Give more details about the equations from each of the following papers. Here are the paper titles and sets of equations. use the context provided to give more details on each of them.

Consensus Equations for Data-Centre Cooling Flexibility

(Re-organised paper-by-paper, with plain-English notes)

Paper 1: D. Chen et al., "Adaptive Physically Consistent Neural Networks for Data-Centre Thermal Modelling", Applied Energy 377 (2025) 124637

Scope. Data-hall air $\leftrightarrow$ IT heat transfer; compares PCNN, G-PCNN, A-PCNN variants.

Eq. ID	Equation	What it captures
(1) PCNN air-temperature update	$T_{\mathrm{DH}}^{k+1} = T_{\mathrm{DH}}^{k} - (\dot{m}_{\mathrm{air}} c_p)(T_{\mathrm{DH}}^{k} - T_{\mathrm{SA}}) + \dot{m}_{\mathrm{air}} c_p PI_{\mathrm{TE}}^{k} + \dot{m}_{\mathrm{air}} c_p \sum_i h_i A_i (T_{\mathrm{si}} - T_{\mathrm{DH}}^{k})$	One-step Euler discretisation of energy balance inside the hall.
(2) G-PCNN	$T_{\mathrm{DH}}^{k+1} = T_{\mathrm{DH}}^{k} - a(T_{\mathrm{DH}}^{k} - T_{\mathrm{SA}}) v_{\mathrm{SA}}^{k} + bPI_{\mathrm{TE}}^{k} + g(T_{\mathrm{DH}}^{k}, T_{\mathrm{OA}}^{k}, H_{\mathrm{OA}}^{k})$	Learns additional nonlinear residual $g(\cdot)$ from data for variable-speed cooling.
(3) A-PCNN split	$T_{\mathrm{DH}}^{k+1} = T_{\mathrm{DH}}^{k} + E_k + D_k$, where $E_k = (-a(\dots) + bPI_{\mathrm{TE}}^{k})$, $D_k = g(\dots)$	Separates physics term E and data-driven residual D for interpretability.
(8) Convective IT-heat term	$\dot{Q}_{\mathrm{ITE}} = h_k A_{\mathrm{ITE}}(T_{\mathrm{ITE}}^{k} - T_{\mathrm{DH}}^{k})$	Couples rack temperature to air temp; feeds Eq.(1–3).

Key constraint

- Server-air ΔT safety: $T_{\mathrm{server}}(t) \leq T_{\mathrm{server,max}}$ (thermal reliability margin).

Paper 2: M. Siltala et al., "Physical & Data-Driven Models for Edge-Data-Centre Cooling Systems" (Pre-print)

Focus on hot-aisle CFD-to-lumped-model reduction.

Hot-aisle energy balance

$$\rho_{\mathrm{air}} V_{\mathrm{hot}} c_p \frac{dT_{\mathrm{hot}}}{dt} = q_{\mathrm{IT}} + \dot{m}_{\mathrm{air}} c_p (T_{\mathrm{cold}} - T_{\mathrm{hot}}) - h_{\mathrm{hot}} A_{\mathrm{hot}} (T_{\mathrm{hot}} - T_{\mathrm{amb}}) - h_{\mathrm{IT}} A_{\mathrm{IT}} (T_{\mathrm{hot}} - T_{\mathrm{IT}})$$

States and parameters in Appendix A explain how airflow set-point ($\dot{m}_{\mathrm{air}}$) becomes a flexibility lever.

Paper 3: IEEE Xplore, "Virtual-Battery Abstraction of Thermostatically Controlled Loads (TCLs)"

Equation	Purpose
$\theta(t+1) = \alpha\theta(t) + (1-\alpha)(\theta_a(t) + cr(t) - bp(t))$	Discrete TCL temperature with control power $p(t)$.
$0 \le p(t) \le p_{\max}$	Compressor power bound.
$\theta_{\text{ref}} - \Delta \le \theta(t) \le \theta_{\text{ref}} + \Delta$	Comfort band (flexibility budget).

Take-away: Same structure can approximate CRAH unit cycling inside a data-centre pod.

