Economic, Fiscal- or Monetary-policy Uncertainty Shocks: What Matters for a Small Open Economy?*

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Abstract

How much do uncertainty shocks contribute to the business cycle fluctuations of a small open economy? Using a Bayesian-estimated structural model, we decompose and quantify which time-varying risk—in domestic demand or supply conditions, in domestic monetary or fiscal policy, or, in international economic and policy spillovers factors—matter for a small open economy like Canada. Our results suggest that the historical movements in Canadian real GDP is due largely to domestic fiscal- and monetary-policy shocks, and, due to non-negligible time variations in the riskiness of these policy shocks.

Keywords: Uncertainty Shocks; Stochastic Volatility; Small Open Economy; Internal vs. External

JEL Classification: D52, E52, E62, F41, F44

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1 Introduction

There has been a recent surge of interest in identifying and measuring time-variations in the riskiness of underlying shocks to a macroeconomy (Bloom et al., 2007; Bloom, 2009; Born and Pfeifer, 2014; Caggiano et al., 2014; Fernández-Villaverde et al., 2015; Baker et al., 2016). The notion of time variation in the riskiness or volatility of shocks is commonly referred to as *uncertainty shocks* in the literature. However, the analyses in this literature tend to focus on the U.S. economy and are silent on the dimension of international economic and policy spillovers.

Purpose and contribution. For small open economies, policy makers are often concerned about external versus internal sources of shocks and uncertainties, and whether their conduct of policy itself may contribute to economic uncertainty. We seek to address this policy-relevant question from the point of view of an estimated model. To the best of our knowledge, our open-economy contribution to the structural macroeconometric uncertainty shocks literature is a first.

In this paper, we ask: How much do uncertainty shocks contribute to the business cycle fluctuations of a small open economy? In particular, we decompose and quantify which time-varying risk—in domestic demand or supply conditions, in domestic monetary or fiscal policy, or, in international economic and policy spillovers factors—matter for a small open economy.

The empirical literature on macroeconomic and policy uncertainty shocks go about measuring this notion in two ways. In the first method, one may interpret economic uncertainty shocks from the perspective of statistical measures of exogenous economic and/or policy volatility constructed directly from observed data (Caggiano et al., 2014; Baker et al., 2016).[1] In the second, one may take a slight more structured approach from the point of view of an economic model and treat as "structural" the statistical processes governing time-varying riskiness in economic and policy shocks (Bloom et al., 2007; Bloom, 2009; Born and Pfeifer, 2014; Fernández-Villaverde et al., 2015).

We take the second approach of posing an explicit theoretical interpretation for what is a demand, a supply, or a fiscal- and a monetary-policy shock—and their distributional heteroskedasticity processes—as part of an equilibrium theory. This allows us to avoid well-known problems of weak identification of impulse dynamics in less theoretically-constrained statistical models (see, e.g., Yao et al., 2017, and the cited references therein). By construction, we will have well-defined (or identified) notions of domestic-versus-foreign economic and policy shocks. We will use Canada as an example small open economy in our analysis.

Results. Our main results are as follows. First, we identify considerable time-varying volatilities in domestic technology, fiscal-policy and monetary-policy shocks. The same holds for the exogenous block representing the foreign economy, in terms of foreign inflation, output-growth, and nominal interest rate shocks. Specifically, we find that there has been a broad decline in economic

[1] Commonly used measures are the Chicago Board Options Exchange Market Volatility Index (i.e., the VIX) and the Economic Policy Uncertainty (EPU) index of Baker et al. (2016).

and policy shock uncertainty for both the Canadian (domestic) and our representation of the U.S. (foreign) economy. However, there has been a noticeable increase in our identified monetary- and fiscal-policy shocks' uncertanties leading up to and after the Great Recession period (2008-2013). Second, we find that a substantial degree of output fluctuation in the Canadian economy can be attributed to domestic monetary policy shocks (around 85 percent). International spillovers playing a smaller role (around 15 percent). When we further decompose the sources of these shocks to uncover the component due to time-varying volatility of the shocks, we find that uncertainty accounts for a significant proportion of output fluctuations (around 13 percent), in which the primary contributor is domestic monetary policy shock uncertainty (around nine percent).

Related literature. In terms of empirical methods, the paper closest to ours is Justiniano and Primiceri (2008). The authors consider a medium-scale DSGE framework (see, e.g., Del Negro et al., 2007; Smets and Wouters, 2003; Christiano et al., 2005) and augment the structural shocks with time-varying volatility in the distributions of the shocks. Using this framework to explore the potential causes of the Great Moderation, they conclude that reductions in the volatility of investment-specific technological shocks were a key driver of the reduction in real GDP volatility (see also, Fernández-Villaverde and Rubio-Ramírez, 2007; Bloom et al., 2007, for similar conclusions).[2]

More recently, research has moved beyond the role of the investment channel by investigating the role of domestic (monetary and fiscal) policy in shaping the business cycle. For instance Mumtaz and Zanetti (2013) find that monetary-policy uncertainty has a negligible effect on real GDP (approximately 0.15%). A similar result which also encompasses the effects of fiscal-policy uncertainty is found by Born and Pfeifer (2014). Interestingly, when exploring a zero lower bound environment, Fernández-Villaverde et al. (2015) find that fiscal uncertainty shocks may decrease real GDP by around 1.5%. Our work here complements this literature by asking how important are time-varying riskiness in fiscal- and monetary-policy shocks for a small open economy, and how

[2] In following Justiniano and Primiceri (2008), the shock-uncertainty (i.e., stochastic-volatility) components can be estimated along with the rest of the structural model using full-information Bayesian methods on a conditionally linear Gaussian state-space representation. However, to facilitate this computationally tractable method, one trades-off with accuracy of model solution and likelihood approximation. In our approach, as in Justiniano and Primiceri (2008), we solve the model to first-order accuracy by a standard perturbation method. We then approximate the originally non-linear and non-Gaussian conditional density of structural shocks by a mixture of Gaussian processes (Kim et al., 1998). Alternatively, one may prefer to trade-off estimation and computational speed in return for model accuracy: This can be done by solving the model using higher-order approximations of equilibrium policies, evaluating the non-Gaussian data likelihood of a resulting non-linear state-space representation and constructing the model's posterior density by sequential Monte Carlo. This latter method is costly, and as a result, researchers tend to use an incomplete-information approach to estimate the model (see, e.g., Fernández-Villaverde et al., 2011; Born and Pfeifer, 2014; Fernández-Villaverde et al., 2015): The authors would separately estimate the stochastic volatility (SV) processes, and then estimate the rest of the structural parameters of the DSGE conditioning on the estimated (SV) block. The second stage estimation is usually done using a partial-information method of simulated moments.

In our application, we think there is not much lost in terms of accuracy, since the decision problems faced by agents in our small open economy example is away —theoretically and in the observed Canadian data—from crucial sources of nonlinear dynamics like the zero lower bound on nominal interest. Hence, we conduct our analyses using the methods similar to Justiniano and Primiceri (2008).

they vary over the recent history of a small open economy.

Our research is also related to recent research on the effects of international spillovers. For example, Mumtaz and Theodoridis (2017) employ a dynamic factor model with stochastic volatility to show that cross-country uncertainty spillovers have real effects among eleven OECD countries. Faccini et al. (2016) show that U.S. government spending has a significant spillover effect on its major trading partners. Our research complements both of these papers from the point of view of a small open economy.

Finally, our paper is also closely related to the literature on monetary policy evaluation using structural small open-economy models. For instance Lubik and Schorfheide (2007) consider whether the monetary authorities of four small open economies—Australia, Canada, New Zealand and the UK—respond to variations in nominal exchange rates. These behavioral responses are expanded upon by Kam et al. (2009) who identify institutionally-defined policy preferences to comment on the similarities and differences of policy design across Australia, Canada and New Zealand. Finally, a similar analysis is conducted by Justiniano and Preston (2010b) who explore the optimal monetary policy design of the same three small open economies when the policy maker is faced with parameter uncertainty. The difference between parameter uncertainty and macroeconomic uncertainty is as follows: The former refers to the degree of spread or imprecision in the 95% credible interval of the estimated parameters posterior distribution, whereas the latter refers to time variation in volatilities of the distributions of structural shocks. Thus, in addition to allowing for parameter uncertainty, our framework offers an additional dimension of uncertainty through which small-open-economy fiscal- and monetary-policy behavior can be empirically assessed.

The outline of the paper is as follows. Section 2 and 4 respectively explain the model and it's associated solution method. Section 5 outlines the Bayesian estimation procedure. Section 6 presents the results and Section 7 concludes.

2 Model

The small-open-economy model is a version of Alonso-Carrera and Kam (2016), which in turn, is an incomplete-markets generalization of the complete-markets model of Gali and Monacelli (2005) or Justiniano and Preston (2010a). We extend this model to include fiscal policy, physical capital and stochastic volatility in economic and policy shocks.

2.1 Representative household

The small open economy is populated by a continuum of identical households. Following McCallum and Nelson (1999) and Benigno and Thoenissen (2008), each household has access to a pair of non-state-contingent domestic and foreign money bonds, denoted B_t and B_t^*, which are respectively denominated in Home and Foreign currency. More precisely, if $\boldsymbol{s}_t$ denotes the $t-$history of aggregate macroeconomic shocks, then $B_{t+1}(\boldsymbol{s}_t)$ and $B^*_{t+1}(\boldsymbol{s}_t)$ respectively denote currency specific unit claims (e.g., one dollar) conditional on $\boldsymbol{s}_t$.[3] Thus, letting r_t and r^*_t respectively denote the domestic and foreign nominal interest rates, the date t cost of each bond in domestic currency terms are given by $(1+r_t)^{-1}$ and $S_t(\boldsymbol{s}_t)\left(1+r^*_t\right)^{-1}$, where $S_t(\boldsymbol{s}_t)$ is the nominal exchange rate expressed as domestic currency per unit of foreign currency. In what follows we reduce the notation on endogenous (random) variables by suppressing the arguments. More precisely, we define $\boldsymbol{X}_t := \boldsymbol{X}_t(s_t)$, where the function $\boldsymbol{s} \mapsto \boldsymbol{X}_t(\boldsymbol{s})$ is vector-valued, consisting of endogenous (i.e., equilibrium-determined) functions. For instance, $B^*_{t+1}(\boldsymbol{s}_t)$ is written more compactly as B^*_{t+1}.

Each household faces the budget constraint:

$$P_tC_t + P_tI_t + \frac{B_{t+1}}{R_t} + \frac{S_tB^*_{t+1}}{(1+r^*_t)} \leq (1-\tau_{W,t})W_tN_t + (1-\tau_{K,t})R_{k,t}K_t + P_t\tau_{K,t}\xi K_tB_t + S_tB^*_t + P_t\Theta_t, \quad (1)$$

where P_t is the domestic consumer price index, I_t is domestic investment in physical capital, C_t is a CES composite index of home and foreign produced consumption goods later defined in (9), R_t is the domestic (gross) nominal return on money holdings, $\tau_{W,t}$ is the marginal labor income tax rate, W_t is the per hour nominal wage rate, N_t is the number of hours of labor supplied , $\tau_{K,t}$ is the marginal capital income tax rate, K_t is the date t level of capital, $R_{k,t}$ is the per unit rental rate of capital, $\xi \in (0,1)$ is the capital depreciation rate and $\Theta \equiv \int_{[0,1]}\Theta(i)\mathrm{d}i$ is the total number of dividend payment received from ownership of all differentiated-product firms indexed by $i \in [0,1]$. The budget constraint (1) requires that the nominal value of consumption investment and new asset purchases must be feasibly financed by post-tax capital and labor income, current holdings of Home and Foreign money claims, and profits from firm ownership.

The introduction of physical capital into the model induces a law of motion for capital:

$$K_{t+1} = (1-\xi)K_t + I_t\left[1-\mathscr{D}\left(\frac{I_t}{I_{t-1}}\right)\right], \quad (2)$$

where we follow Fernández-Villaverde et al. (2015) and assume a convex capital adjustment cost of the form:

$$\mathscr{D}\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g_A)\right), \quad (3)$$

in which κ is the cost of capital adjustment parameter, $\mathscr{D}\left(g_A\right) = \mathscr{D}'\left(g_A\right) = 0$ and $\mathscr{D}''\left(g_A\right) = \kappa$.

[3] We will summarize what $\boldsymbol{s}_t$ comprises later in Section 3.

