Econometrics 710
Midterm Exam
March 11, 2008

1. Take the model

$$\begin{aligned} y_i &= x_{1i}'\beta_1 + x_{2i}'\beta_2 + e_i \\ E\left(e_i \mid x_i\right) &= 0 \\ E\left(e_i^2 \mid x_i\right) &= \sigma^2 \end{aligned}$$

where $x_i = (x_{1i}, x_{2i})$, with x_{1i} $k_1 \times 1$ and x_{2i} $k_2 \times 1$. Consider the short regression

$$y_i = x_{1i}'\hat{\beta}_1 + \hat{e}_i$$

and define the error variance estimator

$$s^2 = \frac{1}{n-k_1}\sum_{i=1}^{n}\hat{e}_i^2.$$

   (a) Find $E\left(s^2 \mid X\right)$
   (b) Find the probability limit of s^2 as $n \to \infty$.

2. Take the model

$$\begin{aligned} y_i &= x_i\beta + e_i \\ E\left(x_i e_i\right) &= 0 \end{aligned}$$

with x_i scalar and $Ex_i^2 = 1$. Let V be the asymptotic variance of the least-squares estimator of β, and let V^0 be the "homoskedastic" form of the asymptotic variance.

   (a) Find V and V^0. (You do not need to re-derive the asymptotic distribution.)
   (b) Find the relationship between V, V^0, and $C = cov\left(x_i^2, e_i^2\right)$.

3. Take the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E\left(e_i \mid x_i\right) &= 0 \\ E\left(e_i^2 \mid x_i\right) &= \sigma_i^2 = z_i'\gamma \end{aligned}$$

where z_i is a (vector) function of x_i. The sample is $i = 1, ..., n$ with iid observations. For simplicity, assume that $z_i'\gamma > 0$ for all z_i. Suppose you are interested in forecasting y_{n+1} given $x_{n+1} = x$ and $z_{n+1} = z$ for some out-of-sample observation $n+1$. Describe how you would construct a point forecast and a forecast interval for y_{n+1}.

4. Take the model

$$
\begin{aligned}
y_i &= x_i'\beta + e_i \\
E(e_i \mid x_i) &= 0 \\
z_i &= \left(x_i'\beta\right)\gamma + u_i \\
E(u_i \mid x_i) &= 0
\end{aligned}
$$

Your goal is to estimate γ. (Note that γ is real-valued.0 You use a two-step estimator:

(a) Estimate $\hat{\beta}$ by least-squares of y_i on x_i

(b) Estimate $\hat{\gamma}$ by least-squares of z_i on $x_i'\hat{\beta}$

Show that $\hat{\gamma}$ is consistent for γ.

Extra Credit (**only** if you have time): Find the asymptotic distribution of $\hat{\gamma}$ when $\gamma = 0/$