Analyzing Operational Flexibility of Electric Power Systems

Andreas Ulbig and Göran Andersson
Power Systems Laboratory, ETH Zurich, Switzerland
ulbig | andersson @ eeh.ee.ethz.ch

Abstract

Operational flexibility is an important property of electric power systems and plays a crucial role for the transition of today's power systems, many of them based on fossil fuels, towards power systems that can efficiently accommodate high shares of variable Renewable Energy Sources (RES). The availability of sufficient operational flexibility is a necessary prerequisite for the effective grid integration of large shares of fluctuating power feed-in from variable RES, especially wind power and Photovoltaics (PV).

This paper establishes the necessary framework for quantifying and visualizing the technically available operational flexibility of individual power system units and ensembles thereof. Necessary metrics for defining power system operational flexibility, namely the power ramp-rate, power and energy capability of generators, loads and storage devices, are presented. The flexibility properties of different power system unit types, e.g. load, generation and storage units that are non-controllable, curtailable or fully controllable are qualitatively analyzed and compared to each other. Quantitative results and flexibility visualizations are presented for intuitive power system examples.

An outlook for the usage of the here proposed methods in power system control centers is given.

Index Terms

Operational Flexibility, Power System Analysis, Grid Integration of Renewable Energy Sources (RES)

I. Introduction

This paper presents a novel approach for analyzing the available operational flexibility of a given power system. In the context of this paper we mean by this the combined available operational flexibility that an ensemble of – potentially very diverse – power system units in a geographically confined grid zone can provide in each time-step during the operational planning, given load demand and Renewable Energy Sources (RES) forecast information, as well as in real-time in case of a contingency. Operational flexibility is essential for mitigating disturbances in a power system such as outages or forecast deviations of either power feed-in, i.e. fluctuating electricity generation from wind turbines or solar units, or power out-feed, i.e. fluctuating load demand. Metrics for assessing the technical operational flexibility of power systems, i.e. power ramp-rate (ρ), power capacity (π), energy capacity (ϵ) and ramp duration (δ) have been proposed by Makarov et al. in [1] and their meaning further discussed by the authors in [2]. In this paper we establish the necessary framework for quantifying and visualizing the technically available operational flexibility of individual power system units and ensembles thereof. The functional modeling of all power system units is accomplished using the Power Nodes modeling framework introduced in [3], [4]. The flexibility properties of different power system unit types, e.g. load, generation and storage units that are non-controllable, curtailable or fully controllable are qualitatively analyzed and compared to each other. Quantitative results as well as flexibility visualizations of the here proposed flexibility assessment framework are presented for intuitive benchmark power systems.

The remainder of this paper is organized as follows: Section II discusses operational flexibility and its role in power system operation. It also introduces necessary metrics for operational flexibility. Section III explains how operational flexibility can be modeled using the Power Nodes modeling framework. This is followed by Section IV, which illustrates how operational flexibility can be quantified and analyzed for individual power system units as well as for ensembles of power system units. Finally, a conclusion and a summary of the contributions are given in Section V.

II. Operational Flexibility in Power Systems

Operational flexibility is an important property of electric power systems and essential for mitigating disturbances in a power system such as outages or forecast deviations of either power feed-in, i.e. from wind turbines or Photovoltaics (PV) units, or power out-feed, i.e. load demand. The availability of sufficient operational flexibility is a necessary prerequisite for the effective grid integration of large shares of fluctuating power feed-in from variable RES.

A. Increasing Need for Operational Flexibility

In recent years power system dispatch optimization and real-time operation have become more and more driven by several major trends, which notably include

1) Wide-spread deployment of variable RES, i.e. wind turbines and PV units, that has led to significant relative and absolute shares of power generation, which is highly fluctuating and neither perfectly predictable nor fully controllable. Variable RES power feed-in causes non-deterministic power imbalances and power flow changes on all grid levels [5], [6].
2) Growing power market activity that has led to operational concerns of its own, i.e. deterministic frequency deviations caused by transient power imbalances due to more frequent changes in the now market-driven operating set-point schedules of power plants as well as more volatile (cross-border) power flow patterns [7].
3) The emergence of a *smart grid* vision as a driver for change in power system operation [8]. Using the reference framework of control theory, the term *smart grid* can be understood as the sum of all efforts that improve observability and controllability over individual power system processes, i.e. power feed-in to the grid and power out-feed from the grid as well as power flows on the demand/supply side, happening on all voltage levels of the electricity grid. An improved observability and controllability of individual power system units should, eventually, also lead to an improved observability and controllability of the entire power system and the processes happening therein.

Altogether, these developments constitute a major paradigm shift for the management of power systems. Operating power systems optimally in this more complex environment requires a more detailed assessment of *available* operational flexibility at every point in time for effectively mitigating the outlined system disturbances.

B. Sources of Operational Flexibility

Different sources of power system flexibility exist as is illustrated in Fig. 1. Operational flexibility can be obtained on the generation-side in the form of dynamically fast responding conventional power plants, e.g. gas- or oil-fueled turbines or rather flexible modern coal-fired power plants and on the demand-side by means of adapting the load demand curve to better absorb fluctuating RES power feed-in. In addition to this, RES power feed-in can also be curtailed or, in more general terms, modulated below its given time-variant maximum output level. Furthermore, stationary storage capacities, e.g. hydro storage, Compressed Air Energy Storage (CAES), stationary Battery Energy Storage System (BESS) or fly-wheels, as well as time-variant storage capacities, e.g. the aggregated battery capacity of electric vehicle fleets, are well-suited for providing different types of operational flexibility.

Operational flexibility can also be obtained from other grid zones via the electricity grid's tie-lines in case that the available operational flexibility in one's own grid zone is not sufficient or more expensive. Readily available power import and export capacity, nowadays facilitated by more and more integrated national and international power markets, is used in daily power system operation to a certain degree as a *slack bus* for fulfilling the active power balance and mitigating power flow problems of individual grid zones by tapping into the flexibility potential of other grid zones. For power system operation, importing needed power in certain situations and exporting undesirable power feed-in in other situations to neighboring grid zones is for the time being probably the most convenient and cheapest measure for increasing operational flexibility. However, power import/export can only be performed within the limits given by the agreed line transfer capacities between the grid zones. In the European context this corresponds to the Net Transfer Capacity (NTC) values [9], which are a rather conservative measure of available grid electricity transfer capacity. This discussion also extends to physical line constraints of the grid topology within a grid zone, which may effectively limit the delivery, i.e. the physical transport, of operational flexibility, i.e. active power, between individual bus nodes (confer [10], [11]).

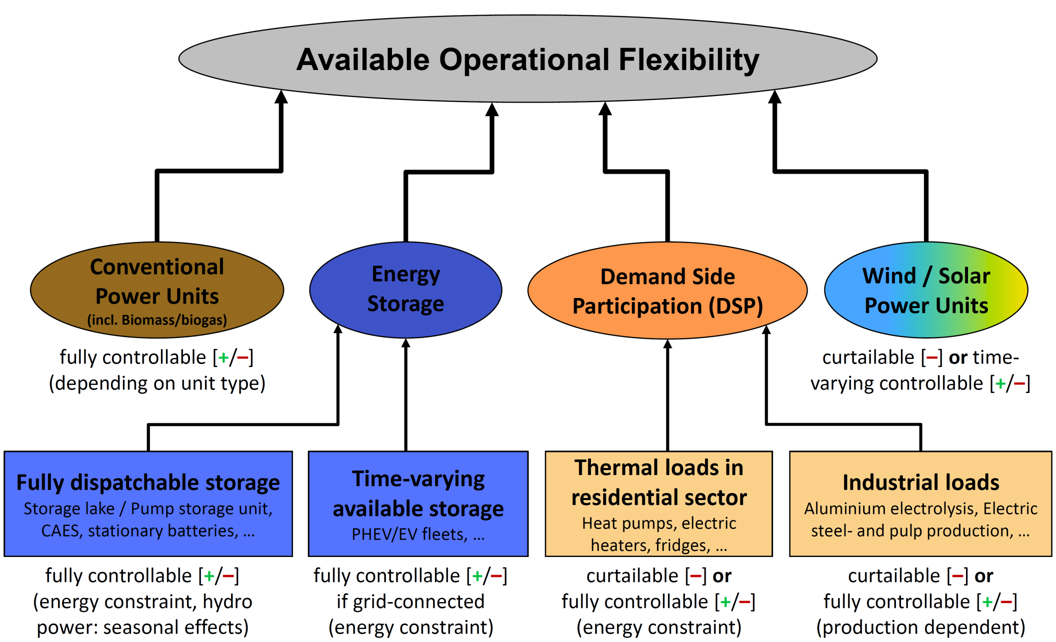

Fig. 1: Sources of Operational Flexibility in Power Systems.

