MECH3496/XJME3496: Thermofluids III
MECH3790: Aero/Aerospace Propulsion

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Revision 2

Revision questions (taken from past papers or the Ward textbook)

1. Covers: 1 x 4 stroke Otto cycle
2. Covers: 2 x ideal rocket cycle
3. Covers:
   - Shock theory for three-shock spike diffuser
   - Non-mixing turbofan engine exhaust system
   - Turbojet with afterburner and VG condi nozzle
4. Covers:
   - Non-mixing turbofan engine exhaust system
   - Turboprop shaft power

Note that these are the same revision questions that I cover every year

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3. This question requires you to demonstrate your knowledge and understanding of rocket engines.

   a) An air-launched missile is fired at an altitude of 10 km. A booster motor first ignites and accelerates the missile for 3 seconds. The missile's solid rocket sustainer motor then fires and provides 3.8 kN of thrust. The missile's sustainer motor has a chamber pressure of 2.8 MPa, and a chamber temperature of 2,800 K. The exhaust gases have a gas constant of R = 355 J/(kg.K) and γ = 1.05. Assuming the nozzle is optimized to achieve full expansion at an altitude of 10 km, where the ambient pressure is 26.5 kPa, determine the following parameters for the sustainer motor:

      i. The velocity of the exhaust gases; **[4 marks]**

      ii. The nozzle throat and exit diameters; **[8 marks]**

      iii. The temperature of the exhaust gases; **[3 marks]**

      iv. The nozzle throat velocity of the combustion gases; **[4 marks]**

   b) Describe why solid rockets are more often used for air-launched missiles than liquid rockets. **[3 marks]**

   c) Describe why liquid rockets are more often used in large, ballistic missiles and space launchers than solid rockets. **[3 marks]**

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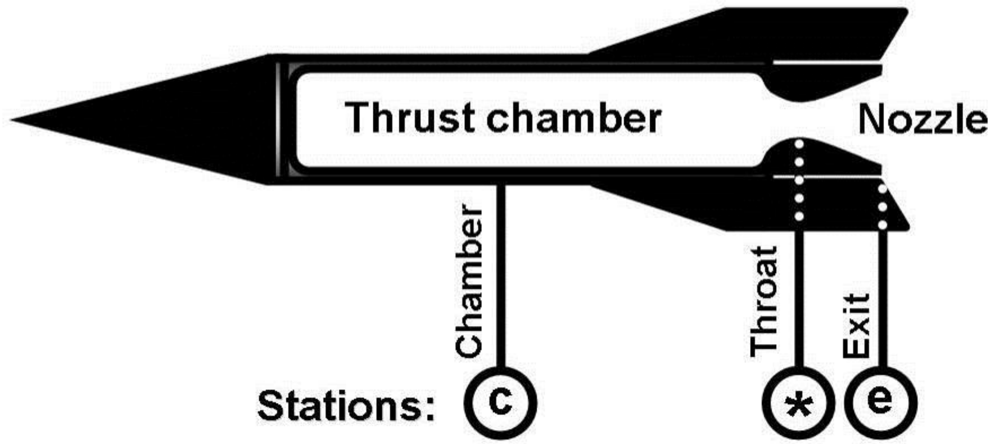
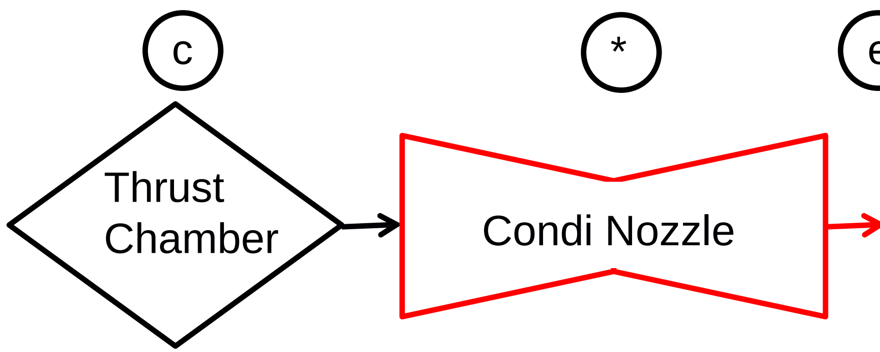
Solution