Household preferences are represented by the total discounted expected utility criterion:

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\delta_t U(C_t,N_t)\right\}, \qquad \delta_t := \begin{cases}\beta(C^a_{t-1}/A_{t-1})\delta_{t-1}, & \text{for } t>0\\ 1 & \text{for } t=0\end{cases} \tag{4}$$

where $\mathbb{E}_t := \mathbb{E}\{\cdot|\boldsymbol{s}_t\}$ is the linear expectations operator conditional on realized public information ($\boldsymbol{s}_t$) at the beginning of date t, A_t is realized total factor productivity (technology), C^a_t/A_t is (detrended) average consumption across households, and, δ_t is an endogenous discount factor.[4] We assume an additively separable utility function of the form:

$$U(C_t,N_t) := \frac{C_t^{1-\rho}}{1-\rho} + v(\tilde{G}_t) - \psi\left(A_t^{1-\rho}\right)\frac{N_t^{1+\varphi}}{1+\varphi},$$

where $\rho>0$ is the intertemporal elasticity of substitution, $\varphi>0$ is the inverse of the Frisch elasticity of labor supply, $\psi>0$ is a scale parameter $\tilde{G}_t$ is the (stationary) level of government spending and $v(\cdot)$ is an increasing, concave and bounded from above function. As in Fernández-Villaverde et al. (2015), the presence of technology in the utility function (i.e., A_t) ensures the existence of a balanced-growth path. Following Ferrero et al. (2010), the endogenous discount factor takes the following parametric form:

$$\beta(C^a_t/A_t) = \frac{\bar{\beta}}{1+\zeta\left[\ln\left(C^a_t/A_t\right)-\vartheta\right)]}; \qquad \bar{\beta}\in(0,1). \tag{5}$$

We will parametrize $\varphi>0$ and ζ so that the endogenous discount factor has a negligible effect on the dynamics of the model, but they will matter enough to ensure the existence of a unique nonstochastic steady-state equilibrium.

The representative household chooses an optimal plan $\{C_t,N_t,B_t,B^*_t\}_{t\in\mathbb{N}}$ to maximize (4) subject to (1), taking the average level of consumption, nominal prices, policy rates and initial bonds holdings—i.e., $\{C^a_t,P_t,W_t,S_t,r_t,r^*_t\}_{t\in\mathbb{N}}, B_0$ and B^*_0—as given. The first-order conditions of this problem at each date t and state $\boldsymbol{s}_t$ are characterized by the functionals:

$$A_t^{1-\rho}\psi N_t^{\varphi}C_t^{\rho} = (1-\tau_{W,t})\frac{W_t}{P_t}, \tag{6}$$

$$C_t^{-\rho} = R_t\mathbb{E}_t\left\{\beta\left(C^a_t/A_t\right)\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\rho}\right\}, \tag{7}$$

$$C_t^{-\rho} = \left(1+r^*_t\right)\mathbb{E}_t\left\{\beta\left(C^a_t/A_t\right)\left(\frac{P^*_t Q_{t+1}}{P^*_{t+1}Q_t}\right)C_{t+1}^{-\rho}\right\}, \tag{8}$$

[4]The Uzawa (1968)-style endogenous discount factor function, $\beta:\mathbb{R}_+\to(0,1)$, ensures that the model exhibits a unique non-stochastic steady state in the presence of incomplete markets and international borrowing and lending. For a survey on different approaches to introducing a deterministic steady state into small open economy models see Schmitt-Grohé and Uribe (2003).

where the aggregate level of consumption is a CES composite index of home and foreign produced consumption goods:

$$C_t = \left[(1-\gamma)^{1/\eta} \left(C_{H,t}^{\frac{\eta-1}{\eta}} \right) + \gamma^{1/\eta} \left(C_{F,t}^{\frac{\eta-1}{\eta}} \right) \right]^{\frac{\eta}{\eta-1}}, \qquad \gamma \in (0,1), \tag{9}$$

in which $\eta > 0$ is the elasticity of substitution between home and foreign goods. Furthermore, these Home and Foreign index goods are Dixit-Stiglitz aggregates over a continuum of differentiated varieties:

$$C_{n,t} = \left[\int_{[0,1]} [C_{n,t}(i)]^{\frac{\epsilon_n - 1}{\epsilon_n}} \mathrm{d}i \right]^{\frac{\epsilon_n}{\epsilon_n - 1}},$$

where $n \in \{H, F\}$ and $\epsilon_n > 1$ is the elasticity of substitution between types of differentiated domestic or foreign goods. Thus, at a given date t, the household faces an associated an expenditure minimization problem, in which they are required to choose varieties of Home and Foreign goods, conditional on their current prices $P_{H,t}$ and $P_{F,t}$. Solving this problem, the optimal consumption demand of each type of good is:

$$C_{H,t} = (1-\gamma) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \qquad C_{F,t} = \gamma \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t,$$

where substitution of these demand functions into (9) yields the consumer price index:

$$P_t = \left[(1-\gamma) P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{10}$$

Finally, given choices $C_{H,t}$ and $C_{F,t}$, the household chooses varieties $C_{n,t}(i)$, conditional on prices $P_{n,t}(i)$, to minimize the expenditure function:

$$\int_{[0,1]} P_{n,t}(i) C_{n,t}(i) \mathrm{d}i + P_{n,t} \left\{ C_{n,t} - \left[\int_{[0,1]} [C_{n,t}(i)]^{\frac{\epsilon_n - 1}{\epsilon_n}} \mathrm{d}i \right]^{\frac{\epsilon_n}{\epsilon_n - 1}} \right\}.$$

Solving this problem results in the demand functions and associated aggregate price levels:

$$C_{n,t}(i) = \left(\frac{P_{n,t}(i)}{P_{n,t}} \right)^{-\epsilon_n} C_{n,t}, \tag{11}$$

$$P_{n,t} = \left(\int_0^1 P_{n,t}(i)^{1-\epsilon_n} di \right)^{\frac{1}{1-\epsilon_n}}. \tag{12}$$

for all $i \in [0,1]$ and $n \in \{H, F\}$.

2.2 Firm

As in Gali and Monacelli (2005), the production side of the economy consists of a continuum of retail firms $i \in [0,1]$, each of whom produce a differentiated product which is sold to the domestic government as well as domestic and foreign households, according to the demand schedule:

$$Y_{H,t+s}(i) = \left(\frac{P_{H,t+s}(i)}{P_{H,t+s}}\right)^{-\epsilon_H} Y_{H,t+s}, \qquad Y_{H,t+s} := C_{H,t+s} + I_{H,t} + G_{H,t+s} + C^*_{H,t+s}, \tag{13}$$

where $P_{H,t}$ is the domestic-goods producer price index, $Y_{H,t}$ is the aggregate level of domestic production, $C_{H,t}$, $I_{H,t}$ and $C^*_{H,t}$ respectively denote total levels of domestic and foreign consumption expenditure on home produced goods and expenditure, investment on domestic capital and $G_{H,t}$ is to total level of government expenditure at any $t, s \in \mathbb{N}$. For simplicity, we assume that the Home government only consumes Home goods. The production technology is (constant returns to scale) Cobb-Douglas:

$$Y_{H,t}(i) = [A_t N_t(i)]^{(1-\alpha)} [K_t(i)]^{(\alpha)}, \tag{14}$$

where $\alpha \in (0,1)$, A_t is a labor-augmenting productivity term (later defined in (29)), $N_t(i)$ is the labor input and $K_t(i)$ is the capital input. Cost minimization with respect to capital and labor inputs implies that, in equilibrium, all intermediate good producing firms have the same capital-to-labor ratio and the same marginal cost of production:

$$MC_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{W_t^{1-\alpha} R_{K,t}^{\alpha}}{A_t^{1-\alpha}}, \tag{15}$$

where MC_t is the nominal marginal cost (i.e., the shadow value of, or Lagrange multiplier on, the firm's technology constraint).

Since firms compete in monopolistically competitive environment, they must also decide the price to charge for their variety of good. Following Rotemberg (1982), we assume that each firm faces a convex price-adjustment cost:

$$AC\left(\frac{P_{H,t+s}(i)}{P_{H,t+s-1}(i)}, Y_{H,t+s}(i)\right) := \frac{\varpi}{2}\left(\frac{P_{H,t+s}(i)}{P_{H,t+s-\Pi}(i)} - \Pi\right)^2 \times Y_{H,t+s}(i),$$

where Π is gross CPI inflation along a deterministic balanced-growth path (i.e., the monetary authority's inflation target). The parameter ϖ controls the degree of price stickiness. If $\varpi = 0$ then prices are fully flexible. Thus, a larger ϖ, implies more stickiness in pricing.

The decision problem for each firm $i \in [0,1]$ is given by:

$$\Theta_t(i) = \max_{\{P_{H,t+s}(i)\}_{s\in\mathbb{N}}} \left\{ \mathbb{E}_t \sum_{s=0}^{\infty} \mathscr{D}_t \left[\frac{P_{H,t+s}(i)}{P_{H,t+s}} Y_{H,t+s}(i) - \frac{W_{t+s}}{P_{H,t+s}} N_{t+s}(i) \right.\right.$$
$$\left.\left. - AC\left(\frac{P_{H,t+s}(i)}{P_{H,t+s-1}(i)}, Y_{H,t+s}(i) \right) \right] : (13), (14). \text{ and } (15) \right\},$$

where $\mathscr{D}_t = \delta_{t+s} \frac{U_C(C_{t+s},N_{t+s})}{U_C(C_t,N_t)}$ is the stochastic discount factor. The first-order conditions for this problem at every date $t \in \mathbb{N}$ and state $\boldsymbol{s}_t$, imply that firm i's optimal pricing strategy satisfies:

$$\begin{aligned} 0 = (1-\epsilon_H)\frac{Y_{H,t}(i)}{P_{H,t}} - \frac{MC_t}{P_{H,t}}\frac{\partial Y_{H,t}(i)}{\partial P_{H,t}(i)} - \frac{\partial AC\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}, Y_{H,t}(i)\right)}{\partial P_{H,t}(i)} \\ - \beta(C_t^a/A_t)\mathbb{E}_t\left\{ \frac{U_C(C_{t+1},N_{t+1})}{U_C(C_t,N_t)} \frac{\partial AC\left(\frac{P_{H,t+1}(i)}{P_{H,t}(i)}, Y_{H,t+1}(i)\right)}{\partial P_{H,t}(i)} \right\}, \end{aligned} \tag{16}$$

where

$$\frac{\partial Y_{H,t}(i)}{\partial P_{H,t}(i)} = -\epsilon_H \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon_H - 1} \left(\frac{1}{P_{H,t}} \right) Y_{H,t},$$

and,

$$\begin{aligned} \frac{\partial AC\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}, Y_{H,t}(i)\right)}{\partial P_{H,t}(i)} = \frac{\varpi}{2} AC\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i), Y_{H,t}(i)} \right) \frac{1}{Y_{H,t}(i)} \frac{\partial Y_{H,t}(i)}{\partial P_{H,t}(i)} \\ - \varpi \left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1 \right) \frac{1}{P_{H,t-1}(i)} Y_{H,t}(i) \end{aligned}.$$

The first term on the RHS of (16) is the current real marginal revenue to the firm with respect to its own price variation. The second term is the real marginal cost associated with the marginal variation in labor hiring, as a consequence of the pricing variation's effect on the demand for firm i's output. The third and fourth term, respectively, give current and (expected) future marginal effects of the pricing strategy variation on the firm's profit via the price-adjustment cost terms.

2.3 Market clearing

There are three types of Walrasian markets in our environment: A continuum of domestic labor markets, a continuum of internationally traded goods market, and the international asset markets trading in non-state-contingent money claims. We consider each in turn.

First, under the assumption that labor is immobile across countries, the domestic labor market has to clear in a competitive equilibrium. Equating labor supply (6) and demand (15) gives:

$$A_t^{1-\rho}\psi N_t^{\varphi} C_t^{\rho} = (1-\tau_{W,t}) mc_{H,t} A_t p_{H,t} \tag{17}$$

where we have defined $p_{H,t} = \frac{P_{H,t}}{P_t}$ and $mc_{H,t} = \frac{MC_t}{P_{H,t}}$.

