School of Mechanical Engineering

FACULTY OF ENGINEERING & PHYSICAL SCIENCES

UNIVERSITY OF LEEDS

# MECH3496/XJME3496: Thermofluids III
# MECH3790: Aero/Aerospace Propulsion


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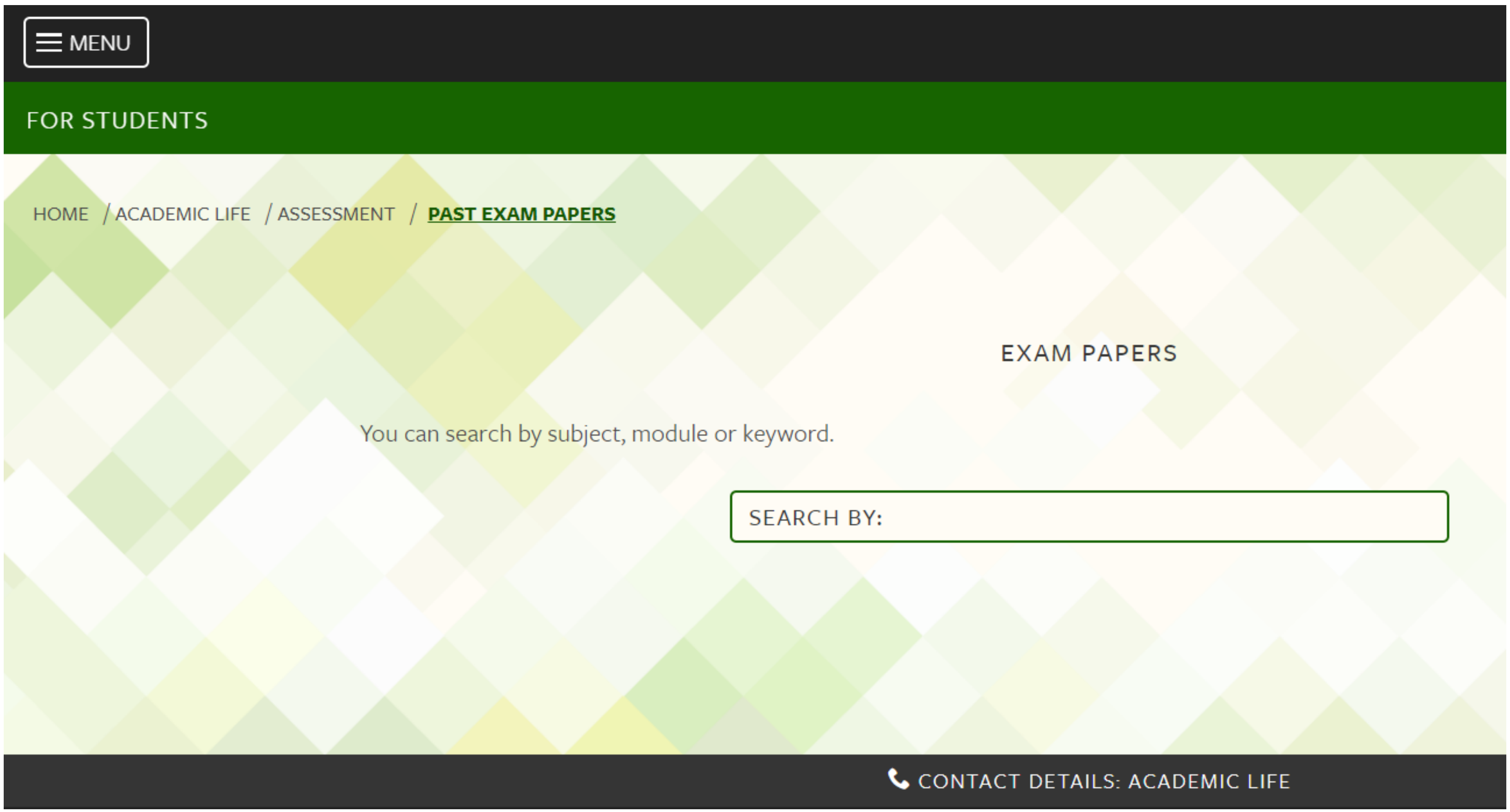
Revision 1