$P_c = 2.8$ MPa
$T_c = 2{,}800$ K

$F_N = 3.8$ kN @ $h = 10$ km
$P_0 = 26.5$ kPa

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Solution

We need to use our knowledge of rocket engines;

1. The thrust chamber velocity is not specified and so assumed zero…

$$T_c = T_{tc} = 2{,}800K$$

$$P_c = P_{tc} = 2.8MPa$$

2. The nozzle flow is assumed isentropic, with no losses in total pressure or temperature, so…

$$T_{tc} = T_t^* = T_{te} = 2{,}800K$$

$$P_{tc} = P_t^* = P_{te} = 2.8MPa$$

3. The rocket uses a Condi-nozzle in order to produce a supersonic exit velocity. Consequently the flow must be sonic at the throat (M* = 1.0)

4. The flow is steady, so…

$$\dot{m}^* = \dot{m}_e$$

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Solution

We are told the nozzle is optimized to achieve full expansion at an altitude of 10 km, where;

$$P_e = P_0 = 26.5\ kPa$$

$$\frac{P_{te}}{P_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$M_e^2 = \frac{2}{\gamma - 1}\left[\left(\frac{P_{te}}{P_e}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right] = \frac{2}{1.05 - 1}\left[\left(\frac{2{,}800{,}000 Pa}{26{,}500 Pa}\right)^{\frac{1.05-1}{1.05}} - 1\right]$$

$$M_e = 3.15$$

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Solution

$$T_e = \frac{T_{te}}{\left(1+\frac{\gamma-1}{2}M_e^2\right)} = \frac{2{,}800\ K}{\left[1+\left(\frac{1.05-1}{2}\right)(3.15)^2\right]} = \boxed{2243.5\ K}$$

$$V_e = M_e\sqrt{\gamma\ R\ T_e}$$

$$= (3.15)\sqrt{(1.05)\left(355\ \tfrac{J}{kg\cdot K}\right)\left(\ 2243.5\ K\ \right)} = \boxed{2{,}882\ \tfrac{m}{s}}$$

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Solution

The rocket thrust equation is:

$$F_N = \underbrace{\dot{m}\, V_e}_{\substack{momentum\\component}} + \underbrace{A_e \left(P_e - P_0\right)}_{\substack{pressure\\component}}$$

But since $P_e = P_0$ at this altitude (10km);

$$\dot{m} = \frac{F_N}{V_e} = \frac{3{,}800\ N}{2{,}882\ \frac{m}{s}} = 1.32\ \frac{Kg}{s}$$

Now that the mass flow rate is known, the rocket dimensions can be calculated

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Solution

$$\dot{m} = \frac{P^* A^* V^*}{R T^*} = \frac{P_e A_e V_e}{R T_e} = 1.32 \frac{Kg}{s}$$

$$A_e = \frac{\dot{m} R T_e}{P_e V_e} = \frac{(1.32 \frac{Kg}{s})\left(355 \frac{J}{kg \cdot K}\right)\left(2243.5 K\right)}{(26{,}500 Pa)(2{,}882 \frac{m}{s})} = 0.014 m^2$$

$$A_e = \frac{\pi d_e^2}{4}, \qquad \boxed{d_e = 0.134\ m}$$

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Solution

We can derive static gas properties at the throat ($M^* = 1$) using isentropic equations

$$T^* = \frac{T_t^*}{\left(1+\frac{\gamma-1}{2}M^{*,2}\right)} = \frac{2{,}800\ K}{\left[1+\left(\frac{1.05-1}{2}\right)(1.0)^2\right]} = 2{,}732\ K$$

$$V^* = M^* \sqrt{\gamma\ R\ T^*}$$

$$= (1.0)\sqrt{(1.05)\left(355\ \tfrac{J}{kg\cdot K}\right)\left(2{,}732\ K\right)} = \boxed{1{,}009\ \tfrac{m}{s}}$$

$$P^* = \frac{P_t^*}{\left(1+\frac{\gamma-1}{2}M^{*,2}\right)^{\frac{\gamma}{\gamma-1}}} = \frac{2{,}800{,}000\,Pa}{\left[1+\left(\frac{1.3-1}{2}\right)(1.0)^2\right]^{\frac{1.3}{1.3-1}}} = 1{,}528\ kPa$$

