



# Operational flexibility of active distribution networks with the potential from data centers

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## HIGHLIGHTS

- The modeling of IT equipment is formulated based on piecewise linearization.
- The spatio-temporal dispatch strategies for workload are proposed.
- An operational flexibility analysis model with data centers is established.
- The potential of data centers is explored to improve the system flexibility.

## ARTICLE INFO

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## ABSTRACT

With the development of information technology, the scale and quantity of internet data centers (IDCs) are expanding rapidly. IDCs have emerged as the major electricity consumers in active distribution networks (ADNs), which dramatically increase the electricity load and have a significant impact on the operational flexibility of ADNs. Geographically distributed IDCs can participate in the operation of ADNs with the potential for spatio-temporal load regulation. This paper proposes flexible dispatch strategies of data centers to improve the operational flexibility of ADNs. **First, a data-power model of IT equipment is proposed based on piecewise linearization to describe the power consumption characteristics of data centers.** The flexible dispatch strategies for the delay-tolerant workload are further proposed from two aspects of temporal transfer and spatial allocation. Then, considering the potential for spatio-temporal load regulation, the operational flexibility analysis model with data centers is formulated to adapt to the operational requirements of ADNs in complex environments. Case studies show that through the spatio-temporal regulation of workload, the energy efficiency of IDCs can be effectively improved. The flexible dispatch of IDCs can also reduce the voltage violation and feeder load imbalance of ADNs, which can facilitate providing the high-quality power supply for IDCs.

## 1. Introduction

With the development of information technology (IT), the scale and quantity of internet data centers (IDCs) are expanding rapidly [1]. IDCs can provide high-performance computing services to a wide range of applications [2]. IDCs have emerged as the major electricity consumers in active distribution networks (ADNs), dramatically increasing the electricity load [3]. In China, the overall energy consumption of IDCs in 2017 exceeded 120 billion kWh, which accounts for about 2% of the total Chinese electricity consumption [4]. By 2025, the total global electricity cost of IDCs will reach 10 billion dollars with an annual

growth rate of 6%. The integration of IDCs brings enormous challenges to the flexible operation of ADNs. Without proper dispatch, the huge energy demand of data centers will lead to a sharp fluctuation of feeder power and voltage violation in ADNs [5].

As workloads with longer service delay tolerance can be flexibly transferred in IDCs, geographically distributed data centers have significant potential for the spatio-temporal load regulation [6]. Thus, IDCs can be utilized as an important demand response (DR) resource to improve the operational flexibility of ADNs [7], which refers to the ability to schedule flexible resources to cope with the complex operation environments in ADNs [8].

Previous works have investigated the characteristics and regulation

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Nomenclature		
<b>Sets</b>		
$\Omega_{ac}, \Omega_{dc}$	Sets of alternating-current nodes and direct-current nodes	
$\mathcal{L}_{ac}, \mathcal{L}_{dc}$	Sets of alternating-current lines and direct-current lines	
<b>Indices</b>		
$i$	Indices of nodes, from 1 to $N_n$	
$f$	Indices of workload types, from 1 to $N_f$	
$\delta$	Indices of front-end servers, from 1 to $N_s$	
$t$	Indices of time periods, from 1 to $N_T$	
<b>Variables</b>		
$\lambda_{i,\delta,f,t}$	Amount of workload of type $f$ allocated from front-end server $\delta$ to data center $i$ in period $t$	
$\lambda_{i,j,f,t}$	Amount of workload of type $f$ allocated from data center $j$ to data center $i$ in period $t$	
$L_{\delta,f,t}, L_{i,f,t}$	Amount of workload of type $f$ received by front-end server $\delta$ or data center $i$ in period $t$	
$\lambda_{i,f,t}^{drop}$	Amount of dropped workload of type $f$ in data center $i$ in period $t$	
$P_{i,t}^{IDC}$	Total power consumption of data center $i$ in period $t$	
$P_{i,t}^{IT}, P_{i,t}^{CO}, P_{i,t}^{LE}$	Power consumption of IT/cooling/electrical equipment in data center $i$ in period $t$	
$m_{i,t}$	Number of active servers in data center $i$ in period $t$	
$\alpha_{i,t,k}, \gamma_{i,t,k}$	Auxiliary variables in piecewise linearization for power consumption curve of IT equipment	
$d_{i,f,t}$	Amount of workload of type $f$ processed in data center $i$ in period $t$	
$E_{i,t}$	Amount of stored workload in data center $i$ in period $t$	
$\mu_i$	Average service rate of servers in data center $i$	
$E_{i,max}$	Data storage capacity in data center $i$	
$M_i$	Total number of servers in data center $i$	
$I_{ij}^{thr}$	Threshold of current magnitude of branch $ij$	
$V_{max}^{thr}, V_{min}^{thr}$	Threshold of voltage magnitude of node $i$	
$I_{ij,max}$	Maximum current magnitude of branch $ij$	
$\Delta\lambda_{i,f,t}$	Amount of changed workload of type $f$ in data center $i$ in period $t$	
$s_{i,t}^{drop}, s_{i,t}^{re}$	Binary variables indicating the sending/receiving status of workload in data center $i$ in period $t$	
$\lambda_{f,t}^{ref}$	Amount of workload of type $f$ that should be processed by the end of period $t$	
$F_{ij,t}^B$	Branch current margin of branch $ij$ in period $t$	
$F_{i,t}^V$	Node voltage margin of node $i$ in period $t$	
$v_{i,t}, l_{ij,t}$	Squared voltage magnitude of node $i$ and current magnitude of branch $ij$ in period $t$	
$P_{ij,t}, Q_{ij,t}$	Active/reactive power flow of branch $ij$ in period $t$	
$P_{i,t}, Q_{i,t}$	Net active/reactive power injection at node $i$ in period $t$	
$P_{i,t}^{DG}, Q_{i,t}^{DG}$	Active/reactive power injection by DG at node $i$ in period $t$	
$P_{i,t}^{VSC}, Q_{i,t}^{VSC}$	Active/reactive power injection by VSC at node $i$ in period $t$	
<b>Parameters</b>		
$N_n, N_L$	Total number of nodes and lines	
$N_T$	Total number of time periods	
$N_s$	Total number of front-end servers	
$N_f$	Total number of workload types	
$D_f$	Delay tolerance time of delay-sensitive workload of type $f$	
$t_f$	Delay tolerance time of delay-tolerant workload of type $f$	
$V_{min}, V_{max}$	Upper/lower limit of node voltage	
$PUE_i$	Power usage effectiveness of data center $i$	
$r_{ij}, x_{ij}$	Resistance/reactance of branch $ij$	
$P_{i,t}^{LD}, Q_{i,t}^{LD}$	Active/reactive power consumption at node $i$ in period $t$	
$A_i$	Loss coefficient of the VSC at node $i$	
$S_{max}^{VSC}$	Capacity of the VSC at node $i$	
$Q_{i,max}^{VSC}, Q_{i,min}^{VSC}$	Upper/lower limit of reactive power of VSC at node $i$	

of data centers to facilitate the optimal operation of ADNs. With the application of dynamic cluster server configuration [9], dynamic voltage and frequency scaling [10] and high-efficient cooling technology [11], the efficiency of workload processing has been greatly improved. Wang et al. in [12] proposed a risk constrained hour-head bidding strategy to minimize the operational costs with various uncertainties. The joint power management of data centers and electric vehicles was considered in [13] to minimize energy costs and maximize regulation service revenue. Yang et al. in [14] proposed a novel data center control framework to smooth the power fluctuation and instability with the integration of DGs. The authors in [15] analyzed the demand response capability of IDCs by shifting the workload temporally. Currently, the researches on IDCs mainly focus on price-based DR with data centers integration [16]. The signals of electricity price guide the temporal distribution of IDC loads to balance regional loads in ADNs [17].