# Past exam papers available from;

http://students.leeds.ac.uk/site/custom_scripts/exam_papers_form.php



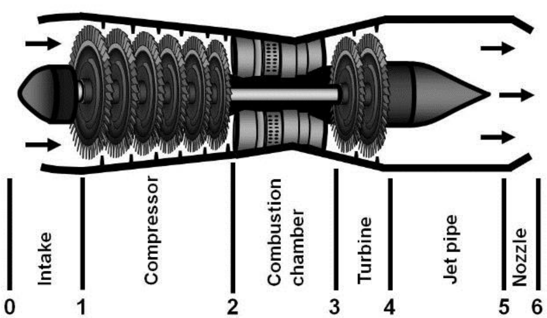
## Lecture topics

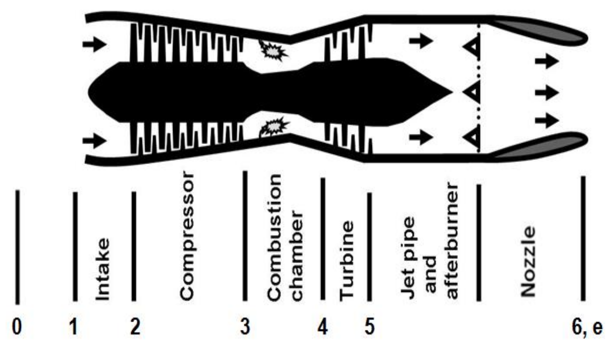
1. Introduction & fundamental theories
2. Compressible aerodynamics
3. Gas turbine engines
4. Aero piston engines
5. Rockets

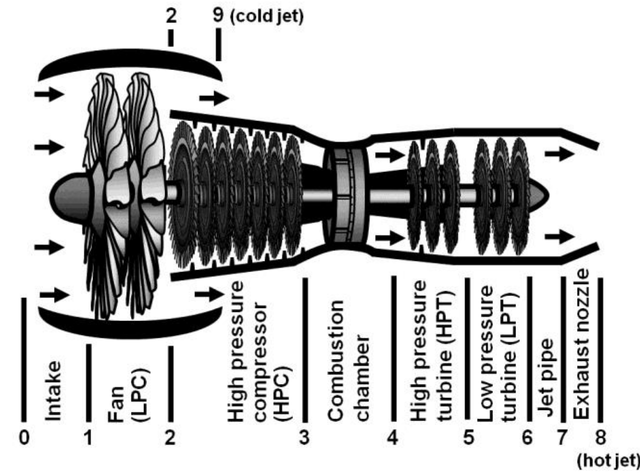
What is NOT in the exam

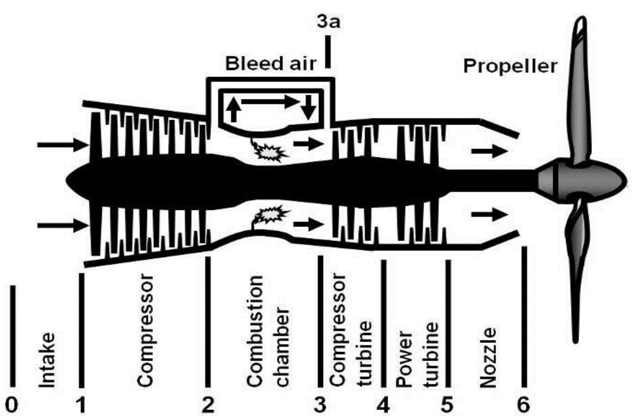
- Historical context
- Propulsion Integration
- Ramjets & Scramjets

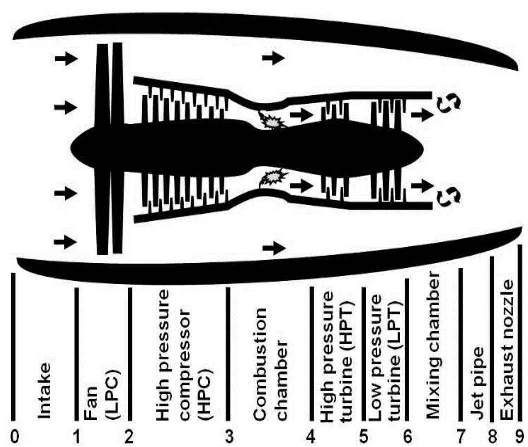
# Review the examples covered in lectures

