Econometrics 710 Midterm Exam March 4, 2015

The exam questions all concern the following setting. The random variables are (y, x) with y scalar and $x \in \mathbb{R}^k$ and there is a random sample $\{y_i, x_i : i = 1, ..., n\}$ from (y, x). The goal is to estimate the parameter vector θ which minimizes the Mean Trimmed Squared Error (MTSE) defined by

$$T(\theta) = E\left[\left(y - x'\theta \right)^2 \tau(x) \right]$$

where $\tau(x)$ is a known, scalar, non-negative, bounded, "trimming" function which downweights large values of x.

Define $e = y - x'\theta$.

- 1. Give an explicit formula for the value of θ which minimizes $T(\theta)$.
- 2. Show that $E(xe\tau(x)) = 0$.
- 3. Under what condition (other than $\tau(x) = 1$) will this θ equal the Best Linear Predictor?
- 4. Write down an estimator $\hat{\theta}$ for θ .
- 5. Under what condition will $\hat{\theta}$ be unbiased for θ ?
- 6. Show that $\hat{\theta}$ is consistent for θ .
- 7. Find the asymptotic distribution of $\sqrt{n}(\hat{\theta} \theta)$.
- 8. Show how this asymptotic distribution simplifies when e satisfies $E(e^2|x) = \sigma^2$.
- 9. Suggest an estimator for the asymptotic covariance matrix.