However, there are still some challenges in the load regulation of data centers for flexibility improvement of ADNs. Due to the requirement of construction environment, the regulation potential of data centers is not easily obtainable as the traditional flexible resources. It is essential to coordinate the regulation of IDCs with ADN operation. Then, the dispatch of workload should be considered from both perspectives of the spatial and temporal transfer. Through the spatio-temporal regulation of workload, the energy efficiency of IDCs can be effectively improved. The flexible dispatch of IDCs can also facilitate the operational flexibility improvement of ADNs, which can provide the high-quantity power supply for IDCs.

Besides, as IT equipment in data centers is generally direct-current (DC)-powered servers, DC power supply can facilitate to improve the operation efficiency of data centers [18]. Soft open points (SOPs) are novel power electronic devices to realize the flexible connection and power flow control of feeders [19]. Through the DC link of SOPs [20], data centers can be effectively integrated with improved power supply efficiency and reliability [21].

Considering the regulation potential of data centers, this paper proposes flexible dispatch strategies of data centers to improve the operational flexibility of ADNs. The main contributions of this paper are summarized as follows:

1) First, a data-power model of IT equipment is proposed based on piecewise linearization to describe the power consumption characteristics of data centers. The flexible dispatch strategies for the delay-tolerant workload are further proposed from temporal transfer and spatial allocation.

2) Then, the operational flexibility analysis model with data centers is formulated to improve the operating performance of ADNs. Through the spatio-temporal regulation of workload, the energy efficiency of IDCs can be effectively improved. The flexible dispatch of IDCs can also reduce the voltage violation and feeder load imbalance of ADNs, which can facilitate providing the high-quantity power supply for IDCs.

The remainder of this paper is organized as follows. Section 2 describes the power consumption of data centers, and flexible dispatch strategies for the workload are also analyzed. Section 3 proposes an operational flexibility analysis model with data center integration, in which the voltage margin and feeder load margin are considered. Case

studies based on the modified IEEE 33-node distribution network and a practical distribution network with IDCs are given in Section 4. Conclusions are stated in Section 5.

## 2. Problem formulation with data center integration

In this section, the power consumption model of data centers is formulated, in which the piecewise linearization method is adopted. Besides, the flexible dispatch strategies for the workload are analyzed from two aspects: temporal transfer and spatial allocation.

### 2.1. Power consumption modeling of data centers

The equipment in data centers is mainly divided into IT equipment, cooling equipment and electrical equipment. IT equipment is used for data processing, data storage and communications. Cooling equipment is used to maintain temperature and humidity. The electrical equipment includes lighting systems, power delivery systems, and auxiliary energy systems, which are used for DC-AC conversion and providing reliable and high-quality power for IT equipment. Constraint (1) indicates the total power consumption of data center  $i$ .

$$P_{i,t}^{\text{IDC}} = P_{i,t}^{\text{IT}} + P_{i,t}^{\text{CO}} + P_{i,t}^{\text{L}} \quad (1)$$

1) Data center equipment

(1) Power consumption of IT equipment

As for the power consumption of IT equipment, a dynamic cluster server configuration [9] is adopted, which means the minimum number of servers is activated to process data workload and the remaining servers are idle. Thus, the power consumption of IT equipment can be evaluated by the number of active servers [14], as shown in (2).

$$P_{i,t}^{\text{IT}} = k_i m_i + \beta_{1,i} \quad (2)$$

where  $k_i$  represents the increasement of power consumption of IT equipment with each server activated.  $m_i$  represents the number of active servers in data center  $i$ .  $\beta_{1,i}$  denotes the fixed power consumption of IT equipment.

In practical operation, the power consumption of IT equipment increases nonlinearly with the growth of server utilization [22]. Assuming that all the servers in IDCs are homogenous with the same service efficiency, the characteristics of IT equipment can be described as that the power consumption of IT equipment will increase faster with more servers activated [23]. Thus, the piecewise linearization method is proposed to formulate the power consumption model of IT equipment. The power consumption curve of IT equipment is divided into 3 sections with 4 points in this paper, as shown in (3).

$$P_{i,t}^{\text{IT}} = \begin{cases} k_1(m_{i,t} - m_1) + \beta_{1,i} & m_{i,t} \in [m_1, m_2) \\ k_2(m_{i,t} - m_2) + k_1(m_2 - m_1) + \beta_{1,i} & m_{i,t} \in [m_2, m_3) \\ k_3(m_{i,t} - m_3) + k_2(m_3 - m_2) + k_1(m_2 - m_1) + \beta_{1,i} & m_{i,t} \in [m_3, m_4] \end{cases} \quad (3)$$

where  $m_k$  ( $\forall k \in \{1, 2, 3, 4\}$ ) denotes the x-axis of the 4 points (0, 0.4 $M_i$ , 0.8 $M_i$ ,  $M_i$ ) in the power consumption curve and  $P_k$  ( $\forall k \in \{1, 2, 3, 4\}$ ) denotes their y-axis.  $k_i$  ( $\forall i \in \{1, 2, 3\}$ ) denotes the increasing slope coefficients.

By introducing the continuous variable  $\alpha_{i,t,k}$  ( $k = 1, 2, 3, 4$ ) and binary variable  $\gamma_{i,t,k}$  ( $k = 1, 2, 3$ ), the power consumption curve of IT equipment can be reformulated as follows:

$$\begin{aligned} m_{i,t} &= \sum_{k=1}^4 \alpha_{i,t,k} m_k, \quad P_{i,t}^{\text{IT}} = \sum_{k=1}^4 \alpha_{i,t,k} P_k \\ \alpha_{i,t,1} &\leq \gamma_{i,t,1}, \quad \alpha_{i,t,4} \leq \gamma_{i,t,3} \\ \alpha_{i,t,k} &\leq \gamma_{i,t,k} + \gamma_{i,t,k-1} \quad k = 2, 3 \\ \alpha_{i,t,k} &\geq 0, \quad \gamma_{i,t,k} \in \{0, 1\} \\ \sum_{k=1}^4 \alpha_{i,t,k} &= 1, \quad \sum_{k=1}^3 \gamma_{i,t,k} = 1 \end{aligned} \quad (4)$$

where  $\gamma_{i,t,k}$  represents which part  $m_{i,t}$  belongs to and the auxiliary variable  $\alpha_{i,t,k}$  is used to describe  $P_{i,t}^{\text{IT}}$  in the selected part.

(2) Power consumption of cooling system

To ensure the equipment can operate at a suitable temperature, the cooling system power should be adjusted timely. The power consumption of cooling equipment and electrical equipment will increase as the workload to be processed increases, as shown in (5).

$$P_{i,t}^{\text{CO}} + P_{i,t}^{\text{L}} = \alpha_{1,i} P_{i,t}^{\text{IT}} + \beta_{2,i} \quad (5)$$

where  $\alpha_{1,i}$  and  $\beta_{2,i}$  are empirical constants. The power usage effectiveness (PUE) is widely used to describe the energy efficiency of data centers [11], which represents the ratio of the total power consumption of data centers to the power consumption of IT equipment. In practical operation, the empirical constraint  $\alpha_{1,i}$  can be set as  $\text{PUE}_i - 1$ .

