Week 1 (3) – Fundamental Data Structures

CST370 – Design & Analysis of Algorithms

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Computer Science

Lecture Objectives

- After completion of this lecture, you will be able to
  - recall the important linear data structures such as an array and a linked list.
  - explain basic terminologies of a graph and represent it using two different representations.

Chapter 1: Introduction

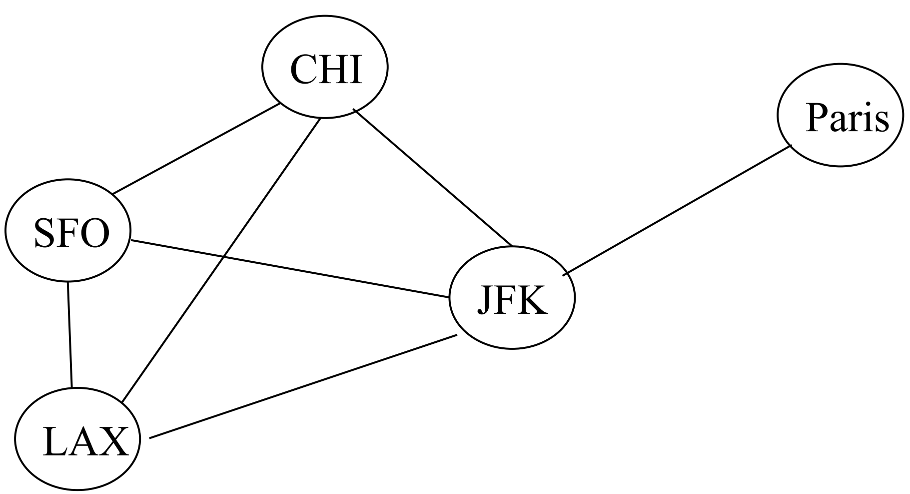
- 1.1 What is an Algorithm?
- 1.2 Fundamentals of Algorithmic Problem Solving
- 1.3 Important Problem Types
- **<u>1.4 Fundamental Data Structures</u>**

<<< Course Instruction >>>

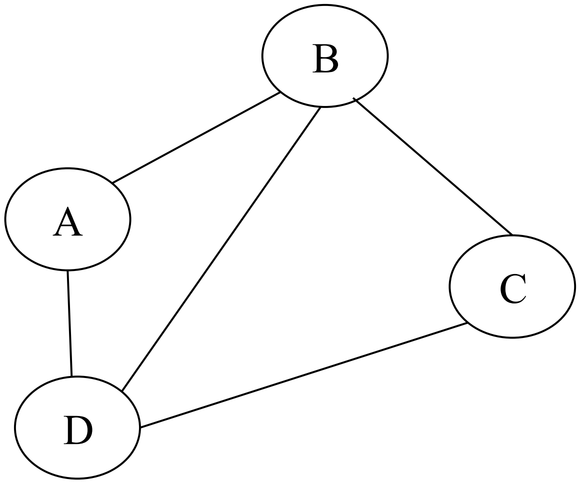
- Before moving on to the next slide, **read page 25 – 28** (before "graph") of the textbook.
  - In this section, you will review the basic linear data structures such as an array, a linked list (= singly linked list and doubly linked list), a stack, and a queue.
  - Because you already took a data structures course before, this topic will be straightforward.

Introduction to Graphs

- Watch the video first
  - https://youtu.be/2HPgDfJDSBY
- A graph is composed of nodes (= vertices) and lines (= edges).
  - They are used for modeling many real-life applications such as transportation, communication networks, project scheduling, etc.
- Example



Graph Example: Undirected Graph

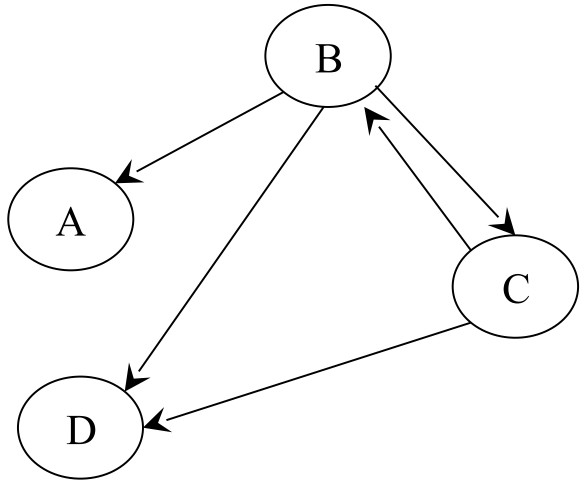


- The graph has 4 vertices and 5 undirected edges.

