_s \lambda^s \qquad \text{for all } s.$$

Dynamic Analysis (III)

- A simple way to solve the master problem is with a gradient/subgradient projection method:

$$\lambda_l(t+1) = \left[\lambda_l(t) - \gamma\theta_l \nabla_{\lambda_l} g\left(\boldsymbol{\lambda}\left(t\right)\right)\right]^+$$

where $\gamma\theta_l$ denotes the stepsize for the lth element, $[\cdot]^+ = \max\left(0, \cdot\right)$, and the gradient/subgradient of the dual function is

$$\nabla_{\lambda_l} g\left(\boldsymbol{\lambda}\left(t\right)\right) = c_l - \sum_{s:l \in \mathcal{L}(s)} x_s^\star\left(\lambda^s\right).$$

Dynamic Analysis (IV)

- The solution to the subproblems for the particular choice of the utilities $U_s(x_s) = \alpha_s d_s \log x_s$ is

$$x_s^\star(\lambda^s) = \frac{\alpha_s d_s}{\lambda^s}.$$

- The gradient update can then be written as

$$\lambda_l(t+1) = \left[\lambda_l(t) + \gamma\theta_l\left(\sum_{s:l\in\mathcal{L}(s)} \frac{\alpha_s d_s}{\lambda^s} - c_l\right)\right]^+.$$

- For sufficiently small γ the algorithm will converge to a primal-dual optimal point $(\mathbf{x}^\star, \boldsymbol{\lambda}^\star)$.

Dynamic Analysis (V)

- Let's now go back to the TCP Vegas algorithm.
- The evolution of the buffer occupancy $b_l(t)$ at link l is

$$b_l(t+1) = \left[b_l(t) + \sum_{s:l \in \mathcal{L}(s)} x_s(t) - c_l \right]^+$$

or

$$\frac{b_l(t+1)}{c_l} = \left[\frac{b_l(t)}{c_l} + \frac{1}{c_l} \left(\sum_{s:l \in \mathcal{L}(s)} x_s(t) - c_l \right) \right]^+ .$$

Dynamic Analysis (VI)

- Recalling that we had interpreted $b_l(t)/c_l$ as the Lagrange multiplier $\lambda_l(t)$, we can rewrite the previous expression as

$$\lambda_l(t+1) = \left[\lambda_l(t) + \frac{1}{c_l}\left(\sum_{s:l\in\mathcal{L}(s)} x_s(t) - c_l\right)\right]^+$$

which is exactly the gradient update that we previously derived for the master dual problem:

$$\lambda_l(t+1) = \left[\lambda_l(t) + \gamma\theta_l\left(\sum_{s:l\in\mathcal{L}(s)} x_s^\star(t) - c_l\right)\right]^+$$

with stepsize $\gamma = 1$ and scaling factor $\theta_l = 1/c_l$.

Dynamic Analysis (VII)

- Thus, the dual update based on the gradient projection algorithm coincides with the TCP Vegas algorithm with the difference that the source rates $x_s(t)$ are updated differently:
  - in the dual method, we use the optimum (one-shot update)

$$x_s^\star(t) = \frac{\alpha_s d_s}{\lambda^s(t)}$$

  - in TCP Vegas algorithm, the window $w_s(t)$ is updated based on whether

$$w_s(t) - x_s(t) d_s < \alpha_s d_s \quad \text{or} \quad w_s(t) - x_s(t) d_s > \alpha_s d_s.$$

Dynamic Analysis (VIII)

- Interestingly, recalling how this quantity is related to the backlog:

$$w_s(t) - x_s(t)d_s = \sum_l \frac{x_s(t)}{c_l} b_l(t) = x_s(t) \sum_l \lambda_l(t) = x_s(t)\lambda^s(t),$$

  the conditions for the update become

$$x_s(t) < \frac{\alpha_s d_s}{\lambda^s(t)} \quad \text{or} \quad x_s(t) > \frac{\alpha_s d_s}{\lambda^s(t)}.$$

- Observe now that, at equilibrium, the previous TCP Vegas update will imply $x_s(t) = \frac{\alpha_s d_s}{\lambda^s(t)}$, which coincides with the one-shot update of the dual-based algorithm $x_s^\star(t) = \frac{\alpha_s d_s}{\lambda^s(t)}$.

Different Congestion Control and Utilities

- We have seen how TCP Vegas algorithm can be interpreted as an approximate dual-based gradient method corresponding to a NUM with a particular choice of utilities.

- TCP Reno:
  - source utility: arctan
  - link price: packet loss

- TCP Vegas
  - source utility: weighted log
  - link price: queuing delay

Summary (I)

- TCP Reno/Vegas algorithms for the Internet were devised based on heuristics, common sense, and trial-and-error in the 1980s-1990s. There was no solid understanding of the convergence and stability properties of the algorithms.

- More recently, in the early 21st century, it was realized that one can reinterpret those algorithms as approximate dual-based gradient methods solving implicitly a NUM problem.

Summary (II)

- In particular,
  - a careful inspection of the KKT conditions shows that a steady-state solution of the TCP Vegas algorithm solves a NUM with logarithmic utilities
  - the window update in TCP Vegas is in fact an approximation of a dual-based gradient projection method solving the NUM.

- Convex optimization has been used to reverse-engineer and understand existing protocols.

- Even more than that, convex optimization is being used to devise better protocols for the Internet.

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