C. Definitions of Operational Flexibility

The term Operational Flexibility in power systems is often not properly defined. In a power systems context, the term *flexibility* may refer to very different things ranging from the quick response times of generation units, e.g. gas turbines, to the degree of efficiency or robustness of a given power market setup. The topic is receiving wide attention [1], [2], [6], [12]–[14]. In the remainder of this paper the focus is solely on the technical aspects of operational flexibility (Def. 1).

> **Definition 1:** – ***Operational Flexibility in Power Systems***
> *Operational flexibility is the technical ability of a power system unit to modulate electrical power feed-in to the grid and/or power out-feed from the grid over time.*
> *This means the technical ability of a grid operator to modulate the power in-flow/outflow on a global scale, i.e. for achieving power balance, and within a grid topology, i.e. to control power flows via the modulation of power injections and outtakes at specific grid nodes.*

D. Classification of Operational Flexibility

In liberalized power systems, operational flexibility is traded in the form of *energy products* via power markets, i.e. day-ahead and intra-day spot markets, as well as *control reserve products*, i.e. primary/secondary/tertiary frequency control reserves, from Ancillary Services (AS) markets.

A classification of operational flexibility in resources and reserves, inspired by established classification system for natural resources, e.g. crude oil, natural gas and coal [15], is presented in the following. The categories for classifying *Available Operational Flexibility* are:

- *Potential Flexibility Resources*, i.e the flexibility resources exist *physically* and could be used. The necessary controllability, and also observability, over the power system units is lacking.
- *Actual Flexibility Resources*, i.e. the part of the potential flexibility resources that can in fact be used because controllability, and also observability, over the power system units exists.
- *Flexibility Reserves*, i.e. the part of the actual flexibility resources can be used economically.
- *Market-Available Flexibility Reserves*, i.e. the part of the flexibility reserves that can be procured from power or Ancillary Services markets. Constraints due to AS product *structuring* may limit the amount of operational flexibility that can in fact be procured in practice.

The procurement of the market-available operational flexibility reserves is accomplished via the market auctioning of the so-called

- Flexibility products, i.e. Ancillary Services, or
- Power products, i.e. adjusting unit generation or load demand profiles via the scheduling mechanism of day-ahead and intra-day spot markets.

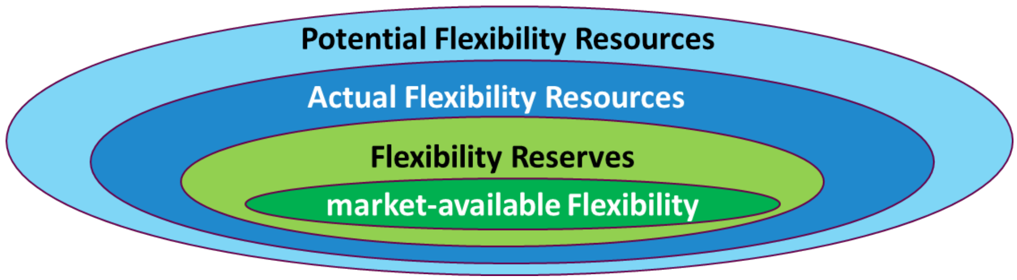

Fig. 2: Classification of Operational Flexibility Resources and Reserves.

E. Metrics for Operational Flexibility

For analysis purposes, the technical capability for the provision of operational flexibility needs to be characterized and categorized by appropriate flexibility metrics. A valuable method for assessing the needed operational flexibility of power systems, for example for accommodating high shares of wind power feed-in, has been proposed by Makarov et al. in [1]. There, the following four metrics have been characterized:

- Power provision capacity π (MW),
- Power ramp-rate capacity ρ (MW/min.),
- Energy provision capacity ϵ (MWh) as well as
- Ramp duration δ (min.).

Since the ramp duration δ is actually dependent on the power ramp rate ρ and power capacity π as $\delta = \pi/\rho$, it is thus sufficient to use the power-related metrics ρ, π and ϵ for describing flexibility.

The role of $\{\rho, \pi, \epsilon\}$ in modulating the operation point of a power system unit, i.e. its nominal power production or load demand level, and with it the relative power injection into the electric grid (> 0) and power outtake from the electricity grid (< 0) with respect to the originally scheduled operation point is illustrated by Fig. 3.

Here, the deliberate deviation between the nominal power plant output trajectory and the actual power output trajectory constitutes the available operational flexibility of the power system unit

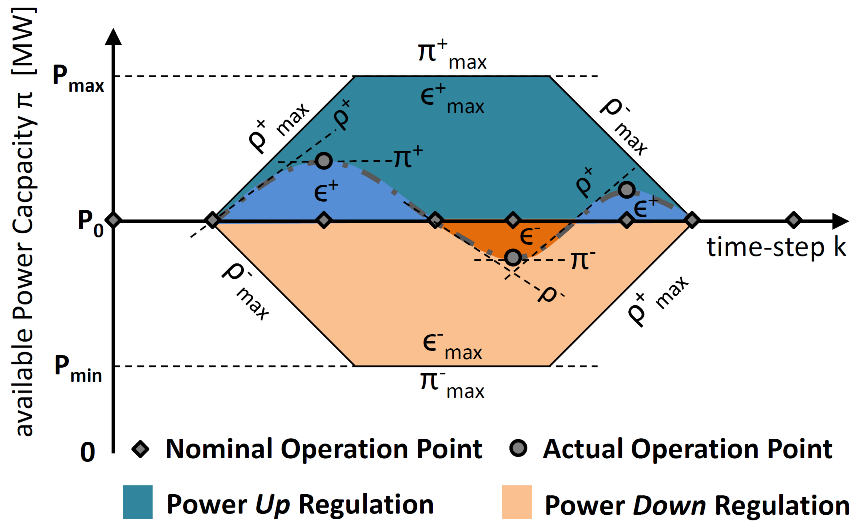

Fig. 3: Flexibility Metrics in Power Systems Operation: Power Ramp-Rate ρ, Power π & Energy ϵ.

in question. Note that for load units the picture would be very similar except that the modulation of power outtake instead of power injection would then be considered.

Operational flexibility is thus the set of all possible operation set-points, i.e. the reach set, which are bounded by the maximum flexibility capability, i.e. the three metrics $\rho^{\pm}_{\max}$, $\pi^{\pm}_{\max}$ and $\epsilon^{\pm}_{\max}$. For the sake of simplicity and clarity we will stick to the same notation as in [1].