  $V = \{A, B, C, D\}$

  $E = \{(A, B), (A, D), (B, C), (B, D), (C, D)\}$

Graph Example: Directed Graph



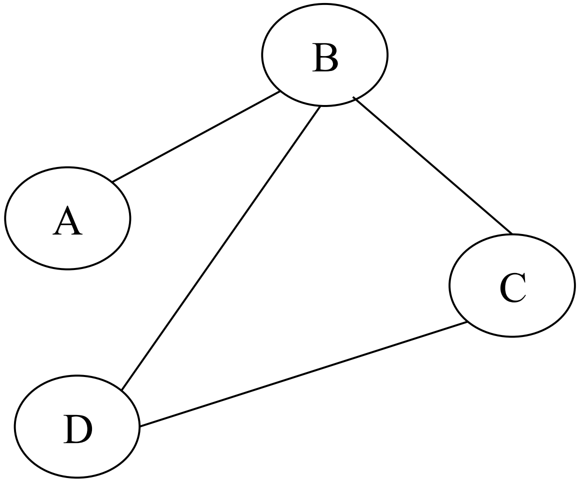
- The graph has 4 vertices and 5 directed edges.

$$V = \{A, B, C, D\}$$

$$E = \{(B, A), (B, C), (B, D), (C, B), (C, D)\}$$

Graph Representation: Adjacency Matrix

- Example

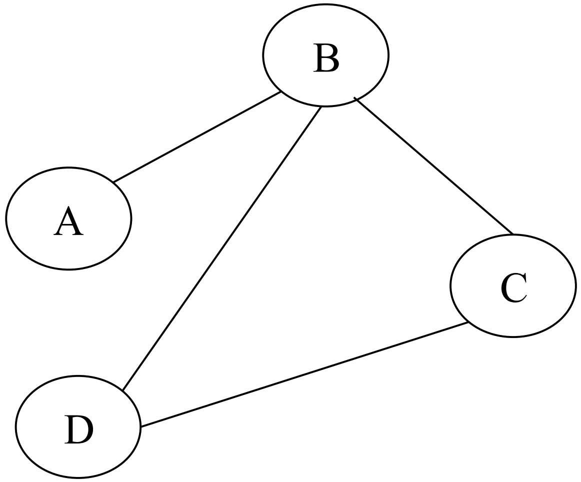


Graph

Adjacency Matrix

Graph Representation: Adjacency List

- Example



Graph

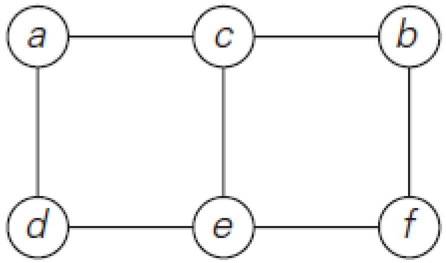
Adjacency List

<<< Course Instruction >>>

- **Read pages 28 – 30** (before "Weighted Graphs") in the textbook before moving on to the next slide.
  - Graphs are an important data structure in Computer Science.
  - We will cover many different graph algorithms for the entire semester.

Graph Definition (1 of 2)

- Formally, a *graph* $G = \langle V, E \rangle$ is defined by a pair of two **sets**: a finite nonempty set V of items called *vertices* and a set E of pairs of these items called *edges*.
- Example: Undirected graph



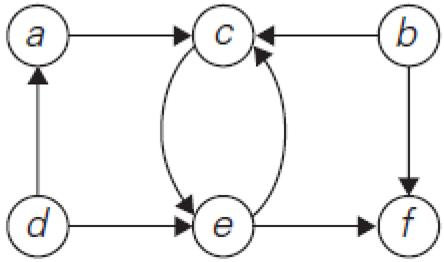
  – It has six vertices and seven undirected edges:

    $V = \{a, b, c, d, e, f\}$

    $E = \{(a, c), (a, d), (b, c), (b, f), (c, e), (d, e), (e, f)\}$

Graph Definition (2 of 2)

- Example: Directed graph (or Digraph)



