Resonance: a driven damped oscillator

The basic differential equation is:

$$\left(\frac{d^2}{dt^2} - \frac{\omega_r}{Q}\frac{d}{dt} + \omega_r^{\ 2}\right) f(t) = g(t)$$

where ω_r is the resonant frequency and Q the dimensionless damping factor. For the LRC circuit one has ω_r^2 = C/L, and $Q = \sqrt{LC/R}$. The driving term is g(t), and the response function f(t). Given a g(t), one solves this equation for f. The usual technique is to assume a single frequency for g, g=exp(-iωt), for which the solution is

$$f(t) = \frac{1}{-\omega^2 + \omega_r^{\ 2} + i\omega\omega_r / Q} \exp(-i\omega t)$$

The amplitude squared is then just the square of the magnitude,

$$|f|^2 = \frac{1}{\left(-\omega^2 + \omega_r^{\ 2}\right)^2 + \left(\omega\omega_r / Q\right)^2}$$

or, normalizing to 1 on resonance, the amplitude is simply

$$A(\omega) = \frac{1}{\sqrt{1 + Q^2\left(\dfrac{\omega_r^{\ 2} - \omega^2}{\omega_r \omega}\right)^2}}$$

$$= \left[1 + Q^2\left(\omega_r / \omega - \omega / \omega_r\right)^2\right]^{-1/2}$$

A function node in LabVIEW: