Econometrics 710 Midterm Exam March 10, 2011

## 1. Take the linear model

$$y_i = \mathbf{x}_i' \mathbf{\beta} + e_i$$

$$E(e_i \mid \mathbf{x}_i) = 0$$

$$E(e_i^2 \mid \mathbf{x}_i) = \sigma^2(\mathbf{x}_i)$$

Consider two approximations to the conditional variance  $\sigma^2(x)$ 

$$\gamma_1 = \operatorname{argmin} E \left(\sigma^2(\boldsymbol{x}_i) - \boldsymbol{x}_i' \boldsymbol{\gamma}\right)^2$$

$$\gamma_2 = \operatorname{argmin} E \left( e_i^2 - x_i' \gamma \right)^2$$

Show that either  $\gamma_1 = \gamma_2$  or derive their difference.

## 2. Consider the short and long projections

$$y_i = x_i \gamma_1 + e_i$$

$$y_i = x_i \beta_1 + x_i^2 \beta_2 + u_i$$

- (a) Under what condition does  $\gamma_1 = \beta_1$ ?
- (b) Now suppose the long projection is

$$y_i = x_i \theta_1 + x_i^3 \theta_2 + v_i$$

Is there a similar condition under which  $\gamma_1 = \theta_1$ ?

## 3. Take the linear model

$$y_i = x_i \beta + e_i$$

$$E(e_i \mid x_i) = 0$$

with n observations and  $x_i$  is scalar (real-valued). Consider the estimator

$$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i^3 y_i}{\sum_{i=1}^{n} x_i^4}$$

Find the asymptotic distribution of  $\sqrt{n} \left( \hat{\beta} - \beta \right)$  as  $n \to \infty$ 

## 4. Take the linear model

$$y_i = \alpha + \mathbf{x}_i' \mathbf{\beta} + e_i$$

$$E(e_i) = 0$$

$$E(\mathbf{x}_i e_i) = \mathbf{0}$$

with n observations. Consider the restriction

$$\beta = 0 \tag{1}$$

- (a) Find the constrained least-squares (CLS) estimator of  $\alpha$  under (1).
- (b) Find an expression for the efficient minimum distance estimator of  $\alpha$  under (1).