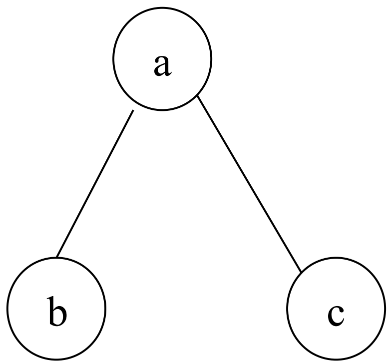
  - It has six vertices and eight directed edges:

    $V = \{a, b, c, d, e, f\}$

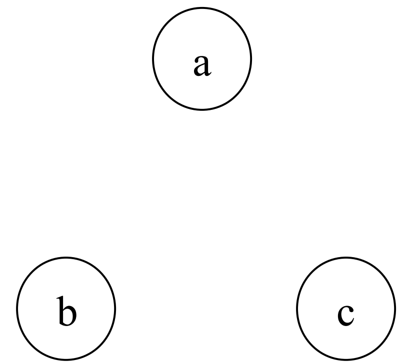
    $E = \{(a, c), (b, c), (b, f), (c, e), (d, a), (d, e), (e, c), (e, f)\}$

Question

- Are the figure (i) and (ii) graphs?



(i)



(ii)

Answer

- Both are graphs.
  - Figure (i) has three vertices and two undirected edges:

    $V = \{a, b, c\}$

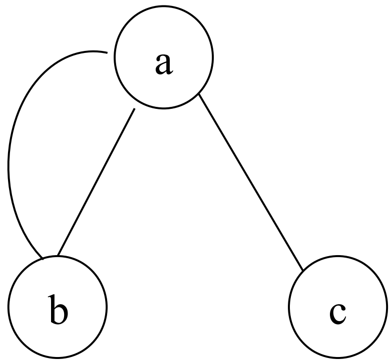
    $E = \{(a, b), (a, c)\}$
  - Figure (ii) has three vertices and zero edges:

    $V = \{a, b, c\}$

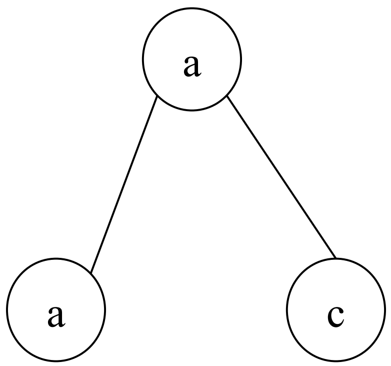
    $E = \{\ \}$

Question

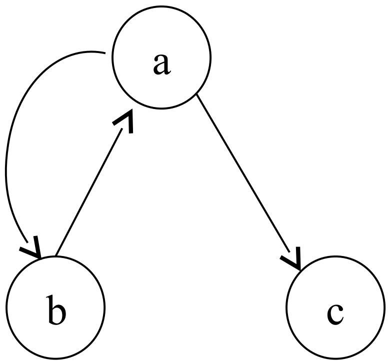
- How about the figure (iii), (iv), and (v)?
  - Are they graphs?



(iii)



(iv)



(v)

Answer

- Figure (v) is a graph. But figure (iii) and (iv) are not graphs.
  - Figure (iii) has duplicated edges:

    $E = \{\boldsymbol{(a, b)}, \boldsymbol{(a, b)}, (a, c)\}$
  - Figure (iv) has duplicated vertices:

    $V = \{\boldsymbol{a}, \boldsymbol{a}, c\}$
  - Since *V* and *E* are sets in the graph definition, they can't have duplication.
  - However, figure (v) is fine because the edge (a, b) and (b, a) indicate different edges in the directed graph.

<<< Course Instruction >>>

- Conventionally, we enumerate a set's elements using a **non-decreasing order**.
  - For example, in the digraph on slide number 12, the sets V and E are shown as follows:

    $V = \{a, b, c, d, e, f\}$

    $E = \{(a, c), (b, c), (b, f), (c, e), (d, a), (d, e), (e, c), (e, f)\}$
  - When expressing these sets, the sequence is not important. However, we enumerate the elements in **alphabetical order** (from a to z) in this class.
  - You **must always follow this convention** when you are doing homework or at exams.

