Exercise 2

Keating (2002) studied the annual numbers of females with cubs-of-the-year in the Yellowstone grizzly bear population, from 1986 to 2001. Shown in the next Table is the number of unique females with cubs-of-the-year that were seen exactly j times during the year 1998.

Table 1: Sights of unique females with cubs-of-the-year (1998).

Sights	1	2	3	4	5	6	7
No. of bears (frequency)	11	13	5	1	1	0	2

This is a multiple capture-recapture problem where the main objective is to know the total number of females.

1. Use Chao's estimator to estimate the total number of females. You can find many information about Chao's estimator in internet. For instance:

   `https://www.idescat.cat/sort/sort411/41.1.1.chao-colwell.pdf`

   or here:

   `https://onlinelibrary.wiley.com/doi/full/10.1111/sjos.12293`

2. A random variable X follows a zero-truncated Poisson distribution when its probability function is,

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{(1 - e^{-\lambda})k!}, \ k = 1, 2, ...$$

   Assuming that the counts shown in Table 1 follow a zero-truncated Poisson distribution, estimate the parameter λ of this distribution (for instance, by maximum likelihood or by the method of the moments). Then, estimate the total number of females using the Horvitz-Thompson estimator,

$$\hat{N} = \frac{n}{1 - e^{-\hat{\lambda}}},$$

where n is the total number of observed females. Use parametric bootstrap to compute a confidence interval of the total number of females N. Compare the results with those obtained with Chao's estimator.