An intriguing feature is that the metric terms ρ, π and ϵ are closely linked via integration and differentiation operations in the time domain. The interactions of the individual metrics clearly exhibit so-called double integrator dynamics: energy is the integral of power, which in turn is the integral of power ramp-rate. Due to their inter-temporal linking, the three metrics constitute a *flexibility trinity* in power system operation (Eq. 1).

$$\begin{array}{ccccc}
\textbf{Ramp-Rate} & & \textbf{Power} & & \textbf{Energy} \\
[\mathrm{MW/min.}] & & [\mathrm{MW}] & & [\mathrm{MWh}] \\
 & \int dt & & \int dt & \\
\rho & \rightleftarrows & \pi & \rightleftarrows & \epsilon \\
 & \frac{d}{dt} & & \frac{d}{dt} & \\
\| & & \| & & \| \\
\rho & & \pi = (\,\rho \cdot t\,) & & \epsilon = (\,\frac{1}{2} \cdot \rho \cdot t^2\,)
\end{array} \tag{1}$$

Using these three flexibility metrics instead of only one, for example the power ramping capability ρ as in [13], allows a more accurate and complete representation of power system flexibility, including the relevant inter-temporal constraints over a given time interval. The power ramp-rate for absorbing a disturbance event, measured in MW/min, in a power system may be abundant at a certain time instant. But for a persistent disturbance, the maximum regulation power that can be provided by a generator is limited as is the maximum regulation energy that can be provided by storage units, which are inherently energy-constrained. As the share of storage units in power systems and their importance for the grid integration of RES feed-in is rising, the inter-temporal links between providing ramping capability and eventually reaching relevant power/energy limits cannot be neglected when assessing the available operational flexibility of a power system.

Having defined these flexibility metrics as well as the causal inter-linking between them (Fig. 4), allows in the following the assessment of the available operational flexibility of an individual power system unit and for whole power systems. Note that the operational constraints, i.e. minimum/maximum ramping, power and energy constraints, of individual power system units have to be considered when assessing their available operational flexibility.

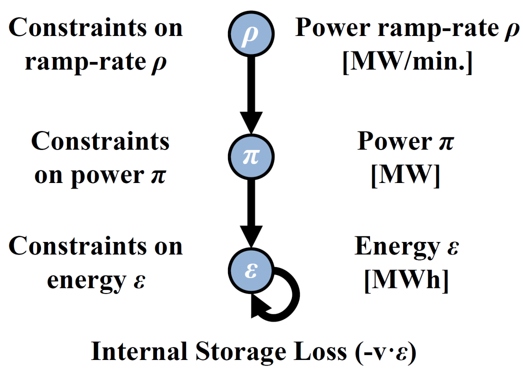

Fig. 4: Inter-temporal linking of flexibility metrics including internal storage losses (dissipation).

III. Modeling of Operational Flexibility

The analysis and assessment of operational flexibility in power system operation first of all necessitates a modeling framework that allows to explicitly include information on the degree of freedom for shifting operation set-points so as to modulate the power feed-in and out-feed patterns of individual power system units. This includes information on whether or not a unit has a storage and is thus energy-constrained, whether or not a unit provides fluctuating power feed-in, and what type of controllability and observability, the latter also includes predictability, that a system operator has over fluctuating power generation and load demand processes (i.e. full, partial or none). The combination of all these properties defines a power system unit's operational flexibility. This is in fact highly related to existing embedded storage capacity inside or behind a power system unit.

For our modeling purposes we use the Power Nodes modeling framework, which allows the detailed functional modeling of power system units such as

- diverse storage units, e.g. batteries, fly-wheels, pumped hydro, CAES, ...,
- diverse generation units, e.g. fully dispatchable conventional generators, variably feed-ining power units, e.g. wind turbine and PV units, and
- diverse load demand units, e.g. conventional (non-controllable), interruptible or thermal (both partially controllable), ...,

including their operational constraints as well as relevant information of their underlying power supply and demand processes. Operational constraints such as minimum/maximum power ramp rates, minimum/maximum power set-points and energy storage operation ranges, information of the underlying power system processes, i.e. fully controllable, curtailable/sheddable or non-controllable, as well as information on observability and predictability of the underlying power system processes, i.e. state measures and/or state-estimation and prediction of fully or only partially observable/predictable system and control input states, can also be included. The workings of the Power Node notation are illustrated by the model representation for an energy storage unit (Fig. 5).

The provided and demanded energies are lumped into an external process termed ξ, with $\xi < 0$ denoting energy use and $\xi > 0$ denoting energy supply. The term u_{gen} describes a conversion corresponding to a power generation with an efficiency η_{gen}, while u_{load} describes a conversion corresponding to consumption with an efficiency η_{load}. The introduction of generic energy storage in the Power Nodes framework adds an important modeling layer to classical power system modeling.

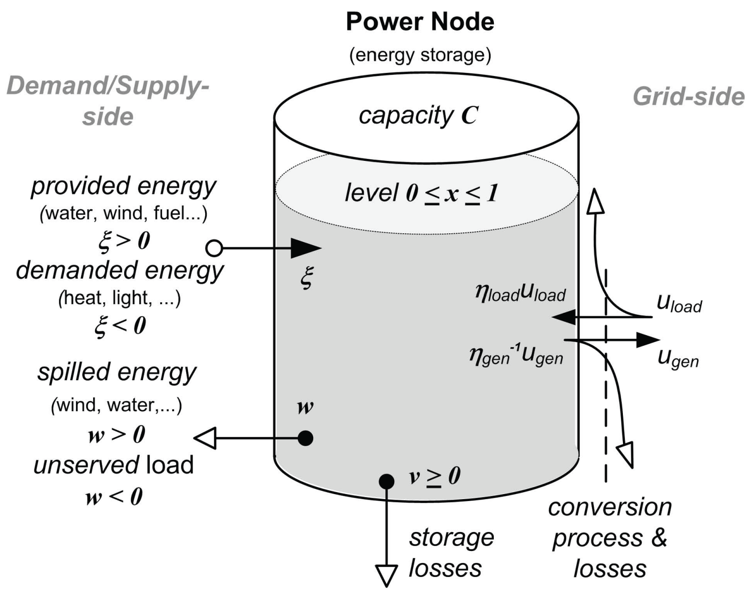

Fig. 5: Power Node model of an energy storage unit with power feed-in (u_{gen}) and out-feed (u_{load}).

Its energy storage level, the State-of-Charge (SOC), is normalized to $0 \leq x \leq 1$ with an energy storage capacity $C \geq 0$. The illustrated storage unit serves as a buffer between the external process ξ and the two grid-related power exchanges $u_{\text{gen}} \geq 0$ and $u_{\text{load}} \geq 0$. Internal energy losses associated with energy storage, e.g. physical, state-dependent dissipation losses, are modeled by the power dissipation term $v(x) \geq 0$, while enforced energy losses, e.g. curtailment/shedding of a power supply or demand process, are denoted by the waste power term w, where $w > 0$ denotes a loss of provided energy and $w < 0$ an unserved load demand.

The dynamics of a power node i, which can be nonlinear in the general case, are:

$$
\begin{aligned}
C_i\,\dot{x}_i \quad &= \quad \eta_{\text{load},i}\,u_{\text{load},i} - \eta_{\text{gen},i}^{-1}\,u_{\text{gen},i} + \xi_i - w_i - v_i, \\
\text{s.t.} \quad (\text{a}) \quad & 0 \leq x_i \leq 1 \quad , \\
(\text{b}) \quad & 0 \leq u_{\text{gen},i}^{\min} \leq u_{\text{gen},i} \leq u_{\text{gen},i}^{\max} \quad , \\
(\text{c}) \quad & 0 \leq u_{\text{load},i}^{\min} \leq u_{\text{load},i} \leq u_{\text{load},i}^{\max} \quad , \\
(\text{d}) \quad & \dot{u}_{\text{gen},i}^{\min} \leq \dot{u}_{\text{gen},i} \leq \dot{u}_{\text{gen},i}^{\max} \quad , \\
(\text{e}) \quad & \dot{u}_{\text{load},i}^{\min} \leq \dot{u}_{\text{load},i} \leq \dot{u}_{\text{load},i}^{\max} \quad , \\
(\text{f}) \quad & 0 \leq \xi_i \cdot w_i \quad , \\
(\text{g}) \quad & 0 \leq |\xi_i| - |w_i| \quad , \\
(\text{h}) \quad & 0 \leq v_i \quad .
\end{aligned}
\tag{2}
$$