Second, goods market clearing for each variety of good $i \in [0,1]$, accounting for the resource cost of price adjustments, yields the condition:

$$\begin{aligned}
\left[1 - \frac{\varpi}{2}\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - \Pi\right)^2\right] Y_{H,t}(i) &= C_{H,t}(i) + I_{H,t}(i) + G_{H,t}(i) + C^*_{H,t}(i) \\
&= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_H} Y_{H,t} \\
&= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_H} \left[(1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}(C_t + I_t + G_t)\right. \\
&\qquad \left. + \gamma\left(\frac{P_{H,t}(i)}{S_t P^*_{F,t}}\right)^{-\epsilon_F}\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t\right],
\end{aligned} \tag{18}$$

where the second equality utilizes the derived demand for good i, by Home households and government, and also by Foreigners. The third equality is derived from the demands for Home index goods by the same agents, which embed the assumption that Foreign and Home agents have symmetric preference representations.

Let us define an aggregate Home output index:

$$Y_t = \left[\int_0^1 Y_{H,t}^{(\epsilon_H-1)/\epsilon_H}(i)\mathrm{d}i\right]^{\epsilon_H/(\epsilon_H-1)}. \tag{19}$$

The aggregate goods market clearing condition is

$$\left[1 - \left(\Pi_{H,t} - \Pi\right)^2\right] Y_t = Y_{H,t} \equiv \left(p_{H,t}\right)^{-\eta}\left[(1-\gamma)(C_t + G_t) + \gamma Q_t^{\eta} C^*_t\right]. \tag{20}$$

2.4 Government behavior and policy shock processes

We close the model by describing monetary and fiscal policies as following simple policy rules. The monetary authority follows a conventional Taylor-type rule:

$$\frac{R_t}{R} = \frac{R_{t-1}}{R}^{\phi_R} \frac{\Pi_t}{\Pi}^{(1-\phi_R)\phi_\Pi} \frac{Y_t}{YA_t}^{(1-\phi_R)\phi_Y} \exp\left\{\sigma_{R,t}\varepsilon_{R,t}\right\}, \qquad \varepsilon_{\tau_W,t} \sim \mathcal{N}(0,1), \tag{21}$$

where $\phi_R \in [0,1)$ models the degree of interest-rate smoothing behavior, while $\phi_\Pi > 0$ and $\phi_Y \geq 0$ respectfully model the monetary authority's response to CPI inflation ($\Pi_t := P_t/P_{t-1}$) and contemporaneous output Y_t (to be defined later). Terms in the denominators without a subscript denote steady-state levels or rates of their respective variables in the numerator. As in Primiceri (2005), the structural shock $\varepsilon_{R,t}$ captures "non-systematic monetary policy", interpretable as "policy mistakes", as well as any policy actions that are left unexplained by the model. The time varying volatility term $\sigma_{R,t}$, thus allows for time-varying uncertainty in the likelihood of non-systematic monetary policy. For instance, institutional changes such as the change from monetary to inflation targeting may result in a lower value for $\sigma_{R,t}$.

For simplicity, we follow Fernández-Villaverde et al. (2015) and assume that the fiscal authority does not accumulate a stock of debt, so the government budget constraint is always balanced:

$$G_t = \tau_{W,t} \frac{W_t N_t}{P_{H,t}} + \tau_{K,t} \frac{R_{K,t} K_t}{P_{H,t}}. \tag{22}$$

Following Fernández-Villaverde et al. (2015), the (capital and labor) income tax rates are modeled as a mean-reverting tax smoothing rule:

$$\tau_{i,t} - \tau_i = \alpha_i \left(\tau_{i,t-1} - \tau_i\right) + \phi_{i,Y} \left(\frac{Y_t}{Y_{t-1}} - 1\right) + \exp\left\{\sigma_{\tau_i,t}\right\} \varepsilon_{\tau_i,t} \qquad \varepsilon_{\tau_i,t} \sim \mathcal{N}(0,1), \tag{23}$$

in which $i \in \{K, W\}$, τ_i is the steady state tax rate, $\alpha_i \in [0,1)$ is a stationary autoregressive coefficient and $\phi_{i,Y} > 0$ is a feedback effect from the current state of the business cycle. As was the case in the Taylor rule, $\varepsilon_{\tau_i,t}$ captures unanticipated changes to fiscal policy. This component can be thought of as exogenous political changes arising in the implementation of fiscal policy. The time-varying volatility term $\sigma_{\tau_i,t}$ specified in (34) captures riskiness in these non-systematic fiscal policy shocks.

2.5 Competitive equilibrium

Asset pricing. The requirement of zero profitable arbitrage in equilibrium is given by the equality between the Euler functionals (7) and (8). This can be rewritten as:

$$R_t \mathbb{E}_t \left\{ \beta\left(C_t^a / A_t\right) \left(\frac{P_t}{P_{t+1}}\right) C_{t+1}^{-\rho} \right\} = \mathbb{E}_t \left\{ \beta\left(C_t^a / A_t\right) \left(\frac{Q_{t+1}}{Q_t}\right) C_{t+1}^{-\rho} \tilde{R}_{t+1}^* \right\}, \tag{24}$$

which implies uncovered interest parity (UIP) condition. Since the process for $\tilde{R}_t^*$ is (exogenously) given as an AR(1)-SV model, this asset pricing condition will exhibit an exogenous time-varying risk component.

Phillips curve. We restrict attention to a symmetric equilibrium: All firms $i \in [0,1]$ will choose a pricing strategy such that at each date t and state $\boldsymbol{s}_t$, $P_{H,t}(i) = P_{H,t}$. Denote $\Pi_{H,t} := P_{H,t}/P_{H,t-1}$. After some algebra, the firms' optimal pricing condition (16) implies an equilibrium "Phillips curve" functional equation:

$$\begin{aligned} &\Pi_{H,t}(\Pi_{H,t} - \Pi) - \frac{\epsilon_H}{2}(\Pi_{H,t} - \Pi)^2 = \\ &\qquad \beta(C_t^a / A_t) \mathbb{E}_t \left\{ \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}} \left(\Pi_{H,t+1} - \Pi\right) \Pi_{H,t+1} \cdot \frac{Y_{H,t+1}}{Y_{H,t}} \right\} + \frac{\epsilon_H}{\varpi} \left[mc_{H,t} - \frac{\epsilon_H - 1}{\epsilon_H} \right]. \end{aligned} \tag{25}$$

This is an expectations-augmented Phillips curve. Also, the greater is the cost of prices adjustment, $\varpi \to \infty$, the gap between expected discounted next-period marginal (profit) value of inflation and current marginal value of inflation goes to zero. That is, prices are expected not to change very much (i.e., are not sensitive to real marginal cost deviations) the more costly is price adjustment. The greater is the elasticity of demand, $\epsilon_H \to +\infty$, the more positive and sensitive is the response of current inflation to real marginal cost (limiting case of perfect competition) deviation. Note that $(\epsilon_H - 1)/\epsilon_H$ is the inverse of a monopolist's static optimal markup, which depends on the firm's demand elasticity ϵ_H.

Useful identities. From the CPI index (10), we can derive the Home final goods price index relative to the CPI index as:

$$p_{H,t} := \frac{P_{H,t}}{P_t} = \left[\frac{1-\gamma(Q_t)^{1-\eta}}{1-\gamma}\right]^{\frac{1}{1-\eta}}, \tag{26}$$

where we have used the definitions $P_{F,t}/P_t = S_t P_t^*/P_t =: Q_t$. As a corollary, we have that $p_{H,t}/p_{H,t-1} = (P_{H,t}/P_{H,t-1})/(P_t/P_{t-1})$, which implies:

$$\Pi_t = \frac{\Pi_{H,t}}{p_{H,t}/p_{H,t-1}} = \Pi_{H,t} \times \left[\frac{1-\gamma(Q_{t-1})^{1-\eta}}{1-\gamma(Q_t)^{1-\eta}}\right]^{\frac{1}{1-\eta}}. \tag{27}$$

Aggregating (14) up, we have

$$Y_{H,t} = [A_t N_t]^{(1-\alpha)} [K_t]^{(\alpha)}. \tag{28}$$

Given these identities, a recursive competitive equilibrium is defined as follows:

Definition 1. Given policies (21) and (23), a *recursive competitive equilibrium* is a system of allocation functions $\boldsymbol{s}_t \mapsto (C_t, N_t, G_t, Y_{H,t}, mc_{H,t})(\boldsymbol{s}_t)$, and pricing functions $\boldsymbol{s}_t \mapsto (\Pi_{H,t}, p_{H,t}, \Pi_t, Q_t)(\boldsymbol{s}_t)$, such that:

1. Households optimize: (6), (7), (8), $C_t^a = C_t$, (26) and (27);
2. Firms optimize: (15), (25) and (28);
3. Markets clear (given agents optimize): (17) and (20);
4. Government budget constraint holds: (22);

and $\lim_{t\to\infty} \delta_t R_t \mathbb{E}_t \left\{C_{t+1}^{-\rho} \Pi_{t+1}^{-1} B_{t+1}\right\} = \lim_{t\to\infty} \delta_t \mathbb{E}_t \left\{C_{t+1}^{-\rho} Q_{t+1} \tilde{R}_{t+1}^* B_{t+1}^*\right\} = 0$, for each date $t \in \mathbb{N}$ and state $\boldsymbol{s}_t$.

Since the labor augmenting technology process A_t has a unit root, consumption, labor, government expenditure and output all evolve along the stochastic growth path. Thus, before solving the model, we first need to solve for the competitive equilibrium in terms of stationary allocation and pricing functions. To do so, we define stationary functions by taking the ratio $\tilde{X}_t = X_t/A_t$, where $X \in \{C, N, G, Y\}$. The characterization of Definition 1 in stationary terms is provided in the online appendix.

3 Exogenous Stochastic Processes

Domestic monetary- and fiscal-policy shock. We have already alluded to two sources of exogenous structural shocks acting through domestic monetary and fiscal policy, respectively, in (21) and (23). Their statistical model will be given below. Before getting there, we will complete the description of the rest of the exogenous shock processes that shift the model economy.

Domestic technology shock. The law of motion for labor-augmenting technology A_t is a mean reverting process:

$$\ln\left(g_{A,t}\right) = \left(1-\rho_A\right) g_A + \rho_A \ln\left(g_{A,t}\right) + \sigma_{A,t}\varepsilon_{A,t}, \qquad \varepsilon_{A,t} \sim \mathcal{N}(0,1), \tag{29}$$

where $g_{A,t} := \frac{A_t}{A_{t-1}}$ is the gross growth rate of technology, g_A is the rate of growth rate along the balanced growth path, and $\rho_A \in (0,1)$ is an AR(1) coefficient. The fact that (29) has time varying volatility means that the degree of uncertainty about the future path of domestic economic growth is permitted to change over the course of the business cycle.

The rest of the world. We assume that the rest of the world can be modeled as the limit of a large closed economy. Thus, $C_t^* = Y_t^*$ is the rest of the world's output. The rest of the world is assumed to follow a recursively identified first-order VAR-SV process:

$$\mathbf{Z}_t^* = \beta \mathbf{Z}_{t-1}^* + \mathbf{w}_t, \qquad \mathbf{w}_t \sim \mathcal{N}\left(\mathbf{0}, \Sigma_{\mathbf{Z},t}\right) \tag{30}$$

where $\mathbf{Z}_t^* := [\Delta \ln\left(Y_t^*\right), \pi_t^*, i_t^*]'$, comprises (de-meaned) percentage growth in foreign real GDP, foreign inflation rate, and, foreign nominal interest rate. The stochastic variance-covariance matrix is defined by $\boldsymbol{\Sigma}_{\mathbf{Z},t} = \left(\mathbf{L}'\mathbf{F}_t^{-1}\mathbf{L}\right)^{-1}$, where

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ \sigma_{\pi^*,Y^*} & 1 & 0 \\ \sigma_{i^*,Y^*} & \sigma_{i^*,\pi^*} & 1 \end{bmatrix}, \text{ and, } \mathbf{F}_t = \begin{bmatrix} \sigma^2_{Y^*,t} & 0 & 0 \\ 0 & \sigma^2_{\pi^*,t} & 0 \\ 0 & 0 & \sigma^2_{i^*,t} \end{bmatrix}. \tag{31}$$

As is standard in the VAR literature, interest-rate (i.e., monetary policy) shocks are assumed to be independent of any other innovations, however the ordering of the non-policy block is somewhat arbitrary (see, e.g., Primiceri, 2005).[5]

[5] For robustness we ensured that the results are not subject to ordering effects.