The total power consumption of IDC can be described as follows.

$$P_{i,t}^{\text{IDC}} = \text{PUE}_i P_{i,t}^{\text{IT}} + \beta_{2,i} \quad (6)$$

2) Modeling of workload

(1) Workload balance constraint

The workload can be allocated by the front-end servers to balance the electricity consuming in data centers. The amount of workload allocated by the front-end servers is shown as follows.

$$\sum_{i=1}^{N_n} \lambda_{i,\delta,f,t} = L_{\delta,f,t} \quad \forall \delta, \forall f, \forall t \quad (7)$$

$$\lambda_{i,\delta,f,t} \geq 0, \quad L_{\delta,f,t} \geq 0$$

In addition to receiving workload from the front-end servers, the data center can also transfer the workload among data centers. The amount of workload received from other data centers can be described as follows.

$$\begin{aligned} \sum_{j=1}^{N_n} \lambda_{i,j,f,t} &= L_{i,f,t} \quad \forall i, \forall f, \forall t \\ \lambda_{i,j,f,t} &\geq 0, \quad L_{i,f,t} \geq 0 \end{aligned} \quad (8)$$

where  $\lambda_{i,j,f,t}$  denotes the amount of workload of type  $f$  allocated from data center  $j$  to data center  $i$  in period  $t$ .

(2) Delay processing constraint

Considering different delay requirements, the workload can be divided into two categories:

- a) The delay-sensitive workload should be processed within tens or hundreds of milliseconds.
- b) The delay-tolerant workload can be scheduled to any time slot before its deadline.

Due to the strict delay requirement, the delay-sensitive workload should be processed at the period when it is received. The amount of workload to be processed is equal to the received workload minus the transferred workload in time period  $t$ , as shown in (9).

$$d_{i,f,t} = \sum_{\delta=1}^{N_s} \lambda_{i,\delta,f,t} + \sum_{j=1}^{N_n} \lambda_{i,j,f,t} - \lambda_{i,f,t}^{\text{drop}} \quad \forall i, j \neq i \quad (9)$$

$$d_{i,f,t} \geq 0$$

where  $\lambda_{i,f,t}^{\text{drop}}$  denotes the amount of dropped workload of type  $f$  in data center  $i$  in time period  $t$ .

Due to the strict delay requirement of the delay-sensitive workload, the queueing models are often adopted to model the queueing delay within one time period. Thus, the M/M/1 model is adopted [24], in which the amount of received workload satisfies the Poisson distribution and the delaying time satisfies the exponent distribution. The constraint of processing delay can be described as follows.

$$0 \leq \frac{1}{\mu_i - d_{i,f,t}/m_{i,t}} \leq D_f \quad (10)$$

Constraint (10) can be further modified to indicate the required number of active servers to satisfy the tolerant service delay.

$$m_{i,t} \geq \frac{d_{i,f,t}}{\mu_i - 1/D_f} \quad (11)$$

### (3) Computing capability constraint

As the number of servers and the computing resources in each data center are limited, sufficient computing resources should be provided to process the workload.

$$\sum_{f=1}^{N_f} d_{i,f,t} \leq CR_{i,t} \quad \forall i, \forall t \quad (12)$$

where  $CR_{i,t}$  denotes the provided computing resources of data center  $i$  in period  $t$ .

$CR_{i,t}$  can be calculated by the number of active servers and the average service rate of servers in data center  $i$ , shown as follows.

$$\begin{aligned} CR_{i,t} &= \mu_i m_{i,t} \quad \forall i, \forall t \\ m_{i,t} &\leq M_i \end{aligned} \quad (13)$$

## 2.2. Flexible regulation of workload

By optimizing the power consumption of data centers, workload regulation can effectively improve the operational state of ADNs. The data centers are with the potential of regulating workload temporally and spatially, as shown in Fig. 1.

### 1) Temporal transfer of workload

In terms of the temporal transfer of workload, delay-tolerant workload can be processed after some time periods of delay. The temporal transfer strategies of workload are described as follows.

$$\Delta \lambda_{i,f,t} = \sum_{\delta=1}^{N_s} \lambda_{i,\delta,f,t} + \sum_{j=1}^{N_n} \lambda_{i,j,f,t} - \lambda_{i,f,t}^{\text{drop}} - d_{i,f,t} \quad (14.a)$$

$$E_{i,f,t} = E_{i,f,t-1} + \Delta \lambda_{i,f,t} \Delta t \quad \forall i, \forall f, \forall t \quad (14.b)$$

$$E_{i,f,T} = E_{i,f,t_0} \quad \forall i, \forall f \quad (14.c)$$

$$0 \leq \sum_{f=1}^{N_f} E_{i,f,t} \leq E_{i,\max} \quad \forall i, \forall t \quad (14.d)$$

$$E_{i,f,t} \geq 0 \quad \forall i, \forall f, \forall t \quad (14.e)$$

Constraint (14.a) indicates the variation of workload in data center  $i$ . Constraint (14.b) indicates the relationship of stored workload in data center  $i$  in different time periods. Constraint (14.c) ensures that data center  $i$  should process all the received workloads within the operating cycle. Constraints (14.d) and (14.e) ensure that the workload stored in data center  $i$  is within its storage capacity.

In practical operation, the data service customers and IDCs should firstly develop the Service level agreement (SLA) to determine the quality of service (QoS), such as the type of data processing service, the deadline of service and the amount of workload. On the basis of satisfying QoS, the workload should be processed within finite periods,

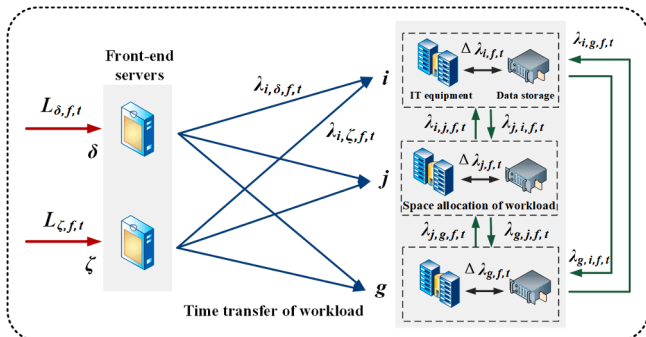


Fig. 1. Flexible regulation of workload in data centers.

shown as (15) and (16).

$$\sum_{t=1}^i \sum_{i=1}^{N_n} d_{i,f,t} \geq \lambda_{f,i}^{\text{ref}} \quad \forall f, \forall t \quad (15)$$

$$\lambda_{f,i}^{\text{ref}} = \begin{cases} \sum_{t=t_0}^{t-t_f} \sum_{\delta=1}^{N_s} L_{\delta,f,t} & \forall f, t - t_0 \geq t_f \\ 0 & \forall f, t - t_0 < t_f \end{cases} \quad (16)$$

where  $\lambda_{f,i}^{\text{ref}}$  denotes the amount of workload of type  $f$  that should be processed at the end of time period  $t$ . And  $t_f$  denotes the delay tolerance time of workload of type  $f$ .