Useful Formula for the Analysis of Algorithms

- Watch the video first
  - https://youtu.be/qrvcWgdkGi8
- You will use the following formula multiple times in the algorithm analysis.
  - As a CS major, you must memorize this formula.

$$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

- Example

  1 + 2 + … + 10 = (10*11)/2 = 55

Puzzle: Add numbers in a table

- A 10 × 10 table is filled with repeating numbers on its diagonals as shown below.
  - **Calculate the total sum** of the table's numbers.

1	2	3			⋯			9	10
2	3						9	10	11
3						9	10	11	
					9	10	11		
				9	10	11			
⋮			9	10	11				⋮
		9	10	11					
	9	10	11						17
9	10	11						17	18
10	11				⋯		17	18	19

Important Note

- **Do not see the answer immediately** on the next page.
  - Also **do not use a calculator**.
- **Hint**
  - It is not a good idea to add all the numbers one by one without a systematic approach.
  - Remember the formula $1 + 2 + 3 + \ldots + n$.
  - There are **at least two different ways** to calculate this sum efficiently.

Solution 1

- You can compute the sum row by row (or column by column).
  - The sum in the first row is equal to (10*11)/2 = 55.
  - The sum of the numbers in second row is 55+10 since each of the numbers is larger by 1 than their counterparts in the row above.
  - The same is true for all the other rows as well.
  - Hence the total sum is equal to

$$55+(55+10)+(55+20)+\cdots+(55+90)$$
$$= 55*10 + (10+20+\cdots+90)$$
$$= 1000.$$

Solution 2

- Another approach is based on the observation that the sum of any two numbers in the squares symmetric with respect to the diagonal connecting the lower left and upper right corners is equal to 20 such as "1 + 19", "2 + 18", "2 + 18", and so on.
  - Since there are (10*10 − 10)/2 = 45 such pairs (we subtracted the number of the squares on that diagonal from the total number of the squares), the sum of the numbers outside that diagonal is equal to 20*45 = 900. With 10*10 = 100 on the diagonal, the total sum is equal to 900 + 100 = 1000.

Remaining Topics in the Graph

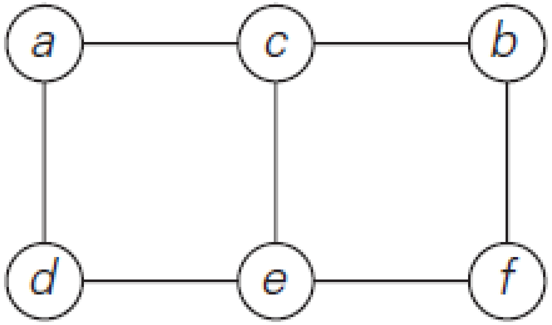
- Study the textbook to understand the following formula:

$$0 \leq |E| \leq |V|(|V| - 1)/2$$

- Also, understand the following terms:
  - **Complete** graph
  - **Dense** graph
  - **Sparse** graph

Graph Representation: Adjacency matrix

- Example



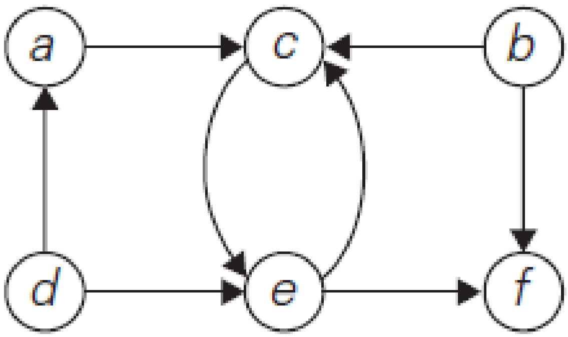
Graph

$$
\begin{array}{c|cccccc|}
 & a & b & c & d & e & f \\
\hline
a & 0 & 0 & 1 & 1 & 0 & 0 \\
b & 0 & 0 & 1 & 0 & 0 & 1 \\
c & 1 & 1 & 0 & 0 & 1 & 0 \\
d & 1 & 0 & 0 & 0 & 1 & 0 \\
e & 0 & 0 & 1 & 1 & 0 & 1 \\
f & 0 & 1 & 0 & 0 & 1 & 0 \\
\hline
\end{array}
$$