3.1 Structural shocks and uncertainty shocks

Let $\tilde{\boldsymbol{u}}_t$ denote an 6×1 vector collecting all the policy and economic disturbances—i.e., the structural shocks:

$$\tilde{\boldsymbol{u}}_t = \boldsymbol{\Sigma}_t^{1/2} \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_6), \tag{32}$$

where $\mathbf{I}_6$ is a (6×6) identity matrix,

$$\boldsymbol{\Sigma}_t = \begin{bmatrix} \mathbf{F}_t & \mathbf{0}_{(3\times3)} \\ \mathbf{0}_{(3\times3)} & \mathbf{D}_t \end{bmatrix}, \text{ and, } \mathbf{D}_t = \begin{bmatrix} \sigma^2_{A,t} & 0 & 0 \\ 0 & \sigma^2_{R,t} & 0 \\ 0 & 0 & \sigma^2_{\tau_W,t} \end{bmatrix}. \tag{33}$$

Note that when we abuse notation and write $\boldsymbol{\Sigma}_t^{1/2}$, it is understood that $\boldsymbol{\Sigma}_t$ is a diagonal matrix. Each element of the stochastic volatilities, $\boldsymbol{\Sigma}_t$, evolves according to the stochastic process:

$$\log \sigma_{i,t} = \log \sigma_{i,t-1} + \nu_{i,t}, \qquad \nu_{i,t} \sim \mathcal{N}(0, \omega_i^2), \tag{34}$$

for $i \in I := \{A, R, \tau_W, Y^*, \pi^*, i^*\}$.[6] Denote the collection of variance parameters as $\boldsymbol{\omega} = \{\omega_i\}_{i \in I}$.

We can make two observations: First, the vector of states relevant to agent decisions is $\boldsymbol{s}_t = \left(B_t, B_t^*, K_{t+1}, A_t, \sigma_{\tau_{W,t}} \varepsilon_{\tau_W,t}, \sigma_{R,t}\varepsilon_{R,t}, \mathbf{Z}_t^*\right)$. Second, from (32) and (34), we can see that there are two sources of variations in structural shocks, $\tilde{\boldsymbol{u}}_t$. For each shock i, one source of innovation, $\varepsilon_{i,t} \in \boldsymbol{\varepsilon}_t$, is a Gaussian shock to the policy or economic variable itself—a *mean* structural shock. Another component of innovation, $\nu_{i,t}$, is *uncertainty shock*, which renders a permanent shock to the riskiness of the distribution of each $\tilde{u}_{i,t} \in \tilde{\boldsymbol{u}}_t$.

4 Solution Method and Observables

From a stationarized version of Definition 1, its implied deterministic steady-state equilibrium is computed by the steps listed in the online appendix. Following Justiniano and Primiceri (2008), the model's recursive competitive equilibrium conditions are approximated by a perturbation method which is accurate to first-order. We then use a standard rational expectations equilibrium (REE) algorithm to find the stable REE solution, represented as a conditionally linear and Gaussian state-space system, and map observed data to it as

$$\begin{aligned} \mathbf{x}_{t+1} &= \mathbf{A}_\theta \mathbf{x}_t + \mathbf{B}_\theta \tilde{\boldsymbol{u}}_t, \\ \mathbf{y}_t^o &= \mathbf{H}^o \mathbf{y}_t, \end{aligned} \tag{35}$$

[6] The reason for modeling the stochastic processes (34) in logarithms, is to ensure that the random levels of the standard deviations, $\sigma_{i,t}$, remain positive almost everywhere, except on measure-zero events.

where $\mathbf{x}_t$ is a vector of all endogenous variables in the system, $\mathbf{y}_t^o$ is a vector of observables, and, $\mathbf{H}^o$ is the linear observation equation. In our empirical application, the vector of observables is given by:

$$\mathbf{y}_t^o = (\Delta \log Y_t, \log R_t, \log \tau_{W,t}, \Delta \log Y_t^*, \pi_t^*, i_t^*),$$

where $\Delta \log X_t$ denotes the first difference $\log X_t - \log X_{t-1}$, Y_t denotes the level of real GDP per capita in the SOE, R_t is the domestic gross nominal interest rate, $\tau_{W,t}$ is the marginal labor tax rate, Y_t^* is the international level of real GDP per capita, π_t^* is the foreign inflation rate, and, i_t^* is the foreign interest rate. Data sources and transformations of each series used in the empirical analysis are provided in the online appendix.

In summary, the implied econometric model is jointly given by equations (29), (30), (31), (32), (33), (34), and, (35).

5 Bayesian Estimation

We now outline a method for evaluating the non-analytical posterior joint distribution of the implied econometric model given observed data.[7] To obtain posterior draws for the model's structural parameters $(\boldsymbol{\theta}, \boldsymbol{\omega}^2)$ and the time varying volatilities $(\{\boldsymbol{\Sigma}_t\}_{t=1}^T)$, our estimation procedure uses a four-step Metropolis-within-Gibbs algorithm. In the first step, given $\boldsymbol{\omega}^2$ and $\{\boldsymbol{\Sigma}_t\}_{t=1}^T$, the model's microeconomic parameters $\boldsymbol{\theta}$ are drawn (updated) through a random walk Metropolis-Hastings algorithm as in Schorfheide (2000). Next, the structural shocks $\{\tilde{\boldsymbol{u}}\}_{t=1}^T$ are simulated using the efficient disturbance smoother developed by Durbin and Koopman (2002). Drawing the time-varying volatilities requires the combination of two procedures: In the first step, we apply the auxiliary mixture sampler of Kim et al. (1998) to approximate the underlying non-linear, non-Gaussian state-space representation as a mixture of linear Gaussian models.[8] Following this, the volatilities $\boldsymbol{\Sigma}_t$ can then be sampled with standard linear Gaussian methods as in Carter and Kohn (1994). However, here we make use of an efficient algorithm by Chan and Hsiao (2014) which takes advantage of the fact that the precision matrices of the underlying state space model are both block-banded and sparse. Conditional on the above blocks, the posterior distributions of the remaining parameters, $(\boldsymbol{\omega}^2)$, have analytical Inverse Gamma density representations. This Gibbs-sampling with conditional blocking method is known to induce the correct posterior density of the model's parameters $(\boldsymbol{\theta}, \boldsymbol{\omega}^2)$ (see Stroud et al., 2003; Del Negro and Primiceri, 2015).

[7] This is similar to the method used in Justiniano and Primiceri (2008), with the exception of the penultimate step where we utilize a more efficient smoother to construct a sequence of stochastic volatilities conditional on other estimated blocks in the Gibbs sampler. More details are presented in Appendix C in the online appendix.

[8] The stochastic-volatility components of the model renders nonlinearity and non-Gaussian (χ^2) distributions in its state-space representation, which can be aproximated as a log-linear Gaussian mixture process.

5.1 Priors

The prior distributions, mean and standard deviations of each parameter in the DSGE-SV model are provided in Table 1. The associated posterior estimates will be discussed in the next section. The prior means and distributions of the domestic economies DSGE parameters follow from Justiniano and Preston (2010a). This includes setting $\epsilon_H = 8$ - which implies a steady state markup of 14 percent.[9]

Consistent with the literature, the Rotemberg price stickiness parameter is centered at a mean of 60, which translates to the standard Calvo price stickiness probability of 0.75 (e.g., Gali and Monacelli (2005)), while the steady state discount factor $\bar{\beta} = 0.99$ and the parameters in the endognenous discount factor function (5) are set to $\varphi = 1.27$ and $\vartheta = 1^{-6}$. Next, to the best of our knowledge there are no studies that estimate similar tax rules for the Canadian economy. For this reason, we set the prior means of the parameters equal to the posterior median values in Table 1 of Fernández-Villaverde et al. (2015). In practice we found that g_A was not well identified by the data. Following Fernández-Villaverde et al. (2015), we therefore set $g_A = 0.005$ which implies a steady state growth rate of technology to 2 percent per annum. Finally, the priors for the AR coefficients and covariance terms in the foreign VAR-SV model are respectively given by truncated and unresricted Gaussian priors. The prior means were obtained by estimating a VARSV model on US data and then taking the posterior means as our prior means in the DSGE-SV model.[10]

[9] To see this, note that the percentage markup is given by $\frac{\epsilon_H}{\epsilon_H - 1} - 1$.

[10] We note that the support of each parameter's density is in line with appropriate restrictions from the economic theory. For instance, the autoregressive term in the labor augmenting technology equation follows a beta distribution on the unit interval. This strategy is also applied in setting the priors for the foreign economies VAR-SV model. In that case, note that the Gaussian priors for the autoregressive terms in the VAR-SV model are truncated at ± 1.

Table 1: Prior and posterior densities for the baseline model with stochastic volatility

Parameter	Description[a]	Family[b]	Prior Mean	Prior Std.	Post. Mean[c]	Post. Std.[c]
ρ	Intertemporal ES	N	1.00	0.10	1.00	0.01
η	Elasticity H-F Goods	N	0.90	0.10	0.90	0.01
ϖ	Price-stickiness	N	60.00	0.32	60.00	0.10
ϕ_R	MP, Smoothing	B	0.90	0.10	0.88	0.01
ϕ_Π	MP, Inflation	N	1.80	0.10	1.80	0.01
ϕ_Y	MP, Output	N	0.25	0.10	0.24	0.01
ϕ_W	FP Output (N)	N	0.04	0.10	0.03	0.01
α_W	FP, Smoothing (N)	B	0.99	0.10	0.99	0.01
ρ_A	TFP, Smoothing	B	0.60	0.10	0.56	0.01
κ	Capital Adjustment Stickiness	N	60.00	0.32	60	0.10
ϕ_K	FP Output (K)	N	0.043	0.10	0.04	0.01
α_K	FP, Smoothing (K)	B	0.98	0.10	0.98	0.01
$\rho_{(Y^*,Y^*)}$	VAR-SV, AR	TN	0.01	0.10	0.43	0.01
$\rho_{(Y^*,\pi^*)}$	VAR-SV, AR	TN	0	0.10	-0.09	0.01
$\rho_{(Y^*,i^*)}$	VAR-SV, AR	TN	0.00	0.10	0.00	0.01
$\rho_{(\pi^*,Y^*)}$	VAR-SV, AR	TN	0.05	0.10	0.05	0.01
$\rho_{(\pi^*,\pi^*)}$	VAR-SV, AR	TN	0.09	0.10	0.09	0.01
$\rho_{(\pi^*,i^*)}$	VAR-SV, AR	TN	0.00	0.10	0.00	0.01
$\rho_{(i^*,Y^*)}$	VAR-SV, AR	TN	0.00	0.10	0.00	0.01
$\rho_{(i^*,\pi^*)}$	VAR-SV, AR	TN	0.04	0.10	0.04	0.01
$\rho_{(i^*,i^*)}$	VAR-SV, AR	TN	0.13	0.10	0.13	0.01
$\sigma_{(\pi^*,Y^*)}$	VAR-SV, SV	N	-0.01	0.10	-0.01	0.01
$\sigma_{(i^*,Y^*)}$	VAR-SV, SV	N	-0.03	0.10	-0.03	0.01
$\sigma_{(i^*,\pi^*)}$	VAR-SV, SV	N	0.03	0.10	0.03	0.01

[a] MP (or FP) stands for Monetary (or Fiscal) Policy rule. TFP denotes Total Factor Productivity. AR denotes autoregressive coefficient and COV denotes covariance.

[b] B stands for Beta, N Normal and TN Truncated Normal.

[c] Posterior moments are generated from a thinned sample of 1,000,000 MCMC draws in which we save 1 in 50 draws after a 50,000 draw burn-in. Convergence diagnostics are presented in presented in Appendix D in the online appendix.

6 Estimation Results

We will address our main question on the accounting of macroeconomic and policy shock uncertainty on a small open economy's real GDP growth in Section 6.3 below. Readers interested in the economic conclusions may go directly there from here. Otherwise, we will discuss the model estimation results here.

In Section 6.1 we first discuss the structural parameter estimates, and then we comment on the stochastic-volatility estimates in Section 6.2. In Section 6.3, we then takes up our primary research question by way of historical decompositions (i.e., an accounting of sources of volatility shocks) over the sample period.

6.1 Structural Parameter Estimates

The posterior mean and standard-deviation statistics of of the estimated parameters are reported alongside their corresponding prior densities' statistics in Table 1. We have tested to ensure that the posteriors plotted in Figure **??** represent an ergodic distribution of the parameter Markov chain (induced by our Metropolis-within-Gibbs sampler). In the interest of brevity, these convergence diagnostics are summarized in our online appendix.

6.2 Behavior of Shock Uncertainties

Now we examine the economic significance of allowing for time varying volatilities in each of the shocks. Figure 1 displays the posterior mean of the estimated stochastic volatilities over the sample period.