Depending on the specific process represented by a Power Node, each term in the Power Node equation may be controllable or not, observable or not, and driven by an external process or not. Internal dependencies, such as a state-dependent loss term $v_i(x_i)$, are possible. Power charge and discharge efficiencies may be non-constant and possibly also state-dependent: $\eta_{\text{load,i}}(x_i)$, $\eta_{\text{gen,i}}(x_i)$. Non-linear conversion efficiencies can be arbitrarily well approximated by a set of Piece-wise Affine (PWA) linear equations [16]. The constraints (a)–(h) denote a generic set of requirements on the variables. They are to express that (a) the SOC is normalized, (b)–(e) the grid power feed-ins and out-feeds as well as their time derivatives (power ramp-rates) are non-negative and constrained, (f) the power supply or demand and the curtailment need to have the same sign, (g) the power supply/demand curtailment cannot exceed the supply/demand itself, and (h) the storage losses are non-negative. The explicit mathematical form of a power node equation depends on the particular modeling case. The notation provides technology-independent categories that can be linked to evaluation functions for energy and power balances. Power Nodes can also represent energy processes that are independent of storage, such as fluctuating RES generation. More details on the Power Node modeling framework, modeling examples and reasoning can be obtained from [3], [4].

IV. Analyzing Operational Flexibility

The functional representation of complex power system interactions using the Power Nodes notation allows a straight-forward analysis of the three power-related operational flexibility metrics, i.e. the power ramp-rate ρ, power π and energy capability ϵ.

A. Quantification of Operational Flexibility

Taking as an illustrative example the operational flexibility of a generation unit i that has an embedded storage function and the possibility for curtailment, e.g. a Hydro Storage Lake (HSL), given by the Power Node model

$$C_i\,\dot{x}_i = -\eta_{\text{gen},i}^{-1}\,u_{\text{gen},i} + \xi_i - w_i - v_i \quad , \tag{3}$$

for providing power regulation is accomplished by calculating the set of all feasible power regulation points $\{\pi_i(k)\}$ based on equation

$$\begin{aligned} \Big\{\pi_i(k)\Big\} &= \Big\{u_{\text{gen},i}^{\text{feasible}}(k)\Big\} - u_{\text{gen},i}^{0}(k) \\ &= \Big\{\eta_{\text{gen}} \cdot (\xi - w - v_x - C\dot{x})\Big\}_{k,i} - u_{\text{gen},i}^{0}(k) \\ \text{s.t.} \qquad & 0 \leq u_{\text{gen},i}^{\min}(k) \leq \{u_{\text{gen}}^{\text{feasible}}(k)\} \leq u_{\text{gen},i}^{\max}(k) \quad . \end{aligned} \tag{4}$$

Here, $u_{\text{gen},i}^{0}(k)$ denotes the nominal set-point of the generation unit and the term $u_{\text{gen},i}^{\text{feasible}}(k)$ represents an arbitrary set-point from the set of all feasible operating points $\{\cdot\}$ to which the unit can be steered to provide operational flexibility. Both terms can be chosen to be time-variant, they are given here for time-step k. The set of all feasible operation points thus depends upon the internal status of the generation unit, as defined by the terms $\xi_i(k)$, $w_i(k)$, $v_i(x_i(k))$ and $C_i x_i(k)$, and bounded by the unit's power rating constraints (confer Eq. 2 (b–d)).

The maximum available operational flexibility for up/down power regulation is given as

$$\begin{aligned} \pi_{\max,i}^{+}(k) &= \min\left[\eta_{\text{gen}}\left(\xi^{\max} - w^{\min} - v_x - C\dot{x}\right),\ u_{\text{gen}}^{\max}\right]_{k,i} - u_{\text{gen},i}^{0}(k) \quad , \\ \pi_{\min,i}^{-}(k) &= \max\left[\eta_{\text{gen}}\left(\xi^{\min} - w^{\max} - v_x - C\dot{x}\right),\ u_{\text{gen}}^{\min}\right]_{k,i} - u_{\text{gen},i}^{0}(k) \quad , \end{aligned} \tag{5}$$

in which $w_i^{\min}(k)$ and $w_i^{\max}(k)$ define the minimum/maximum allowable curtailment for generation unit i at time-step k. In case the primary fuel supply is controllable, the terms $\xi_i^{\min}(k)$ and $\xi_i^{\max}(k)$ define the minimum/maximum allowable primary power provision. The sign of the storage power term $C\dot{x}$ is negative when providing positive power, i.e. discharging ($C\dot{x} < 0$), and positive when providing negative power, i.e. charging ($C\dot{x} > 0$). Please note that in the time-discrete case the term $C\dot{x}$ becomes $C\delta x = C(\,x(k) - x(k-1)\,)$.

The set of all feasible operation points, i.e. $\Big\{\pi_i(k)\Big\}$, thus depends upon the internal status of the generation unit, as defined by the terms $\xi_i(k)$, $w_i(k)$, $v_i(x_i(k))$ and $C_i(x_i(k))$, and bounded by the unit's power rating constraints (Eq. 5, see also Eq. 2 (b–d)). Furthermore, it makes sense to split up the set of all feasible operation points, i.e. $\Big\{\pi_i(k)\Big\}$: Let power regulation up/down be denoted by '+/−', then one can define the subset of feasible operation points that can provide power *up* regulation as $\Big\{\pi_i^{+}(k)\Big\} = \Big\{\forall\, \pi_i(k) \parallel \pi_i(k) > 0\Big\}$. The term $\Big\{\pi_i^{+}(k)\Big\}$ thus represents the positive part of the set of power output flexibility. Equivalently, one can define the subset of feasible operation point that can provide power *down* regulation as $\left\{\pi_i^-(k)\right\} = \left\{\forall\, \pi_i(k) \parallel \pi_i(k) < 0\right\}$. The term $\left\{\pi_i^-(k)\right\}$ thus represents the negative part of the set of power output flexibility.

The assessment approach for metric π (Eq. 4–5) can be extended to the other two flexibility metrics, ρ and ϵ, via time-differentiation and integration, respectively. The flexibility assessment can be accomplished for all conceivable power system unit types in a similar fashion. Please note that the maximum available flexibility calculated in this way is without any consideration of how long a certain power system unit would need to *reach* a new operation set-point that allows the provision of this operational flexibility.

B. Visualization of Operational Flexibility

The three thereby calculated metrics span a so-called *flexibility volume*, which can be represented in its simplified form as a *flexibility cube* for a generic power system unit i. Here, the vertices or extreme points are defined by the set of metric terms $\{\rho^+_{\max}, \rho^-_{\max}, \pi^+_{\max}, \pi^-_{\max}, \epsilon^+_{\max}, \epsilon^-_{\max}\}$ as is qualitatively shown in Fig. 6 The flexibility volume is cut into eight separate sectors.

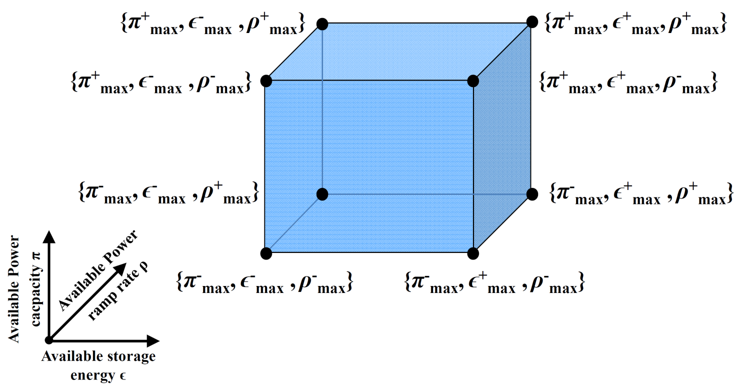

Fig. 6: Flexibility cube of maximum available operational flexibility of a generic power system unit.