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Solution

We can derive static gas properties at the throat ($M^* = 1$) using isentropic equations

$$A^* = \frac{\dot{m} R\, T^*}{P^* V^*} = \frac{(1.32\,\frac{Kg}{s})\left(355\,\frac{J}{kg \cdot K}\right)\left(2{,}732\ K\right)}{(1{,}528{,}000\ Pa)(1{,}009\ \frac{m}{s})} = 0.00083 m^2$$

$$A^* = \frac{\pi\, d^{*2}}{4}, \qquad \boxed{d^* = 0.028\, m}$$

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Solution

b) Describe why solid rockets are more often used for air-launched missiles than liquid rockets.

Some advantages of solid rockets are: ease of maintenance (since they have no moving parts like pumps or complex feed systems); only simple igniters are required; they can generally be stored for up to 20+ years; and they are generally smaller than liquid rockets. For these reasons, solid rockets are more suitable for air-launched missiles than liquid rockets.

c) Describe why liquid rockets are more often used in large, ballistic missiles and space launchers than solid rockets.

Liquid rockets are more powerful than solid rockets in terms of both gross thrust and specific impulse, and can be throttled.

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A rocket engine has a chamber pressure and temperature of 46 bar and 2,300 K, respectively. Combustion gases enter a convergent-divergent nozzle at a very low velocity and the nozzle is designed to maintain a mass flow rate of 3 kg/s with an atmospheric exit pressure of 0.5 bar. The exhausted gaseous products of combustion have properties: γ = 1.3 and R = 390.4 J/(kg·K). Determine;

- The critical (throat) section area **[15 marks]**
- The exit area **[5 marks]**
- The thrust developed at this condition **[5 marks]**

Note: 1 bar = 10^5 Pa

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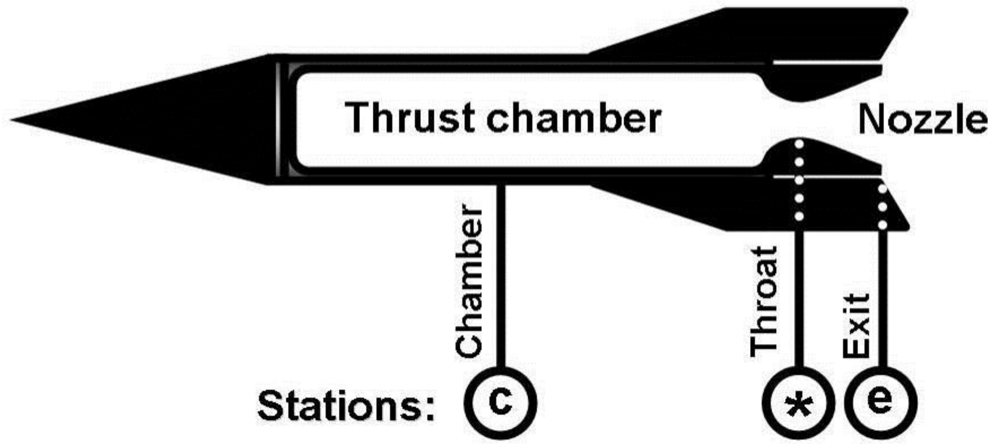
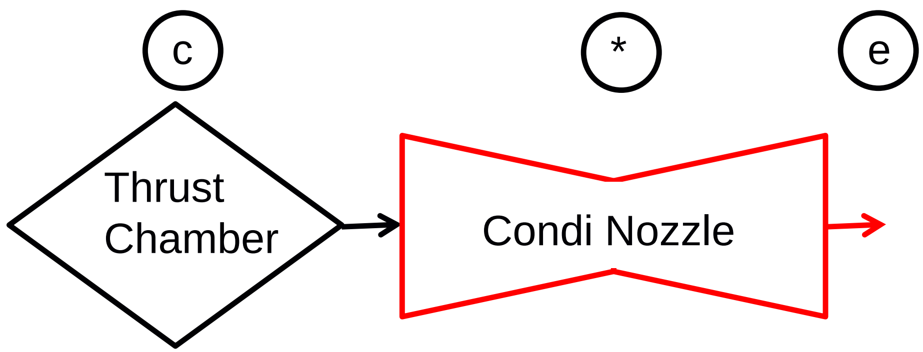
Solution