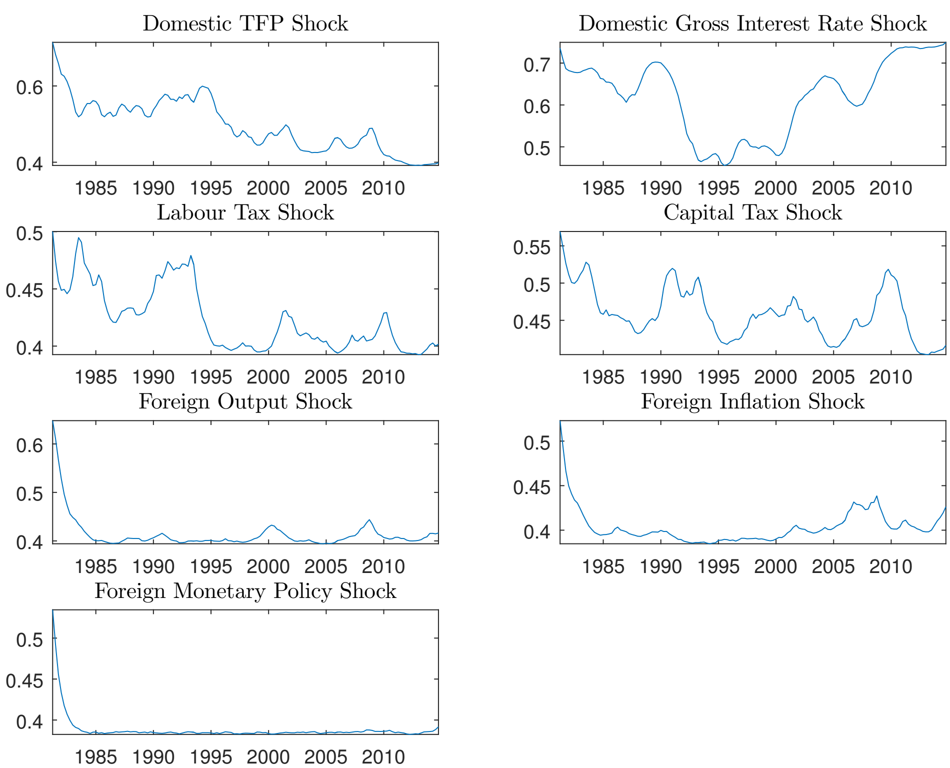

Figure 1: Estimated Uncertainty: $(\sigma_{i,t})_{i \in \{A,R,\tau_W,Y^*,i^*,\pi^*\}}$

The initial decline in volatility across most variables following 1980 is consistent with both

DSGE and VAR evidence on the Great moderation period (Primiceri, 2005; Justiniano and Primiceri, 2008). The results for the US shocks differ from those found in VAR studies (Primiceri, 2005; Mumtaz and Zanetti, 2013). Aside from the difference in sample size, one possible explanation for these differences is the fact that both of these studies utilize a time-varying-coefficient VAR-SV model, compared to our DSGE model which implies a constant-coefficient VAR-SV reduced form. We note that the choice of a VAR-SV for the foreign economy is in line with the wider literature. For instance, Primiceri (2005) shows that the posterior model probabilities are highest for a VAR with tight prior variance on the time-varying coefficients. This result is corroborated by Chan and Eisenstat (2017) who develop importance sampling based model comparison methods for time varying VARs. Their show that the VAR-SV model outperforms its various model counterparts - e.g. a VAR-SV with time-varying-coefficient and constant VAR.

With respect to the domestic economy, there is a steady decline in Canadian output volatility, however noticeable spikes do occur around the crisis periods of 2000 and 2008. The domestic gross interest rate volatility is more interesting. After the initial decline, there are then three distinct changes in the volatility trend. The spike and subsequent reduction around 1990 coincides with the adoption of inflation targeting in 1991, while the subsequent spikes in 2000 and 2008 correspond with the two US recessions. While Canada did not go through a recession in 2000, the increase in domestic gross interest rate volatility is in line with the strong bank rate movements at the time. Finally, the fluctuations in labor tax volatility are similar to those seen in the domestic output volatility. One difference is the large spike in capital tax volatility following the 2008 crisis.

6.3 Volatility Shock Accounting

We address the main question in the paper here by decomposing and quantifying which time-varying risk—in domestic demand or supply conditions, in domestic monetary or fiscal policy, or, in international economic and policy spillovers factors—matter for a small open economy.

To this end, historical decompositions of real GDP growth are constructed as follows: First, using the draws from the posterior distribution the model parameters, we obtain a simulated sample $\{\tilde{\boldsymbol{u}}_t\}_{t=2}^{T}$ of (compound) structural shocks—defined in (32) and (34)—which contains i.i.d. and stochastic-volatility components, for each type of shock $i \in I$.

Second, we simulate the path of real GDP growth, using the Wold representation of the VAR representation of the theoretical model; i.e., given $\mathbf{y}_1^o$, and $\{\tilde{\boldsymbol{u}}_t\}_{t=2}^{T}$, we have:

$$\mathbf{y}_t^o(i) = \mathbf{H}^o \left[(\mathbf{A}_\theta)^{t-1} \mathbf{y}_1 + \sum_{j=0}^{t-2} (\mathbf{A}_\theta)^j \mathbf{B}_\theta \mathbf{S}_i \tilde{\boldsymbol{u}}_{t-j} \right], \qquad \forall t > 1$$

where $\mathbf{S}_i$ selects a particular structural shock $\tilde{u}_{i,t-j} \in \tilde{\boldsymbol{u}}_{t-j}$.

Third, we construct historical decomposition statistics, $\left\{HD_{n,t}(i) : i \in I\right\}_{t=2}^{T}$, which account for the contribution to each date-t observable variable, $\mathbf{y}_{n,t}^o$, by the cumulative effect of the structural shock i, i.e., $\tilde{\boldsymbol{u}}_i$, from the beginning of the sample period until date t. The accounting formula for this is:

$$HD_{n,t}(i) := \frac{\left|\mathbf{y}^o_{n,t}(i)\right|}{\sum_{i\in I}\left|\mathbf{y}^o_{n,t}(i)\right|}. \tag{36}$$

Note that we define this statistic in terms of absolute sizes or magnitudes of contributions, since we are not *per se* interested in the directions or net effects of these shocks.

A rough cut. We plot the historical decomposition statistics defined by (36) for each combined structural shock $\tilde{u}_i$, $i \in I$, in Figure 2. (Note that this is a coarser analysis for now as each combined shock contains variations from an i.i.d. shocks component and an stochastic-volatility shock component.) Each shock i has a unique color-coded patch in the figue. From Figure 2, we can deduce that domestic shocks have contributed to majority of the output fluctuations over the sample period. On average, international spillovers have contributed to around 15 percent of domestic real GDP fluctuations, with the majority of contributions stemming from foreign monetary policy shocks - around 13.5 percent. In contrast, domestic policy shocks have contributed around 85 percent of all output fluctuations, with monetary policy being the predominant shock - around 70 percent. Interestingly, the contribution of foreign shocks seems to have diminished since the 1990s.

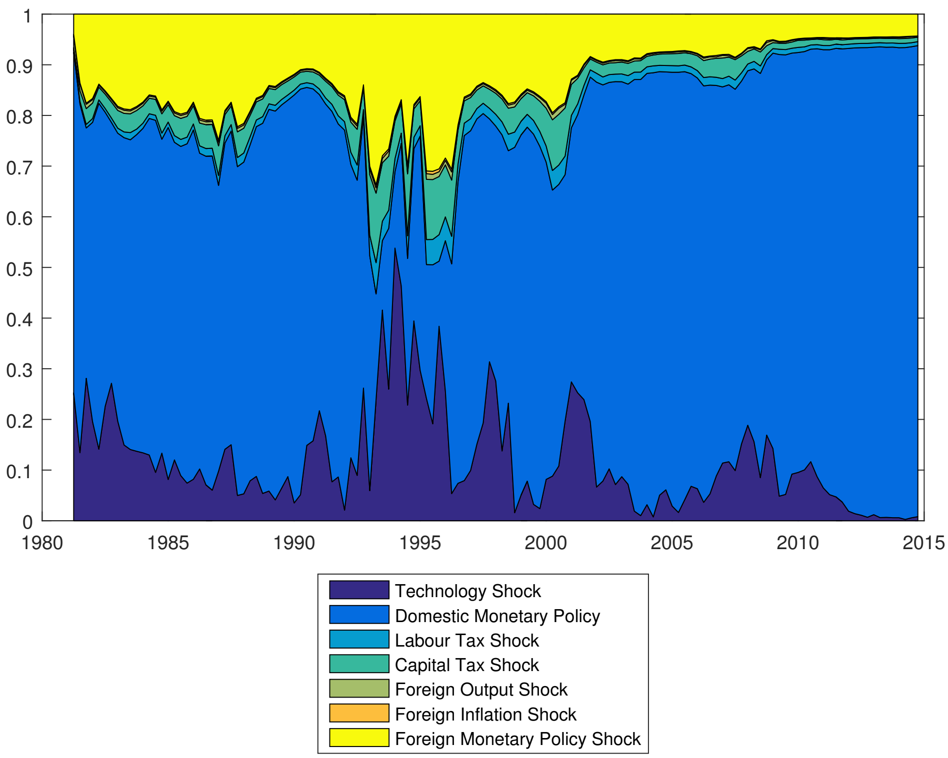

Figure 2: Historical Decomposition by Combined Structural Shocks ($\tilde{\boldsymbol{u}}$)

A finer cut. To analyze the relative contribution of uncertainty shocks, we further decompose the contribution of the two stochastic elements of the structural shocks $\tilde{\boldsymbol{u}}_t$—i.e., changes in the i.i.d. innovations ($\boldsymbol{\varepsilon}_t$) versus changes in the riskiness of structural shocks ($\boldsymbol{\Sigma}_t$). More precisely, given $\tilde{u}_{i,t}$ from $\tilde{\boldsymbol{u}}_t$, we can square the definition (32) and take logarithms on both sides to obtain:

$$\log(\tilde{u}_{i,t}^2) = \log(\sigma_{i,t}^2) + \log(\varepsilon_{i,t}^2). \tag{37}$$

We can then define the (absolute) proportion of $\log(\tilde{u}_{i,t}^2)$ explained by the stochastic volatility component as:

$$\xi_{i,t} = \frac{|\log(\sigma_{i,t}^2)|}{|\log(\tilde{u}_{i,t}^2)|}. \tag{38}$$

What this refined measure tell us is as follows: A larger magnitude of $\xi_{i,t}$ implies that the distributional riskiness of a particular shock i is more important in accounting for the variations in the compound structural shock i, i.e., $\tilde{u}_{i,t}$. (The complement of this statistic, which accounts for the share of the i.i.d. component, can be readily deduced as well.)

We present a refinement of the statistics (36) using (38), and its complement, for each shock i

across the sample periods t, in Figure 3. Figure 3 is basically Figure 2 decomposed further using the formula (38). Recall from Figure 2, we learned that monetary policy shocks account for most of the variations in the Canadian data's real GDP growth. Now, in Figure 3 we further deduce that such policy shocks are largely accounted for by their respective i.i.d. components, $\varepsilon_{R,t}$ and $\varepsilon_{\tau_W,t}$. However, there is also non-negligible contributions to real GDP growth by domestic output and foreign monetary policy-shock uncertainties.