The evolution over time of the (maximum) available operational flexibility from a generic storage unit with both load and generation terms, $u_{\text{load}}(k)$ and $u_{\text{gen}}(k)$, is illustrated in Fig. 7. The plots show that the available operational flexibility is highly time-variant due to the actual storage usage over time. However, when taking into account the internal double-integrator dynamics, the flexibility volume becomes a significantly more complex polytope object. An illustration of this more realistic polytope flexibility volume is given by Fig. 8. Here, the information of how long it takes to reach a certain new operation point providing a required set $\{\rho, \pi, \epsilon\}$ of operational flexibility is explicitly given. The set of reachable operation points providing additional flexibility (green) becomes larger

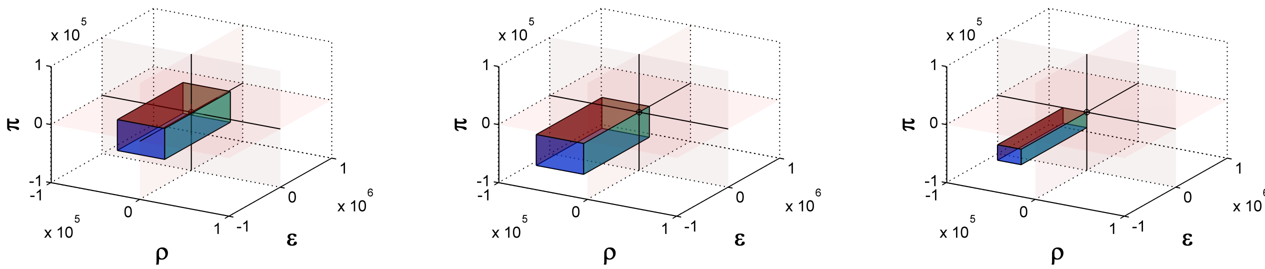

Fig. 7: Time-evolution of maximum available operational flexibility ($k = 36\,\mathrm{h},\ 48\,\mathrm{h},\ 60\,\mathrm{h}$).

when the available time span is longer. The flexibility set converges towards the set of maximal flexibility (red) as defined by the underlying technical constraints of a given power system unit. Calculating the available set of operational flexibility that is achievable after a given number of time-steps k is equivalent to a classical *reach set* calculation. This approach is more exact but also more computationally expensive than the previously presented simpler analytic approach Eq. (4–5).

For the reach-set calculations, the reachability functions of the `MPT Toolbox` [17] have been used in `Matlab`. There a so-called polytope method is employed that involves besides other things the calculation of the *Controllability Gramian* W_C. See [18, p. 19 ff.] for a general discussion of reachability analysis. The advantage of the `MPT Toolbox` is that it explicitly allows the usage of box constraints for inputs and states of dynamical systems. In power systems, a typical example of a box constraint are the limitations on minimum/maximum power ramp-rate, e.g. $\dot{u}_{\text{gen.}}^{\text{min}} \leq \dot{u}_{\text{gen.}}(k) \leq \dot{u}_{\text{gen.}}^{\text{max}}$, and minimum/maximum power output, e.g. $u_{\text{gen.}}^{\text{min}} \leq u_{\text{gen.}}(k) \leq u_{\text{gen.}}^{\text{max}}$. Other approaches for calculating gramians and the corresponding reach-sets include Linear Matrix Inequalities (LMI) methods, as explained in [19], as well as so-called ellipsoidal methods, which have been implemented for example in the `Ellipsoidal Toolbox` [20]. The ellipsoidal methods have a potential disadvantage as they only allow ellipsoidal constraints on system input and states. On the other hand they are computationally much less expensive than `MPT`'s polytope method when it comes to larger system sizes as is nicely illustrated in [18, p. 63 ff.]. Please note that the theoretical maximum reachability volume calculated by the analytic approach may in fact never be fully reached by the power system unit, when using the reach set approach (Fig. 8). This gap is due mainly to the non-infinite discrete sampling time in combination with somewhat pathological operation points at some of the flexiblity cube's vertices, e.g. discharging a storage unit (π^-) while at the same time trying to keep it at its maximum energy storage level (ϵ^+).

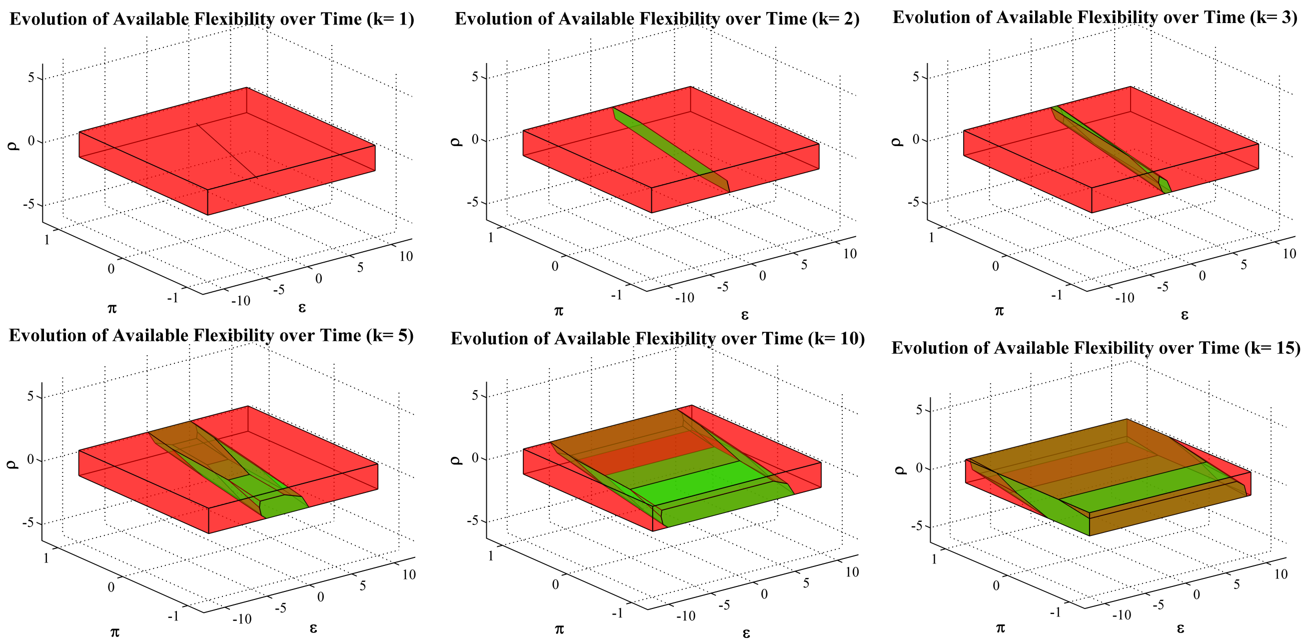

Fig. 8: Time-evolution of available operational flexibility from a storage unit at its nominal set-point $u^0 = \left[u^0_{\text{gen}},\, u^0_{\text{load}}\right]_k$ at $k = 0$. Green: Available flexibility at k = 1h, 2h, 3h, 5h, 10h, 15h (reach set calculation). Red: Maximum available flexibility at $k \to \infty$ (calculated using Eq. (4–5)).

C. *Aggregation of Operational Flexibility*

An important question in power system analysis is how a group or pool of power system units act together in achieving a given objective, i.e. delivering a scheduled power trajectory or providing ancillary services by tracking a control signal. Combining different power system units to provide a service that the units cannot provide individually is an active research topic [21], [22].