### 2) Spatial allocation of workload

The spatial allocation strategies of workload can be described as follows.

$$\lambda_{i,f,t}^{\text{drop}} = \sum_{j=1}^{N_n} \lambda_{j,i,f,t} \quad \forall i, j \neq i, \forall f, \forall t \quad (17)$$

In addition, as the data centers cannot send and receive workload at the same time, the constraints of (18) are added.

$$\begin{aligned} L_{i,f,t} &\leq s_{i,t}^{\text{re}} \sum_{\delta=1}^{N_s} L_{\delta,f,t} \quad \forall i, \forall f, \forall t \\ \lambda_{i,f,t}^{\text{drop}} &\leq s_{i,t}^{\text{drop}} \sum_{\delta=1}^{N_s} L_{\delta,f,t} \quad \forall i, \forall f, \forall t \\ s_{i,t}^{\text{drop}} + s_{i,t}^{\text{re}} &= 1 \quad \forall i, \forall t \end{aligned} \quad (18)$$

where  $s_{i,t}^{\text{re}}$  and  $s_{i,t}^{\text{drop}}$  denote the receiving and sending status of workload in data center  $i$  in period  $t$ . When  $s_{i,t}^{\text{re}}$  is 1, it means that data center  $i$  receives workload from other data centers. When  $s_{i,t}^{\text{drop}}$  is 1, it means that data center  $i$  transfers workload to other data centers.

## 3. Operational flexibility improvement with IDC regulation

As a new type of DR, data centers can adjust feeder loads by transferring workload across different geographical locations. Data centers can also mitigate load fluctuation by delaying the processing of data workload. The adjustment ability of data centers will effectively improve the operational flexibility of ADNs.

### 3.1. Index of operational flexibility

To quantify the operational flexibility of ADNs, branch current margin and node voltage margin are adopted as indexes [25]. Branch load margin  $F_{ij,t}^B$  is described as follows.

$$f_B = \sum_{t=1}^{N_T} \sum_{ij=1}^{N_L} F_{ij,t}^B \quad (19)$$

The related constraints of branch current margin  $F_{ij,t}^B$  are added as follows.

$$\begin{aligned} F_{ij,t}^B &\geq l_{ij,t} - (I_{ij}^{\text{thr}})^2 \\ F_{ij,t}^B &\geq 0 \end{aligned} \quad (20)$$

where  $l_{ij,t}$  denotes the square of current value on branch  $ij$  in period  $t$ , and  $I_{ij}^{\text{thr}}$  denotes the threshold of branch current. When the branch current exceeds the expected range  $[0, I_{ij}^{\text{thr}}]$ ,  $f_B$  will take effect to adjust the workload of data centers to mitigate the feeder load fluctuation.

The constraints of voltage margin are shown as follows.

$$\begin{aligned} f_V &= \sum_{t=1}^{N_T} \sum_{i=1}^{N_n} F_{i,t}^V \\ F_{i,t}^V &\geq v_{i,t} - (V_{\max}^{\text{thr}})^2, \quad F_{i,t}^V \geq -v_{i,t} + (V_{\min}^{\text{thr}})^2 \\ F_{i,t}^V &\geq 0 \end{aligned} \quad (21)$$

where  $v_{i,t}$  denotes the square of the voltage value of node  $i$ .  $[V_{\min}^{\text{thr}},$

$V_{\max}^{\text{thr}}$  denotes the expected range of the node voltage value. When the node voltage exceeds the expected range  $[V_{\min}^{\text{thr}}, V_{\max}^{\text{thr}}]$ , the margin index  $f_v$  will take effect to reduce the voltage deviation.

Branch current margin, node voltage margin, power consumption of IDCs, and network power loss are used as the objectives of the flexibility analysis model.

$$\begin{aligned} \min f &= \omega_1 f_B + \omega_2 f_V + \omega_3 f_P + \omega_4 f_I \\ f_P &= \sum_{i=1}^{N_T} \sum_{j=1}^{N_B} P_{i,j}^{\text{IDC}} \\ f_I &= \sum_{i=1}^{N_T} \sum_{j=1}^{N_L} r_{ij} l_{ij,t} \end{aligned} \quad (22)$$

where  $\omega_1, \omega_2, \omega_3$  and  $\omega_4$  are the weight coefficients of the objective terms, satisfying that  $\omega_1 + \omega_2 + \omega_3 + \omega_4 = 1$ . The weight coefficient of power losses  $\omega_4$  is set as a relatively small positive value in this paper. Thus, when the branch current and node voltage deviation exceed the expected range, the objective terms  $f_B, f_V$  and  $f_P$  will take the primary effect to reduce the voltage violation, feeder load imbalance and power consumption of IDCs. Otherwise, the objective terms  $f_B$  and  $f_V$  will take no effect and the objective function only involves  $f_P$  and  $f_I$  to improve the system operation economy.

### 3.2. Operational flexibility analysis model with data centers

By fully exploring the potential of workloads in temporal and spatial regulation, the power consumption in data centers can be rapidly adjusted to adapt to the operational requirements of ADNs in complex environments.

#### 1) Operation constraints of distribution networks

Operation constraints of distribution networks mainly include power flow constraints and secure operation constraints [26]. The DistFlow model is used to describe the power flow constraints of distribution networks. By variable substitution and convex relaxation, the original model is converted into a second-order cone programming form as follows.

$$\begin{aligned} \sum_{ij \in \mathcal{L}_{ac}} (P_{ij,t} - r_{ij} l_{ij,t}) + P_{j,t} &= \sum_{jh \in \mathcal{L}_{ac}} P_{jh,t} \\ \sum_{ij \in \mathcal{L}_{ac}} (Q_{ij,t} - x_{ij} l_{ij,t}) + Q_{j,t} &= \sum_{jh \in \mathcal{L}_{ac}} Q_{jh,t} \\ v_{i,t} - v_{j,t} + (r_{ij}^2 + x_{ij}^2) l_{ij,t} &= 2(r_{ij} P_{ij,t} + x_{ij} Q_{ij,t}) \quad i, j \in \Omega_{ac}, ij \in \mathcal{L}_{ac} \\ \|2P_{ij,t} - 2Q_{ij,t} l_{ij,t} - v_{i,t}\| &\leq l_{ij,t} + v_{i,t} \quad i \in \Omega_{ac}, ij \in \mathcal{L}_{ac} \\ P_{j,t} &= P_{j,t}^{\text{DG}} - P_{j,t}^{\text{IDC}} - P_{j,t}^{\text{LD}} + P_{j,t}^{\text{VSC}} \quad j \in \Omega_{ac} \\ Q_{j,t} &= Q_{j,t}^{\text{DG}} - Q_{j,t}^{\text{LD}} + Q_{j,t}^{\text{VSC}} \quad j \in \Omega_{ac} \end{aligned} \quad (23)$$

As for DC links, the related power flow constraints are described as follows.

$$\begin{aligned} \sum_{ij \in \mathcal{L}_{dc}} (P_{ij,t} - r_{ij} l_{ij,t}) + P_{j,t} &= \sum_{jh \in \mathcal{L}_{dc}} P_{jh,t} \\ v_{i,t} - v_{j,t} + r_{ij}^2 l_{ij,t} &= 2r_{ij} P_{ij,t} \quad i, j \in \Omega_{dc}, ij \in \mathcal{L}_{dc} \\ \|2P_{ij,t} l_{ij,t} - v_{i,t}\| &\leq l_{ij,t} + v_{i,t} \quad i \in \Omega_{dc}, ij \in \mathcal{L}_{dc} \\ P_{j,t} &= P_{j,t}^{\text{DG}} - P_{j,t}^{\text{IDC}} - P_{j,t}^{\text{LD}} + P_{j,t}^{\text{VSC}} \quad j \in \Omega_{dc} \end{aligned} \quad (24)$$