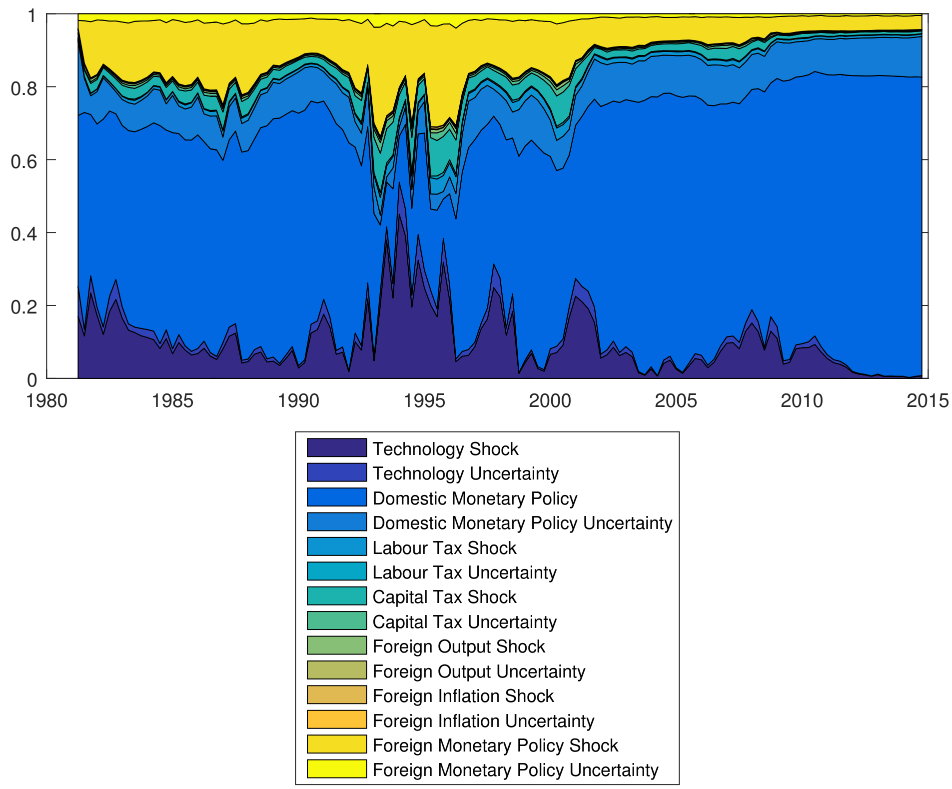

Figure 3: Historical Decomposition by Uncertainty Shocks ($\sigma_{i,t}$)

In Figure 4 we plot the contributions of the stochastic volatility to variations in Canadian real GDP directly. The contribution of uncertainty shocks, averaged over the data sample, is around 14 percent, with domestic monetary-policy-shock uncertainty explaining most of the effects - around 9 output growth percent. The next two largest uncertainty contributors are domestic output uncertainty - around two percent - and foreign monetary-policy-shock uncertainty - around one and a half percent. The relatively large domestic monetary-policy-shock uncertainty are are much larger than closed-economy studies on the US economy. For instance Mumtaz and Zanetti (2013) find that monetary-policy-shock uncertainty has a negligible effect on US real GDP (approximately 0.15 percent). A similar result which also encompasses the effects of fiscal-policy-shock uncertainty is found by Born and Pfeifer (2014). Interestingly, when exploring a zero lower bound environment, Fernández-Villaverde et al. (2015) find that fiscal uncertainty shocks may decrease real GDP by around 1.5 percent.

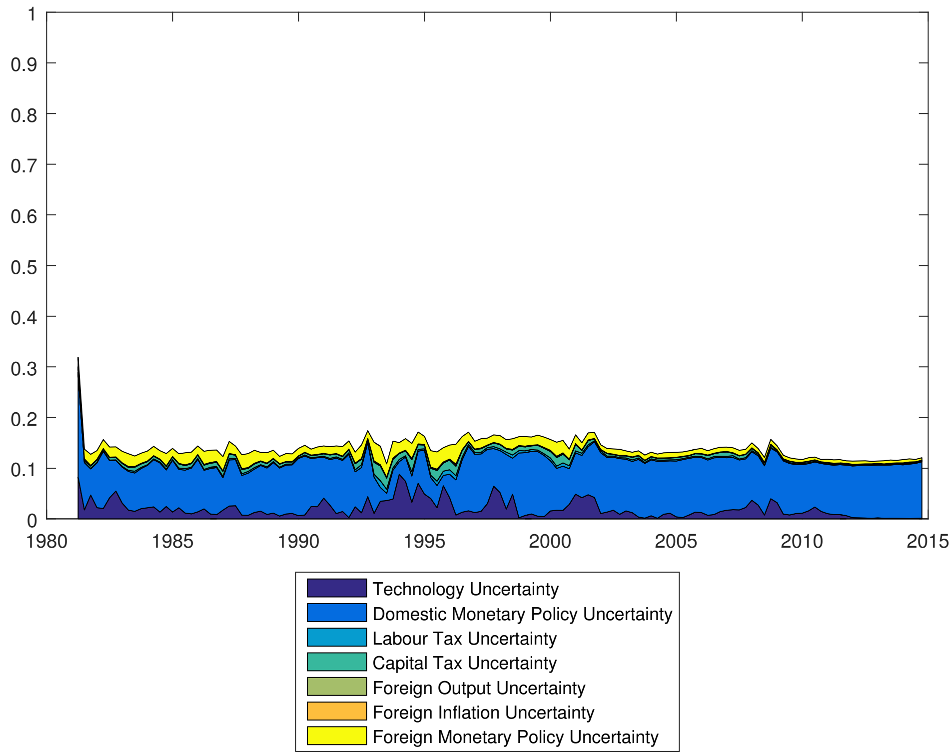

Figure 4: Historical Decomposition by IID ($\epsilon_{i,t}$) and Uncertainty Shocks ($\sigma_{i,t}$)

In summary, our results suggest that the historical movements in Canadian real GDP is due largely to domestic monetary-policy shocks, and, due to non-negligible time variations in the riskiness of such policy shocks.

7 Conclusion

In this paper, our main goal was to understand which of unexpected variations in international economic uncertainty, or domestic economic and policy uncertainties, are the more dominant drivers of a small open economy's business cycle.

To answer this question, we extended a version of a well-known small-open-economy DSGE model to allow for the volatilities of the structural disturbances to change over time. Using this model structure as a yardstick for interpretation and quantification, we identified and accounted for both domestic versus international, and, market-driven versus policy-driven, sources of uncertainty shocks.

First, we identify considerable time-varying volatilities in both domestic and foreign shock processes. Specifically, we find that there has been a broad decline in economic and policy shock uncertainty for both the Canadian (domestic) and our representation of the U.S. (foreign) economy. However, there has been a noticeable increase in our identified monetary policy shock uncertainties leading up to and after the Great Recession period.

Finally, in contrast to the closed-economy literature on the US economy, we found that uncertainty accounts for a significant proportion of output fluctuations (around 13 percent). The primary contributor is domestic monetary-policy-shock uncertainty (around 9 percent), with international uncertainty spillovers also a smaller, yet significant effect (around two percent). Taken together results highlight the importance of allowing for stochastic volatility when examining the Canada/US relationship.

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— SUPPLEMENTARY (ONLINE) APPENDIX —

Economic, Fiscal- or Monetary-policy Uncertainty Shocks: What Matters for a Small Open Economy?

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This document has public access: https://github.com/phantomachine/soerisky

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A Data

Domestic macroeconomic variables. Canadian real GDP and population data are sourced from the Canadian National Statistical Agency (Tables: 380-0064 and 051-0005 respectively). Real GDP per capita growth is calculated by first taking the ratio of real GDP to population, and then computing the log-difference of the resulting quarterly series. Nominal interest rate data is sourced from the FRED Database maintained by the St.Louis Fed (series IRSTCB01CAQ156N). We transform the series by converting to gross rates, taking the (natural) log and then subtracting the implied the long-run first moment. In addition to these observable series we also use data from World Bank to pin down consumption and government expenditure shares of output (Household final consumption expenditure and General government final consumption expenditure respectively (% of GDP 1980-2015.)).

Foreign macroeconomic variables. US real GDP per capita, GDP deflator and nominal interest rate data were sourced from the FRED Database maintained by the St.Louis Fed (series A939RX0Q048SBEA, GDPCTPI and FEDFUNDS respectively). Real GDP growth and inflation were respectively calculated as the log-difference of the real GDP and CPI series, while the monthly nominal interest rate was converted to a quarterly rate by geometric mean and dividing by 400. Before entering the model each series was demeaned by its implied long-run first moment.

Labor income tax rates. Following Born and Pfeifer (2014) and Fernández-Villaverde et al. (2015), our approach to calculating an average tax rates closely follows the works of Mendoza et al. (1994), Jones (2002), and Leeper et al. (2010). For completeness, we list the details of this two-step procedure. In the first instance, average personal income tax is computed as:

$$\tau_p = \frac{PIT}{WS + PI + RI + CP + NFI}, \tag{39}$$

where PIT denotes the level of (aggregate) personal income tax, WS is income from wage and salaries, PI is property income, RI is rental income, CP is corporation profits and NFI is income from non-farm entities. In our calculations we exclude property taxes due to a lack of available data. Next, given (39), the average labor tax rates are computed as:

$$\tau_N = \frac{\tau_p WS + SS}{CE}, \tag{40}$$

where SS is the total Social Security benefits and CE is the Compensation for Employees. Finally, given (39), the average capital tax rates are computed as:

$$\tau_K = \frac{\tau_p CI + CT}{CI}, \tag{41}$$

All data was sourced from the Canadian National Statistical Agency (Tables: 380-0072 and 380-0074). We note that the average tax rates enter the model in demeaned (natural) logs. The resulting series are plotted Figure 5.

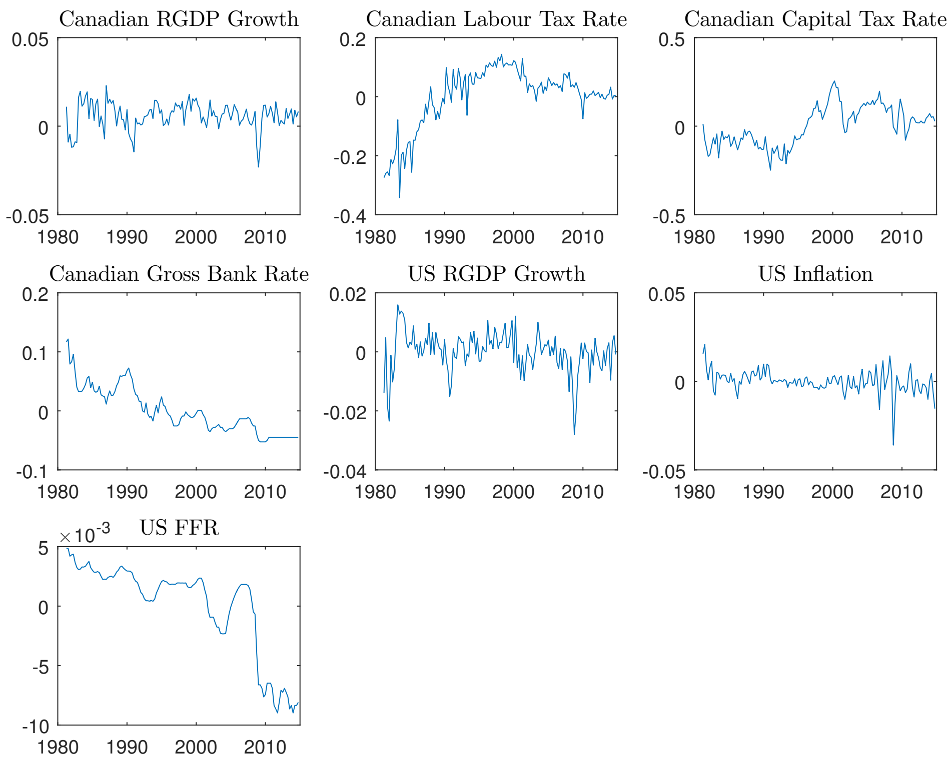

Figure 5: Observed Data

B Stationary RCE characterization

We transform the necessary conditions from Definition 1 into stationary form as follows, where a variable with a "tilde" denotes a stationarized ratio between its original level and the level of domestic total factor productivity, i.e., $\tilde{X}_t := X_t / A_t$.