The key idea behind an aggregation or *pooling* of several power system units is that this leads to the addition of individual flexibility metrics. In turn, a potentially existing individual flexibility deficiencies with respect to one or more of the metrics $\{\rho,\, \pi,\, \epsilon\}$ can be *"masked"* (Y. Makarov) within an appropriately chosen unit pool. An illustrative example is to combine a dynamically slow power plant with a dynamically fast, but energy-constrained storage unit to provide fast frequency regulation that neither of the units could provide individually [23] due to the lack of one flexibility metric, i.e. the missing fast ramp-rate capability ρ of the power plant, or another, i.e. the small energy capability ϵ of the storage unit:

- A dynamically slow unit, e.g. a thermal/hydro power plant (ρ small, π large, ϵ only limited by fuel provision), is pooled with
- A dynamically fast but energy-constrained storage unit, e.g. a fly-wheel or battery (ρ large,

π small, ϵ limited and small).

- The aggregation of these two units will feature the sum of the individual flexibility capabilities. The energy constraint of the storage unit will be masked by the resulting unit pool (Fig. 9).

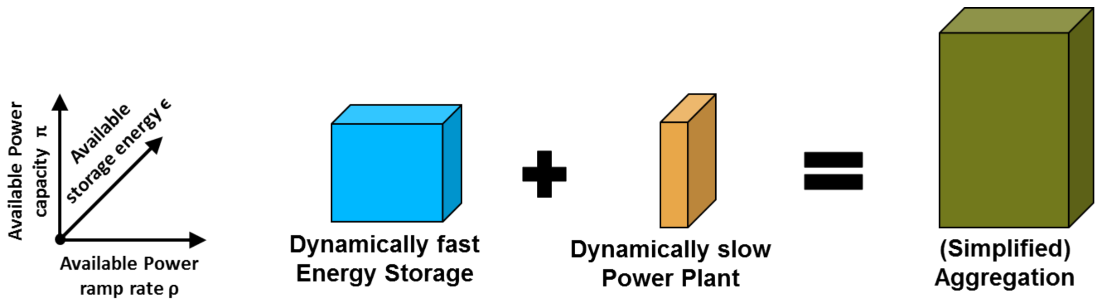

Fig. 9: Aggregation of Operational Flexibility by Pooling of Power System Units.

Obtaining the aggregated operational flexibility that a pool of different power system units provides, is equivalent to aggregating the flexibility volumes of the individual units. Since these are given by more or less complex polytope sets, depending on the chosen calculation approach presented in the previous section, a well-known polytope operation, the *Minkowski Summation*, can be employed for calculating the aggregated flexibility of the unit pool. In the following, we illustrate the aggregation of a slow-ramping power plant together with a fast-ramping but energy-constrained storage unit in Fig. 9. We simplify the task by assuming that within the grid zone of a unit pool, grid constraints are minor and not of practical relevance for the quantification of aggregated flexibility. Although this is clearly a simplifying assumption, it is often used in practice, e.g. in power markets operation.

The aggregation of power system units leads to the addition of individual flexibility metrics, i.e.

$$\{\rho, \pi, \epsilon\}_{\text{agg}} = \{\rho, \pi, \epsilon\}_{\text{slow}} + \{\rho, \pi, \epsilon\}_{\text{fast}} \quad . \tag{6}$$

The aggregation of the operational flexibility of both units, given individually by their respective polytope objects, is accomplished via *Minkowski Summation*

$$\begin{aligned} \rho^+_{\text{agg}} &= \sum_i \rho^+_i \, , \quad \rho^-_{\text{agg}} = \sum_i \rho^-_i \quad , \\ \pi^+_{\text{agg}} &= \sum_i \pi^+_i \, , \quad \pi^-_{\text{agg}} = \sum_i \pi^-_i \quad , \\ \epsilon^+_{\text{agg}} &= \sum_i \epsilon^+_i \, , \quad \epsilon^-_{\text{agg}} = \sum_i \epsilon^-_i \quad . \end{aligned} \tag{7}$$

The slow-ramping unit, e.g. a thermal power plant, with $\{\rho, \pi, \epsilon\}_{\text{slow}}$, is assumed to have an unlimited fuel supply, which implies that no energy constraints exist and that the energy provision capability is infinite ($\epsilon_{\text{slow}} \to \infty$). Also, the potential power output π is large. Dynamically slow means in this context that the power ramp-rate ρ is small. The fast-ramping storage unit, e.g. a fly-wheel or battery system, with $\{\rho, \pi, \epsilon\}_{\text{fast}}$, has a limited run-time bounded by energy constraints of the storage unit and thus only a limited energy storage capability exists ($0 < \epsilon_{\text{fast}} \ll \epsilon_{\text{slow}}$).

As is often the case for storage units, ramp-rate ρ is large whereas power capability π is comparatively small. Depending on storage technology, time-dependent storage losses, $v(x)$, can be significant. This is notably the case of fly-wheel energy storage systems, where storage losses due to bearing friction become large when going beyond a storage cycle duration of a few minutes.

D. *Needed versus Available Operational Flexibility*

There are two sides to operational flexibility in power system operation. First, the *needed* flexibility that system operators require for coping with a wide range of power imbalances. Second, the *available* flexibility that system operators can obtain from various flexibility sources (Fig. 1–2).

In general, operational flexibility is needed for balancing out schedule deviations and disturbances coming from load demand as well as conventional and renewable generation units, and of course, all kinds of outages that cause power and power flow imbalances. Matter of fact, the original work of Makarov et al. was concerned with quantifying the necessary operational flexibility for mitigating the feed-in uncertainty of high energy shares of wind power (see [1] as an illustration of needed flexibility for balancing wind forecast errors in the CAISO grid). There, the needed operational flexibility in order to allow operators to always re-balance power feed-in disturbances was quantified by using probabilistic worst-case scenarios and capturing their flexibility requirements using the previously established three, respectively four, flexibility metrics.

The available operational flexibility that a power system unit i can provide depends on this unit's ability to modulate it's power output, i.e. for a generation unit, or power input, i.e. for a consumption unit or both for a two-way storage unit. The flexibility type, i.e. ρ, π and/or ϵ, as well as the actual amount that can be provided is defined by the operation constraints and nominal operating point of the unit i, i.e. $\left[u^0_{\text{gen},i},\ u^0_{\text{load},i}\right]_{k,i}$ at time-step k. What type of operational flexibility can be provided depends clearly on the characteristics of the power system unit in question.

The relationship between needed operational flexibility, as assessed by using deterministic or probabilistic worst-case scenarios etcetera, and the available operational flexibility, as given by an assessment of the capabilities and constraints of the available power system unit pool, is straightforward. The operational flexibility that is available to system operators should be at least as large as the operational flexibility they need for mitigating the (expected) worst-case disturbance as is illustrated by Fig. 10 (a). Clearly, this condition needs to be fulfilled individually for every time-step k and not just on average. Figuratively this means that the cube of needed operational flexibility needs to *fit nicely* into the cube of available operational flexibility as is shown by Fig. 10 (b). In mathematical terms this corresponds to the following six conditions for the flexibility metrics:

$$\rho^{+}_{\text{needed}} \leq \rho^{+}_{\text{available}} \,, \quad \rho^{-}_{\text{needed}} \leq \rho^{-}_{\text{available}} \,, \tag{8}$$

$$\pi^{+}_{\text{needed}} \leq \pi^{+}_{\text{available}} \,, \quad \pi^{-}_{\text{needed}} \leq \pi^{-}_{\text{available}} \,, \tag{9}$$

$$\epsilon^{+}_{\text{needed}} \leq \epsilon^{+}_{\text{available}} \,, \quad \epsilon^{-}_{\text{needed}} \leq \epsilon^{-}_{\text{available}} \,. \tag{10}$$

In case that one of these conditions is violated, figuratively, one of the sides of the (smaller) cube of needed flexibility *sticks* out of the (larger) cube of available flexibility (Fig. 10).