Secure operation constraints of distribution networks are described as follows.

$$(V_{\min})^2 \leq v_{i,t} \leq (V_{\max})^2 \quad (25)$$

$$l_{ij,t} \leq (I_{ij,\max})^2 \quad (26)$$

#### 2) Operation constraints of data centers

Considering the power consumption and flexible dispatch strategies of workloads, the operational constraints of data centers are described as constraints (4), (6) - (9) and (11) - (18). The adjustable power range of

IDC can be expressed as:

$$P_{i,t,\min}^{\text{IDC}} \leq P_{i,t}^{\text{IDC}} \leq P_{i,t,\max}^{\text{IDC}} \quad (27)$$

#### 3) Operation constraints of SOP

The direct-current power supply of data centers is achieved through the DC link of SOPs. Considering the AC/DC conversion [27], the operation constraints of voltage source converter (VSC) in SOPs are described as follows.

$$P_{i,t}^{\text{VSC}} + P_{j,t}^{\text{VSC}} + A_i |P_{i,t}^{\text{VSC}}| = 0 \quad i \in \Omega_{ac}, j \in \Omega_{dc} \quad (28)$$

$$\left\| \frac{P_{i,t}^{\text{VSC}}}{Q_{i,t}^{\text{VSC}}} \right\| \leq S_{\max}^{\text{VSC}} \quad i \in \Omega_{ac} \quad (29)$$

$$Q_{i,\min}^{\text{VSC}} \leq Q_{i,t}^{\text{VSC}} \leq Q_{i,\max}^{\text{VSC}} \quad i \in \Omega_{ac} \quad (30)$$

$U_{dc}$ -Q-PQ control is selected as the control mode of SOPs. The converter VSC1 of SOP stabilizes the DC voltage, and the other converter VSC2 controls the power flow of the connected feeder.

In summary, the flexibility improvement model with data center integration can be established as follows. The overall model is formulated as a mixed-integer second-order cone programming (MISOCP) model to ensure the feasibility and optimality of the solution. The proposed MISOCP model can be effectively solved by commercial solvers, such as CPLEX.

$$\begin{aligned} \min f &= \omega_1 f_B + \omega_2 f_V + \omega_3 f_P + \omega_4 f_I \\ \text{s.t. } &(4), (6) - (9), (11) - (30) \end{aligned} \quad (31)$$

In practical operation, through regulating the workload temporally and spatially, the power demand of data centers can be adjusted to improve the operational flexibility of ADNs. Specifically, from the perspective of temporal transfer, through storing and deferring the workload with longer service delay tolerance, the peak load can be shifted in time scales to avoid the voltage violation and feeder load imbalance. From the perspective of spatial allocation, the data center loads can be adjusted to balance the regional load and reduce the power loss of ADNs. Through the flexible dispatch of workload, the voltage violation and feeder load imbalance are effectively reduced to improve the operational flexibility of ADNs.

## 4. Case studies and analysis

In this section, the effectiveness of the flexible dispatch strategy of data centers is verified on the modified IEEE 33-node distribution network. The proposed model is implemented in the YALMIP optimization toolbox with MATLAB R2014a and solved by CPLEX 12.6. Numerical experiments were carried out on a computer with an Intel Xeon CPU E5-1620 processor running at 3.70 GHz with 32 GB of RAM.

### 4.1. Modified IEEE 33-node distribution network

The test case is based on the modified IEEE 33-node distribution network, as shown in Fig. 2. The rated voltage level is 12.66 kV. Total active and reactive power demands are 3715.0 kW and 2300.0 kvar. Detailed parameters can refer to [28].

Two VSCs of SOP with a capability of 3.0 MVA are installed to connect nodes 28 and 29, as well as nodes 18 and 33. The data centers are integrated in the 10 kV-DC links of SOPs, located at nodes 29, 30 and 33. The workload is delivered by two front-end servers to data centers, of which the typical daily curve is shown in Fig. 3 [29]. The detailed parameters of IDC are listed in Table 1.

To consider the impact of distributed generators (DGs) integration [30], two photovoltaics (PV) units with 800.0 kW and two wind turbines (WT) units with 500.0 kVA are integrated into the network at nodes 14, 17, 15 and 32, respectively. The power factor of DGs is assumed to be



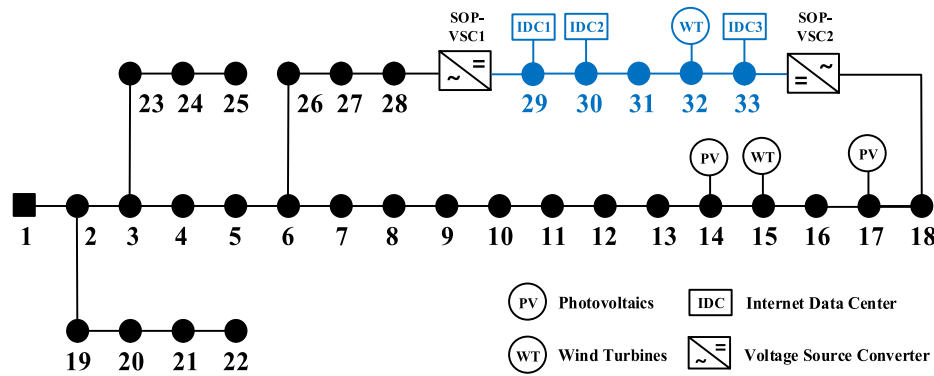


Fig. 2. Structure of the modified IEEE 33-node network.

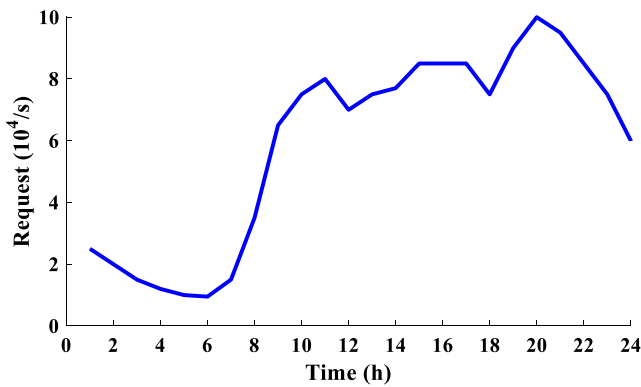


Fig. 3. Typical daily workload curve.

**Table 1**  
Parameter of Internet data centers.

Parameter	Value
Number of servers in each sub-data center	2500
Power consumption of each active server in different segments of curve	0.25/ 0.35/ 0.4 kW
Fixed power consumption of IT equipment	50 kW
PUE of data centers	1.2
Workload processing efficiency of each server	30 requests/s
Maximum stored workload in data centers	$1.5 \times 10^9$ requests

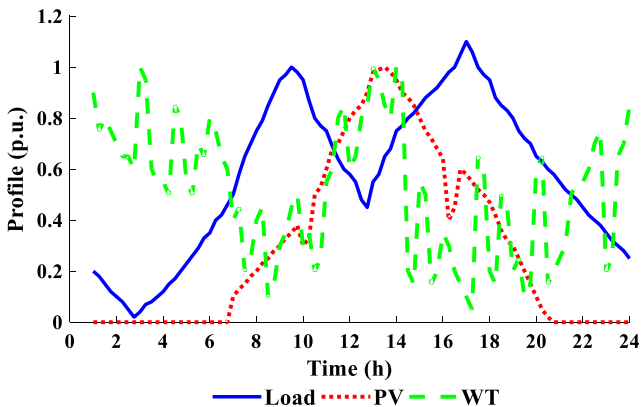


Fig. 4. Operation curves of load and DG.