Household optimal portfolio choices:

$$\tilde{C}_t^{-\rho} = R_t \mathbb{E}_t \left\{ \beta(\tilde{C}_t) \frac{\tilde{C}_{t+1}^{-\rho}}{\Pi_{t+1}} \exp\left[g_A + \sigma_A \varepsilon_{A,t+1}\right]^{-\rho} \right\}, \tag{42a}$$

$$\tilde{C}_t^{-\rho} = \mathbb{E}_t \left\{ \tilde{R}_{t+1}^{*} \beta(\tilde{C}_t) \frac{\tilde{C}_{t+1}^{-\rho} Q_{t+1}}{Q_t} \exp\left[g_A + \sigma_A \varepsilon_{A,t+1}\right]^{-\rho} \right\}. \tag{42b}$$

Capital accumulation equation and convex adjustment costs:

$$\tilde{K}_{t+1} \exp(g_{A,t+1}) = (1-\xi)\tilde{K}_t + \tilde{I}_t \left[1 - \mathscr{D}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} \right) \right], \tag{42c}$$

$$\mathscr{D}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\exp(g_{A,t+1}) - \exp(g_A)\right), \tag{42d}$$

Firm optimal pricing and hiring:

$$\Pi_{H,t}(\Pi_{H,t}-\Pi) - \frac{\epsilon_H}{2}(\Pi_{H,t}-\Pi)^2 = \frac{\epsilon_H}{\theta}\left[mc_{H,t} - \frac{\epsilon_H - 1}{\epsilon_H}\right] + \beta(\tilde{C}_t^a)\mathbb{E}_t\left\{\frac{\tilde{C}_{t+1}^{-\rho}}{\tilde{C}_t^{-\rho}}\left(\Pi_{H,t+1}-\Pi\right)\Pi_{H,t+1}\cdot\frac{\tilde{Y}_{H,t+1}}{\tilde{Y}_{H,t}}\exp\left[g_A + \sigma_A\varepsilon_{A,t+1}\right]^{1-\rho}\right\}, \tag{42e}$$

$$\tilde{Y}_{H,t} = N_t^{1-\alpha}\tilde{K}_t^{\alpha}. \tag{42f}$$

Labor market and goods market clearing:

$$\psi N_t^{\varphi}\tilde{C}_t^{\rho} = (1-\tau_{W,t})mc_{H,t}p_{H,t} \tag{42g}$$

$$\left[1-\left(\Pi_{H,t}-\Pi\right)^2\right]\tilde{Y}_t = \tilde{Y}_{H,t} \equiv \left(p_{H,t}\right)^{-\eta}\left[(1-\gamma)(\tilde{C}_t+\tilde{I}_t+\tilde{G}_t)+\gamma Q_t^{\eta}\tilde{C}_t^{*}\right]. \tag{42h}$$

Government budget constraint:

$$\tilde{G}_t = \left((1-\alpha)\tau_{W,t}+\alpha\tau_{K,t}\right)mc_{H,t}\tilde{Y}_{H,t}, \tag{42i}$$

Identities:

$$p_{H,t} = \left[\frac{1-\gamma\left(Q_t\right)^{1-\eta}}{1-\gamma}\right]^{\frac{1}{1-\eta}}, \tag{42j}$$

$$\Pi_t = \Pi_{H,t}\times\frac{p_{H,t-1}}{p_{H,t}}. \tag{42k}$$

Note that $mc_{H,t} := MC_t/P_{H,t}$, N_t, $p_{H,t} := P_{H,t}/P_t$, Q_t, Π_t and $\Pi_{H,t}$ are already stationary variables, as are R_t, $\tilde{R}_t^*$ and $\tau_{W,t}$.

Given the exogenous stochastic process s_t, and policy behaviors, (21), (23), and stochastic volatility processes, (30), (32) and (34), the system above characterizes a bounded stochastic process for allocation $\{\tilde{C}_t, N_t, \tilde{G}_t, \tilde{Y}_{H,t}, mc_{H,t}\}_{t\in\mathbb{N}}$ and pricing functions $\{\Pi_{H,t}, p_{H,t}, \Pi_t, Q_t\}_{t\in\mathbb{N}}$.

B.1 Steady state and model calibrations

In this section we describe the model's non-stochastic steady state (hereinafter we will refer to this as the unique "steady state"). It is easy to check that the steady state results in an under identified system. Thus, to get a unique solution, we need to use a combination moment matching to first-order observable (i.e., long-run) data as well as calibration. In what follows, we denote a variable without explicit time subscript as its steady state point.

First, given the data's long-run foreign-output and consumption shares, respectively, $\tilde{C}^*/\tilde{Y} := \tilde{Y}^*/\tilde{Y}$ and $\tilde{C}/\tilde{Y}$, we estimate (pin down) the share γ, from matching the first moment using (42h).

Next, Equation (42e), implies that the steady state marginal cost is given by $mc_H = (\epsilon_H - 1)/\epsilon_H$. Thus, given parameter estimates (ρ, φ) we can choose ψ in (42g) to calibrate the proportion of hours worked to $N = 0.33$. Given N and mc_H, the steady state level of capital can then be pinned down from (15) along with the well known marginal product of capital and labor equations. These two inputs are then use in equation (42f) to pin down $\tilde{Y}_H$, and the first equality of (42h), pins down Y. Domestic consumption and foreign output are then derived by multiplying the implied steady state level of output by the first moment statistics in the respective time series. Similarly, we pin down the capital and labor tax by using the implied steady state from the first moment statistics from the derived rates (0.32 and 0.21 respectively). Given the steady state capital and labor tax rates, we then choose α to match government expenditure share of output in (42i) to it's observed first moment of 0.21 - thsi equates to $\alpha = 0.31$. The stationary capital accumulation equation (42c) is then used to pin down steady state investment.

Next, we normalize the steady state real exchange rate to unity (i.e. $Q = 1$). From (42j), this implies that $p_H = 1$, and from (42k), we will also have $\Pi = \Pi_H$. To pin down the steady state level of inflation we first pin down both domestic and foreign real gross interest rates (i.e., R and R^* by making use of first moment statistics from their respective time series data. More precisely, using quarterly data on Canada's bank rate we set $R = 1.02$. Similarly, using quarterly data on the federal funds rate and US CPI inflation rates, the steady state international nominal interest and inflation rates are respectively set to $i^* = 1.27$ and $\pi^* = 0.67$ percent. Using the well known Fisher relationship, these values imply a real gross international interest rate of $R^* = 1.00596$. Using these two results along with the steady state domestic and foreign Euler conditions (42a) and (42b), implies that $\Pi = \frac{R}{R^*}$. Similarly, (42b) implies that $\delta = \frac{g_A}{R^*}$, conditional on g_A. These values can then be used to solve for the steady state value of ζ in the endogenous discount factor (where $\vartheta \approx 0$ is a calibrated value).

C The Estimation Algorithm

C.1 Standard DSGE Model with Homoskedastic Disturbances

To fix ideas, we begin with the standard linear DSGE framework and discuss its Bayesian estimation problem. To draw from the posterior distribution of the standard DSGE model's parameters—i.e., the model without stochastic volatility—we can follow the algorithm set out in Appendix A of Justiniano and Primiceri (2008).

Let the vector $\boldsymbol{\theta}^{(g)}$ denote the saved draw of all parameters of the baseline DSGE model at iteration $g > 0$. Using Dynare (version 4.4.3), the first-order Taylor approximation of the stationary RCE conditions has a linear Gaussian state-space representation:

$$\mathbf{y}_t^o = \mathbf{H}^o \mathbf{x}_t, \tag{43}$$

$$\mathbf{x}_t = \mathbf{A}\left(\boldsymbol{\theta}^{(g)}\right)\mathbf{x}_{t-1} + \mathbf{B}\left(\boldsymbol{\theta}^{(g)}\right)\mathbf{u}_t, \tag{44}$$

where $\mathbf{y}_t^o$ is a $N_Y \times 1$ vector of observable variables, $\mathbf{x_t}$ is a $N_x \times 1$ vector of endogenous/state variables in log-deviation from the deterministic steady state, $\mathbf{H}^o$ is a $N_Y \times N_x$ selection matrix that maps the data to their model counterpart, $\mathbf{A}\left(\boldsymbol{\theta}^{(\mathbf{g})}\right)$ and $\mathbf{B}\left(\boldsymbol{\theta}^{(\mathbf{g})}\right)$ are respectively $N_x \times N_x$ and $N_x \times N_u$ matrices that contain (implicit) cross-equation restrictions involving the model's deep parameters, and $\mathbf{u}_t \sim N(\mathbf{0}, \Sigma)$ is a $N_u \times 1$ vector of (independent) structural shocks.

Since the posterior distribution of the DSGE models parameters does not belong to a standard class of distributions, we follow the now standard practice of implementing a random walk Metropolis-Hastings (RW-MH) MCMC procedure through which a new candidate parameter vector; $\theta^{(c)}$, is drawn from a proposal density, and accepted with probability:

$$\alpha = \min\left\{1, \frac{\mathscr{L}\left(\mathbf{Y}|\theta^{(c)}\right) p\left(\theta^{(c)}\right)}{\mathscr{L}\left(\mathbf{Y}|\theta^{(g)}\right) p\left(\theta^{(g)}\right)}\right\}, \tag{45}$$

where $\mathbf{Y} := \{\mathbf{y}_t^o\}_{t=1}^T$ is the matrix of data, $\mathscr{L}\left(\mathbf{Y}|\theta^{(i)}\right)$ is the model likelihood and $p\left(\theta^{(i)}\right)$ is the prior distribution where $i \in \{c, g\}$. If $\theta^{(c)}$ is accepted then $\theta^{(g+1)} = \theta^{(c)}$, otherwise $\theta^{(g+1)} = \theta^{(g)}$.[12]

C.2 Modeling Heteroskedastic Disturbances

As discussed in Appendix B of Justiniano and Primiceri (2008), when the structural shocks exhibit heteroskedastic disturbances in the form of latent stochastic volatilities, then the above algorithm must be modified from a Metropolis MCMC to a Metropolis-within-Gibbs MCMC algorithm. To this end, equation (44) is augmented as:

$$\mathbf{x}_t = \mathbf{A}\left(\boldsymbol{\theta}^{(g)}\right)\mathbf{x}_{t-1} + \mathbf{B}\left(\boldsymbol{\theta}^{(g)}\right)\tilde{\mathbf{u}}_t, \tag{46}$$

[12] For a textbook treatment of the RW-MH algorithm in DSGE models see Chapter 4 of Herbst and Schorfheide (2015).

where $\mathbf{x}_t$, $\mathbf{A}\left(\boldsymbol{\theta}^{(\mathbf{g})}\right)$ and $\mathbf{B}\left(\boldsymbol{\theta}^{(\mathbf{g})}\right)$ are as defined in the previous section and $\tilde{\mathbf{u}}_t$ is a $N_u \times 1$ vector of structural shocks with a time-varying covariance matrix, $N_u = \text{card}(I)$ and I is defined as the finite set of structural shock indexes in the paper. Let each of the structural shocks be indexed by i, the associated i stochastic volatilities are modeled as a non-linear state space model with measurement and state equations respectively defined by a random-walk-plus-noise model:

$$\tilde{u}_{i,t} = \sigma_{i,t}\varepsilon_{i,t}, \tag{47}$$

$$h_{i,t} = h_{i,t-1} + v_{i,t}, \tag{48}$$

where $h_{i,t} = \log\sigma_{i,t}$, $\varepsilon_{i,t} \sim N(0,1)$ and $v_{i,t} \sim N\left(0,\omega_i^2\right)$ for $i = 1,\ldots,N_u$. In what follows we simplify the notation by letting $\tilde{\mathbf{u}} = (\tilde{\mathbf{u}}_1,\ldots,\tilde{\mathbf{u}}_T)$, $\mathbf{h}_i = \left(h_{i,1},\ldots,h_{i,T}\right)'$, $\mathbf{H} = \left(\mathbf{h}_1,\ldots,\mathbf{h}_{N_u}\right)$, and $\boldsymbol{\omega} = \left(\omega_1^2,\ldots,\omega_{N_u}^2\right)$. To illustrate the Metropolis-within-Gibbs MCMC algorithm, let $\boldsymbol{\theta}^{(g)}$,$\mathbf{H}^{(g)}$ and $\boldsymbol{\omega}^{(g)}$ denote the last saved draw of all parameters of the baseline DSGE model, stochastic volatilities and associated parameters. Estimation of the DSGE model parameters, stochastic volatilities and associated parameters in iteration $(g+1)$ involves the following five steps.

C.2.1 Draw the structural shocks

In order to get a draw of the stochastic volatilities we must first obtain a sample of the structural shocks, $\tilde{\mathbf{u}}^{(g+1)}$, associated with the approximate solution of the model (46). This is completed with the efficient disturbance simulation smoother as developed by Durbin and Koopman (2002).

C.2.2 Draw the stochastic volatilities

Conditional on $\tilde{\mathbf{u}}^{(g+1)}$, the associated stochastic volatilities can be estimated with the two stage auxiliary mixture sampling approach developed by Kim et al. (1998). In the first stage, equation (47) can be made linear in $\sigma_{i,t}$ by first squaring both sides and then taking the logarithm:

$$\tilde{u}_{i,t}^{*} = 2h_{i,t} + \tilde{\varepsilon}_{i,t}^{*}, \tag{49}$$

where $\tilde{u}_{i,t}^{*} = \log\left(\tilde{u}_{i,t}^{2}\right)$ and $\tilde{\varepsilon}_{i,t}^{*} = \log\left(\varepsilon_{i,t}^{2}\right)$. In practice $\tilde{u}_t^{*} = \log\left(\tilde{u}_{i,t}^{2} + c_1\right)$ where c_1 is a small constant that makes the estimation procedure more robust - in practice it is common to set $c_1 = 10^{-4}$. The cost of linearizing the measurement equation in this manner is that the innovations are no longer Gaussian, but instead follow a $\log-\chi_1^2$ distribution. This means that standard estimation algorithms for linear Gaussian state space models can not be directly applied.