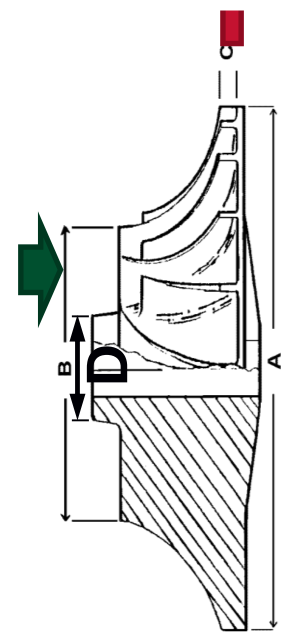


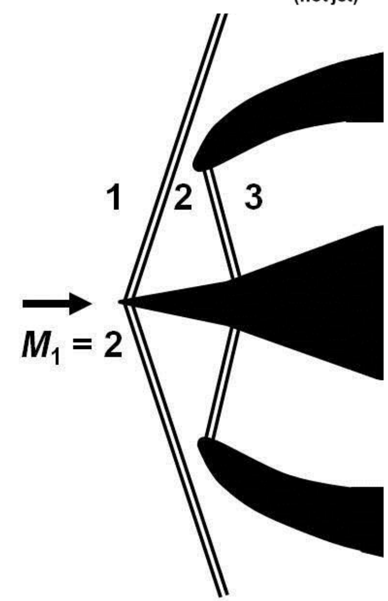



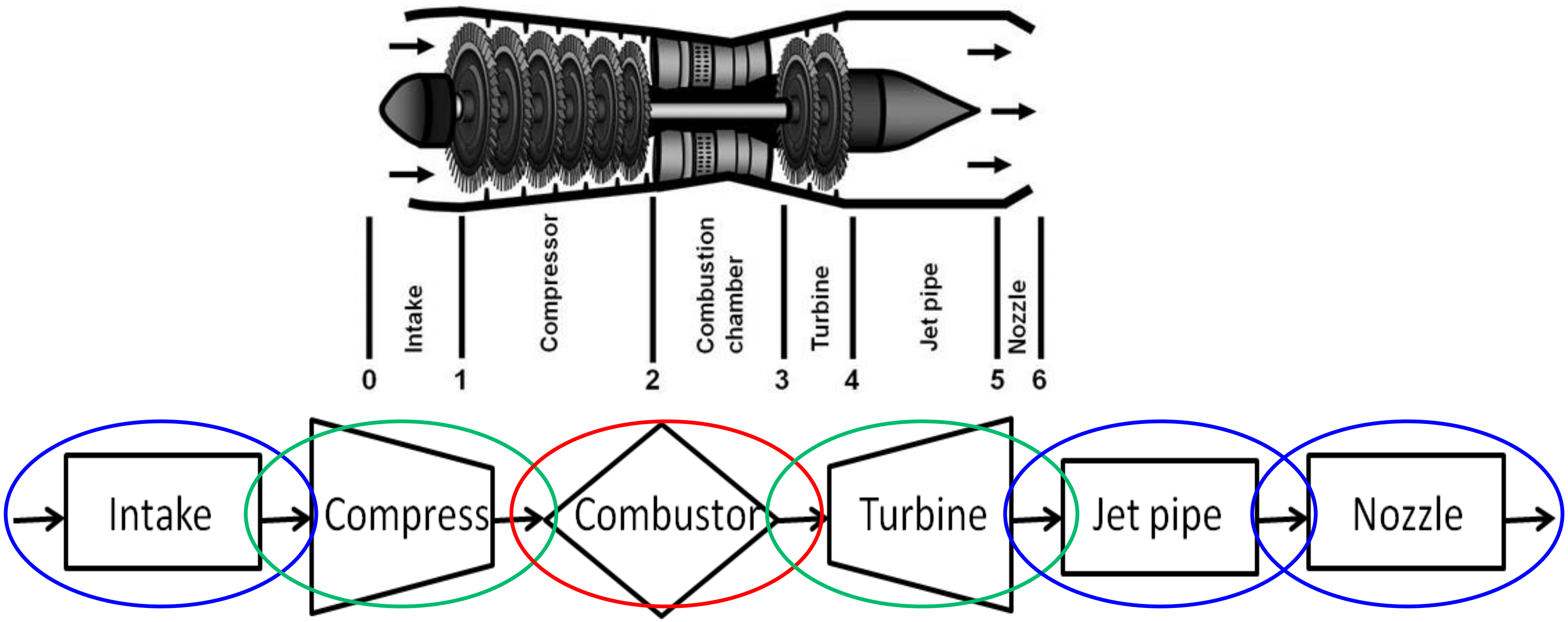




Two-shock inlet

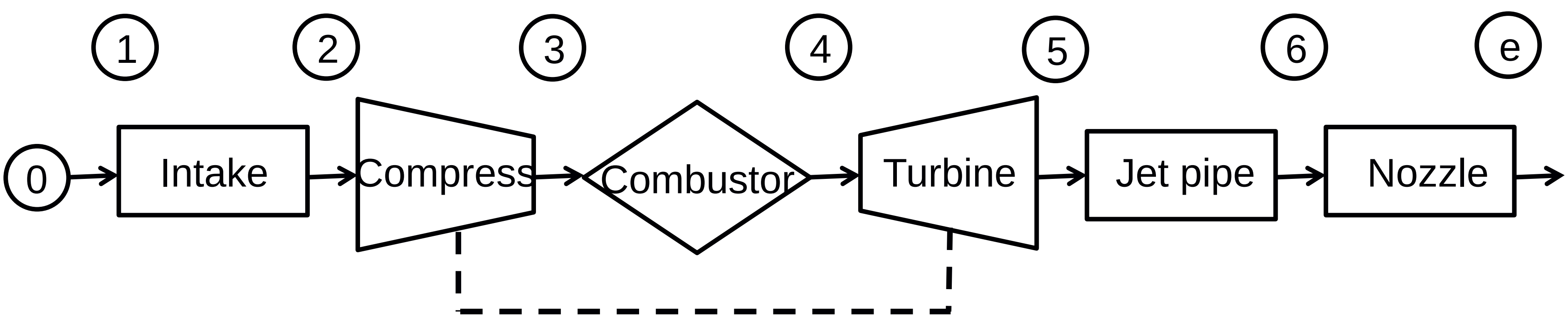
## Suggested analysis steps

1. Draw a stage diagram corresponding to the given engine description. For example, the diagram below represents a single spool turbojet engine without reheat (afterburner). Use shapes to distinguish between **FLOW**, **WORK** and **HEAT** processes.