1.0. The daily DG and load operation curves are given in Fig. 4 [25], in which 15 min is used as the unit time period.

The upper and lower limits of the secure voltage range are set as 1.07 p.u. and 0.93 p.u. The coefficients  $\omega_1$ ,  $\omega_2$ ,  $\omega_3$  and  $\omega_4$  are set as 0.4, 0.4, 0.19 and 0.01, which can be adjusted based on the requirement of distribution system operators (DSO).

The lower and upper limits of the desired voltage range are set to 0.97 p.u. and 1.03 p.u., respectively. It is assumed that the threshold of desired current load level is 0.5 p.u., which can be adjusted according to the risk assessment of DSO.

In distribution networks, the branches near substations have greater current limits than those far from substations. Based on the branch current in normal operation, Table 2 lists the current limits of the branches.

#### 4.2. Result analysis of data center integration

Through the flexible dispatch of workloads in data centers, the voltage violation and feeder load imbalance are effectively reduced to improve the operational flexibility of ADNs.

Two scenarios are used to verify the effectiveness of the dispatch strategy of data centers.

a) Scenario I: The initial state of ADNs is obtained without regulation of data centers and SOP.

b) Scenario II: The flexible dispatch strategy of data centers is conducted to improve the operational flexibility of ADNs.

##### 1) Flexible dispatch of data centers

With the rapid increase of workload, active servers can be flexibly adjusted in data centers to achieve load regulation, as shown in Fig. 5. It can be seen that the number of active servers is relatively larger between the periods 11 a.m.–2 p.m., corresponding to the high level of workload.

Fig. 6 shows the stored workload in IDCs in Scenario II. It shows that the workload processing can be deferred to adapt to the system operation requirements. Take the period between 6 p.m. and 12 p.m. as an example. Without proper regulation, the high workload demand leads to an increase of the power consumption of IDCs. By shifting the peak load demand to the off-peak period between 1 a.m. and 6 a.m., the workload can be redistributed to avoid sharp fluctuation of feeder power and undesirable voltage violation.

The system voltage profiles at 6 p.m. are shown in Fig. 7. In Scenario

**Table 2**  
Branch current limitation configurations.

Rated branch current (A)	Branch current in normal operation (A)
100	<60
200	60 – 120
400	120 – 240
800	240 – 480
1200	≥ 480

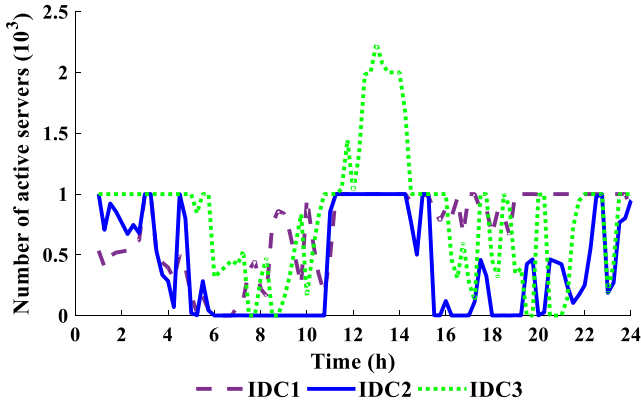


Fig. 5. Active servers in IDCs in Scenario II.

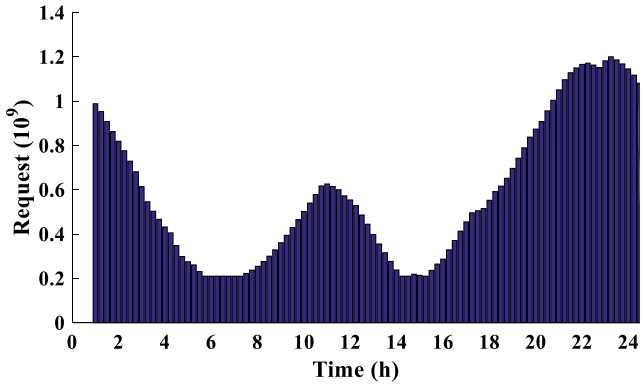


Fig. 6. Stored workload of IDCs in Scenario II.

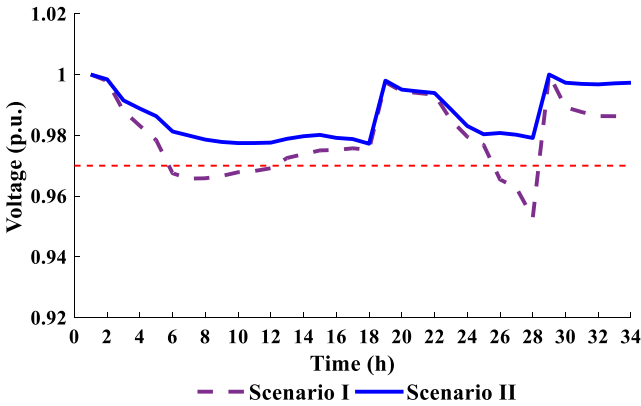


Fig. 7. System voltage profiles at 6 p.m. in Scenarios I and II.

II, the voltage deviation is effectively reduced. The fluctuation conditions of voltage and feeder loading are shown in Fig. 8 and Fig. 9. As node 28 and line 27 are located at the end of the feeder, the voltage deviation and feeder load imbalance are more obvious. The results show that through regulating the workload, the voltage can be maintained within the desired level and the unbalanced feeder loading can be significantly reduced.

Besides, the operation of SOPs can be coordinated with the regulation of IDCs, rapidly responding to the changes of system states. The operation strategies of SOPs are shown in Fig. 10.

## 2) Operational flexibility improvement of ADNs

The optimization results of Scenarios I and II are listed in Table 3. It can be seen that through the flexible dispatch of workload in IDCs, the

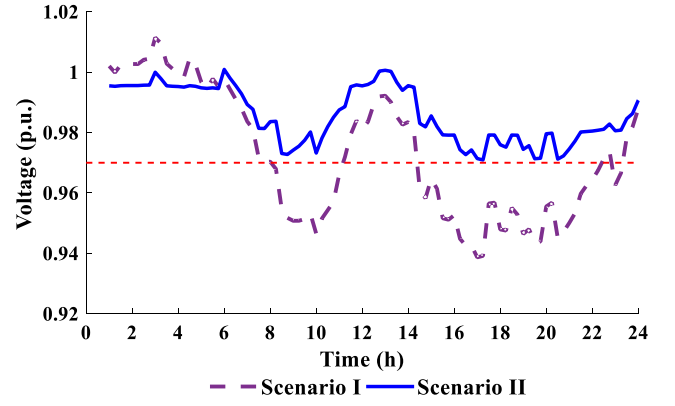


Fig. 8. Voltage profiles of node 28 in Scenarios I and II.