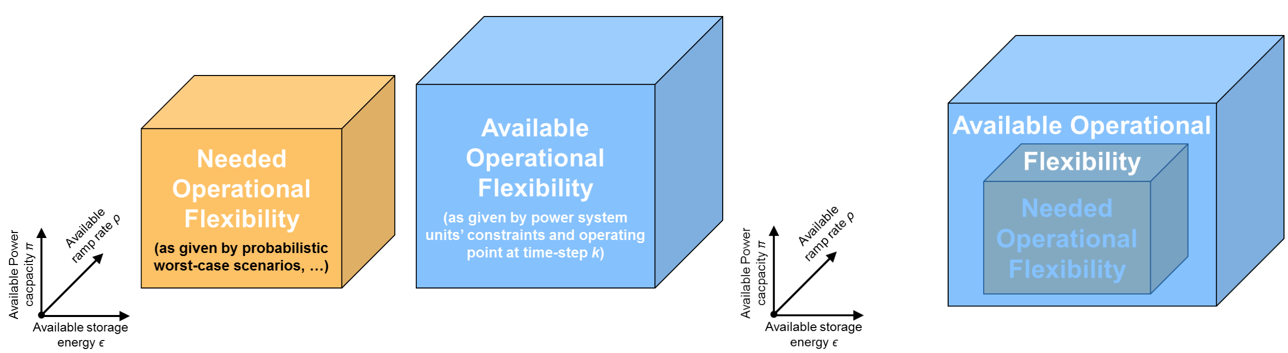

Fig. 10: Needed Operational Flexibility versus Available Operational Flexibility.
(a) Comparison, (b) Necessary condition for robust power system operation.

Effectively balancing disturbances requires the ability to follow steep power ramps and to provide large amounts of regulating power and energy over time. For a given power system to successfully accommodate such disturbance events, the *available* flexibility volume should always envelope the *needed* flexibility volume (Fig. 11). If this is not the case, flexibility is lacking along at least one axis of the flexibility metrics and a disturbance event could not be fully accommodated. Calculating the polytope of the still remaining operational flexibility after mitigating a disturbance event (Fig. 12) requires another polytope operation, the *Pontryagin Difference*.

V. Conclusion

The contributions of this paper are the presented modeling and analysis techniques for the quantitative assessment and visualization of operational flexibility in electric power systems.

These techniques allow in a first phase the modeling and definition of operational flexibility of individual power system units by building up on our previous work on the Power Nodes modeling framework [3], [4] and combining it with the valuable work of others, notably in [1]. In a second phase, the analysis and visualization of the operational flexibility of individual power system units is presented for some illustrative examples. The approaches are, however, also applicable for more complex, larger-scale power system setups. For the later, the illustrated method of aggregation of the operational flexibility from several and different individual power system units is useful. It allows the analysis of the combined flexibility properties of unit pools, in which different power system units are aggregated and work together to achieve a common control objective. The calculation of the *remaining* operational flexibility in a power system after having subtracted the *needed* from the originally *available* operational flexibility was shown for an illustrative case study.

The outlined methods can help power system operators to evaluate the needed flexibility for coping with system disturbances as well as to assess the available flexibility that the currently dispatched unit portfolio can provide for them. We envision that these techniques will become useful tools for system operators, allowing the aggregation of the available – often too plentiful – power system state information into intuitive visual charts, i.e. 3D images of *available* and *needed* operational flexibility cubes, and straight-forward flexibility quantification, i.e. the flexibility metrics $\{\rho, \pi, \epsilon\}$, for the current system state as well as for predicted future system states.

This would notably allow the real-time analysis of the overall flexibility properties of unit pools, in which different power system units are aggregated and work together to achieve a common control objective, e.g. frequency and power balance regulation, but also the calculation of the *remaining* operational flexibility set in a power system after having subtracted the *needed* flexibility for mitigating a disturbance, e.g. forecast error, from the originally *available* operational flexibility. Thereby operators in control centers could better assess in real-time *how close to the limits* the power system is currently operated or would be operated in case of a disturbance event and from which sources flexibility could be drawn in such a scenario for system balancing and restoration [24].

We hope that the implementation of such flexibility visualization and analysis methods will lead to a qualitatively and quantitatively improved (power) system awareness, control and operation.

References

[1] Y. Makarov, C. Loutan, J. Ma, and P. de Mello, "Operational Impacts of Wind Generation on California Power Systems," *Power Systems, IEEE Transactions on*, vol. 24, no. 2, pp. 1039 –1050, May 2009.

[2] A. Ulbig and G. Andersson, "On Operational Flexibility in Power Systems," in *IEEE PES General Meeting, San Diego, USA*, July 2012.

[3] K. Heussen, S. Koch, A. Ulbig, and G. Andersson, "Energy Storage in Power System Operation: The Power Nodes Modeling Framework," in *IEEE PES Conference on Innovative Smart Grid Technologies (ISGT) Europe, Gothenburg*, 2010.

[4] ——, "Unified system-level modeling of intermittent renewable energy sources and energy storage for power system operation," *Systems Journal, IEEE*, vol. 6, no. 1, pp. 140 –151, March 2012.

[5] T. Gül and T. Stenzel, *Variability of Wind Power and other Renewables – Management Options and Strategies.* IEA, Paris, 2005.

[6] L. E. Jones, Ed., *Renewable Energy Integration – Practical Management of Variability, Uncertainty, and Flexibility in Power Grids.* Academic Press (Elsevier), 2014.

[7] T. Weissbach and E. Welfonder, "Improvement of the Performance of Scheduled Stepwise Power Programme Changes within the European Power System," in *17th IFAC World Congress, The International Federation of Automatic Control (IFAC)*, Seoul, Korea, 2008.

[8] US Deptartment of Energy, "Smartgrid.gov," 2014. [Online]. Available: www.smartgrid.gov/the_smart_grid

[9] ENTSO-E, "Operation Handbook," 2011.

[10] S. Backhaus and M. Chertkov, "Getting a grip on the electrical grid," *Physics Today*, vol. 66, no. 5, pp. 42–48, 2013.

[11] M. Bucher, S. Chatzivasileiadis, and G. Andersson, "Managing flexibility in multi-area power systems," *CoRR*, vol. abs/1409.2234, 2014. [Online]. Available: http://arxiv.org/abs/1409.2234

[12] J. Ma, V. Silva, R. Belhomme, D. Kirschen, and L. Ochoa, "Evaluating and planning flexibility in sustainable power systems," *Sustainable Energy, IEEE Transactions on*, vol. 4, no. 1, pp. 200–209, 2013.

[13] E. Lannoye, D. Flynn, and M. O'Malley, "Evaluation of power system flexibility," *Power Systems, IEEE Transactions on*, vol. 27, no. 2, pp. 922–931, 2012.

[14] M. Huber, D. Dimkova, and T. Hamacher, "Integration of wind and solar power in europe: Assessment of flexibility requirements," *Energy*, vol. 69, pp. 236–246, 2014.

[15] C. Besson, *Resources to Reserves.* IEA, Paris, 2005.

[16] E. D. Sontag, "Nonlinear regulation: The piecewise linear approach," *Automatic Control, IEEE Transactions on*, vol. 26, no. 2, pp. 346–358, 1981.

[17] M. Kvasnica, P. Grieder, and M. Baotić, "Multi-Parametric Toolbox (MPT)," 2004. [Online]. Available: http://control.ee.ethz.ch/~mpt/

[18] P. Gagarinov and A. A. Kurzhanskiy, "Ellipsoidal Toolbox Manual (Release 2.0 beta 1)," Tech. Rep., 19 November 2013.

[19] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear matrix inequalities in system and control theory.* SIAM, 1994, vol. 15.