To overcome this computational difficulty, the second step in the auxiliary mixture sampling approach is make the transformed measurement equation conditionally Gaussian by defining a (conditionally) Gaussian auxiliary mixture random variable that matches the moments of the $\log-\chi_1^2$

Table 2: A seven-component Gaussian mixture for approximating the $\log \chi_1^2$ distribution

Component	p_j	m_j	r_j^2
1	0.00730	-10.12999	5.79596
2	0.10556	-3.97281	2.61369
3	0.00002	-8.56686	5.17950
4	0.04395	2.77786	0.16735
5	0.34001	0.61942	0.64009
6	0.24566	1.79518	0.34023
7	0.25750	-1.08819	1.26261

distribution. More precisely, Kim et al. (1998) show that:

$$f\left(\tilde{\varepsilon}_{i,t}^{*}\right) \approx \sum_{j=1}^{7} p_j f\left(\tilde{\varepsilon}_{i,t}^{*} | s_{i,t} = j\right), \tag{50}$$

where $s_{i,t} \in \{1,\ldots,7\}$ is an auxiliary random variable that serves as the mixture component indicator for the ith innovation at date t, $p_j = Pr\left(s_i = j\right)$ and $f(\cdot)$ is a (conditional) Gaussian density with mean $m_j - 1.2704$ and variance r_j^2. For completeness, the parameters for each of the seven Gaussian distributions in (10) are reported in Table 2.

Note that the Gaussian mixture does not have any unknown parameters. Thus, conditional on $\mathbf{s}^{(g)} = \left(\mathbf{s}_1,\ldots,\mathbf{s}_{N_u}\right)$, where $\mathbf{s}_i = (s_1,\ldots,s_T)'$, the system has an approximate linear Gaussian state space form, from which a new draw $\mathbf{H}^{(g+1)}$ can be obtained with standard linear Gaussian sampling algorithms. In our application $\mathbf{H}^{(g+1)}$ are efficiently sampled with the precision-sampling based algorithm explained in Chan and Hsiao (2014).

C.2.3 Draw the indicators of the mixture approximation

Given the draws $\tilde{\mathbf{u}}^{(g+1)}$ and $\mathbf{H}^{(g+1)}$, the components of the auxiliary mixture component indicator: $s_{i,t}^{(g+1)}$, can be independently sampled from the following seven point discrete distribution:

$$Pr\left(s_{i,t}^{(g+1)} = j | \tilde{u}_{i,t}^{(g+1)}, h_{i,t}^{(g+1)}\right) = \frac{1}{c_t} p_j f\left(\tilde{\varepsilon}_{i,t}^{*} | 2h_{i,t}^{(g+1)} + m_j - 1.2704, r_j^2\right), \tag{51}$$

where $c_t = \sum_{j=1}^{7} p_j f\left(\tilde{\varepsilon}_{i,t}^{*} | 2h_{i,t}^{(g+1)} + m_j - 1.2704, r_j^2\right)$ is a normalization constant.

C.2.4 Draw the associated parameters of the stochastic volatilities

Having generated $\tilde{\mathbf{u}}^{(g+1)}$ and $\mathbf{H}^{(g+1)}$, elements of the vector $\boldsymbol{\omega}^{(g+1)}$ can be sampled with the usual Normal inverse-Gamma distributions. For instance, under the conjugate prior $\omega_i^2 \sim IG\left(\nu_{\omega_i^2}, S_{\omega_i^2}\right)$, the independent variances of the state equations in (48), can be sampled from:

$$\left(\omega_i^{2(g+1)} | \tilde{u}_{i,t}^{(g+1)}, h_{i,t}^{(g+1)}\right) \quad \sim \quad IG\left(\nu_{\omega_i^2} + \frac{T-1}{2}, S_{\omega_i^2} + \sum_{t=2}^{T} (h_t - h_{t-1})^2\right). \tag{52}$$

C.2.5 Draw the DSGE parameters

Finally, as in the baseline model, the DSGE model parameters are sampled using a random walk Metropolis MCMC procedure in which the new candidate parameter vector $\theta^{(c)}$ is drawn from a proposal density. Since the model now includes stochastic volatility the likelihood ratio must be adjusted such that $\theta^{(c)}$ is now accepted with probability:

$$\alpha = \min\left\{1, \frac{\mathscr{L}\left(\mathbf{Y}|\theta^{(c)}, \mathbf{H}^{(g+1)}\right) p\left(\theta^{(c)}\right)}{\mathscr{L}\left(\mathbf{Y}|\theta^{(g)}, \mathbf{H}^{(g+1)}\right) p\left(\theta^{(g)}\right)}\right\} \tag{53}$$

where $\mathbf{Y}$ is the matrix of data, $\mathscr{L}\left(\mathbf{Y}|\theta^{(j)}, \mathbf{H}^{(g+1)}\right)$ is the model likelihood and $p\left(\theta^{(j)}\right)$ is the prior distribution where $i \in \{c, g\}$. If $\theta^{(c)}$ is accepted then $\theta^{(g+1)} = \theta^{(c)}$, otherwise $\theta^{(g+1)} = \theta^{(g)}$.

D Convergence Diagnostics

To assess convergence of the Markov chain to its ergodic distribution we conduct both formal and informal diagnostic checks. All test statistics were computed with a thinned sample of 10^6 MCMC draws in which 1 in 100 draws was stored after a 50,000 draw burn-in. As an informal check Figures 6 and 7 show the stored MCMC draws of the DSGE micro-parameters after thinning the chain. Since the draws resemble a white noise process, there is informal evidence to support the hypothesis that the draws are in fact independent. Next, Figure 8 shows the inefficiency factors (IFs) for both the DSGE micro-parameters and the log-volatilities. The IF is the inverse of the well known relative numerical efficiency (RNE) measure of Geweke (1992). In each case, the IFs are computed by comparing the first 10% of draws to the final 50%. For interpretation purposes IFs of approximately 20 or less are indicative of convergence. Overall, the evidence suggests that the Markov chains for each parameter have converged to their stationary distributions.

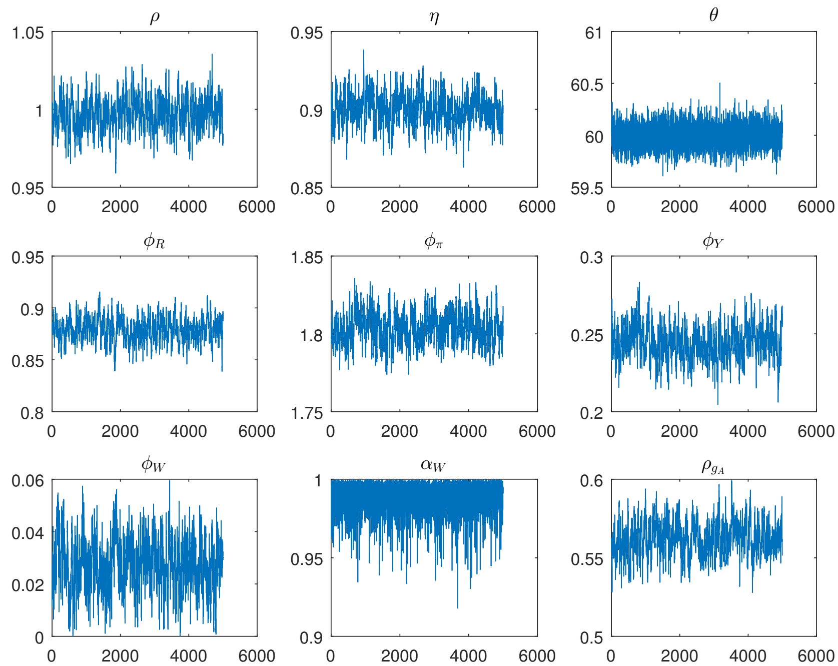

Figure 6: Markov chain of DSGE micro parameters

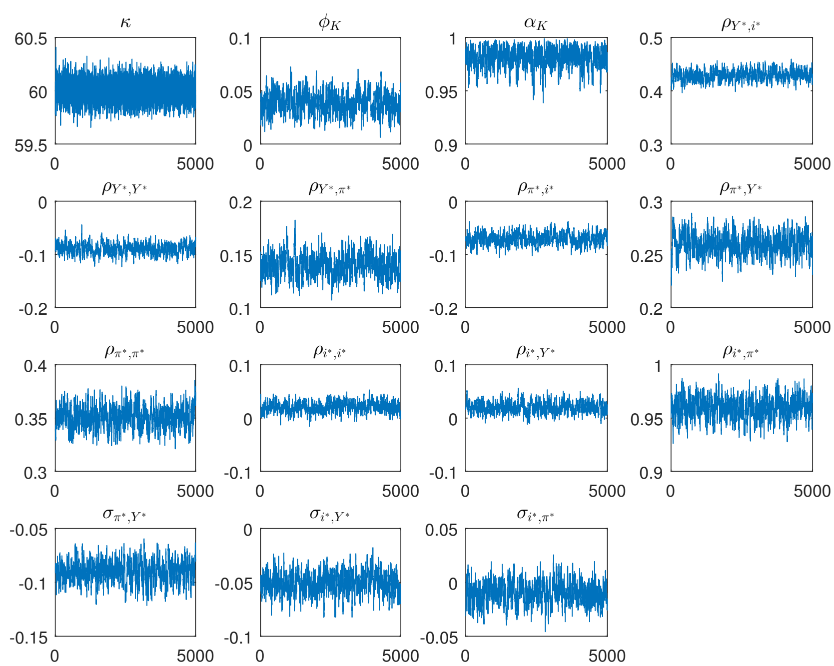

Figure 7: Markov chain of DSGE exogenous-processes' parameters

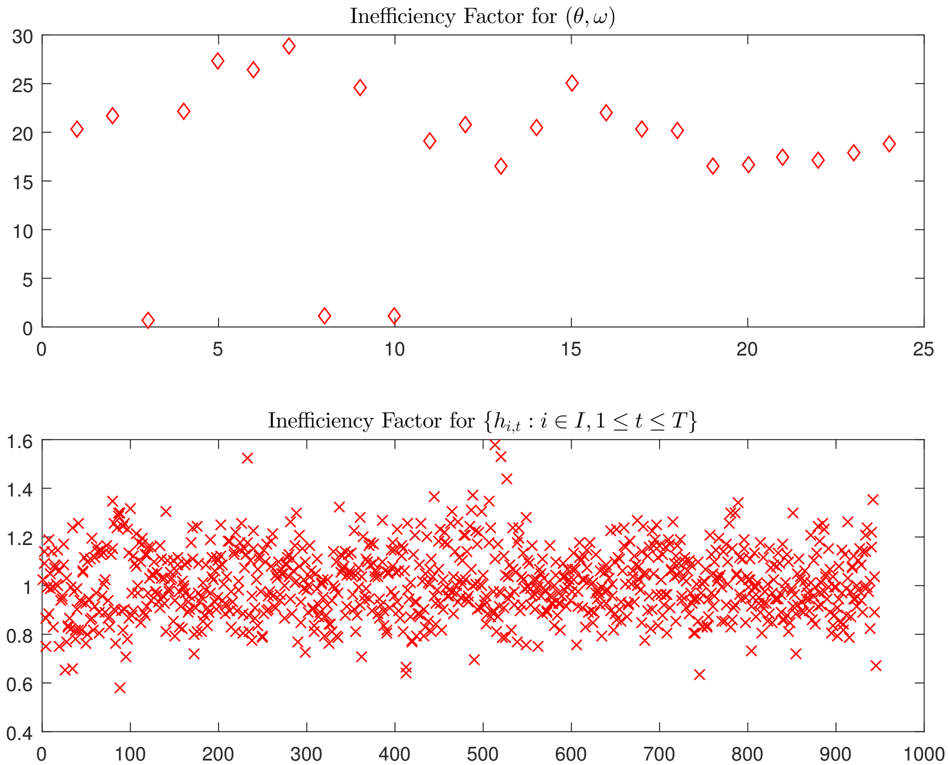

Figure 8: Parameter Markov Chain Convergence Statistics. The top panel reports the IF statistics each of the 21 the DSGE parameters, $(\boldsymbol{\theta}, \boldsymbol{\omega})$. The bottom panel reports that of the 6 stochastic volatilities over a length-T ($T = 135$) sample history, $\mathbf{H}$.