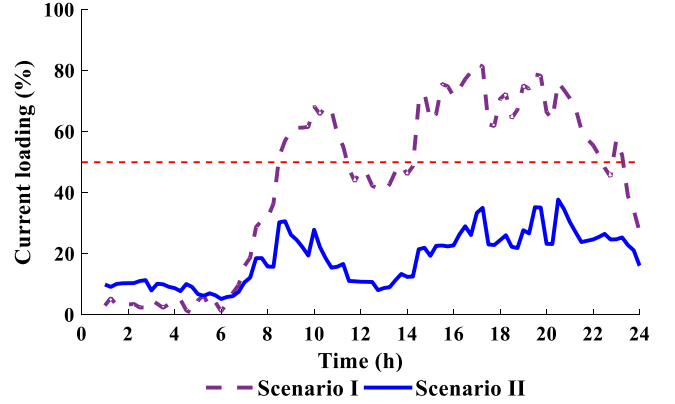


Fig. 9. Current loading levels of line 27 in Scenarios I and II.

voltage is maintained within the desired level and the maximum current loading is reduced by 48.53% relative to that of Scenario I. The total power loss of the system is reduced by 62.50%, which is a considerable improvement of operational flexibility. With flexible workload dispatch spatially and temporally, the power consumption of IDCs is reduced by 15.23%. Fig. 11 shows the power consumption of IDCs over a day under different scenarios and the network power losses are shown in Fig. 12.

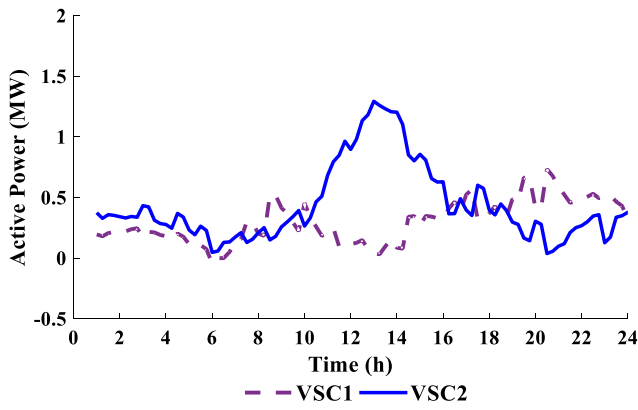
Fig. 13 shows the maximum current loading of each line over a day for Scenarios I and II. Feeder load imbalance is exacerbated in Scenario I, due to the high penetration of DGs and integration of IDCs. The data centers flexibly adjust the workload through time transfer and space allocation in Scenario II, which maintains the current loading within the desired level and significantly mitigates the feeder load imbalance.

Compared with Scenario I, the regulation of workload in IDCs in Scenario II can also effectively alleviate the system voltage deviation. The maximum and minimum system voltages in each time period are shown in Fig. 14. It can be seen that a fairly flat voltage profile is attained by the flexible dispatch of workload in IDCs.

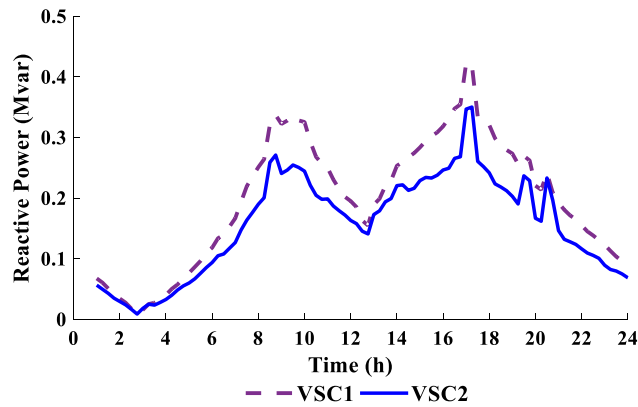
The above analysis shows that the flexible dispatch of workload in IDCs can effectively reduce the power consumption of data centers and alleviate the feeder load imbalance and voltage deviation. The operational flexibility of ADNs is significantly improved with the regulation of IDCs.

## 4.3. Practical distribution network with IDC integration

The structure of a practical distribution network with data centers is shown in Fig. 15. Three data centers are integrated in the DC links. Two SOPs are installed to provide power supply to the data centers. The capability of converters in SOPs is 10.0 MVA. A PV station with 2500.0



(a) Active power transmission



(b) Reactive power compensation

Fig. 10. Operation strategies of SOP in Scenario II.

**Table 3**  
Operation results of the modified IEEE 33-node distribution network.

Scenario	Minimum voltage (p. u.)	Maximum voltage (p. u.)	Maximum current loading (%)	Power losses (MWh)	IDC power consumption (MWh)
I	0.9387	1.0734	92.97	1.9988	20.7265
II	0.9700	1.0255	47.85	0.7496	17.5707

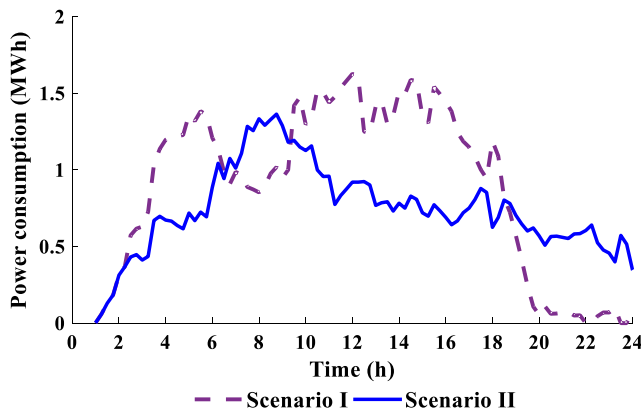


Fig. 11. Power consumption of IDCs in Scenarios I and II.

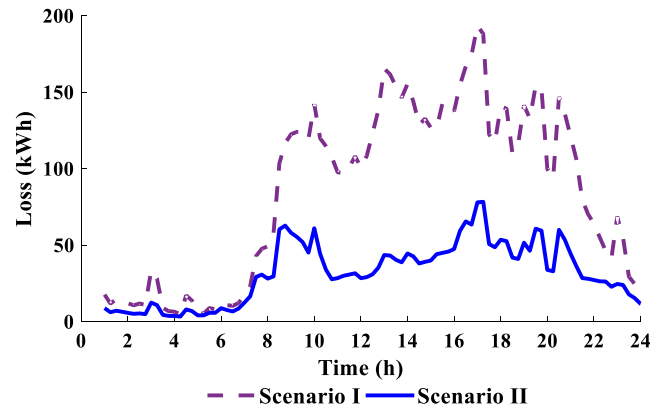


Fig. 12. Active power losses in Scenarios I and II.

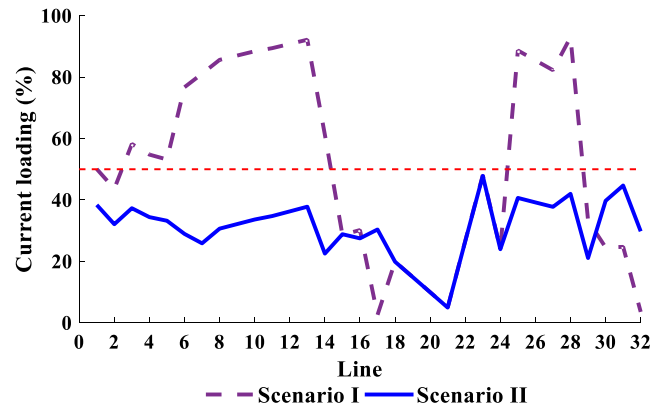


Fig. 13. Maximum current loading of each line in Scenarios I and II.