$P_c = 46$ bar

$T_c = 2{,}300$ K

$\dot{m} = 3$ kg/s

$P_0 = 0.5$ bar

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Solution

We need to use our knowledge of rocket engines;

1. We are told the thrust chamber velocity is very low hence….

$$T_c = T_{tc} = 2{,}300K$$

$$P_c = P_{tc} = 46bar$$

2. The nozzle flow is assumed isentropic, with no losses in total pressure or temperature, so…

$$T_{tc} = T_t^* = T_{te} = 2{,}300K$$

$$P_{tc} = P_t^* = P_{te} = 46bar$$

3. The rocket uses a Condi-nozzle in order to produce a supersonic exit velocity. Consequently the flow must be sonic at the throat (M* = 1.0)

4. The flow is steady, so…

$$\dot{m}^* = \dot{m}_e = 3kg/s$$

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Solution

Consequently if *M** = 1.0, the mass flow rate can be expressed as;

$$T^* = \frac{T_t^*}{\left(1 + \frac{\gamma - 1}{2} M^{*,2}\right)} = \frac{2{,}300\ K}{\left[1 + \left(\frac{1.3 - 1}{2}\right)(1.0)^2\right]} = 2{,}000\ K$$

$$V^* = M^* \sqrt{\gamma\ R\ T^*}$$

$$= (1.0)\sqrt{(1.3)\left(390.4\ \tfrac{J}{kg \cdot K}\right)\left(2{,}000\ K\right)} = 1{,}007\ \tfrac{m}{s}$$

$$P^* = \frac{P_t^*}{\left(1 + \frac{\gamma - 1}{2} M^{*,2}\right)^{\frac{\gamma}{\gamma - 1}}} = \frac{46\ bar}{\left[1 + \left(\frac{1.3 - 1}{2}\right)(1.0)^2\right]^{\frac{1.3}{1.3-1}}} = 25.1\ bar = 2.51\ MPa$$

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Solution

Consequently if *M** = 1.0, the mass flow rate can be expressed as;

$$\dot{m}^* = \frac{P^* A^* V^*}{R\, T^*}$$

Rearranging;

$$A^* = \frac{\dot{m}^* R T^*}{P^* V^*} = \frac{(3\,\frac{Kg}{s})(390.4\,\frac{J}{kg\cdot K})\left(2000\,K\right)}{(25{,}100{,}000\,Pa)(1{,}007\,\frac{m}{s})} = \boxed{0.00093 m^2}$$

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Solution

Since we are told that;

$$P_e = P_0 = 0.5bar = 50kPa$$

$$\frac{P_{te}}{P_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$M_e^2 = \frac{2}{\gamma - 1}\left[\left(\frac{P_{te}}{P_e}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = \frac{2}{1.3-1}\left[\left(\frac{46bar}{0.5bar}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$M_e = 3.5$$

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Solution

$$T_e = \frac{T_{te}}{\left(1+\frac{\gamma-1}{2}M_e^2\right)} = \frac{2{,}300\ K}{\left[1+\left(\frac{1.3-1}{2}\right)(3.5)^2\right]} = 810.6\ K$$

$$V_e = M_e\sqrt{\gamma\ R\ T_e}$$

$$= (3.5)\sqrt{(1.3)\left(390.4\ \tfrac{J}{kg\cdot K}\right)\left(810.6\ K\right)} = 2{,}245\ \tfrac{m}{s}$$

$$A_e = \frac{\dot{m}R\ T_e}{P_e\ V_e} = \frac{(3\,\tfrac{Kg}{s})\left(390.4\ \tfrac{J}{kg\cdot K}\right)\left(810.6\ K\right)}{(50{,}000\,Pa)(2{,}245\ \tfrac{m}{s})} = \boxed{0.0085m^2}$$

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Solution

The thrust equation is;

$$F_N = \underbrace{\dot{m}\, V_e}_{\substack{momentum\\component}} + \underbrace{A_e \left(P_e - P_0\right)}_{\substack{pressure\\component}}$$

But since full expansion occurs and $P_e = P_0$;

$$F_N = \left(3\,\frac{Kg}{s}\right)\left(2{,}245\,\frac{m}{s}\right) = \boxed{6.73\,kN}$$

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