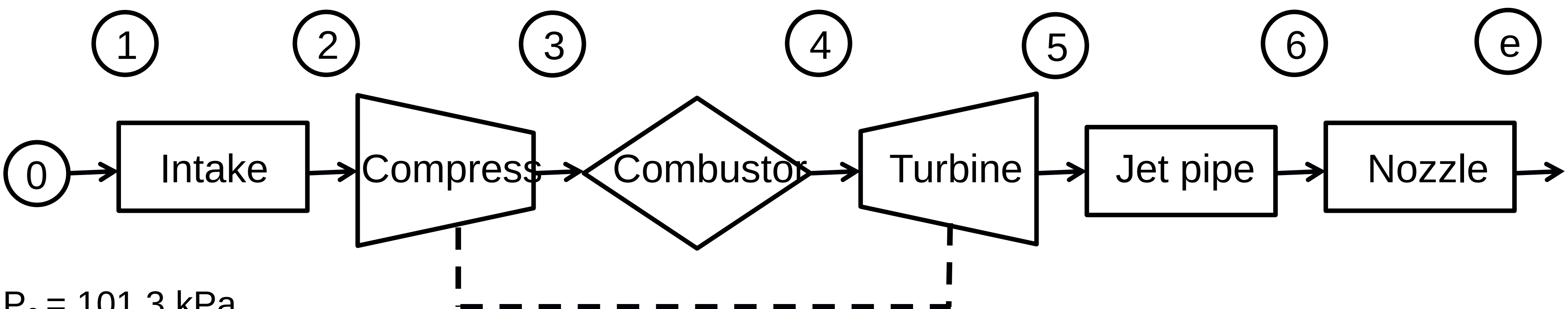
## Suggested analysis steps

2. Add mechanical linkages between stages (i.e. Compressor → Turbine). For a single-spool engine, there will only be one driveshaft linking a compressor and turbine, but more complex engines can have up to three spools.
3. Number the stages, remembering to start with ‘0’, representing freestream conditions ahead of the engine. Use ‘e’ to denote exit conditions.