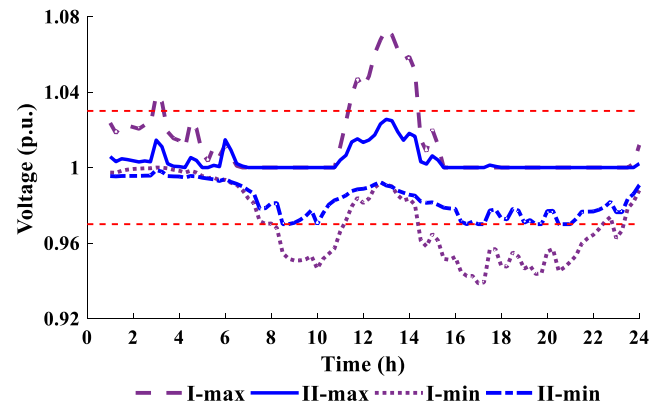


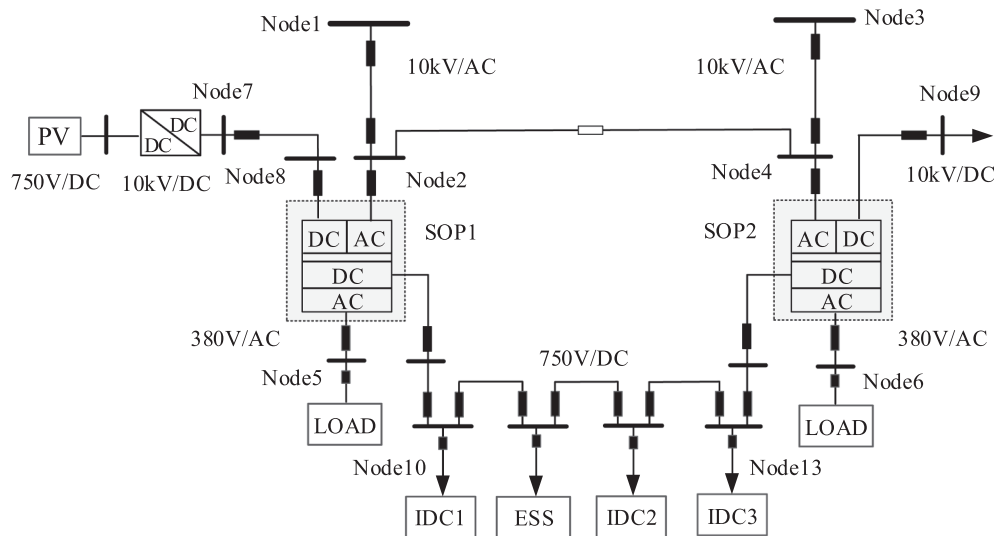
Fig. 14. Maximum and minimum system voltages in Scenarios I and II.

kVA capacity and an ESS with the capacity of 3 MWh are integrated into the distribution networks. The detailed parameters of IDCs are identical with those in the modified IEEE 33-node network. Two scenarios in subsection 4.2 are also adopted to verify the effectiveness of the flexible dispatch strategy of data centers.

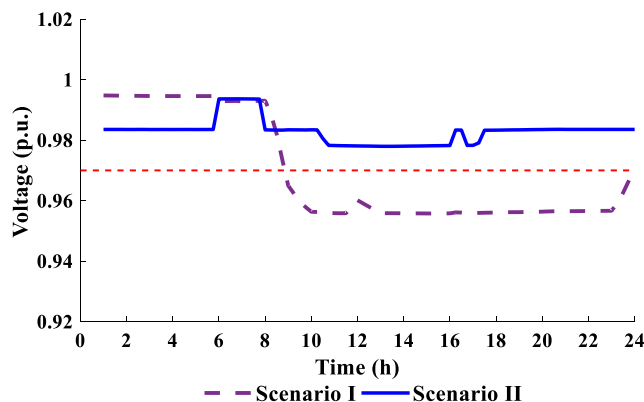
The voltage of node 12 and the maximum current loading of each period are shown in Fig. 16 and Fig. 17. It can be seen that through regulating the workload in IDCs, the voltage violation and feeder load imbalance can be effectively reduced to improve the operational flexibility of ADNs.

The operation results of the practical distribution network with IDCs are listed in Table 4. It can be seen that the proposed regulation strategies of IDCs have a good performance in the practical distribution

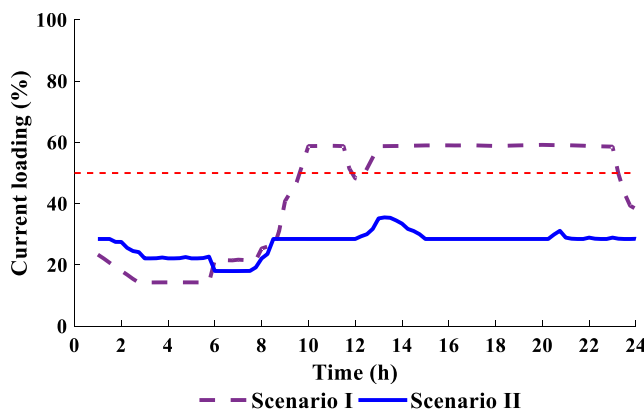




**Fig. 15.** Structure of distribution network with data centers.



**Fig. 16.** Node voltage profiles of node 12 in Scenarios I and II.



**Fig. 17.** Maximum current loading of each period in Scenarios I and II.

**Table 4**  
Operation results of the practical distribution network with IDCs.

Scenario	Minimum voltage (p. u.)	Maximum voltage (p. u.)	Maximum current loading (%)	Power losses (MWh)	IDC power consumption (MWh)
I	0.9558	1.0247	59.19	2.1416	39.7260
II	0.9780	1.0247	35.52	0.8766	35.8705

network. The power consumption of IDCs and the total power loss are reduced by 9.71% and 59.07%, respectively. Through the spatio-temporal regulation of workload, the energy efficiency of IDCs and operational flexibility of ADNs can be effectively improved.

## 5. Conclusions

With the increasing integration of data centers, the huge energy consuming of IDCs puts forward a higher requirement for the operational flexibility in ADNs. Considering the spatio-temporal regulation potential of workload, this paper proposes flexible dispatch strategies of data centers to improve the operational flexibility of ADNs. First, a data-power model of IT equipment is proposed based on piecewise linearization to describe the power consumption characteristics of data centers. The flexible dispatch strategies for the delay-tolerant workload are further proposed from temporal transfer and spatial allocation. Then, considering the potential for spatio-temporal load regulation, the operational flexibility analysis model with data centers is formulated to adapt to the various operation requirements of ADNs. Results show that through the spatio-temporal regulation of workload, the energy efficiency of IDCs can be effectively improved. The flexible dispatch of IDCs can also reduce the voltage violation and feeder load imbalance of ADNs, which can facilitate providing the high-quality power supply for IDCs.

There are several directions for our future works. With the rapid growth of diverse information services, the regulation of data centers should consider the impact of cyber systems, which have a significant influence on data dispatch. Considering the uncertainties of workloads, another issue is to investigate the robust operation strategy of data centers. The aim is to further improve the adaptability of data center regulation.

## CRedit authorship contribution statement

**Sirui Chen:** Data curation, Writing - original draft. **Peng Li:** Conceptualization, Methodology. **Haoran Ji:** Visualization, Writing - review & editing. **Hao Yu:** Investigation, Software. **Jinyue Yan:** Supervision. **Jianzhong Wu:** Validation. **Chengshan Wang:** Project administration.

### Declaration of Competing Interest

The authors declare that they have no known competing financial

interests or personal relationships that could have appeared to influence the work reported in this paper.

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