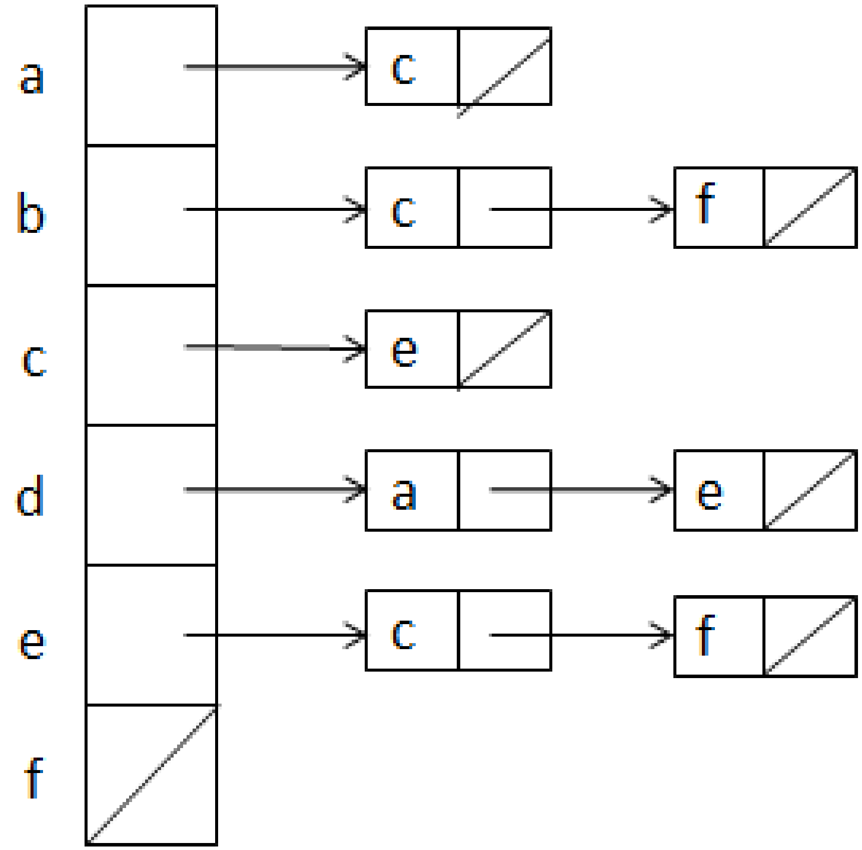
Adjacency Matrix

Graph Representation: Adjacency list

- Example



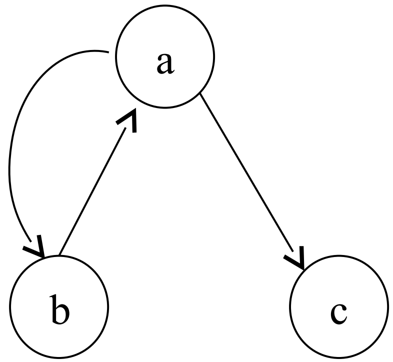
Graph



Adjacency List

Exercise

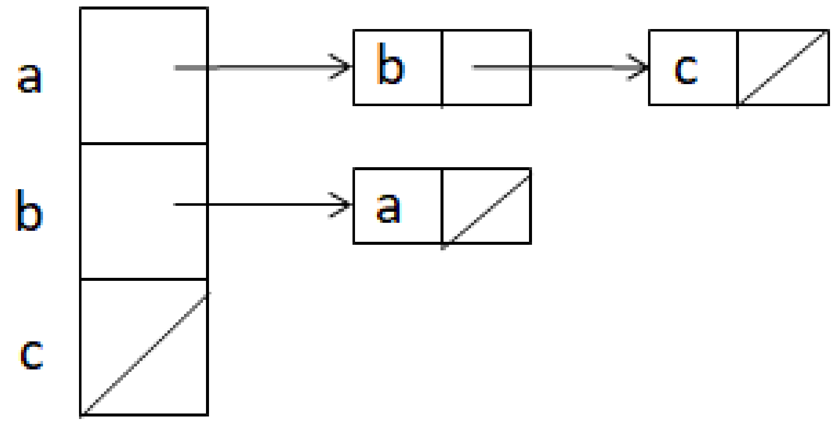
- Represent the following graph in adjacency matrix and adjacency list.
  - Again, do not see the answer immediately. Try to solve it first.



Solution

	a	b	c
a	0	1	1
b	1	0	0
c	0	0	0

Adjacency Matrix



Adjacency List

<<< Course Instruction >>>

- This lesson is over.
  - If you have any questions, please contact your instructor.
- When you are done, study the next lecture (week_1_4.ppt) on the Canvas.