Paper 4: Z. Li et al., "Equivalent Thermal-Parameter Model for Building-Scale Storage", Proc. SPIE 12918 (2023)

Equations used in grid-interactive DC microgrids:

- Temperature ODE: $\frac{dT_{\text{in}}}{dt} = \alpha(Q_h - Q_c) - \beta(T_{\text{in}} - T_{\text{out}})$
- Storage power bounds: $-P_{\text{bc,max}} \le P_{b,t} \le P_{\text{bd,max}}$
- SOC update: $L_t = L_{t-1} + \eta_b P_{b,t}$
- SOC range: $L_{\min} \le L_t \le L_{\max}$

These map 1-for-1 to chilled-water TES scheduling.

Paper 5: Antal et al., "Data-Centre Thermal-Energy Flexibility Model for Waste-Heat Re-use", E2DC 2017

Cooling loop & TES equations

Operational constraints

- ΔT_{rad}, ΔT_{evap}, ΔT_{exch} constant (assumes fixed HX UA).
- $TES_{\text{cold,min}} < TES_{\text{cold}}(t) < TES_{\text{cold,max}}$ (state-of-charge window).

Paper 6: IES-VE White-Paper, "Data-Centre Power-Consumption Sensitivity to Free-Cooling"

Rule-based chiller dispatch:

$$\Upsilon = \begin{cases} \text{Outdoor air} & \text{if } T_{\text{amb}} \le T_{\text{set}} + \delta \\ \text{Mechanical chiller} & \text{otherwise} \end{cases}$$

This binary switch is a hard flexibility constraint tying cooling capacity to weather bins.

#	Equation	Comment
(R1)	$\dot{m}_{w,\text{rad}}(t) = \dot{Q}_{\text{removed}}(t)/(c_w \Delta T_{\text{rad}})$	Water flow needed in radiator loop.
(HP-COP-c)	$\text{COP}_{\text{cool}} = \int \dot{m}_{w,\text{evap}} c_w \Delta T_{\text{evap}}\, dt / E_{\text{compressor}}$	Efficiency under cooling mode.
(HP-COP-h)	$\text{COP}_{\text{heat}} = \int \dot{m}_{w,\text{exch}} c_w \Delta T_{\text{exch}}\, dt / E_{\text{compressor}} + 1$	Adds compressor work to hot side.
(HP-HX)	$\dot{Q}_{\text{exch}}(t) = (\text{COP}_{\text{heat}} - 1)\dot{Q}_{\text{absorbed}}(t)$	Heat rejected to hot-TES loop.
(SR-acc)	$Q_{\text{acc}}(t) = mc(T_{\text{serv}}(t) - T_{\text{serv,ref}})$	Thermal debt stored in servers.
(TES-cold)	$TES_{\text{cold}}(t) = \cdots \pm \int \dot{m}_w c_w \Delta T\, dt$	Cold-tank energy balance (charging by evap).
(TES-hot)	$TES_{\text{hot}}(t) = \cdots \pm \int \dot{m}_w c_w \Delta T\, dt$	Hot-tank counterpart.

Paper 7: Y. Fu et al., "Renewable- & Cooling-Aware Workload Management", Applied Energy 277 (2020) 115621

Chilled-water supply reset for demand response:

$$T_{\text{chw,s,set}}(t) = T_{\text{chw,s}}(t) - r(t)\Delta T,$$

where $r(t) \in [0, 1]$ is the flexibility ratio.

Paper 8: J. Jerez Monsalves et al., "Heat-to-Cold Ratio Optimisation in DC Heat-Pumps", Energy 281 (2023) 128112

$$\chi_{\text{HT,e,HP}} = \alpha_{T,\text{HP}} \chi_{\text{CT,o,HP}}$$

Links hot- and cold-side demands through a geometry factor α, useful when co-optimising waste-heat export.

Variable Legend (Common Symbols)

How This Structure Helps Consensus Queries

- The per-paper blocks let the LLM (or you) trace conflicting assumptions—e.g., constant ΔT vs variable-speed fans.
- Copy-paste preserves live LaTeX: select the equation in Word → Insert → Equation → LaTeX to render.
- You can now add your own "Comparative notes" bullet under each paper.

Symbol	Meaning
$\dot{m}_{\mathrm{air}}$, $\dot{m}_w$	Mass-flow rate of air / water
c_p, c_w	Specific heat of air / water
T_{SA}, T_{OA}	Supply-air, outdoor-air temperatures
T_{DH}, T_{hot}, T_{cold}	Data-hall, hot-aisle, cold-aisle temps
PI_{TE}	IT electrical power
COP	Coefficient of performance of heat-pump
$\dot{Q}$	Instantaneous heat flow
ΔT	Temperature lift across HX / radiator