## Suggested analysis steps

4. Below the diagram add the given data.




$P_0 = 101.3$ kPa
$M_0 = 0.0$
$\dot{m}_0 = 81.0$ kg/s

$PR_{2\rightarrow3} = 3.2$

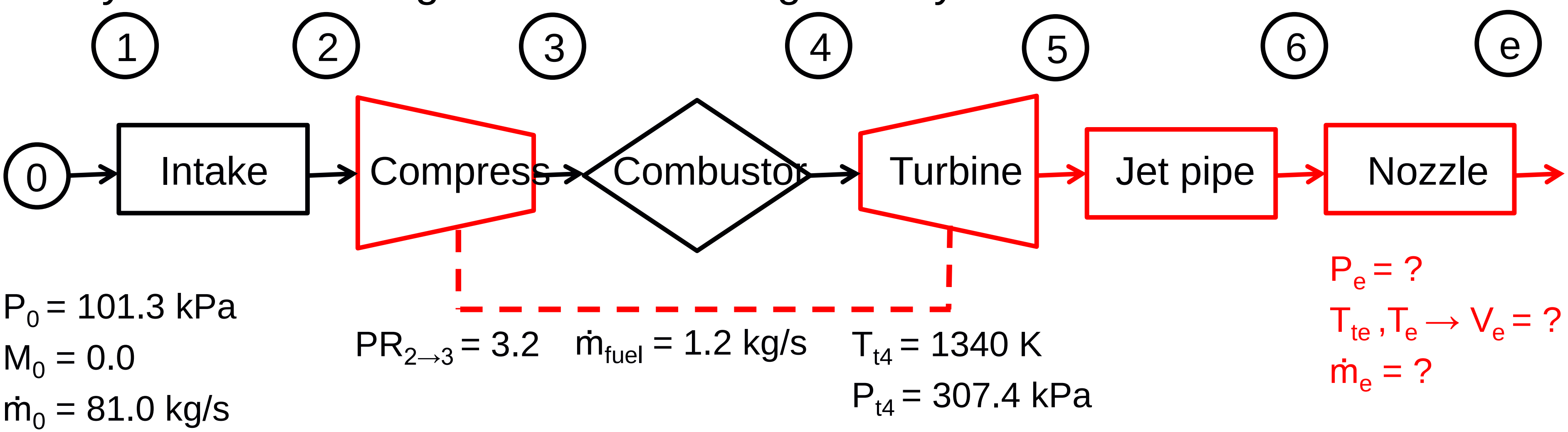
$\dot{m}_{fuel} = 1.2$ kg/s

$T_{t4} = 1340$ K
$P_{t4} = 307.4$ kPa

## Suggested analysis steps

5. An exam question will **likely not** ask for a full-engine cycle analysis !

So, determine what properties are required to answer the problem and highlight the 'analysis route'. Bear in mind however that the analysis route may involve working backwards through the cycle.




6. Use appropriate formulae for each process, considering **FLOW**, **WORK** and **HEAT** processes (i.e. Isentropic, work or thermal energy balance).

| **Process:** | **FLOW** | **WORK** | **HEAT** |
|---|---|---|---|
| **Examples:** | • Intakes<br>• Nozzles<br>• Jet pipes<br>• Mixing chambers | • Compressors<br>• Turbines<br>• Propellers<br>• Rotors | • Combustors<br>• Afterburners |
| **Formulae:** | • Isentropic<br>• Choke test for nozzles<br>• Shock analysis for supersonic intakes | • Isentropic<br>• Work energy balance | • Thermal energy balance |
| **Loss factors:** | • Isentropic efficiency<br>• Total pressure loss<br>• Mixing coefficient | • Isentropic efficiency<br>• Mechanical efficiency<br>• Propeller efficiency | • Combustion efficiency<br>• Total pressure loss |