[20] A. A. Kurzhanskiy and P. Varaiya, "Ellipsoidal toolbox," EECS Department, University of California, Berkeley, Tech. Rep. UCB/EECS-2006-46, May 2006. [Online]. Available: http://code.google.com/p/ellipsoids

[21] M. D. Galus, S. Koch, and G. Andersson, "Provision of load frequency control by phevs, controllable loads, and a cogeneration unit," *Industrial Electronics, IEEE Transactions on*, vol. 58, no. 10, pp. 4568–4582, 2011.

[22] N. Lu, M. R. Weimar, Y. V. Makarov, F. Rudolph, S. Murthy, J. Arseneaux, and C. Loutan, "Evaluation of the flywheel potential for providing regulation service in california," in *Power and Energy Society General Meeting, 2010 IEEE.* IEEE, 2010, pp. 1–6.

[23] C. Jin, N. Lu, S. Lu, Y. Makarov, and R. Dougal, "A coordinating algorithm for dispatching regulation services between slow and fast power regulating resources," *Smart Grid, IEEE Transactions on*, vol. PP, no. 99, pp. 1–8, 2013.

[24] D. Kirschen and F. Bouffard, "Keeping the lights on and the information flowing," *Power and Energy Magazine, IEEE*, vol. 7, no. 1, pp. 50–60, 2009.

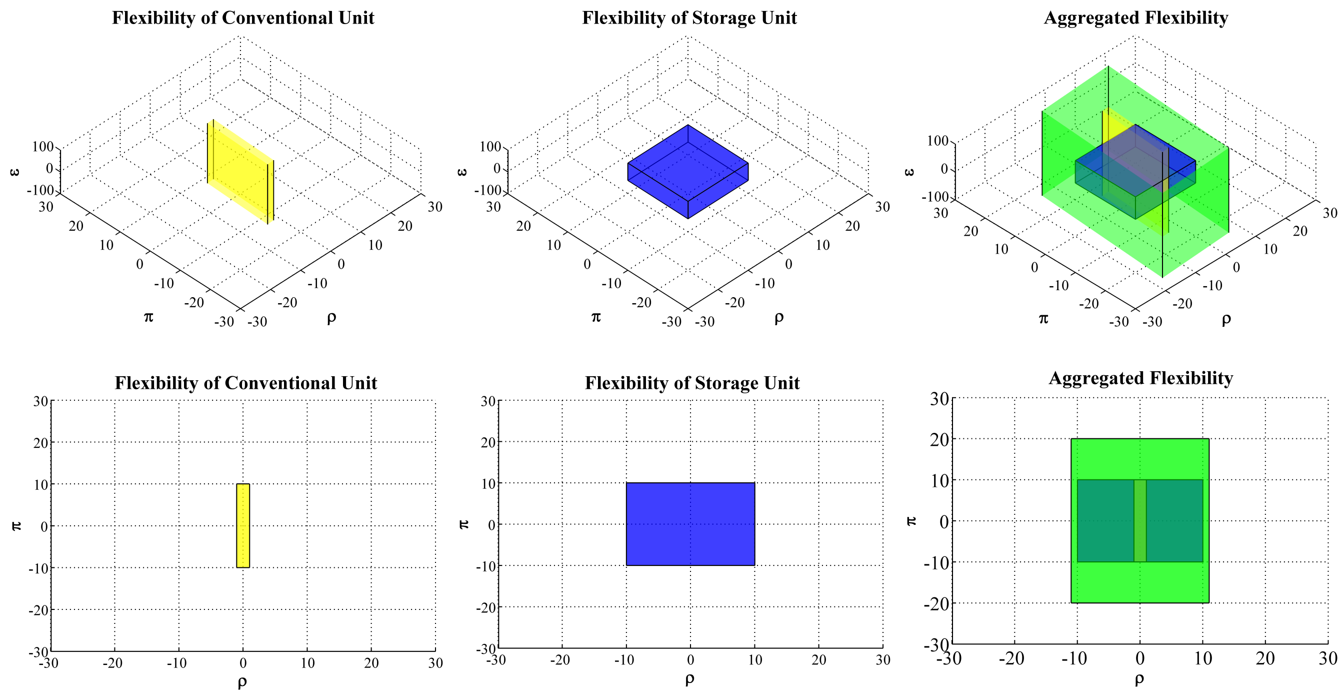

Fig. 11: Aggregation of maximum operational flexibility of individual power system units. Flexibility of conventional unit with no energy constraint (yellow), flexibility of energy-constrained storage (blue) and aggregated flexibility of both units (green).

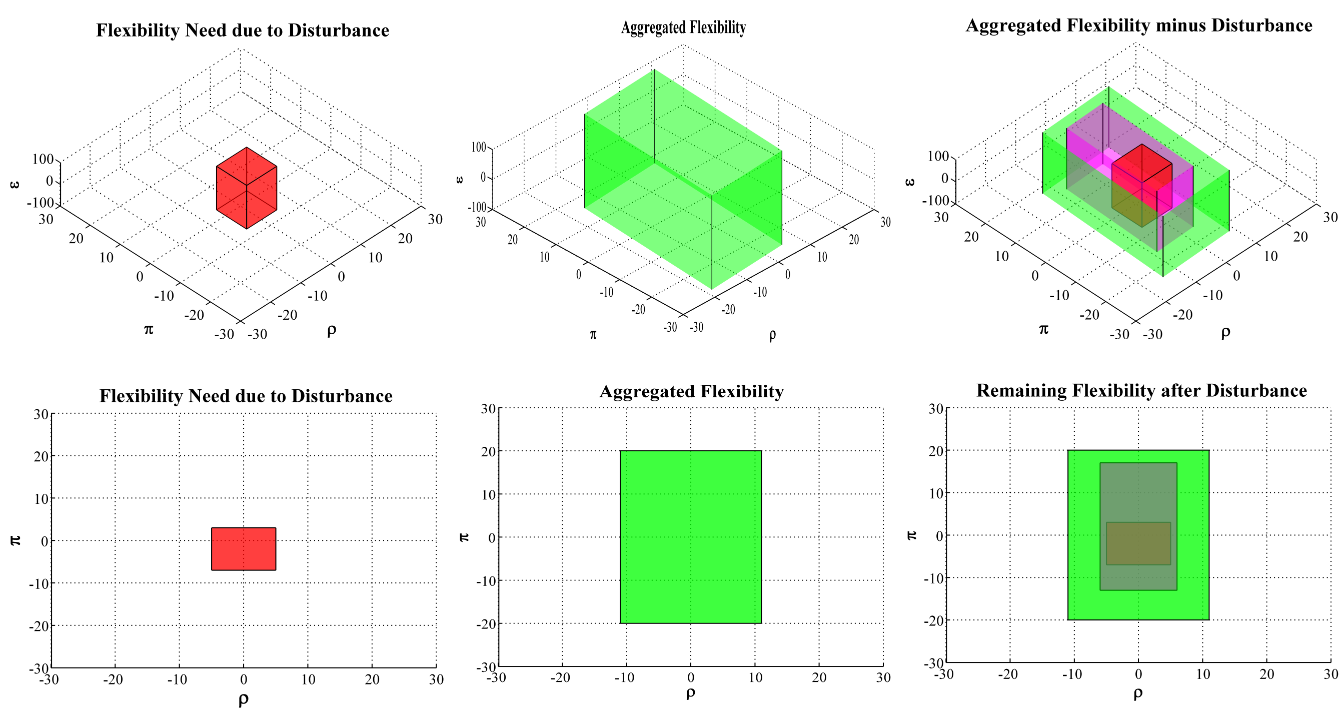

Fig. 12: *Needed* operational flexibility versus *available* operation flexibility.
Needed flexibility volume for balancing a disturbance (red), available flexibility volume (green) and remaining flexibility volume after subtracting the needed flexibility volume (magenta).