General comments when answering questions:

- Show your working clearly
- Use an annotated process diagram
- Write down equations, not just numerical solution
- Explain the source of equations i.e. energy balance (explaining what energy transfer is occuring) or isentropic
- Explain any relevant assumptions
- Use correct notation (i.e. to differentiate total and static properties)
- Be familiar with using tables/charts provided: Normal & Oblique shocks

Format of Propulsion questions for open-book Gradescope exams in 2020/21:

- You will have 2 hours to complete the exam/OTLA
- The paper has 2 parts with 2 compulsory questions in each part (so each question is worth 25%)
- Questions will include a numerical analysis of aerodynamic and thermodynamic processes and cycles, similar to examples covered in the module
- Questions can include an iterative or multipoint calculation, requiring you to use a computer-calculator (i.e. Excel, Matlab, Python etc) to generate a trendline for example, and include a graph in your solution
- Questions will require an element of critical analysis, for example by describing a trendline with reference to the underlying physics

# Past exam questions

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Revision questions (taken from past papers or the Ward textbook)

1. Covers: 1 x 4 stroke Otto cycle
2. Covers: 2 x ideal rocket cycle
3. Covers:
    - Shock theory for three-shock spike diffuser
    - Non-mixing turbofan engine exhaust system
    - Turbojet with afterburner and VG condi nozzle
4. Covers:
    - Non-mixing turbofan engine exhaust system
    - Turboprop shaft power

Note that these are the same revision questions that I cover every year

5. This question requires you to demonstrate your knowledge and understanding of aero piston engines.

An aircraft equipped with two, four-stroke, horizontally-opposed engines flies at an altitude where the intake manifold pressure (*IMP*) and temperature (*IMT*) are 54.05 kPa and 255.7 K, respectively. The engines are throttled to give a shaft speed of 2,575 rpm. Each engine has four cylinders with a stroke ($s$) of 11.1 cm, bore ($b$) of 13 cm, and compression ratio of 8.5. The propulsive efficiency is 0.82 and the mechanical efficiency is 0.7. The air-to-fuel ratio (*AFR*) of the Avgas/air mixture is 15.1 (by mass) and the chemical energy released in 1 kg of this Avgas fuel (*LHV*) is $4.354 \times 10^7$ J. Assume that the engine operates in a 4 stroke Otto cycle and $\gamma = 1.4$, $C_p = 1{,}008$ J/(kg·K) and $R = 287$ J/(kg·K) for air.

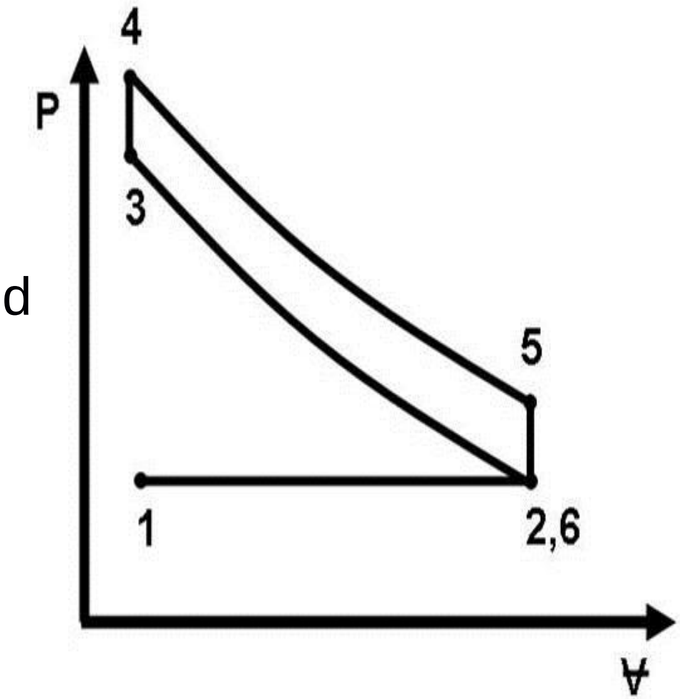
a) Calculate the available power for the two engines combined; **[15 marks]**

b) Determine the mean effective pressure in a single cylinder; **[5 marks]**

c) Explain why most propeller-driven commercial aircraft or transports use turboprop engines instead of piston aerodynamic engines. **[5 marks]**

## Solution: Compression stroke (stages 2-3)

Assume an ideal four-stroke Otto cycle. This means that the compression and power strokes behave isentropically. Therefore the pressure and temperature at position 3 is determined from the compression ratio using isentropic relations:




$$T_2 = T_1 = 255.7K$$

$$P_2 = P_1 = 54.05kPa$$

$$P_3 = P_2 \left( \frac{\forall_2}{\forall_3} \right)^{\gamma} = (54.05\,kPa)(8.5)^{1.4} = 1{,}081\,kPa$$

$$T_3 = T_2 \left( \frac{\forall_2}{\forall_3} \right)^{\gamma-1} = (255.7\,K)(8.5)^{0.4} = 601.9\,K$$

## Solution: Combustion (stages 3-4)

The mass of fuel in the air-fuel mixture when mixed with one kilogram of air is;

Since; $AFR = \frac{m_{air}}{m_{fuel}} = 15.1$ , we require 15.1 kg of Air for 1 kg of Fuel

$$m_{tot} = m_{fuel} + m_{air} = 1\,kg + 15.1\,kg = 16.1\,kg\ of\ \ mixture$$

Therefore, the heat released per kilogram of air-to-fuel mixture is;

$$Q = LHV\ \frac{m_{fuel}}{m_{total}} = \left(4.354\times10^{7}\ \tfrac{J}{kg\,(fuel)}\right)\left(\frac{1\,kg\,(fuel)}{16.1\,kg\,(total\ \ mix)}\right)$$

$$= 2.696\times10^{6}\,\frac{J}{kg\,(mix)}\ \sim Heat\ released\ \ per\ kg\ mixture$$

## Solution: Combustion (stages 3-4)

Energy conservation gives;

$$Q = C_v\left(T_4 - T_3\right) \qquad\qquad T_4 = \frac{Q}{C_v} + T_3$$

Since there is only a small amount of fuel in the mixture (*AFR* = 15.1), we can assume that the mixture approximately has the gas properties of air. Therefore $C_p$ = 1,008 J/(kg·K) and *R* = 287 J/(kg·K).

Hence: $$C_v = C_p - R = 1{,}008\,\tfrac{J}{kg\cdot K} - 287\,\tfrac{J}{kg\cdot K} = 721\,\tfrac{J}{kg\cdot K}$$

## Solution: Combustion (stages 3-4)

Therefore the temperature after combustion ($T_4$) can be found:

$$T_4 = \frac{Q}{C_v} + T_3 = \frac{2.696 \times 10^6 \ \frac{J}{kg}}{721 \ \frac{J}{kg}} + 601.9\ K = 4{,}341.2\ K$$

Since this is air (which approximates an ideal gas), the ideal gas equation of state can be used. [Note:  and $\rho$ = constant.]

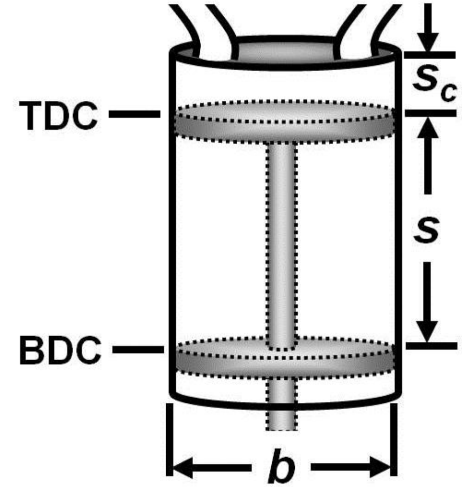
$$\frac{P_3}{RT_3} = \frac{P_4}{RT_4}$$

Therefore the pressure after combustion ($P_4$) can be determined:

$$P_4 = P_3 \left( \frac{T_4}{T_3} \right) = 1{,}081.4\ kPa \left( \frac{4{,}341.2\ K}{601.9\ K} \right) = 7{,}799.6\ kPa$$

## Solution: Power stroke (stages 4-5)

The power stroke can be considered isentropic (since no heat is added), so;




$$P_5 = P_4 \left( \frac{\cancel{V}_4}{\cancel{V}_5} \right)^{\gamma} = \left(7{,}799.6\, kPa\right) \left( \frac{1}{8.5} \right)^{1.4} = 389.8\, kPa$$

The vales of $\cancel{V}_2$ and $\cancel{V}_3$ now need to be determined;

$$\frac{s_c + s}{s_c} = \frac{\cancel{V}_2}{\cancel{V}_3} = 8.5 \rightarrow 8.5\, s_c = s_c + 11.1cm \rightarrow s_c = 1.5cm$$

$$\cancel{V}_2 = \cancel{V}_5 = \frac{\pi\, b^2}{4} \left(s + s_c\right) = \frac{\pi \left(13\, cm\right)^2}{4} \left(11.1\, cm + 1.5\, cm\right) = 1{,}672.4\, cm^3$$

$$\cancel{V}_3 = \cancel{V}_4 = \frac{\pi\, b^2}{4} \left(s_c\right) = \frac{\pi \left(13\, cm\right)^2}{4} \left(1.5\, cm\right) = 196.8\, cm^3$$

## Solution

We can now calculate the work done to compress the gas;

$$\boxed{W_{compress} = \frac{P_2 \cancel{V}_2 - P_3 \cancel{V}_3}{1-\gamma}}$$

$$W_{comp} = \frac{\left(54.05\times10^3\ Pa\right)\left(1.672\times10^{-3}\ m^3\right)-\left(1{,}081.3\times10^3\ Pa\right)\left(1.968\times10^{-4}\ m^3\right)}{1-1.4} = 306J$$

and the work gained in the power stroke is given by;

$$\boxed{W_{power} = \frac{P_5 \cancel{V}_5 - P_4 \cancel{V}_4}{1-\gamma}}$$

$$W_{power} = \frac{\left(389.8\times10^3\ Pa\right)\left(1.672\times10^{-3}\ m^3\right)-\left(7{,}799.4\times10^3\ Pa\right)\left(1.968\times10^{-4}\ m^3\right)}{1-1.4} = 2{,}207.9J$$

Solution:

Consequently, the net work per cycle (of a single cylinder) is therefore:

$$W_{cycle} = W_{power} - W_{compress} = 2{,}207.9\ J - 306\ J = 1{,}901.9\ J$$

The total available power from a single engine then is

$$\boxed{\dot{W}_A = \frac{1}{120}\,\eta_P\,\eta_{mech}\,n_{\substack{shaft\\rpm}}\,N\,W_{cycle}}$$

$$\dot{W}_A = \frac{1}{120}\Big[\,(0.82)(0.7)(2{,}575\ rpm)(4\ cylinders)(1{,}901.9\ J)\,\Big]$$

$$= 93.7\ kW$$

And the total available power from both engines is

$$\dot{W}_A = (2)(93.7) = \boxed{187.4\ kW}$$

Solution

Now calculate the mean effective pressure ($P_{me}$), which is the average representation of pressure in a cylinder in the power stroke, using;

$$\dot{W}_A = \frac{1}{120} \eta_p \, \eta_{mech} \, n_{\substack{shaft \\ rpm}} \, C \, P_{me} \qquad W_{cycle} = \frac{CP_{me}}{N}$$

The capacity is given by:

$$C = \frac{\pi b^2}{4} sN = \frac{\pi \left(0.13\, m\right)^2 \left(0.111\, m\right)\left(4\ cylinders\right)}{4} = 0.006\, m^3$$

$$P_{me} = \frac{W_{cycle} N}{C} = \frac{\left(1{,}901.9\ J\right)\left(4\ cylinders\right)}{\left(0.006\, m^3\right)} = \boxed{1.27 Mpa}$$

Solution

**c) Explain why most propeller-driven commercial aircraft or transports use turboprop engines instead of piston aerodynamic engines.**

Compared with a piston aerodynamic engine, a turboprop can produce a much higher shaft power so it can propel aircraft to much higher speeds. Turboprops are also more fuel efficient and can operate at higher altitudes.